



ML - Data Plotting and Visualization

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PhD IIT Kharagpur

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Quotes

- ▶ Data, like a constellation in the night, finds its brilliance in the art of visualization, revealing the hidden stories of the universe it holds within.

Quotes

- ▶ Data, like a constellation in the night, finds its brilliance in the art of visualization, revealing the hidden stories of the universe it holds within.
- ▶ Dimensionality reduction, where data's symphony finds its graceful, concise melody

Data Plots

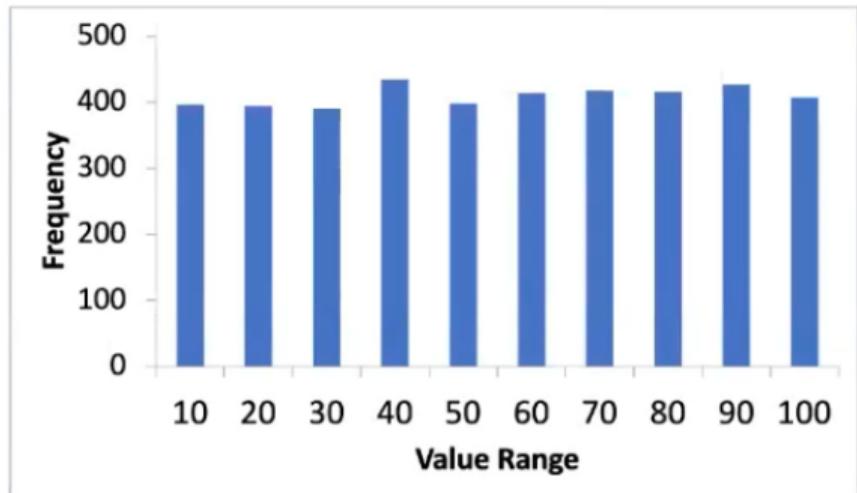
Diagrams to Gain Insights

Why Plot Data

22.65	42.12	67.24	59.13	81.49
23.03	53.42	40.54	89.97	21.85
12.07	93.43	51.93	49.30	43.76
47.68	51.91	13.12	73.88	60.29
86.20	41.28	66.24	62.15	46.87
20.02	92.09	26.50	83.53	70.99
48.38	46.21	10.85	29.61	62.15
55.23	84.90	15.37	35.00	83.23
65.30	26.56	5.78	72.59	12.47
75.71	93.15	3.67	49.80	43.05
69.73	53.77	82.80	43.59	32.35
77.95	14.94	63.71	9.30	1.31
58.90	42.53	62.74	99.91	53.17
6.45	46.29	67.34	32.65	23.94
32.39	57.39	10.61	54.07	53.28
74.35	60.10	2.25	77.55	12.05
82.87	17.02	80.73	29.60	9.96
47.05	97.01	19.84	45.90	1.24
34.26	86.80	19.11	55.53	58.25
12.54	30.40	67.94	73.13	0.23

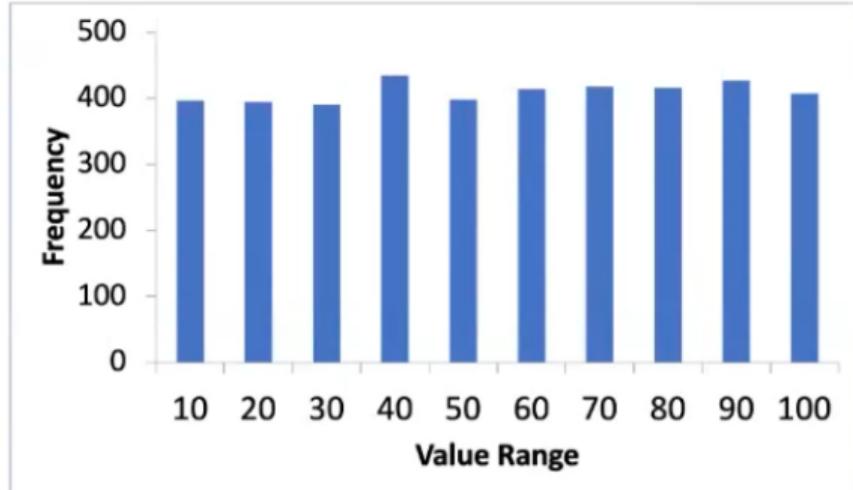
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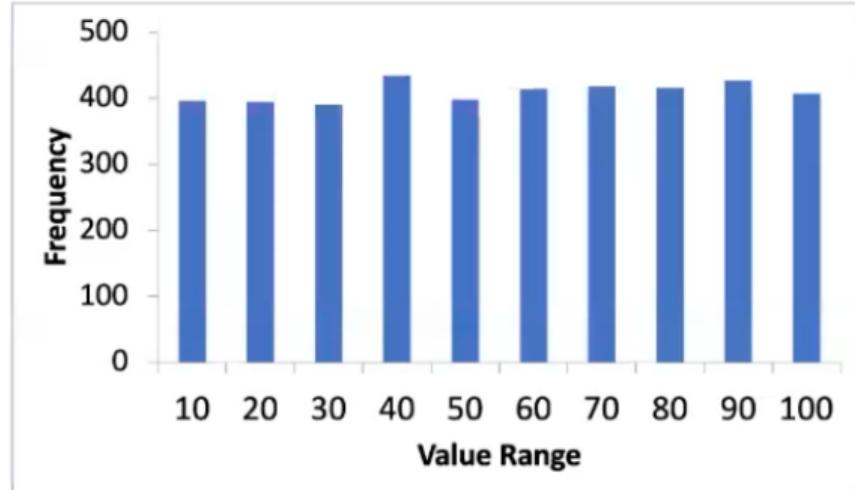
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- Visualization of data provides specific insights into the nature of the data.

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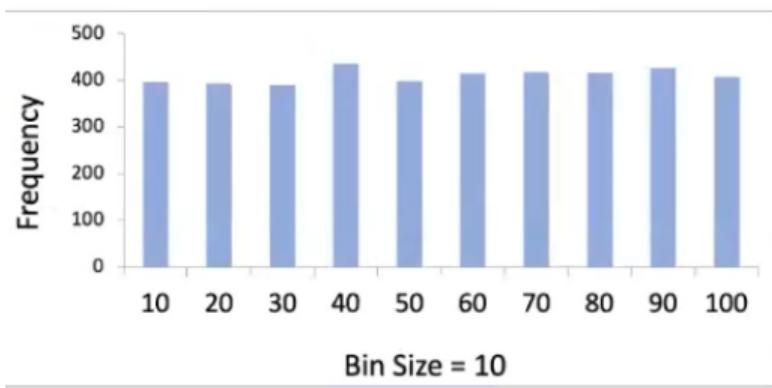
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- Visualization of data provides specific insights into the nature of the data.
- Depending on the plot, we gain different insights

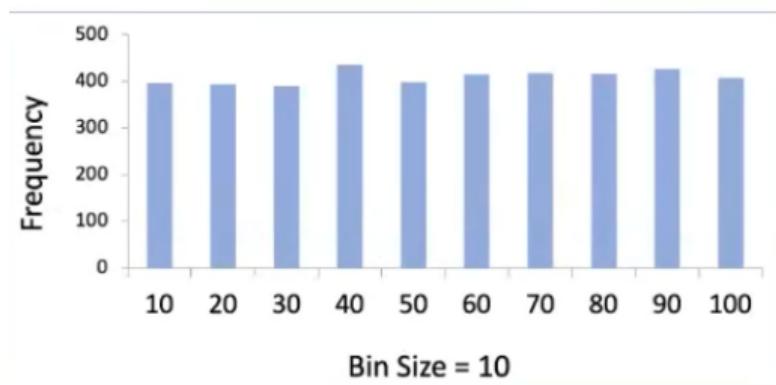
Histogram

- Count of items in each bin



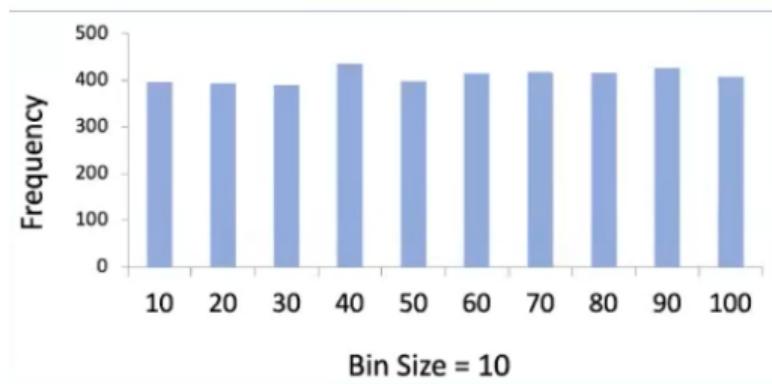
Histogram

- Count of items in each bin
 - Not a bar chart of Data



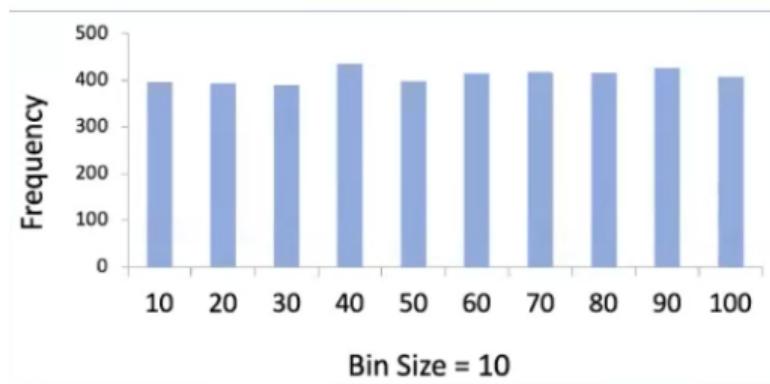
Histogram

- Count of items in each bin
 - Not a bar chart of Data
 - Approximation of Distribution



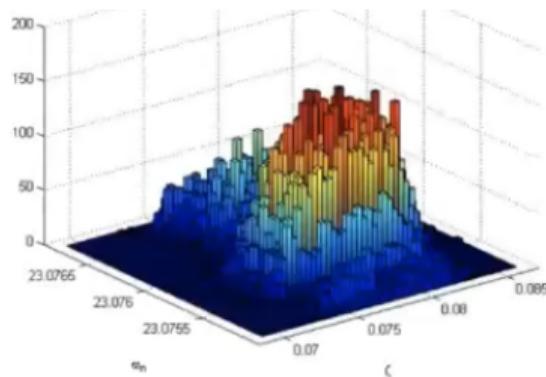
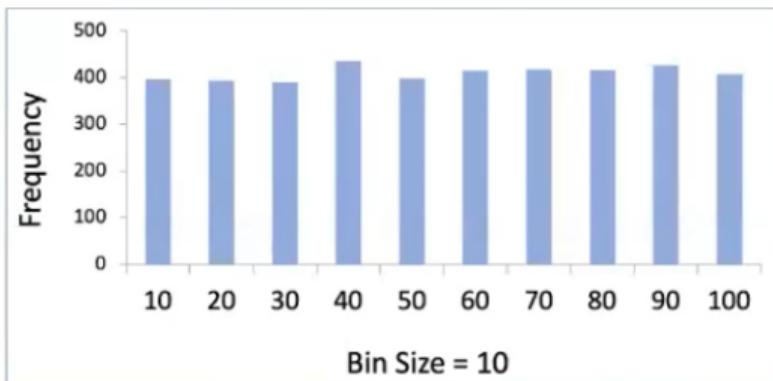
Histogram

- Count of items in each bin
 - Not a bar chart of Data
 - Approximation of Distribution
- Visualize one feature at a time



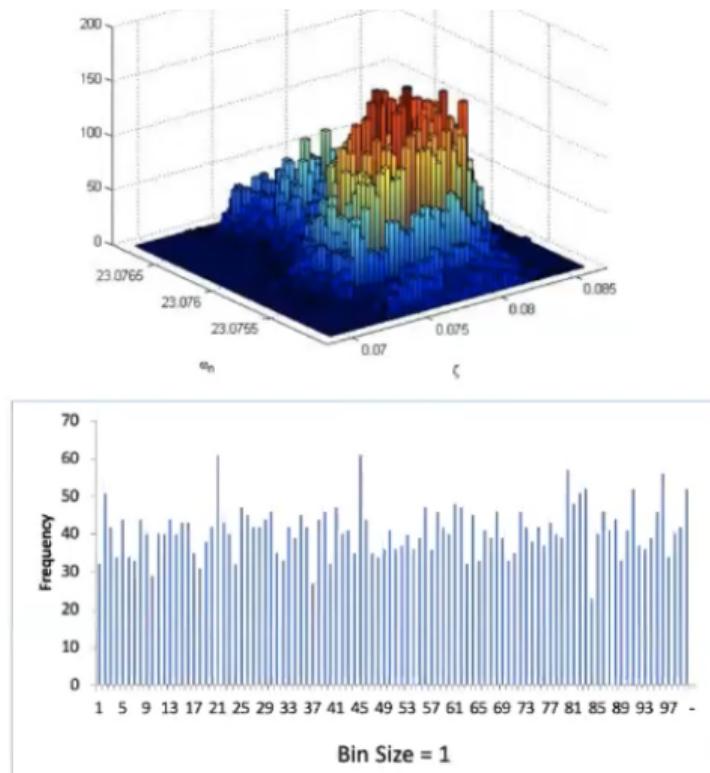
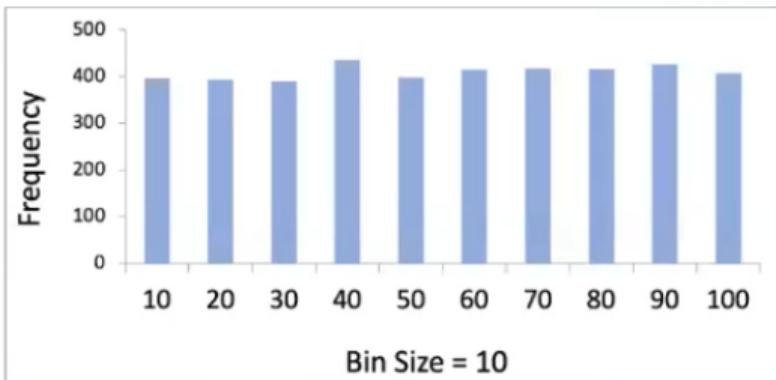
Histogram

- Count of items in each bin
 - Not a bar chart of Data
 - Approximation of Distribution
- Visualize one feature at a time
- Possible to extend to two



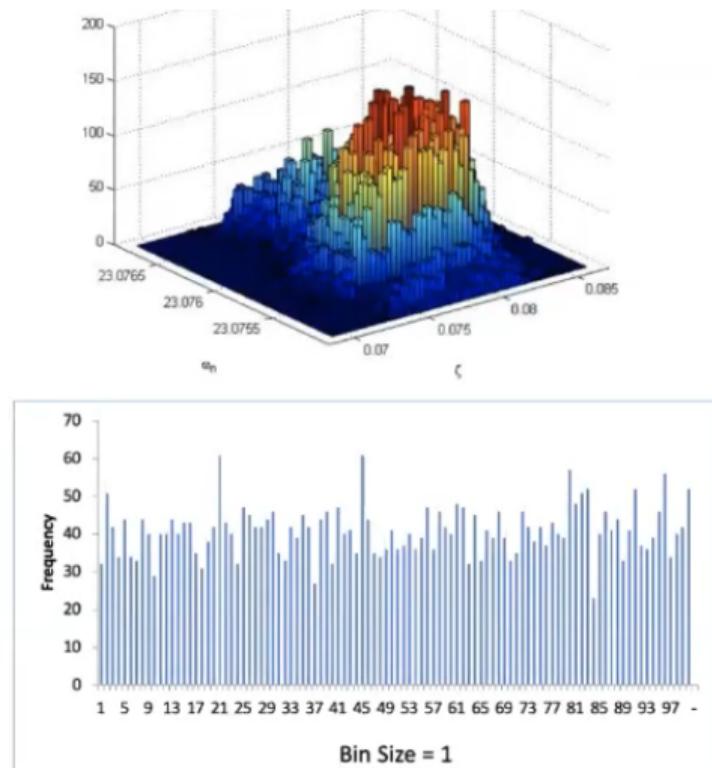
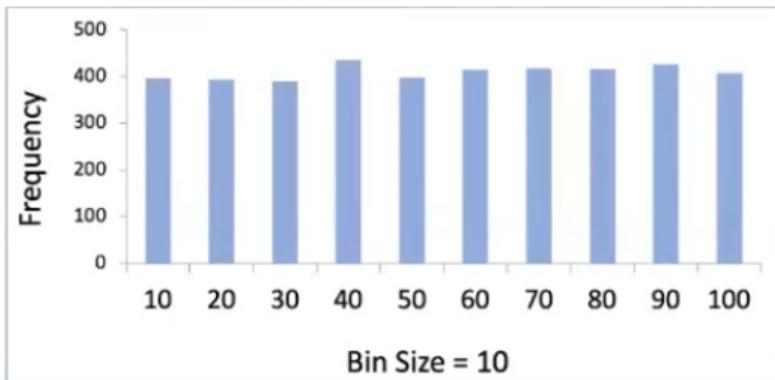
Histogram

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- Visualize one feature at a time
- Possible to extend to two
- Dependency on bins (\sqrt{n} ; $2\sqrt[3]{n}$)



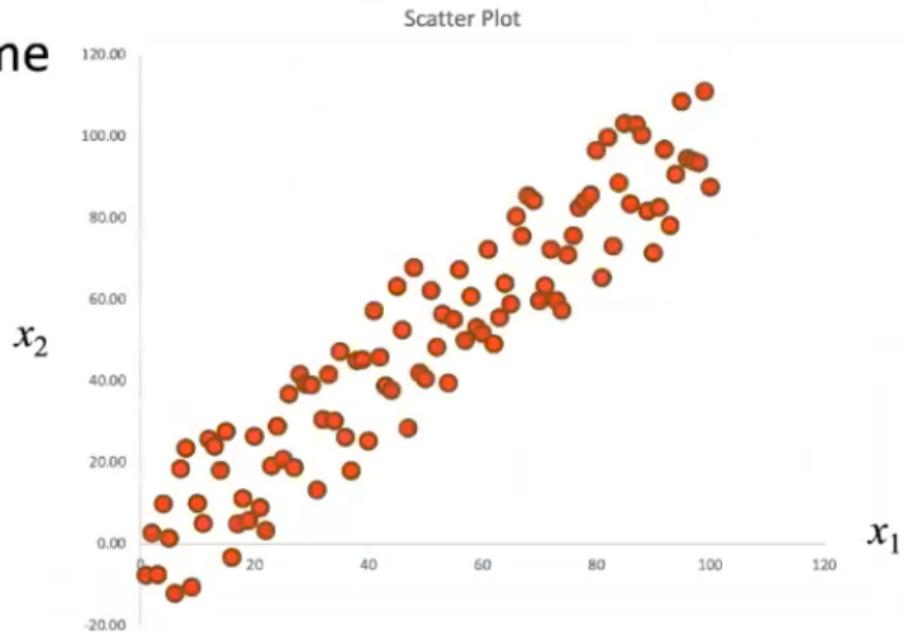
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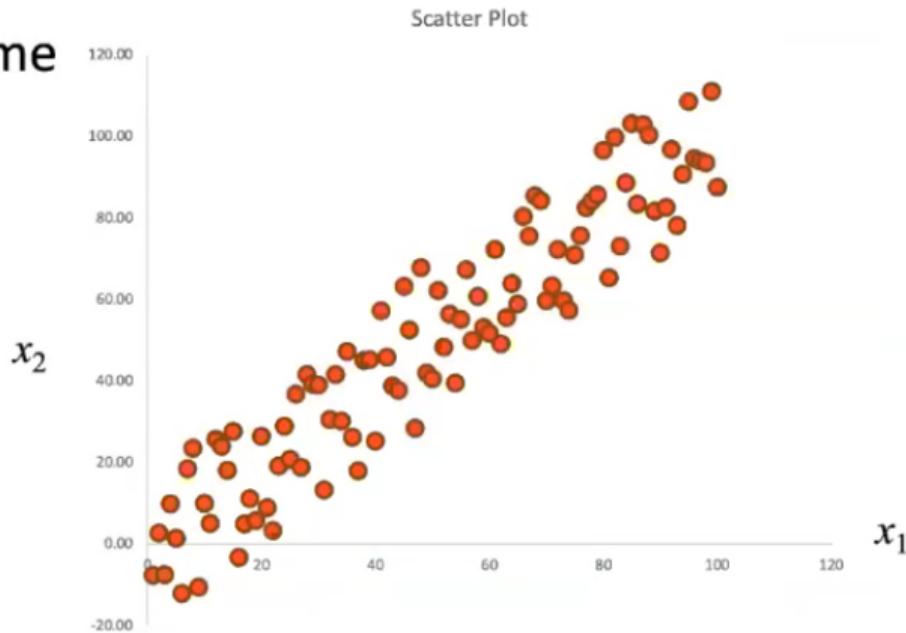
Scatter Plot

- Plots two features at a time



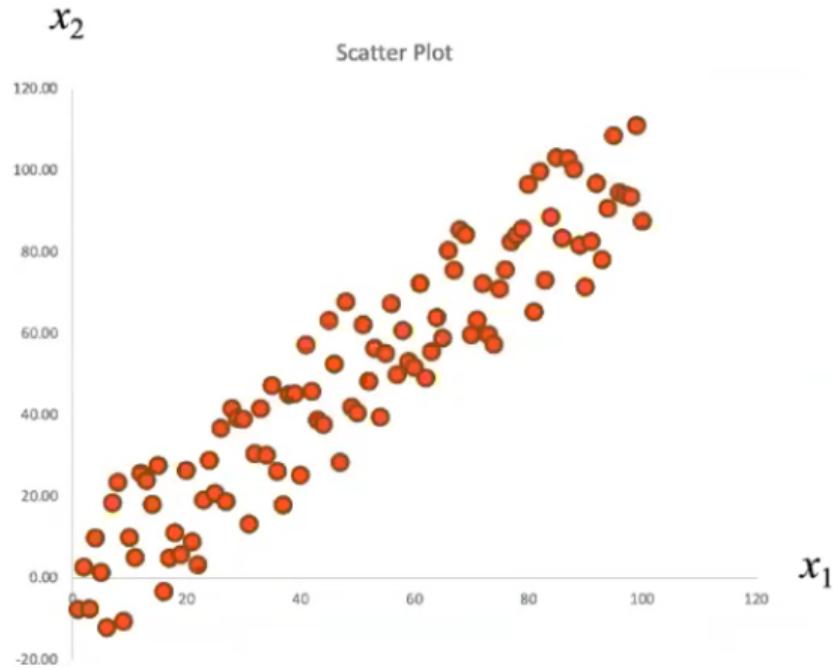
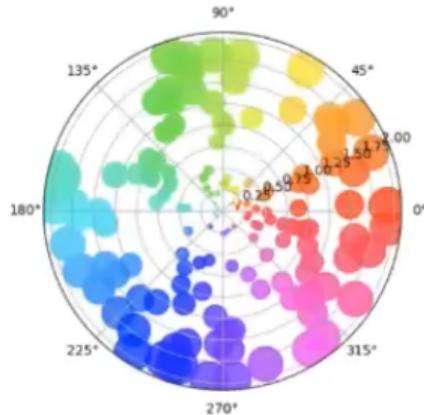
Scatter Plot

- Plots two features at a time
- Captures the correlation between the two



Scatter Plot

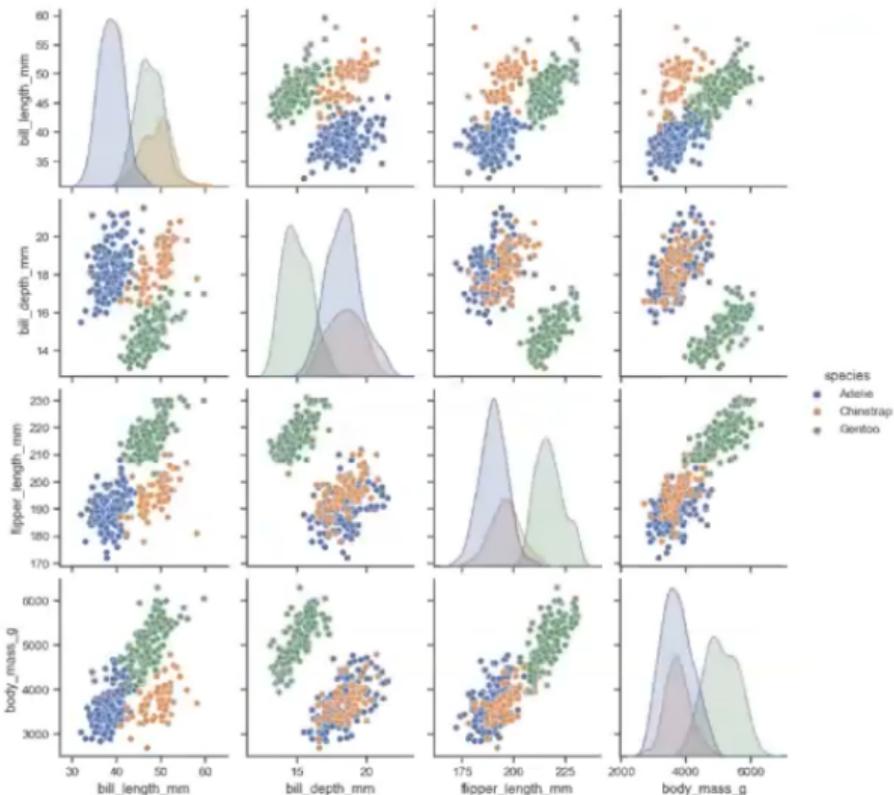
- Plots two features at a time
- Captures the correlation between the two
- Other formats possible



Polar Plot Courtesy: Scatterplot Documentation [matplotlib.org]

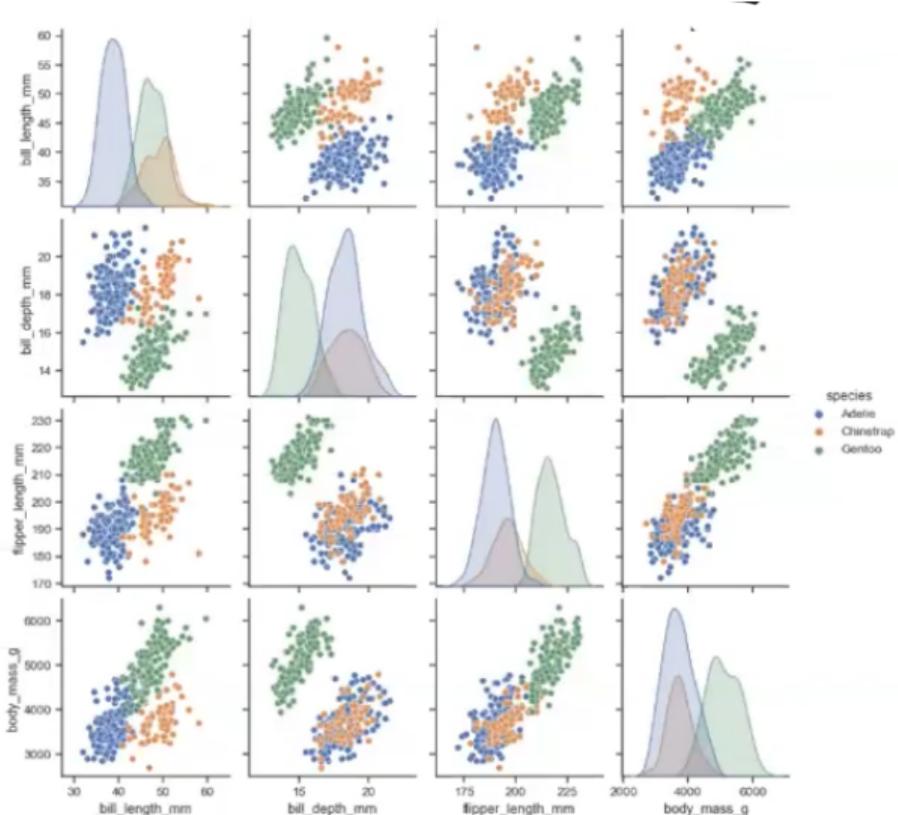
Pair Plot

- Plot each pair of features as a matrix
 - Diagonal entries are histograms (densities)
 - Off-diagonal entries are scatter plots



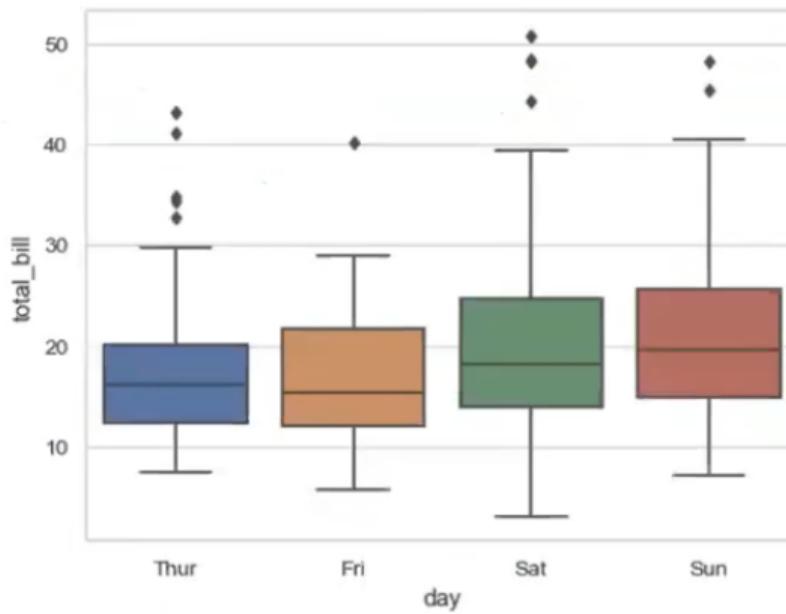
Pair Plot

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 - Off-diagonal entries are scatter plots
- Can use other plots at each cell.



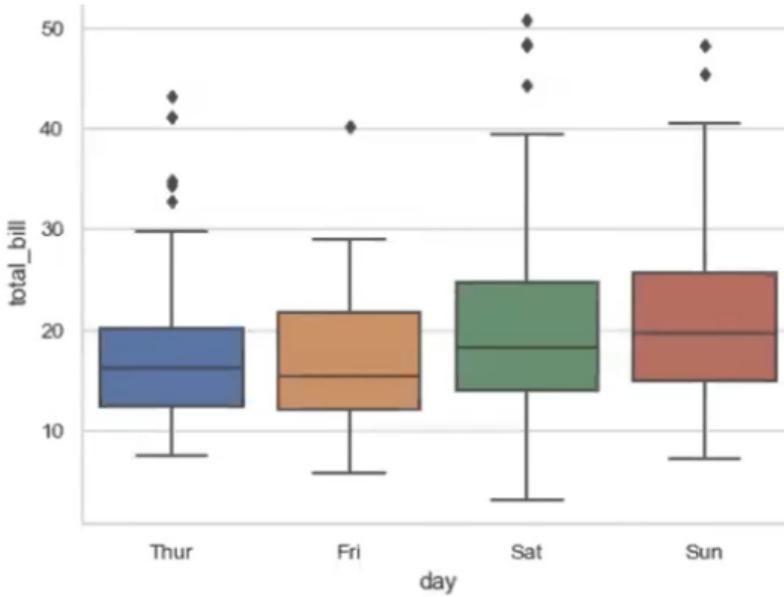
Box Plot

- Show median and quartiles of each feature



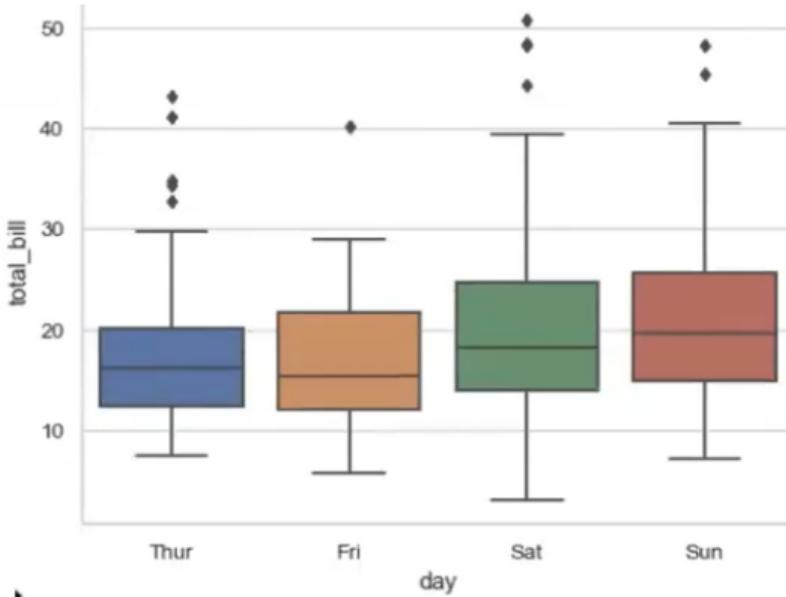
Box Plot

- Show median and quartiles of each feature
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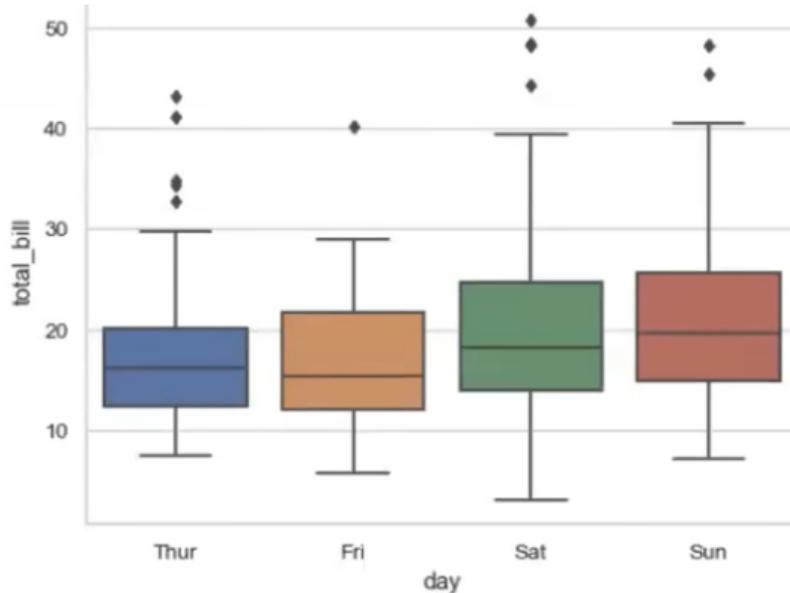
Box Plot

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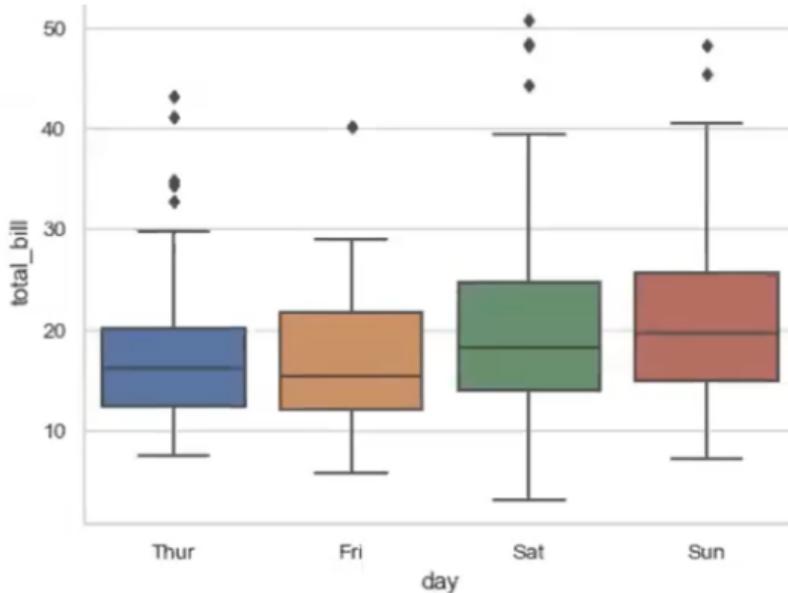
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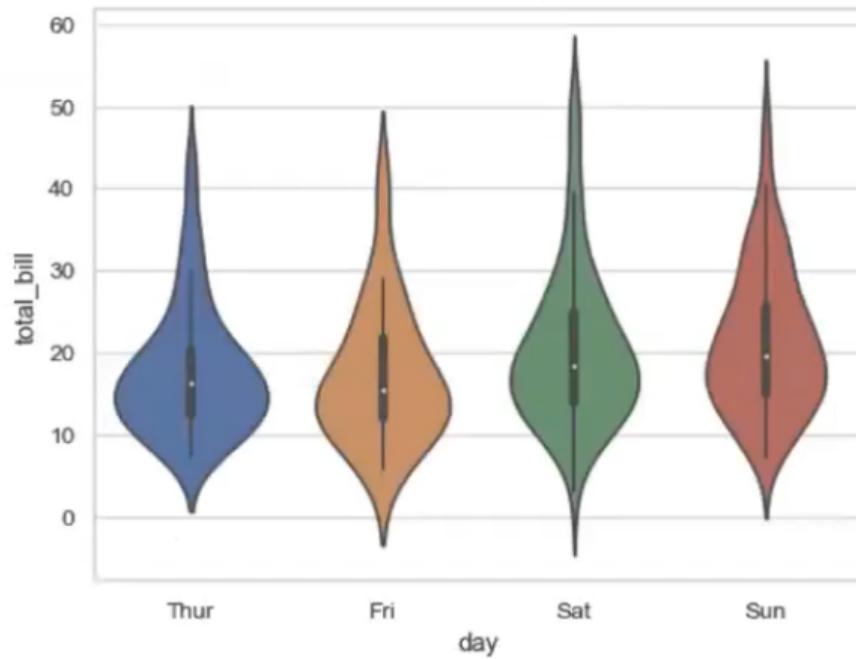
Box Plot

- Show **median** and **quartiles** of each feature
 - Outliers are removed
 - Box-and-whisker plot
- Whiskers can represent other percentiles/data
- Simpler than histograms of each feature



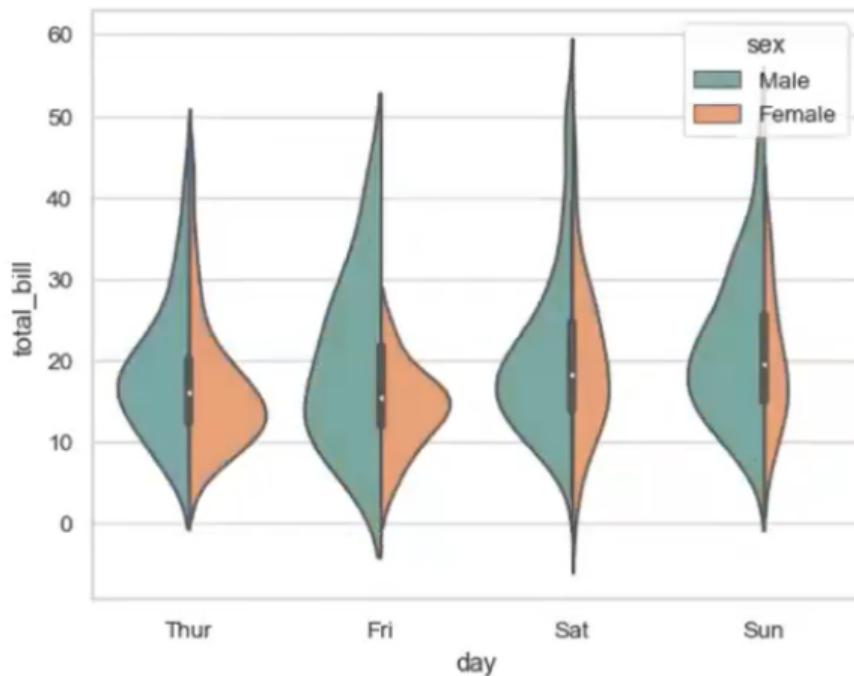
Violin Plot

- Shows the density plot of each feature



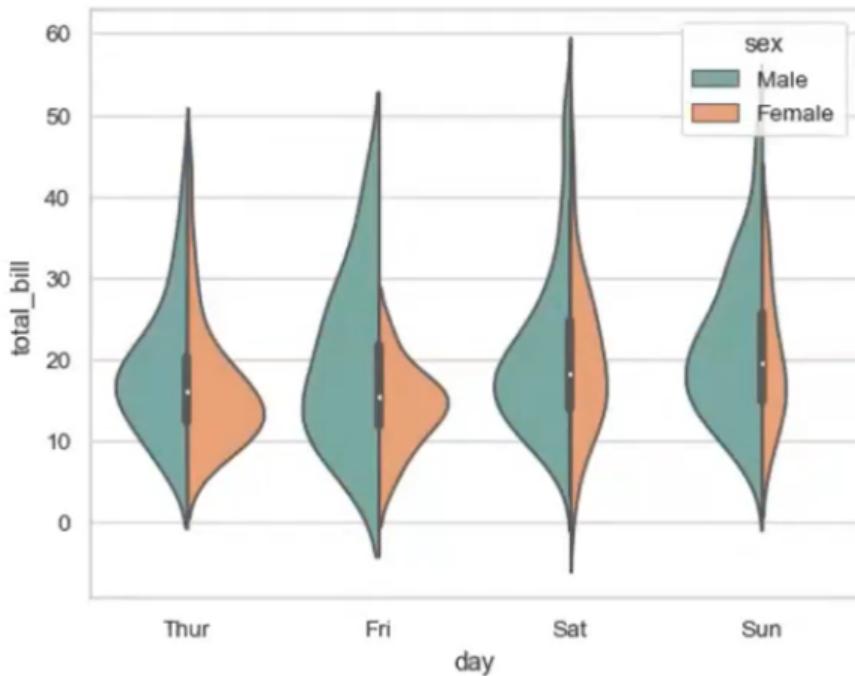
Violin Plot

- Shows the density plot of each feature
 - Similar to Box Plot
- Either side can represent different densities



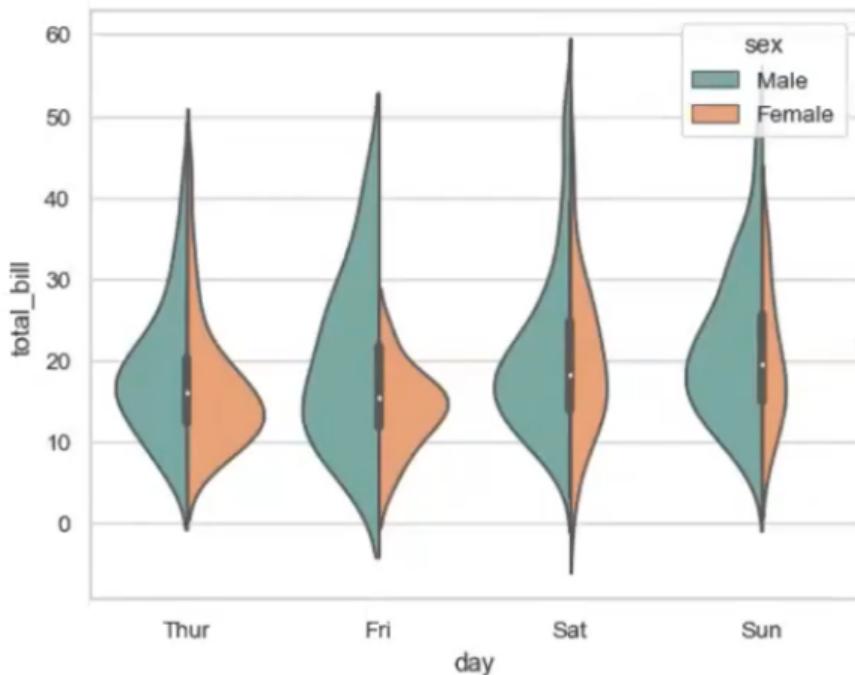
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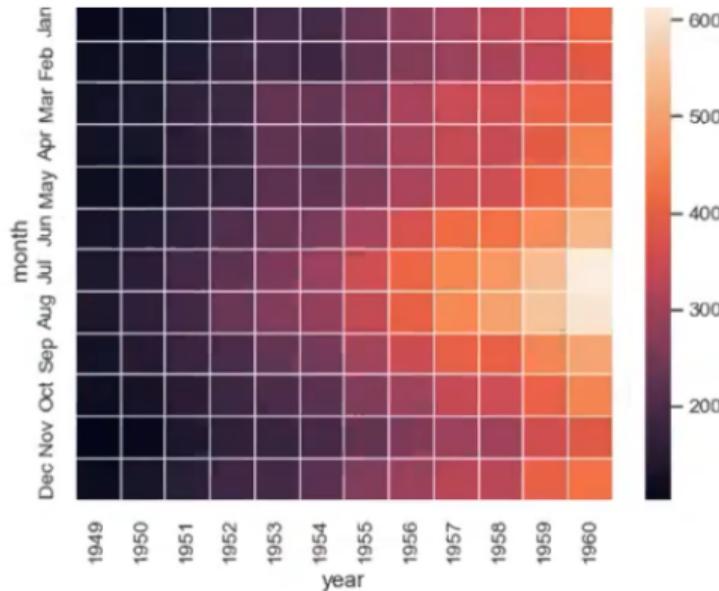
Violin Plot

- Shows the **density plot** of each feature
 - Similar to Box Plot
- Either side can represent different densities
- Densities are smoothed estimates from data



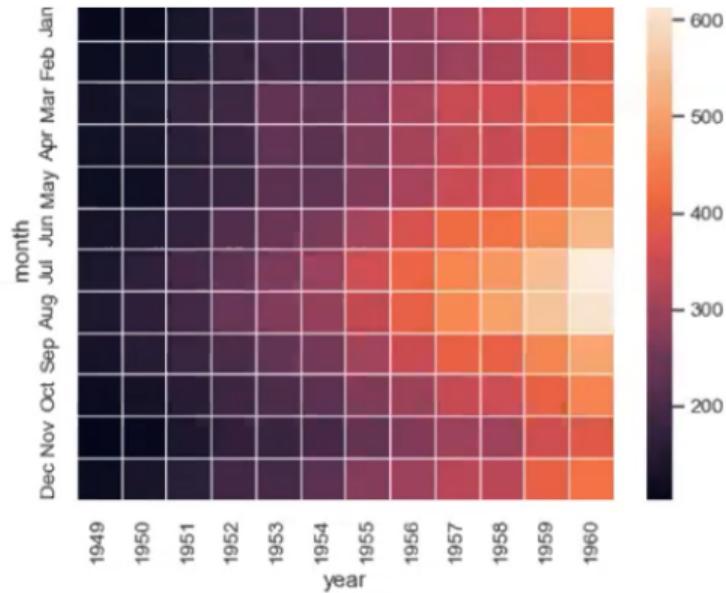
Heat Map

- A color-coded representation of 2D data
- Can be raw data, 2D histogram or any other function of 2 variables



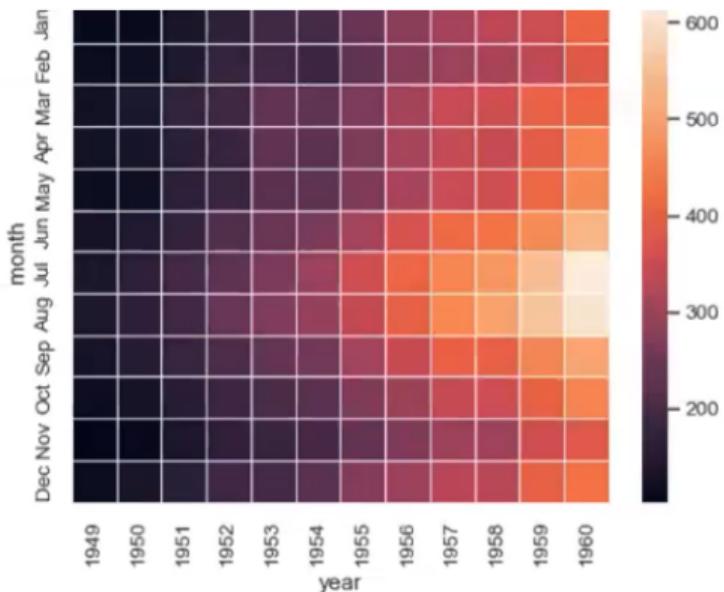
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Heat Map

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- A color map accompanies the heat map
- We will learn other metrics in future that may be visualized as a heat map



Dimensionality Reduction

Representing data using fewer features

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$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1.2 & -0.4 & 2.1 & 0.3 \\ -2 & 1.1 & 0.1 & 5.8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Feature Selection: Forward Selection

Algorithm:

```
 $z \leftarrow$  null vector;  $x$  is the original feature vector  
for each new feature to be added to  $z$ :  
    find  $x_i$  that gives highest accuracy when added to  $z$   
    if adding  $x_i$  to  $z$  improves accuracy:  
        move  $x_i$  to  $z$   
    else  
        break
```

Feature Selection: Backward Elimination

Algorithm:

$z \leftarrow x$

for each new feature to be removed from z :

find z_i that gives highest accuracy when removed from z

if removing z_i improves accuracy:

 remove z_i from z

else

break

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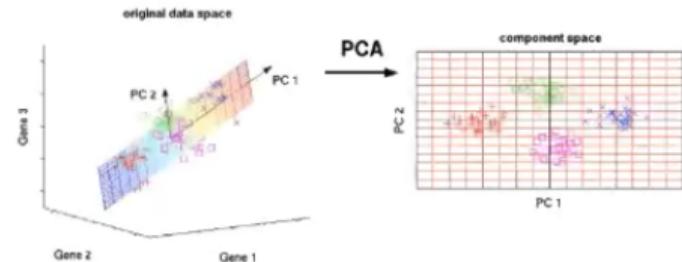
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- The final subset may be written as a binary matrix

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Feature Extraction

- Goal: Identify a matrix \mathbf{W} , such that $\mathbf{z} = \mathbf{Wx}$.

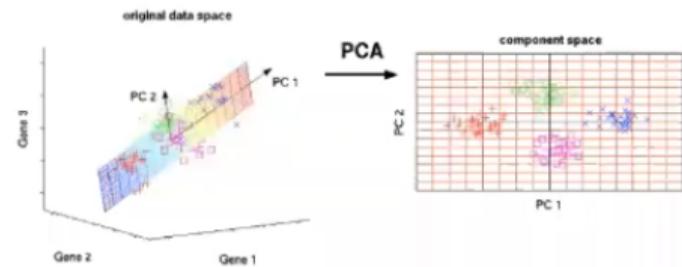


Courtesy: "PCA in Neuroscience", [math.waterloo.ca]

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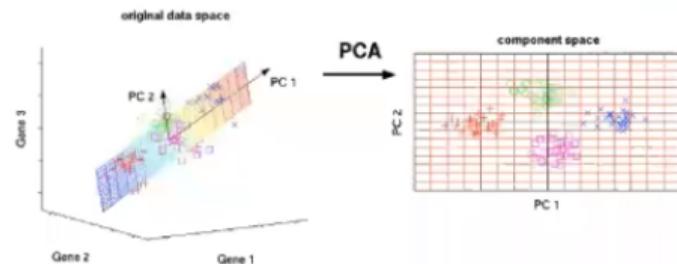


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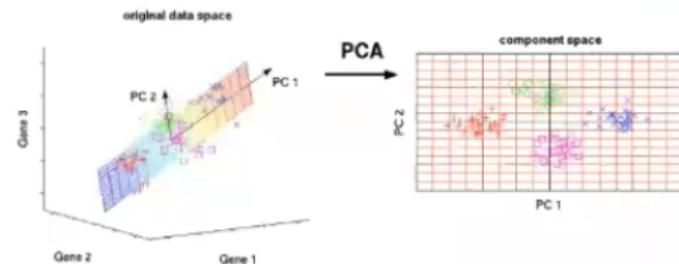
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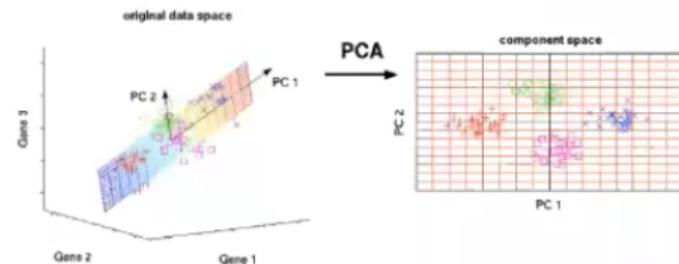
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 - Visualization (in a few dimensions)

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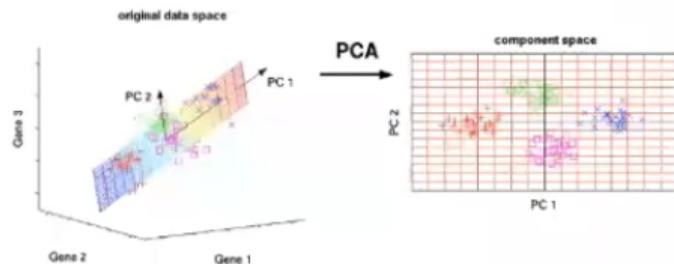
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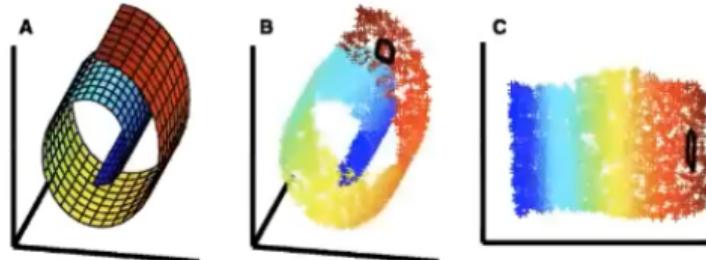
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- The above model is linear.
Non-linear methods exists as well
 - $\mathbf{z} = f_w(\mathbf{x})$



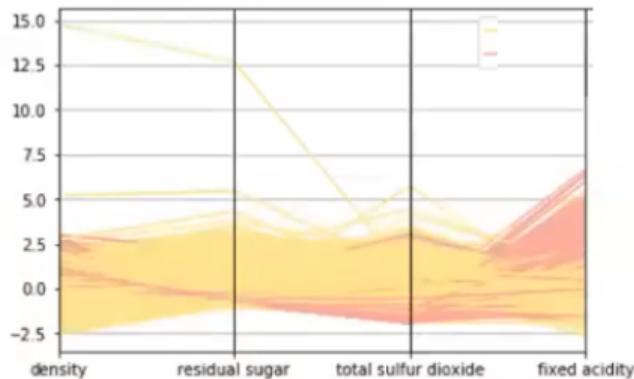
Courtesy: "DR by LLE", by Roweis and Saul

High Dimensional Data Visualisation

Direct Methods and Dimensionality Reduction

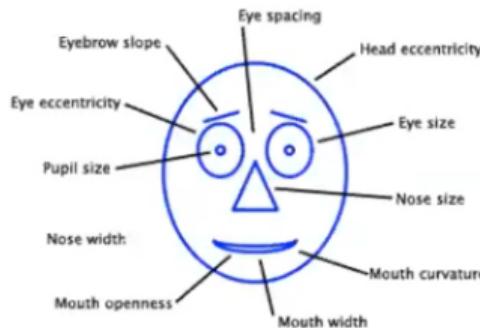
Direct Visualisation

- Parallel Co-ordinates
 - Each vertical line is a dimension
 - A data item is connected by line segments
 - Large number of samples clutters the visualization



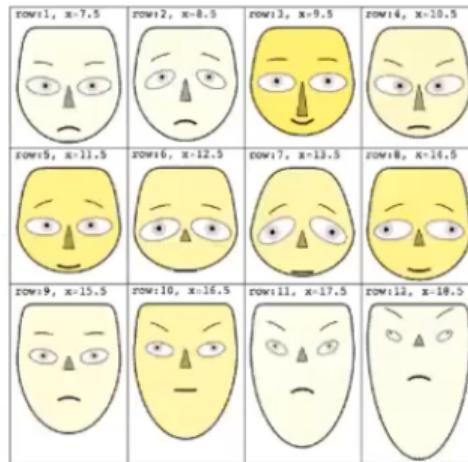
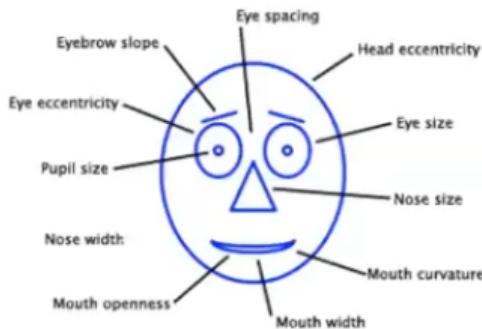
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- A few approaches have been proposed to improve upon Chernoff Faces
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 - Improve visual similarity of similar vectors



Image: The Empathic Visualization Algo: Chernoff Faces Revisited
<http://www0.cs.ucl.ac.uk/staff/a.loizides/218.pdf>

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 - Improve visual similarity of similar vectors
- Metaphoric 3D glyphs
 - Use metaphors other than faces (say Trees)



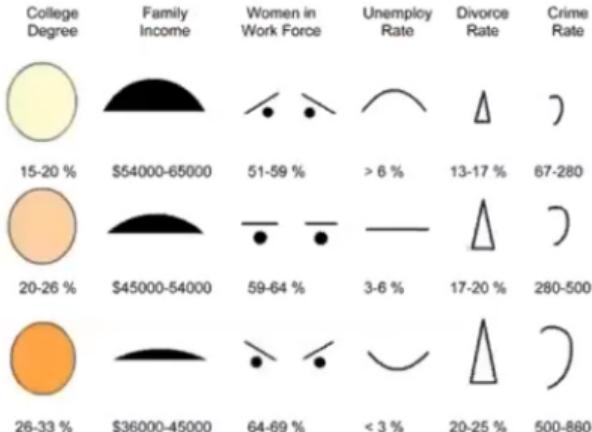
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Image: "Beyond Chernoff faces: Multivariate visualization with metaphoric 3D glyphs", Marco Lardelli

Challenges in HD Visualisation

- Direct Methods does not preserve ordinal nature of features
 - e.g., In Chernoff faces, emotions are not ordinal in eyebrow slant



15-20 % \$54000-65000 51-59 % > 6 % 13-17 % 67-280

20-26 % \$45000-54000 59-64 % 3-6 % 17-20 % 280-500

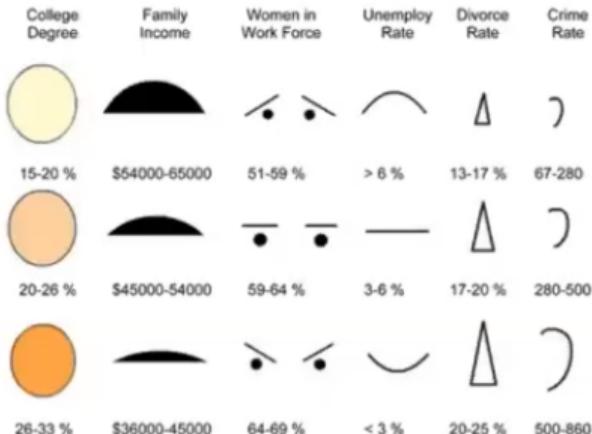
26-33 % \$36000-45000 64-69 % < 3 % 20-25 % 500-880



Courtesy: "The trouble with Chernoff"
[\[http://maphugger.com/post/44499755749/the-trouble-with-chernoff\]](http://maphugger.com/post/44499755749/the-trouble-with-chernoff)

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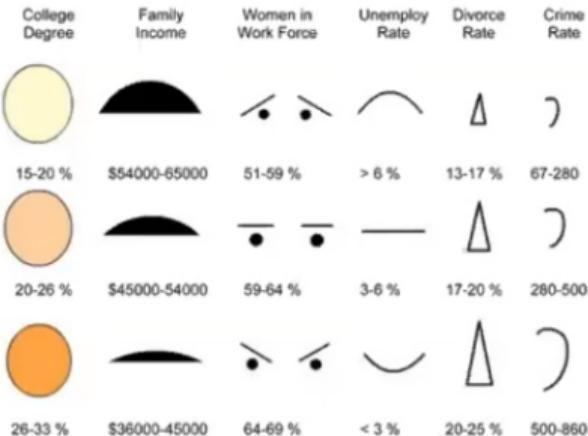
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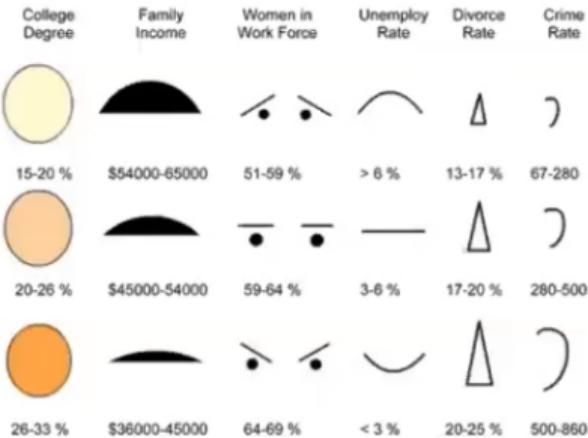
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 - Class Separation
 - Clusters in data