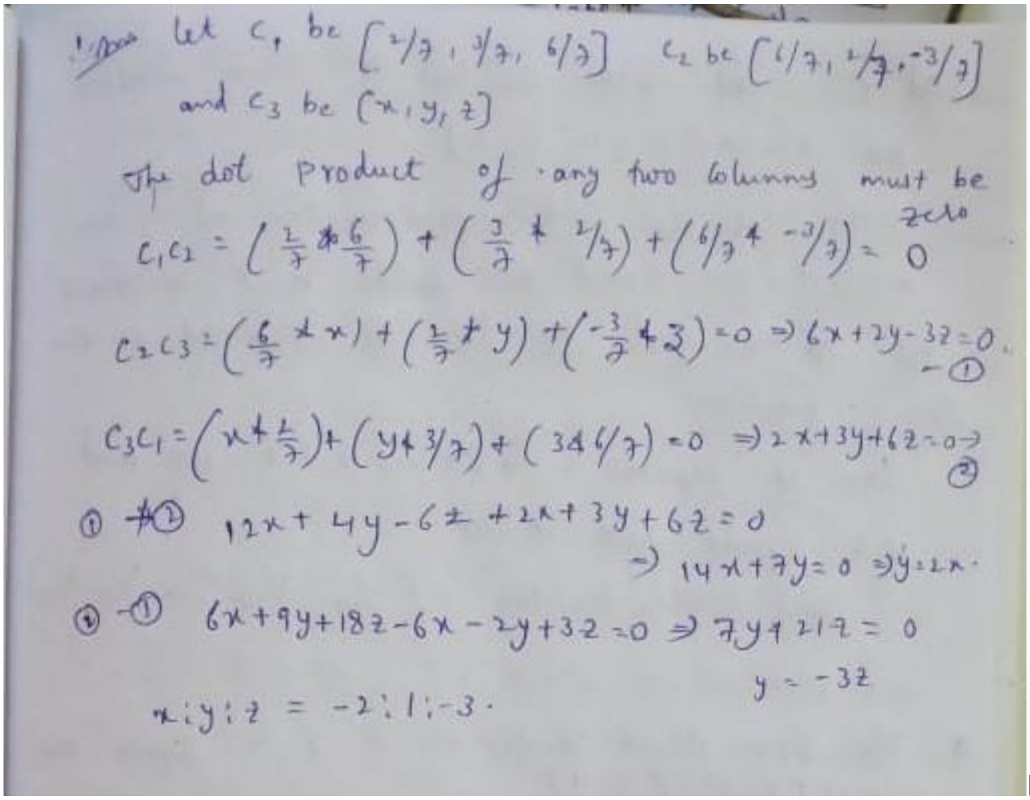
Dimensionality Reduction

Question 1: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7, 2/7, -3/7].



1/ans let c_1 be $[2/7, 3/7, 6/7]$ c_2 be $[6/7, 2/7, -3/7]$
and c_3 be $(x, y, z]$

The dot product of any two columns must be zero

$c_1c_2 = \left(\frac{2}{7} * \frac{6}{7}\right) + \left(\frac{3}{7} * 2/7\right) + \left(6/7 * -3/7\right) = 0$

$c_2c_3 = \left(\frac{6}{7} * x\right) + \left(\frac{2}{7} * y\right) + \left(\frac{-3}{7} * z\right) = 0 \Rightarrow 6x+2y-3z=0$ — (1)

$c_3c_1 = \left(x * \frac{2}{7}\right) + \left(y * 3/7\right) + \left(3 * 6/7\right) = 0 \Rightarrow 2x+3y+6z=0 \rightarrow$ (2)

(1) *(2) $12x+4y-6z+2x+3y+6z=0$
$\Rightarrow 14x+7y=0 \Rightarrow y=2x$.

(2) -(1) $6x+9y+18z-6x-2y+3z=0 \Rightarrow 7y+21z=0$
$y=-3z$

$x:y:z = -2:1:-3$.

Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that $x^2+y^2+z^2 = 1$. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

Question 2: Find the eigenvalues and eigenvectors of the following matrix:

2	3
3	10

You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

① Let the given matrix be $A = \begin{matrix} 2 & 3 \\ 3 & 10 \end{matrix}$ and the eigen vector be of the form $\begin{matrix} 1 \\ e \end{matrix}$.

$$Ax = \lambda x \rightarrow \begin{matrix} 2 & 3 \\ 3 & 10 \end{matrix} = \begin{matrix} 1 \\ e \end{matrix} = \lambda \times \begin{matrix} 1 \\ e \end{matrix} \Rightarrow 2+3e - \lambda$$

and $2 + 3e - \lambda e \rightarrow 3+10e = (2+3e)e$.

$$3e^2 - 8e + 3 = 0 \Rightarrow c = 3, -1/3.$$

The eigen vector are $1/3$ and -3.

The eigen values are $2+3e = \lambda \rightarrow \lambda = 2 + 3\times 3 = 11$ and

$$\lambda = 2+3\left(\frac{-1}{3}\right) = 1$$

Question 3: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

3) Given the Eigen vector of some matrix be $M = [1, 3, 4, 5, 7]$

To get the unit eigen vector of given matrix, we need to divide each component by square root of sum of squares in the same direction.

Sum of squares $= 1^2 + 3^2 + 4^2 + 5^2 + 7^2 = 100$ and it's square root is 10

Unit Eigen vector $= (1/10, 3/10, 4/10, 5/10, 7/10)$

Question 4: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

4, The given three points in a 2-D space are
(4,1) (2,2), (3,4)

we should construct matrix whole rows correspond to points and columns correspond to dimensions
Then the given matrix will be

$$a = \begin{matrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{matrix} \qquad a^T \cdot a = \begin{bmatrix} 1\,2 & 3 \\ 1\,2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 17 \\ 17 & 21 \end{bmatrix}$$

Question 5: Consider the diagonal matrix M =

1	0	0
0	2	0
0	0	0

Compute its Moore-Penrose pseudoinverse.

5/ Moore-Penrose pseudo inverse mean the matrix having diagonal elements replaced by 1 and divided by corresponding elements of given matrix and the other elements will be zero.

Moore-Penrose pseudo inverse of given matrix

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{matrix}$$

Question 6: When we perform a CUR dcomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:

1	2	3
4	5	6
7	8	9
10	11	12

Calculate the probability distribution for the rows.

6, Probability with which we choose row

$$= \frac{\text{Sum of square of elements in row}}{\text{Sum of squares of elements in the matrix}}$$

Sum of squares of elements in the matrix

$$= 12 \times 13 \times 25/6 = 3900/6 = 650.$$

$$P(R_1) = \frac{1^2+2^2+3^2}{650} = 14/650 = 0.62.$$

$$P(R_2) = \frac{4^2+5^2+6^2}{650} = 77/650 = 0.12$$

$$P(R_3) = \frac{7^2+8^2+9^2}{650} = 194/650 = 0.298.$$

$$P(R_4) = \frac{10^2+11^2+12^2}{650} = \frac{365}{650} = 0.56.$$