

MapReduce and PageRank

Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer i and produces the list of pairs (p,i) such that p is a prime divisor of i . For example, $\text{map}(12) = [(2,12),(3,12)]$.

The reduce function is addition. That is, $\text{reduce}(p,[i_1,i_2,\dots,i_k])$ is $(p,i_1+i_2+\dots+i_k)$.

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

Answer 1: The output of map function is

$\text{map}(15) = [(3, 15), (5, 15)]$

$\text{map}(21) = [(3, 21), (7, 21)]$

$\text{map}(24) = [(2, 24), (3, 24)]$

$\text{map}(30) = [(2, 30), (3, 30), (5, 30)]$

$\text{map}(49) = [(7, 49)]$

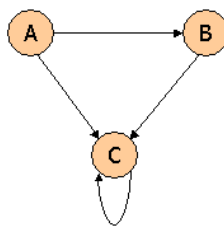
These are the respective prime divisors of inputs

The output of reduce function is

$\text{reduce}(2, 54), \text{reduce}(3,90), \text{reduce}(5,45), \text{reduce}(7, 70)$.

Question 2:

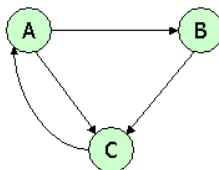
Consider three Web pages with the following links:



Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRank's of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRank's a , b , and c of the three pages A, B, and C, respectively.

2) ans) for $a = \beta(a) + (1-\beta)$
 $b = \beta(a/2) + (1-\beta)$
 $c = \beta(a/2 + b + c) + (1-\beta)$
 Here $\beta = 0.7$, $a + b + c = 3$
 $a = 0.7(0) + (1-0.7) = 0.3$
 $b = 0.7(0.3/2) + (1-0.7) = 0.405$
 $c = 0.7(0.3/2 + 0.405 + c) + (1-0.7)$
 $= 0.405 + 0.7(0.405) + (0.7)(c)$
 $c = 1.7(0.405) + 0.7(c) \Rightarrow (1-0.7)c = 1.7(0.405)$
 $= 0.3c = 0.6885$
 $c = \frac{0.6885}{0.3}$
 $c = 2.295$

Question 3:



Suppose we compute PageRank with $\beta=0.85$. Write the equations for the PageRanks a , b , and c of the three pages A, B, and C, respectively.

Q3
ans

Here

$$\beta = 0.85$$

$$a = 0.85 + c(1 - 0.85)^{1/3}$$

$$a = 0.85c + 0.05$$

$$b = 0.85 + 0.5 \times a + 0.05$$

$$b = 0.425a + 0.05$$

$$c = 0.85 \left[0.5 \times a + b \right] + 0.05$$

$$= 0.425a + 0.85b + 0.05$$

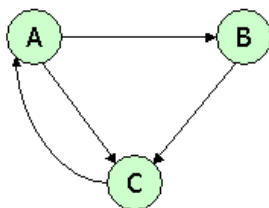
formulas:

$$a = \beta \times c + (1 - \beta)^{1/3}$$

$$b = \beta \times a/2 + (1 - \beta)^{1/3}$$

$$c = \beta \times (a/2 + b) + (1 - \beta)^{1/3}$$

Question 4:



Assuming no "taxation," compute the PageRanks a , b , and c of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank $a = b = c = 1$. compute as far as the 5th iteration, and also determine what the PageRanks are in the limit

General formulae:

$$a = c \quad b = a/2 \quad c = a/2 + b$$

0th iter:

$$a = 1 \quad b = 1 \quad c = 1$$

1st iter:

$$a = 1 \quad b = 1/2 \quad c = 1/2 + 1 = 3/2$$

2nd iter:

$$a = c = 3/2; \quad b = a/2 = 1/2; \quad c = 1/2 + 1/2 = 1$$

3rd iter:

$$a = c = 1; \quad b = a/2 = \frac{3}{2/2} = 3/4; \quad c = 3/4 + 1/2 = 5/4$$

4th iter:

$$a = c = 5/4; \quad b = a/2 = 1/2; \quad c = 5/4$$