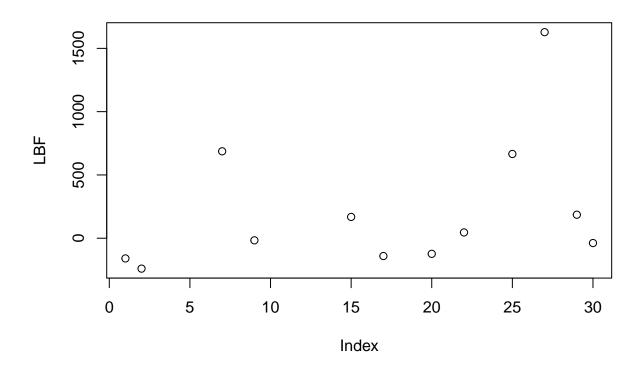
MedianandGeoMeantest

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We're curious to see if our proof has the wrong idea. We examine two functions that are completely different but have the same mean and variance.

We draw from a normal and a normal mixture which have the same variance and same mean, but clearly are not the same function.

```
gm_mean = function(x, na.rm=TRUE){
  exp(sum(log(x[x > 0]), na.rm=na.rm) / length(x))
}
set.seed(10000)
#install.packages("rmutil")
library(matrixStats)
library(rmutil)
##
## Attaching package: 'rmutil'
## The following object is masked from 'package:stats':
##
       nobs
## The following objects are masked from 'package:base':
##
       as.data.frame, units
source("MarginalLikIntfunctions.R")
set.seed(10000)
dlength <- 600
LBF <- c()
LBF2 \leftarrow c()
dataset1 <- rcauchy(dlength)</pre>
dataset2 <- rcauchy(dlength)</pre>
for(i in 1:30)
  dataset1 <- sample(dataset1)</pre>
  dataset2 <- sample(dataset2)</pre>
  XT1 <- dataset1[1:(dlength*.3)]</pre>
  XV1 <- dataset1[-(1:dlength*.3)]</pre>
  XT2 <- dataset2[1:(dlength*.3)]</pre>
  XV2 <- dataset2[-(1:dlength*.3)]</pre>
  ExpectedKernML <- logmarg.kernMCimport(XT1,XV1,iter = 1, importsize = 100) + logmarg.kernMCimport(XT2
  ExpectedKernML2 <- logmarg.kernMCimport(c(XT1,XT2),c(XV1,XV2),iter = 1,importsize = 100)</pre>
  LBF[i] <- ExpectedKernML - ExpectedKernML2
plot(LBF)
```



```
LBF
    [1] -159.63528 -240.09071
                                                                               {\tt NaN}
##
                                          {\tt NaN}
                                                     -Inf
                                                                   Inf
    [7]
          686.69067
                                   -16.59037
                                                     -Inf
                                                                  -Inf
##
                            -Inf
                                                                               NaN
   [13]
                             Inf
                                   168.23056
                                                      NaN -140.69837
                                                                               NaN
##
                 Inf
   [19]
                -Inf -123.29432
                                          Inf
                                                45.67443
                                                                  NaN
                                                                              -Inf
   [25]
          665.68649
                             Inf 1627.94933
                                                     -Inf
                                                            185.72644
                                                                        -38.02012
##
median(sort(LBF))
```

[1] -27.30525

The log BF is interesting at least...

Geometric mean has problems.

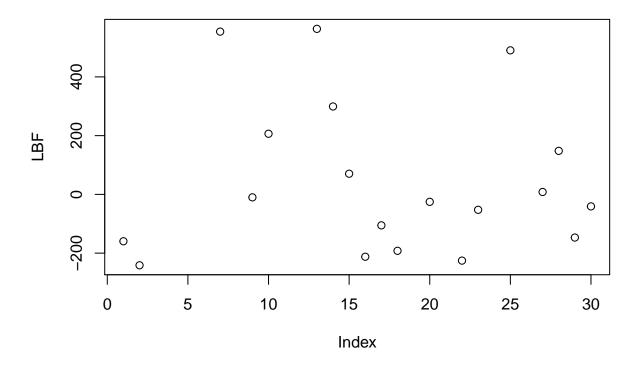
Median doesn't.

We didn't change sample size for this, all we did was shuffle training and validation.

Also using R's integrate seems to fail and just suggests that the integral is diverging. Importance sampling doesn't do anything with this, so it'll work fine, but its slow.

```
set.seed(10000)
#install.packages("rmutil")
library(rmutil)
source("MarginalLikIntfunctions.R")
set.seed(10000)
dlength <- 600
LBF <- c()</pre>
```

```
LBF2 \leftarrow c()
dataset1 <- rcauchy(dlength)</pre>
dataset2 <- rcauchy(dlength)</pre>
trainprop \leftarrow seq(from = .3, to = .8, length = 30)
for(i in 1:30)
{
  dataset1 <- sample(dataset1)</pre>
  dataset2 <- sample(dataset2)</pre>
  XT1 <- dataset1[1:(dlength*trainprop[i])]</pre>
  XV1 <- dataset1[-(1:dlength*trainprop[i])]</pre>
  XT2 <- dataset2[1:(dlength*trainprop[i])]</pre>
  XV2 <- dataset2[-(1:dlength*trainprop[i])]</pre>
  ExpectedKernML <- logmarg.kernMCimport(XT1,XV1,iter = 1, importsize = 100) + logmarg.kernMCimport(XT2</pre>
  ExpectedKernML2 <- logmarg.kernMCimport(c(XT1,XT2),c(XV1,XV2),iter = 1,importsize = 100)</pre>
  LBF[i] <- ExpectedKernML - ExpectedKernML2
}
plot(LBF)
```



```
LBF
    [1] -159.635283 -241.402547
                                          NaN
                                                      -Inf
                                                                   Inf
    [6]
                NaN 554.108342
                                               -10.148233
                                                            206.458301
##
                                         -Inf
##
  [11]
               -Inf
                             {\tt NaN}
                                  563.599046
                                               299.326822
                                                             70.387064
## [16] -212.309634 -105.413988 -192.082173
                                                      -Inf
                                                            -25.365914
                Inf -225.500411 -52.542851
                                                            490.376438
## [21]
                                                      -Inf
```

```
## [26] Inf 8.219317 148.179788 -146.961187 -40.923881
median(sort(LBF))
```

[1] -40.92388

This is still with a pretty pathological distribution (none of its moments are defined!).

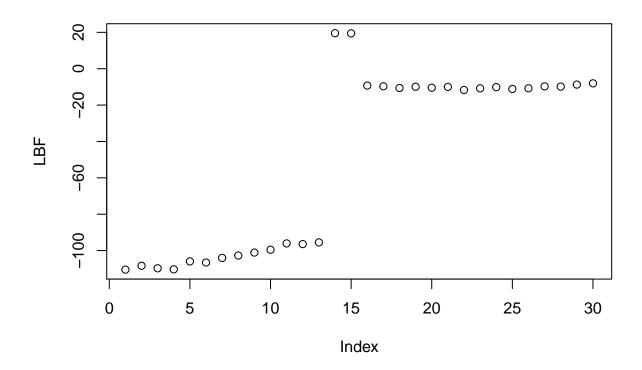
Things are behaving as expected again suprisingly? The median LBF is essentially 0...

Again computing geometric mean is essentially impossible.

I'm not sure where the NaN is coming from?

What about the pareto?

```
#Finite mean but infinite variance
set.seed(10000)
dlength <- 300
dataset1 <- rpareto(dlength, m=2.5, s=2.5)
dataset2 <- rpareto(dlength, m=2.5, s=2.5)
trainprop <- seq(from = .3, to = .8, length = 30)
for(i in 1:30)
  XT1 <- dataset1[1:(dlength*trainprop[i])]</pre>
  XV1 <- dataset1[-(1:(dlength*trainprop[i]))]</pre>
  XT2 <- dataset2[1:((dlength*trainprop[i]))]</pre>
  XV2 <- dataset2[-(1:(dlength*trainprop[i]))]</pre>
  ExpectedKernML <- logmarg.kern(XT1,XV1)[[2]] + logmarg.kern(XT2,XV2)[[2]]</pre>
  ExpectedKernML2 <- logmarg.kern(c(XT1,XT2),c(XV1,XV2))[[2]]</pre>
  LBF[i] <- ExpectedKernML - ExpectedKernML2
}
plot(LBF)
```



```
mean(LBF)

## [1] -48.61682

median(LBF)
```

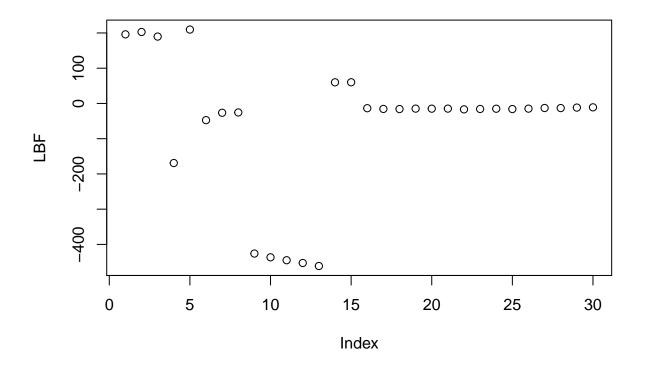
[1] -10.94106

Taking the mean of the logBF is the same as examining the log of the geometric mean of the BF...

What's suprising is that the logBF seems to be dependent on how many samples are being chosen for training for this problem?

We let more moments be 0 and see what happens. We used the integrate function this time as we didn't have computational issues...

```
XV2 <- dataset2[-(1:(dlength*trainprop[i]))]
ExpectedKernML <- logmarg.kern(XT1,XV1)[[2]] + logmarg.kern(XT2,XV2)[[2]]
ExpectedKernML2 <- logmarg.kern(c(XT1,XT2),c(XV1,XV2))[[2]]
LBF[i] <- ExpectedKernML - ExpectedKernML2
}
plot(LBF)</pre>
```

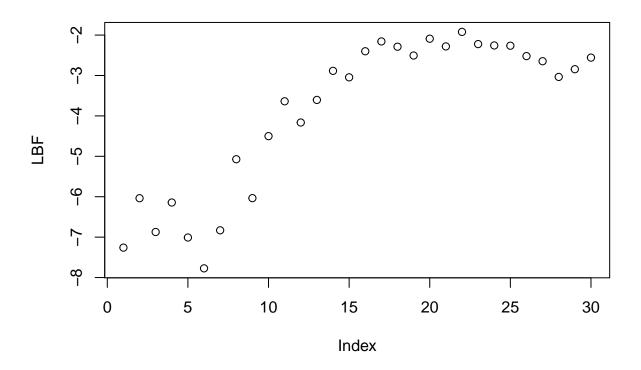


```
LBF
                    202.81022 189.77193 -169.08943 209.67786
##
    [1]
         196.12228
                                                                 -47.41013
##
    [7]
         -26.43591
                    -25.57979 -425.93403 -436.63184 -444.87965 -452.74082
## [13]
       -461.28962
                     59.97783
                                 60.11329
                                           -13.51910
                                                      -15.54137
                                                                  -15.84968
         -14.66632
## [19]
                    -14.79153
                               -14.64685
                                          -16.76680
                                                      -15.70944
                                                                 -14.96856
## [25]
         -15.88894
                    -14.71320
                               -12.94612
                                          -13.14477
                                                      -11.72664
                                                                 -10.99363
log(gm_mean(exp(LBF)))
## [1] -59.57969
mean(LBF)
## [1] -59.57969
median(LBF)
```

This favors the null hypothesis for most choices of training size, but not all. Lets check the normal distribution.

[1] -14.88004

```
set.seed(10000)
dlength <- 300
dataset1 <- rnorm(dlength)</pre>
dataset2 <- rnorm(dlength)</pre>
LBF <-c()
trainprop <- seq(from = .3, to = .8, length = 30)
for(i in 1:30)
  XT1 <- dataset1[1:(dlength*trainprop[i])]</pre>
  XV1 <- dataset1[-(1:(dlength*trainprop[i]))]</pre>
  XT2 <- dataset2[1:((dlength*trainprop[i]))]</pre>
  XV2 <- dataset2[-(1:(dlength*trainprop[i]))]</pre>
  ExpectedKernML <- logmarg.kern(XT1,XV1)[[2]] + logmarg.kern(XT2,XV2)[[2]]</pre>
  ExpectedKernML2 <- logmarg.kern(c(XT1,XT2),c(XV1,XV2))[[2]]</pre>
  LBF[i] <- ExpectedKernML - ExpectedKernML2
}
plot(LBF)
```



```
LBF

## [1] -7.259560 -6.036764 -6.872874 -6.143237 -7.009277 -7.772346 -6.831376

## [8] -5.072312 -6.035072 -4.500693 -3.638267 -4.163574 -3.604568 -2.885059

## [15] -3.045327 -2.400169 -2.157126 -2.287655 -2.505402 -2.089583 -2.280554

## [22] -1.921290 -2.222880 -2.256415 -2.263121 -2.520348 -2.646635 -3.035001

## [29] -2.844375 -2.557447
```

log(gm_mean(exp(LBF)))

[1] -3.895277

mean(LBF)

[1] -3.895277

median(LBF)

[1] -2.96003

I'm not sure if there's a trend I should be aware of?

If things are normal things are fine.