

Vector Algebra

1. **Problem statement :** The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one, Find the value of λ .

Solution: Let

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \quad (1)$$

$$\mathbf{b} + \mathbf{c} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} + \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} \quad (2)$$

Let \mathbf{r} be the unit vector along with $\mathbf{b} + \mathbf{c}$

$$\hat{\mathbf{r}} = \frac{\mathbf{b} + \mathbf{c}}{\|\mathbf{b} + \mathbf{c}\|} = \frac{\mathbf{b} + \mathbf{c}}{\sqrt{(2 + \lambda)^2 + (6)^2 + (-2)^2}} \quad (3)$$

$$\Rightarrow \hat{\mathbf{r}} = \frac{1}{\sqrt{\lambda^2 + 4\lambda + 4}} \Rightarrow \hat{\mathbf{r}} = \frac{1}{\sqrt{\lambda^2 + 4\lambda + 4}} \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} \quad (4)$$

Given $\mathbf{a}^\top (\hat{\mathbf{r}}) = 1$

$$(1 \ 1 \ 1) \left(\frac{1}{\sqrt{\lambda^2 + 4\lambda + 4}} \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} \right) = 1 \quad (5)$$

$$\Rightarrow (1 \ 1 \ 1) \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} = \sqrt{\lambda^2 + 4\lambda + 4} \quad (6)$$

$$\Rightarrow 2 + \lambda + 6 - 2 = \sqrt{\lambda^2 + 4\lambda + 4} \quad (7)$$

$$\Rightarrow \lambda + 6 = \sqrt{\lambda^2 + 4\lambda + 4} \quad (8)$$

$$(9)$$

Squaring on both sides

$$(\lambda + 6)^2 = \left(\sqrt{\lambda^2 + 4\lambda + 4} \right)^2 \quad (10)$$

$$\lambda^2 + 12(\lambda) + 4 = \lambda^2 + 4\lambda + 4 \quad (11)$$

$$8(\lambda) = 8 \quad (12)$$

$$\lambda = 1 \quad (13)$$