

# CS 310: Algorithms

**Instructor:** Naveed Anwar Bhatti



# Who am I? Dr. Naveed Anwar Bhatti

**Hometown:** Islamabad

## Postdoc:

2019

Senior Researcher  
**RISE**, Stockholm, Sweden



## Education:

**PhD**

2018

Computer Science  
Politecnico di Milano, Italy  
*System Support for Transiently  
Powered Embedded Systems*

**MS**

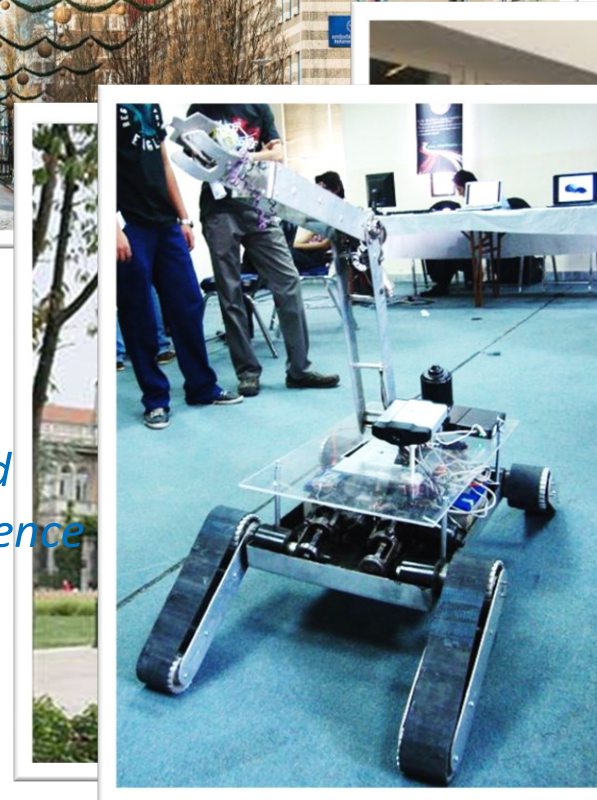
2013

Computer Science  
FAST-NUCES, Islamabad, Pakistan  
*Long range RFID System: Decoupling sensing and  
energy in sensor networks using energy transference*

**BS**

2011

Telecom  
FAST-NUCES, Islamabad, Pakistan  
*Internet Controlled Unmanned Ground Vehicle*







# Long range RFID-like System



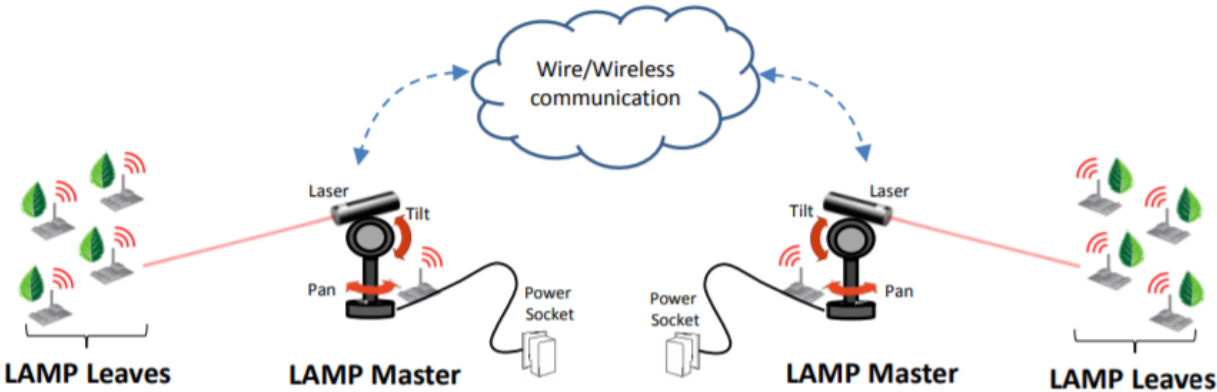
Laser Module



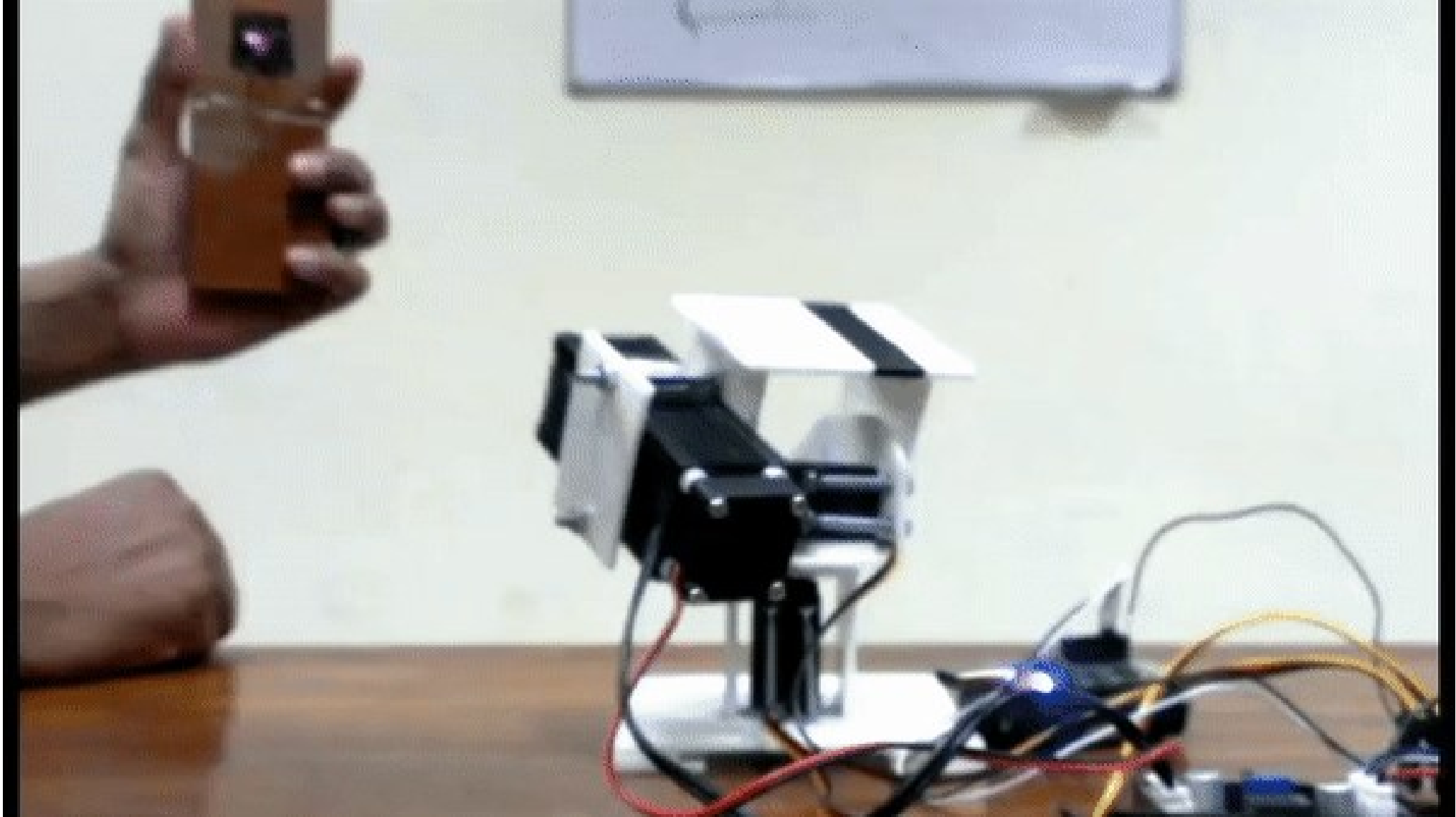
TelosB mote



Solar Panel



# Long range RFID-like System







# Other Sensor Deployments



Wasp mote





# How to reach me?

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**Room:** 9 - G23A

**Office Hours:** Thursday, 11:00 am - 12:00 pm

**Webpage:** [naveedanwarbhatti.github.io](http://naveedanwarbhatti.github.io)

**Class page and slides:** LUMS LMS



# Why are YOU here? - Outline

Week 1 and 2

Introduction to Algorithms, Analysis and Asymptotic growth

Week 3

Graphs

Week 4 and 5

Greedy Algorithms

Week 6 and 7

Divide and Conquer

Week 8, 9 and 10

Dynamic Programming

Week 11 and 12

Network Flow

Week 13 and 14

NP Completeness

# GRADING BREAKUP AND POLICY

Assessment	Weight (%)	Related CLOs
Assignments	10%	CLO1- CLO4
Quizzes	25% (We will have N - 1 policy for quizzes. 1 quizzes will be dropped. No petition for makeup quizzes will be accepted if you have missed only 1 quiz. If you have missed more than 1 quiz for genuine reasons, you may file a petition for the 2nd and subsequent missed quizzes. However, the petition may not be accepted if there is no substantial reason.)	CLO1 - CLO3
Mid-term	27%	CLO1 - CLO3
Final	38%	CLO1 - CLO4





# Lecture slides

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- Bad experience from previous courses
- Lecture slides are **not** the course content
  - Discussions and explanations in between are equally important
  - To extract maximum benefit, take notes (like the good old days!).
  - I will selectively show/use slides

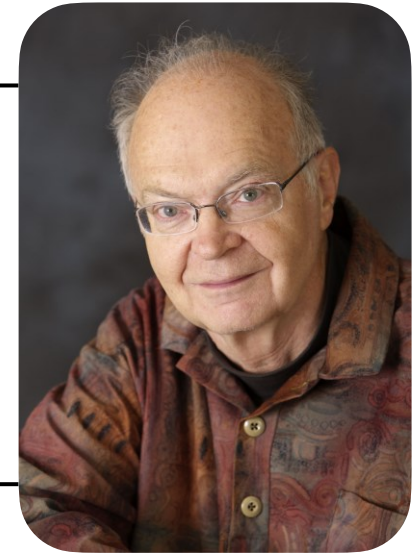
# Algorithm Definition

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*“ An **algorithm** is a finite, definite, effective procedure,  
with some input and some output. ”*

*— Donald Knuth*

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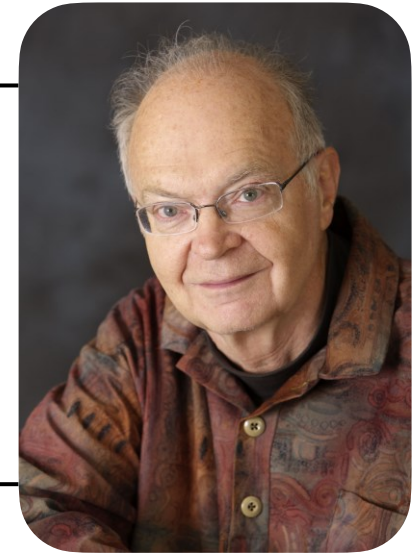
# Algorithm Definition

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*“ An **algorithm** is a **finite, definite, effective** procedure,  
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*— Donald Knuth*

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# Why study algorithms?

To become a proficient programmer.

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*“I will, in fact, claim that the difference between a bad programmer and a good one is whether he considers his code or his data structures more important. Bad programmers worry about the code. Good programmers worry about **data structures** and their **relationships**. ”*

*— Linus Torvalds (creator of Linux)*

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# Why study algorithms?

For intellectual stimulation.


*“For me, great algorithms are the poetry of computation. Just like verse, they can be allusive, dense, and even mysterious. But once unlocked, they cast a brilliant new light on some aspect of computing.”*

— Francis Sullivan

FROM THE EDITORS

THE JOY OF ALGORITHMS

Francis Sullivan, Associate Editor-in-Chief



THE THEME OF THIS FIRST-OF-THE-CENTURY ISSUE OF *COMPUTING IN SCIENCE & ENGINEERING* IS ALGORITHMS. IN FACT, WE WERE BOLD ENOUGH—AND PERHAPS FOOLISH ENOUGH—TO CALL THE 10 EXAMPLES WE’VE SELECTED “THE TOP 10 ALGORITHMS OF THE CENTURY.”

Computational algorithms are probably as old as civilization. Sumerian cuneiform, one of the most ancient written records, consists partly of algorithm descriptions for reckoning in base 60. And I suppose we could claim that the Druid algorithm for estimating the start of summer is embodied in Stonehenge. (That’s really hard hardware!)

Like so many other things that technology affects, algorithms have advanced in startling and unexpected ways in the 20th century—at least it looks that way to us now. The algorithms we chose for this issue have been essential for progress in communications, health care, manufacturing, economics, weather prediction, defense, and fundamental science. Conversely, progress in these areas has stimulated the search for ever-better algorithms. I recall one late-night huff session on the Maryland Shore when someone asked, “Who first ate a crab? After all, they don’t look very appetizing.” After the usual speculations about the observed behavior of sea gulls, someone gave what must be the right answer—namely, “A very hungry person first ate a crab.”

The flip side to “necessity is the mother of invention” is “invention creates its own necessity.” Our need for powerful machines always exceeds their availability. Each significant computation brings insights that suggest the next, usually much larger, computation to be done. New algorithms are an attempt to bridge the gap between the demand for cycles and the available supply of them. We’ve become accustomed to gaining the Moore’s Law factor of two every 18 months. In effect, Moore’s Law changes the constant in front of the estimate of running time as a function of problem size. Important new algorithms do not come along every 1.5 years, but when they do, they can change the exponent of the complexity!

For me, great algorithms are the poetry of computation. Just like verse, they can be terse, allusive, dense, and even mysterious. But once unlocked, they cast a brilliant new light on some aspect of computing. A colleague recently claimed that he’d done only 15 minutes of productive work in his whole life. He wasn’t joking, because he was referring to the 15 minutes during which he’d sketched out a fundamental optimization algorithm. He regarded the previous years of thought and investigation as a sunk cost that might or might not have paid off.

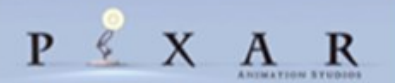
Researchers have cracked many hard problems since 1 January 1900, but we are passing some even harder ones on to the next century. In spite of a lot of good work, the question of how to extract information from extremely large masses of data is still almost untouched. There are still very big challenges coming from more “traditional” tasks, too. For example, we need efficient methods to tell when the result of a large floating point calculation is likely to be correct. Think of the way that check sums function. The added computational cost is very small, but the added confidence in the answer is large. Is there an analog for things such as huge, multidisciplinary optimizations? At an even deeper level is the issue of reasonable methods for solving specific cases of “impossible” problems. Instances of NP-complete problems crop up in attempting to answer many practical questions. Are there efficient ways to attack them?

I suspect that in the 21st century, things will be ripe for another revolution in our understanding of the foundations of computational theory. Questions already arising from quantum computing and problems associated with the generation of random numbers seem to require that we somehow tie together theories of computing, logic, and the nature of the physical world.

The new century is not going to be very restful for us, but it is not going to be dull either! ❏

# Why study algorithms?

For fun and profit.

The Google logo, consisting of the word "Google" in its multi-colored font.The Apple Computer logo, featuring a black silhouette of an apple with a bite taken out of it, and the text "Apple Computer" below it.The Facebook logo, consisting of the word "facebook." in white lowercase letters on a blue rectangular background.The Cisco Systems logo, featuring the text "CISCO SYSTEMS" in red above a stylized bridge graphic.The IBM logo, consisting of the letters "IBM" in a blue, horizontally-striped font.The Nintendo logo, featuring the word "Nintendo" in white inside a red rounded rectangle.The Jane Street logo, featuring a circular gold-colored emblem on the left and the text "JANE STREET" in gold on a dark blue background.The Morgan Stanley logo, consisting of the text "Morgan Stanley" in white on a dark blue rectangular background.The Netflix logo, consisting of the word "NETFLIX" in white capital letters on a red rectangular background.The Adobe logo, featuring a stylized red "A" with a white triangle inside, and the word "Adobe" below it.The RSA Security logo, featuring the letters "RSA" in white on a red square background, with the word "SECURITY" below it.The DE Shaw & Co logo, featuring a green line graphic above the text "DE Shaw & Co" in blue.The Oracle logo, consisting of the word "ORACLE" in red capital letters.The Pandora logo, featuring a large blue letter "P" inside a square frame, with the word "PANDORA" below it.The Akamai logo, featuring a stylized blue wave graphic on the left and the word "Akamai" in yellow on a white background.The Yahoo! logo, consisting of the word "YAHOO!" in red capital letters.The Amazon.com logo, featuring the word "amazon.com" in black with a yellow curved arrow underneath it.The Microsoft logo, consisting of the word "Microsoft" in a bold, black, sans-serif font.The Pixar Animation Studios logo, featuring the word "PIXAR" in large letters with a small character in the dot of the 'i', and "ANIMATION STUDIOS" below it.



# Why study algorithms?

**Internet.**

Web search, packet routing, distributed file sharing, ...

**Biology.**

Human genome project, protein folding, ...

**Computers.**

Circuit layout, databases, caching, networking, compilers, ...

**Computer graphics.**

Movies, video games, virtual reality, ...

**Security.**

Cell phones, e-commerce, voting machines, ...

**Multimedia.**

MP3, JPG, DivX, HDTV, face recognition, ...

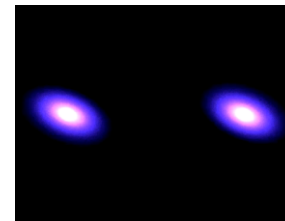
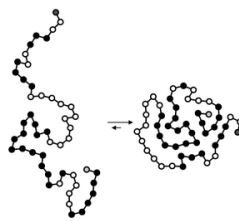
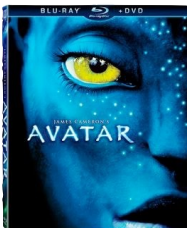
**Social networks.**

Recommendations, news feeds, advertisements, ...

**Physics.**

Particle collision simulation,  $n$ -body simulation, ...

⋮





# A First Problem: **Stable Matching**

- Nicely illustrates many of the **themes we will be emphasizing**
- Target Applications:
  - Selecting teaching assistants (TA) for courses – TA-course matching problem
  - Matching employers to applicants for job hiring
  - College admission – matching students to colleges
  - Content delivery networks – assigning users to web servers



# A First Problem: **Stable Matching**

- Nicely illustrates many of the **themes we will be emphasizing**
- Target Applications:
  - **Selecting teaching assistants (TA) for courses – TA-course matching problem**
  - Matching employers to applicants for job hiring
  - College admission – matching students to colleges
  - Content delivery networks – assigning users to web servers



# Stability Matching – Formulating the Problem

## TA-course matching problem

- Suppose:
  - $n$  courses are being offered by the CS Department in the current semester
  - Due to budget constraints **only one TA can be assigned to each course** –  $n$  TAs for  $n$  courses
  - Each course instructor ranks all the TA applicants in the order of his/her preference (**preference list of the course instructor**)
  - Each TA applicant ranks all the courses in the order of his/her preference (**preference list of TA applicant**)
- Issues
  - An applicant accepting TAship offer for a course may later quit it for a different course that is ranked higher in his/her preference list
  - A course instructor may withdraw an offer accepted by an applicant to hire another TA who is ranked higher in the instructor's preference list
  - **Unstable matching**

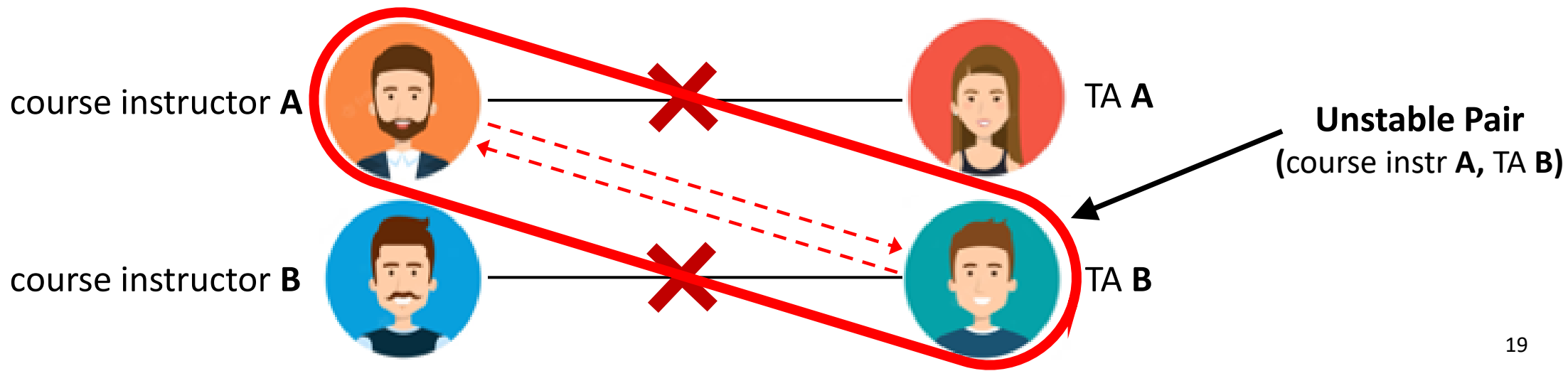
# TA-course matching problem

**Goal.** Given a set of preferences among course instructors and TA applicants, design a **self-reinforcing** TA selection process

**Unstable pair.** Course instructor  $c$  and TA applicant  $a$  form an **unstable pair** if both:

$c$  prefers  $a$  to an already selected TA

$a$  prefers  $c$  to the already assigned course instructor





# TA-course matching problem

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- $c$  prefers  $a$  to an already selected TA

- $a$  prefers  $c$  to the already assigned course instructor

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**Stable assignment.** Assignment with **no unstable pairs**.

- Prevents TAs quitting the assigned courses

- Prevent instructor withdrawing TAship offer



# TA course matching problem - Example

**Input.** A set  $C$  of  $n$  course instructors and a set  $A$  of  $n$  TA applicants

- Each course instructor  $c \in C$  ranks TA applicants
- Each TA applicant  $a \in A$  ranks courses

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
CS 100	Alice	Bob	Charlie
CS 200	Bob	Alice	Charlie
CS 300	Alice	Bob	Charlie

Course instructors' preference list

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
Alice	CS 200	CS 100	CS 300
Bob	CS 100	CS 200	CS 300
Charlie	CS 100	CS 200	CS 300

TA applicants' preference list

# Matching

- **Def.** A **matching**  $M$  is a set of ordered pairs  $c - a$  with  $c \in \mathcal{C}$  and  $a \in A$  such that:
  - Each course instructor  $c \in \mathcal{C}$  appears **in at most one pair** of  $M$
  - Each TA applicant  $a \in A$  appears **in at most one pair** of  $M$

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
CS 100	Alice	Bob	Charlie
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Course instructors' preference list

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Alice	CS 200	CS 100	CS 300
Bob	CS 100	CS 200	CS 300
Charlie	CS 100	CS 200	CS 300

TA applicants' preference list

$$M = \{(CS100 - Charlie)\}$$

# Matching

- **Def.** A **matching**  $M$  is a set of ordered pairs  $c - a$  with  $c \in \mathcal{C}$  and  $a \in A$  such that:
  - Each course instructor  $c \in \mathcal{C}$  appears **in at most one pair** of  $M$
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	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
CS 100	Alice	Bob	Charlie
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CS 300	Alice	Bob	Charlie

Course instructors' preference list

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
Alice	CS 200	CS 100	CS 300
Bob	CS 100	CS 200	CS 300
Charlie	CS 100	CS 200	CS 300

TA applicants' preference list

$M = \{(CS100 - Charlie)\}$

$M = \{(CS100 - Charlie), (CS200 - Bob)\}$

# Perfect matching

**Def.** A matching  $M$  is perfect if  $|M| = |C| = |A| = n$

- Each course instructor  $c \in C$  appears **in one pair** of  $M$
- Each TA applicant  $a \in A$  appears **in one pair** of  $M$

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
CS 100	Alice	Bob	Charlie
CS 200	Bob	Alice	Charlie
CS 300	Alice	Bob	Charlie

Course instructors' preference list

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
Alice	CS 200	CS 100	CS 300
Bob	CS 100	CS 200	CS 300
Charlie	CS 100	CS 200	CS 300

TA applicants' preference list

A perfect matching  $M = \{(CS100 - Charlie), (CS200 - Bob), (CS300 - Alice)\}$



# Unstable pair

- **Def.** Given a perfect matching  $M$ , course instructor  $c$  and a TA applicant  $a$  form an **unstable pair** if both
  - $c$  prefers  $a$  to the assigned TA
  - $a$  prefers  $c$  to the assigned course

$M = \{(CS100 - Charlie), (CS200 - Bob), (CS300 - Alice)\}$

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
CS 100	Alice	Bob	Charlie
CS 200	Bob	Alice	Charlie
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Course instructors' preference list

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Alice	CS 200	CS 100	CS 300
Bob	CS 100	CS 200	CS 300
Charlie	CS 100	CS 200	CS 300

TA applicants' preference list

Can you find any unstable pair?

# Unstable pair

- **Def.** Given a perfect matching  $M$ , course instructor  $c$  and a TA applicant  $a$  form an **unstable pair** if both
  - $c$  prefers  $a$  to the assigned TA
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$M = \{(CS100 - Charlie), (CS200 - Bob), (CS300 - Alice)\}$

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CS 100	Alice	Bob	Charlie
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Course instructors' preference list

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Alice	CS 200	CS 100	CS 300
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Charlie	CS 100	CS 200	CS 300

TA applicants' preference list

**(CS100 – Alice)** are unstable pair

# Unstable pair – Through Bipartite Graph

- **Def.** Given a perfect matching  $M$ , course instructor  $c$  and a TA applicant  $a$  form an **unstable pair** if both
  - $c$  prefers  $a$  to the assigned TA
  - $a$  prefers  $c$  to the assigned course

$M = \{(CS100 - Charlie), (CS200 - Bob), (CS300 - Alice)\}$

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
CS 100	Alice	Bob	Charlie
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Course instructors' preference list

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Alice	CS 200	CS 100	CS 300
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Charlie	CS 100	CS 200	CS 300

TA applicants' preference list

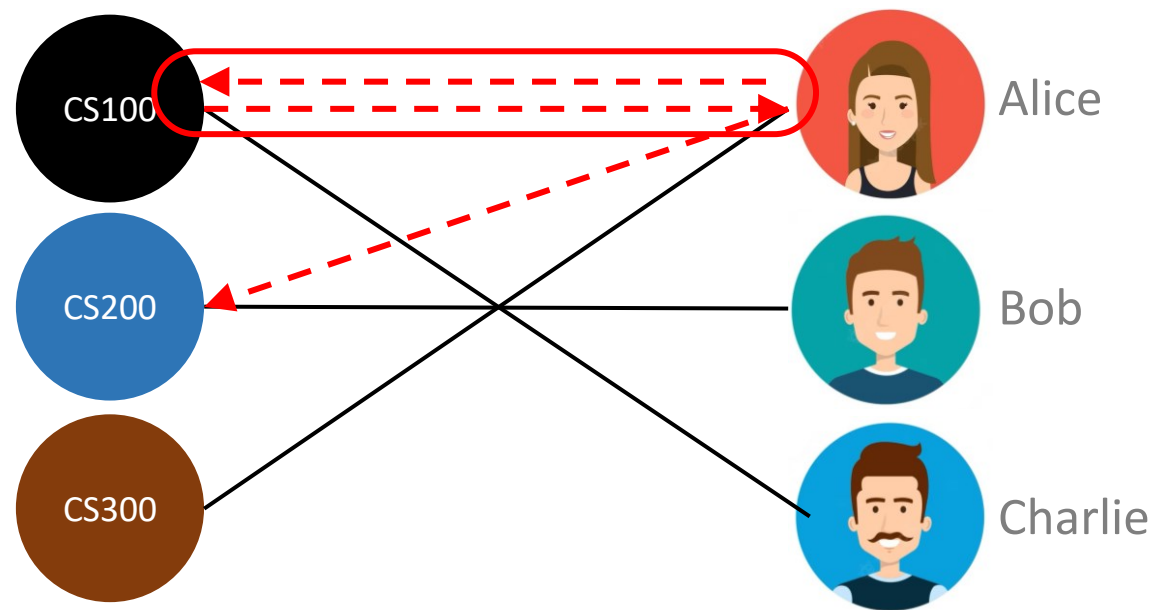
**(CS100 – Alice)** are unstable pair

# Unstable pair – Through Bipartite Graph

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Charlie	CS 100	CS 200	CS 300

$M = \{(CS100 - Charlie), (CS200 - Bob), (CS300 - Alice)\}$



**(CS100 – Alice)** are unstable pair

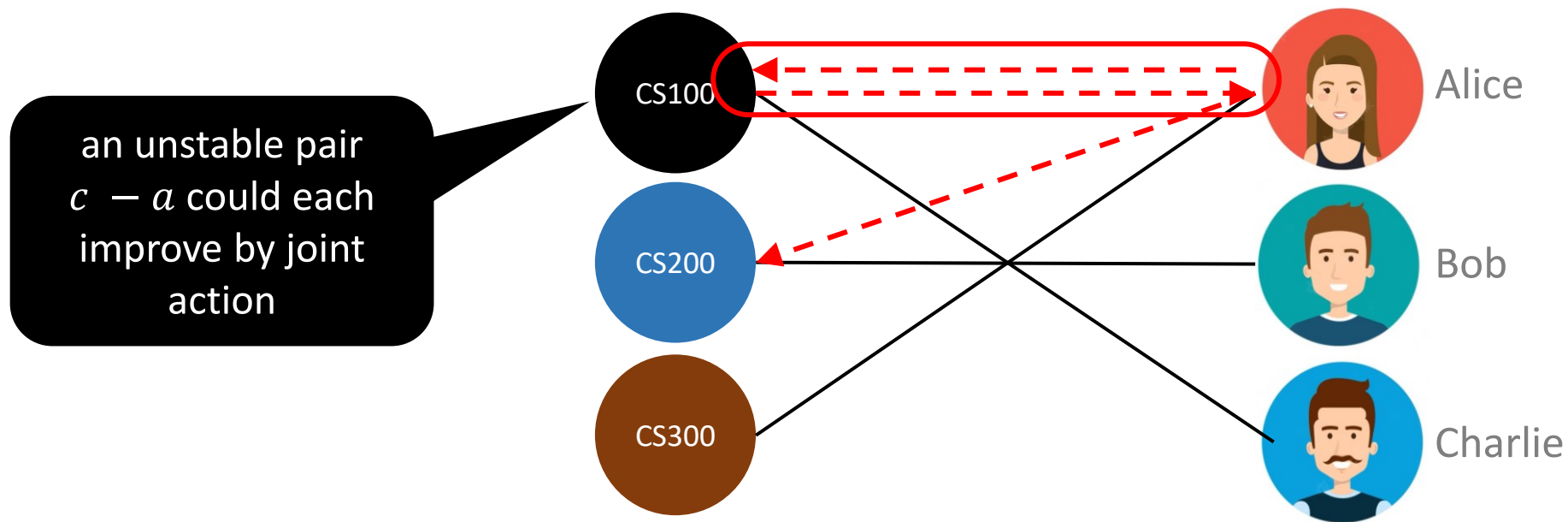


# Unstable pair – How to resolve?

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
CS 100	Alice	Bob	Charlie
CS 200	Bob	Alice	Charlie
CS 300	Alice	Bob	Charlie

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Charlie	CS 100	CS 200	CS 300

$M = \{(CS100 - Charlie), (CS200 - Bob), (CS300 - Alice)\}$



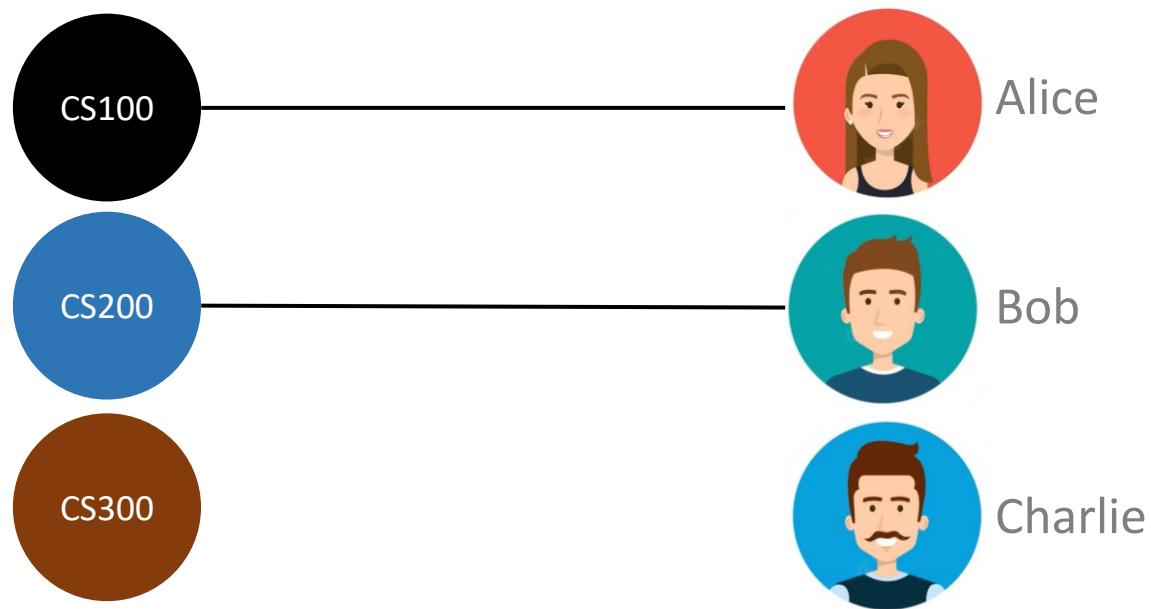
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# Unstable pair – How to resolve?

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CS 100	Alice	Bob	Charlie
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CS 300	Alice	Bob	Charlie

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
Alice	CS 200	CS 100	CS 300
Bob	CS 100	CS 200	CS 300
Charlie	CS 100	CS 200	CS 300

$M = \{(CS100 - Alice), (CS200 - Bob)\}$

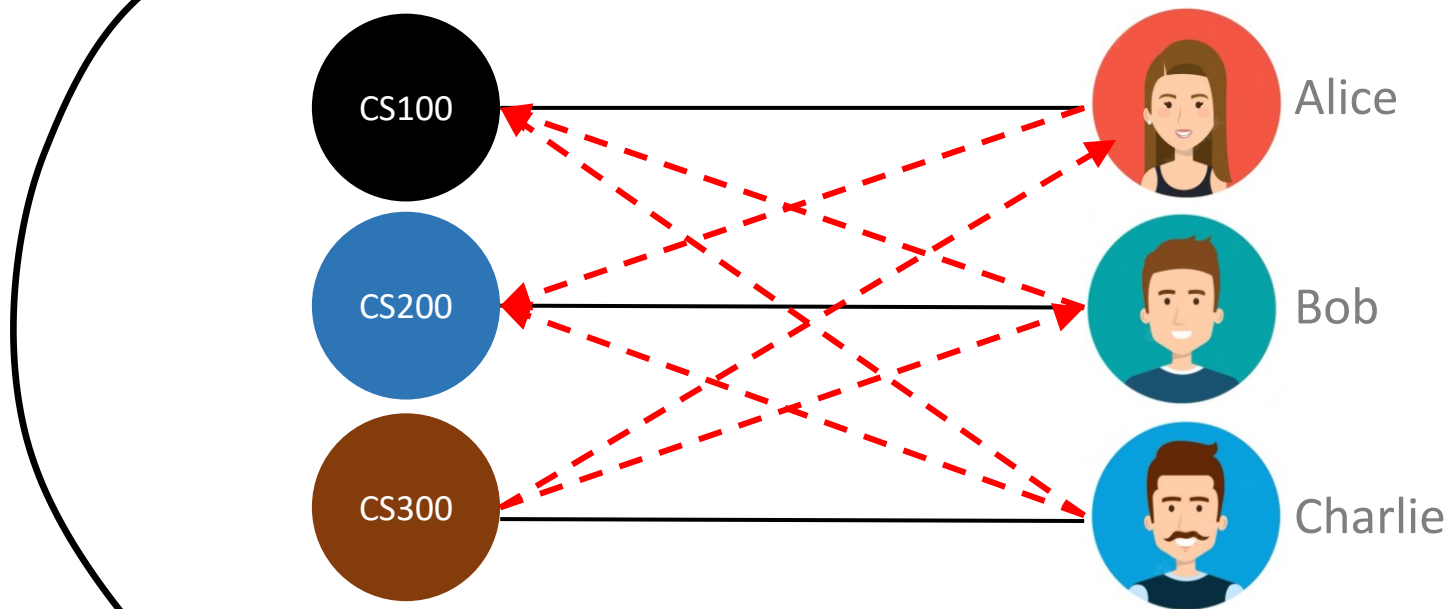


# Unstable pair – How to resolve?

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Alice	CS 200	CS 100	CS 300
Bob	CS 100	CS 200	CS 300
Charlie	CS 100	CS 200	CS 300

$M = \{(CS100 - Alice), (CS200 - Bob), (CS300 - Charlie)\}$



No unstable pair

**Perfect Matching and Stable Assignment**

# Stable matching – Live Poll 1



Scan the QR code to  
vote or go to  
<https://forms.office.com/r/mwS1SuBX0B>

Which pair(s) is unstable in the matching  $M = \{(CS100 - Alice), (CS200 - Charlie), (CS300 - Bob)\}$

- A. (CS100 - Bob)
- B. (CS200 - Alice)
- C. (CS200 - Bob)
- D. None of the above

	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
CS 100	Alice	Bob	Charlie
CS 200	Bob	Alice	Charlie
CS 300	Alice	Bob	Charlie

Course instructors' preference list

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Alice	CS 200	CS 100	CS 300
Bob	CS 100	CS 200	CS 300
Charlie	CS 100	CS 200	CS 300

TA applicants' preference list

## Stable matching – Live Poll 1

Only people in my organization can respond, Record name

1. Which pair(s) is unstable in the matching  $M = \{(CS100-Alice), (CS200-Charlie), (CS300-Bob)\}$

- (CS100 - Bob) 0%
- (CS200 - Alice) 0%
- (CS200 - Bob) 0%
- None of Above 0%



Scan the QR code to  
vote or go to

72 responses

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	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>
CS 100	Alice	Bob	Charlie
CS 200	Bob	Alice	Charlie
CS 300	Alice	Bob	Charlie

Course instructors' preference list

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Alice	CS 200	CS 100	CS 300
Bob	CS 100	CS 200	CS 300
Charlie	CS 100	CS 200	CS 300

TA applicants' preference list



# Thanks a lot



If you are taking a Nap, **wake up**.....Lecture Over