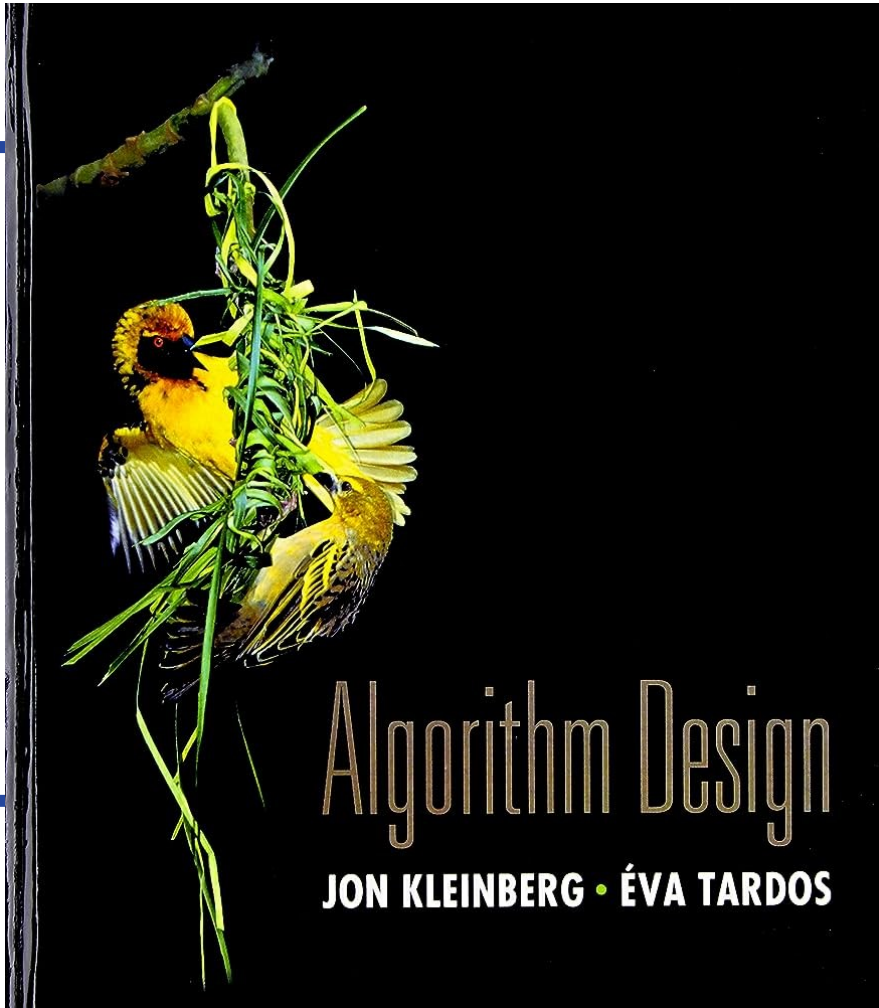


CS 310: Algorithms

Lecture 20

Instructor: Naveed Anwar Bhatti



Chapter 6: Dynamic Programming

Section :
Segmented Least Squares

Segmented Least Squares

- **Least squares:**

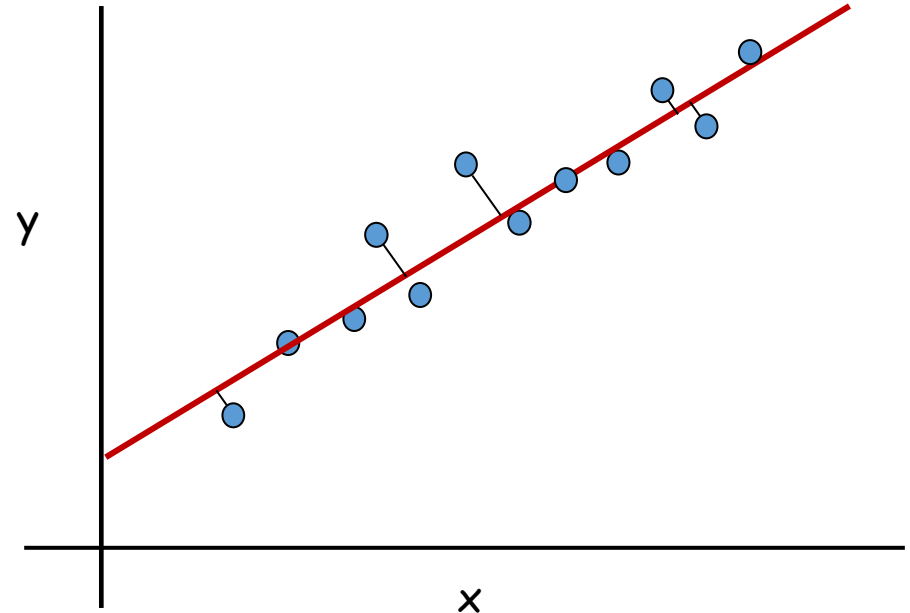
- Foundational problem in statistic and numerical analysis.
- Given n points in the plane: $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$.
- Find a line $y = ax + b$ that minimizes the sum of the squared error:

$$E = \sum_{i=1}^n (y_i - ax_i - b)^2$$

- **Solution:** Calculus \Rightarrow min error is achieved when

$$a = \frac{n \sum_i x_i y_i - (\sum_i x_i)(\sum_i y_i)}{n \sum_i x_i^2 - (\sum_i x_i)^2}$$

$$b = \frac{\sum_i y_i - a \sum_i x_i}{n}$$



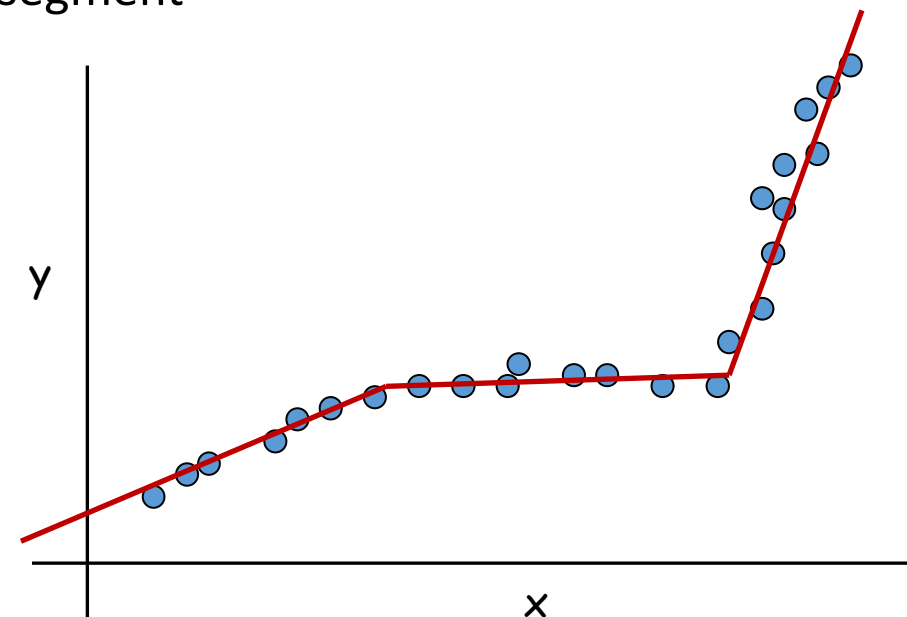
Segmented Least Squares

- **Segmented least squares:**

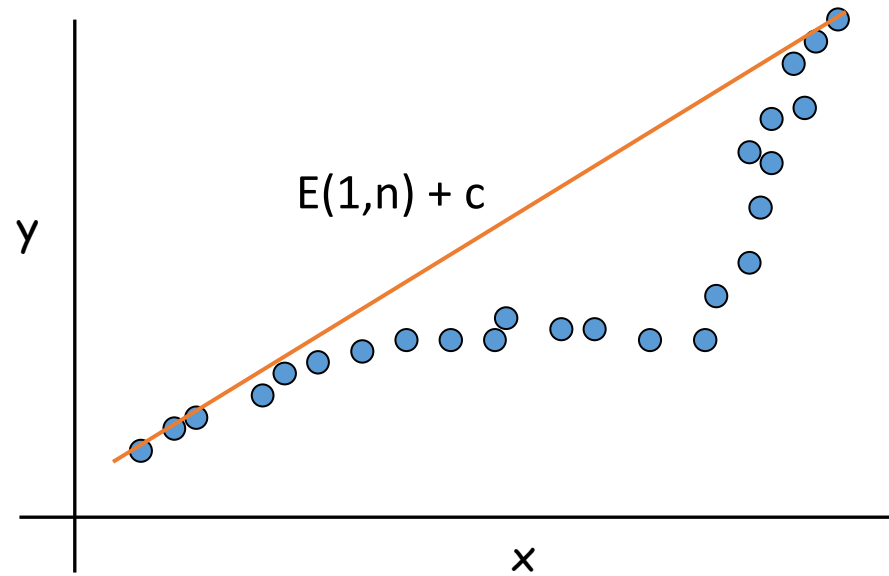
- Points lie roughly on a sequence of **several line segments**
- Given n points in the plane $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$
- find a sequence of lines that minimizes $f(x)$:
 - the sum of the sums of the squared errors E in each segment
 - the number of lines L

Q: What's a reasonable choice for $f(x)$ to balance accuracy and parsimony?

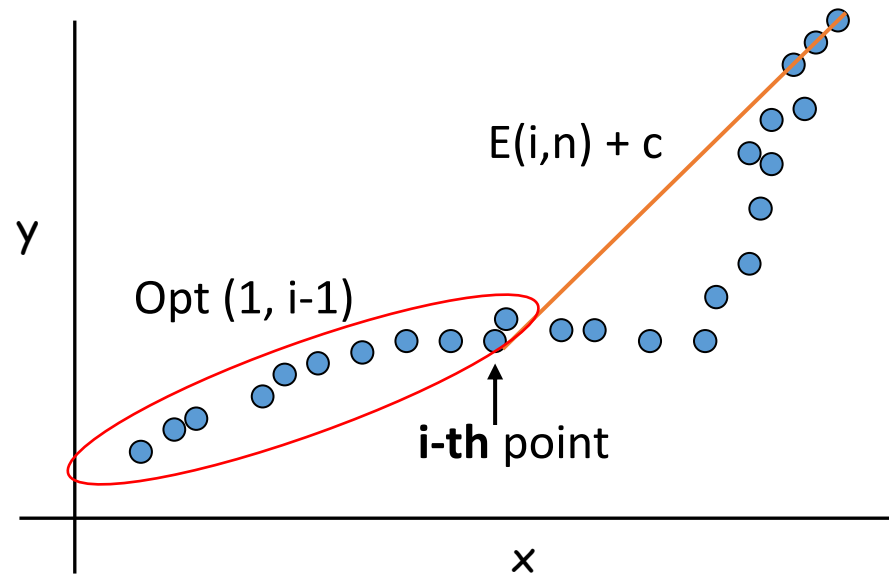
Tradeoff function: $E + c L$, for some constant $c > 0$



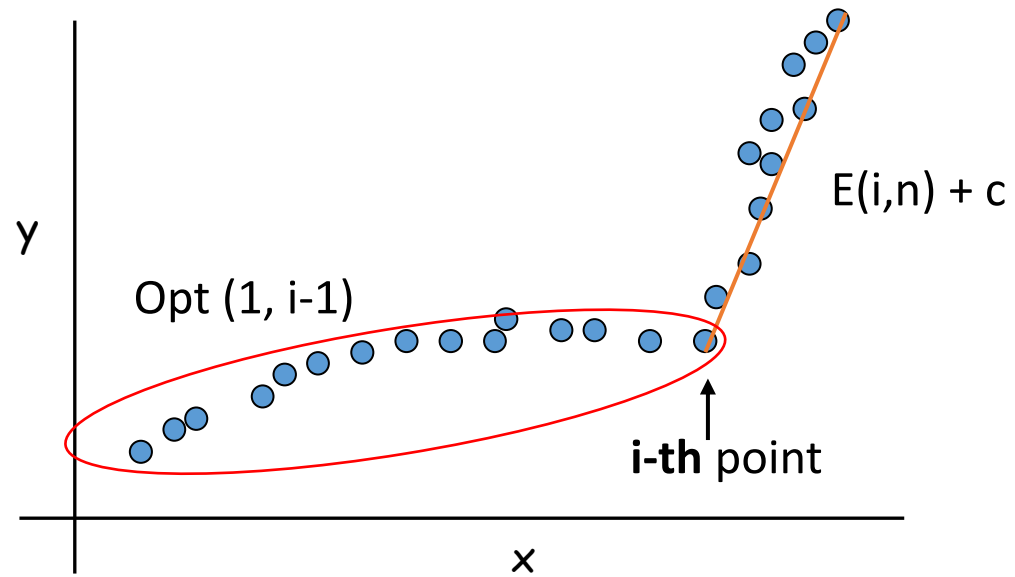
Segmented Least Squares



Segmented Least Squares



Segmented Least Squares



Dynamic Programming: Multiway Choice

- Notation.
 - $OPT(n)$ = minimum cost for points p_1, p_{i+1}, \dots, p_n .
- To compute $OPT(n)$:
 - Last segment uses points p_i, p_{i+1}, \dots, p_n for some i .
 - $e(i, n)$ = minimum sum of squares for points p_i, p_{i+1}, \dots, p_n .
 - Cost = $e(i, n) + c + OPT(i-1)$.

$$OPT(n) = \begin{cases} 0 & \text{if } n = 0 \\ \min_{1 \leq i \leq n} \{e(i, n) + c + OPT(i-1)\} & \text{otherwise} \end{cases}$$

Dynamic Programming: Multiway Choice

$$OPT(n) = \begin{cases} 0 & \text{if } n = 0 \\ \min_{1 \leq i \leq n} \{e(i, n) + c + OPT(i - 1)\} & \text{otherwise} \end{cases}$$

Let say we have 4 points i.e., **n=4**

$$OPT(4) = \min\{e(1, 4) + c + OPT(0), e(2, 4) + c + OPT(1), e(3, 4) + c + OPT(2), e(4, 4) + c + OPT(3)\}$$

$$OPT(3) = \min\{e(1, 3) + c + OPT(0), e(2, 3) + c + OPT(1), e(3, 3) + c + OPT(2)\}$$

$$OPT(2) = \min\{e(1, 2) + c + OPT(0), e(2, 2) + c + OPT(1)\}$$

$$OPT(1) = \min\{e(1, 1) + c + OPT(0)\}$$

$$OPT(0) = 0$$

Segmented Least Squares: Algorithm

```
INPUT:  $n, p_1, \dots, p_N, c$ 

for  $i=0$  to  $n$ :
     $M[n] = -1$ 
 $M[0] = 0$ 

OPT() {
    if  $n == 0$ :
        return 0

    if  $M[0] != -1$ :
        return  $M[n]$ 

    min_cost = 0;

    for  $i=1$  to  $n$ :  $\leftarrow O(n)$ 
        cost =  $e(i, n) + c + \text{OPT}(i - 1)$ 
        if cost < min_cost:
            min_cost = cost
    }

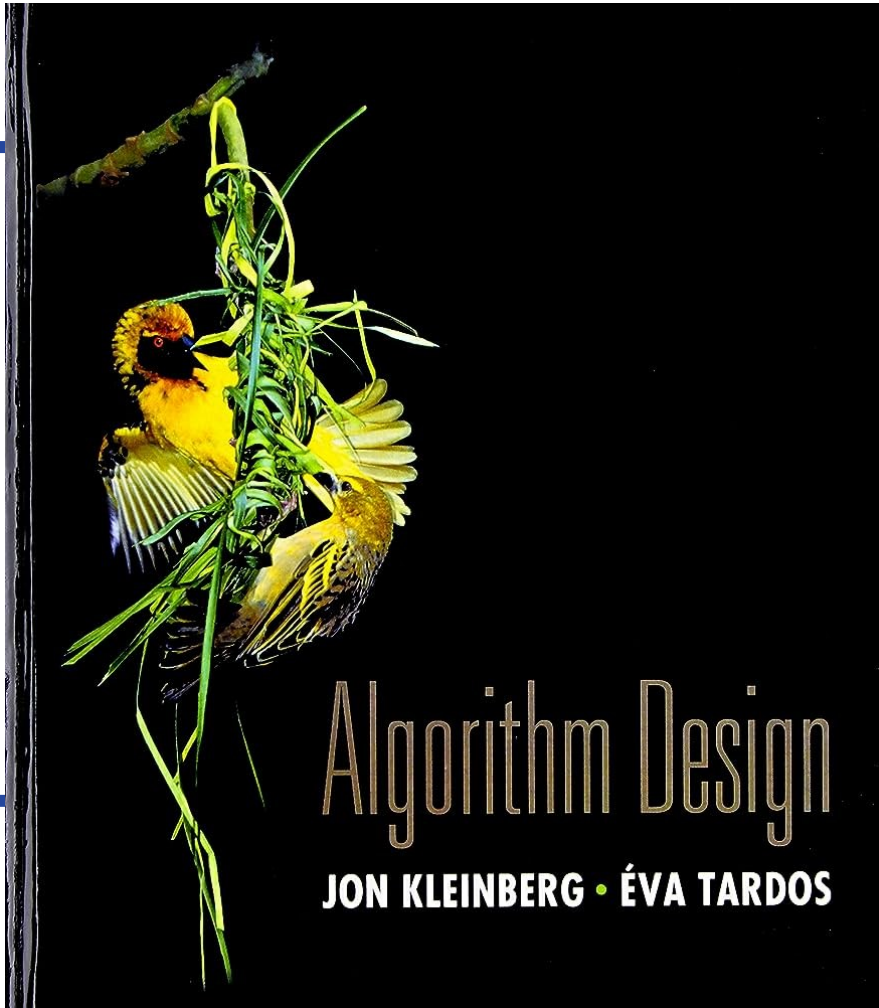
     $M[n] = \text{min\_cost}$ 
    return min_cost
}
```

$O(n)$

$O(n)$

Running time: $O(n^3)$.

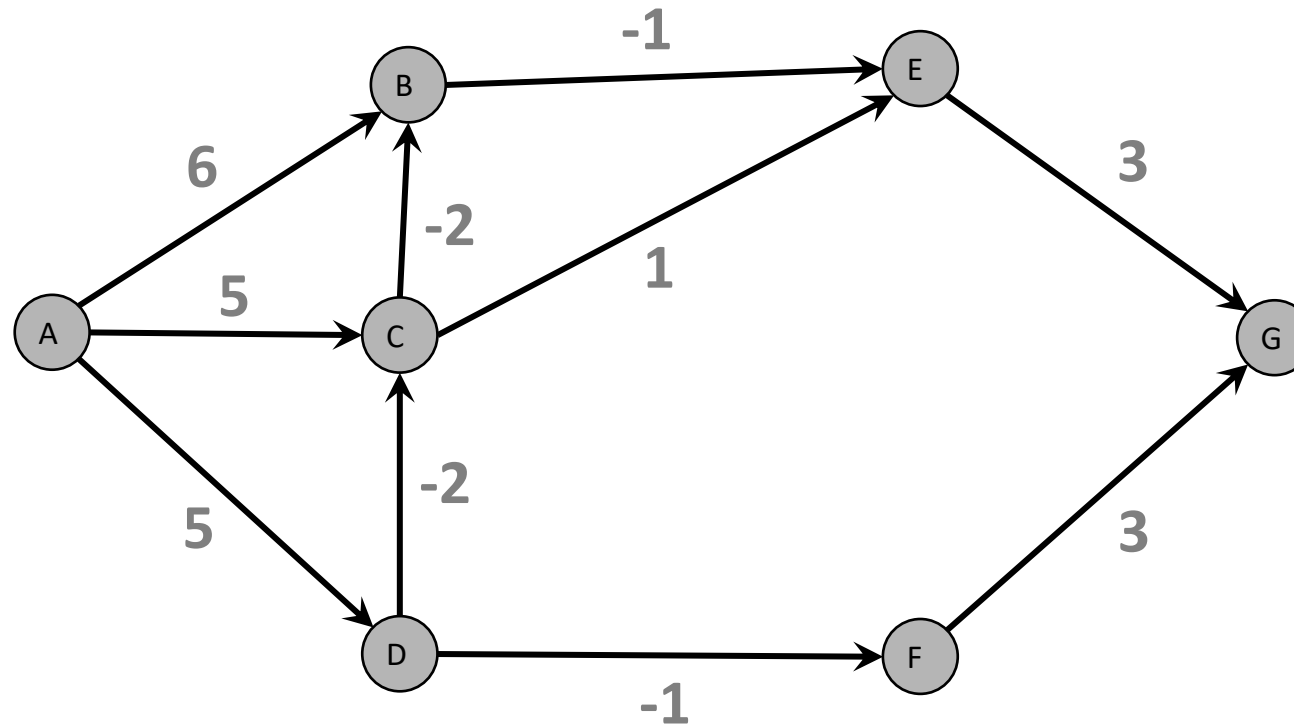
- Bottleneck = computing $e(i, j)$ for $O(n^2)$ pairs, $O(n)$ per pair using previous formula.



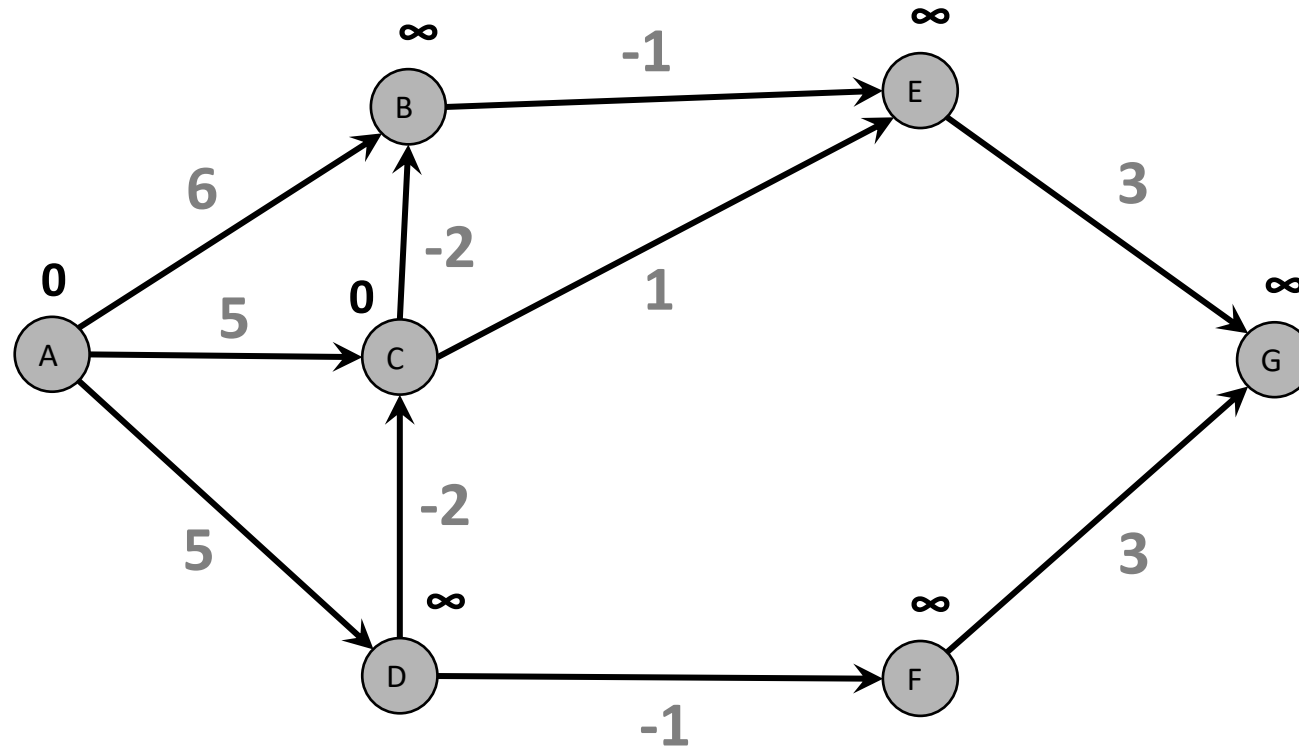
Chapter 6: Dynamic Programming

Section : Bonus Topic
Bellman-ford Algorithm

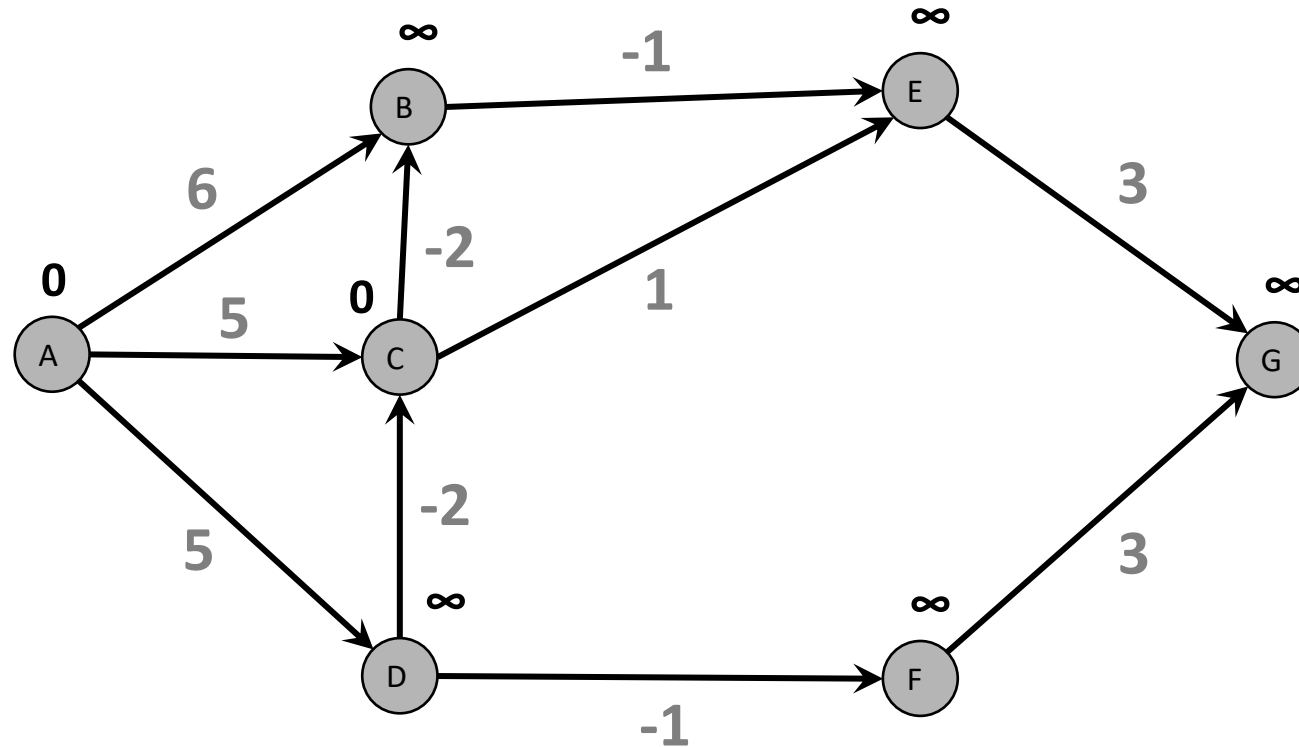
Bellman-ford: Single Source Shortest Path



Bellman-ford: Single Source Shortest Path



Bellman-ford: Single Source Shortest Path

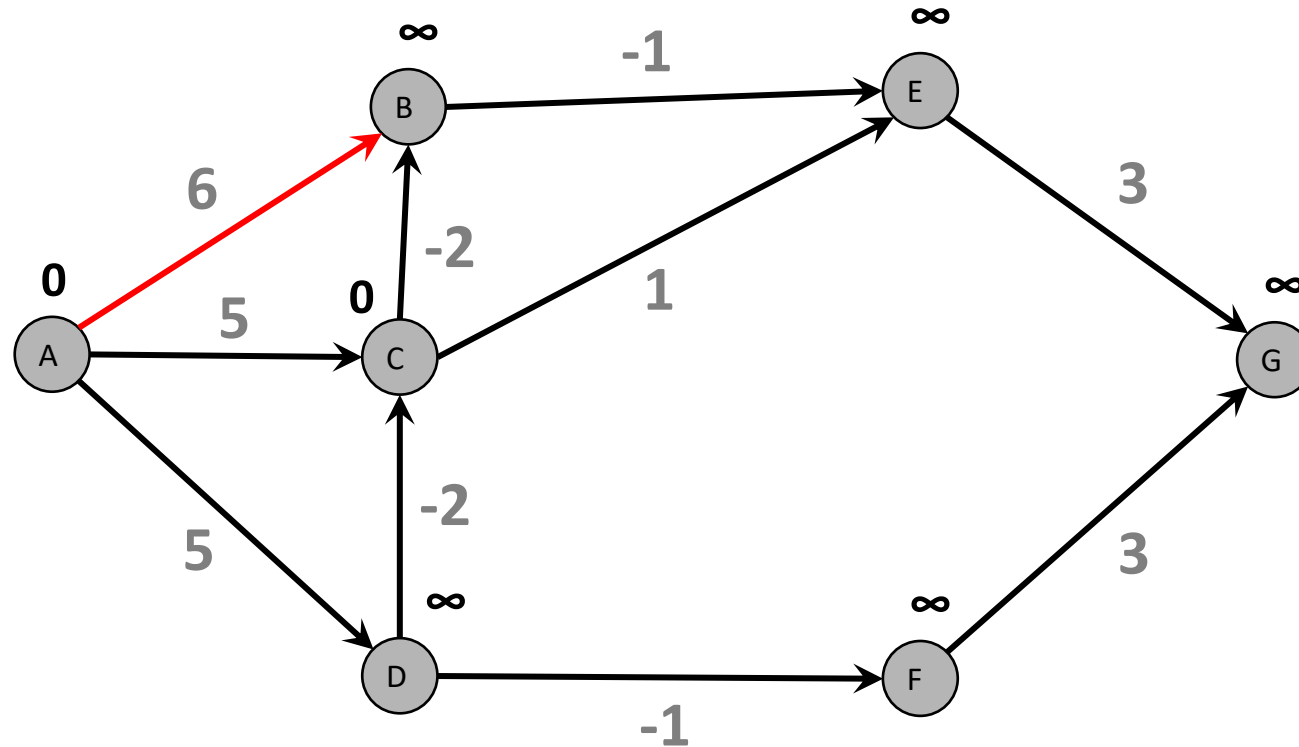


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) (D,C) (D,F) (C,E) (B,E) (F,G) (E,G)

Bellman-ford: Single Source Shortest Path

1st Iteration

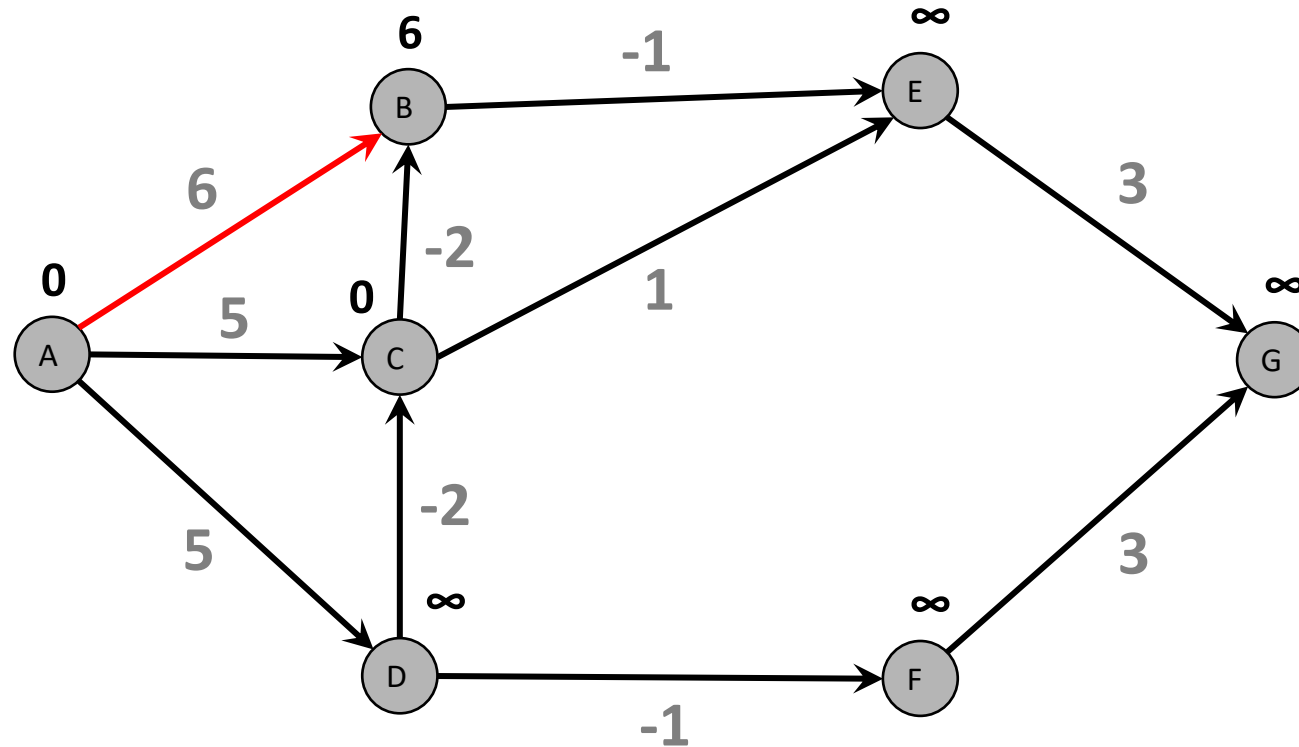


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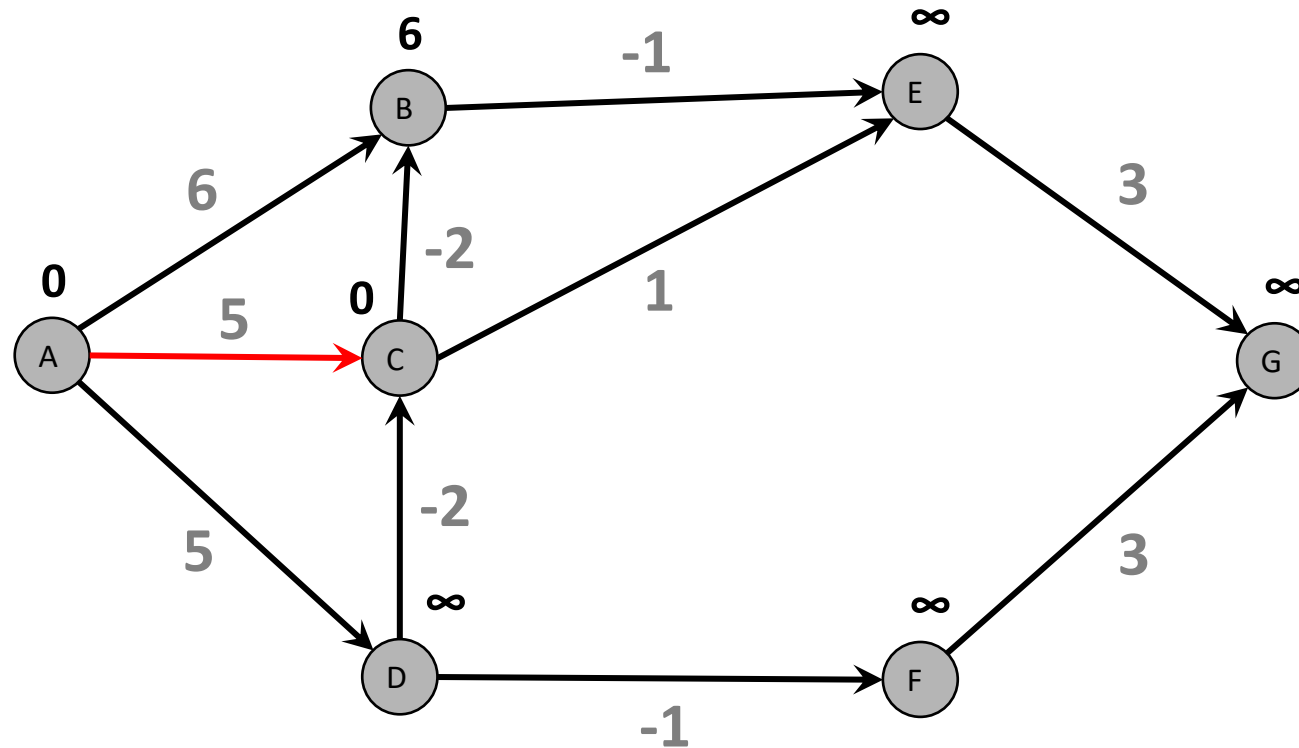


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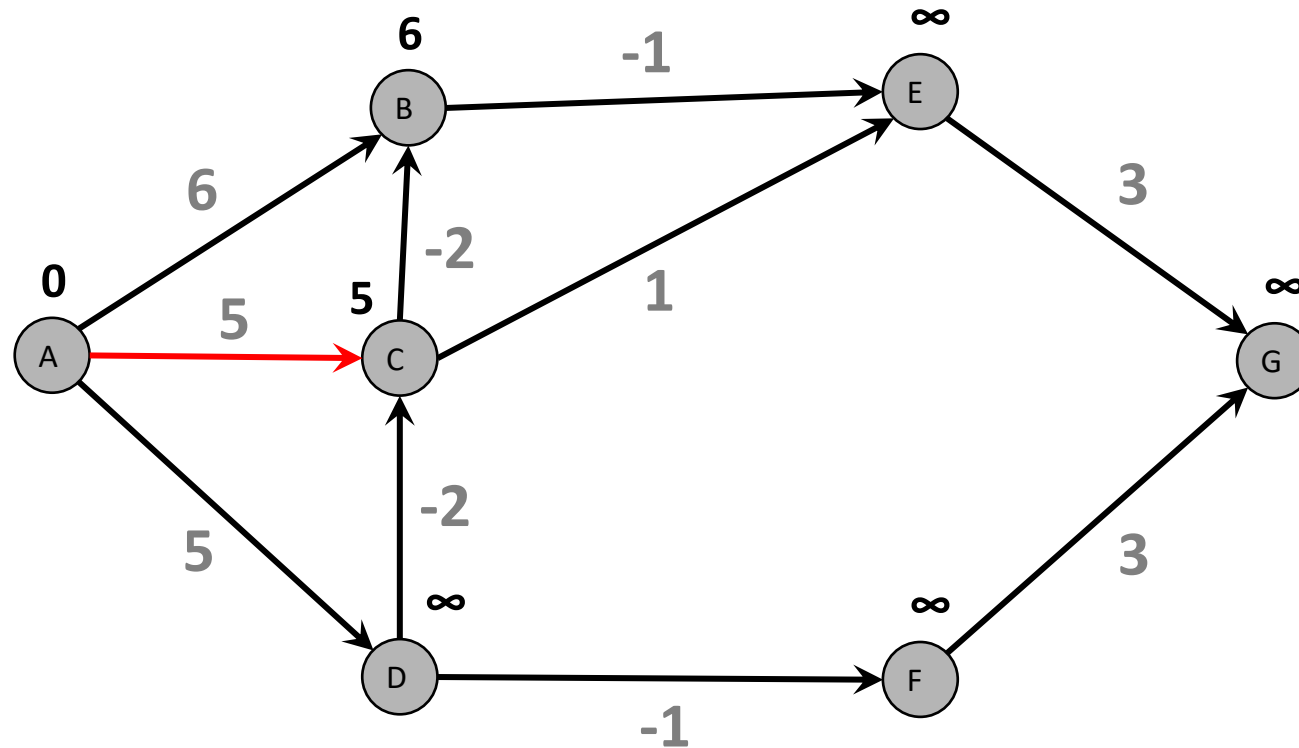


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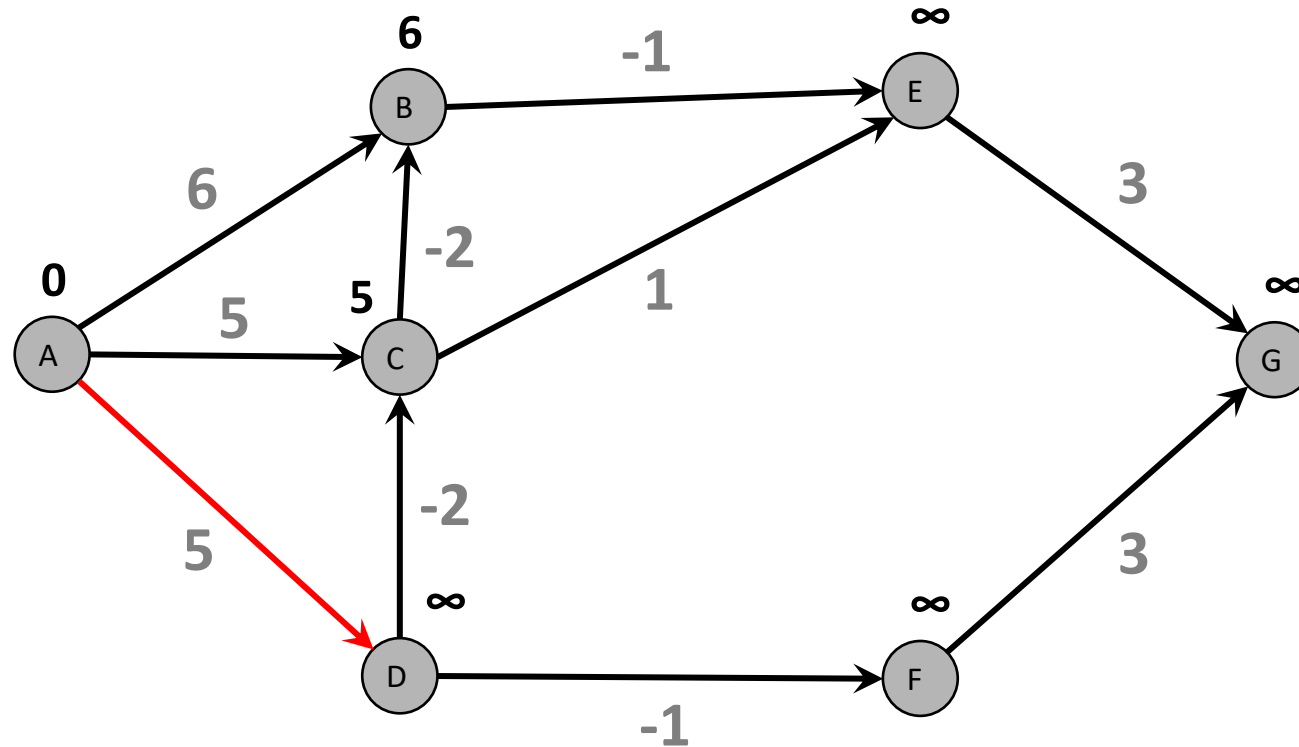


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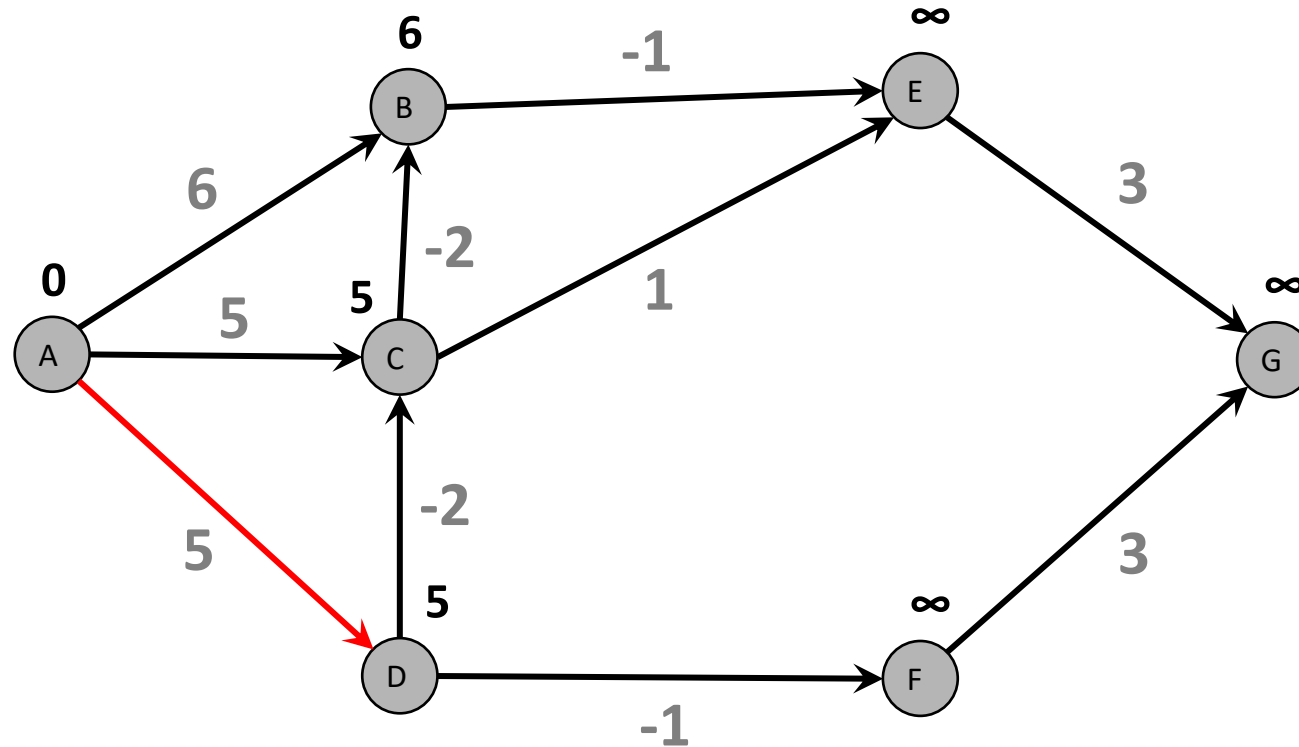


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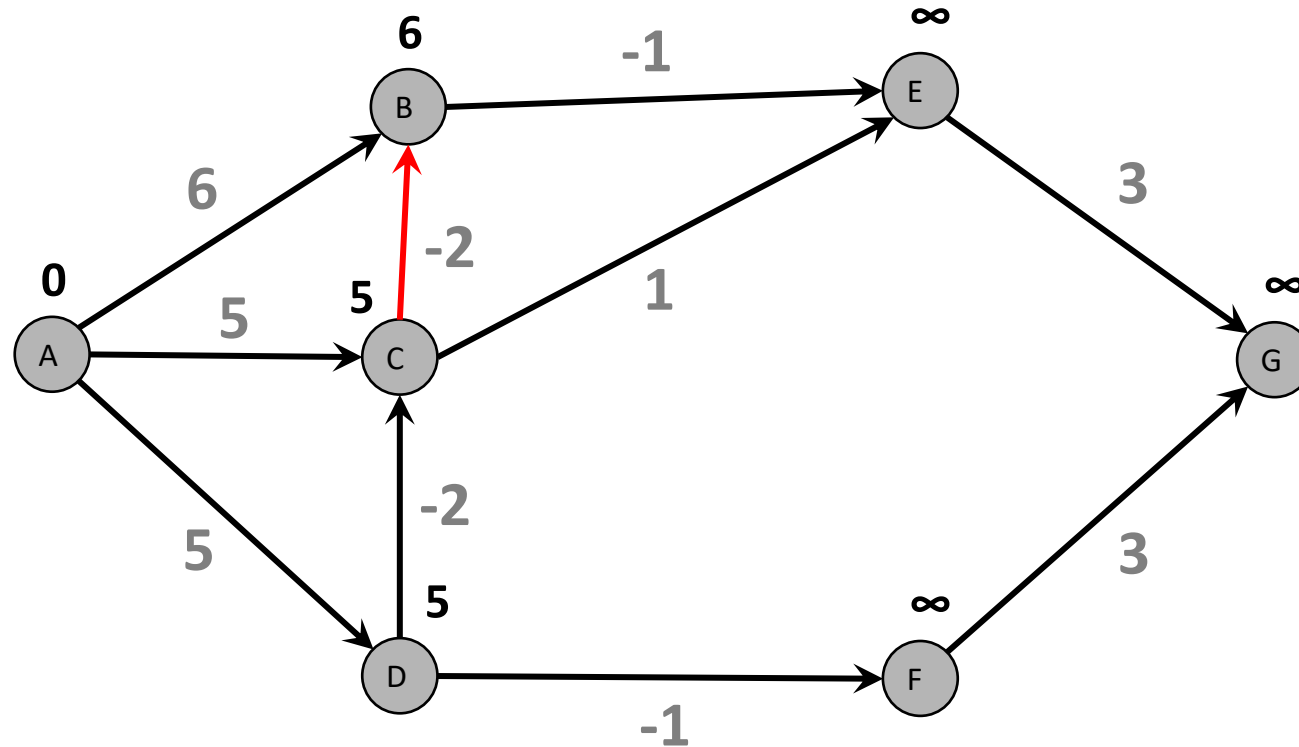


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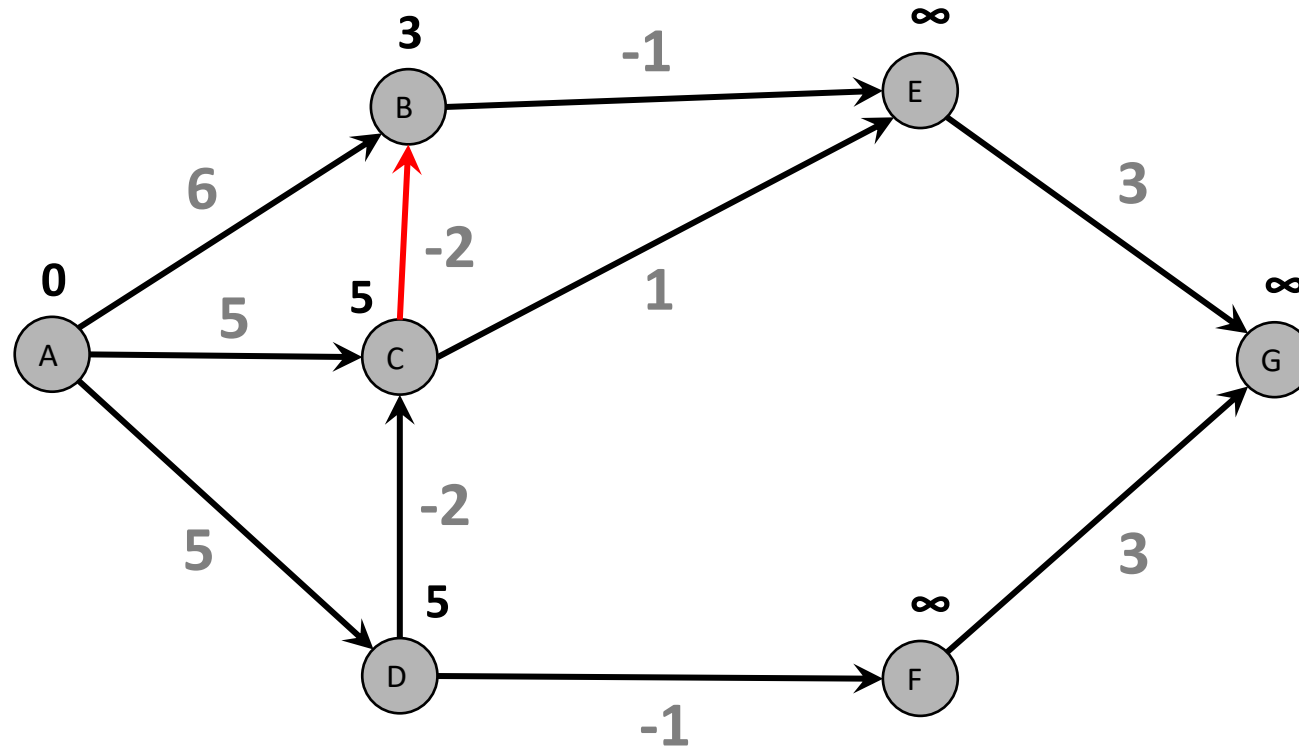


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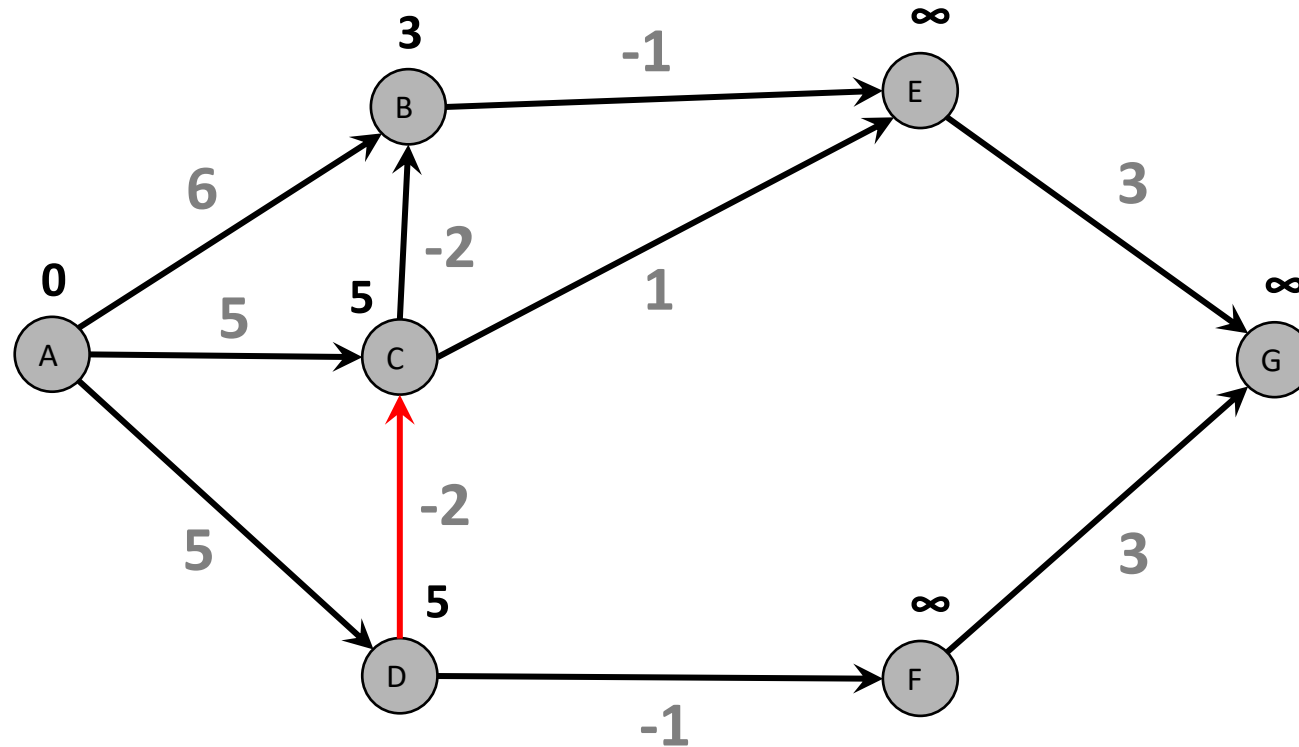


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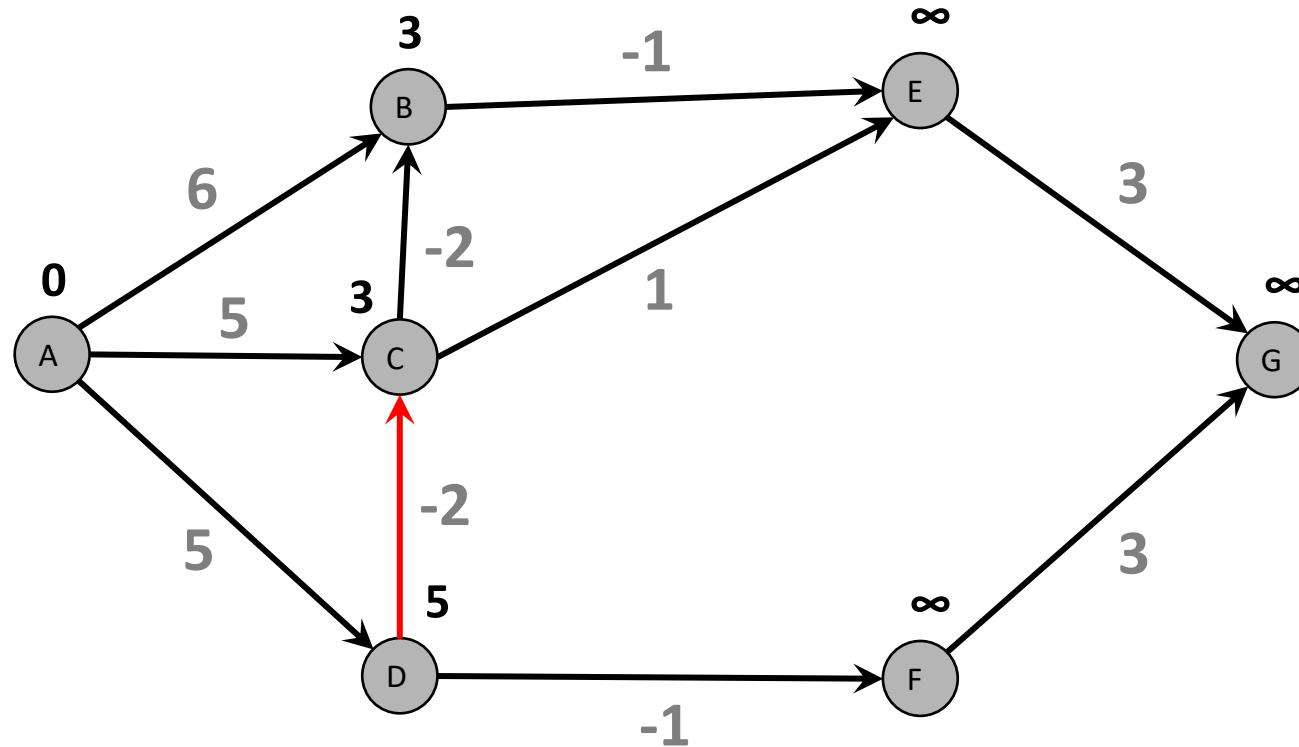


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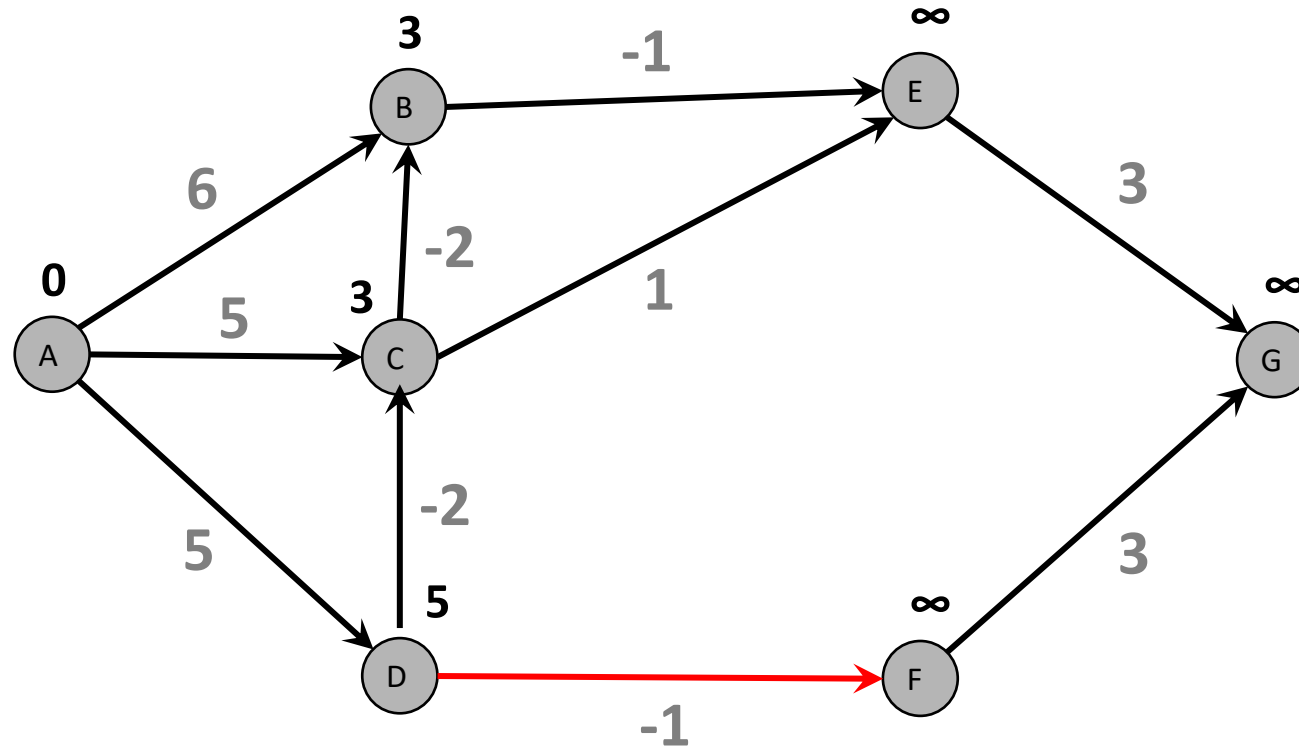


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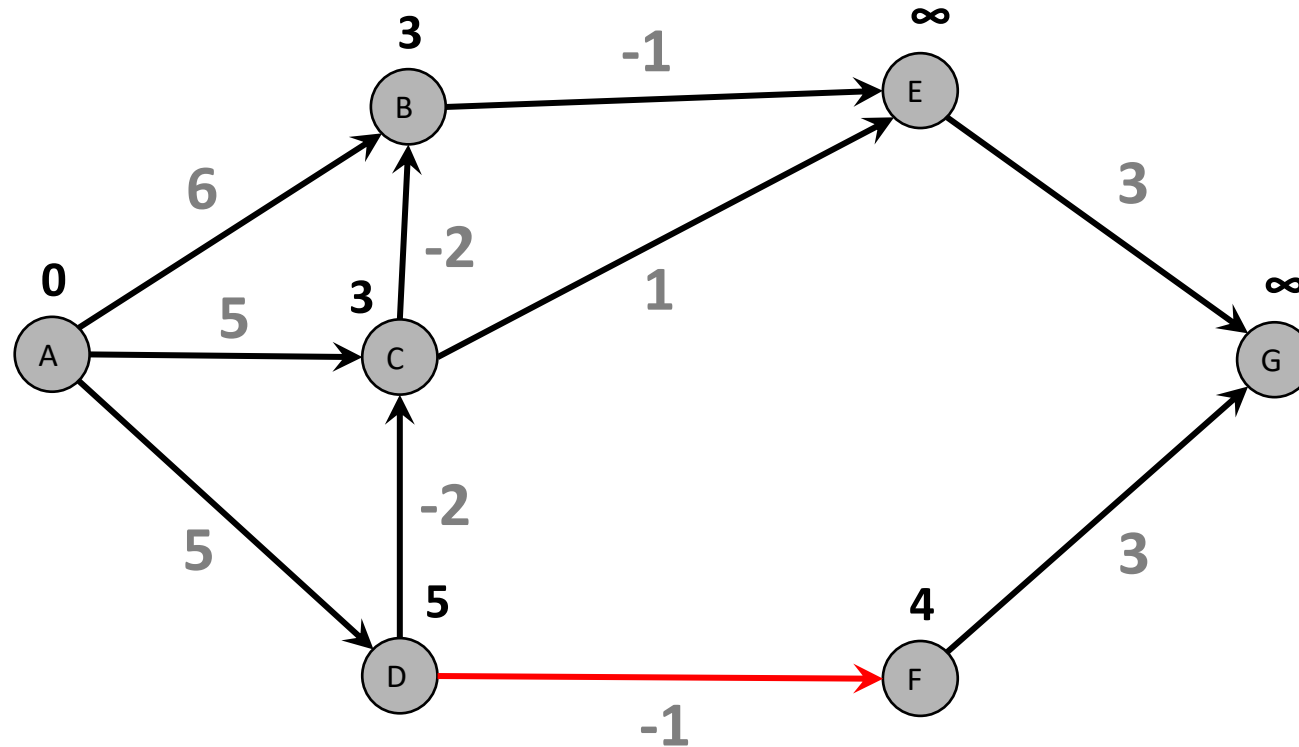


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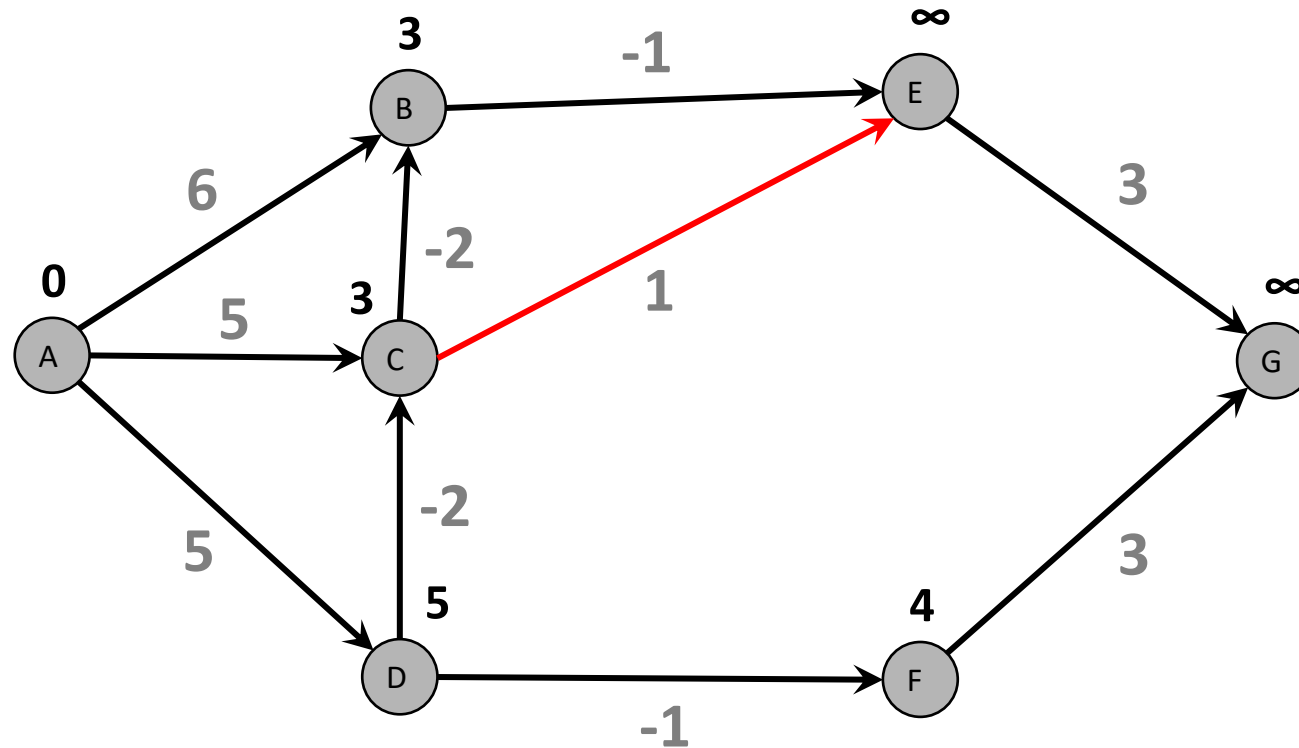


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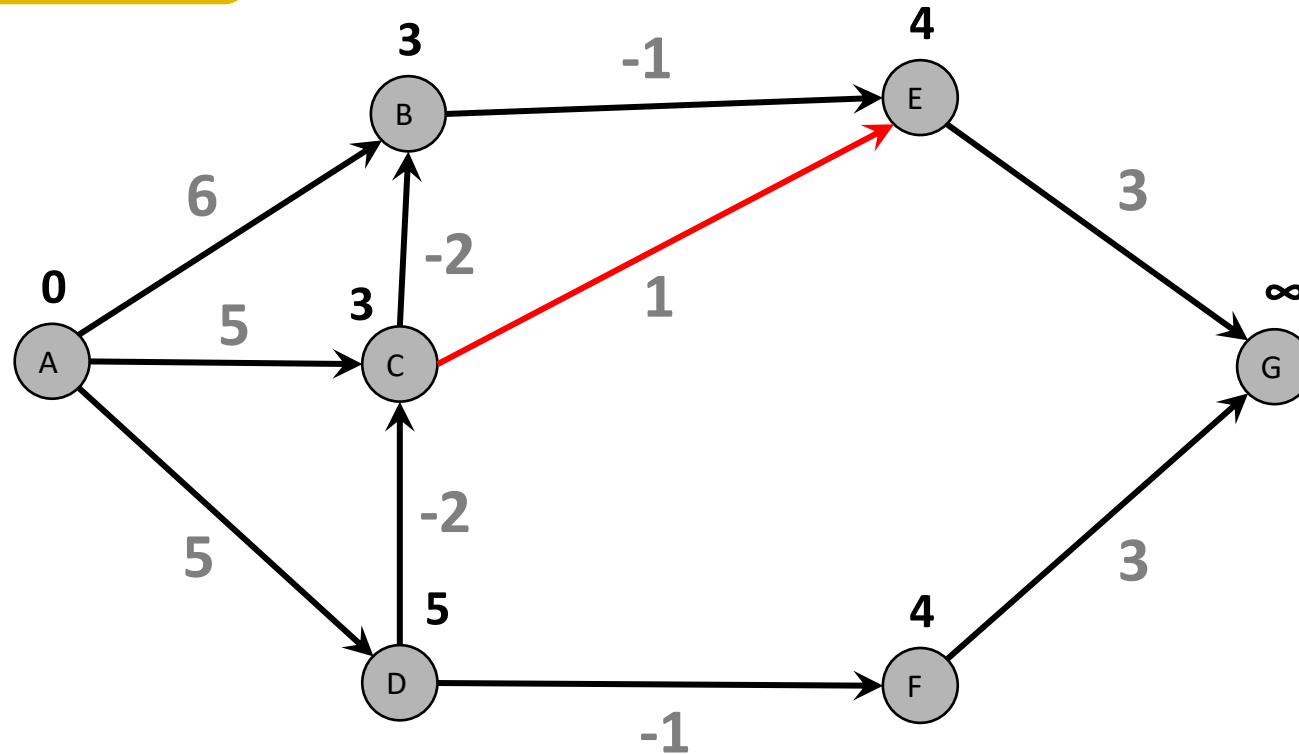


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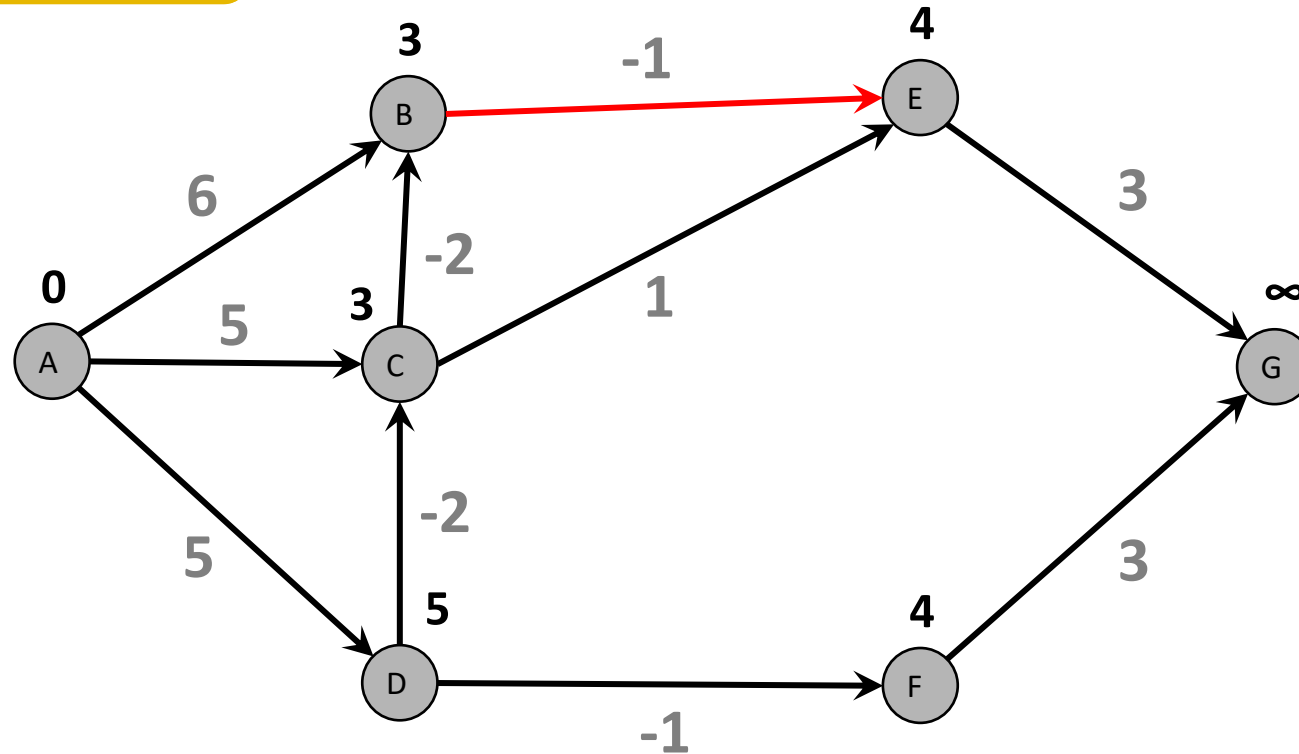


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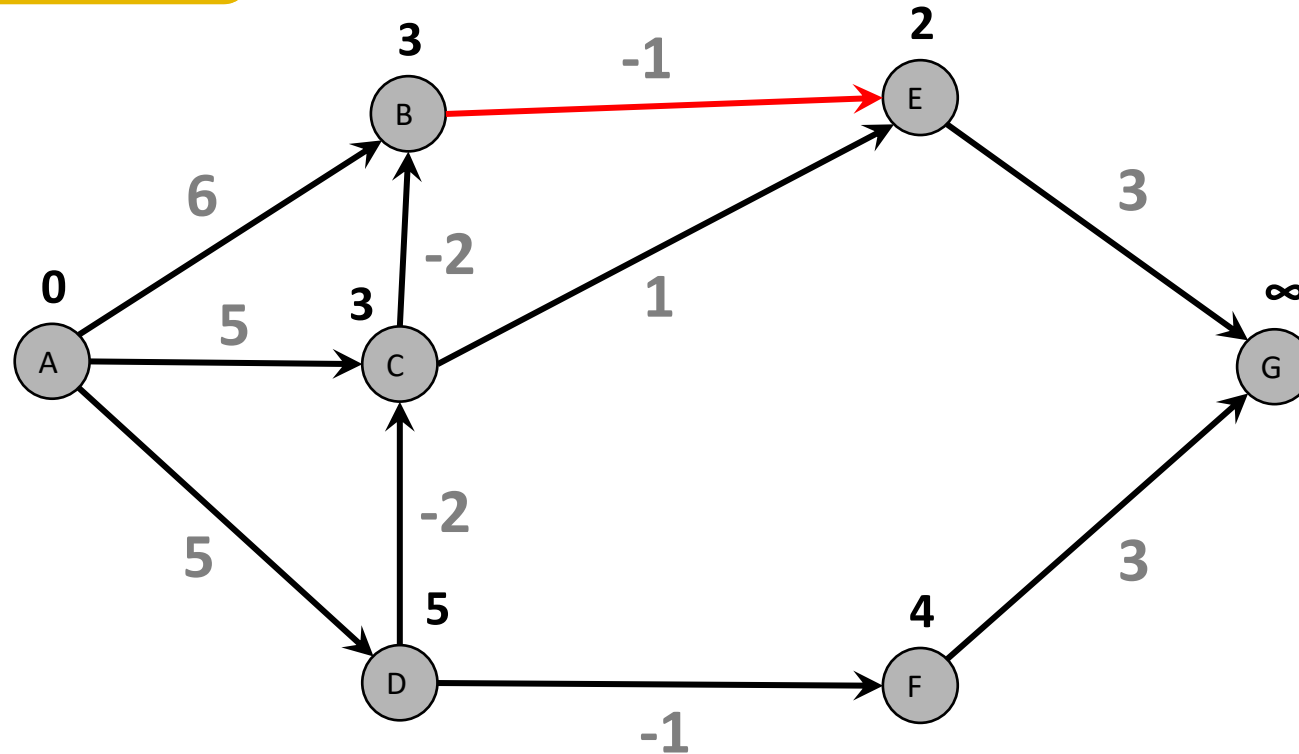


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Bellman-ford: Single Source Shortest Path

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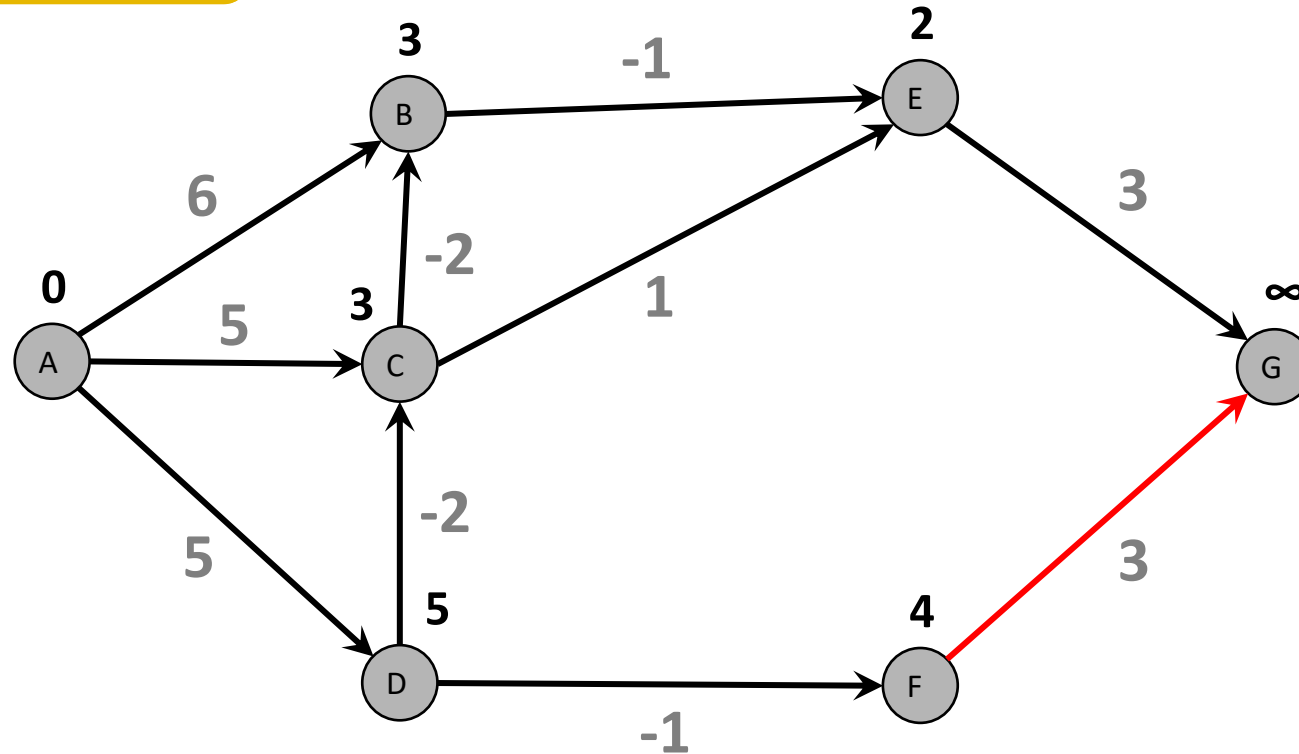


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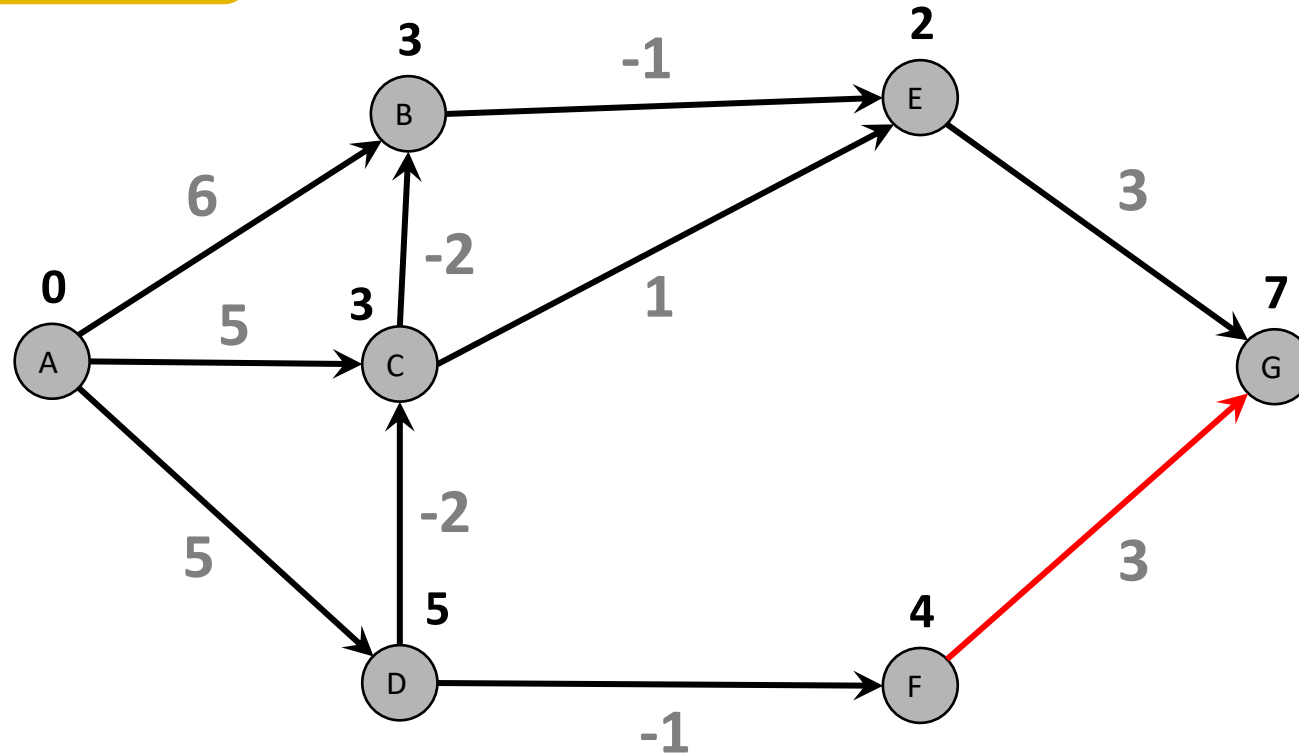


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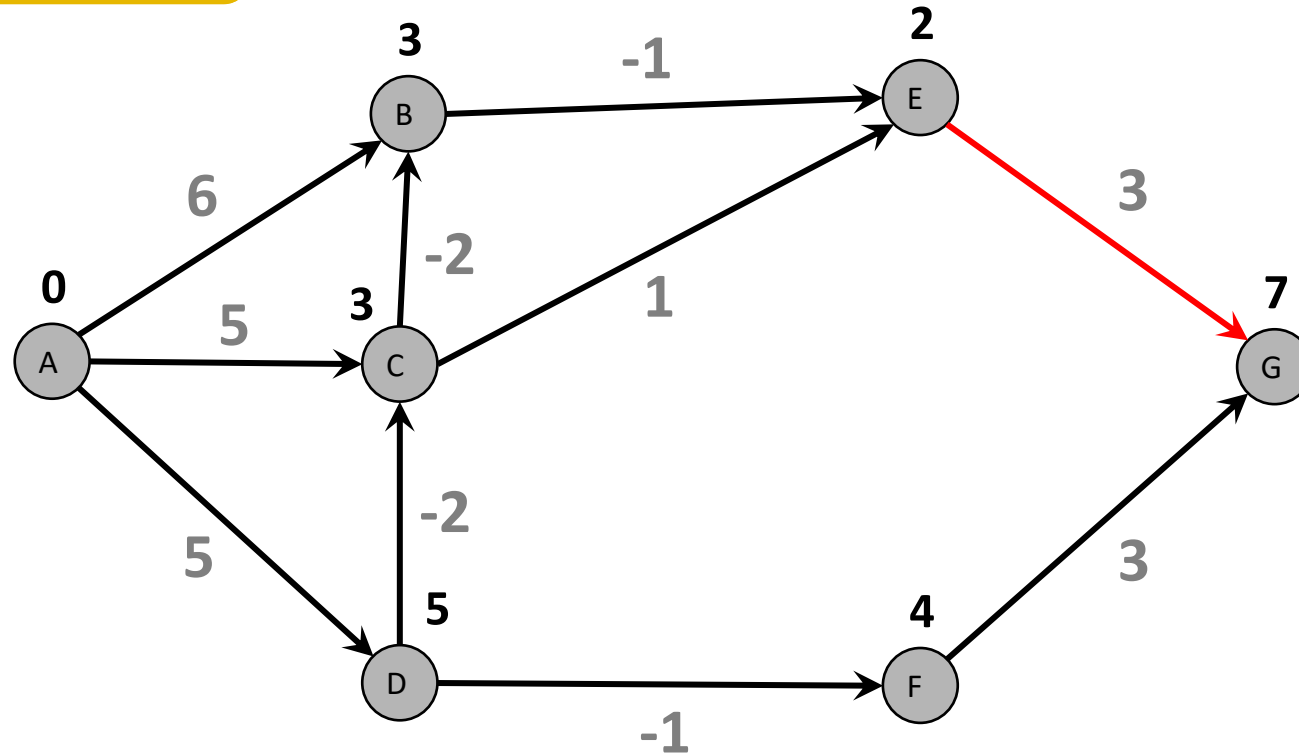


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Bellman-ford: Single Source Shortest Path

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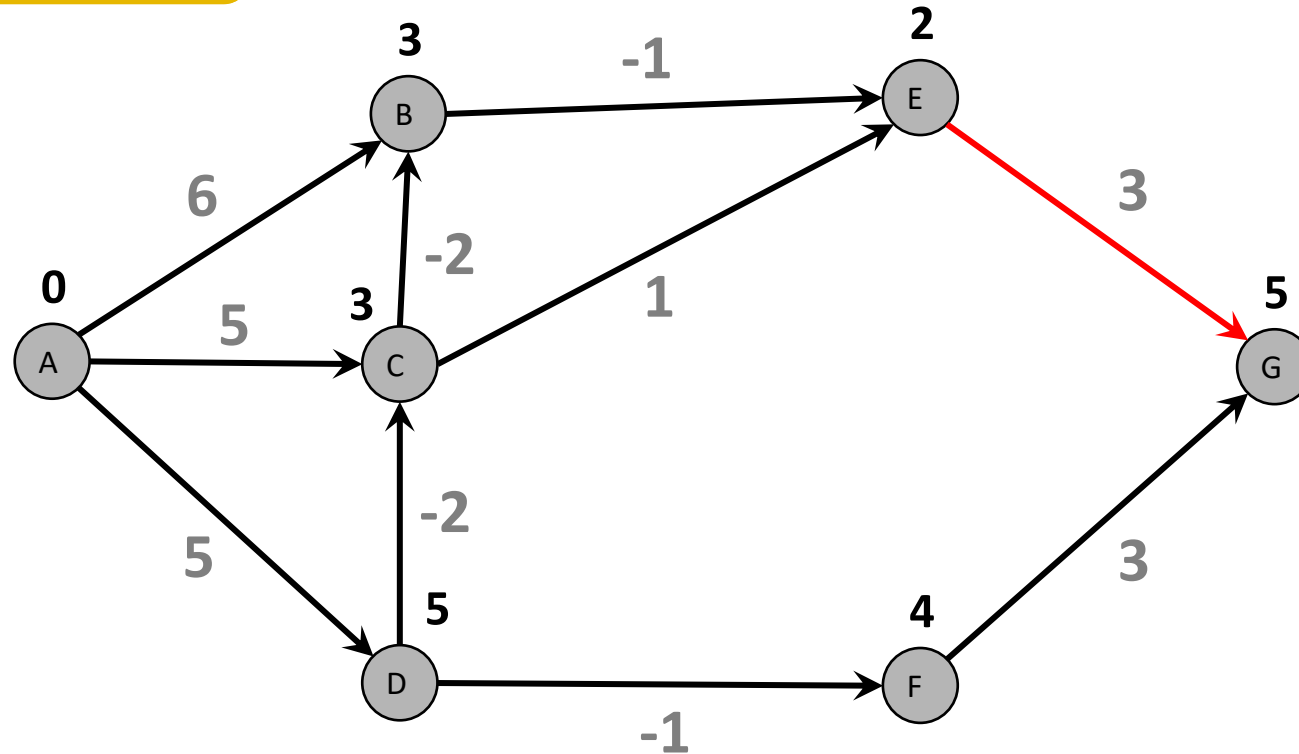


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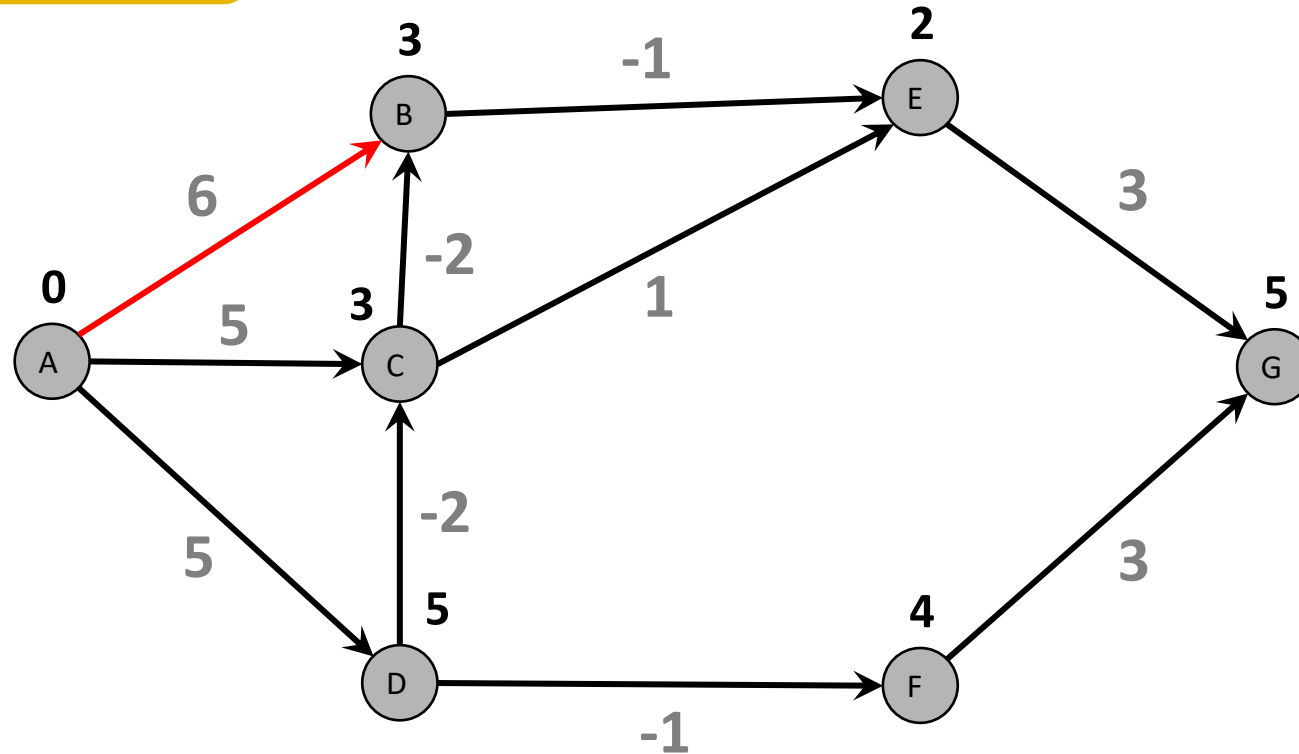


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Bellman-ford: Single Source Shortest Path

2nd Iteration

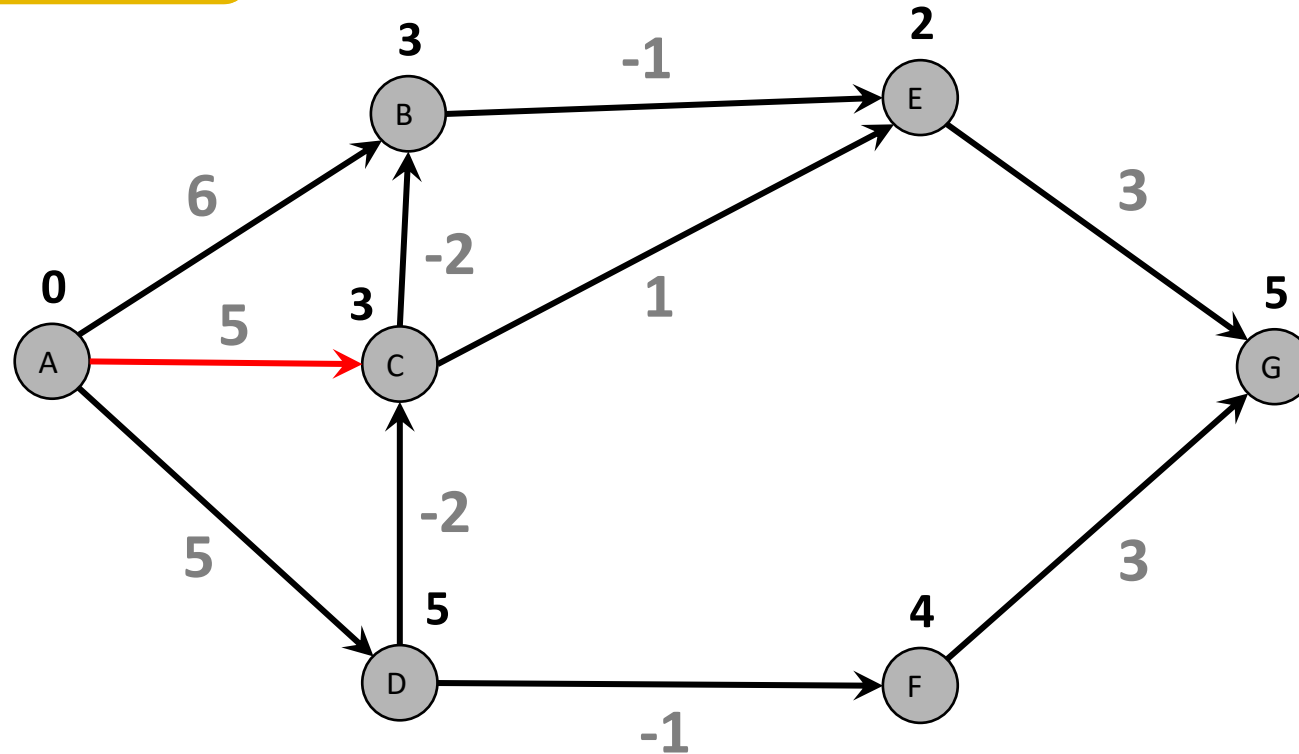


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Bellman-ford: Single Source Shortest Path

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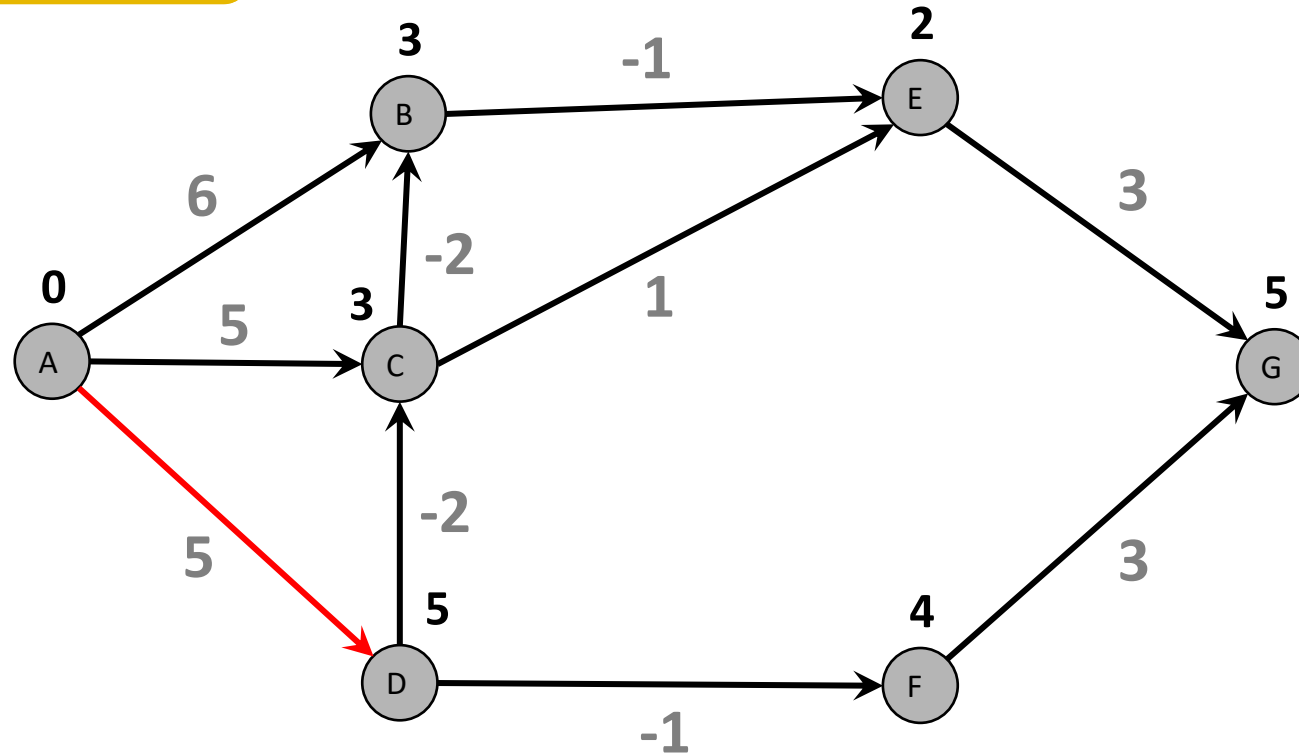


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Bellman-ford: Single Source Shortest Path

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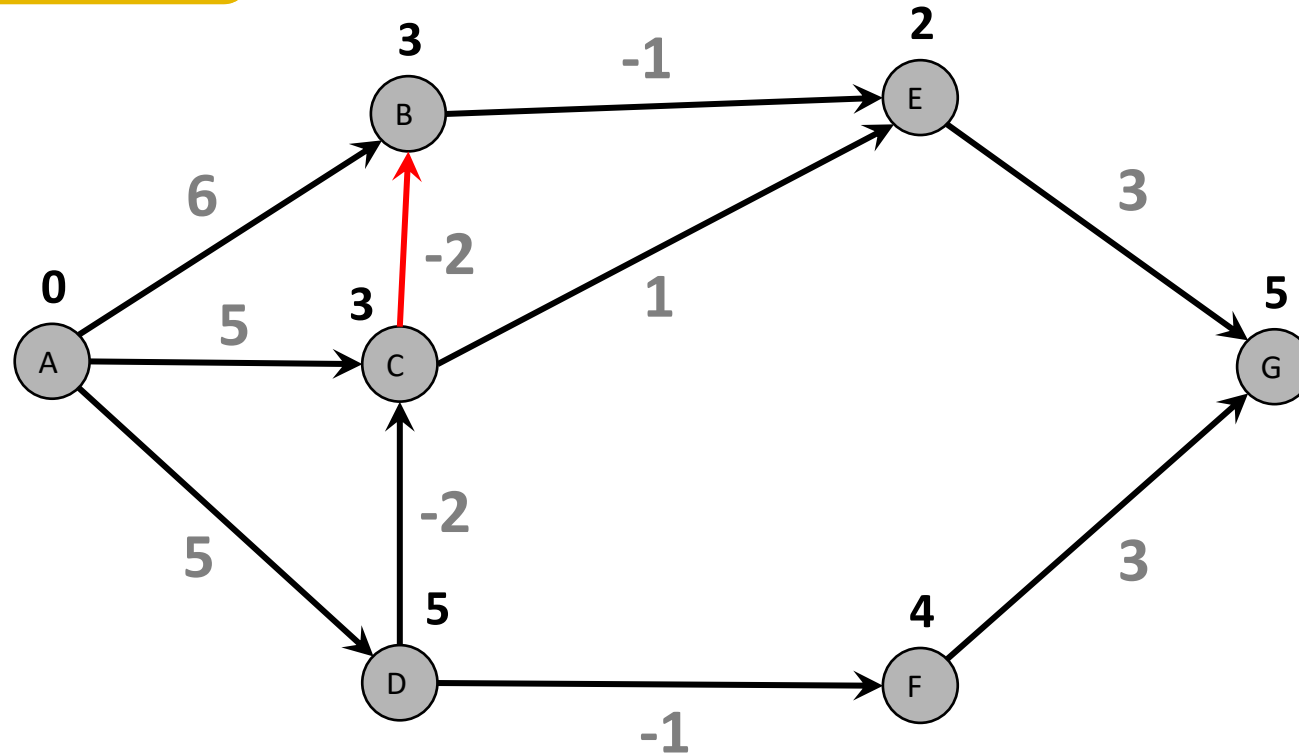


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Bellman-ford: Single Source Shortest Path

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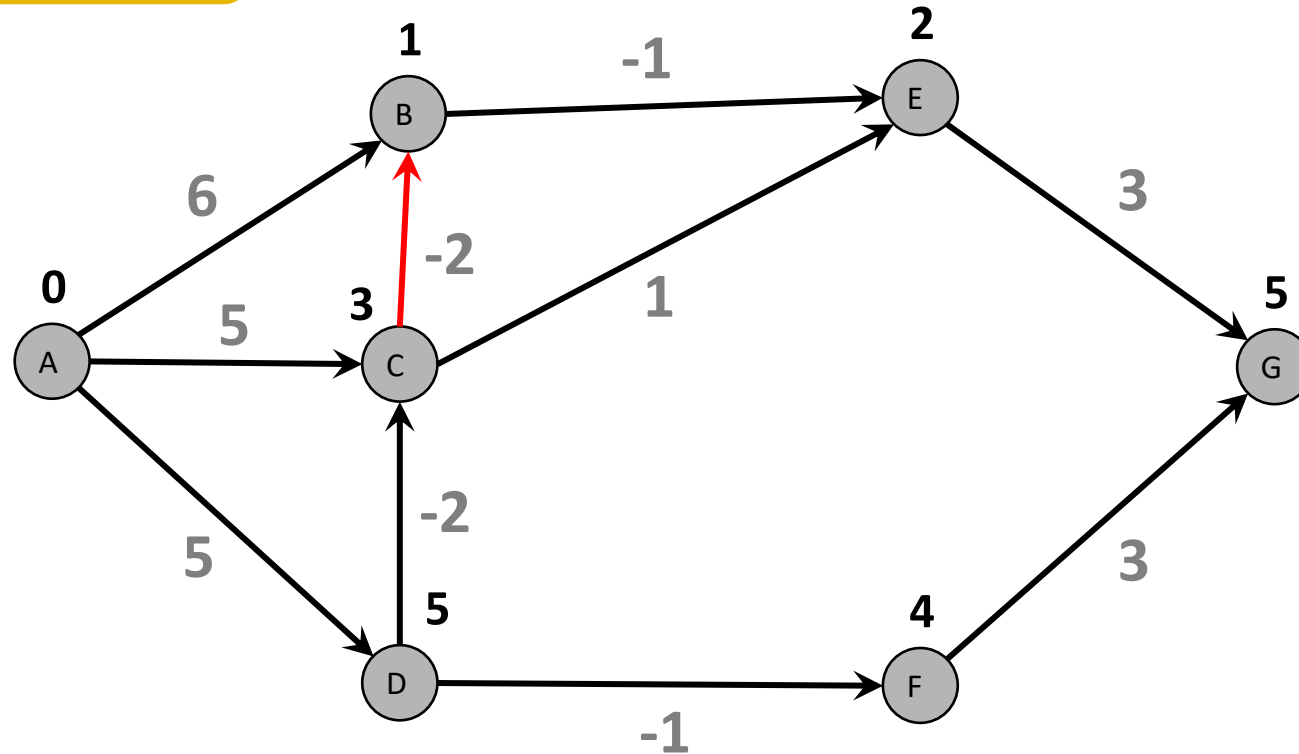


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Bellman-ford: Single Source Shortest Path

2nd Iteration

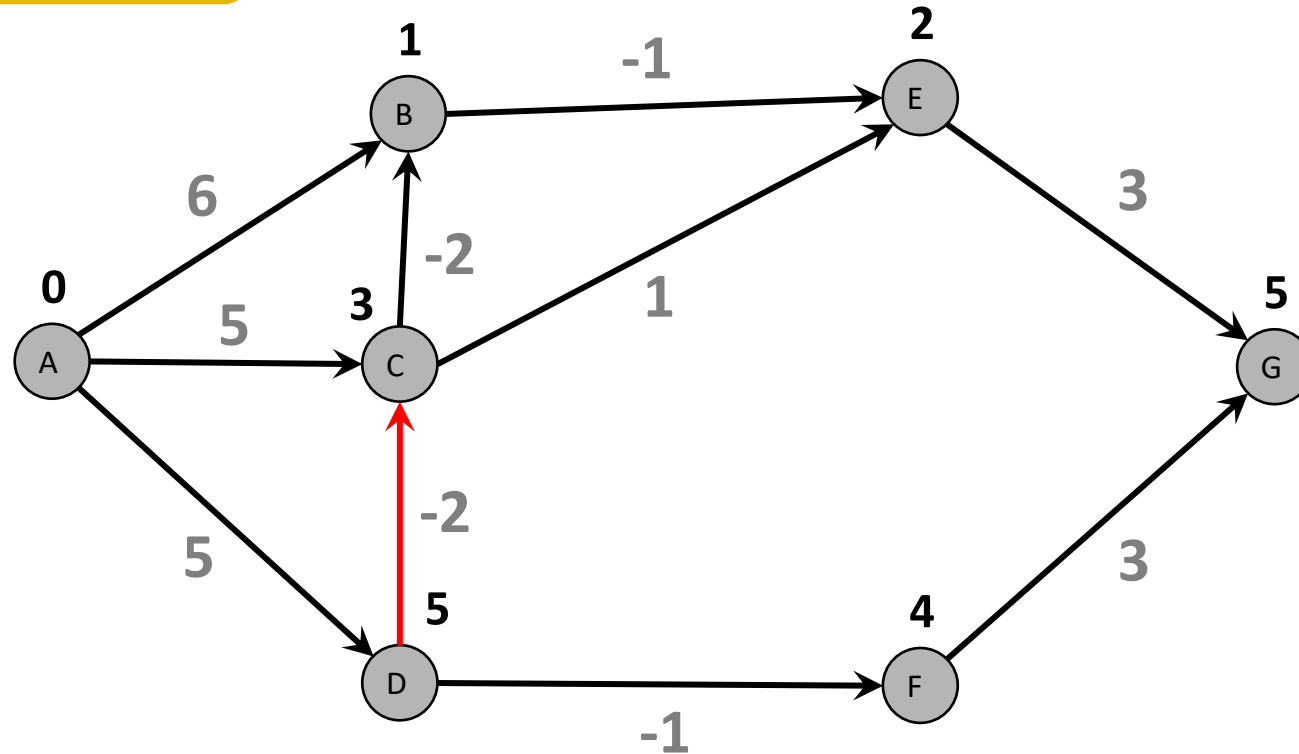


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Bellman-ford: Single Source Shortest Path

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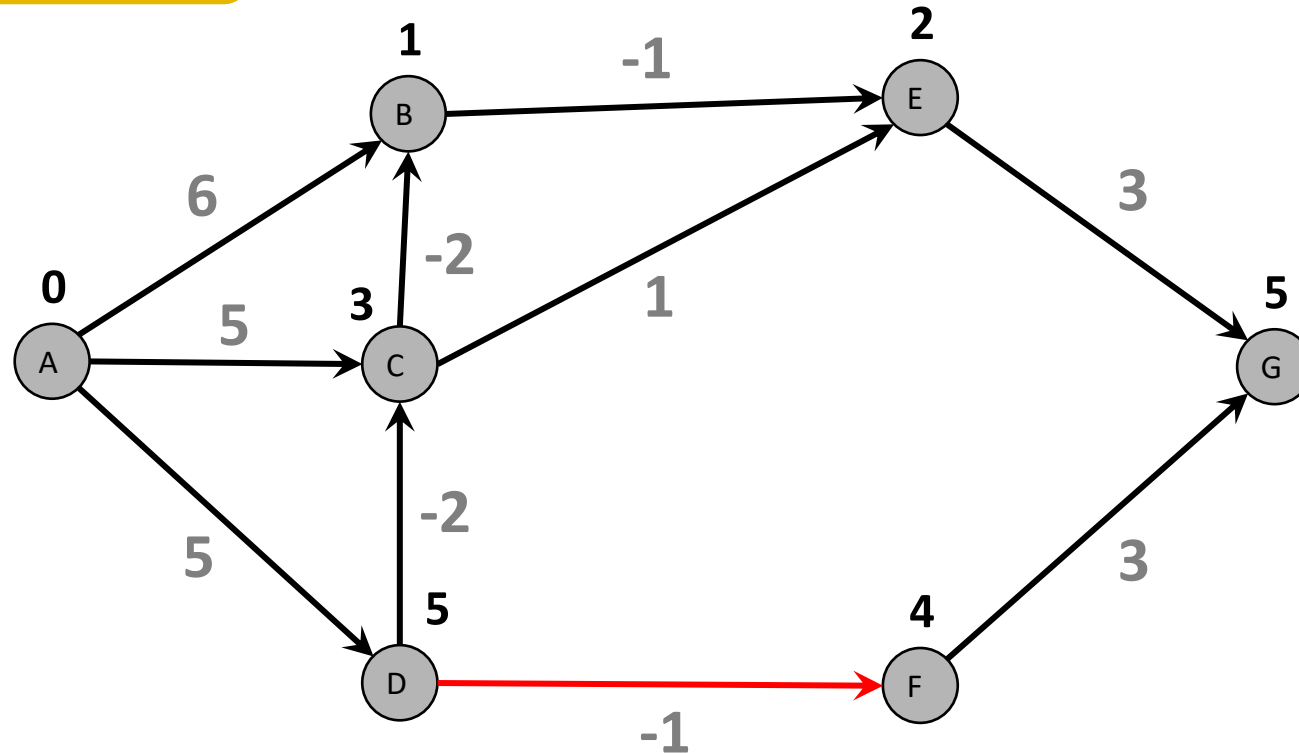


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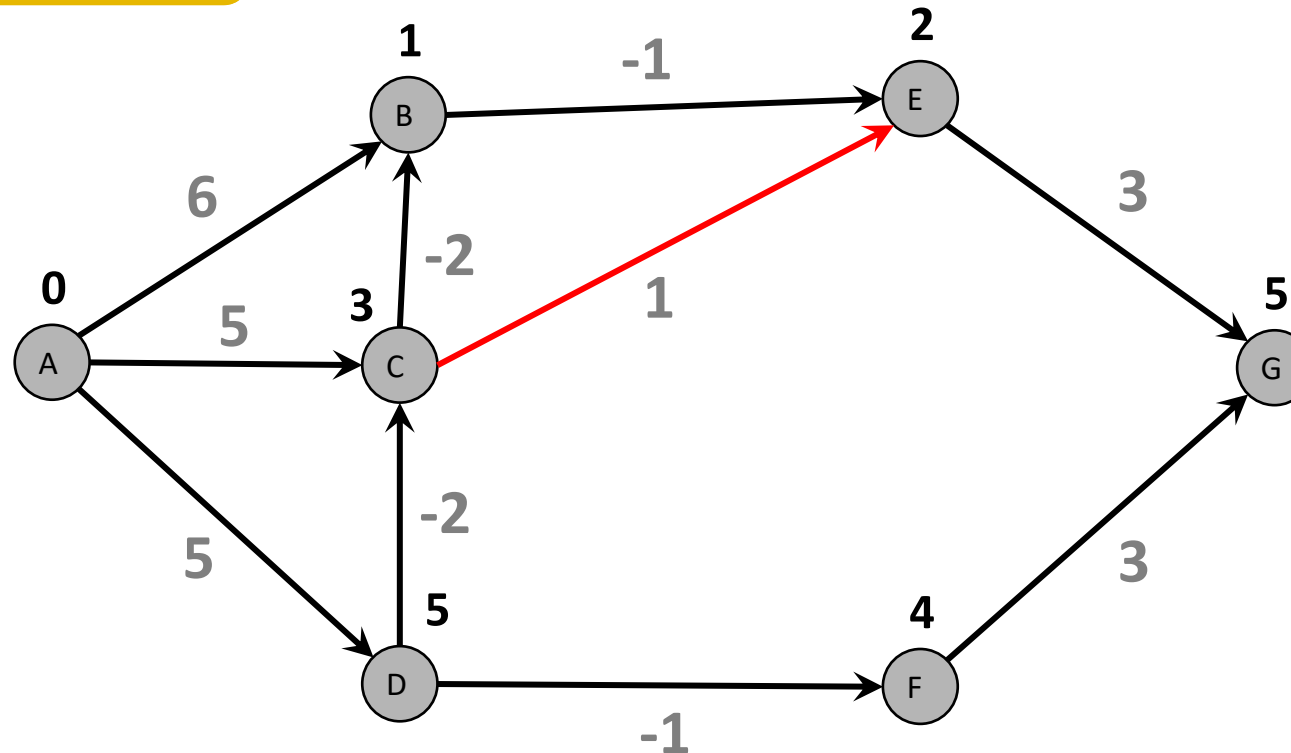


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Bellman-ford: Single Source Shortest Path

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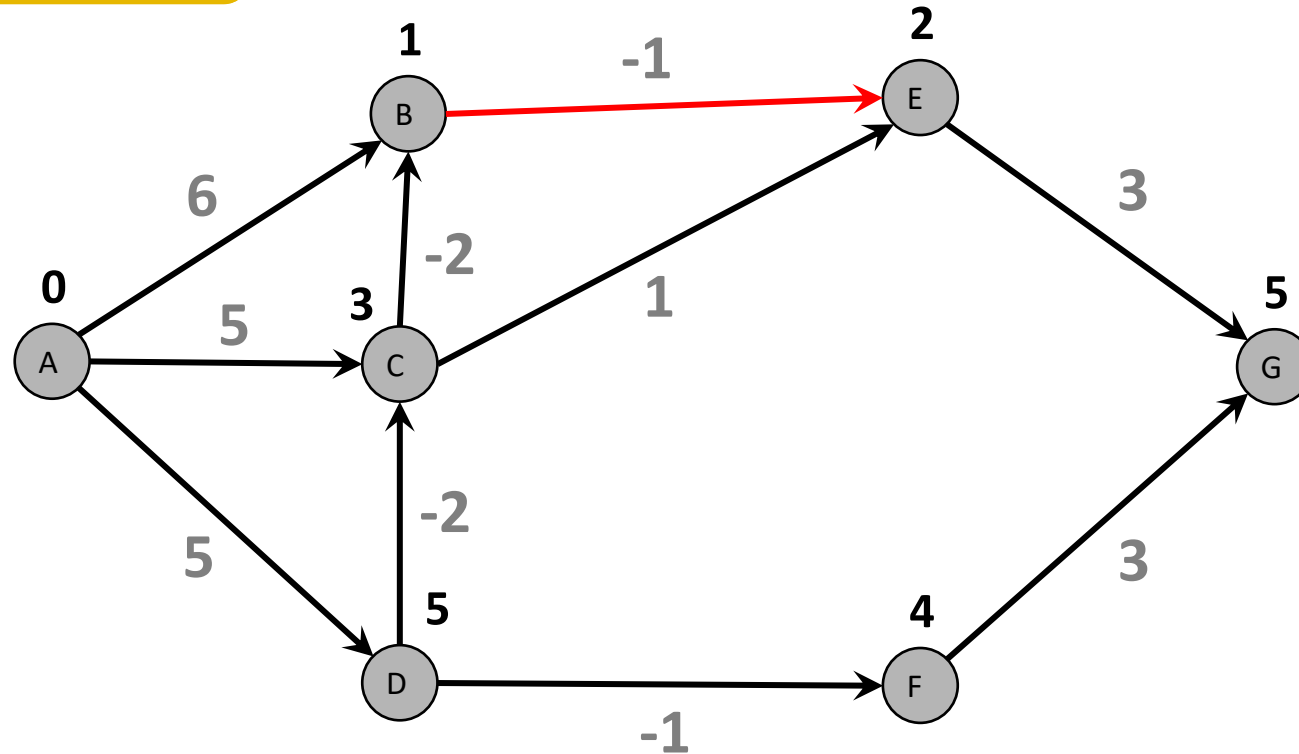


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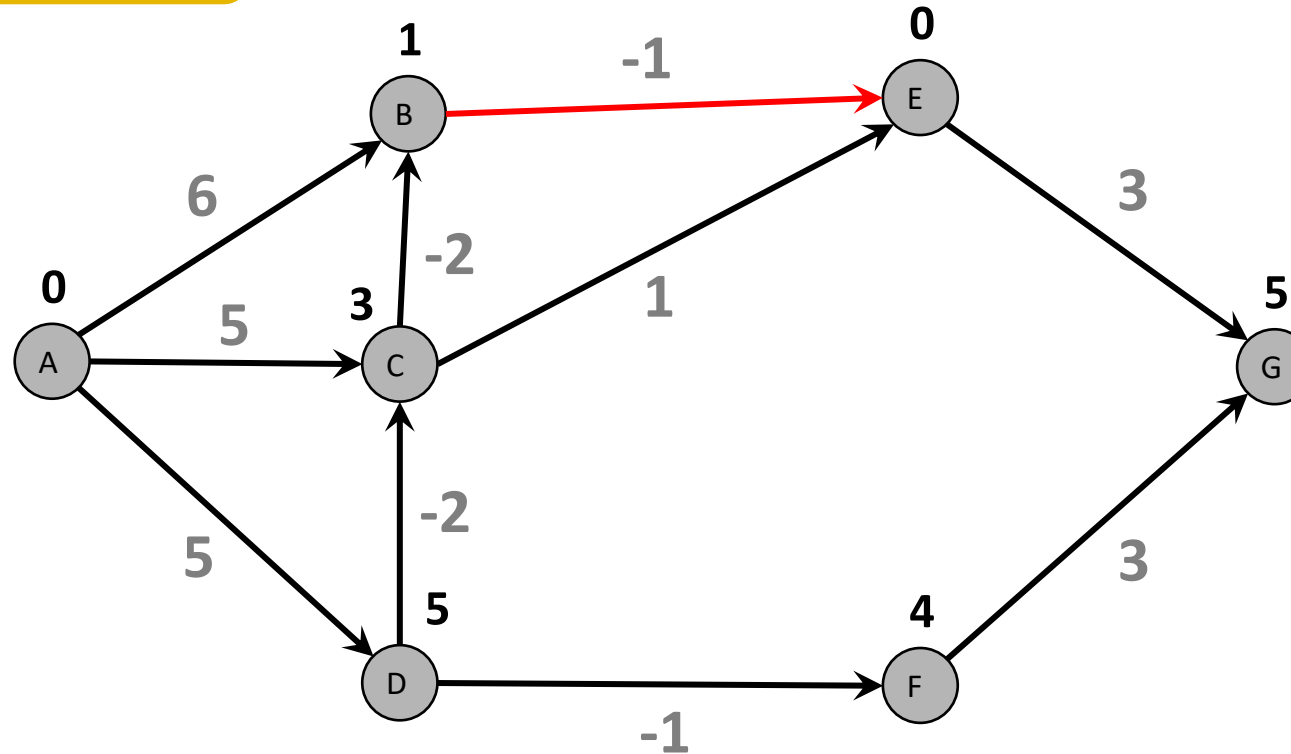


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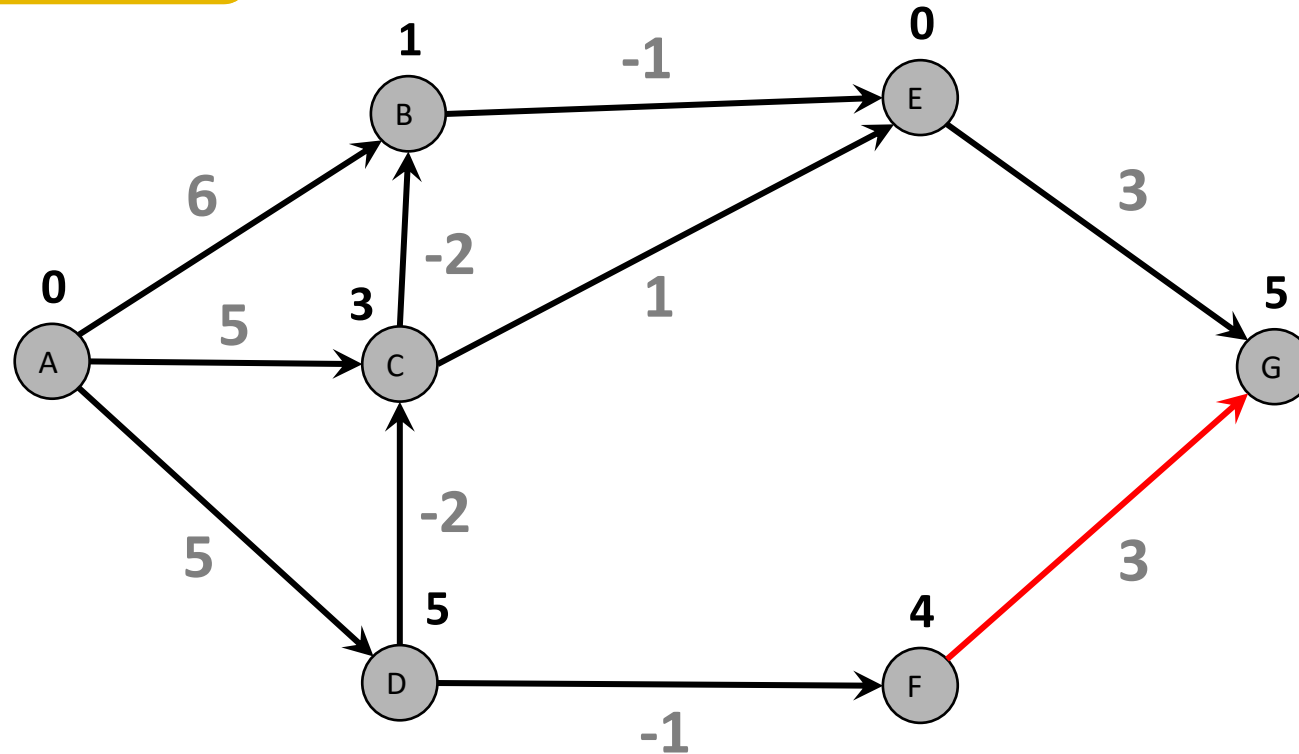


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2nd Iteration

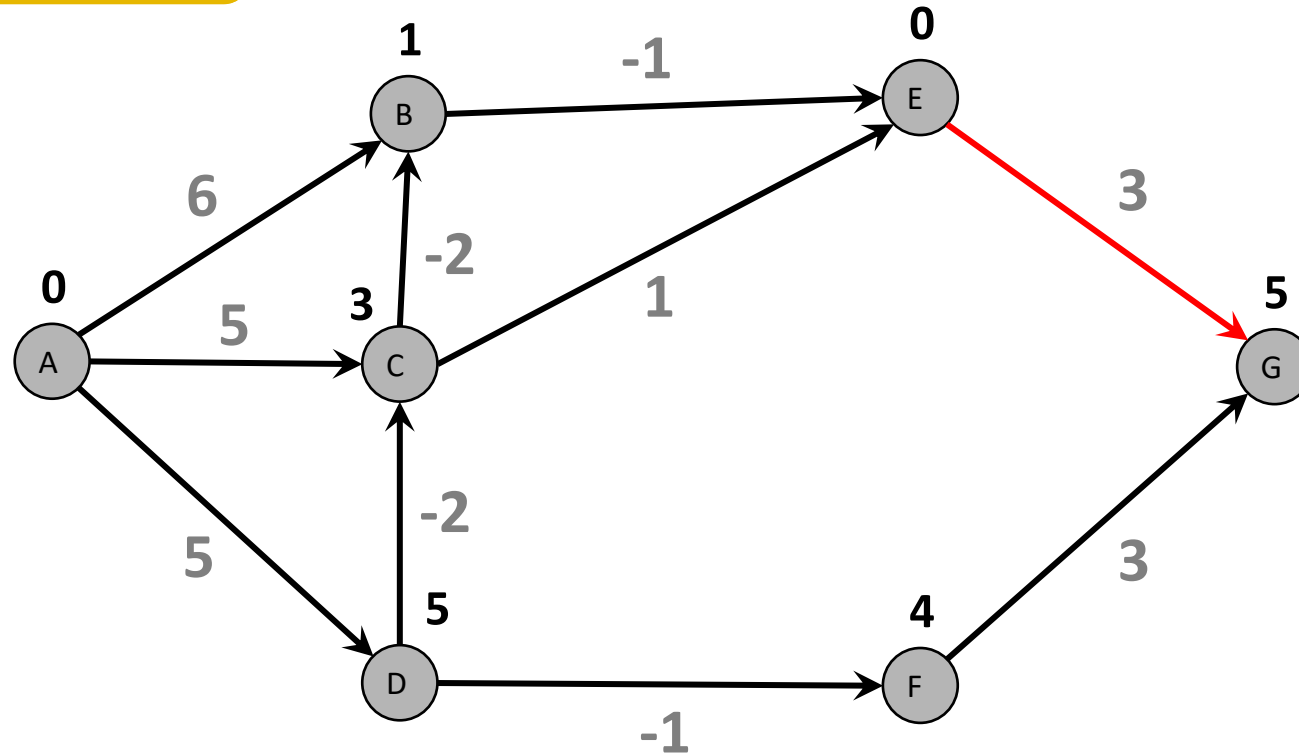


If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) (D,C) (D,F) (C,E) (B,E) **(F,G)** (E,G)

Bellman-ford: Single Source Shortest Path

2nd Iteration

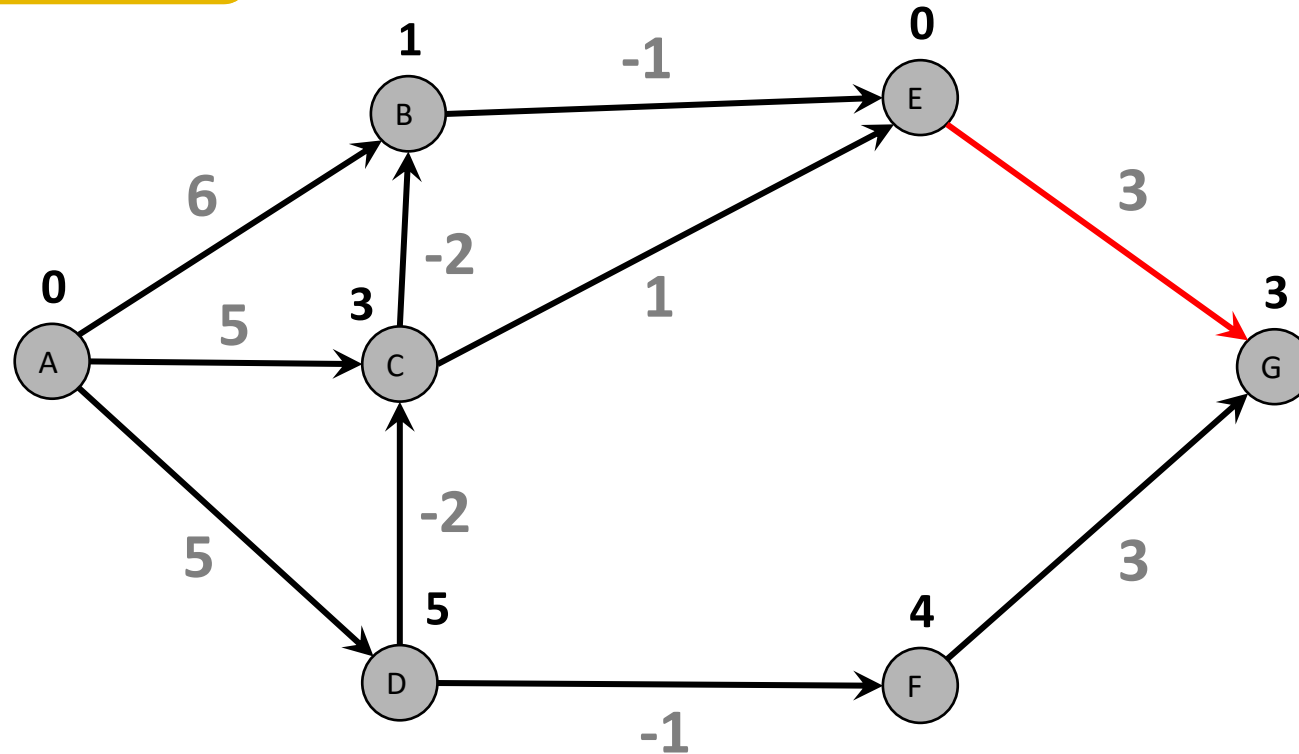


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) (D,C) (D,F) (C,E) (B,E) (F,G) **(E,G)**

Bellman-ford: Single Source Shortest Path

2nd Iteration

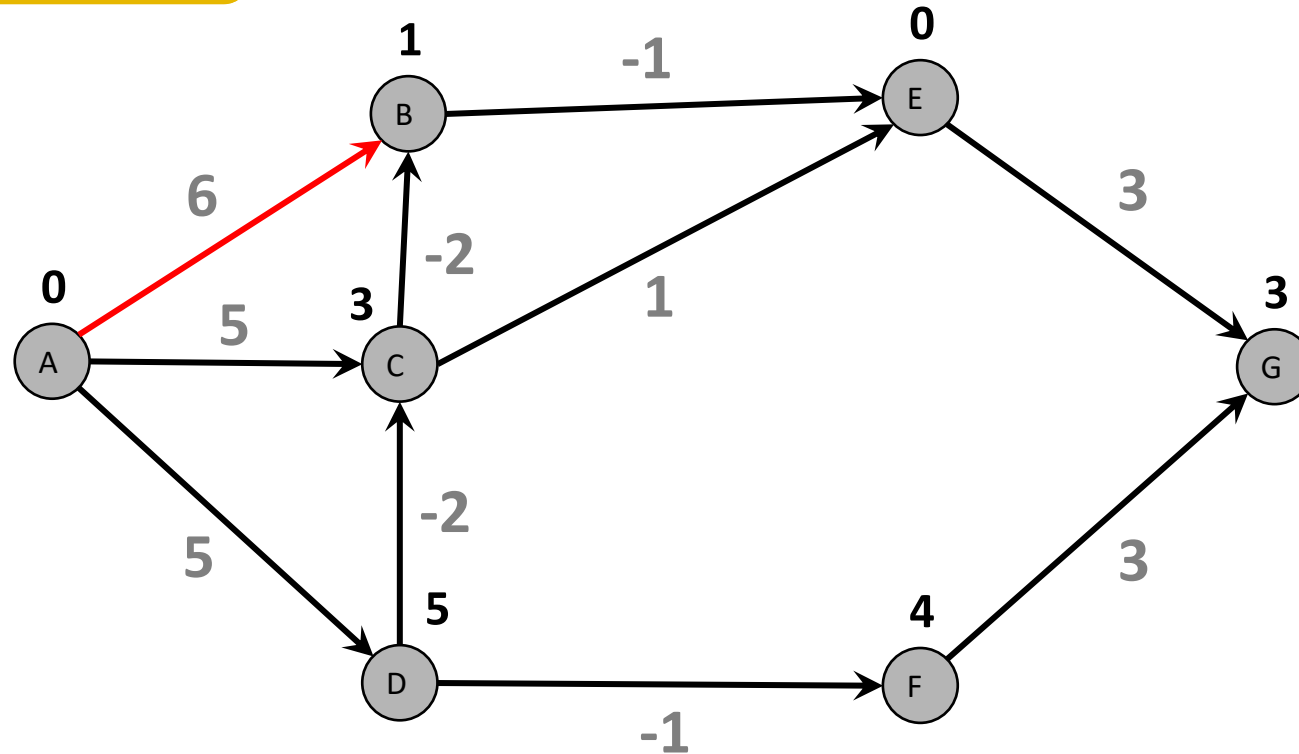


If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) (D,C) (D,F) (C,E) (B,E) (F,G) **(E,G)**

Bellman-ford: Single Source Shortest Path

3rd Iteration

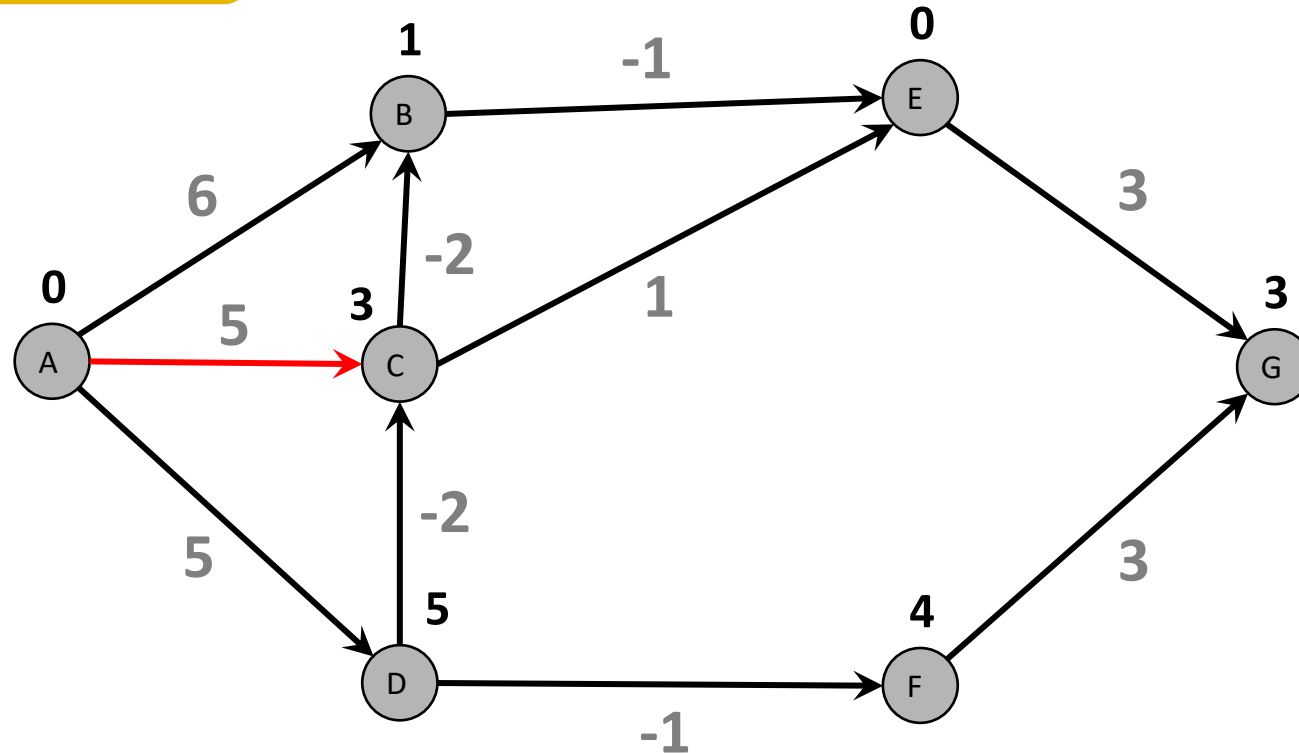


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) (D,C) (D,F) (C,E) (B,E) (F,G) (E,G)

Bellman-ford: Single Source Shortest Path

3rd Iteration

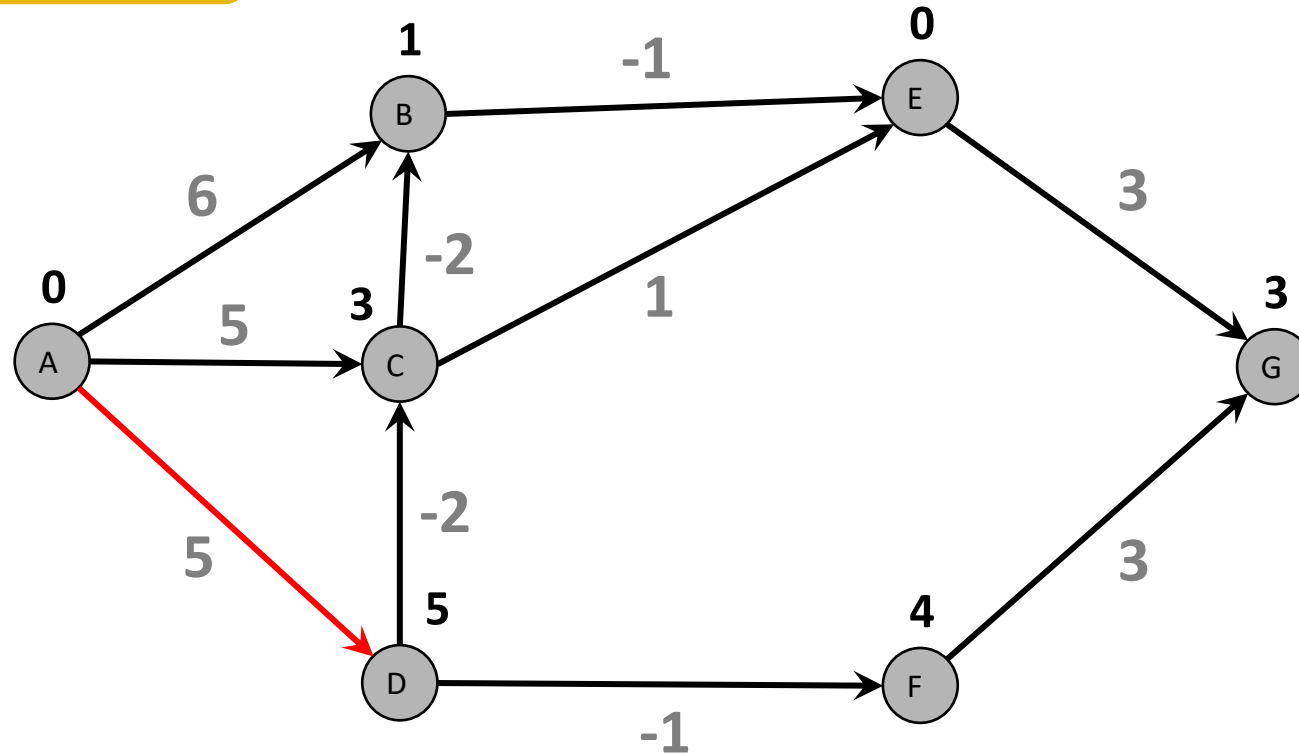


If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (**A,C**) (A,D) (C,B) (D,C) (D,F) (C,E) (B,E) (F,G) (E,G)

Bellman-ford: Single Source Shortest Path

3rd Iteration

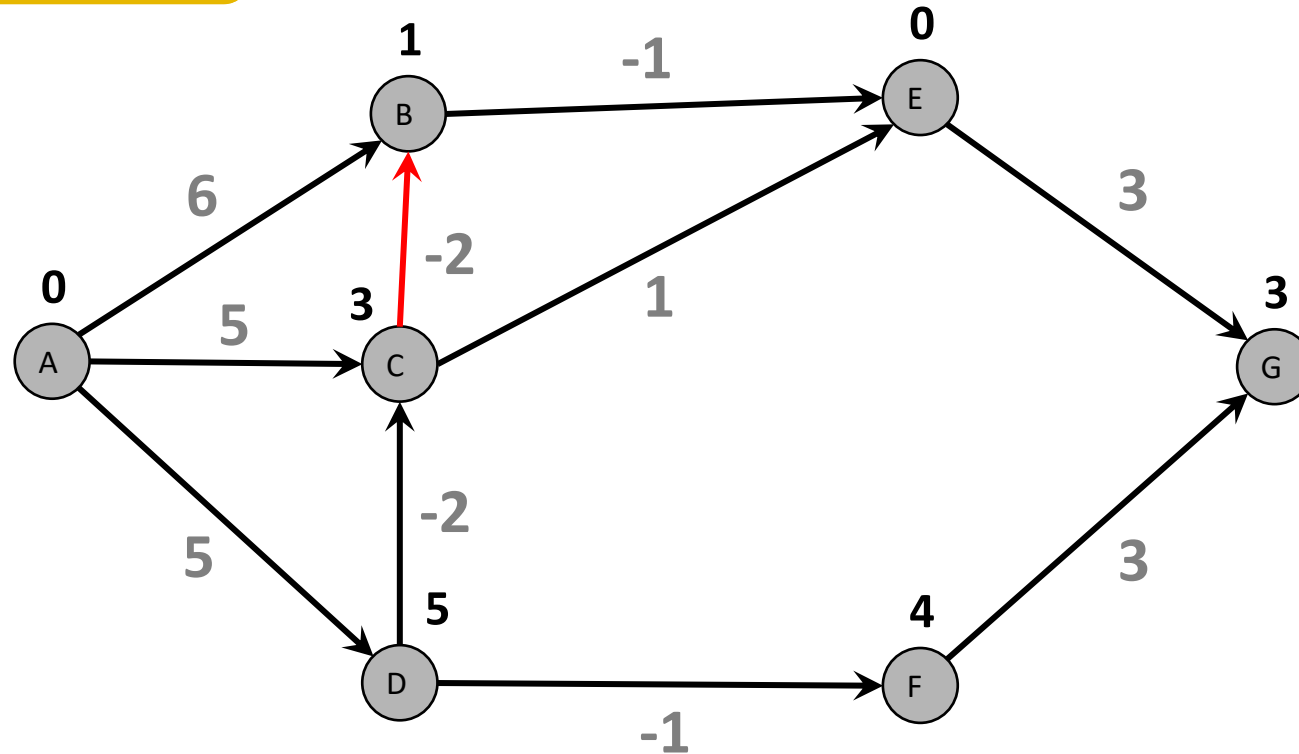


If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) **(A,D)** (C,B) (D,C) (D,F) (C,E) (B,E) (F,G) (E,G)

Bellman-ford: Single Source Shortest Path

3rd Iteration

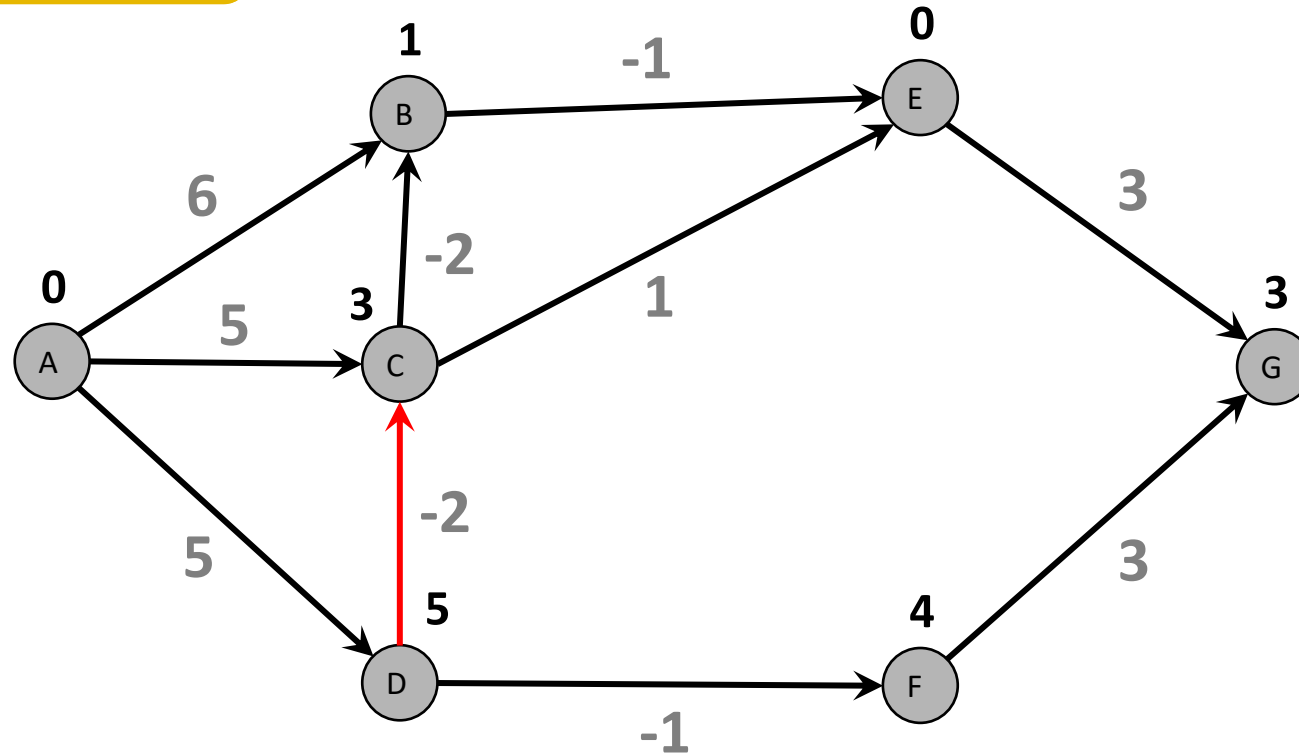


If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) **(C,B)** (D,C) (D,F) (C,E) (B,E) (F,G) (E,G)

Bellman-ford: Single Source Shortest Path

3rd Iteration

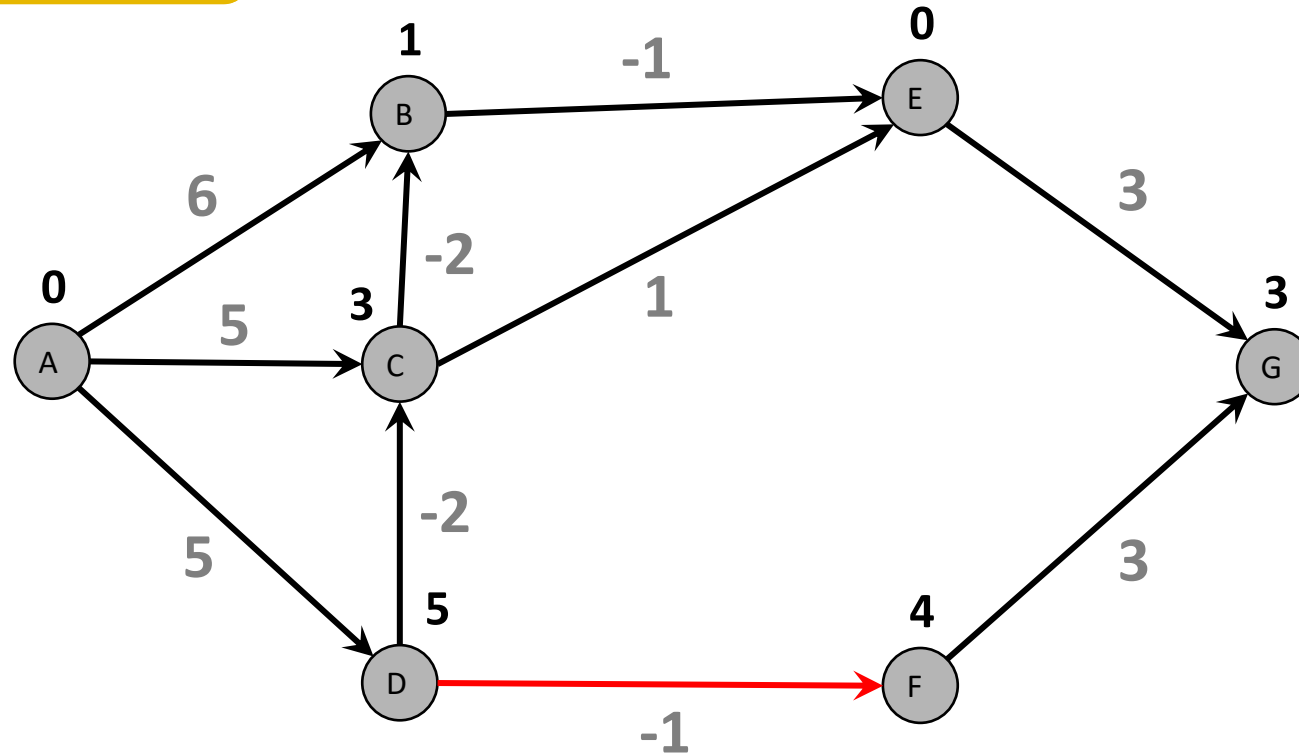


If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) **(D,C)** (D,F) (C,E) (B,E) (F,G) (E,G)

Bellman-ford: Single Source Shortest Path

3rd Iteration

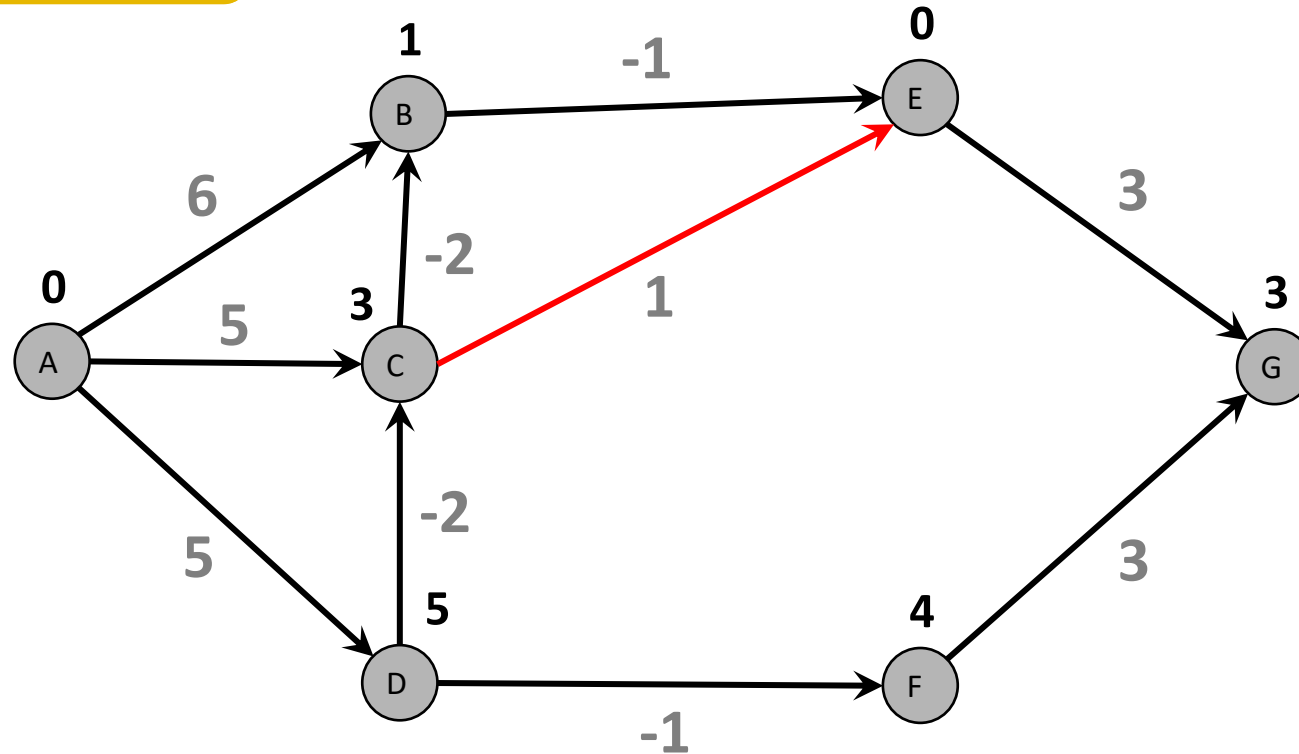


If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) (D,C) **(D,F)** (C,E) (B,E) (F,G) (E,G)

Bellman-ford: Single Source Shortest Path

3rd Iteration

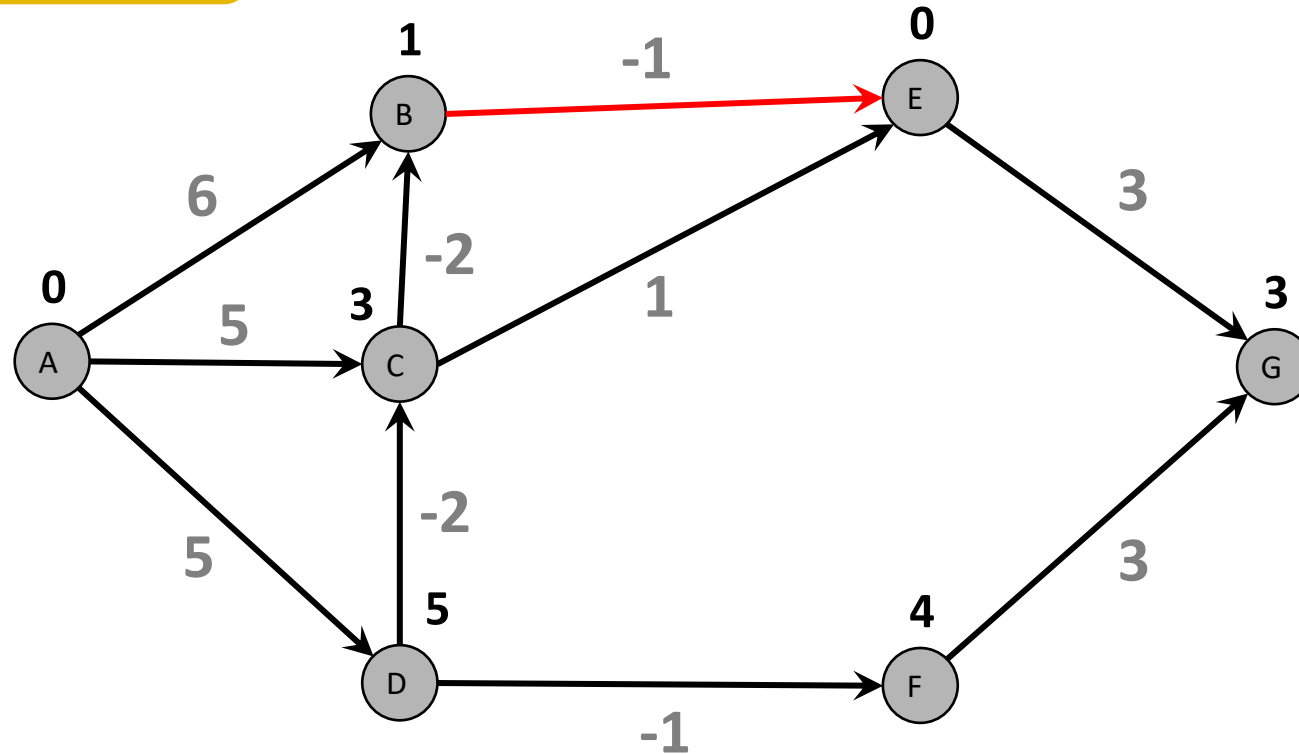


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) (D,C) (D,F) **(C,E)** (B,E) (F,G) (E,G)

Bellman-ford: Single Source Shortest Path

3rd Iteration

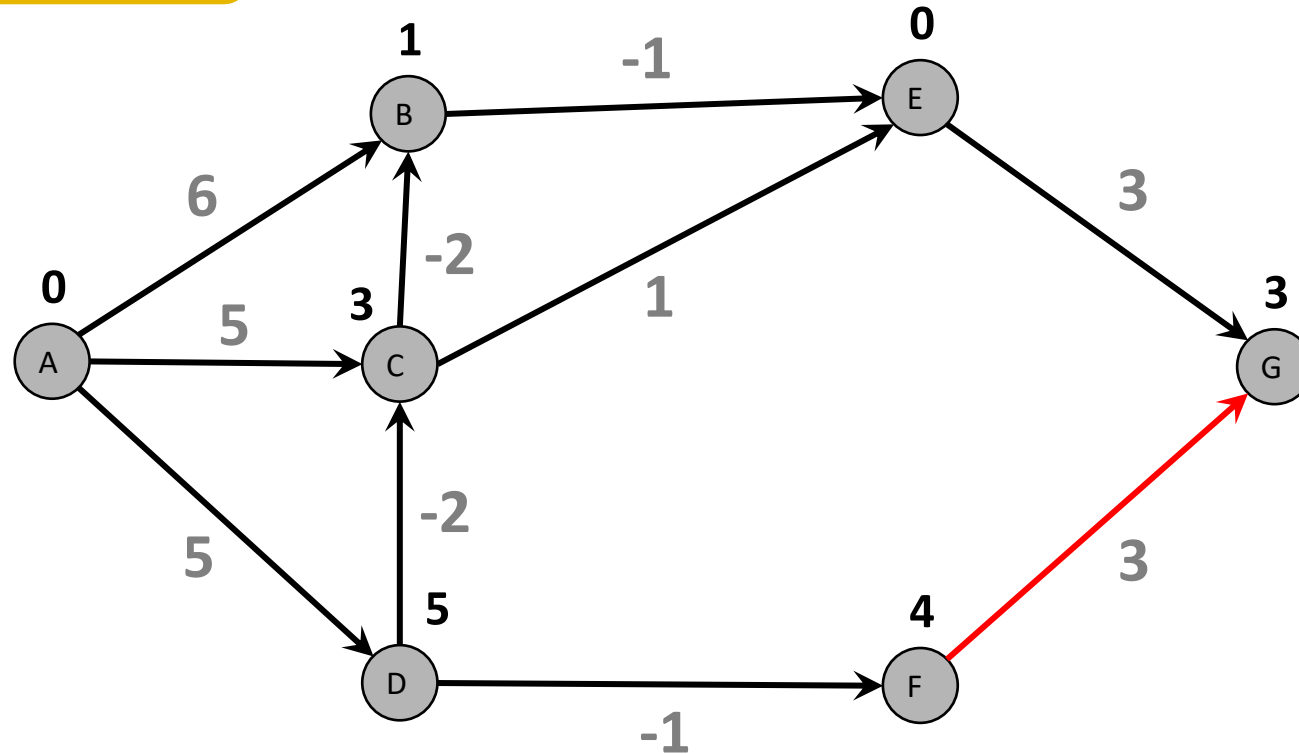


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) (D,C) (D,F) (C,E) **(B,E)** (F,G) (E,G)

Bellman-ford: Single Source Shortest Path

3rd Iteration

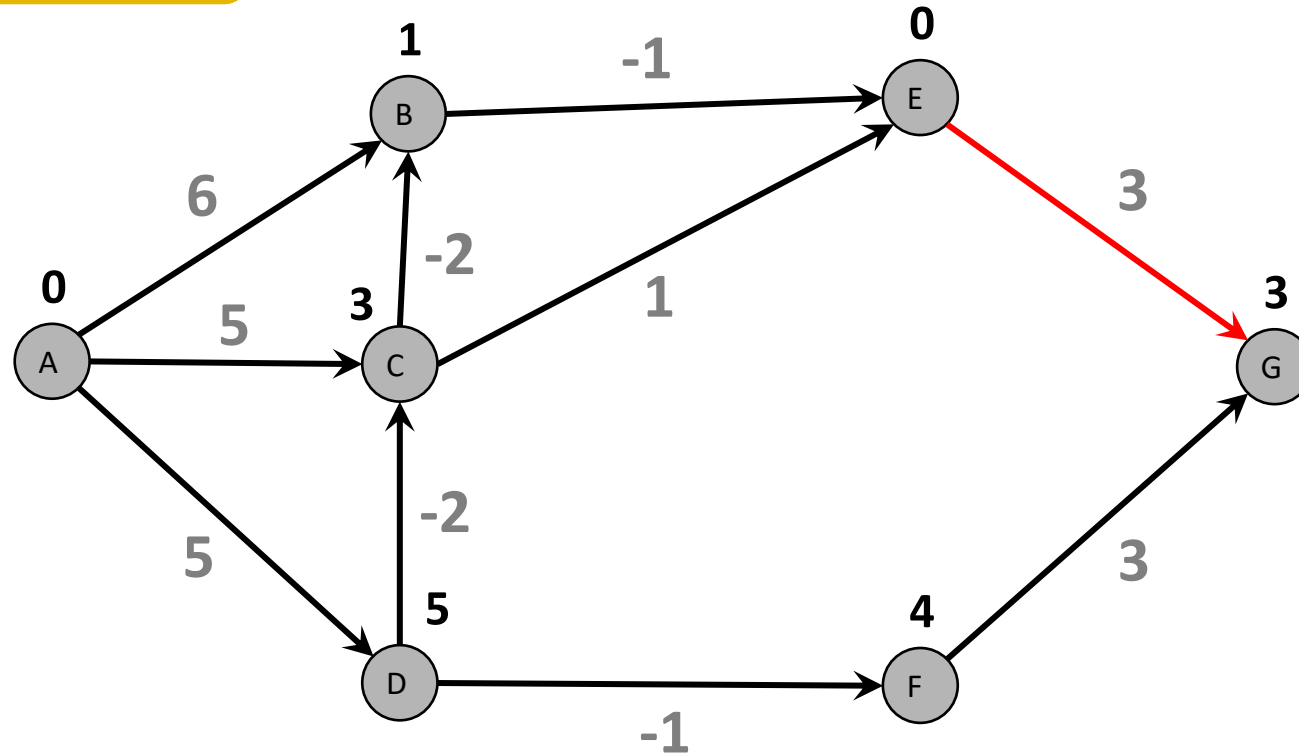


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (A,D) (C,B) (D,C) (D,F) (C,E) (B,E) **(F,G)** (E,G)

Bellman-ford: Single Source Shortest Path

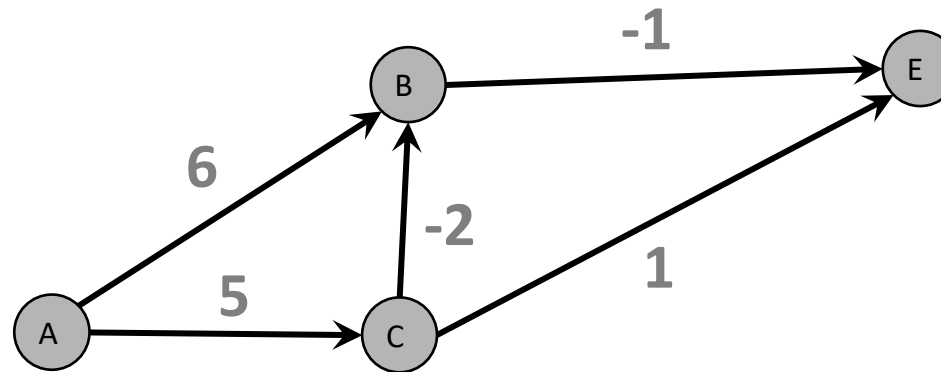
3rd Iteration



If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

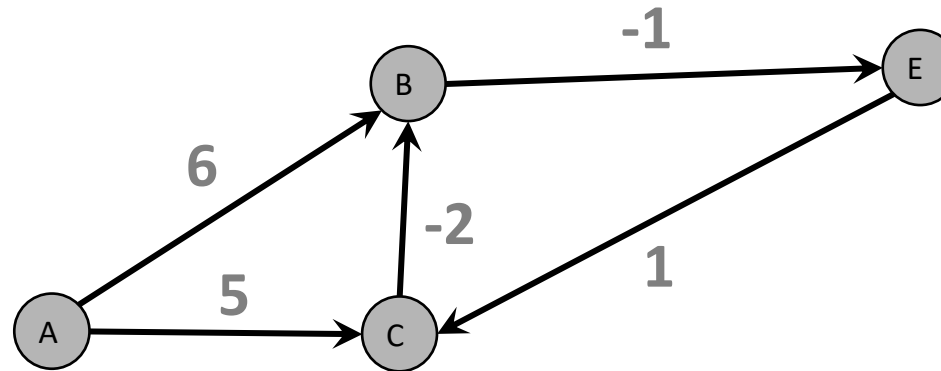
(A,B) (A,C) (A,D) (C,B) (D,C) (D,F) (C,E) (B,E) (F,G) **(E,G)**

Bellman-ford: Single Source Shortest Path



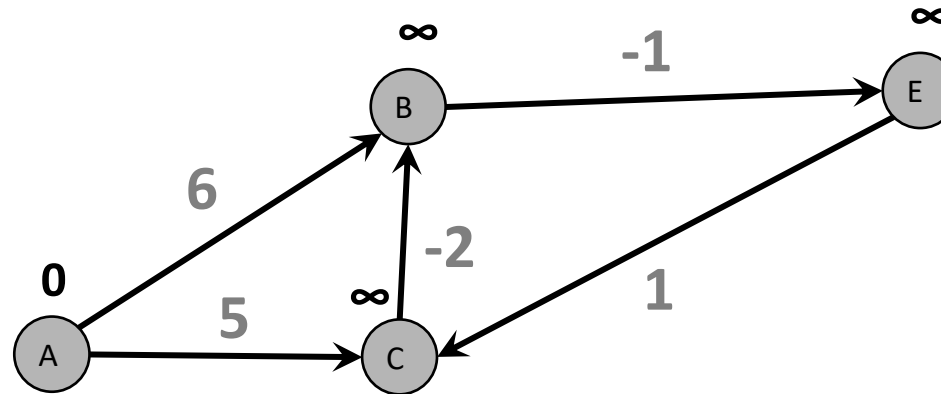
If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

Bellman-ford: Single Source Shortest Path



If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

Bellman-ford: Single Source Shortest Path

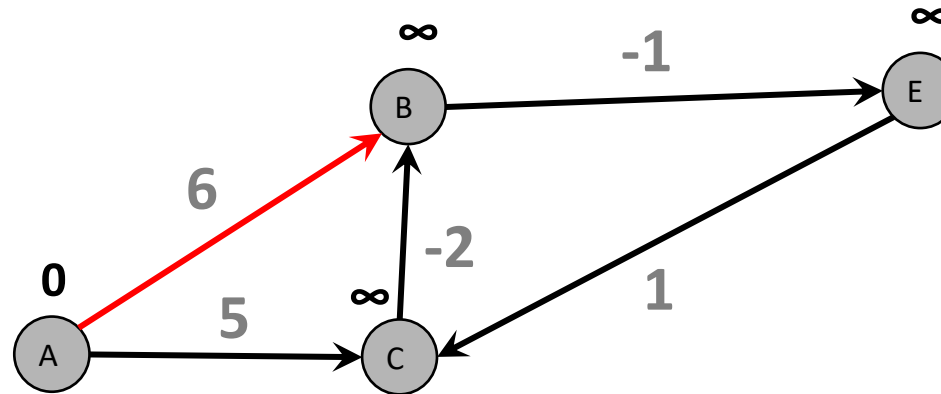


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

1st Iteration

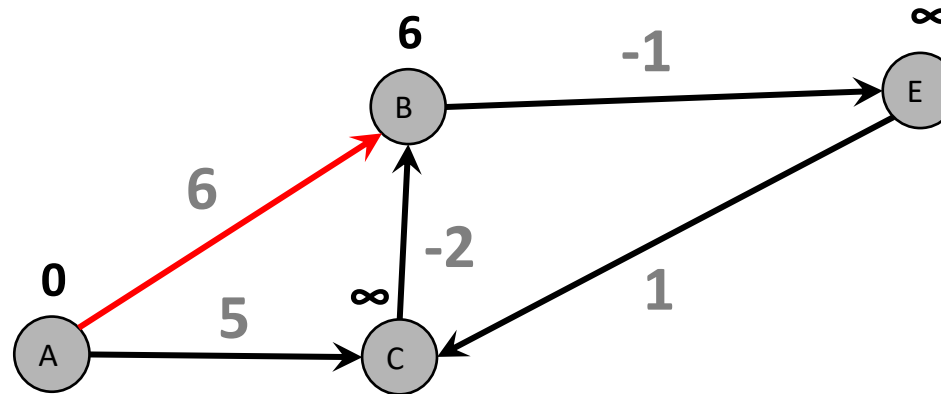


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

1st Iteration

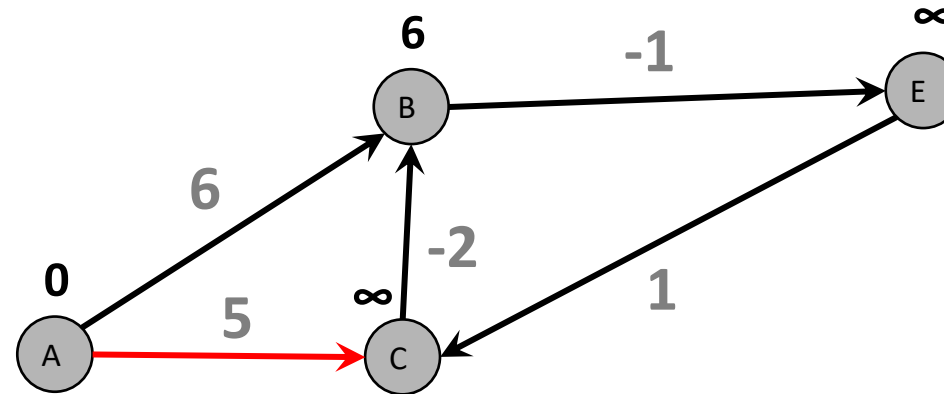


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

1st Iteration

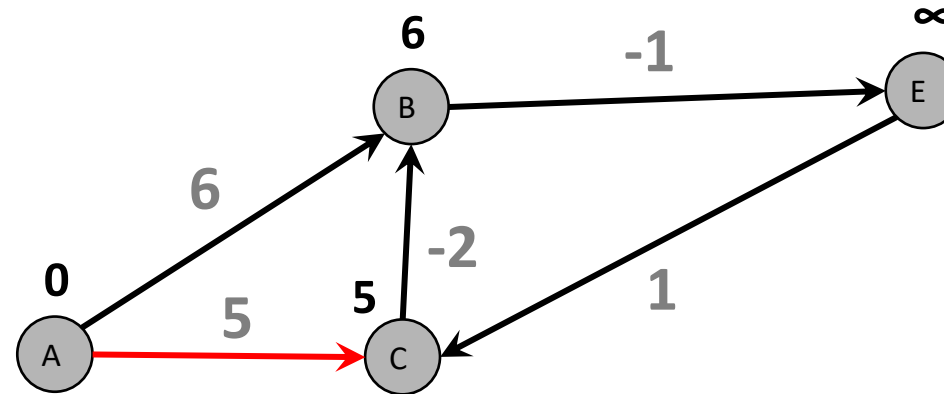


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) **(A,C)** (C,B) (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

1st Iteration

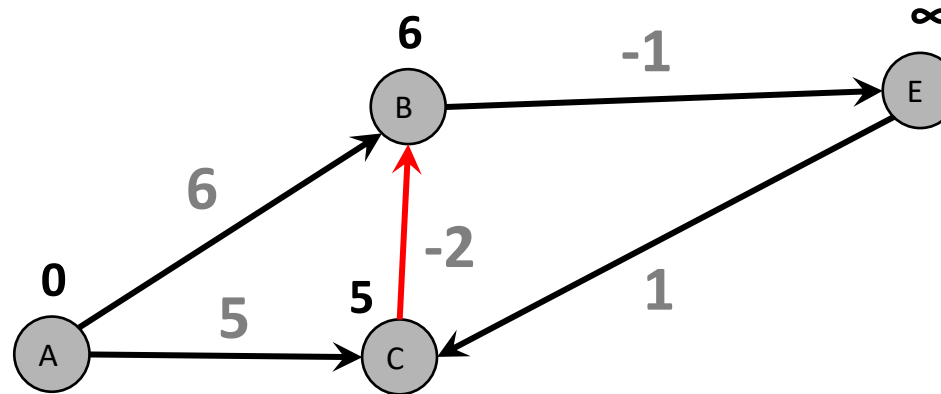


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) **(A,C)** (C,B) (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

1st Iteration

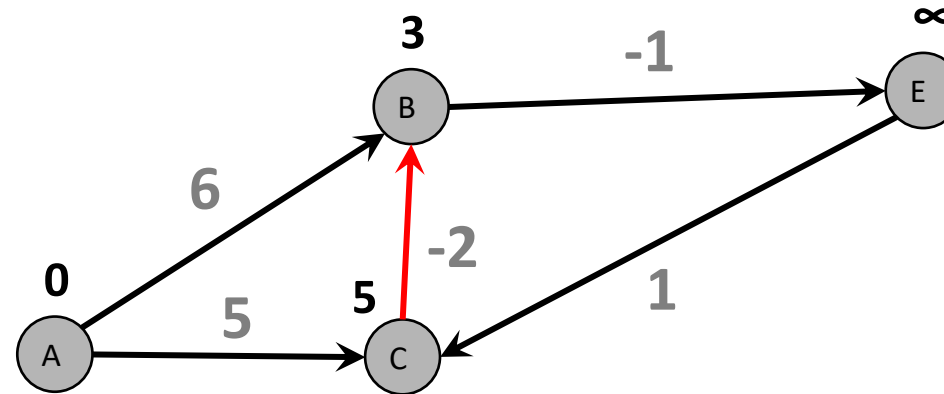


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) **(C,B)** (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

1st Iteration

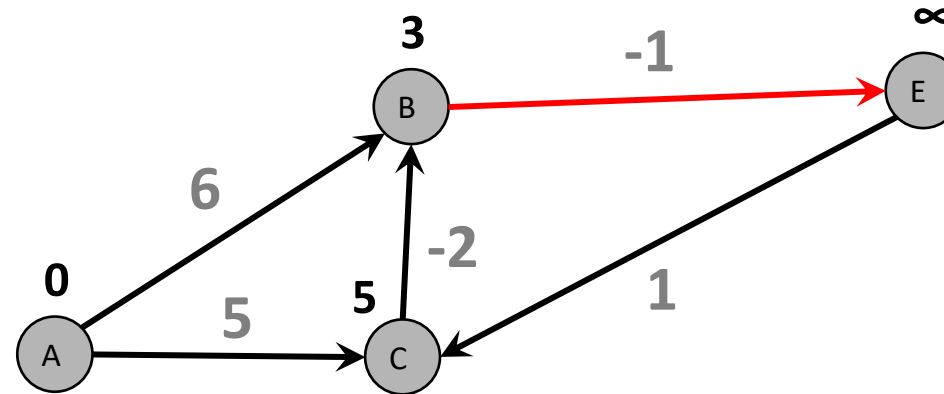


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) **(C,B)** (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

1st Iteration

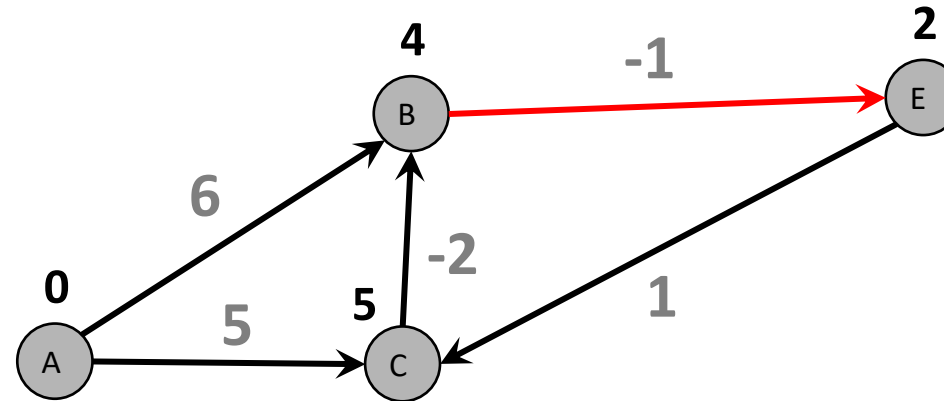


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) **(B,E)** (E,C)

Bellman-ford: Single Source Shortest Path

1st Iteration

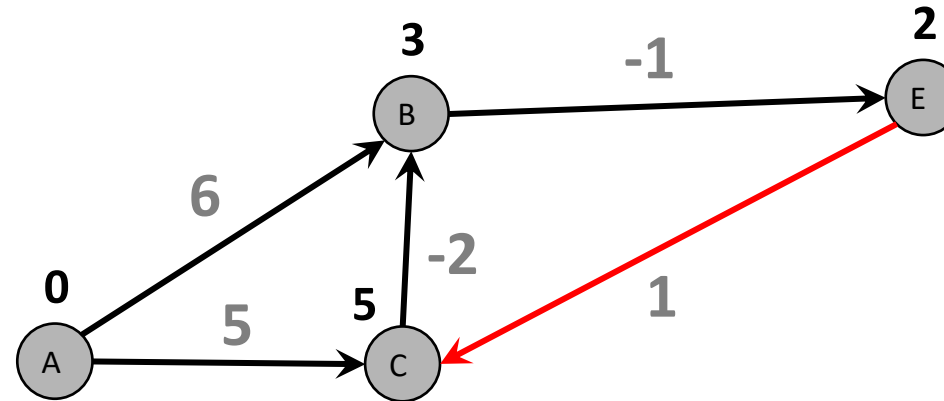


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) **(B,E)** (E,C)

Bellman-ford: Single Source Shortest Path

1st Iteration

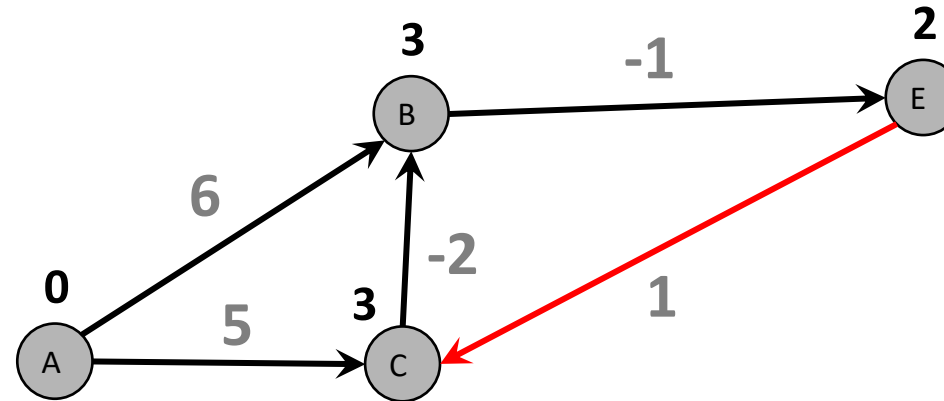


If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) **(E,C)**

Bellman-ford: Single Source Shortest Path

1st Iteration

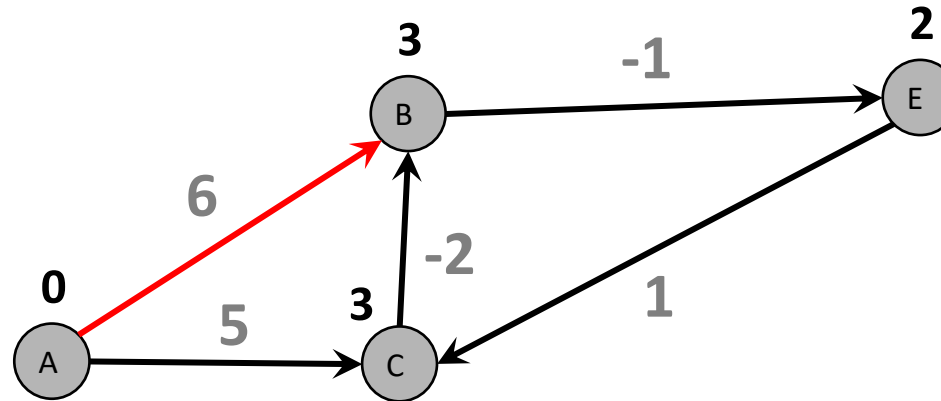


If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) **(E,C)**

Bellman-ford: Single Source Shortest Path

2nd Iteration

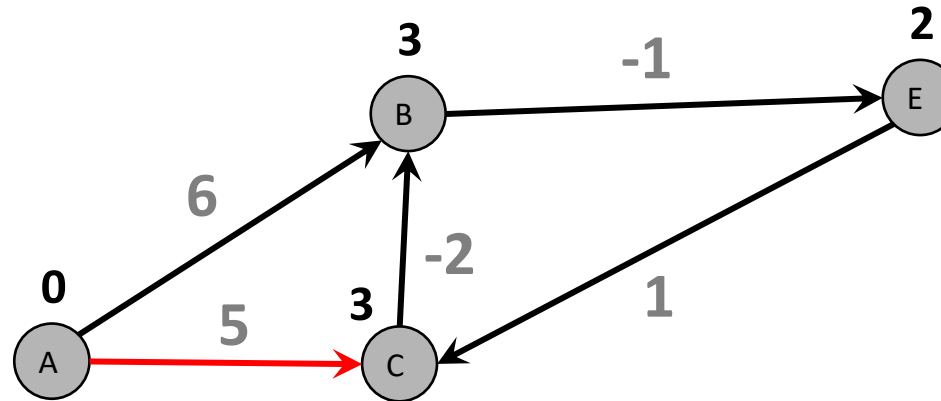


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

2nd Iteration

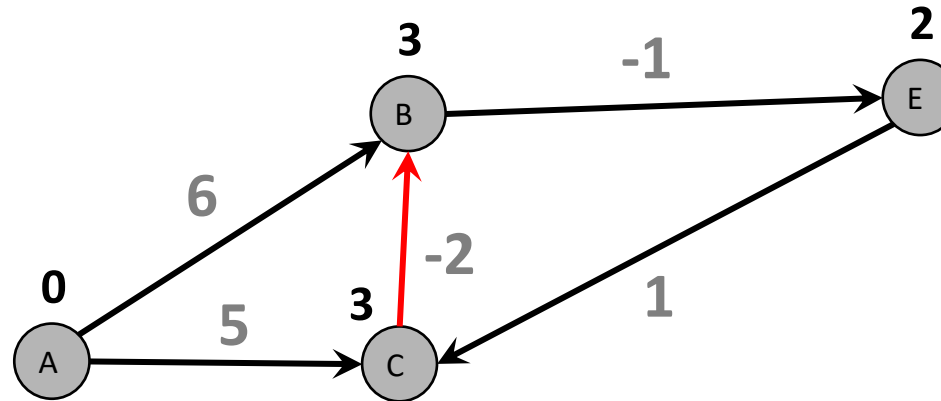


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) **(A,C)** (C,B) (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

2nd Iteration

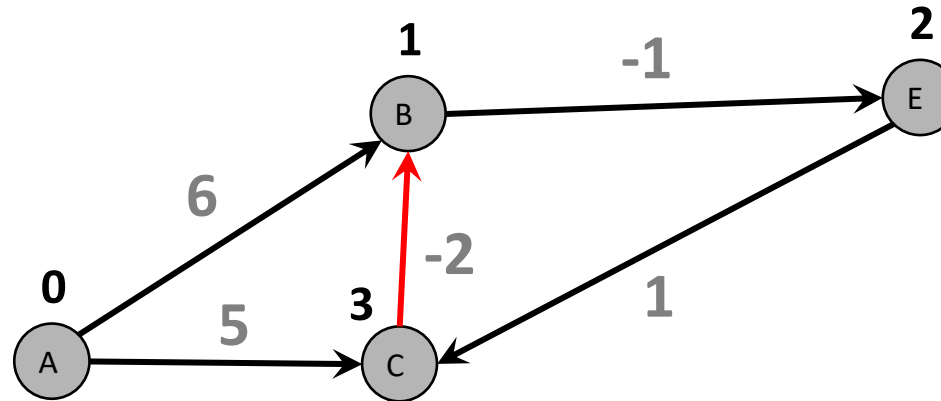


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) **(C,B)** (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

2nd Iteration

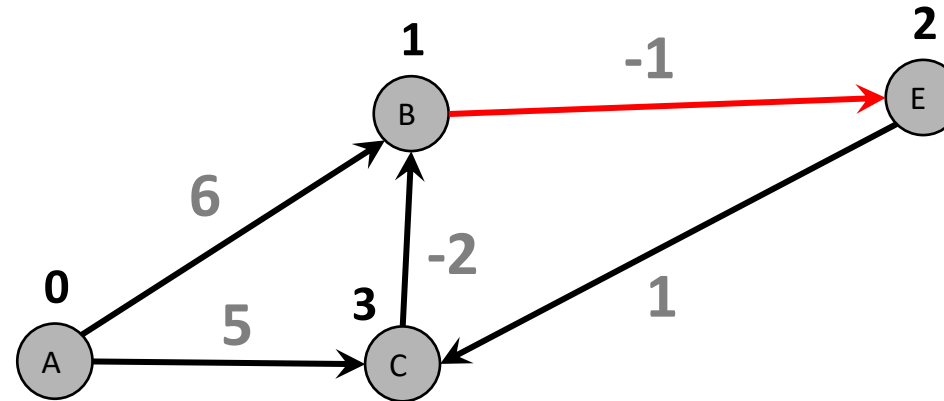


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) **(C,B)** (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

2nd Iteration

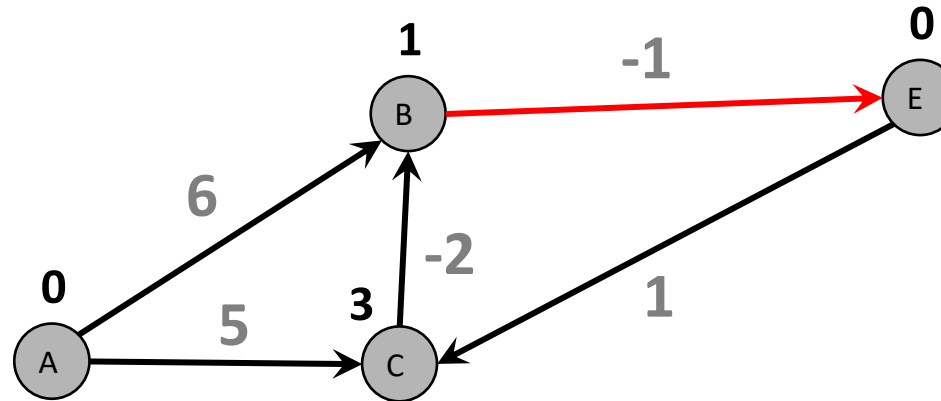


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) **(B,E)** (E,C)

Bellman-ford: Single Source Shortest Path

2nd Iteration

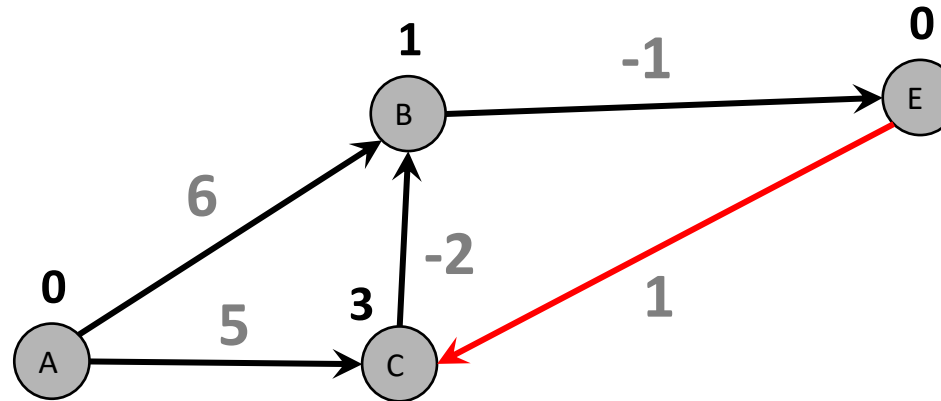


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) **(B,E)** (E,C)

Bellman-ford: Single Source Shortest Path

2nd Iteration

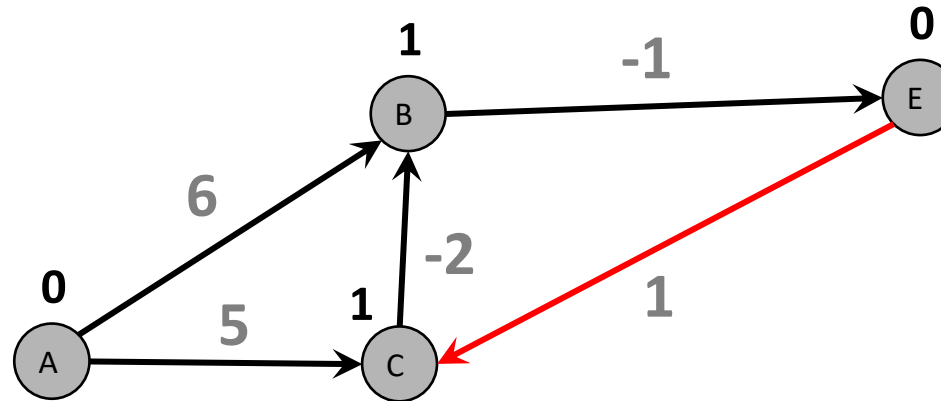


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) **(E,C)**

Bellman-ford: Single Source Shortest Path

2nd Iteration

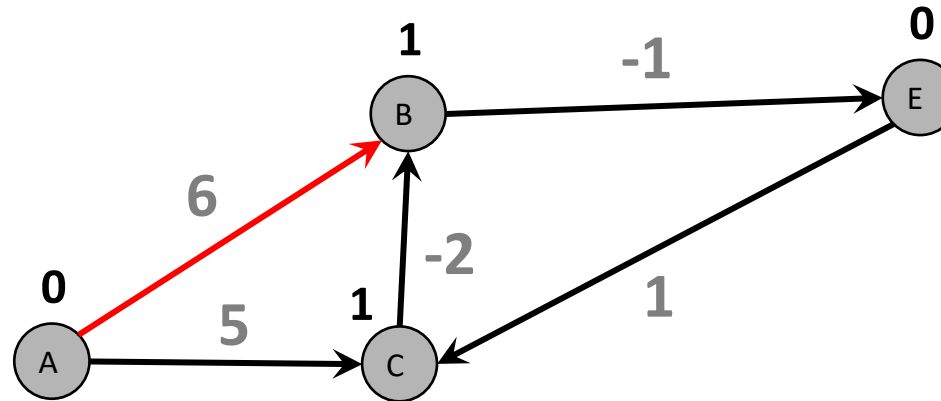


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) **(E,C)**

Bellman-ford: Single Source Shortest Path

3rd Iteration

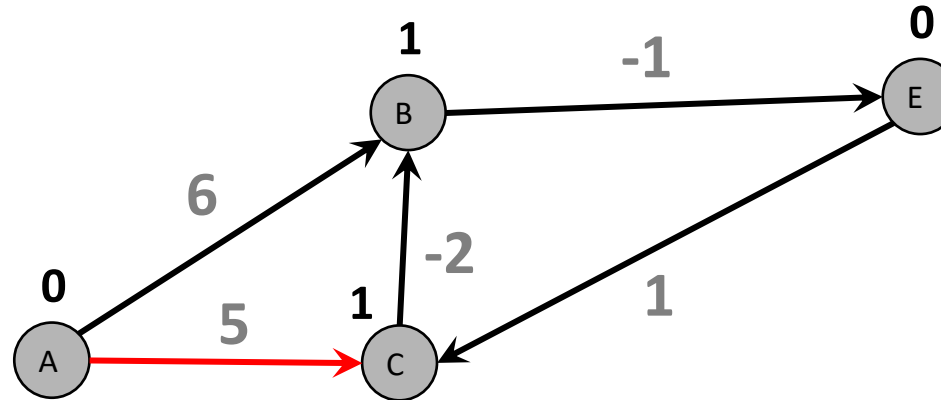


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

3rd Iteration

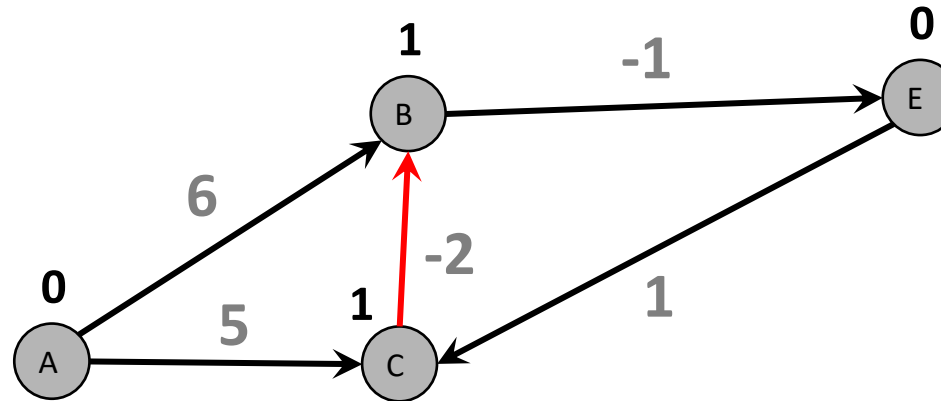


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) **(A,C)** (C,B) (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

3rd Iteration

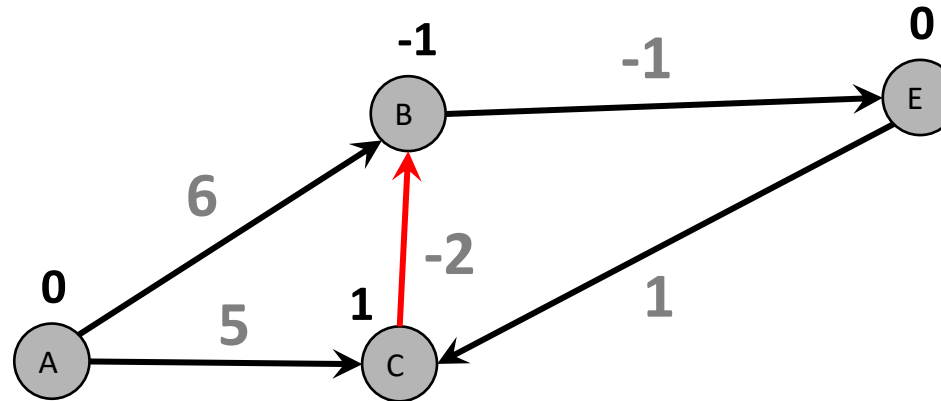


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) **(C,B)** (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

3rd Iteration

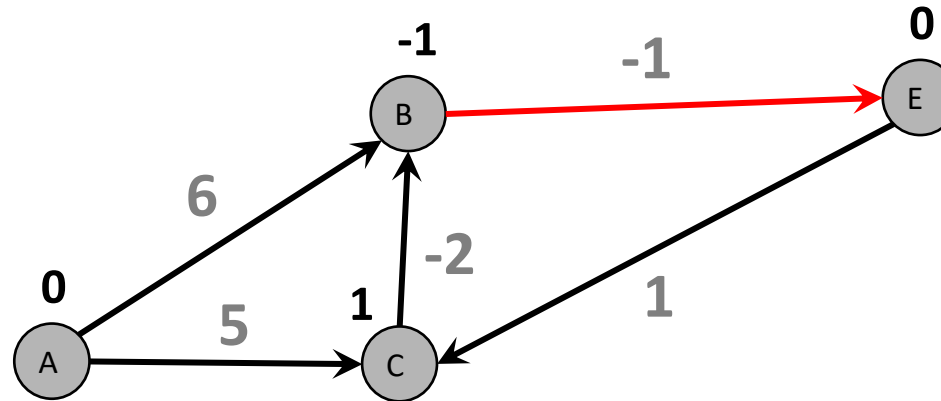


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) **(C,B)** (B,E) (E,C)

Bellman-ford: Single Source Shortest Path

3rd Iteration

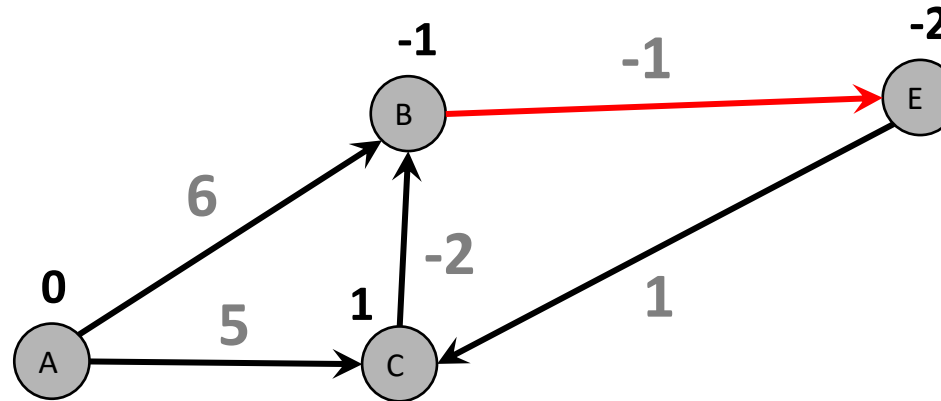


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) **(B,E)** (E,C)

Bellman-ford: Single Source Shortest Path

3rd Iteration

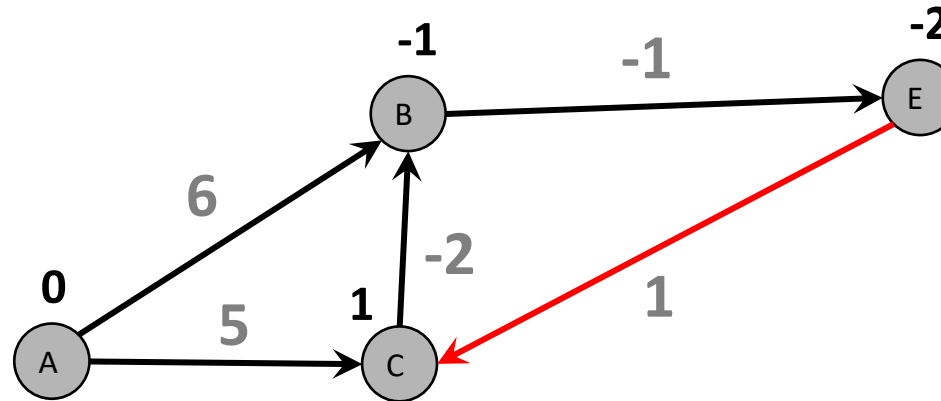


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) **(B,E)** (E,C)

Bellman-ford: Single Source Shortest Path

3rd Iteration

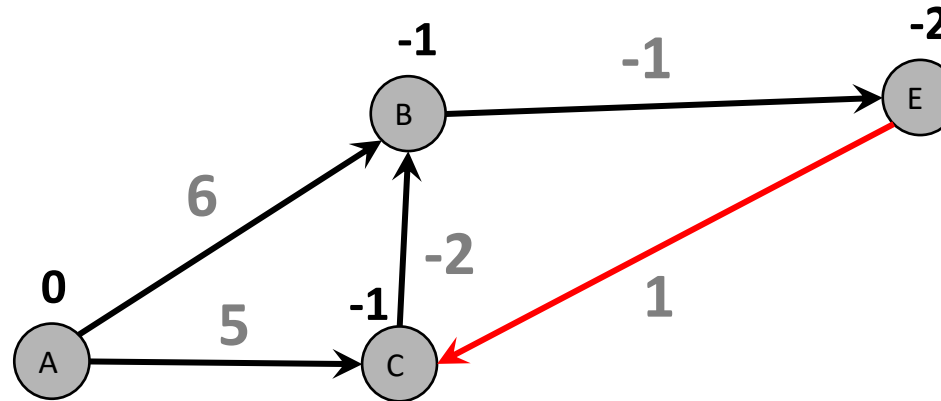


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) **(E,C)**

Bellman-ford: Single Source Shortest Path

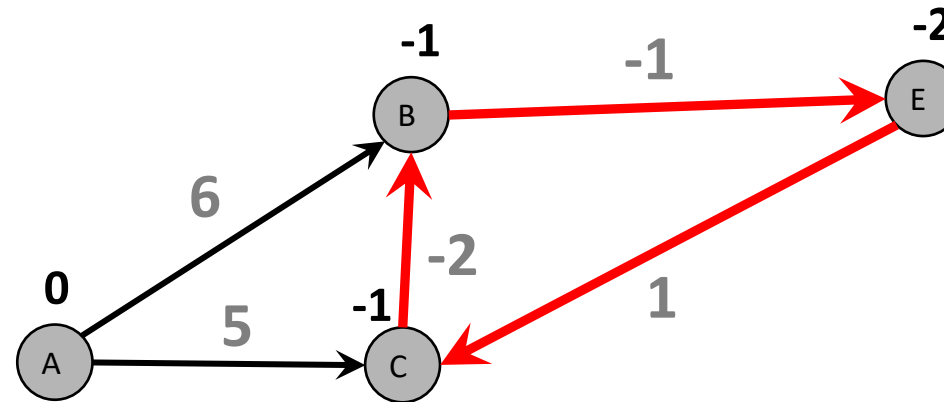
3rd Iteration



If ($d[u] + e(u,v) < d[v]$)
 $d[v] = d[u] + e(u,v)$

(A,B) (A,C) (C,B) (B,E) **(E,C)**

Bellman-ford: Single Source Shortest Path

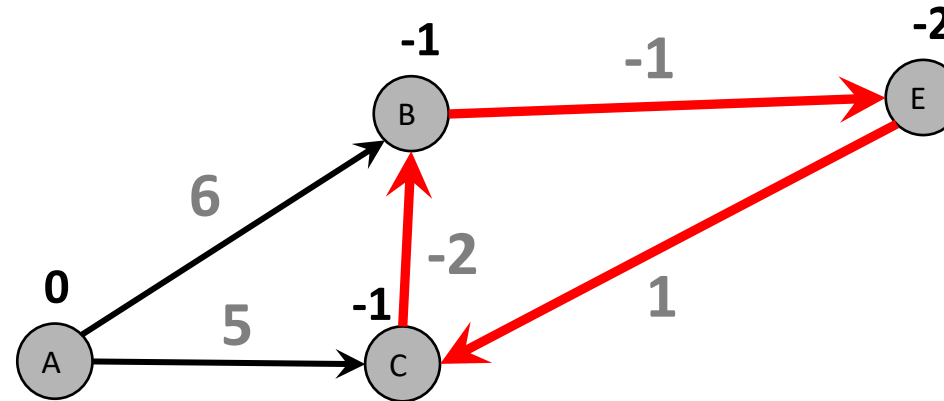


Negative Cycle

If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

Bellman-ford: Single Source Shortest Path

$$(-2) + (-1) + 1 = -2$$

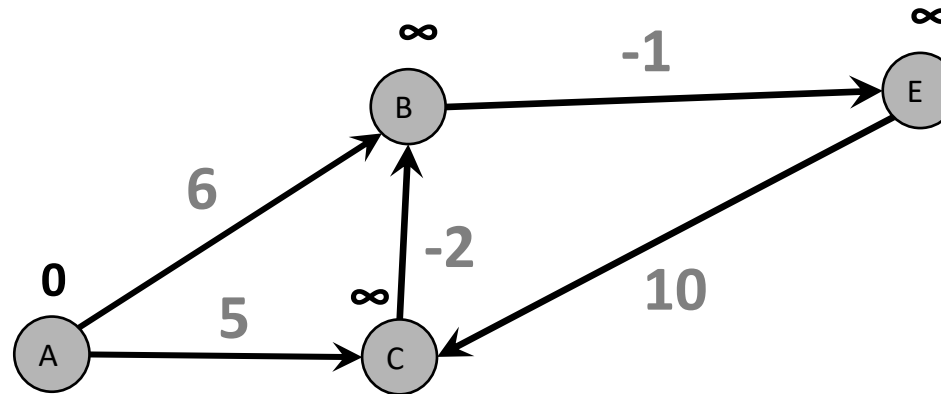


If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

Negative Cycle

Bellman-ford: Single Source Shortest Path

$$(-2) + (-1) + 10 = 7$$



If $(d[u] + e(u,v) < d[v])$
 $d[v] = d[u] + e(u,v)$

Thanks a lot



If you are taking a Nap, wake up.....***Lecture OVER***