Statistics Assignment

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

b.) Calculate the required probability.

Answer:

n = 10 i.e. 10 drugs

QA checks on the previous batches found 4 times the drug produces a satisfactory result

Here based on QA results the Hypothesis test could be to state the number of times the drug is satisfactory, so based on this the null hypothesis $H_0 = 4$ and alternate Hypothesis $H_1 \neq 4$

Note: For this question we will be not doing Hypothesis test.

For question a i.e. the three conditions for a scenario out of 10 sample drugs, at most 3 drugs are not able to do a satisfactory job would mean that less than or equal to 3 are not able to do satisfactory job.

Let's define X = number of drugs not able to do a Satisfactory job

1. Condition 1 that the distribution follows would be P(X=3) which is 3 drugs are not able to do a Satisfactory job
2. Condition 2 that the distribution follows would be P(X=2) which is 2 drugs are not able to do a Satisfactory job

3. Condition 3 is that the distribution follows would be P(X=1) which is 1 drug is not able to do a Satisfactory job

b. The probability for this scenario of X<=3 i.e. at most drugs not able to do a satisfactory job can be calculated as

Applying Additional law, the Probability can be calculated as below

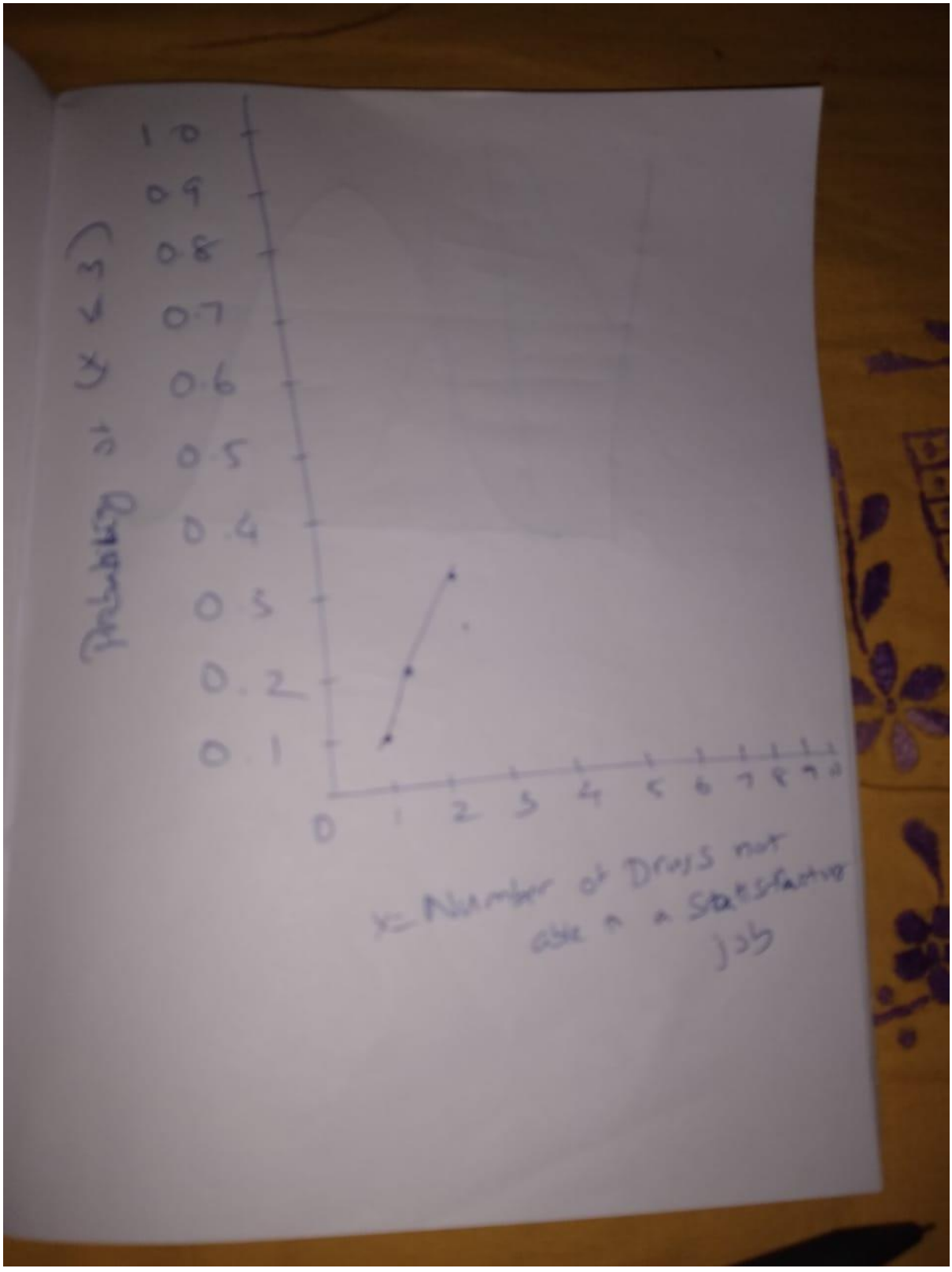
Probability (X<=3) = P(X = 1) + P(X = 2) + P(X = 3)

= (1 / 10) + (2 /10) + (3/10)

= 0.1 + 0.2 + 0.3

= 0.6

That mean 60% Probability of Drugs are not able to do a Satisfactory job….Considering the QA results have shown that 4 drugs likely are able to do satisfactory job 4/10 i.e. 0.4 or 40% are able to do Satisfactory job. This shows clearly the results of the sample size of 10 drugs i.e. 0.6 + 0.4 = 1 and 60% + 40% = 100% and proves the findings theoretically.



The plot above shows x-axis as “Number of Drugs not able to a satisfactory job” and y-axis as the Probability of X<=3 where X is “Number of Drugs not able to a satisfactory job”. This shows it as a “Linear Distribution”

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.)Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

b.)Find the required range.

Step 1 :

Understand the given inputs for this question

n = 100 i.e. Sample size of 100 drugs

The mean time of effect $\bar{x}$ = 207

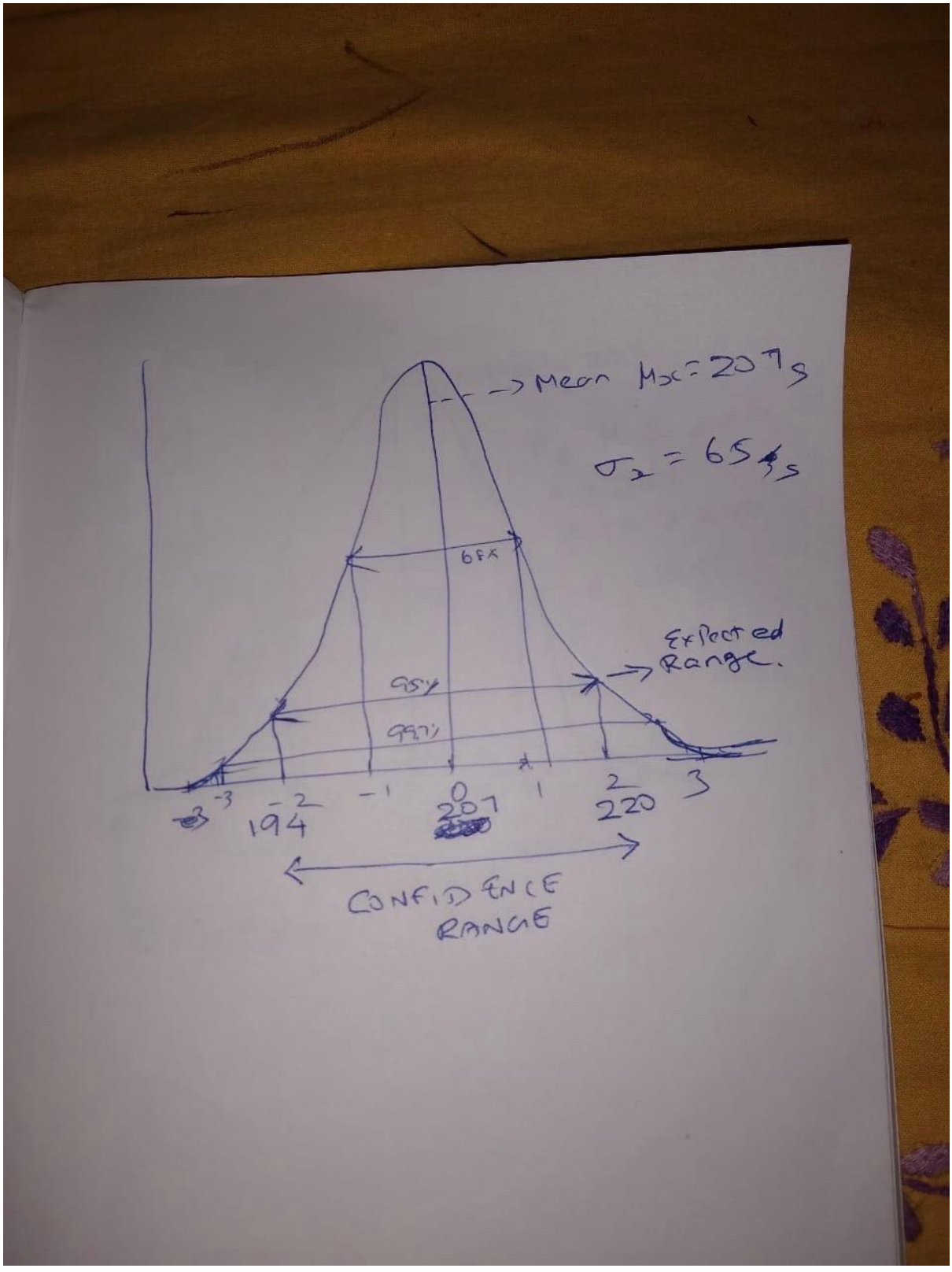
Standard deviation σ_x = 65

Confidence level = 95%

Since n > 30, this would be a normal distribution…

Let's plot this normal distribution…see below

In the normal distribution , the distribution of time effect should be at 95% i.e. between $\mu - 2\sigma$ and $\mu + 2\sigma$


Mean Mx = 207
σx = 6.5
6FX
Expected Range.
95%
99.7%
-3
-2
-1
0
1
2
3
194
207
220
CONFIDENCE RANGE

Step 2:

Let's now with the given Confidence level as 95%, we will find the z value for the Confidence interval. The z-value for 95% is 1.960

Step 3:

Need to find the Standard Error S.E = Standard Deviation / √n

$$\sigma_x = \sigma \div \sqrt{n}$$

$$= 65 / \sqrt{100}$$

$$= 65 / 10$$

$$= 6.5$$

Step 4:

The formula of Confidence Interval = Mean ± Z * S.E

$$= 207 \pm (1.960 * 6.5)$$

$$= 207 \pm 12.74$$

12.74 is also called as the Margin of Error

i.e. adding 12.74 to Mean gives 219.74 seconds rounding off to 220 seconds
and subtracting 12.74 gives 194.26 seconds rounding off to 194 seconds

Confidence range lies between 194 seconds to 220 seconds

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current hypothesis test conditions (sample size, mean, and standard deviation), the value of α and β come out to 0.05 and 0.45 respectively.

Now, a different sampling procedure is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other.

Answer:

a) The mean effect is at most 200 seconds required to do a satisfactory job

Null Hypothesis i.e Status Quo. $H_0 <= 200$
Alternate Hypothesis $H_1 > 200$

S.E was calculated as 6.5

We can call it as Upper Tail Test → Rejection region on right side of distribution as 207 seconds falls under $H_1 > 200$ seconds

Now that we have formulated the hypothesis, we need to decide based on 2 methods.

Critical Value method:

Calculate the value of Zc from the given value of Alpha (significance level) which is given as 5%.

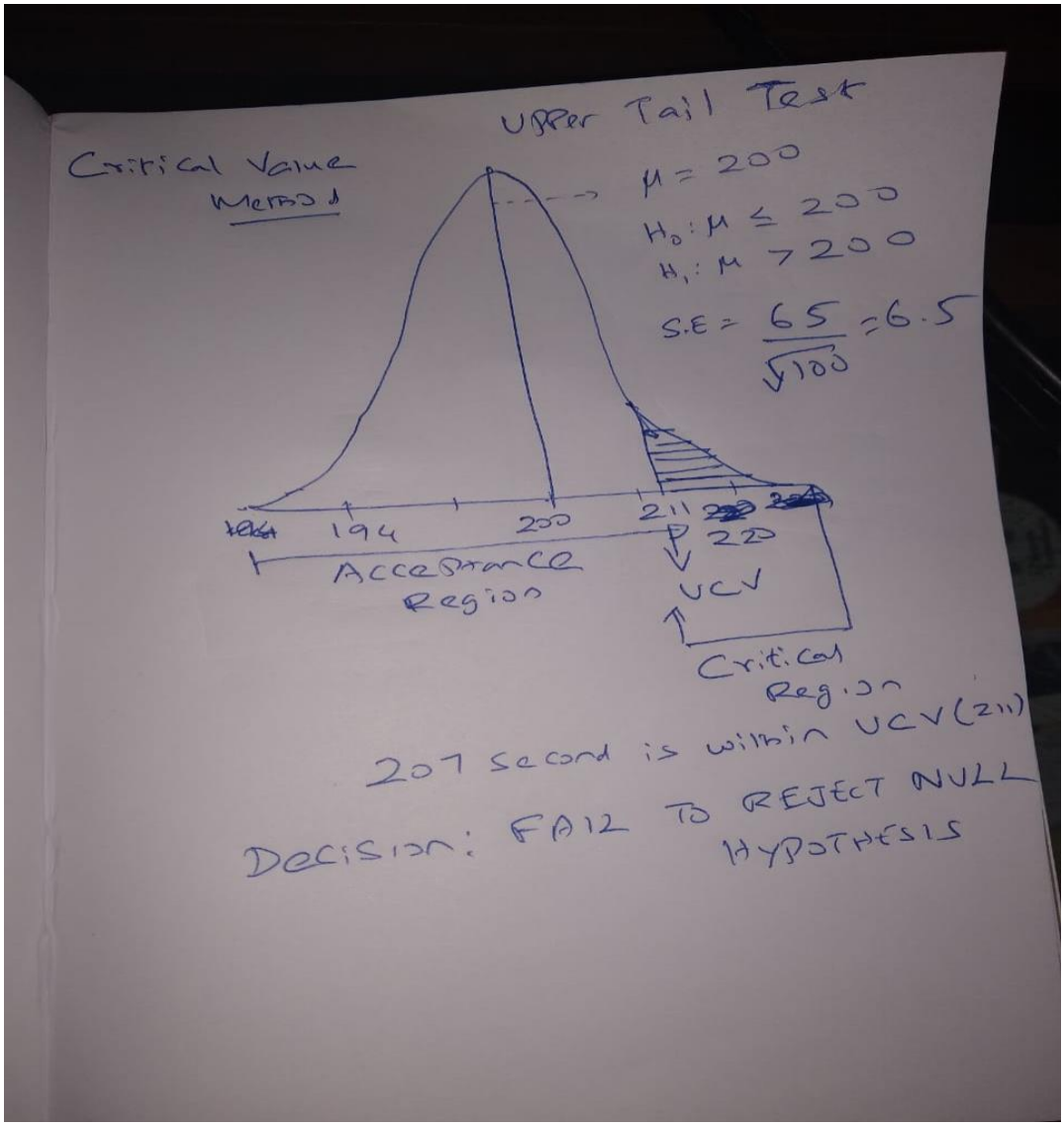
Alpha $\propto$ = 5 % = 5 /100 = 0.05

Cumulative Probability = 1 – 0.05 = 0.95 -→ as it is one tail test subtracting only once

From the z-table based on above Cumulative probability – Zc = 1.64

Calculate CV = μ + (Zc * $\sigma \div \sqrt{n}$)
UCV = 200 + (1.64 * 6.5) = 200 + 10.66 = 210.66 rounding off = 211 seconds

Now the Sample mean 207 seconds is less than UCV and it's in the acceptance region. So, the decision is Fail to reject the NULL Hypothesis.



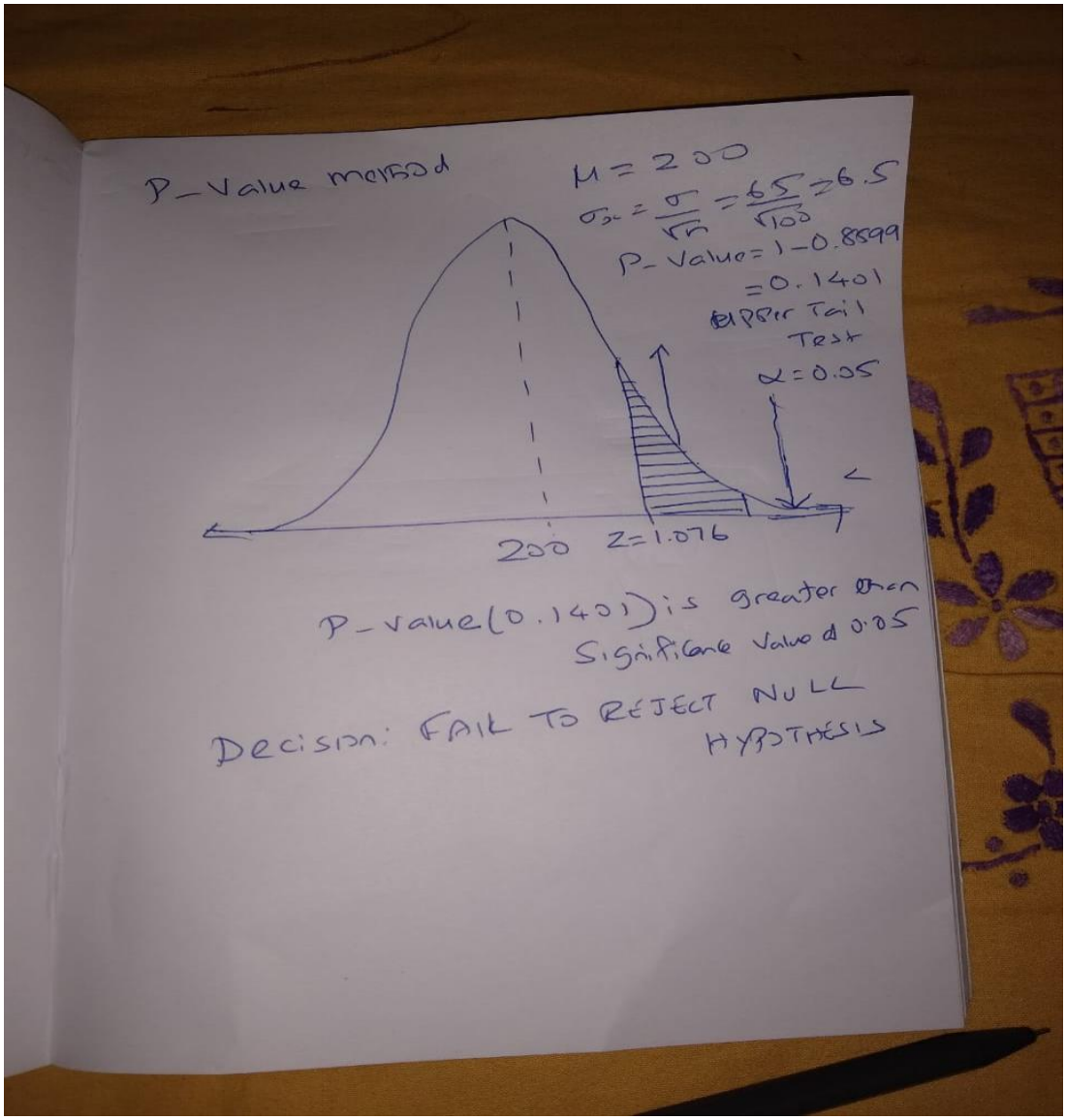
p-value method:

1. Start by finding the z-value for the given sample mean
   For Upper Tail test
   Z = Sample Mean – Mean / S.E
   = 207 - 200 / 6.5 = 1.076

2. The cumulative probability for the above z value is 0.8599

3. The p value is calculated p = 1 - 0.8599= 0.1401

Now since p-value is greater than the significance value of 0.05 the Decision is Fail to reject the Null Hypothesis.



b) The Alpha = 0.05 and Beta = 0.45

The probability of Type 1 error is denoted by Alpha i.e. reject Null Hypothesis H_0 when its true

The probability of Type 2 error is denoted by Beta i.e. Null Hypothesis is false but we fail to reject it.

Using the p-value we can find the probability of statistics i.e if H_0 is true,
p-value < ∝ ➔ Reject H_0
p-value <= ∝ ➔ Fail to Reject H_0

∝ = 0.15

The P-value we got was 0.1401 which is less than 0.15. i.e. we need to Reject H_0

In fact , H_0 is True in our case but we are rejecting H_0 so this is a Type 1 error as we are Rejecting H_0 when its true. And β is false as p-value is > $\propto$, Type 2 error is not

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign for its existing subscribers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer:
Considering all the above questions and the problem statement the key success factor for releasing a new batch of 80000 drug products is the time effect for curing the pain which has been base-lined as <= 200 seconds which is used by the Quality Assurance team to certify if the drug has done a satisfactory job. This information and test findings in the earlier questions can be used by the marketing team to plan an effective online ad campaign for its existing subscribers. The following steps can be followed

1. Figure out what to test
   With the above-mentioned deciding factors there can be 2 ways we can create an ad campaign
   a. We can ask Subscribers how many seconds / minutes it took to Cure your Pain. If we ask this way chances could be that subscriber might not have exactly know the exact time taken to cure the Pain. So better way could be ask directly provide the time taken figures. In Question 2 we had found that the Confidence range was 194 seconds to 220 seconds. We can use this numbers. Again Seconds measure might be trivial to use. Let's convert this to minutes so it can be like Option 1 – less than 3 minutes , Option 2 – = 3 minutes , Option 3 – More than 3 minutes

   b. We can Subscribers if you are Satisfied with the Drug just ask Yes / No

2. Setting Goal
   Goal of this test is to identify how people are forced to respond to this ad campaign i.e. which mode they choose the time effect or just prefer to say satisfactory or not

3. Finalize Hypothesis:

At the conclusion of this test, we expect how many maximum entries we get for Variation A i.e. time effect and Variation B i.e. Satisfactory or not. Hypothesis here can be defined as “Time effect mode ad campaign could deter the Subscribers from participating or responding to this question”. Hypothesis can also be changed by observations on this test.

4. A/B Testing Variation Creations:
   We could use many online available A/B testing tools to validate this Variations A and B. We can set this both variations of 500 each out of 1000 subscribers to this ad campaign and check the response, hits and no responses to this question.

5. Performing A/B Testing:
   We could conduct this test say for a week but there is not any fixed duration. If need be, we could either shorten or lengthen. This would depend on the way this ad campaign is been targeted. Say for example, we could have some Consumer Health items fair been conducted on a weekend, may be then we can market this in that 2 days and perform A/B test.

6. Interpreting Results:
   Once testing done, we would need interpret the results and see which Variation has got a better response in terms of hits and respond. If no proper response in either, we would need to start the process all over again. Variation B could be a better option as the users would tend and relate much better like drug is satisfactory or not instead of telling how long it took to cure the pain.