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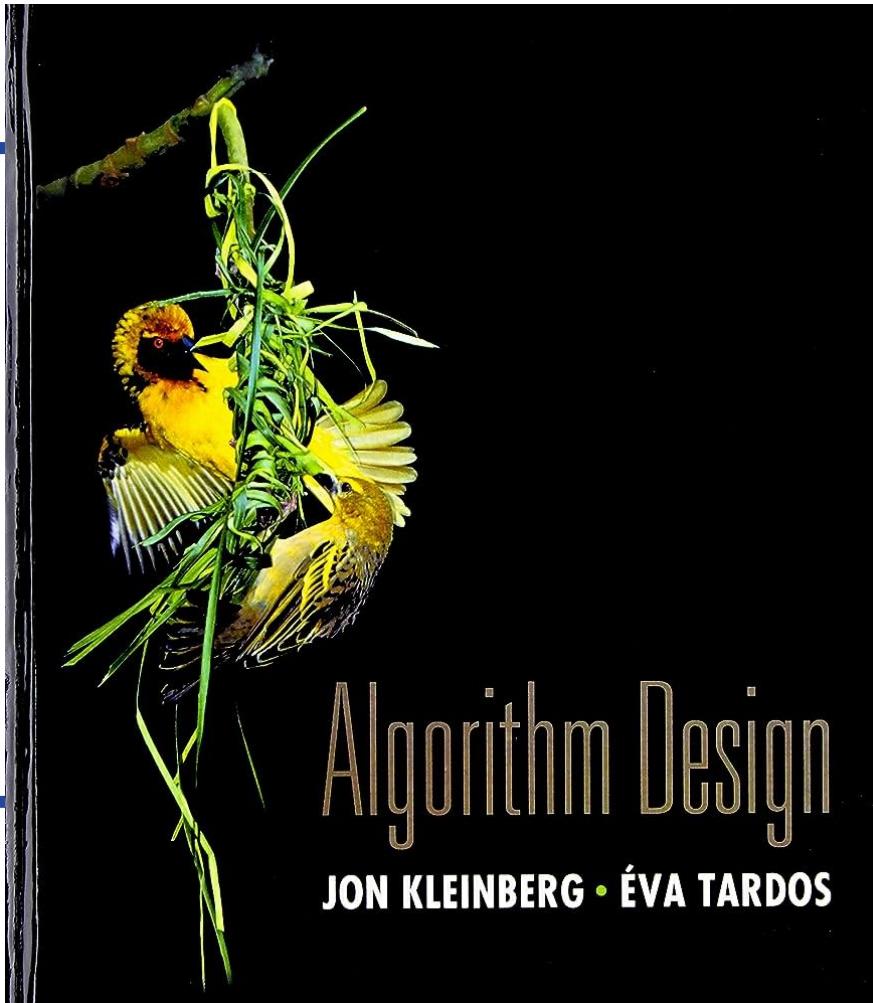
CS 310: Algorithms

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# Lecture 8

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Instructor: Naveed Anwar Bhatti



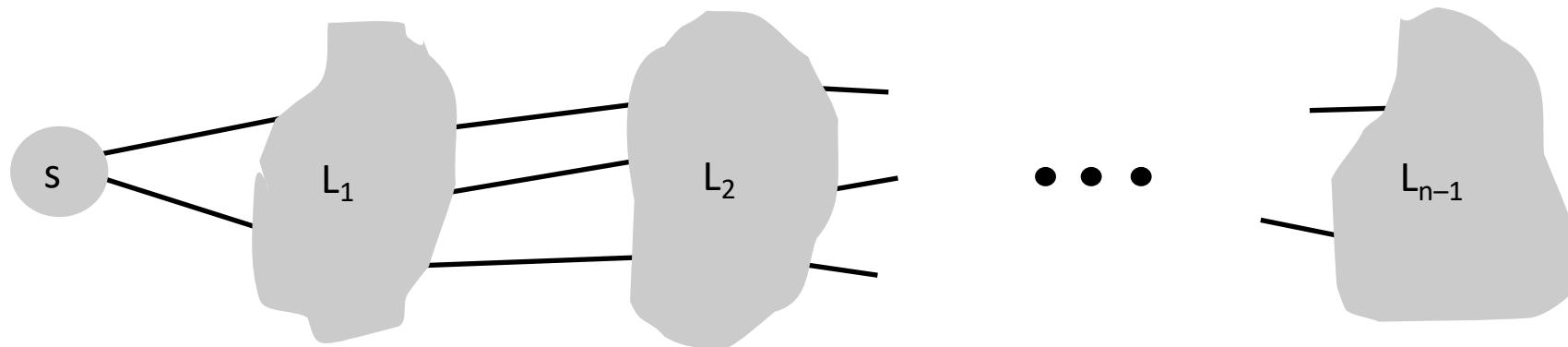
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# Chapter 3: Graphs

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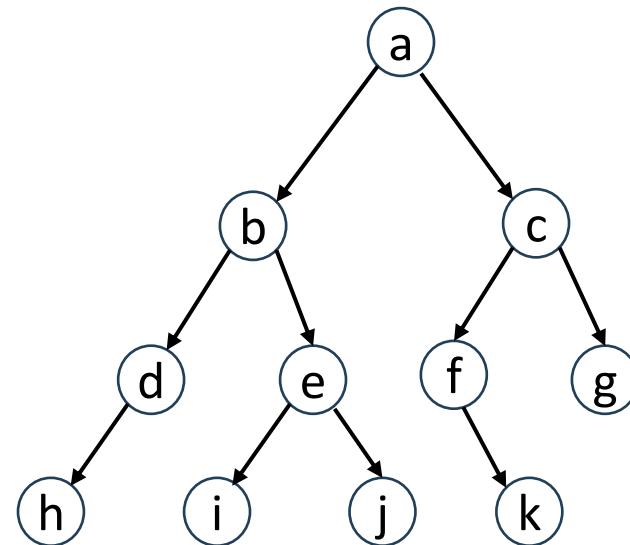
# Breadth First Search (BFS)

- BFS traverses the vertices of a graph from a given source node  $s$  in the order of the distances from the source
  - Discovers all vertices at distance  $k$  from  $s$  before discovering any vertices at distance  $k + 1$
  - Outputs a BFS tree with root  $s$

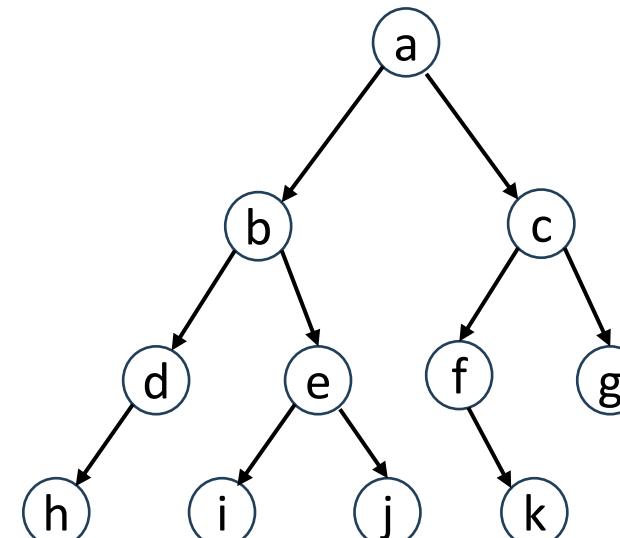


# Depth First Search (DFS)

- DFS follows the strategy of going deep first and then wide
  - DFS starts at the source node  $s$  and explores as far as possible along each branch before backtracking



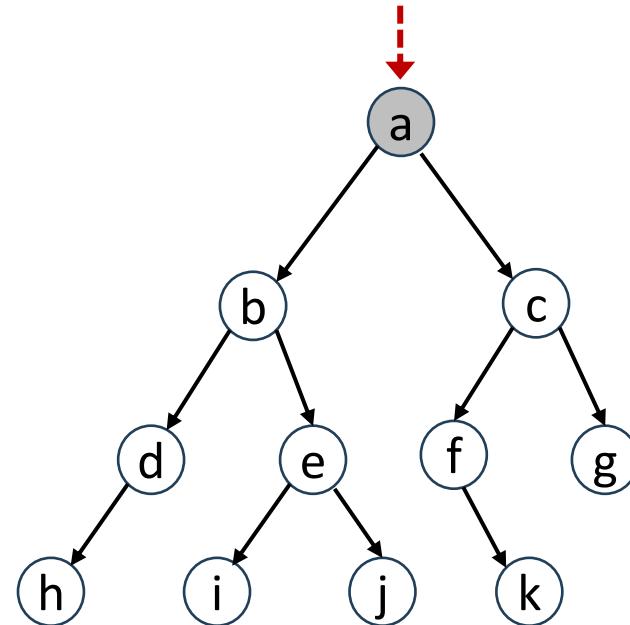
Depth First Search



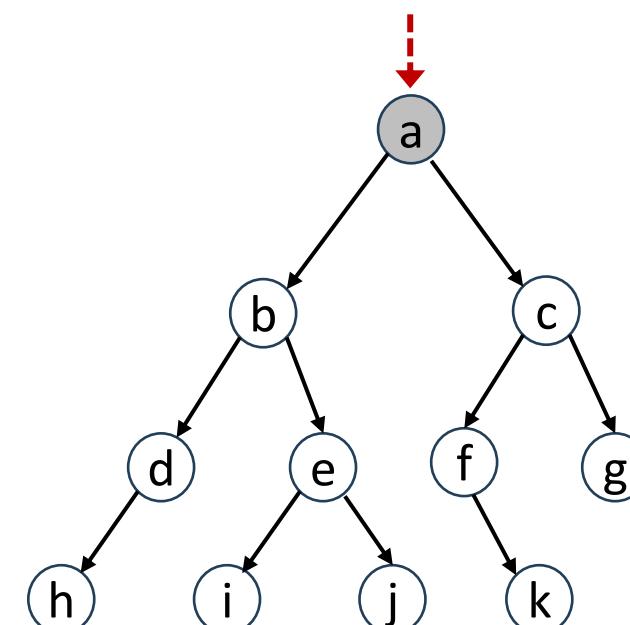
Breadth First Search

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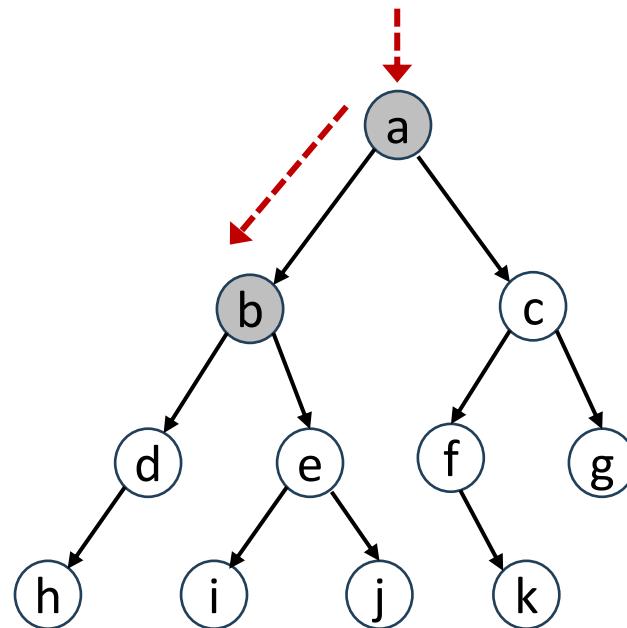
Depth First Search



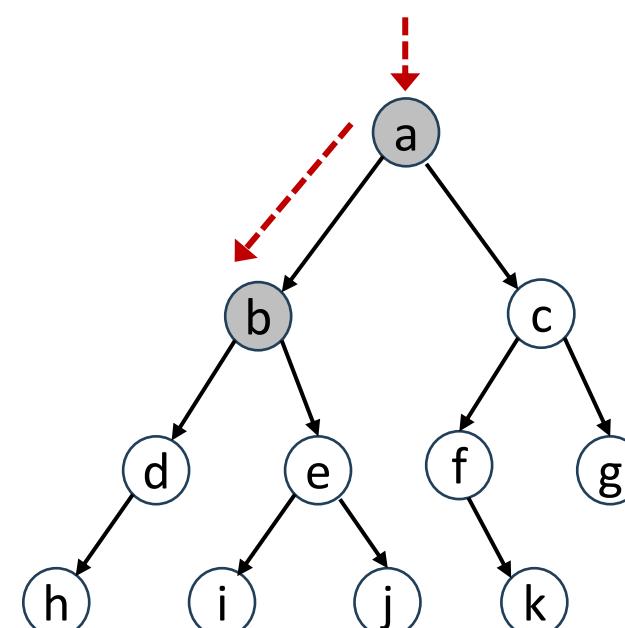
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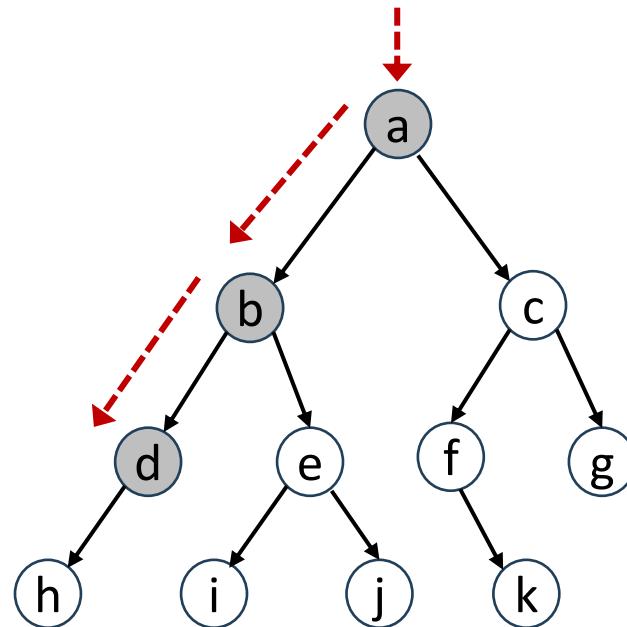
Depth First Search



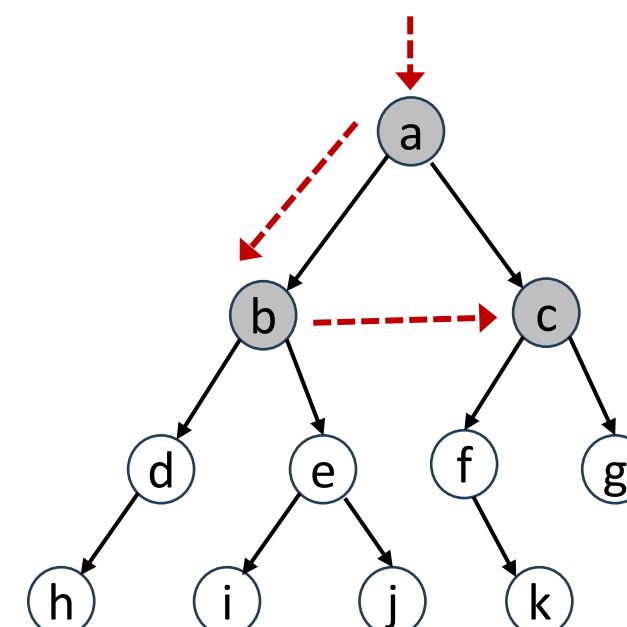
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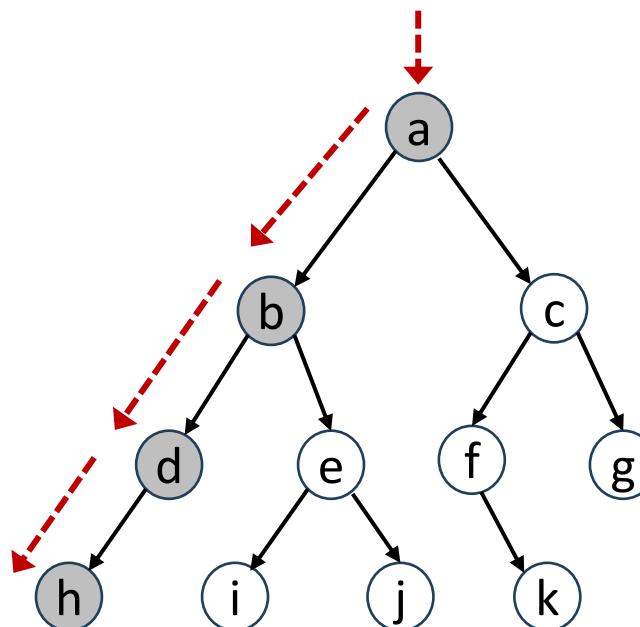
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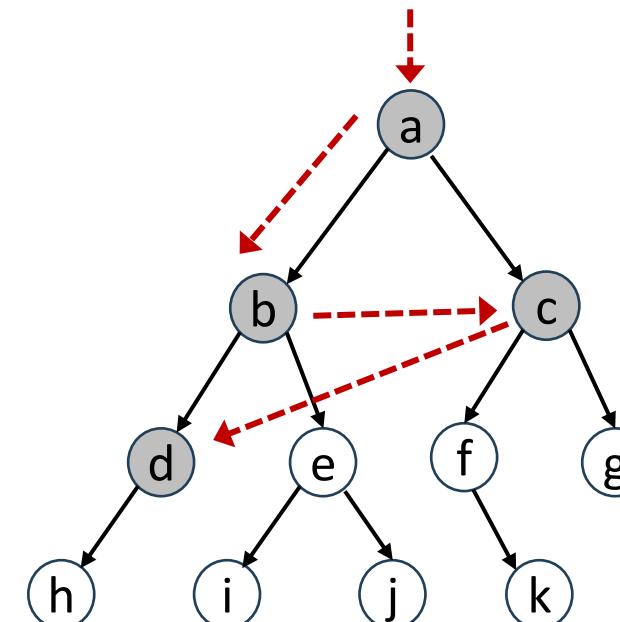
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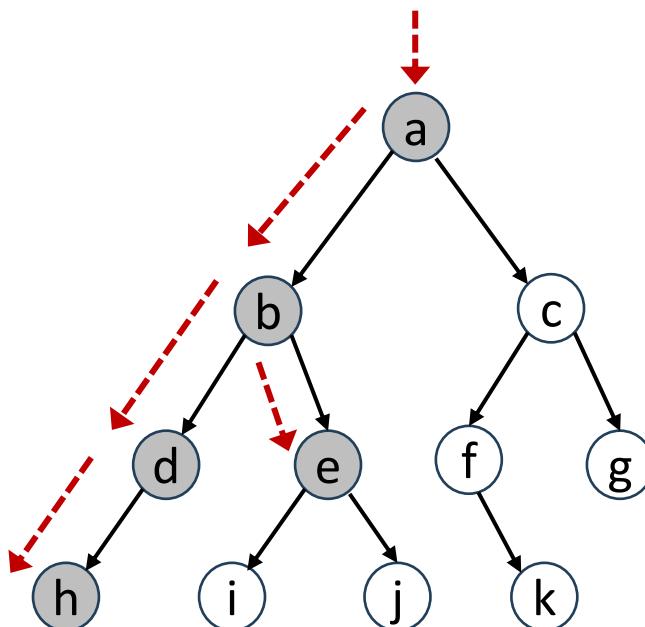
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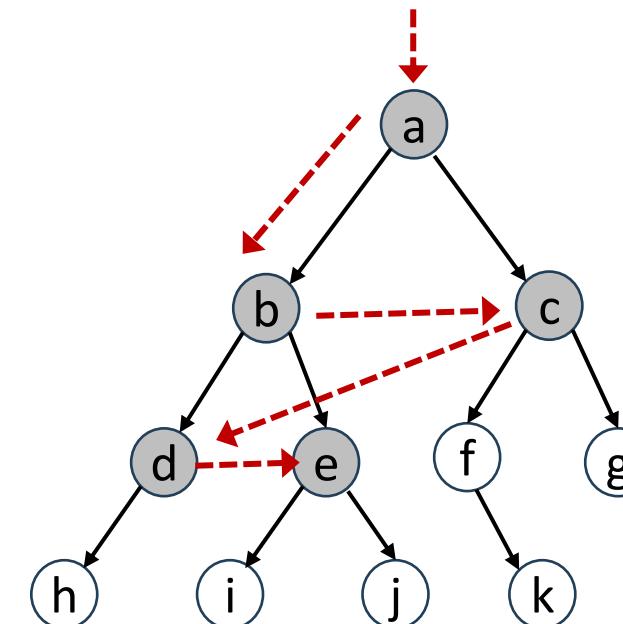
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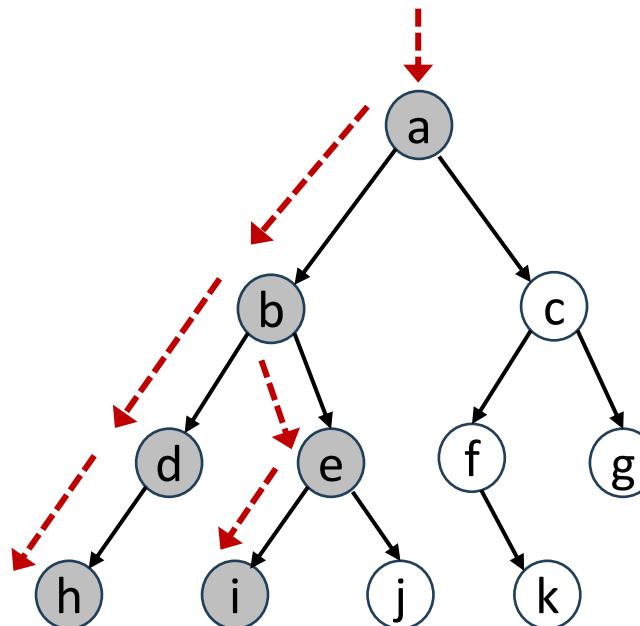
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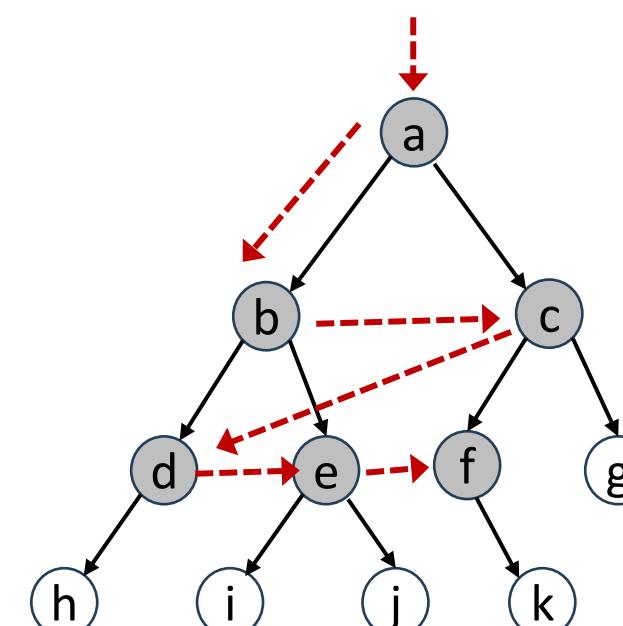
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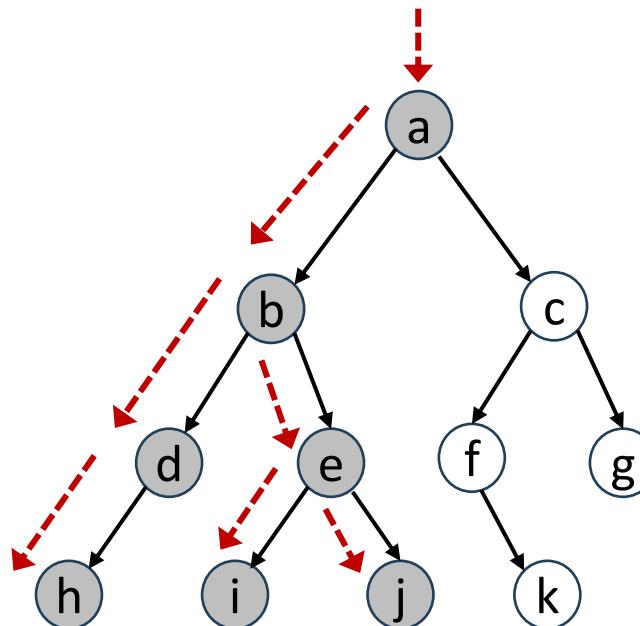
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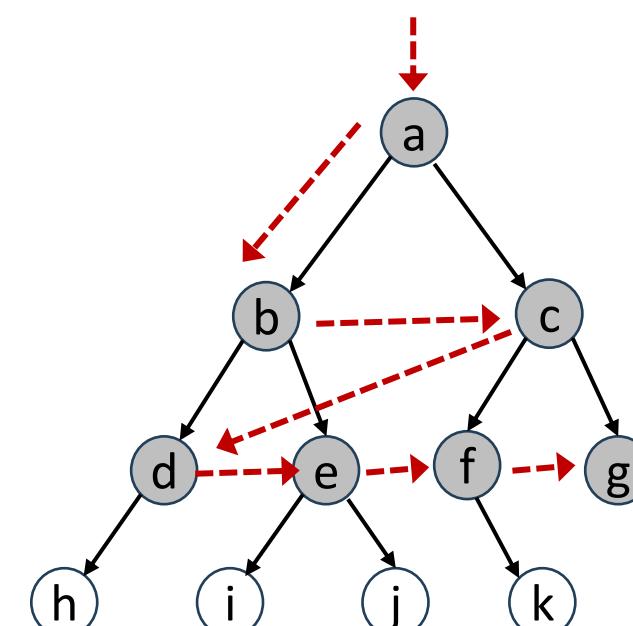
Breadth First Search

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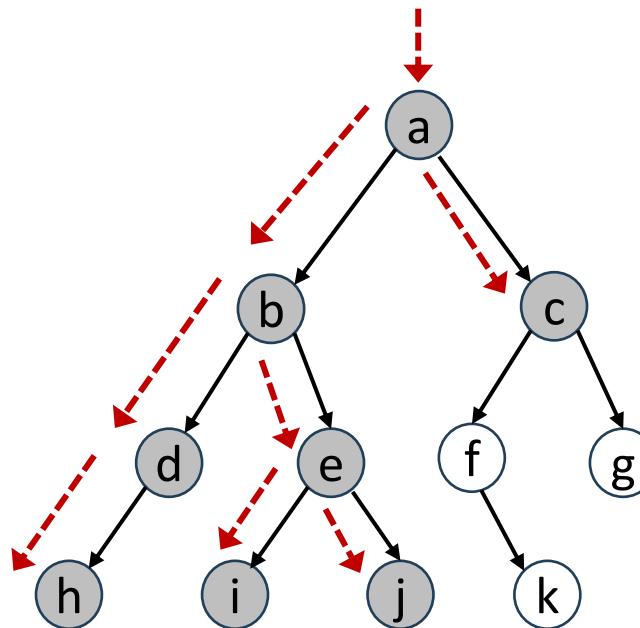
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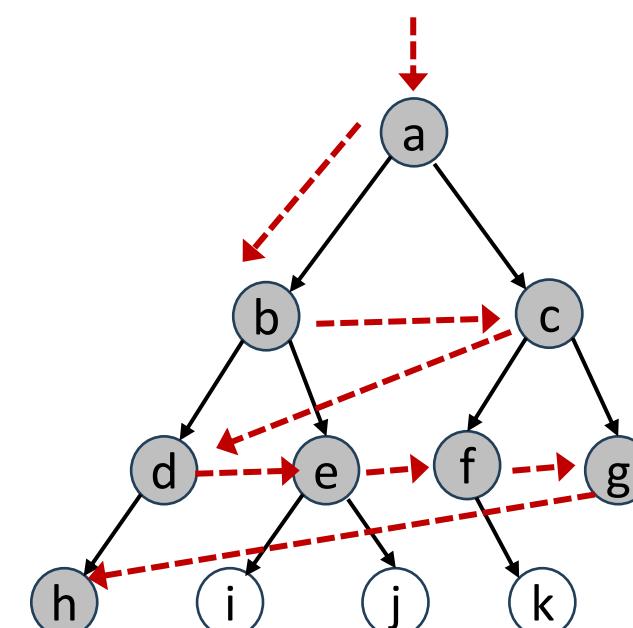
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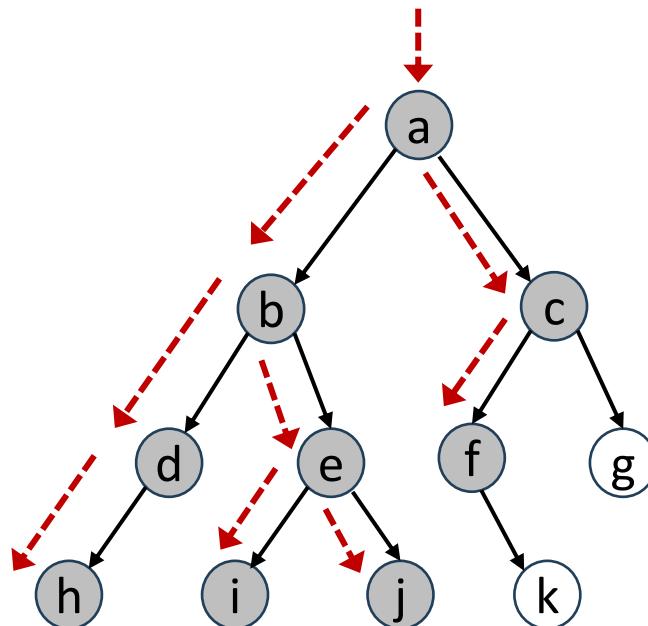
Depth First Search



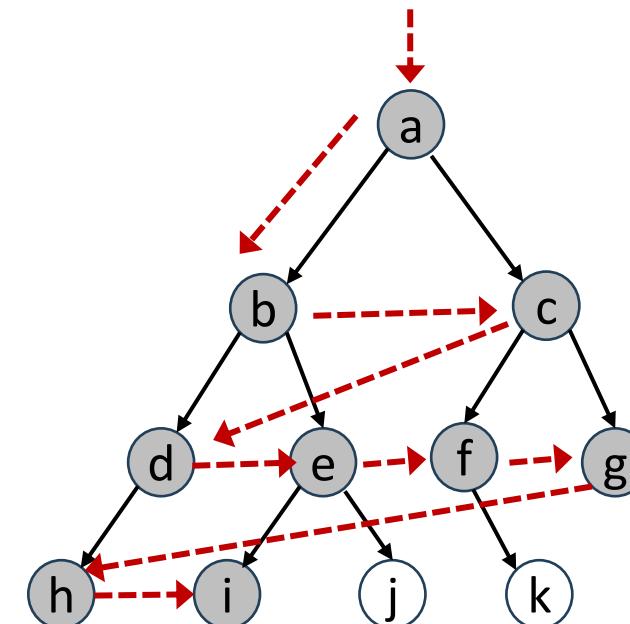
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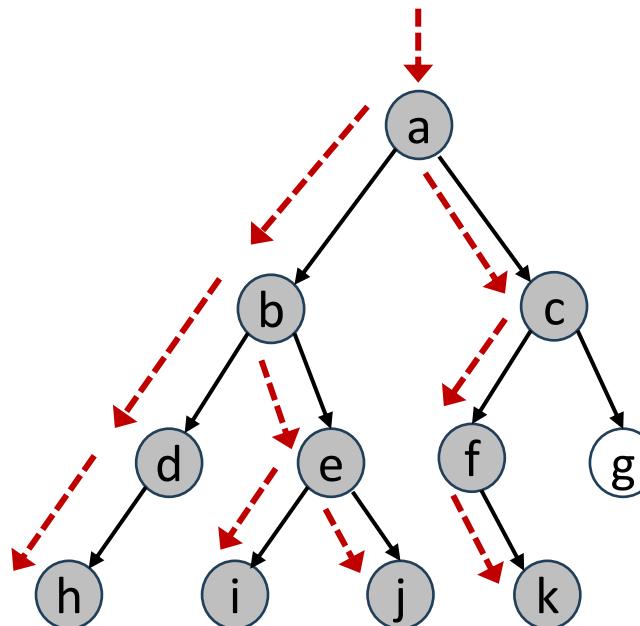
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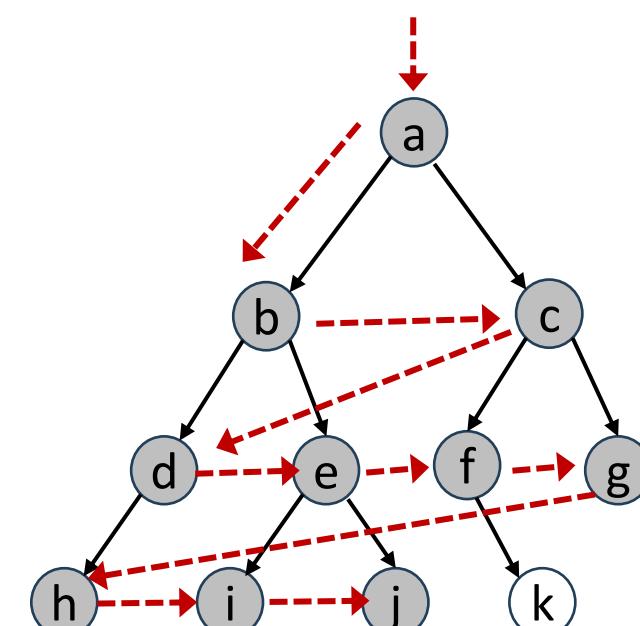
Breadth First Search

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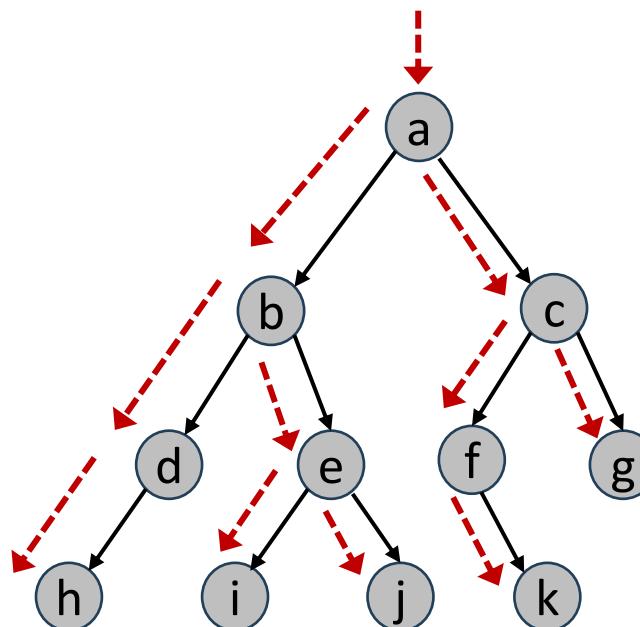
Depth First Search



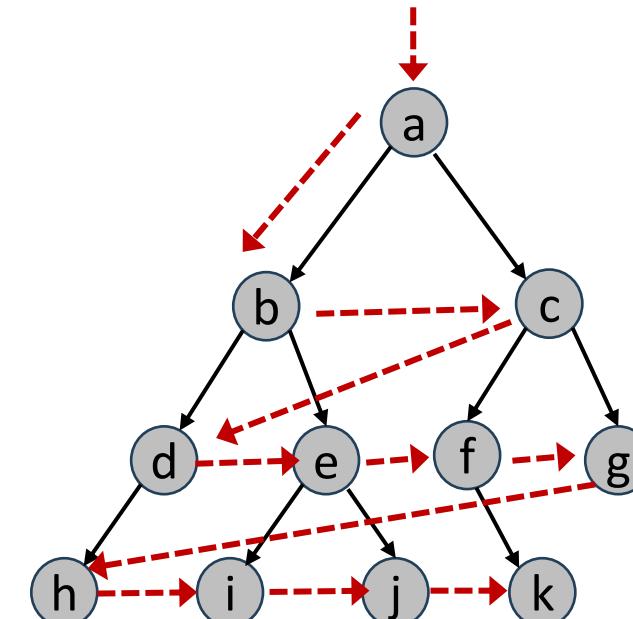
Breadth First Search

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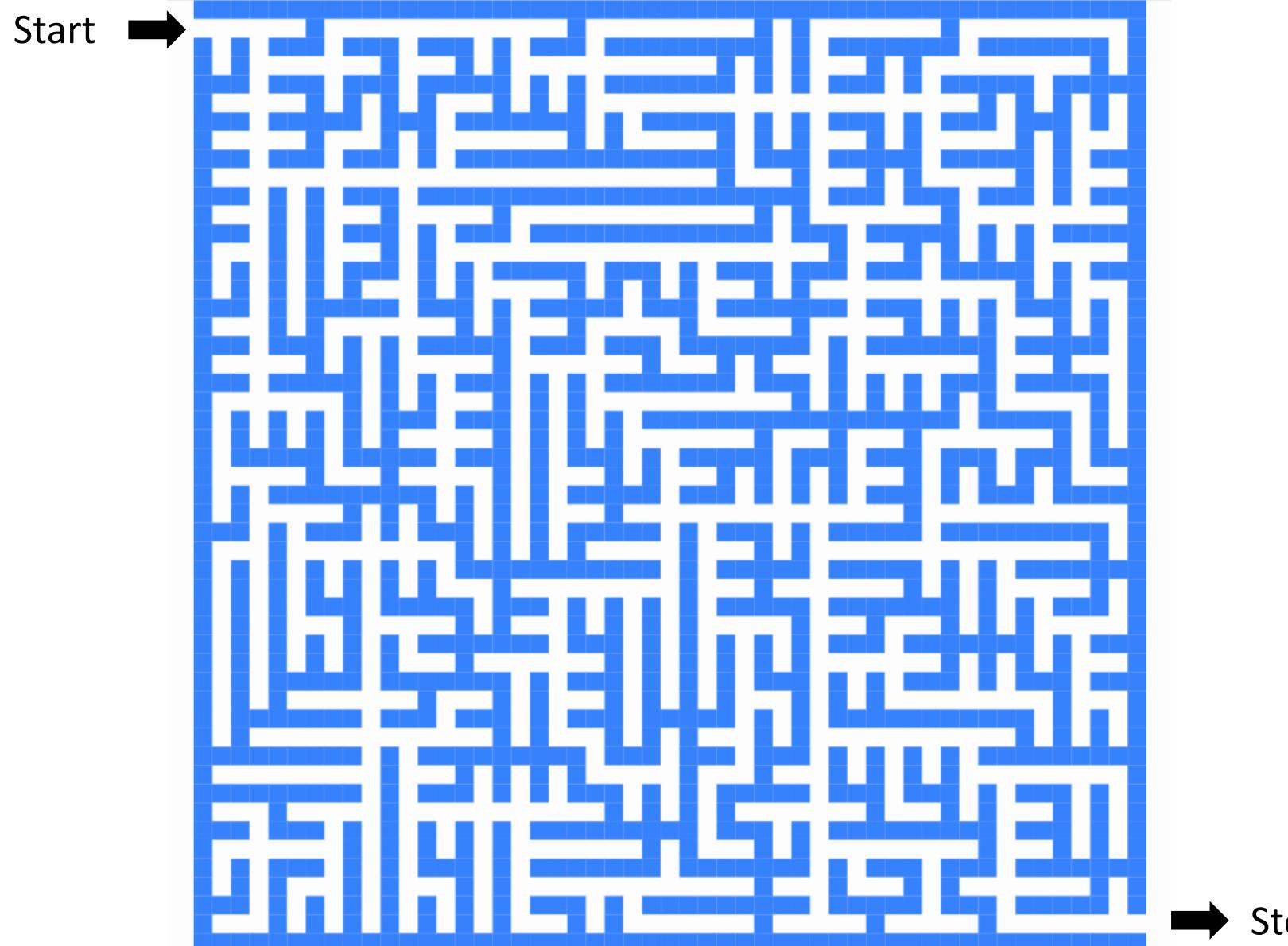
Depth First Search



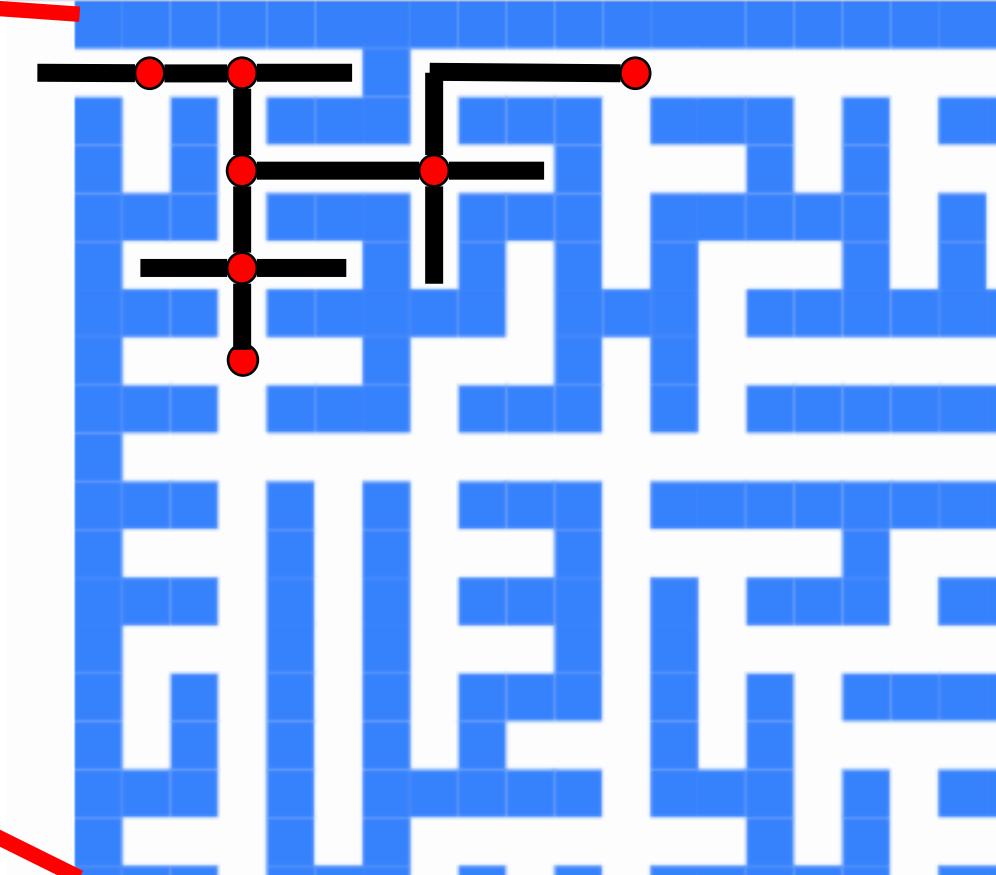
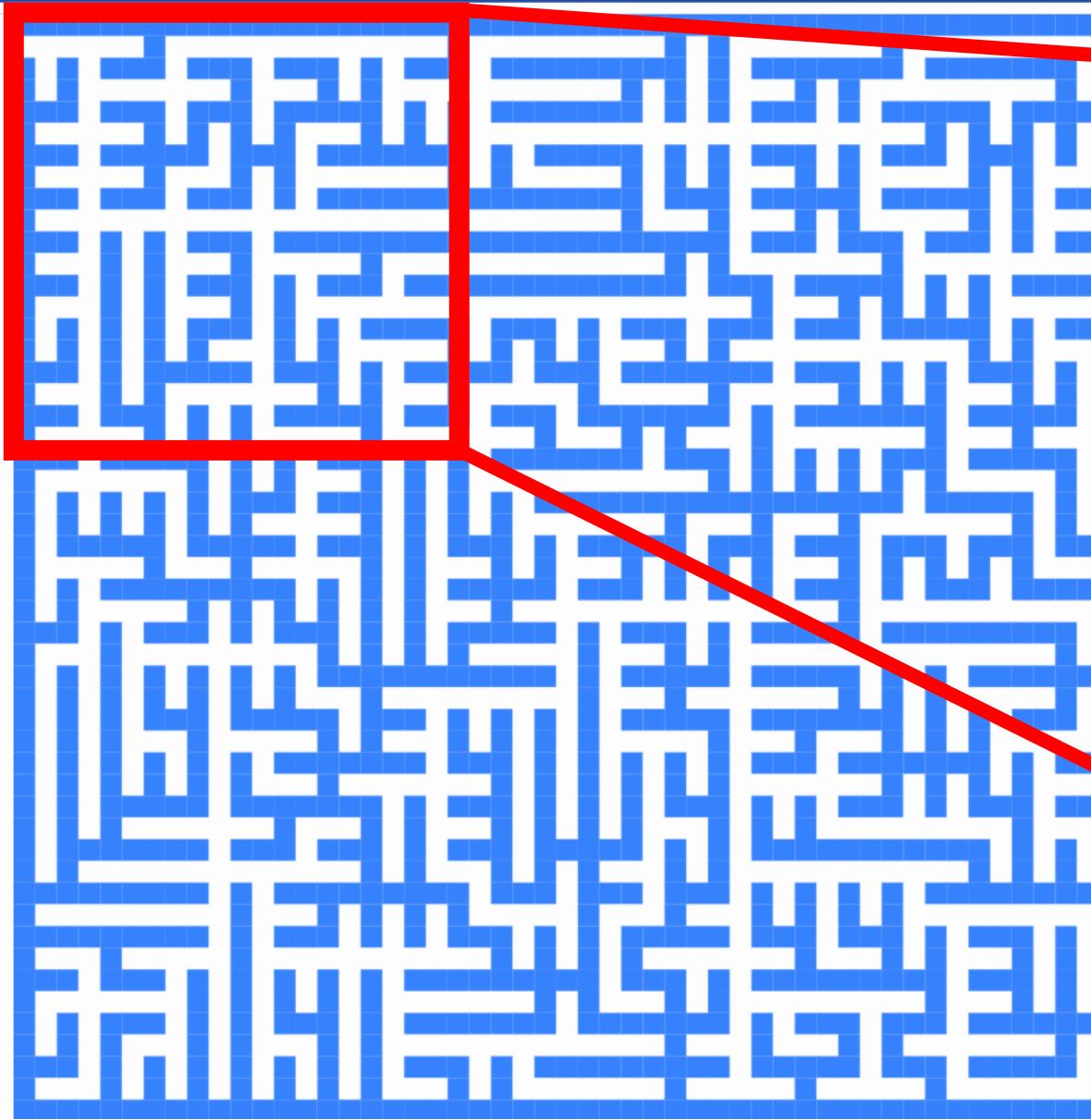
Breadth First Search



# Who can solve this MAZE?

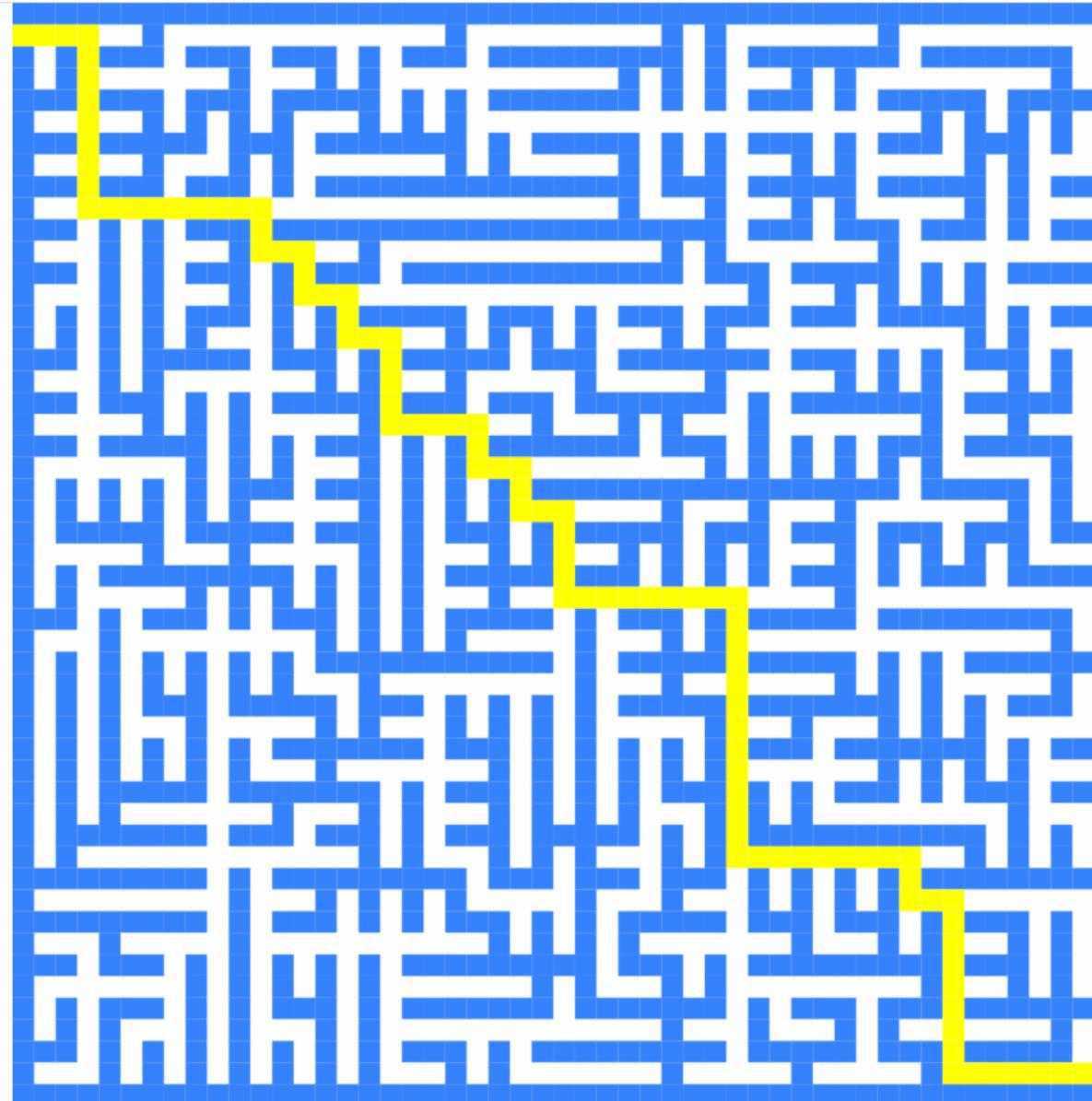


# It's a Tree

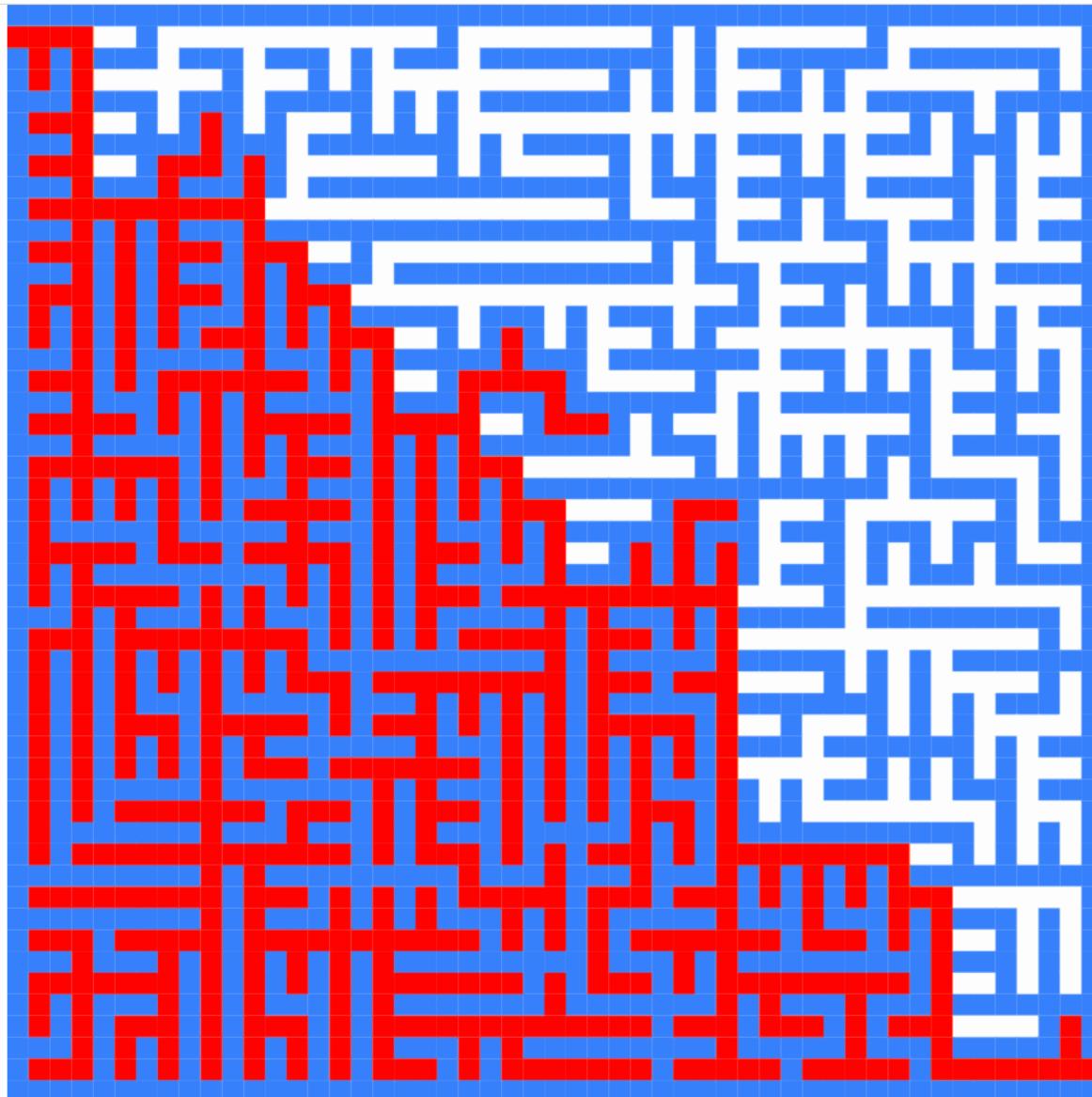




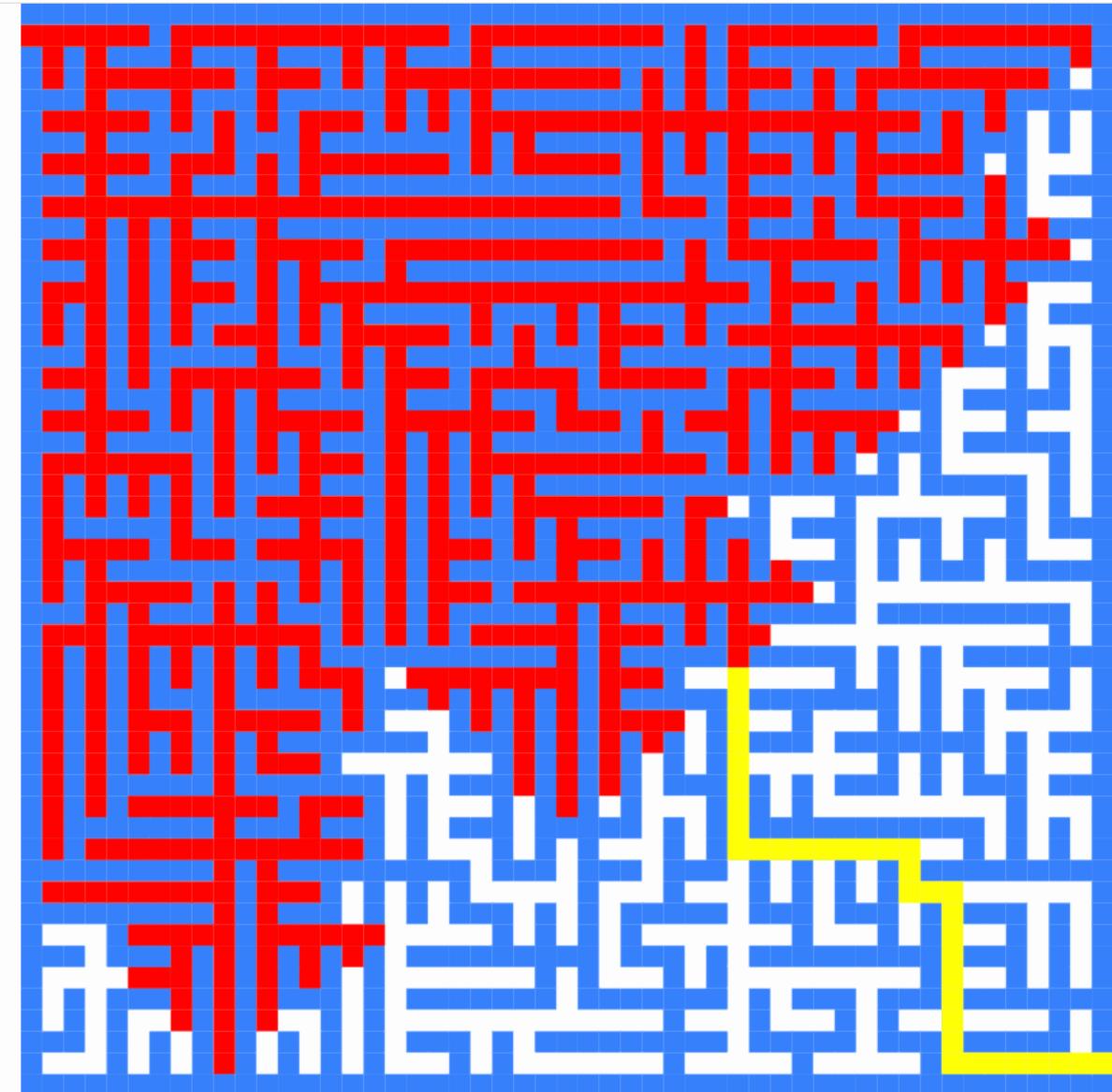
# Here is the solution...



DFS



BFS



# Few interesting videos...



Can water solve a maze?

Steve Mould ✓

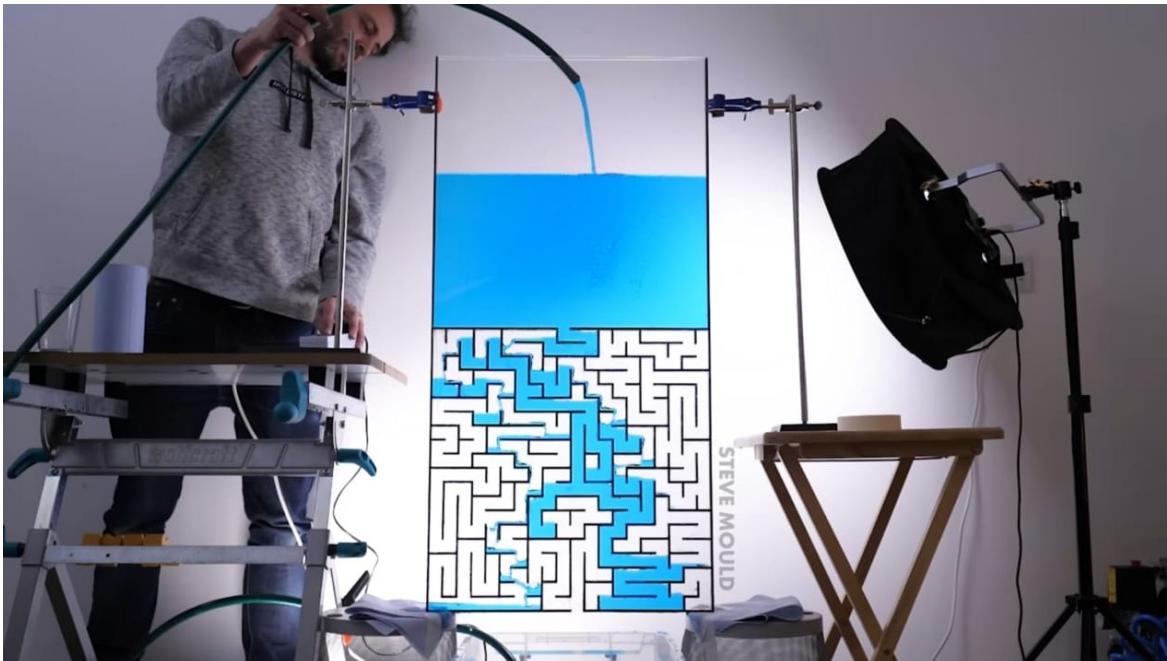
10M views • 5 months ago



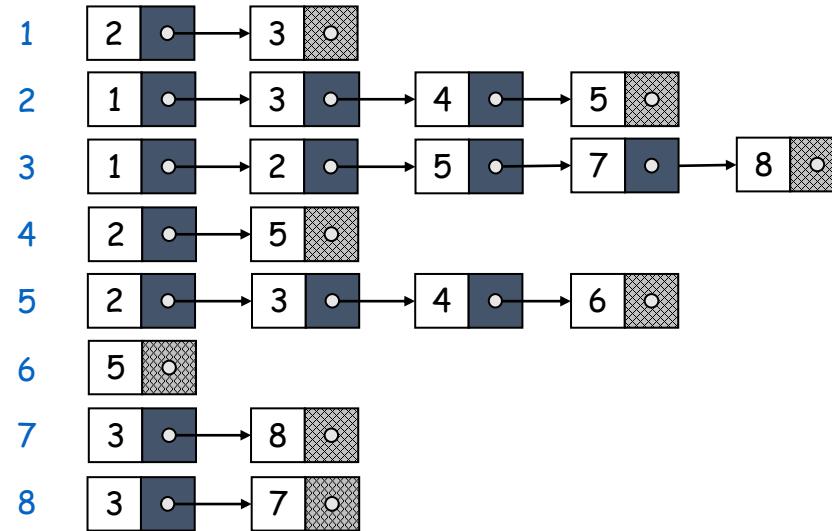
The Fastest Maze-Solving Competition On Earth

Veritasium ✓

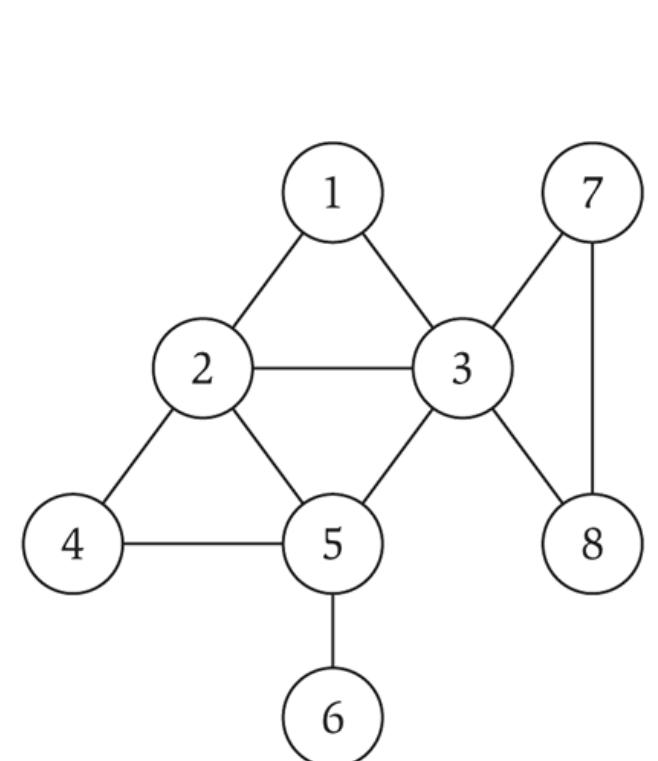
12M views • 4 months ago



## Adjacency List

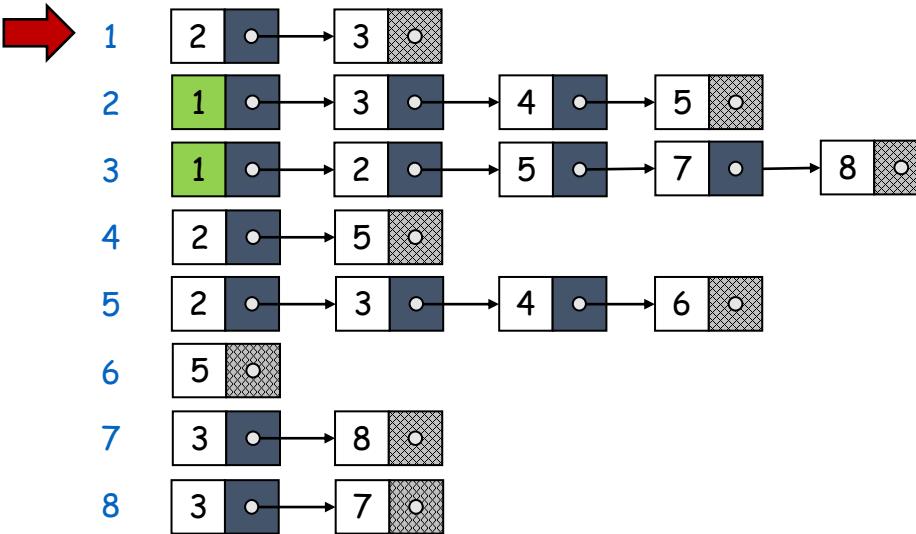


## Graph

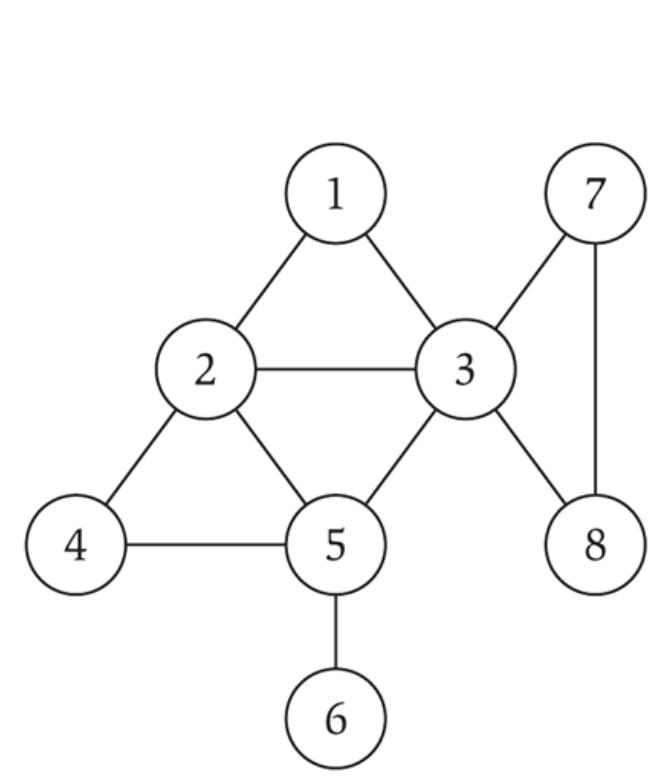


## DFS Tree

Adjacency List



Graph



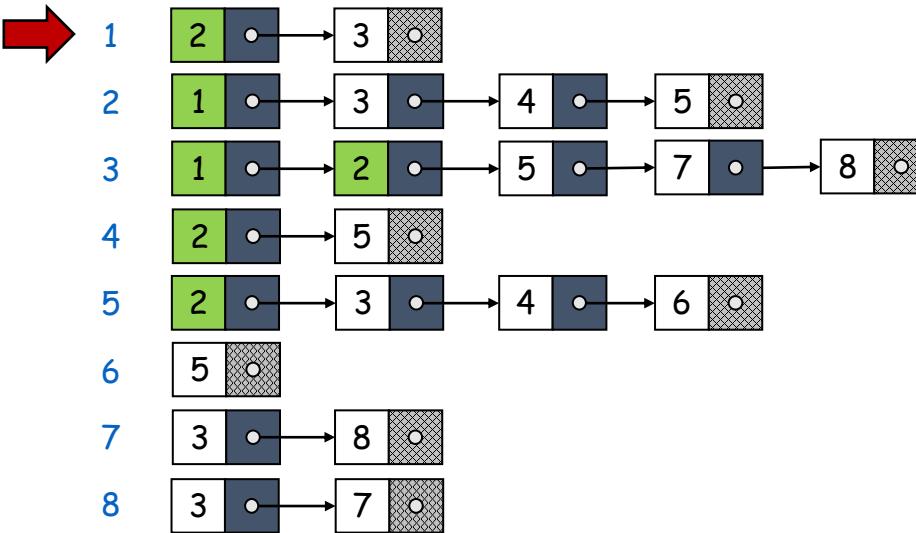
DFS Tree



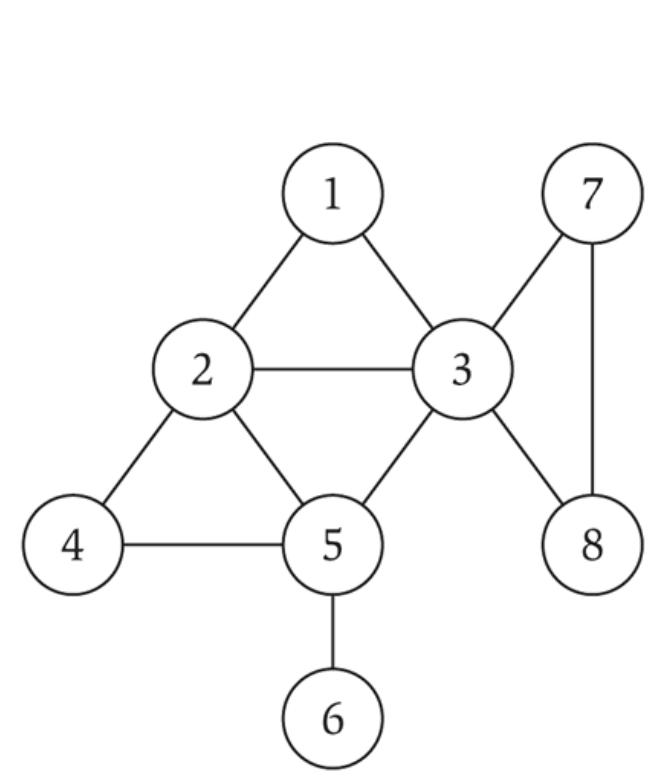
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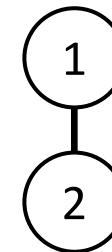
Adjacency List



Graph



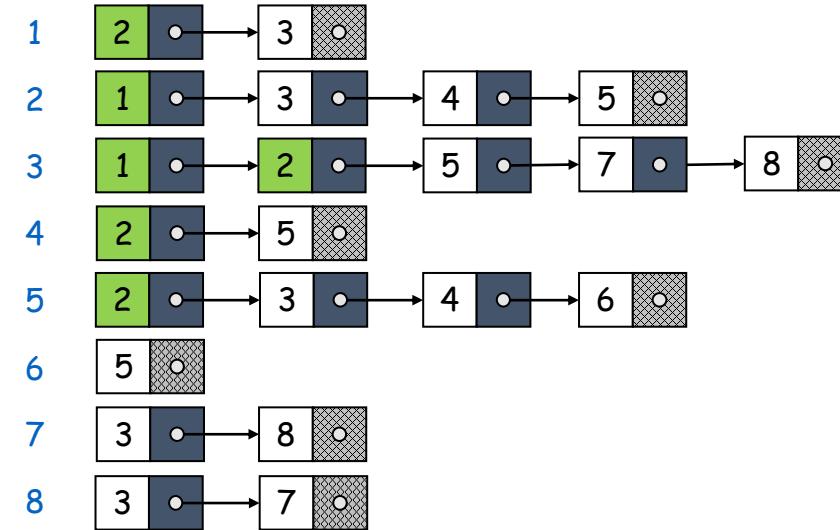
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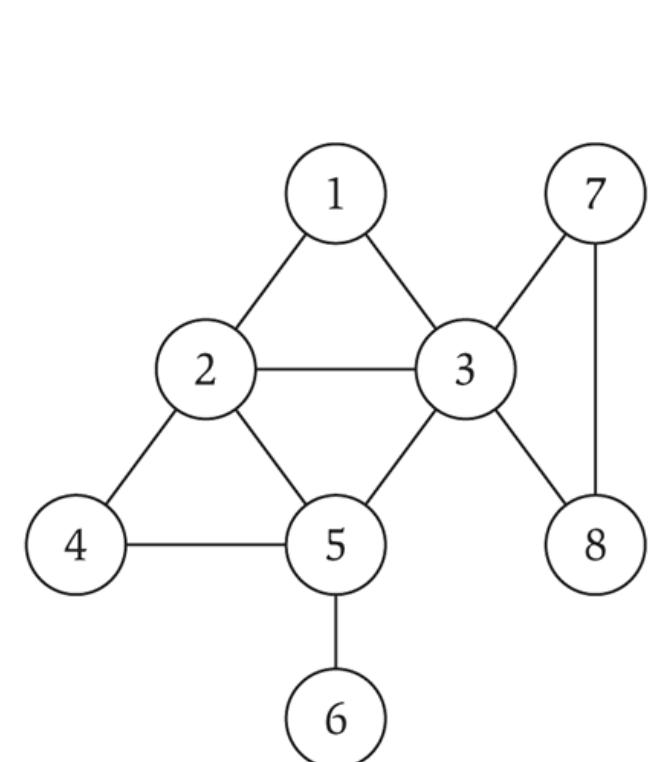
Stack



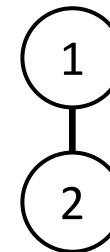
Adjacency List



Graph

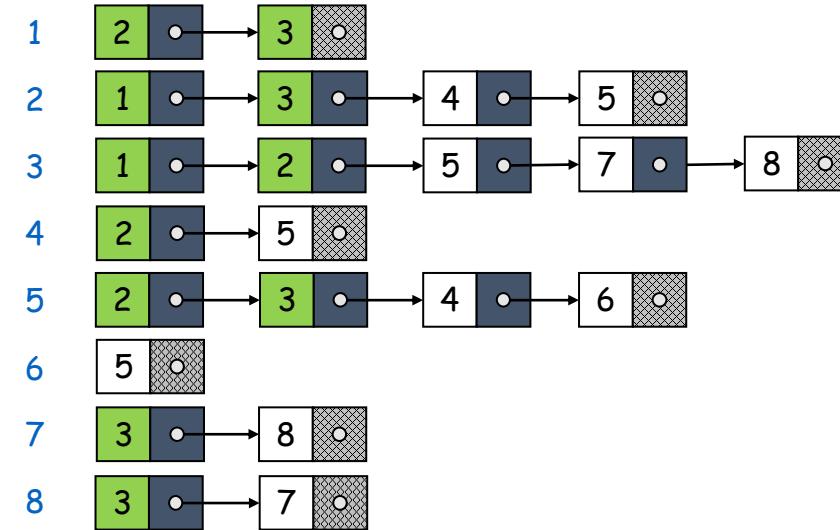


DFS Tree

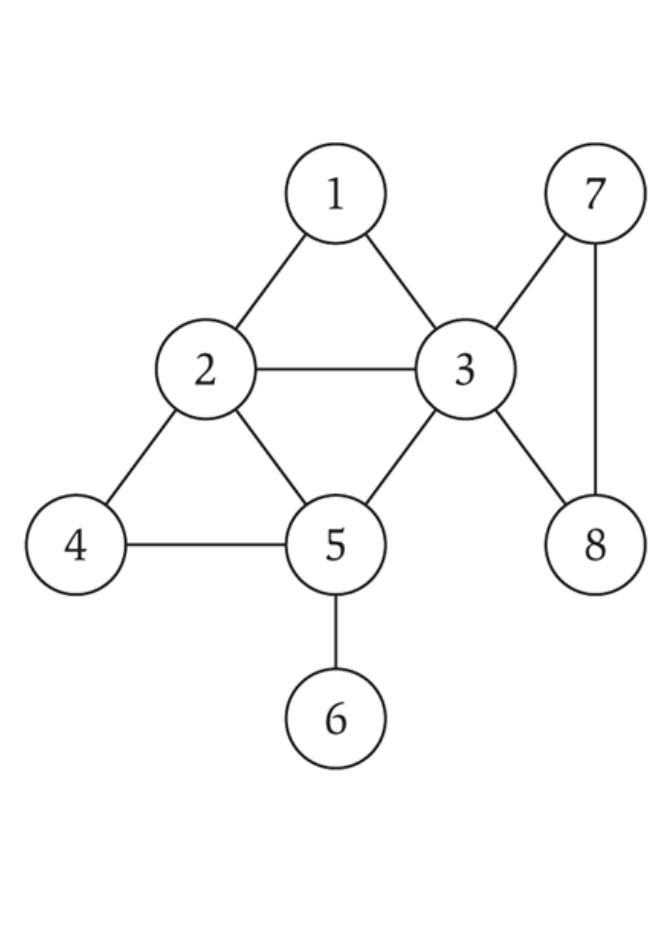


Stack    1 | 2 | 3

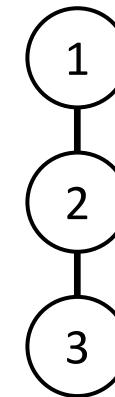
Adjacency List



Graph



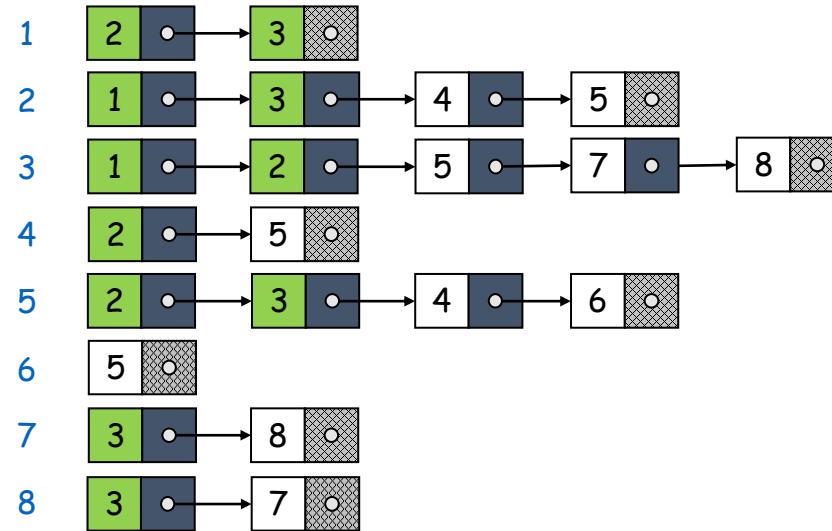
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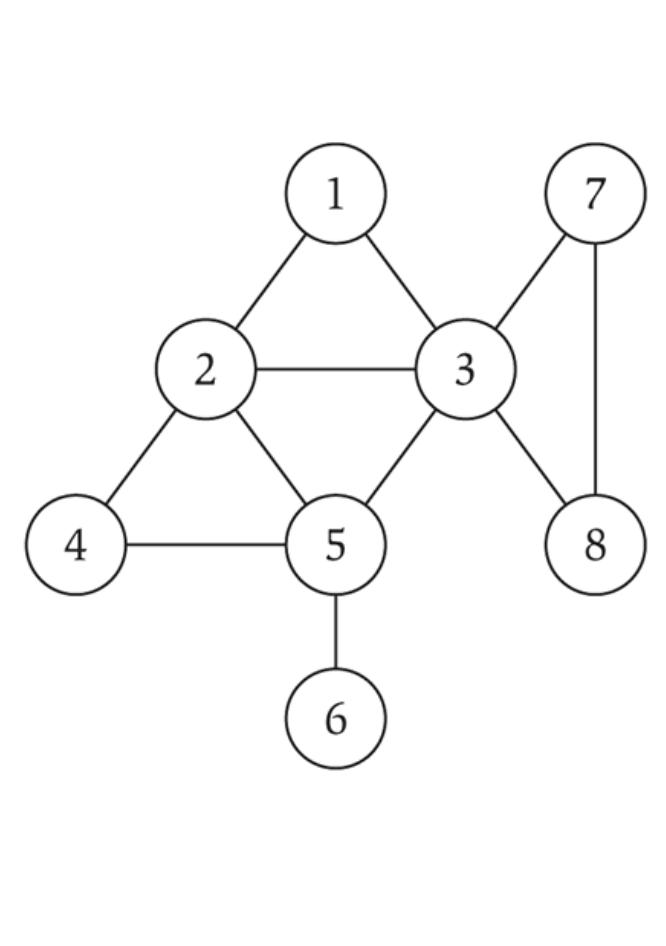
Stack

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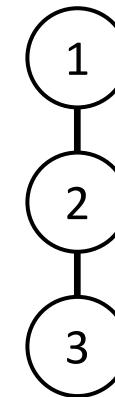
Adjacency List



Graph



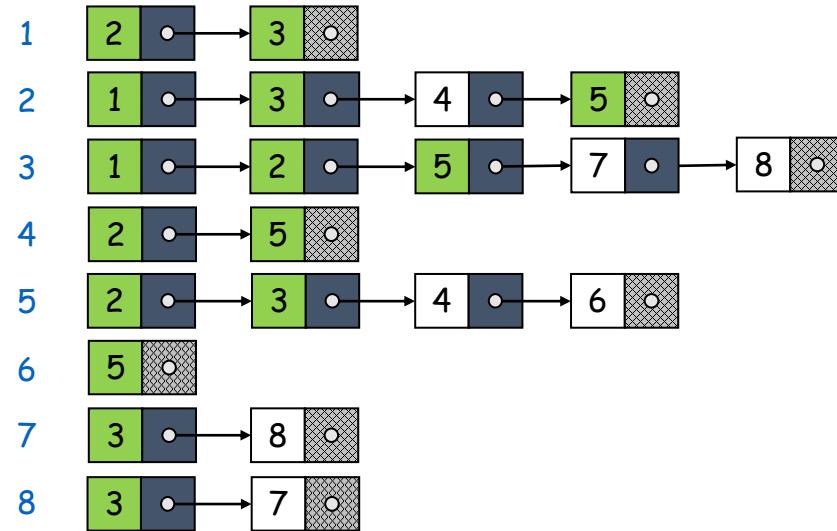
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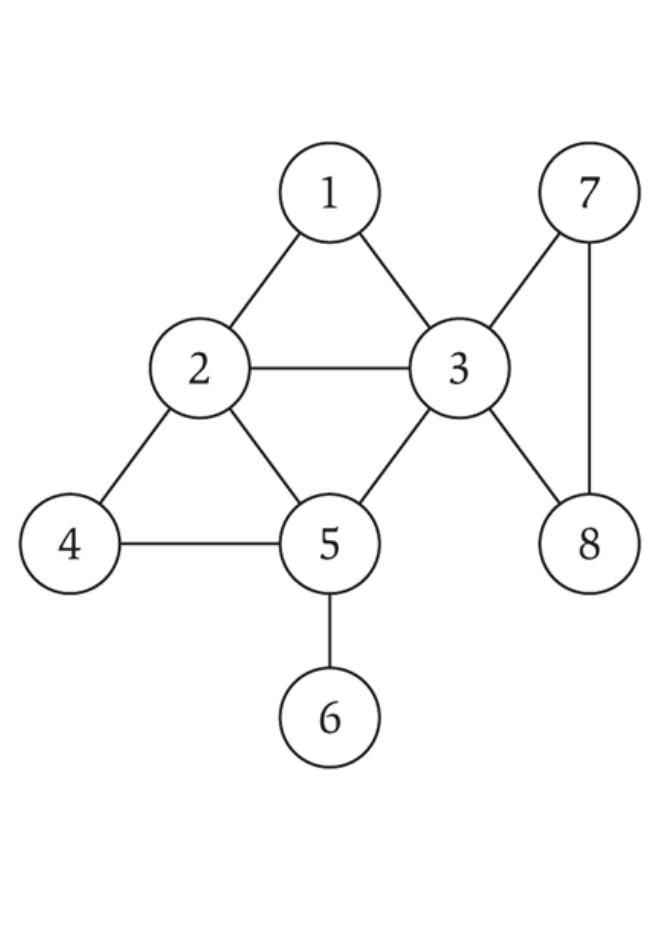
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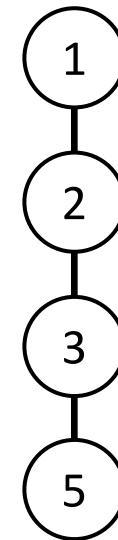
Adjacency List



Graph

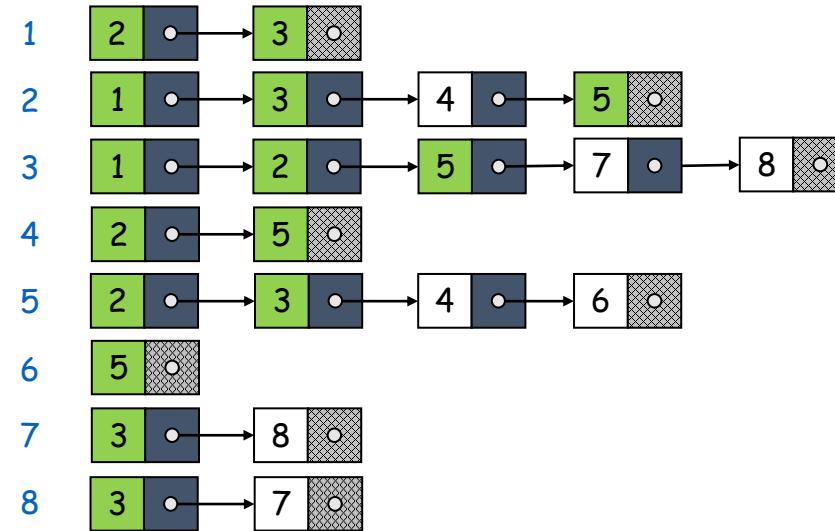


DFS Tree

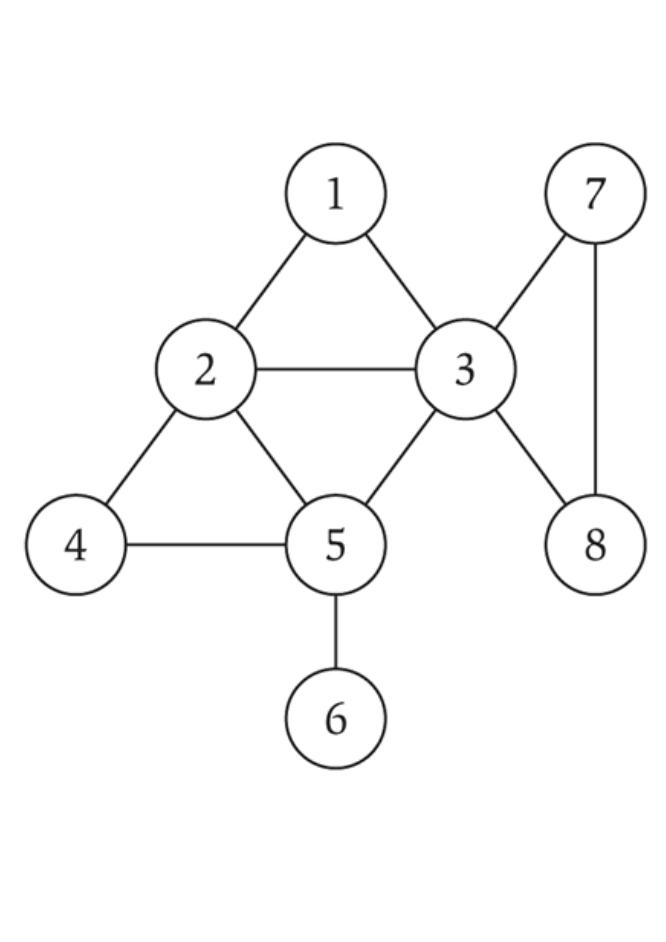


Stack    1 | 2 | 3 | 5

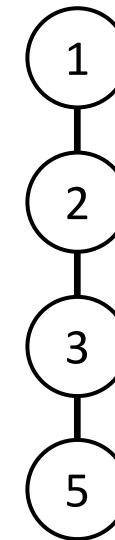
Adjacency List



Graph

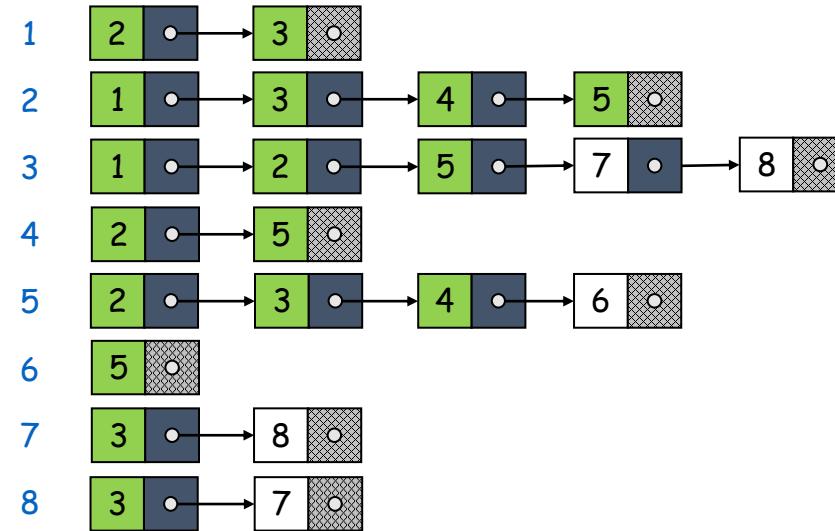


DFS Tree

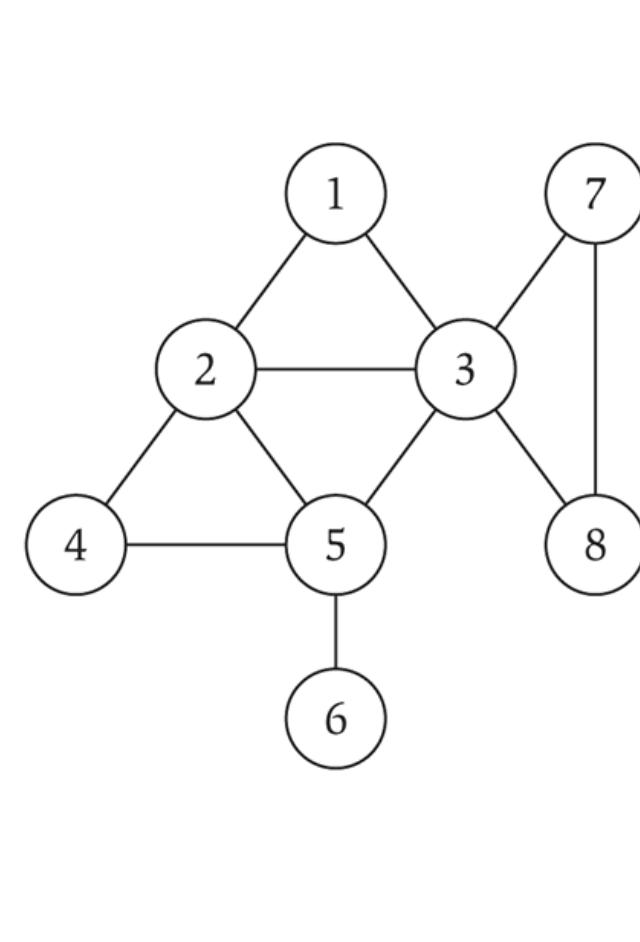


Stack    1 | 2 | 3 | 5 | 4

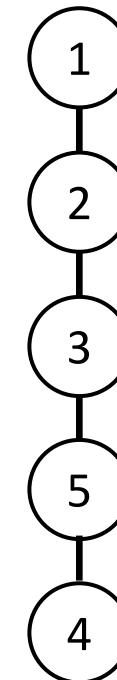
Adjacency List



Graph

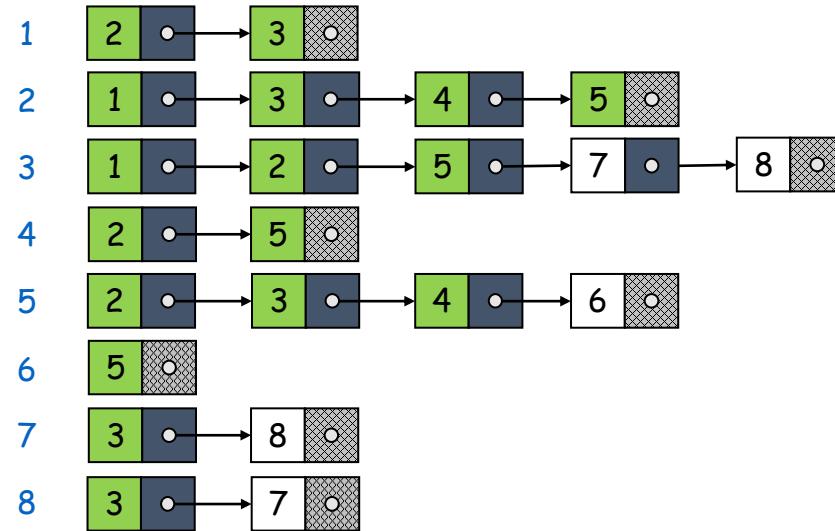


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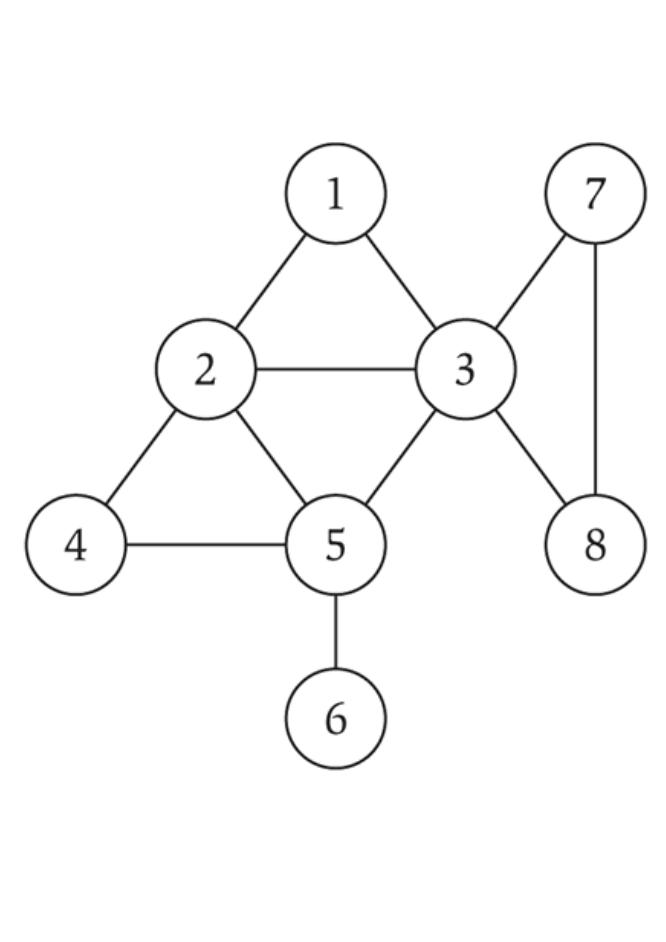


Stack    1 | 2 | 3 | 5 | 4

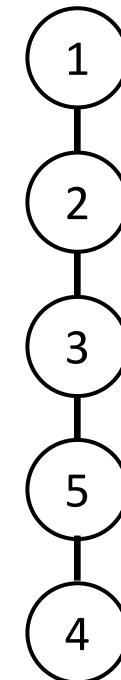
Adjacency List



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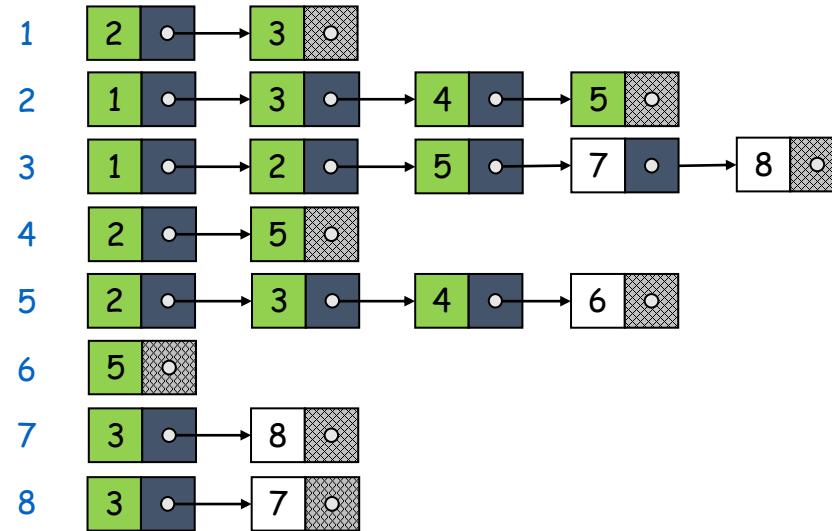


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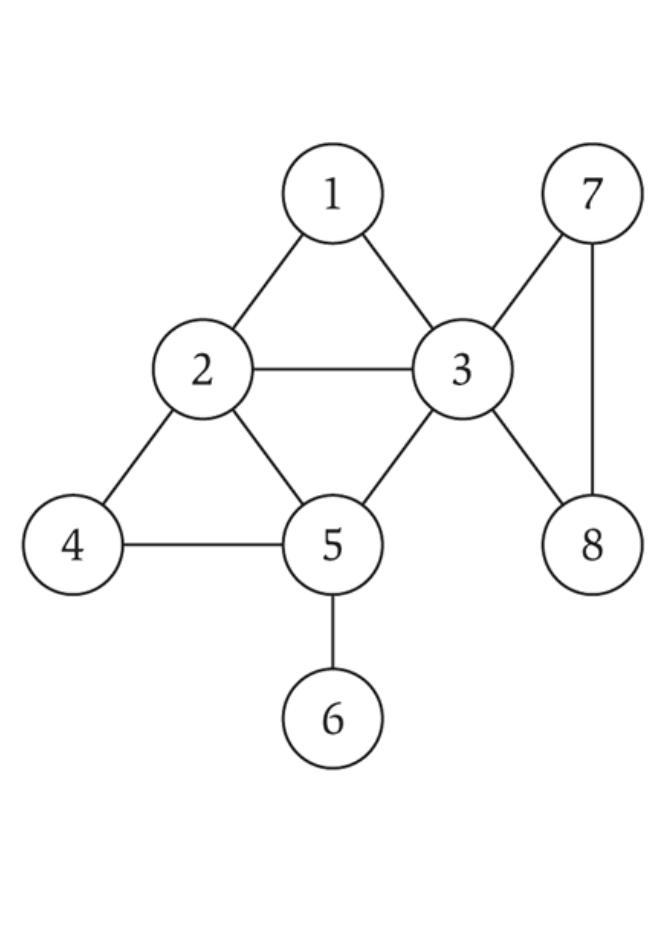


Stack    1 | 2 | 3 | 5

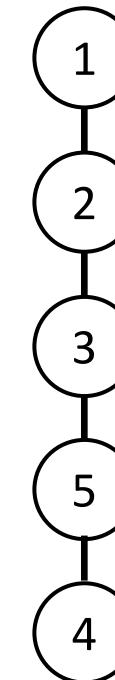
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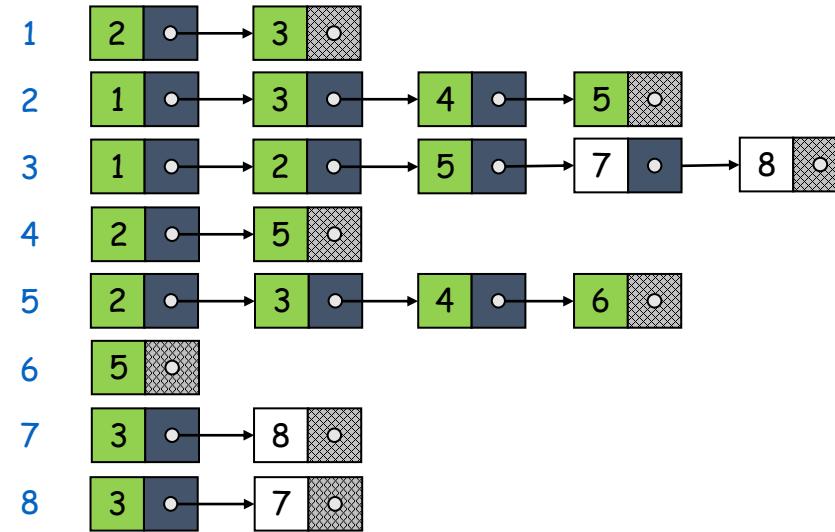
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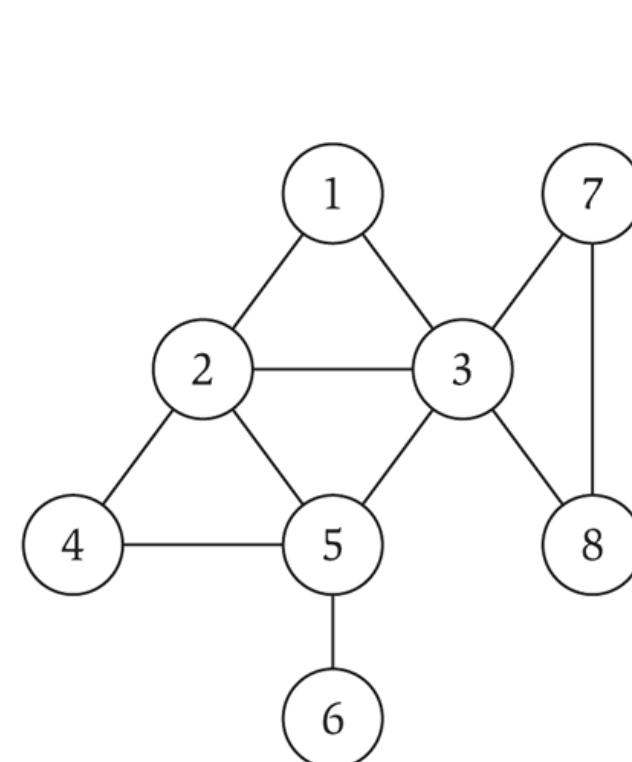
Stack    

1	2	3	5	6
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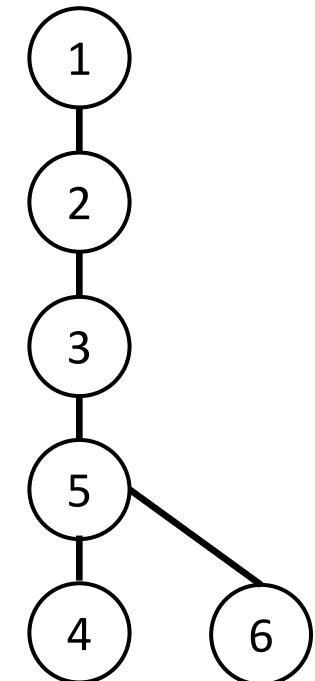
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Graph



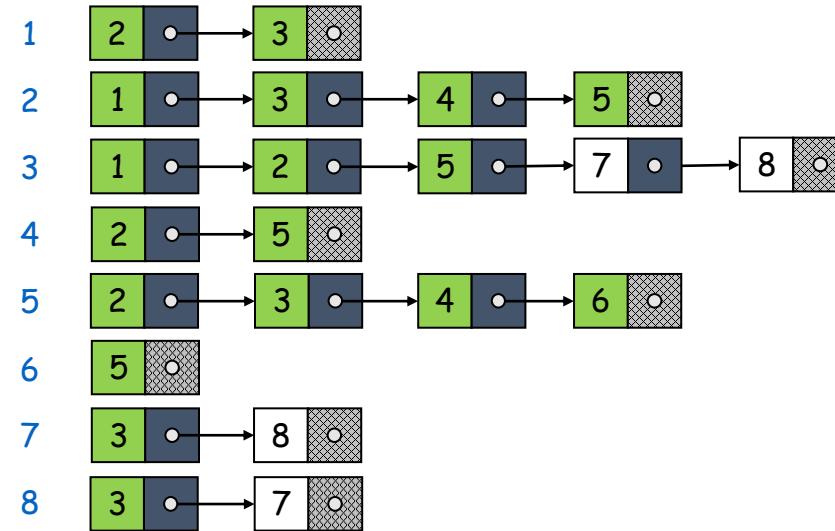
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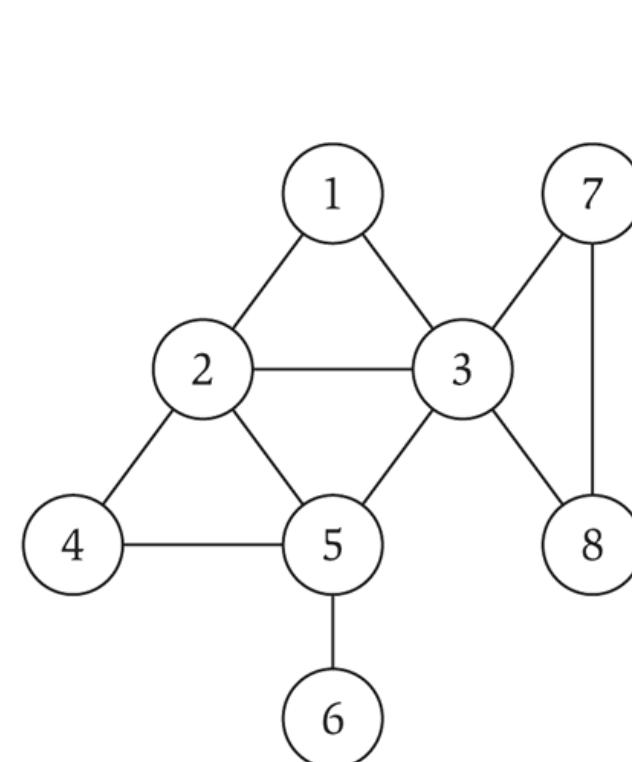
Stack    

1	2	3	5	6
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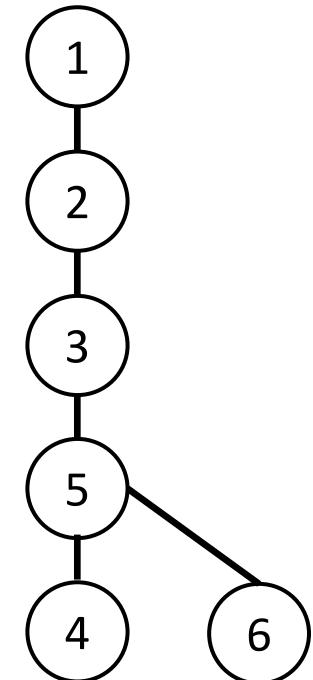
Adjacency List



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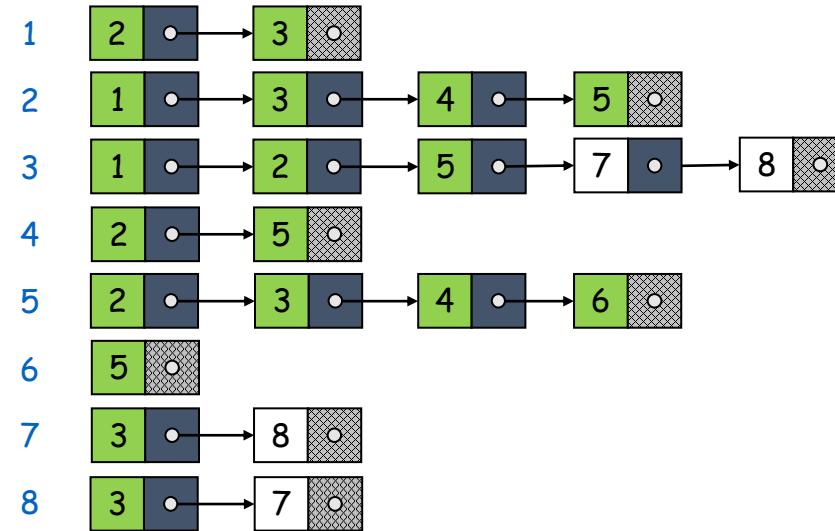


DFS Tree

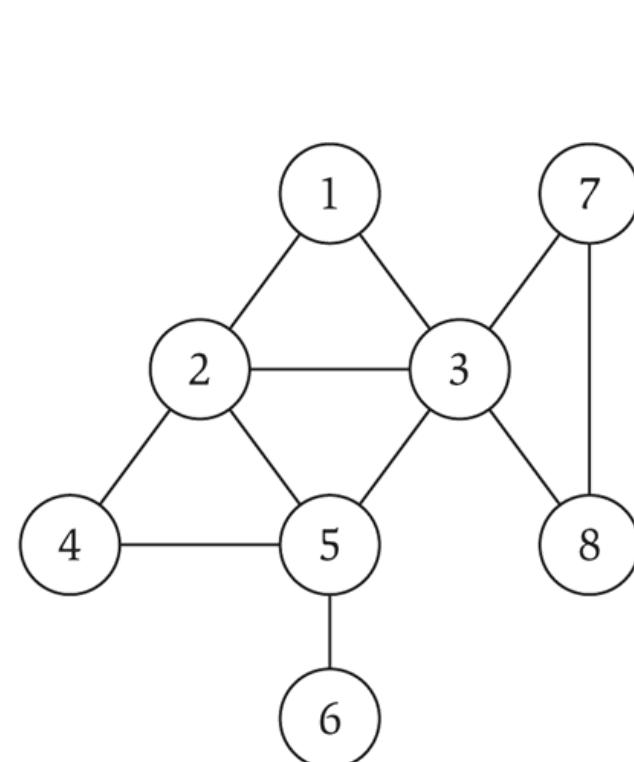


Stack    1 | 2 | 3 | 5

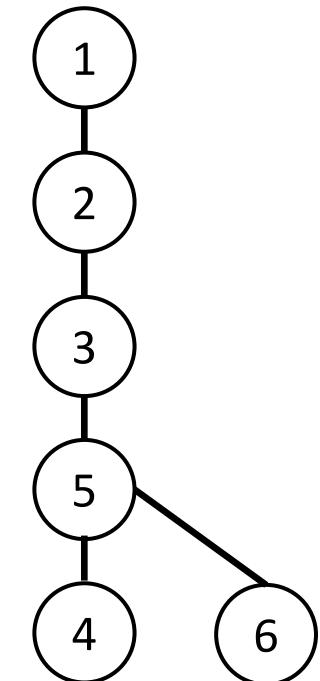
Adjacency List



Graph

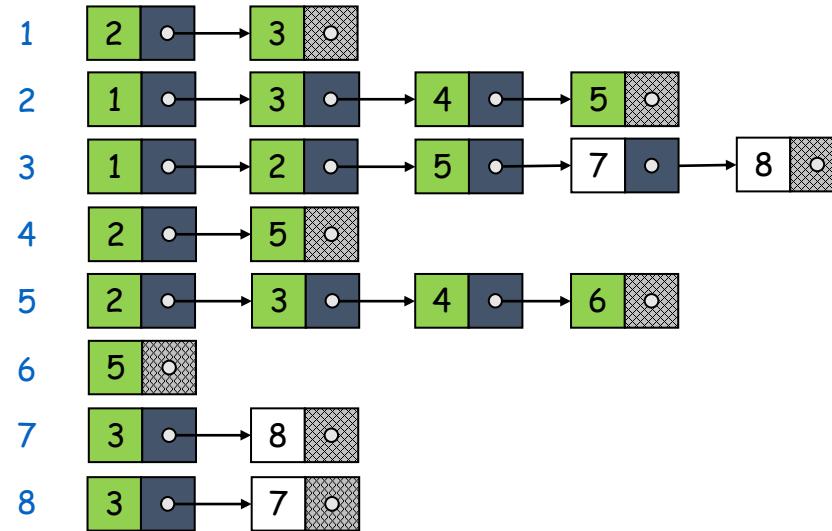


DFS Tree

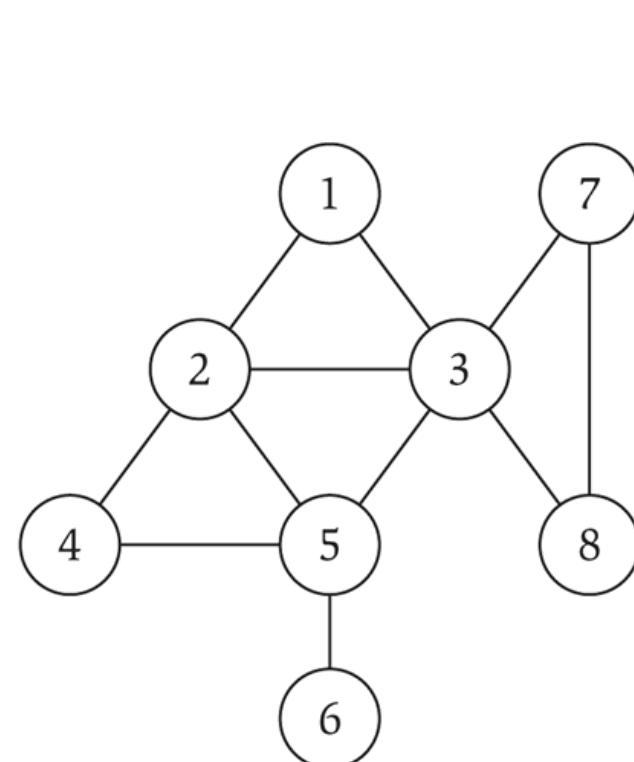


Stack    1 | 2 | 3

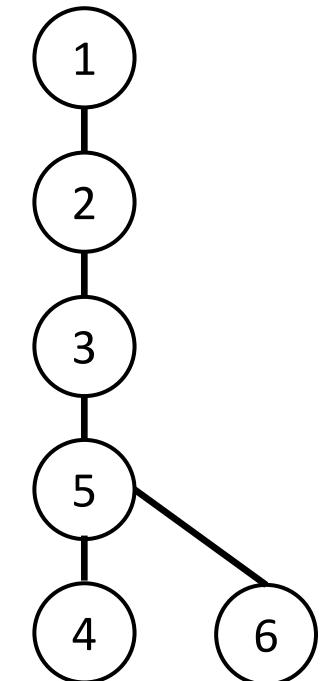
Adjacency List



Graph



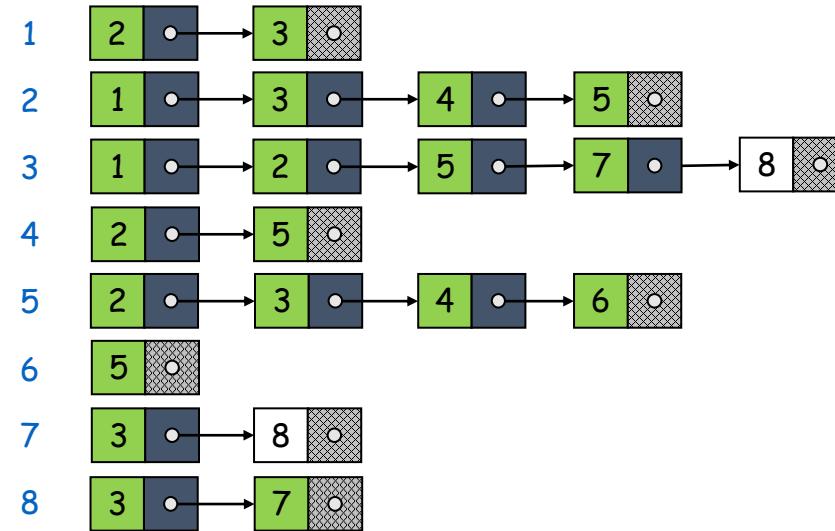
DFS Tree



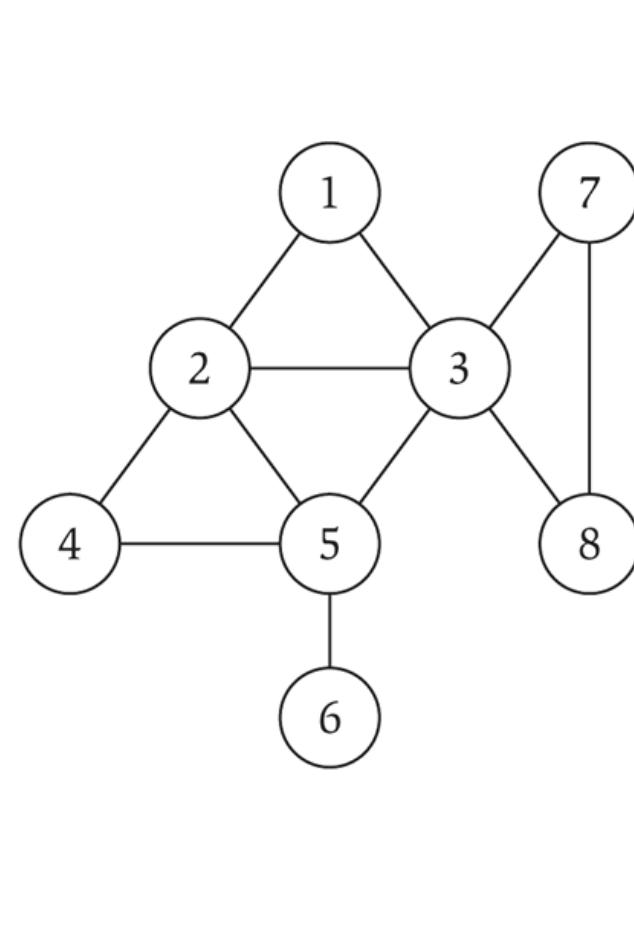
Stack    

1	2	3	7
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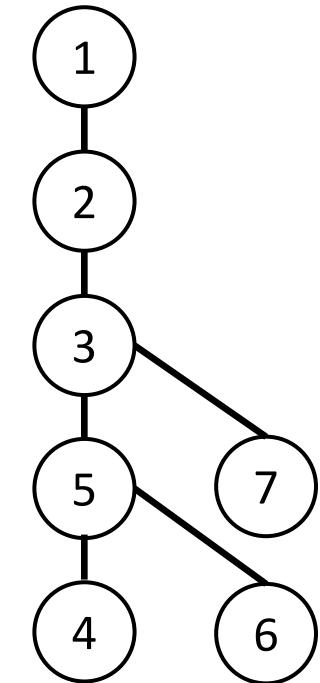
Adjacency List



Graph



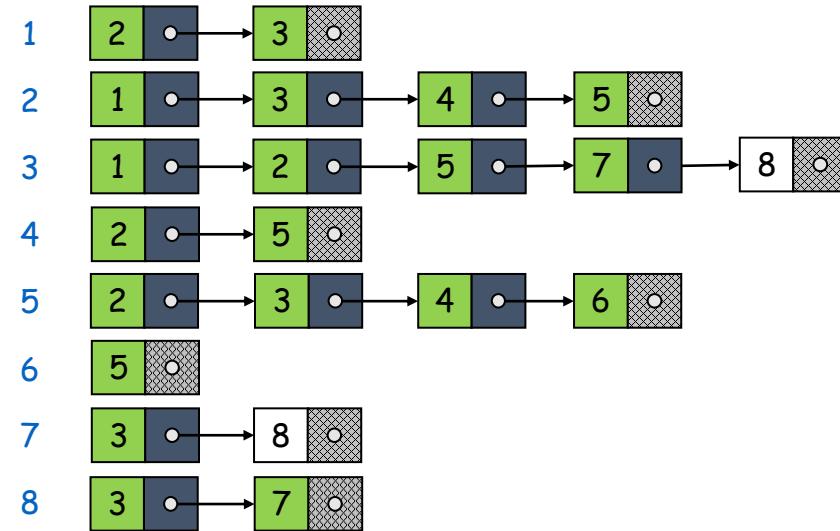
DFS Tree



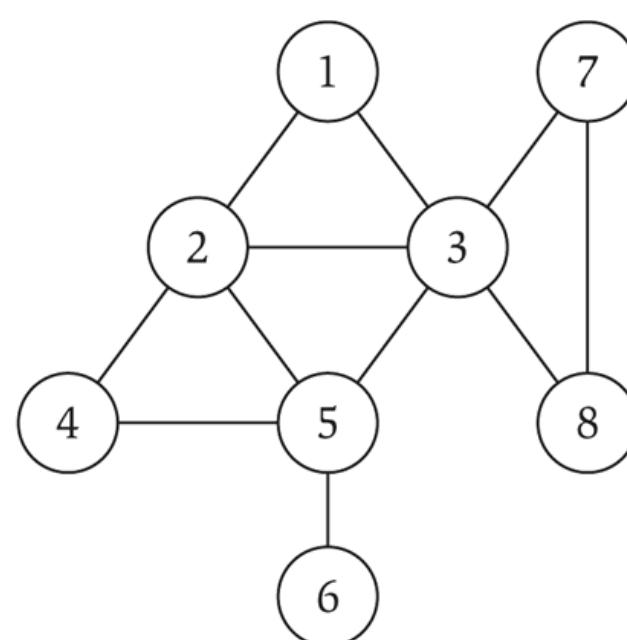
Stack



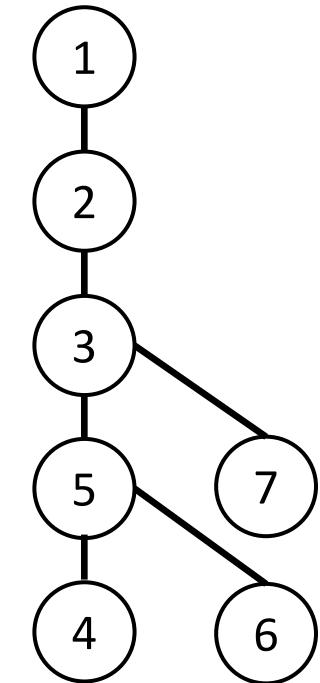
Adjacency List



Graph



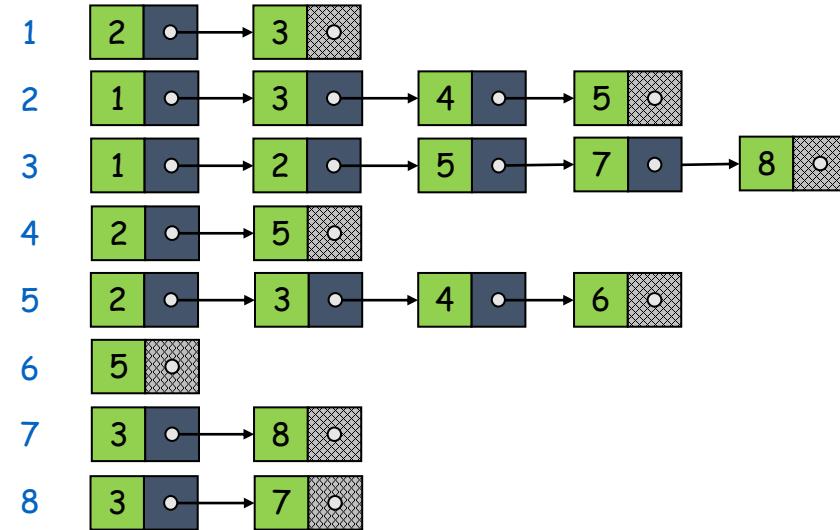
DFS Tree



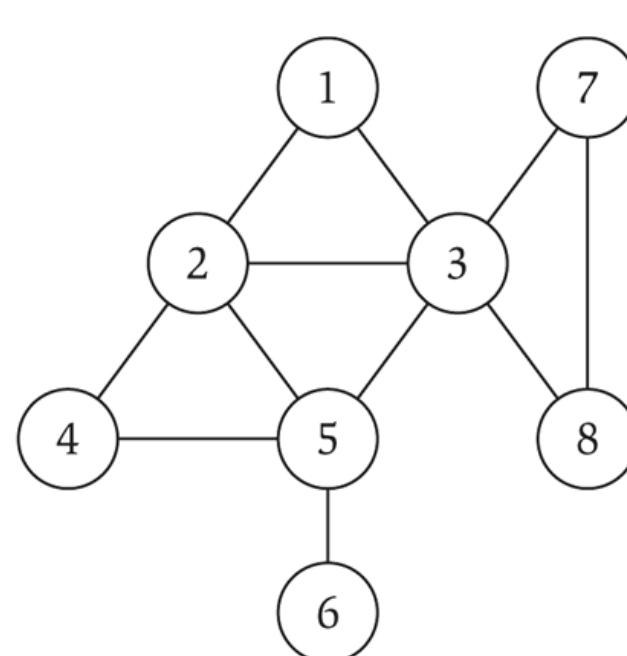
Stack    

1	2	3	7	8
---	---	---	---	---

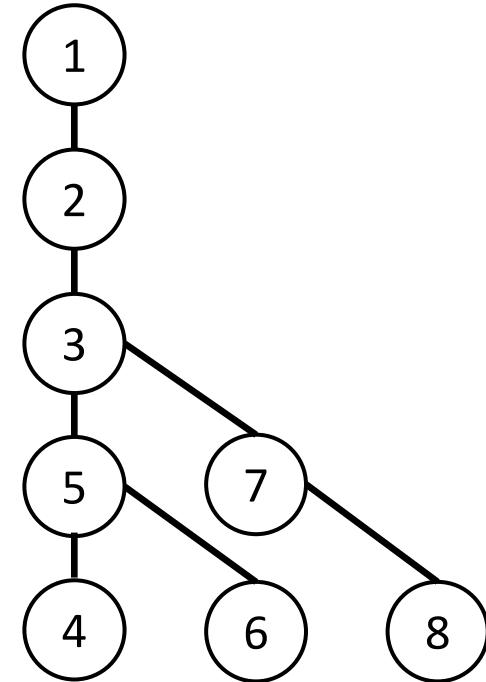
Adjacency List



Graph



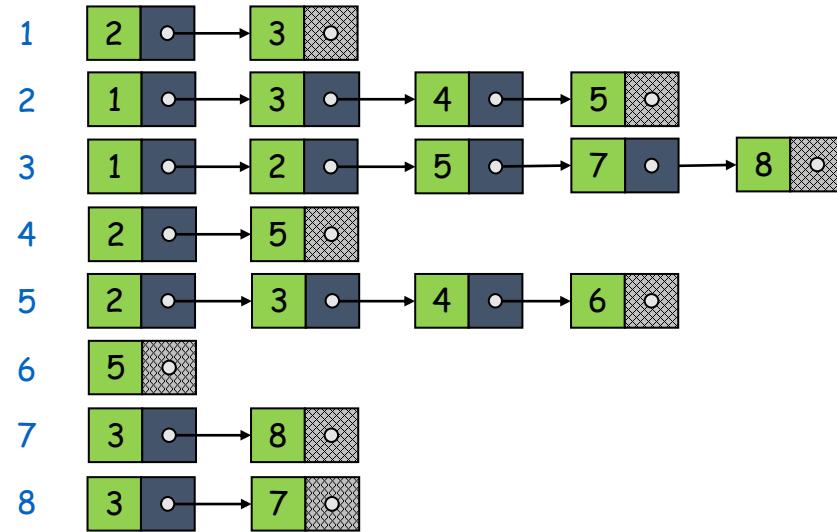
DFS Tree



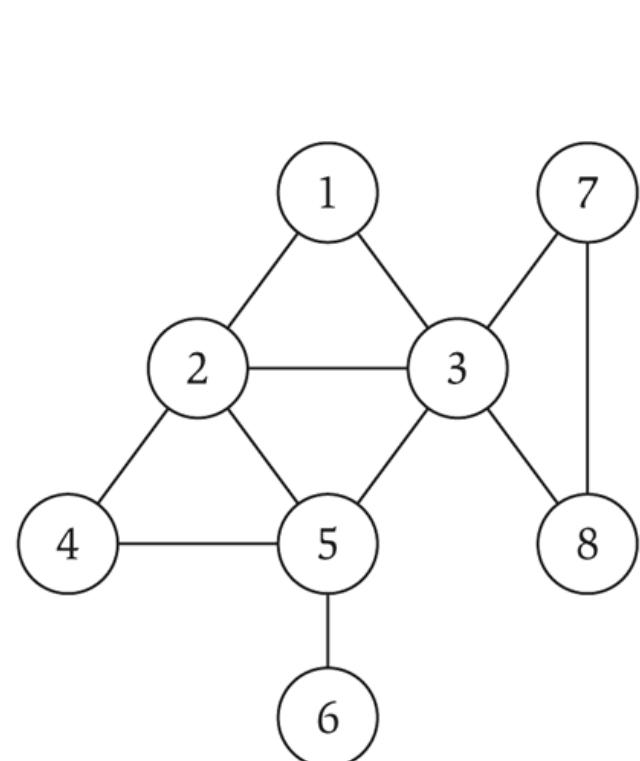
Stack    

1	2	3	7	8
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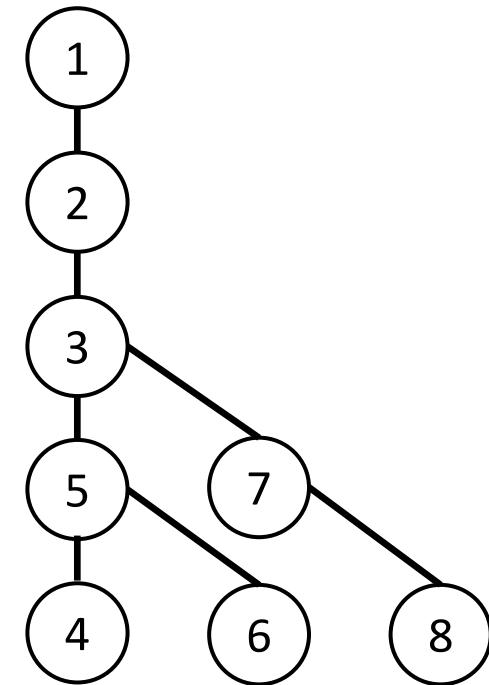
Adjacency List



Graph



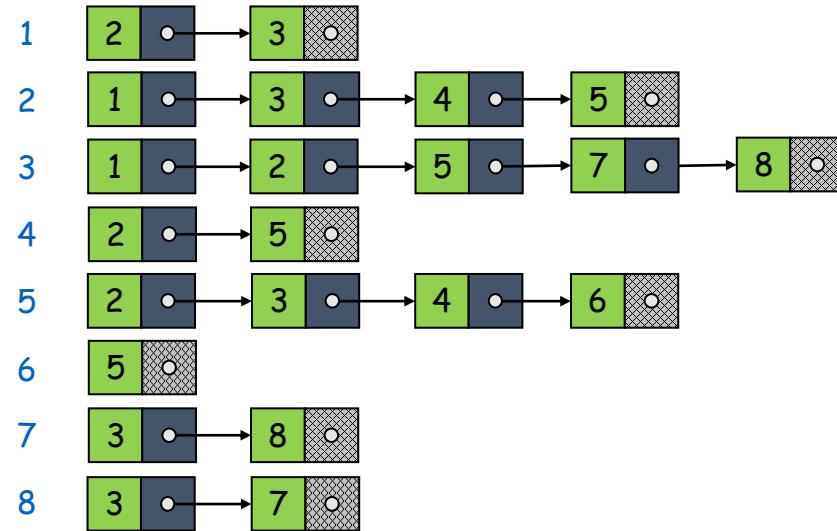
DFS Tree



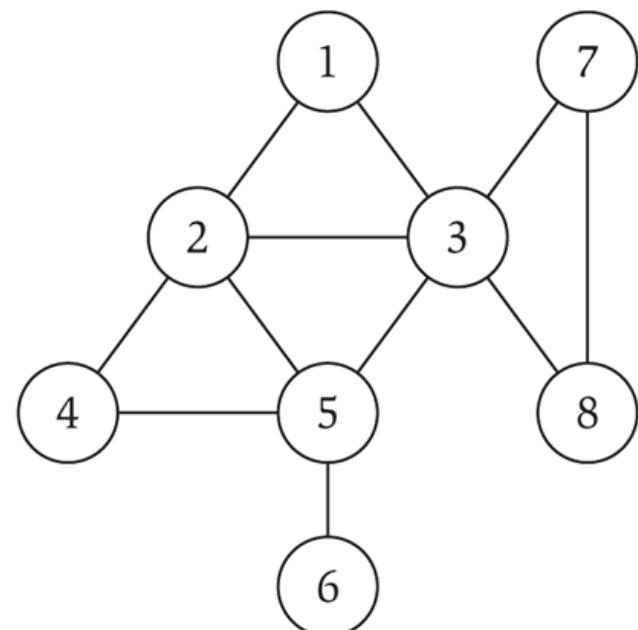
Stack    

1	2	3	7
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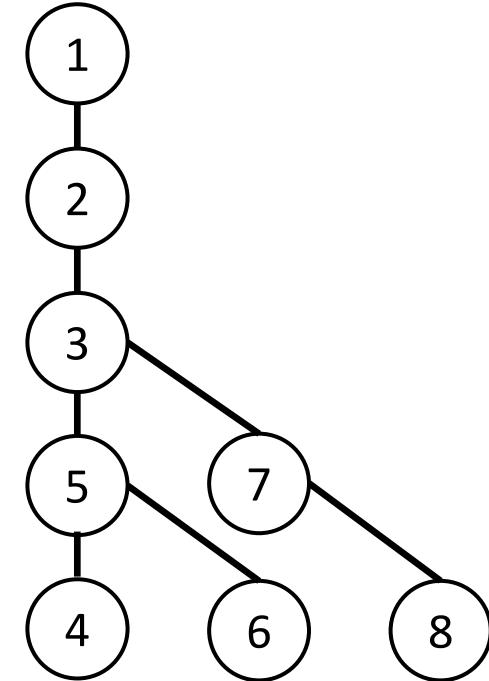
Adjacency List



Graph



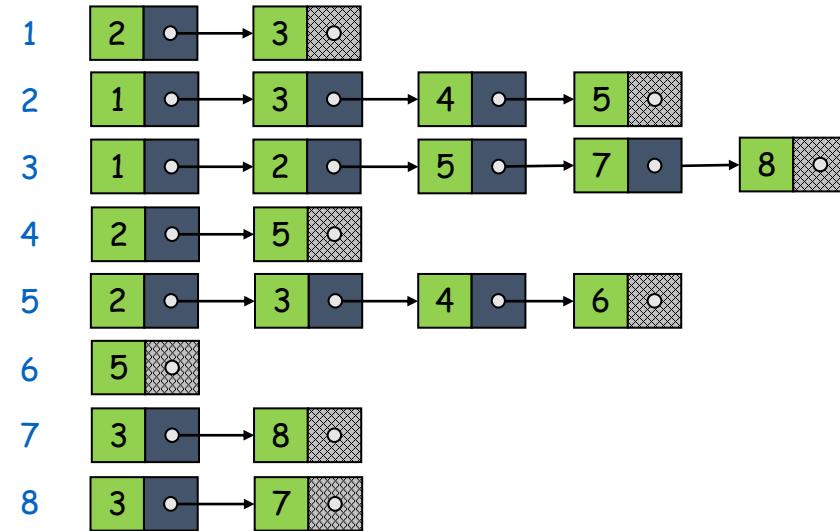
DFS Tree



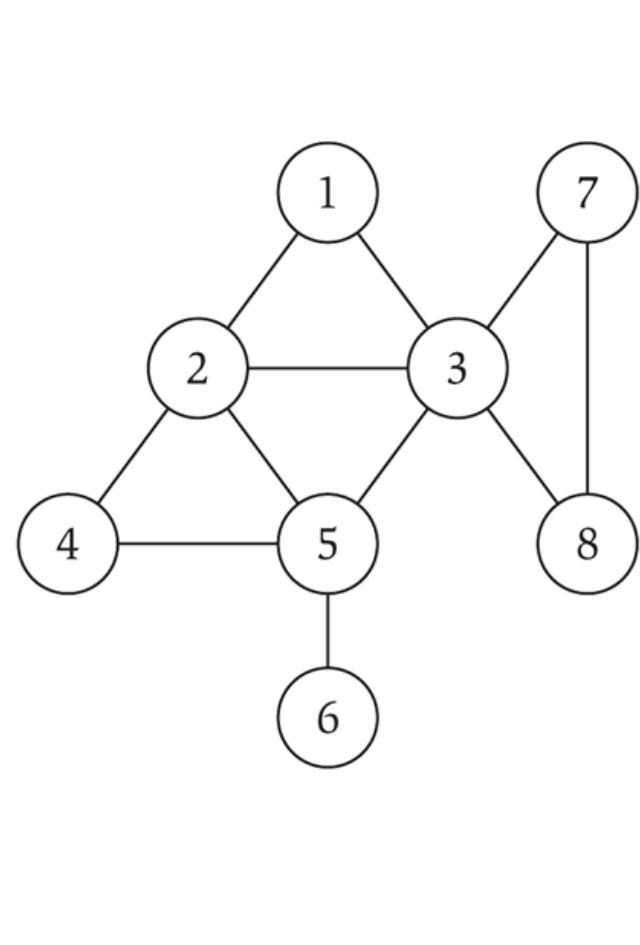
Stack



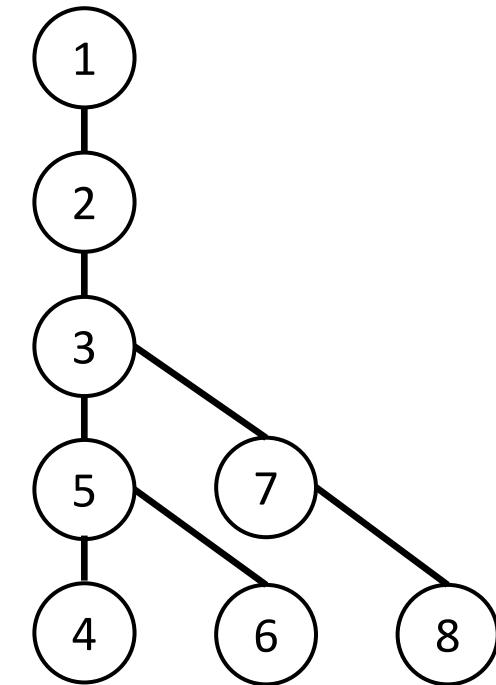
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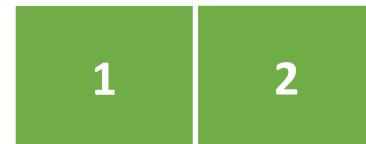
Graph



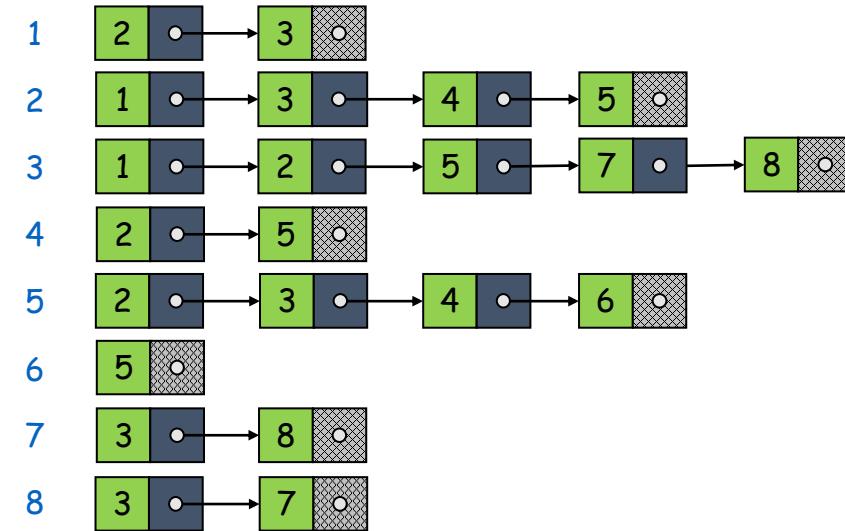
DFS Tree



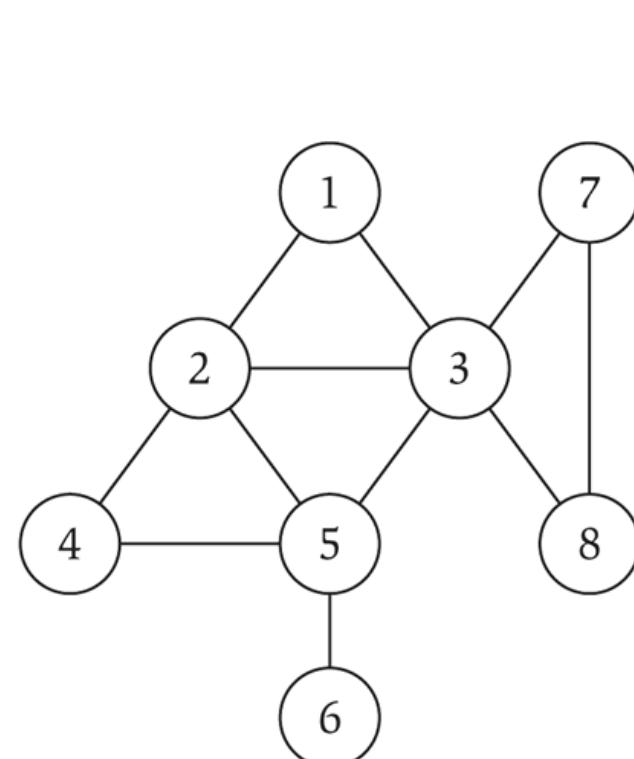
Stack



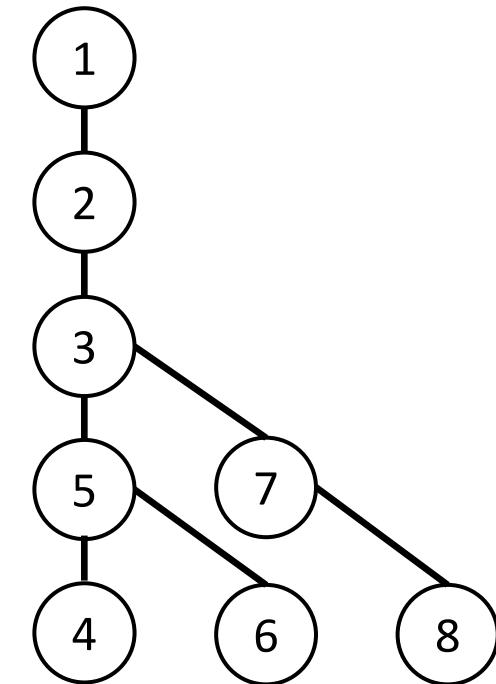
Adjacency List



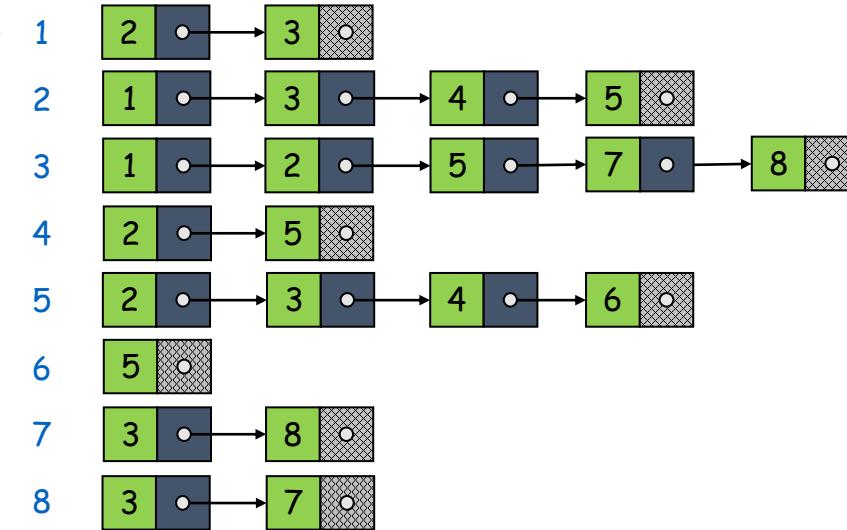
Graph



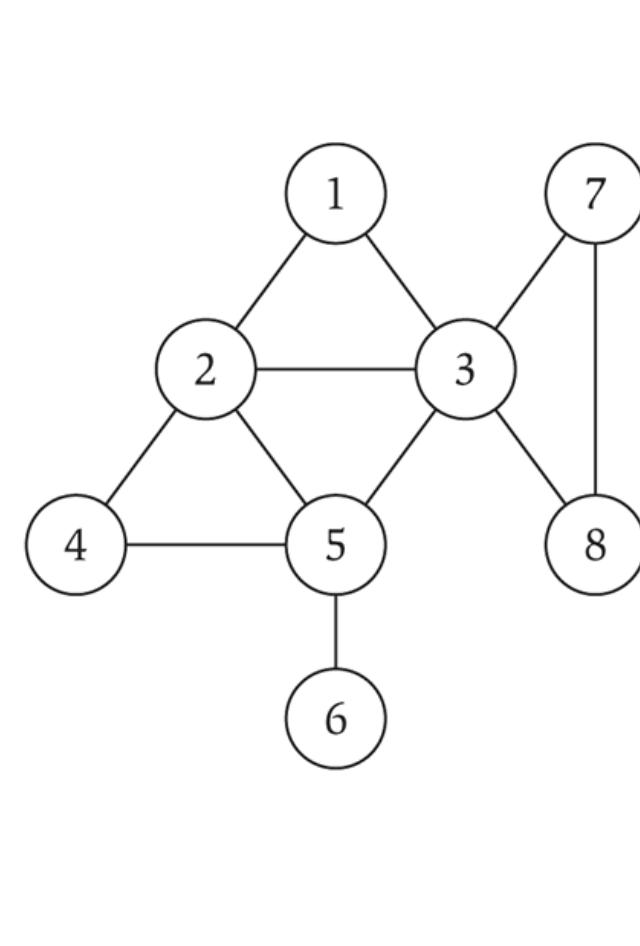
DFS Tree



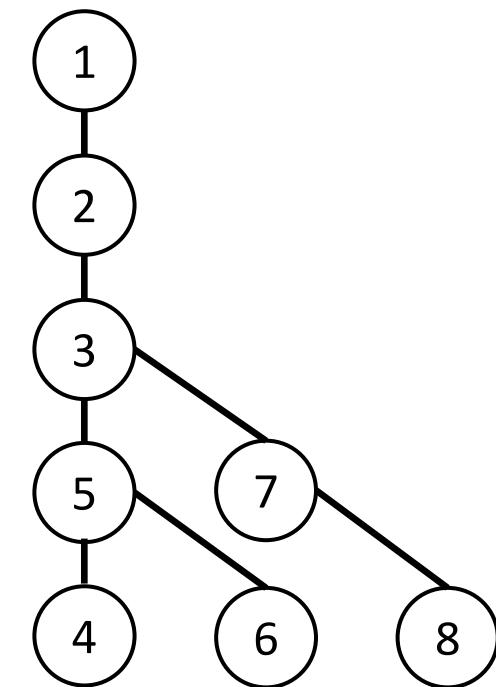
Adjacency List



Graph

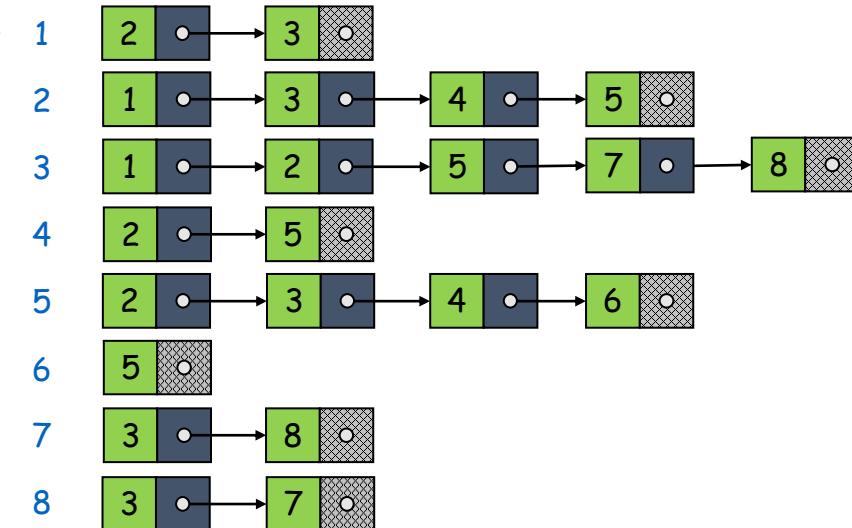


DFS Tree

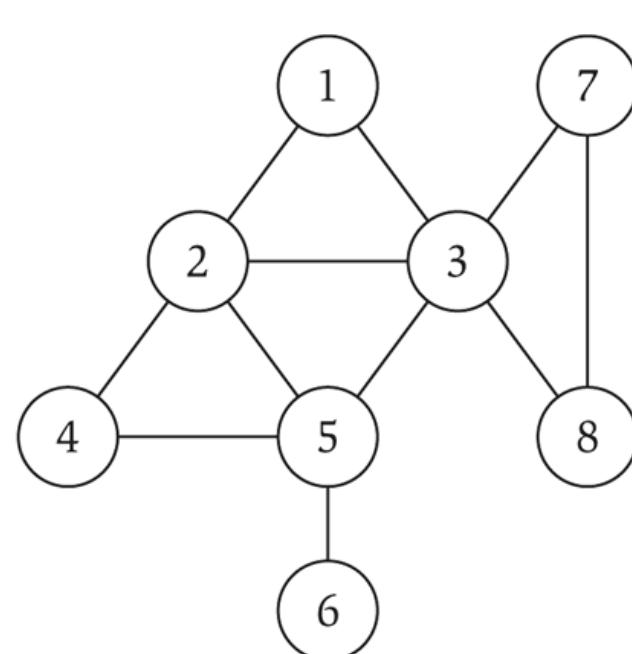


# Stack

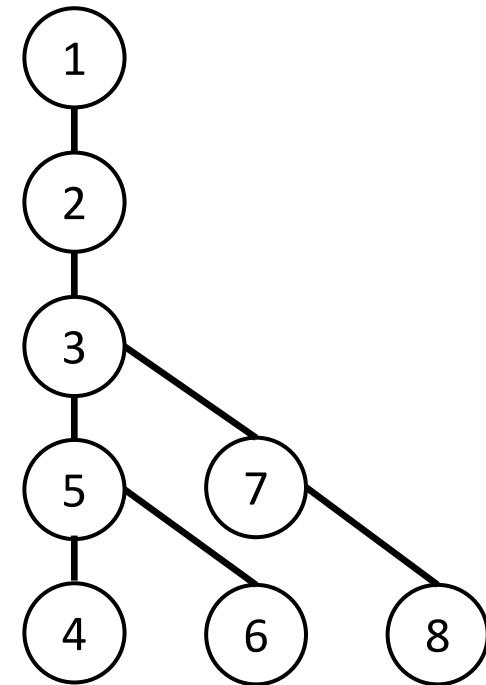
## Adjacency List



## Graph



## DFS Tree



# Depth First Search (DFS) – via Recursion

- Given a directed Graph,  $G = (V, E)$ , DFS traverses all vertices of  $G$ 
  - Each vertex  $v$  is labelled with a discovery time  $d.v$  and finish time  $f.v$ 
    - $d.v$  denotes the time when vertex  $v$  is first processed
    - $f.v$  denotes the time when all of the descendants of  $v$  are finished, i.e., when all the paths originating from  $v$  are processed.
- DFS colors the vertices during the search to indicate their state
  - Initially all vertices are colored **white**
    - White* vertices are not discovered yet
  - When a vertex is discovered, it is colored **gray**
    - Gray* vertices are discovered, however all paths originating from a gray vertex are not explored yet
  - When a vertex is finished it is colored **black**
    - All paths from a *black* vertex are explored

Extra  
Bookkeeping



# DFS Algorithm

$$G = (V, E)$$

**DFS( $G$ )**

```
1  for each vertex  $u \in V$ 
2     $u.\text{color} = \text{WHITE}$ 
3     $u.\pi = \text{NIL}$ 
4   $\text{time} = 0$ 
5  for each vertex  $u \in V$ 
6    if  $u.\text{color} == \text{WHITE}$ 
7      DFS-VISIT( $G, u$ )
```

**DFS-VISIT( $G, u$ )**

```
1   $time = time + 1$ 
2   $u.d = time$ 
3   $u.\text{color} = \text{GRAY}$ 
4  for each  $v \in Adj[u]$ 
5    if  $v.\text{color} == \text{WHITE}$ 
6       $v.\pi = u$ 
7      DFS-VISIT( $G, v$ )
8   $u.\text{color} = \text{BLACK}$ 
9   $time = time + 1$ 
10  $u.f = time$ 
```

$$G = (V, E)$$

**DFS( $G$ )**

```

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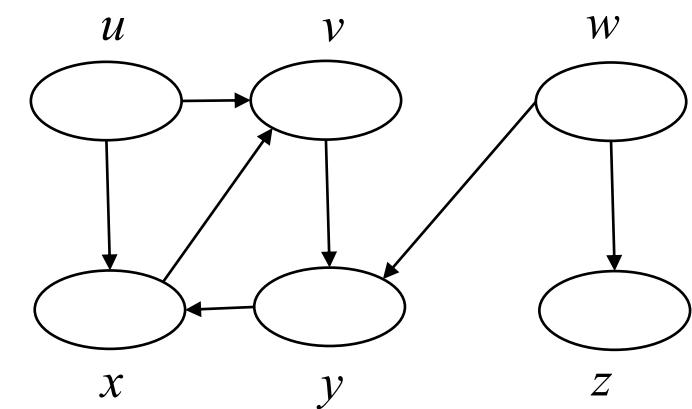
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```

**Example**



# DFS Algorithm

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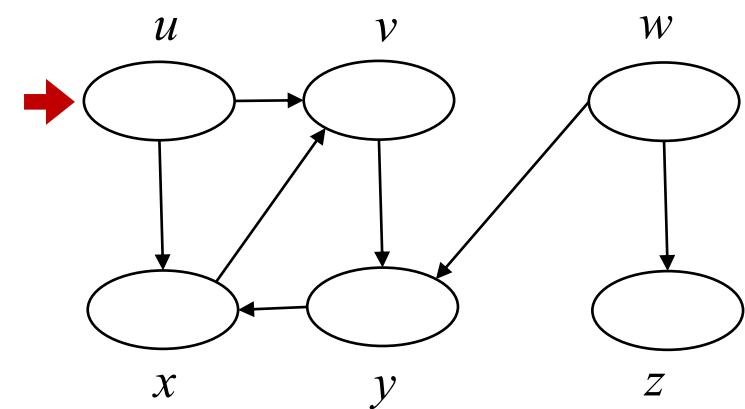
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```

**Example**



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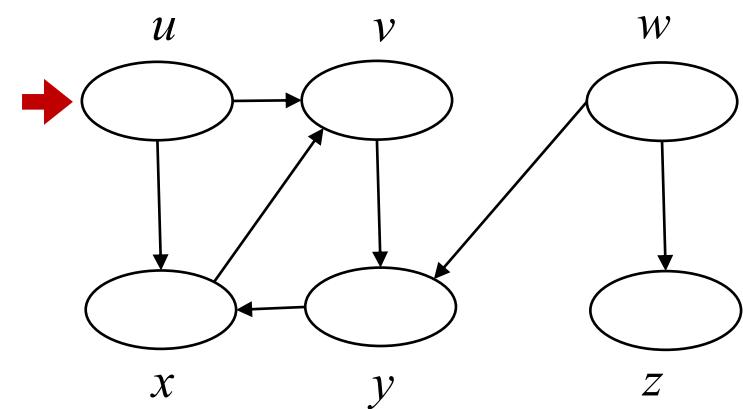
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**Example**



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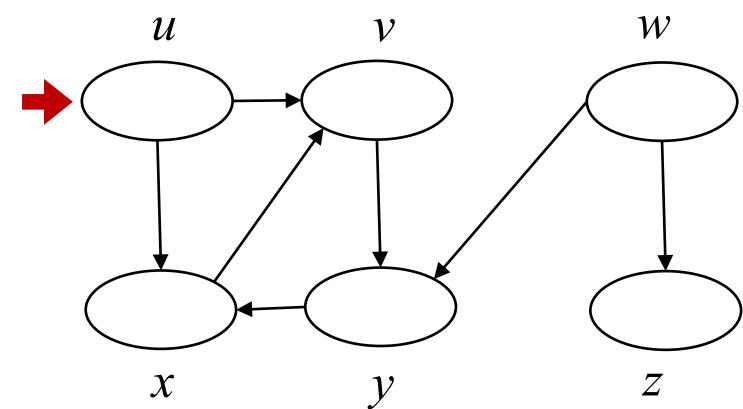
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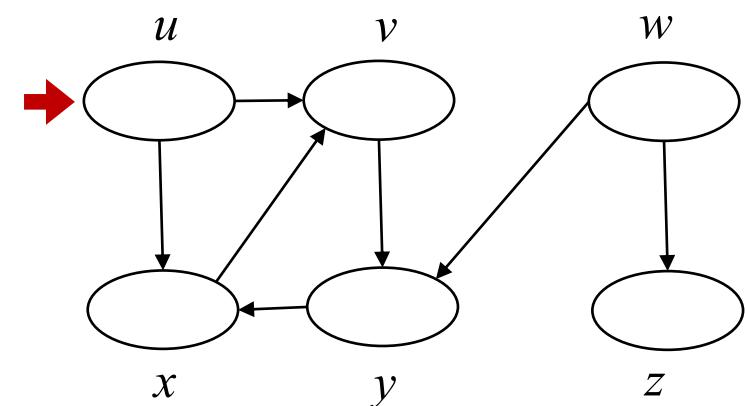
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**Example**



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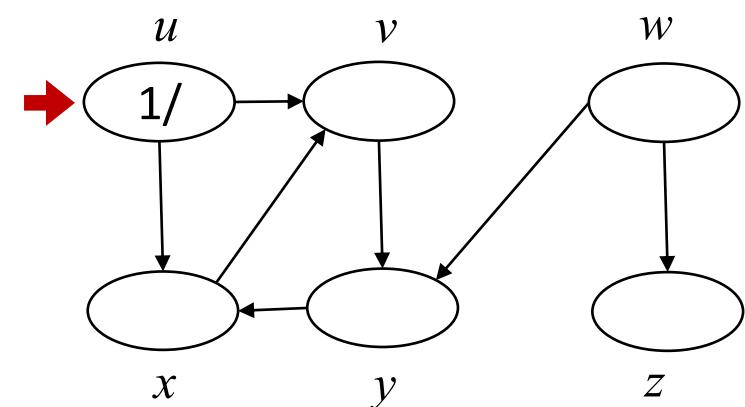
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**Example**



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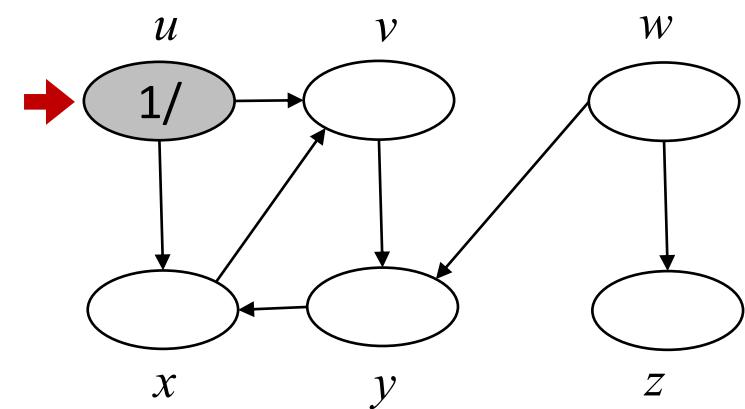
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```

**Example**



$$G = (V, E)$$

**DFS( $G$ )**

```

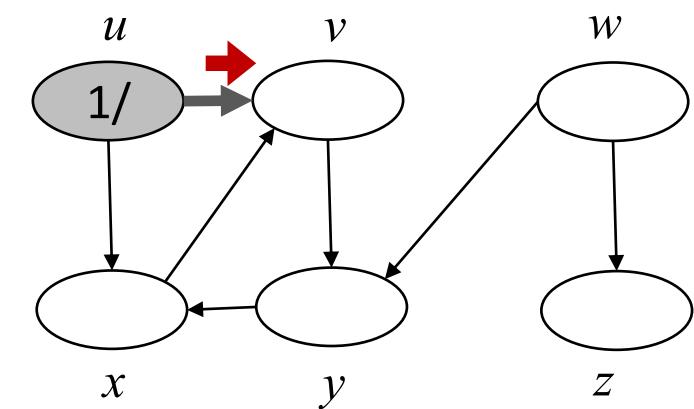
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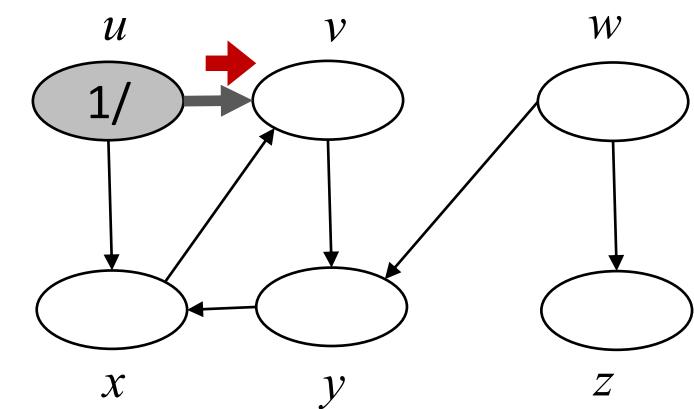
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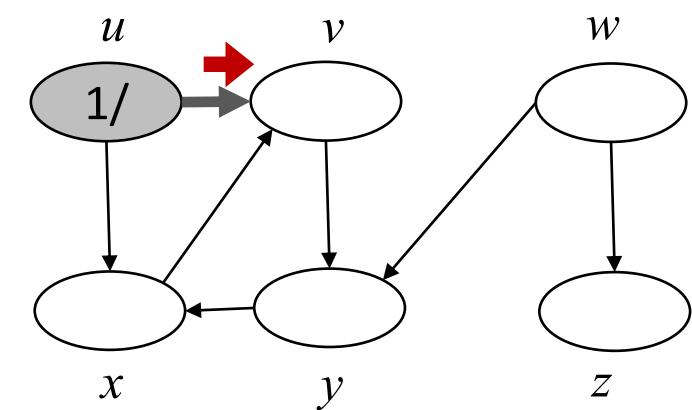
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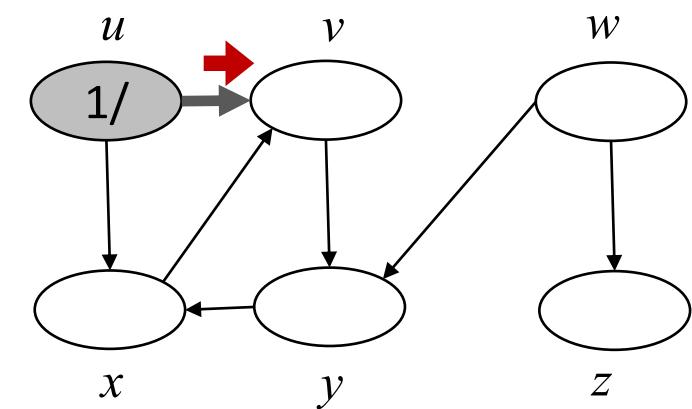
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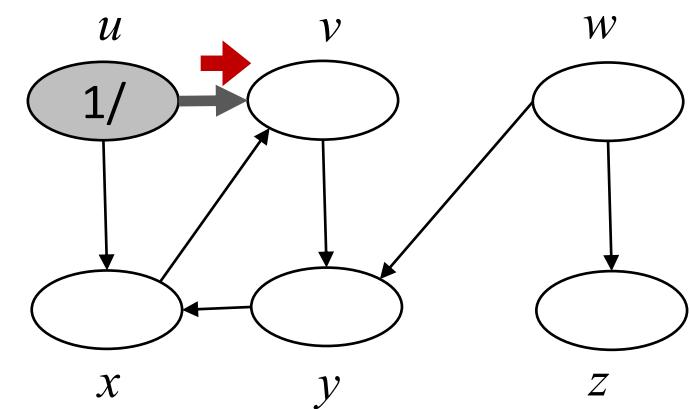
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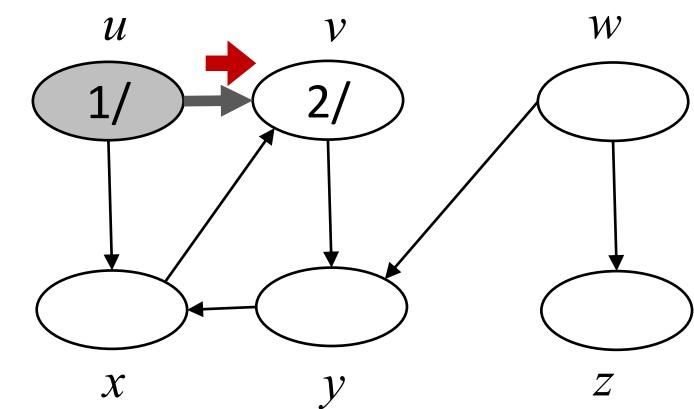
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4  for each  $v \in \text{Adj}[u]$ 
5    if  $v.\text{color} == \text{WHITE}$ 
6       $v.\pi = u$ 
7      DFS-VISIT( $G, v$ )
8   $u.\text{color} = \text{BLACK}$ 
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```

**Example**



$$G = (V, E)$$

**DFS( $G$ )**

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1  for each vertex  $u \in V$ 
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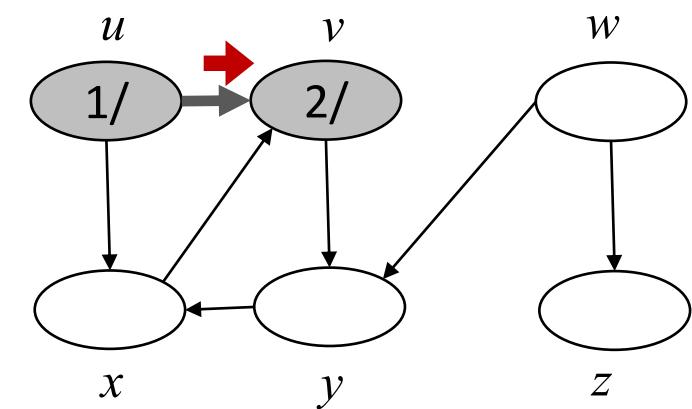
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**Example**



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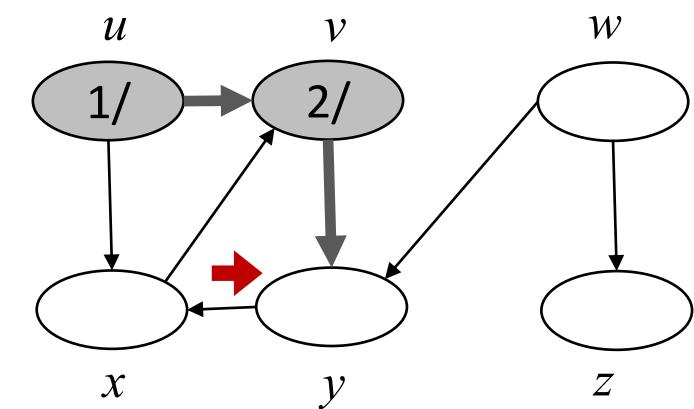
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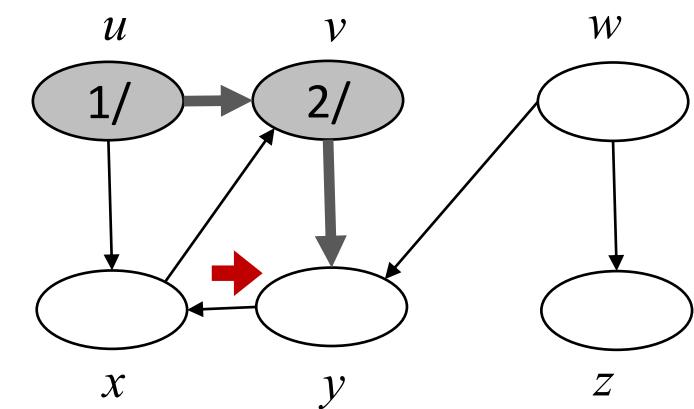
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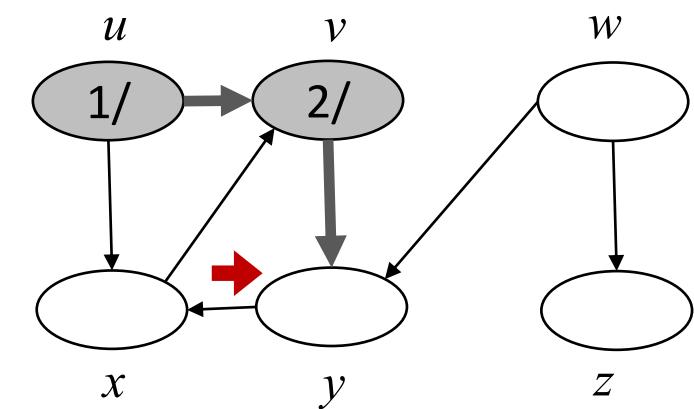
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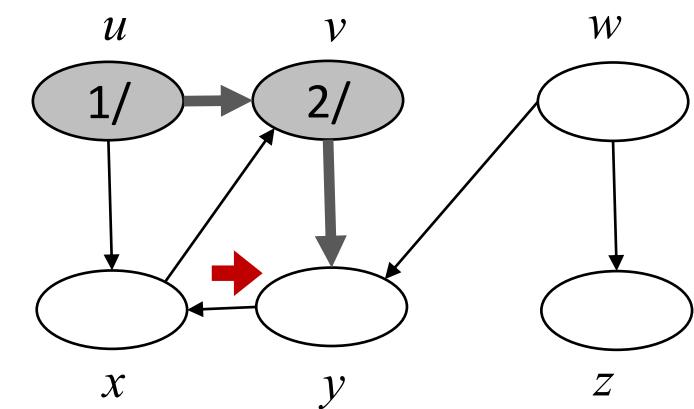
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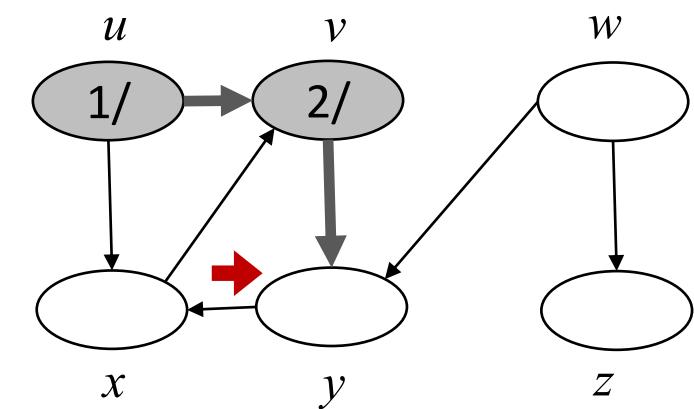
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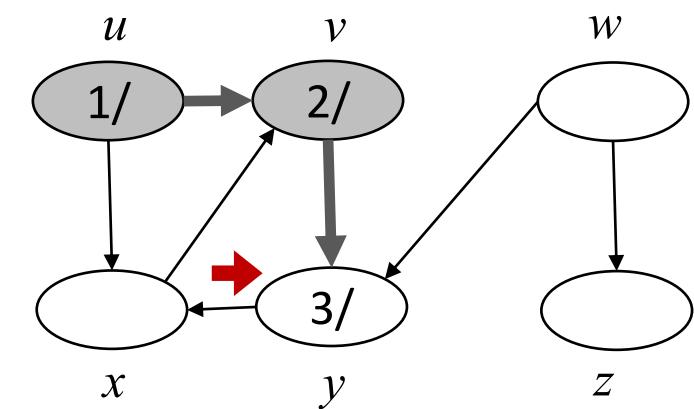
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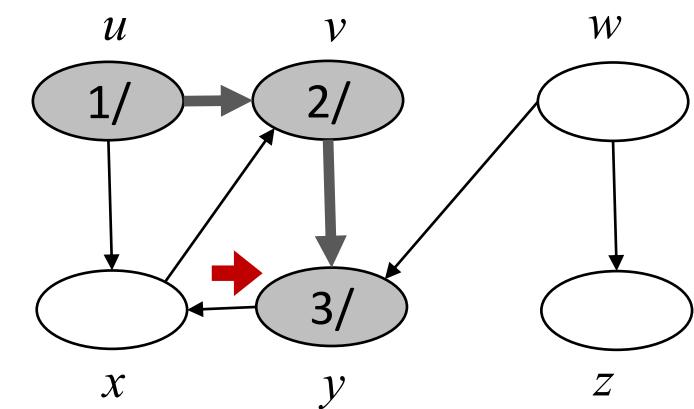
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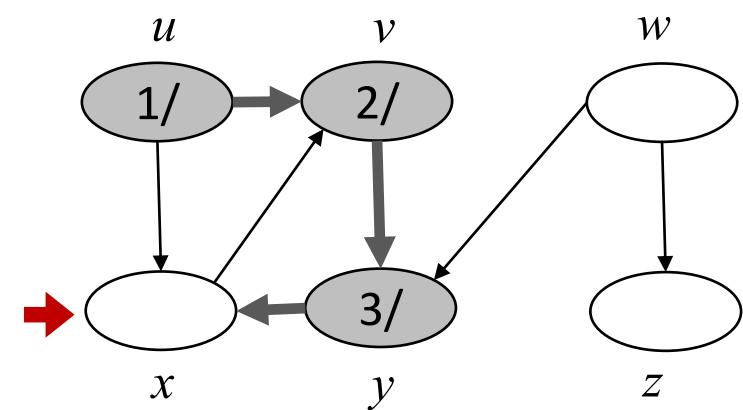
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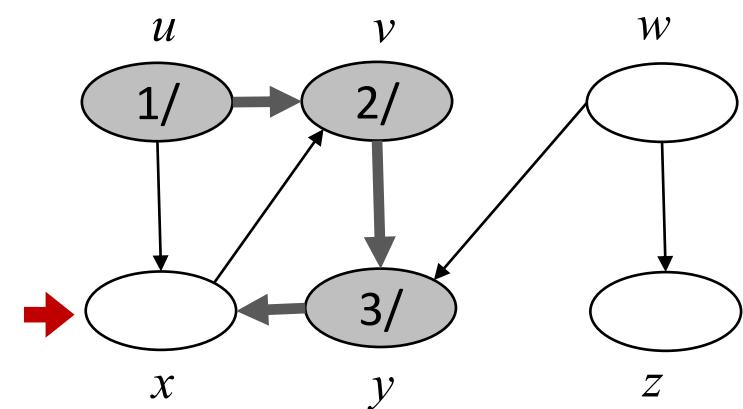
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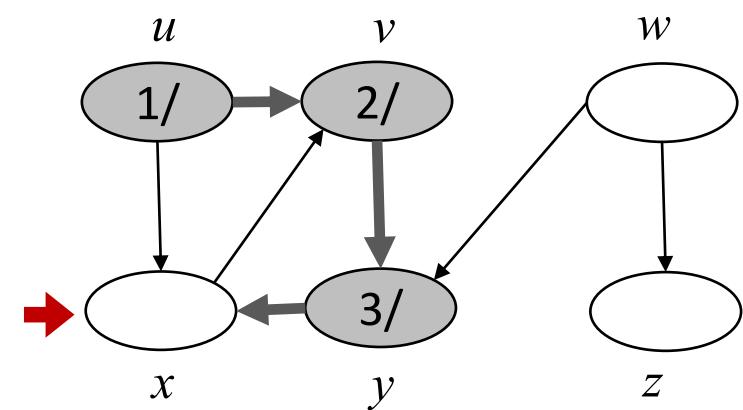
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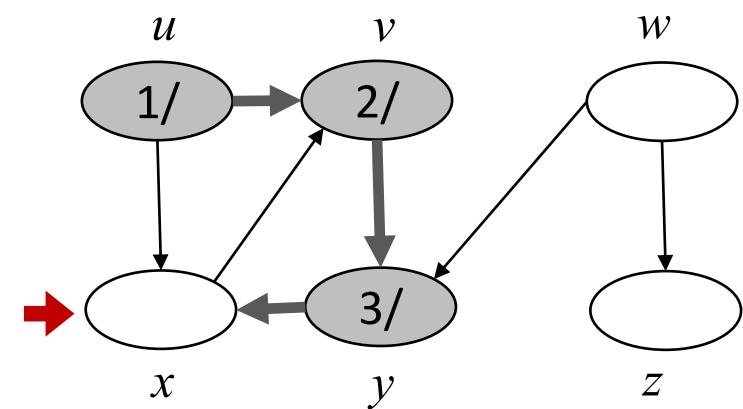
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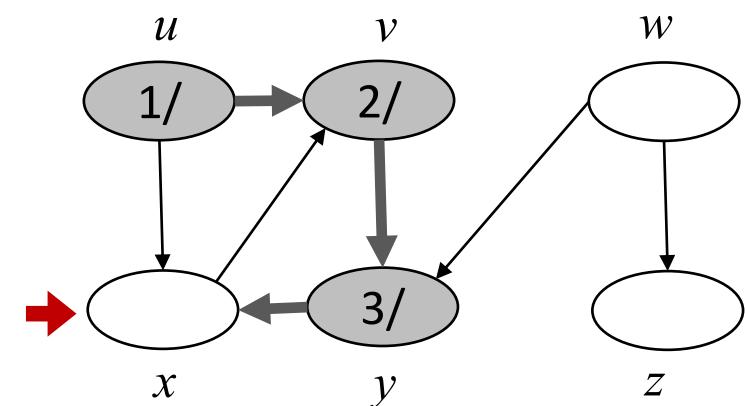
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**Example**



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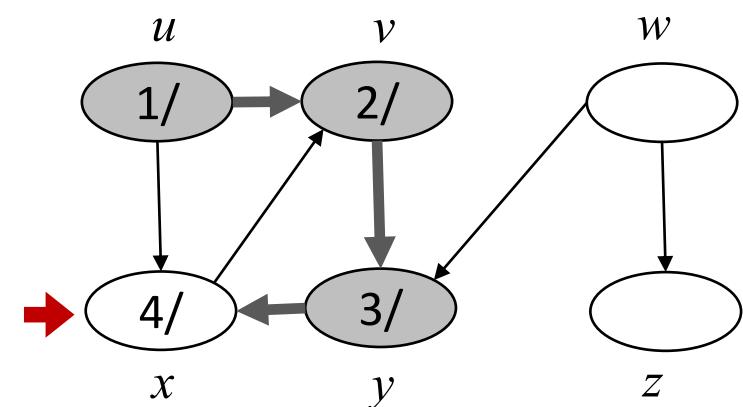
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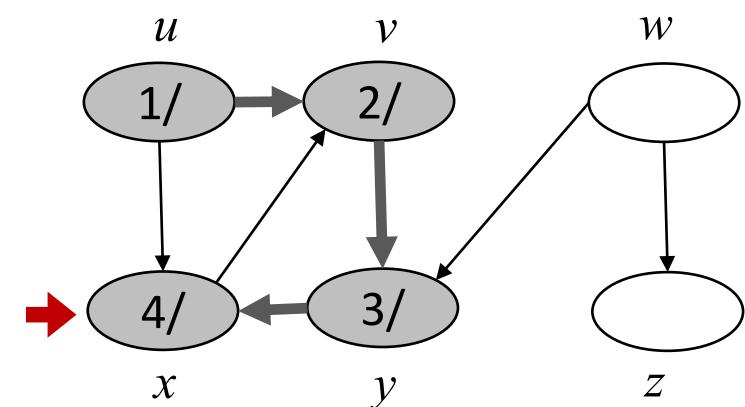
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**Example**



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**DFS( $G$ )**

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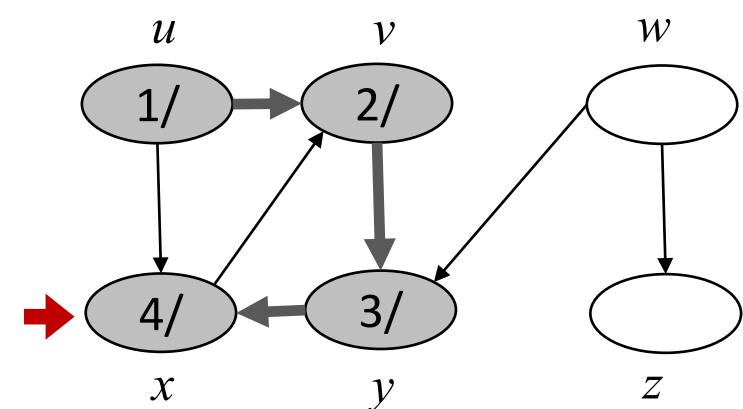
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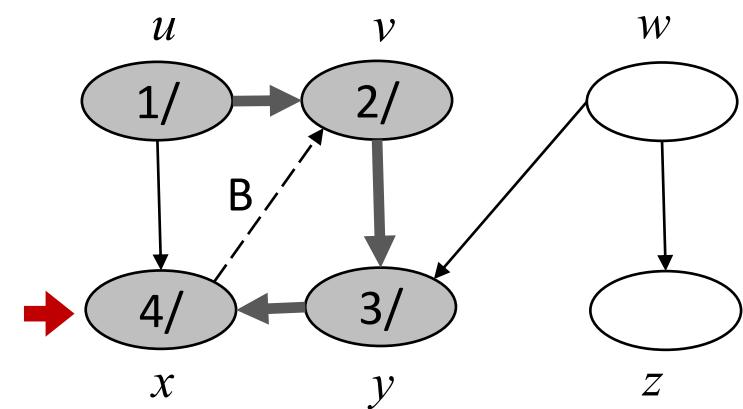
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```

**Example**



**Back edges** are those edges  $(u, v)$  connecting a vertex  $u$  to an ancestor vertex  $v$  in a depth-first tree. Self-loop (edge  $(u, u)$ ), is also considered as Back edge

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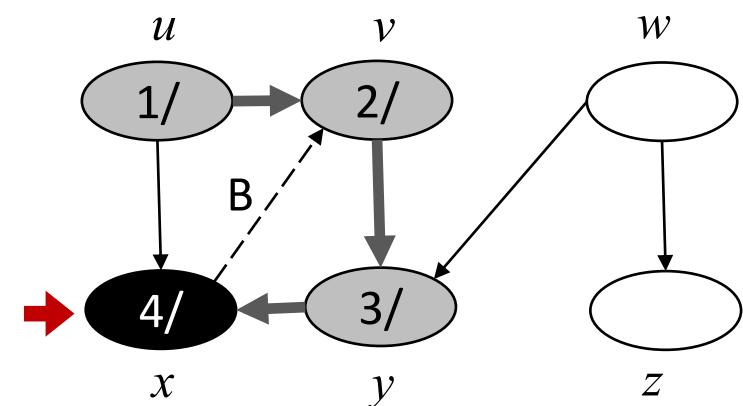
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**Example**



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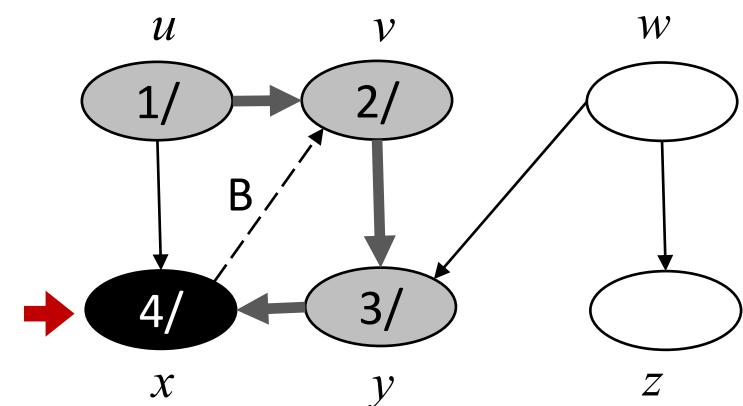
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**Example**



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7    →      DFS-VISIT( $G, u$ )

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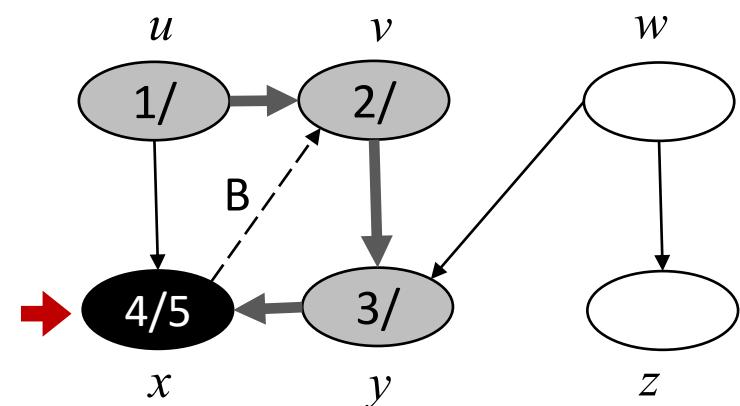
**DFS-VISIT( $G, u$ )**

```

1   $\text{time} = \text{time} + 1$ 
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4  for each  $v \in \text{Adj}[u]$ 
5    if  $v.\text{color} == \text{WHITE}$ 
6       $v.\pi = u$ 
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8   $u.\text{color} = \text{BLACK}$ 
9   $\text{time} = \text{time} + 1$ 
10 →  $u.f = \text{time}$ 

```

**Example**



$$G = (V, E)$$

**DFS( $G$ )**

```

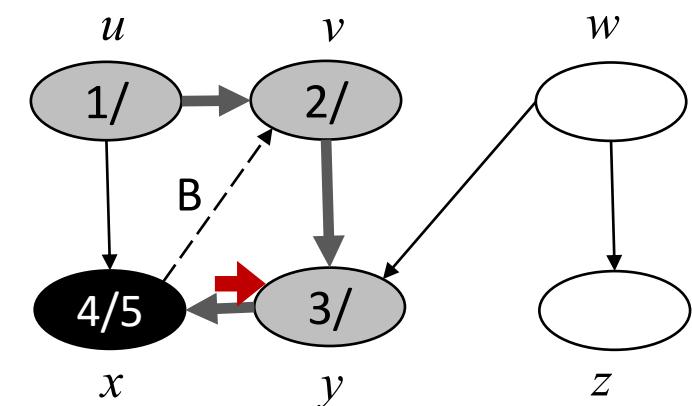
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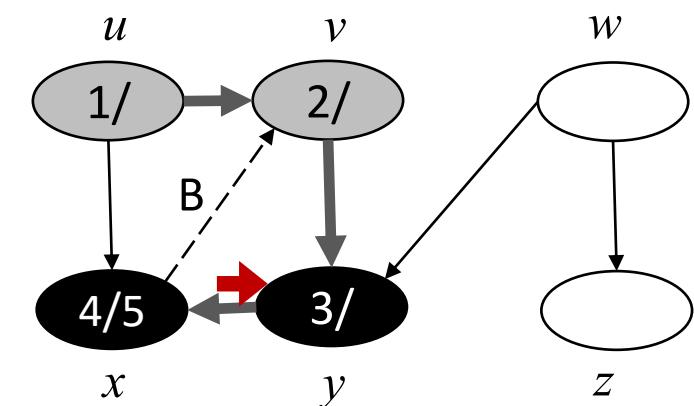
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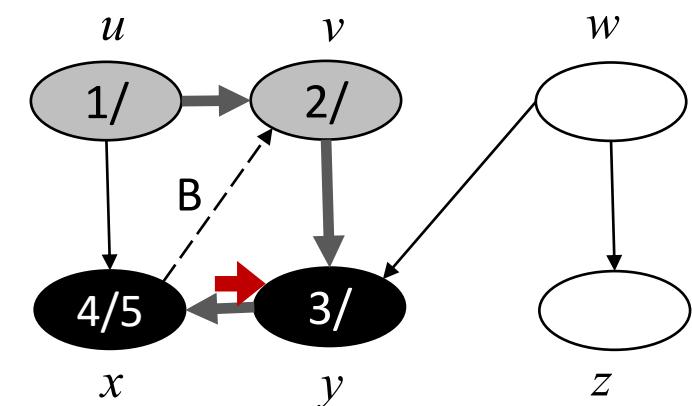
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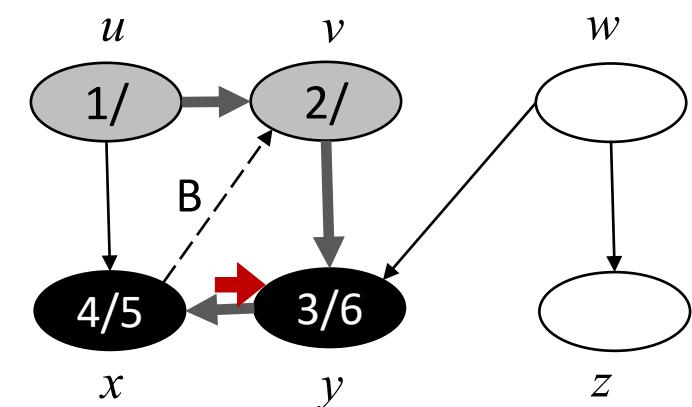
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**Example**



$$G = (V, E)$$

**DFS( $G$ )**

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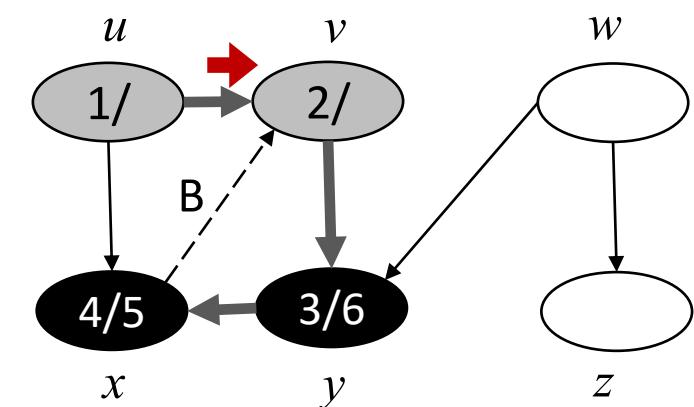
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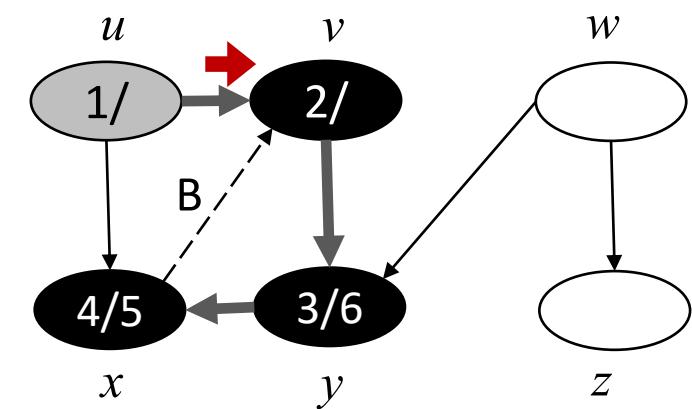
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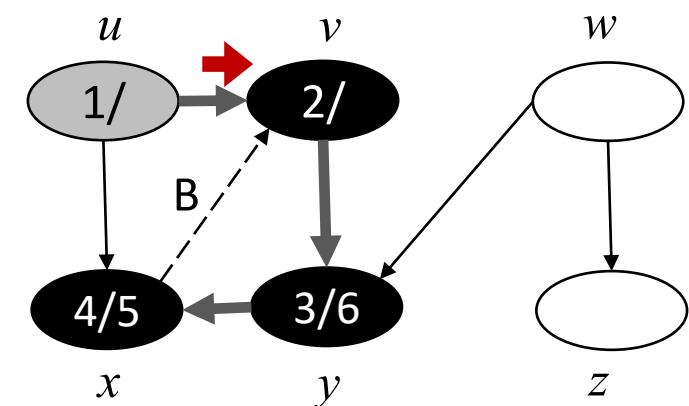
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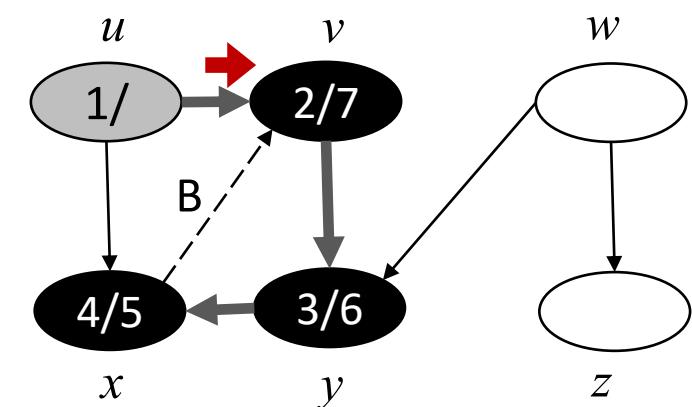
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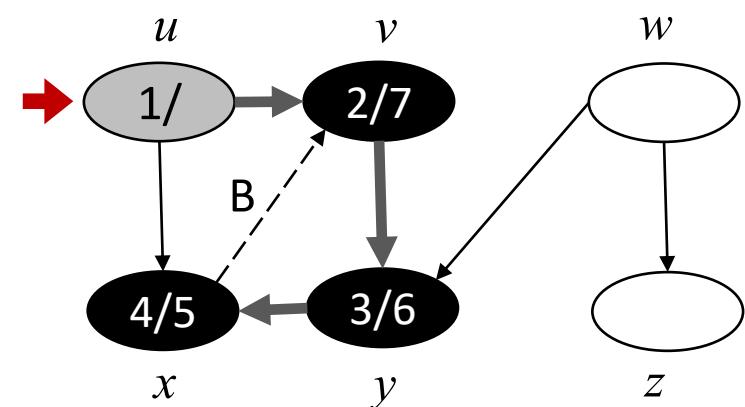
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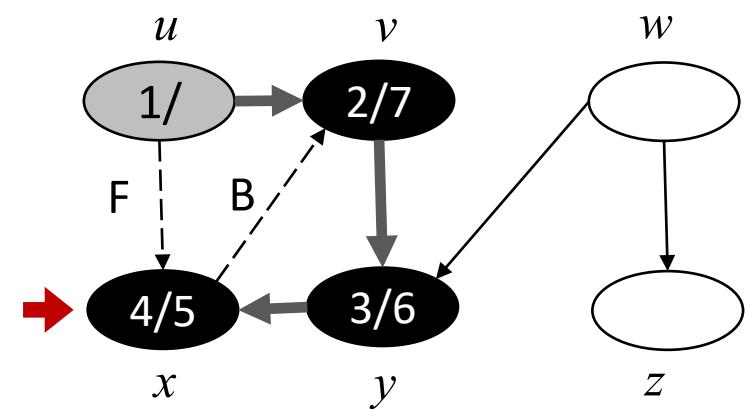
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```

**Example**



**Forward edges** are those edges  $(u,v)$  connecting a vertex  $u$  to a descendent vertex  $v$  that is already discovered and finished .

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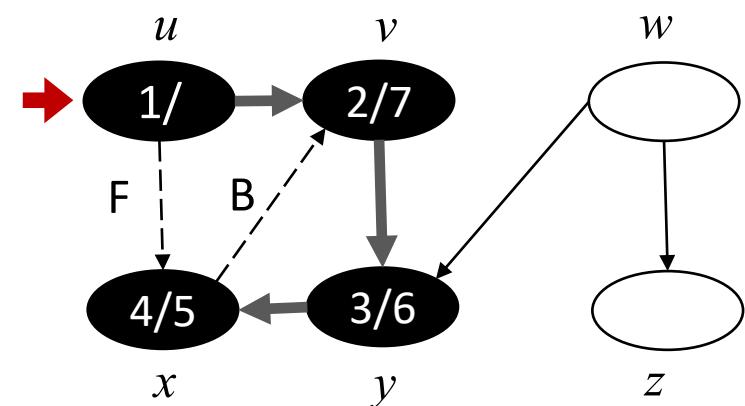
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**Example**



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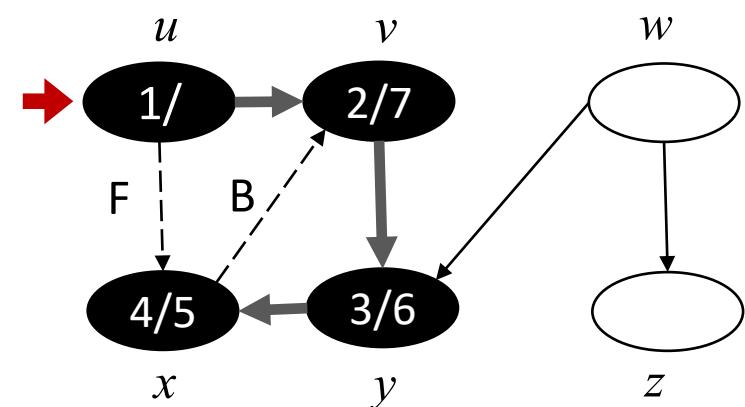
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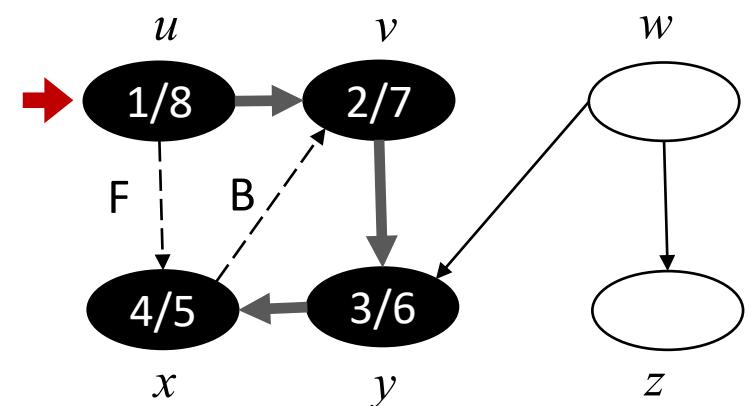
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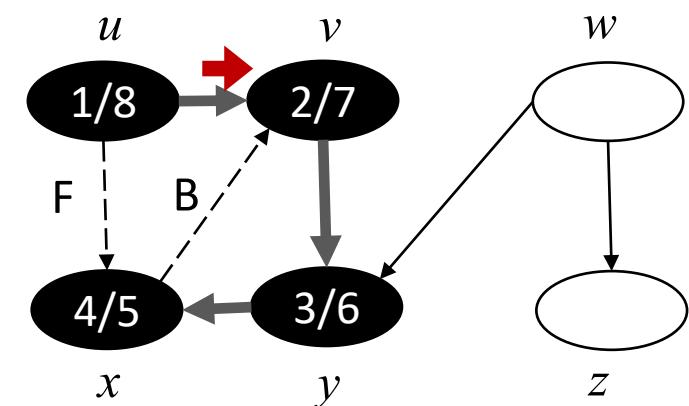
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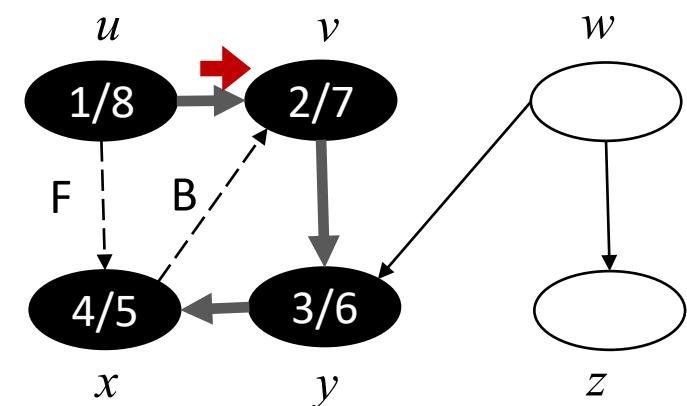
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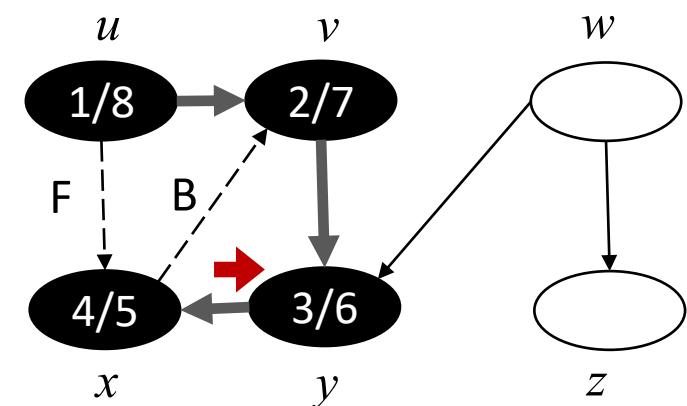
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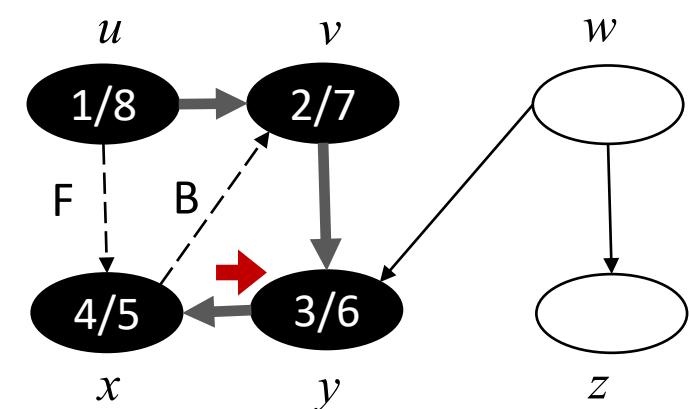
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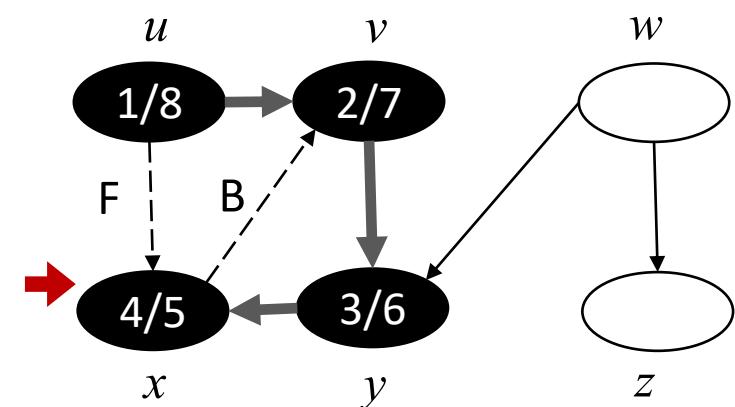
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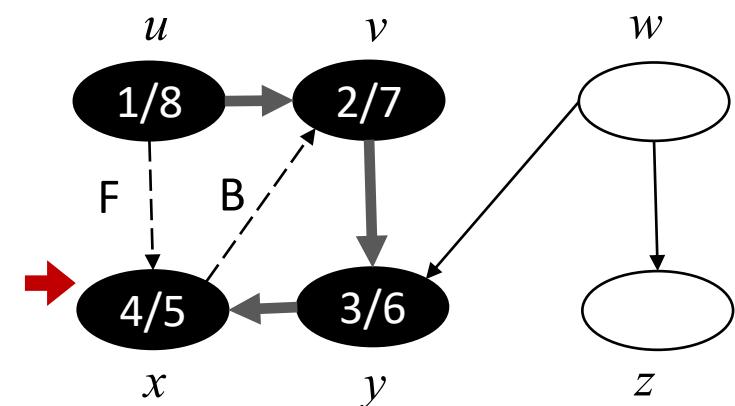
**DFS-VISIT( $G, u$ )**

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1   $\text{time} = \text{time} + 1$ 
2   $u.d = \text{time}$ 
3   $u.\text{color} = \text{GRAY}$ 
4  for each  $v \in \text{Adj}[u]$ 
5    if  $v.\text{color} == \text{WHITE}$ 
6       $v.\pi = u$ 
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8   $u.\text{color} = \text{BLACK}$ 
9   $\text{time} = \text{time} + 1$ 
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```

**Example**



$$G = (V, E)$$

**DFS( $G$ )**

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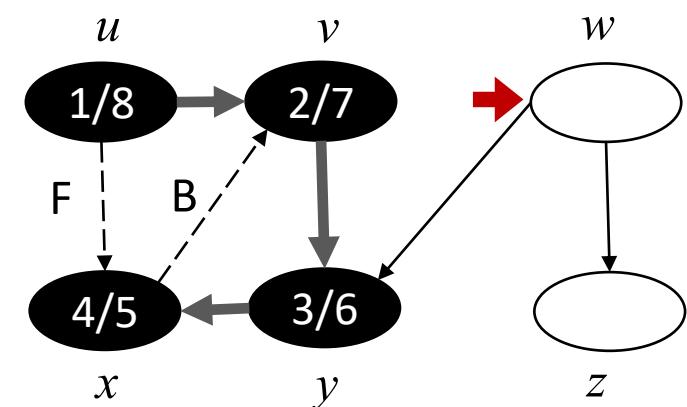
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**Example**



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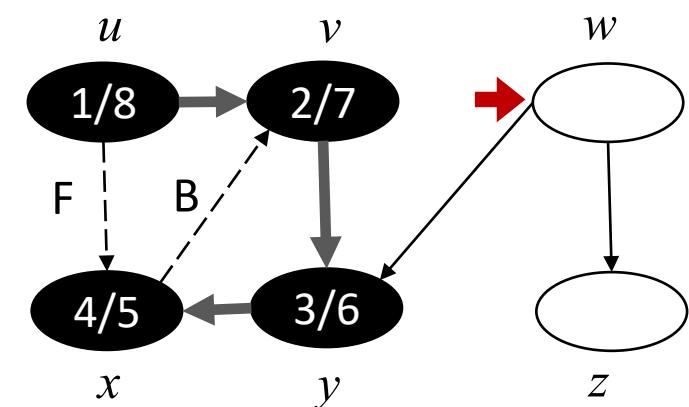
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**Example**



# DFS Algorithm

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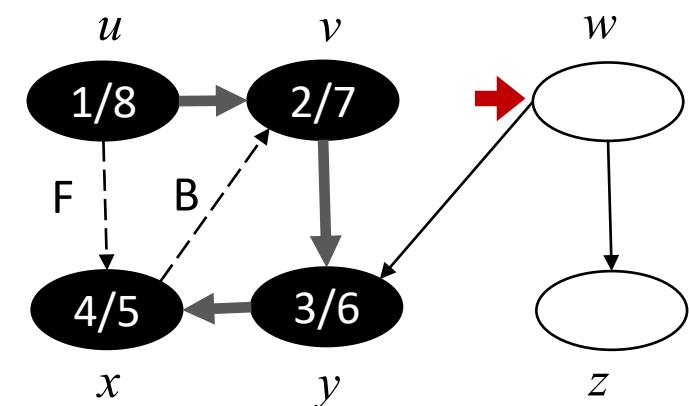
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**Example**



# DFS Algorithm

$$G = (V, E)$$

**DFS( $G$ )**

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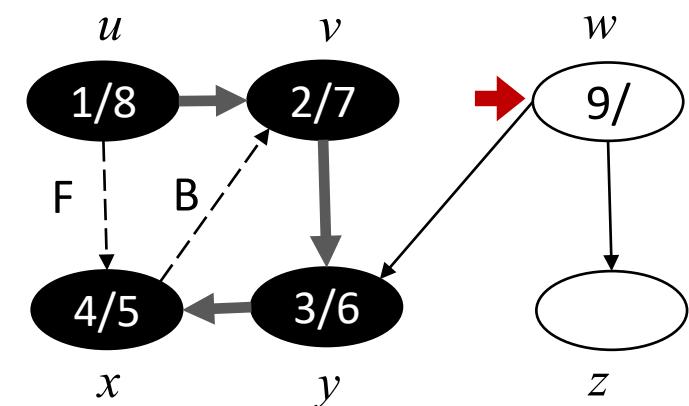
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**Example**



# DFS Algorithm

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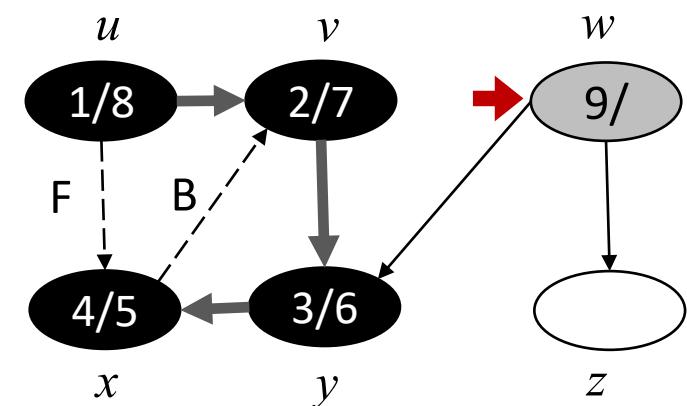
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**Example**



# DFS Algorithm

$$G = (V, E)$$

**DFS( $G$ )**

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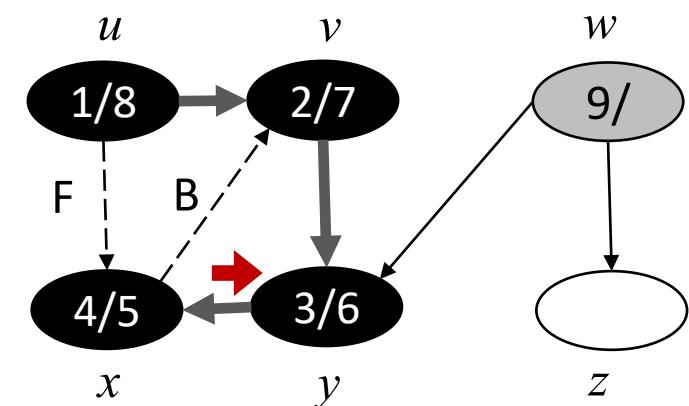
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**Example**



$$G = (V, E)$$

**DFS( $G$ )**

```

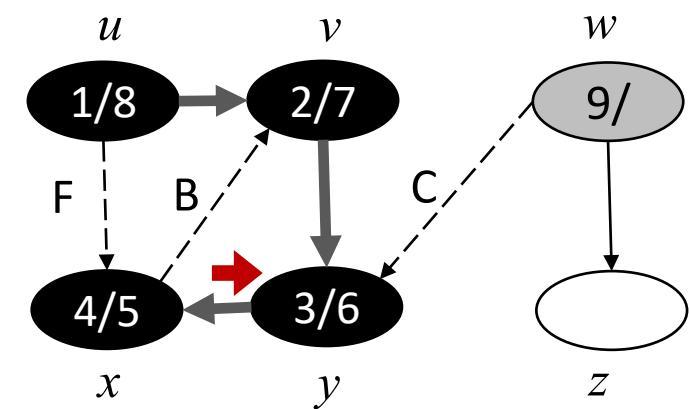
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**Example**



**Cross edges** It connects  $u$  with already discovered/finished  $v$ , but there is no ancestor-descendant relationship between them

# DFS Algorithm

$$G = (V, E)$$

**DFS( $G$ )**

```

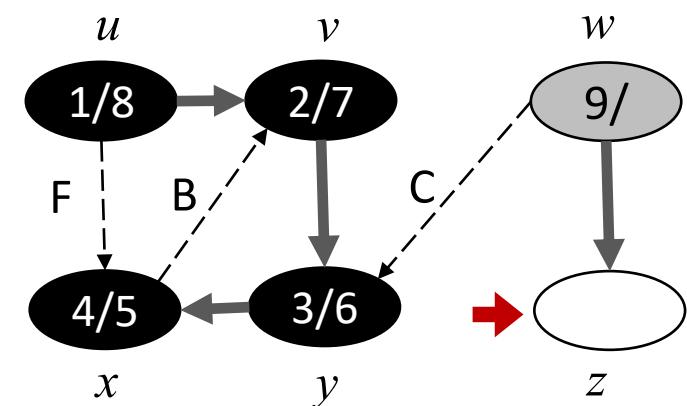
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**Example**



$$G = (V, E)$$

**DFS( $G$ )**

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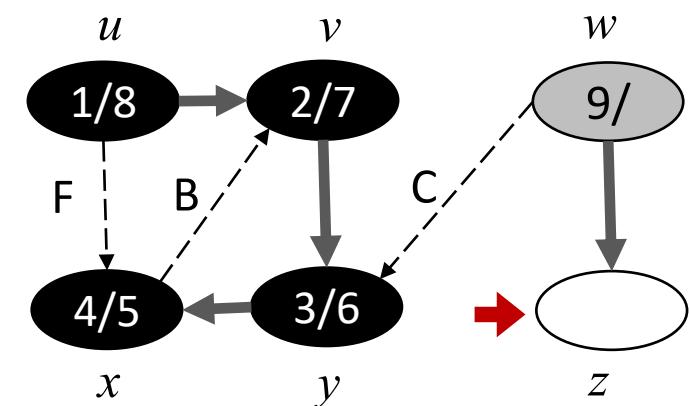
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**Example**



$$G = (V, E)$$

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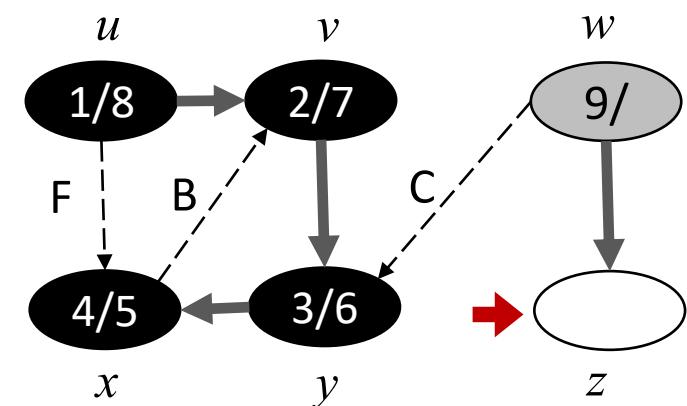
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**Example**



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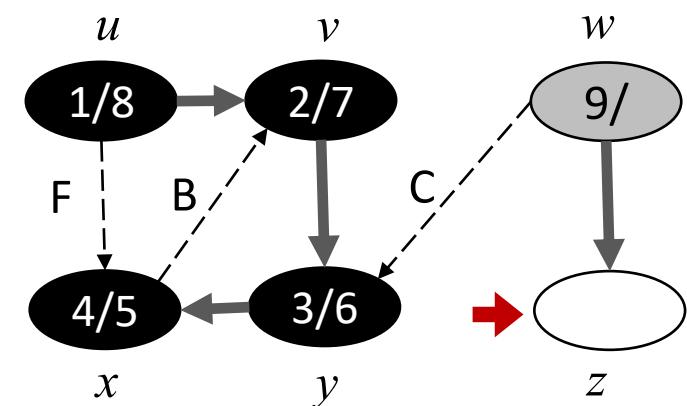
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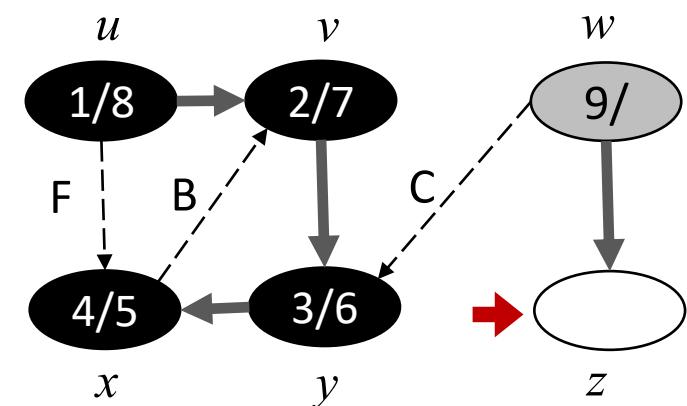
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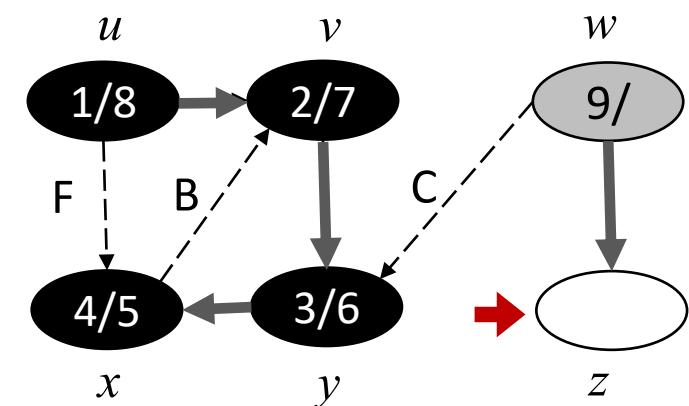
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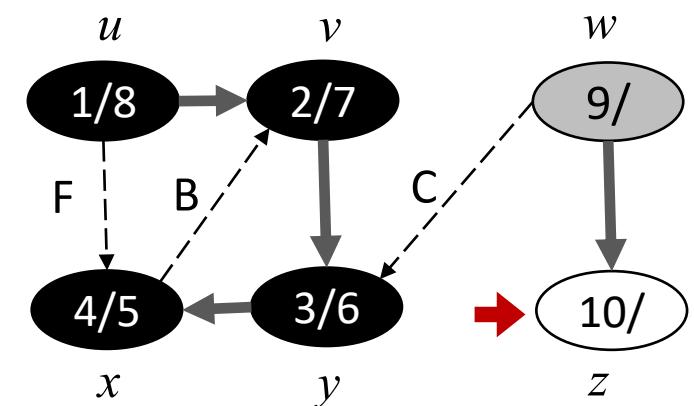
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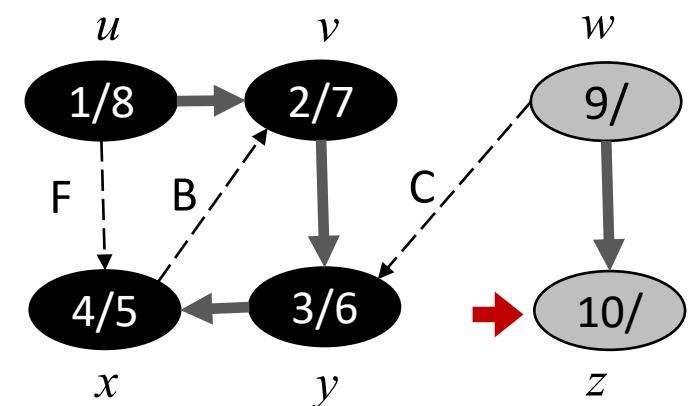
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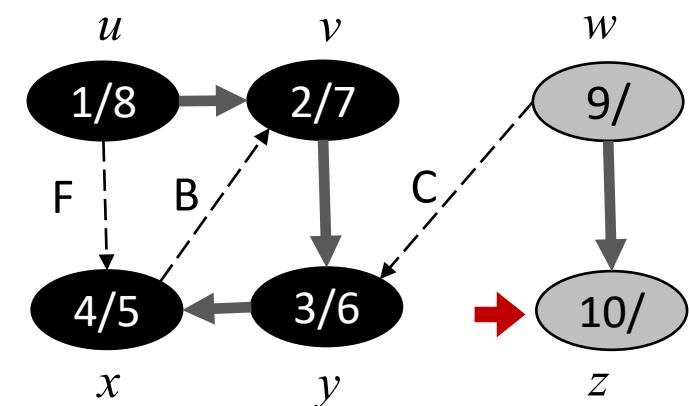
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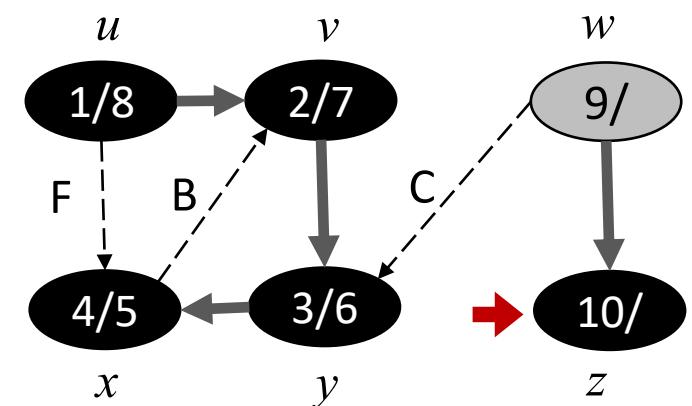
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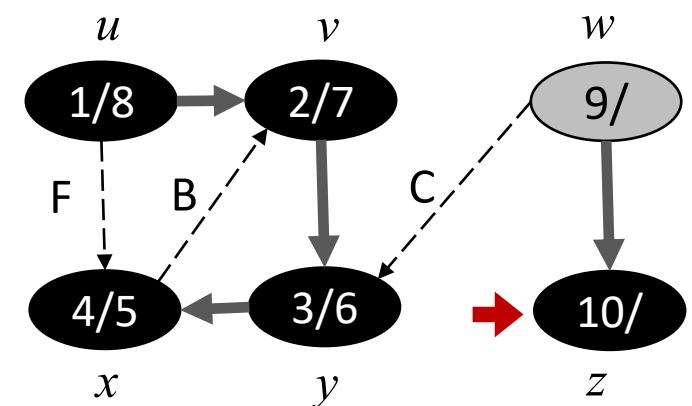
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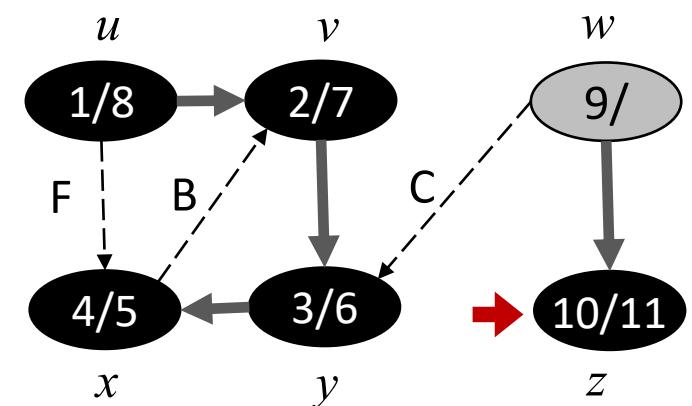
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10 →  $u.f = \text{time}$ 

```

**Example**



# DFS Algorithm

$$G = (V, E)$$

**DFS( $G$ )**

```

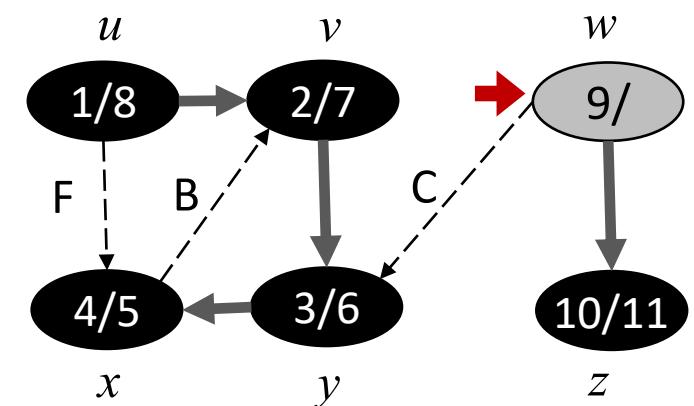
1  for each vertex  $u \in V$ 
2       $u.\text{color} = \text{WHITE}$ 
3       $u.\pi = \text{NIL}$ 
4   $\text{time} = 0$ 
5  for each vertex  $u \in V$ 
6      if  $u.\text{color} == \text{WHITE}$ 
7      →      DFS-VISIT( $G, u$ )
    
```

**DFS-VISIT( $G, u$ )**

```

1   $\text{time} = \text{time} + 1$ 
2   $u.d = \text{time}$ 
3   $u.\text{color} = \text{GRAY}$ 
4 → for each  $v \in \text{Adj}[u]$ 
5      if  $v.\text{color} == \text{WHITE}$ 
6           $v.\pi = u$ 
7          DFS-VISIT( $G, v$ )
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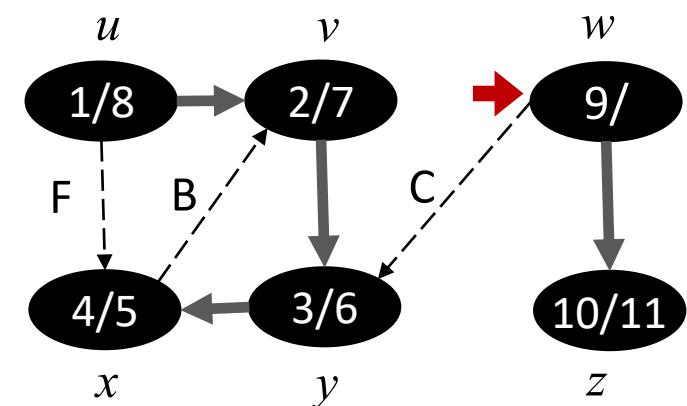
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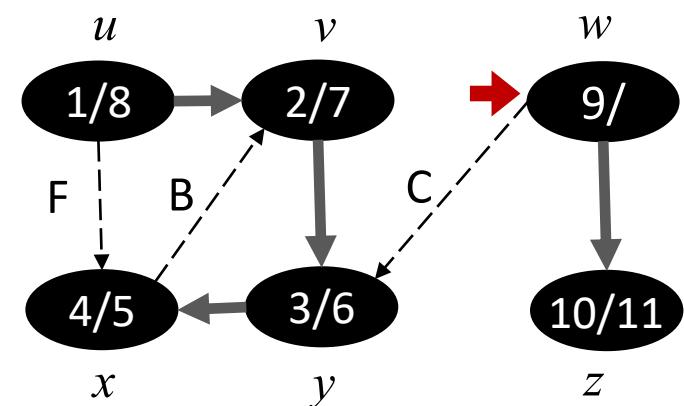
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**Example**



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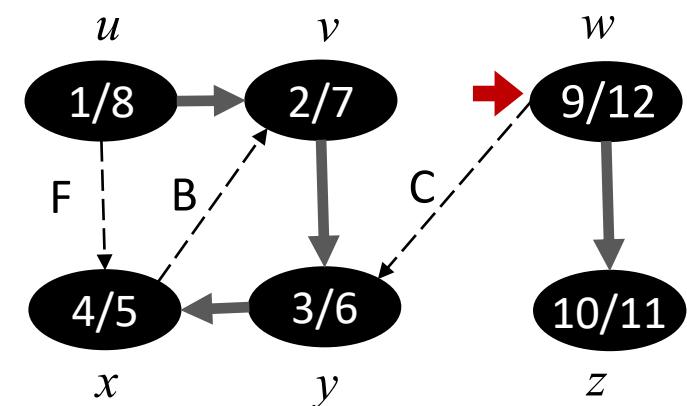
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**Example**



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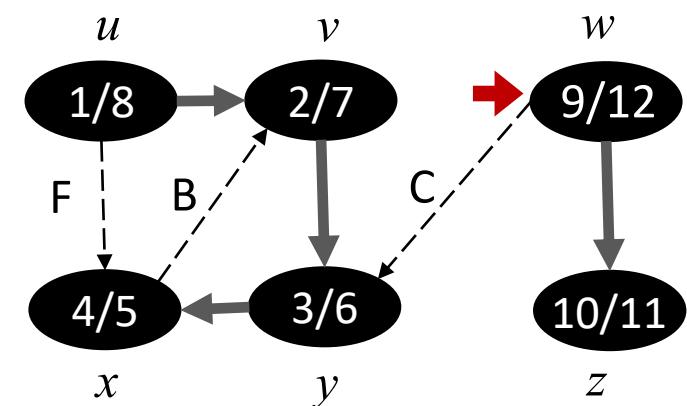
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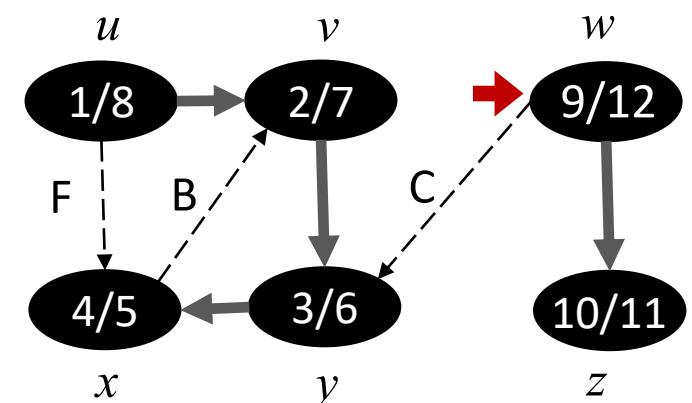
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**Example**



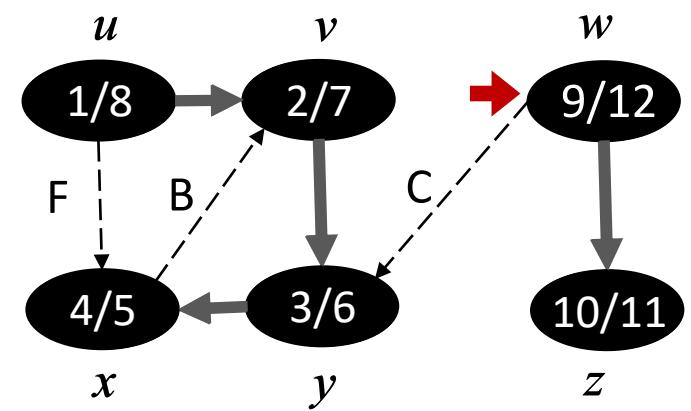
- DFS creates a forest  $G_\pi = (V, E_\pi)$ , a collection of **rooted trees**, where
  - $E_\pi = \{(v.\pi, v) : v \in V \text{ and } v.\pi \neq \text{NIL}\}$
  - All the edges in  $E_\pi$  are tree edges

$$G_\pi = (V, E_\pi)$$

$$V = \{u, v, w, x, y, z\}$$

$$E_\pi = \{(u, v), (v, y), (y, x), (w, z)\}$$

## Example



# DFS Running time Analysis

$$G = (V, E)$$

**DFS( $G$ )**

```

1  for each vertex  $u \in V$            } V
2     $u.\text{color} = \text{WHITE}$       }
3     $u.\pi = \text{NIL}$             }
4   $\text{time} = 0$ 
5  for each vertex  $u \in V$            } V
6    if  $u.\text{color} == \text{WHITE}$     }
7      DFS-VISIT( $G, u$ )          }
```

**DFS-VISIT( $G, u$ )**

```

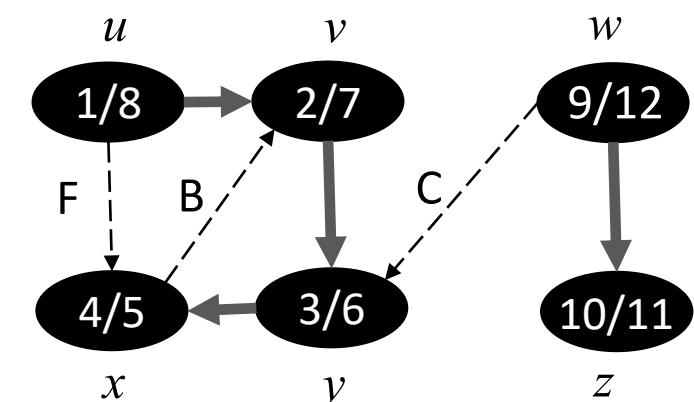
1   $\text{time} = \text{time} + 1$ 
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4  for each  $v \in \text{Adj}[u]$         } V
5    if  $v.\text{color} == \text{WHITE}$       }
6       $v.\pi = u$ 
7      DFS-VISIT( $G, v$ )          }
```

**Deg(v)**

```

8   $u.\text{color} = \text{BLACK}$ 
9   $\text{time} = \text{time} + 1$ 
10  $u.f = \text{time}$ 
```

**Example**



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```

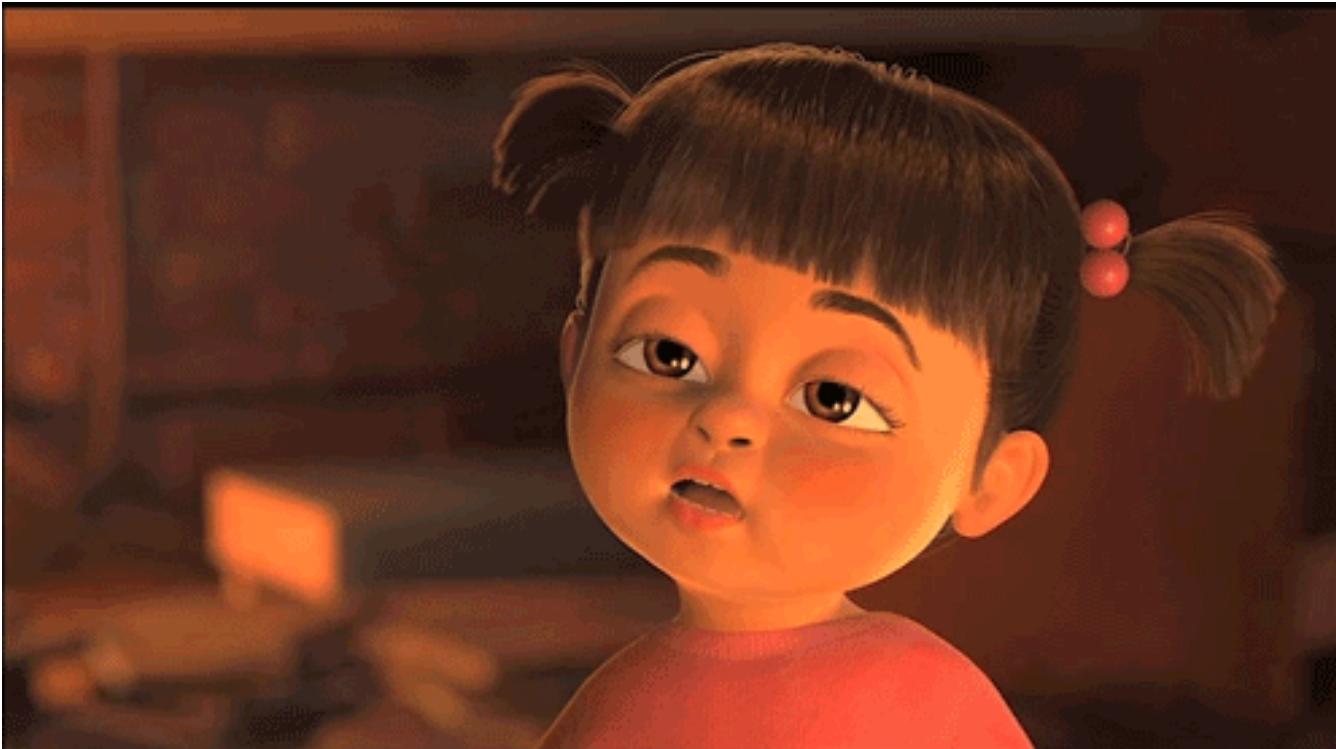
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5    if  $v.\text{color} == \text{WHITE}$       } V
6       $v.\pi = u$ 
7      DFS-VISIT( $G, v$ )          } Deg(v)
8   $u.\text{color} = \text{BLACK}$ 
9   $\text{time} = \text{time} + 1$ 
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```

- Loops on lines 1-3 and lines 5-7 take time  $\Theta(V)$ , exclusive of the time to execute the calls to DFS-VISIT
- During an execution of DFS-VISIT( $G, v$ ), the loop on lines 4-7 executes  $|\text{Adj}[v]|$  times.
- Since  $\sum_{v \in V} |\text{Adj}[v]| = \Theta(E)$ , the total cost of executing lines 4-7 of DFS-VISIT is  $\Theta(E)$
- Therefore, the running time of DFS is  $\Theta(|V| + |E|)$

# Thanks a lot



If you are taking a Nap, **wake up.....Lecture Over**