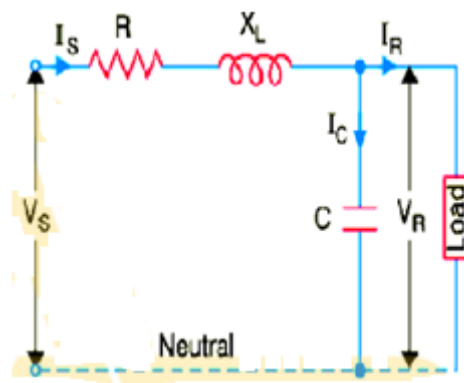


MODULE 3

MODELLING AND PERFORMANCE OF TRANSMISSION LINES

Classification of lines - short line, medium line and long line - equivalent circuits, phasor diagram, attenuation constant, phase constant, surge impedance; transmission efficiency and voltage regulation, real and reactive power flow in lines, Power - circle diagrams, surge impedance loading, methods of voltage control; Ferranti effect.



CLASSIFICATION OF LINES

The important considerations in the design and operation of a transmission line are the determination of voltage drop, line losses and efficiency of transmission. These values are greatly influenced by the line constants R , L and C of the transmission line. For instance the voltage drop in the line depends upon the values of above three line constants. Similarly, the resistance of transmission line conductors is the most important cause of power loss in the line and determines the transmission efficiency. In this chapter, we shall develop formulas by which we can calculate voltage regulation, line losses and efficiency of transmission lines. These formulas are important for two principal reasons. Firstly, they provide an opportunity to understand the effects of the parameters of the line on bus voltages and the flow of power. Secondly, they help in developing an overall understanding of what is occurring on electric power system.

CLASSIFICATION OF OVERHEAD TRANSMISSION LINES

A transmission line has three constants R , L and C distributed uniformly along the whole

length of the line. The resistance and inductance form the series impedance. The capacitance existing between conductors for 1-phase line or from a conductor to neutral for a 3-phase line forms a shunt path throughout the length of the line. Therefore, capacitance effects introduce complications in transmission line calculations. Depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as :

(i) Short transmission lines. When the length of an overhead transmission line is upto about 50 km and the line voltage is comparatively low (< 20 kV), it is usually considered as a short transmission line. Due to smaller length and lower voltage, the capacitance effects are small and hence can be neglected. Therefore, while studying the performance of a short transmission line, only resistance and inductance of the line are taken into account.

(ii) Medium transmission lines. When the length of an overhead transmission line is about 50-150 km and the line voltage is moderately high (>20 kV < 100 kV), it is considered as a medium transmission line. Due to sufficient length and voltage of the line, the capacitance effects are taken into account. For purposes of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more points.

(iii) Long transmission lines. When the length of an overhead transmission line is more than 150 km and line voltage is very high (> 100 kV), it is considered as a long transmission line. For the treatment of such a line, the line constants are considered uniformly distributed over the whole length of the line and rigorous methods are employed for solution. It may be emphasised here that exact solution of any transmission line must consider the fact that the constants of the line are not lumped but are distributed uniformly throughout the length of the line. However, reasonable accuracy can be obtained by considering these constants as lumped for short and medium transmission lines.

Important Terms

While studying the performance of a transmission line, it is desirable to determine its voltage regulation and transmission efficiency. We shall explain these two terms in turn.

(i) Voltage regulation. When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage (VR) of the line is generally less than the sending end voltage (VS). This voltage drop ($V_s - V_R$) in the line is expressed as a percentage of receiving end voltage V and is called

voltage regulation.

The difference in voltage at the receiving end of a transmission line **between conditions of no load and full load is called **voltage regulation** and is expressed as a percentage of the receiving end voltage.

(ii) **Transmission efficiency.** The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance. The ratio of receiving end power to the sending end power of a transmission line is known as the **transmission efficiency** of the line

PERFORMANCE OF SINGLE PHASE SHORT TRANSMISSION LINES

As stated earlier, the effects of line capacitance are neglected for a short transmission line. Therefore, while studying the performance of such a line, only resistance and inductance of the line are taken into account. The equivalent circuit of a single phase short transmission line is shown in Fig.

Here, the total line resistance and inductance are shown as concentrated or lumped instead of being distributed. The circuit is a simple a.c. series circuit.

Let I = load current

R = loop resistance i.e., resistance of both conductors

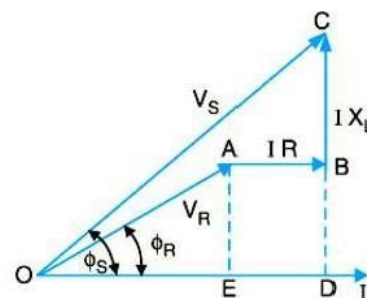
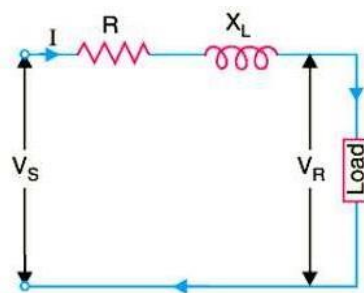
X_L = loop reactance

V_R = receiving end voltage

$\cos\phi_R$ = receiving end power factor (lagging)

V_S = sending end voltage

$\cos\phi_S$ = sending end power factor



The *phasor diagram of the line for lagging load power factor is shown in Fig. From the right angled triangle ODC, we get,

$$\begin{aligned}
 (OC)^2 &= (OD)^2 + (DC)^2 \\
 \text{or } V_S^2 &= (OE + ED)^2 + (DB + BC)^2 \\
 &= (V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2 \\
 \therefore V_S &= \sqrt{(V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2} \\
 (i) \text{ \%age Voltage regulation} &= \frac{V_S - V_R}{V_R} \times 100 \\
 (ii) \text{ Sending end p.f., } \cos \phi_S &= \frac{OD}{OC} = \frac{V_R \cos \phi_R + IR}{V_S} \\
 (iii) \text{ Power delivered} &= V_R I_R \cos \phi_R \\
 \text{Line losses} &= I^2 R \\
 \text{Power sent out} &= V_R I_R \cos \phi_R + I^2 R \\
 \text{\%age Transmission efficiency} &= \frac{\text{Power delivered}}{\text{Power sent out}} \times 100 \\
 &= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I^2 R} \times 100
 \end{aligned}$$

An approximate expression for the sending end voltage V_s can be obtained as follows. Draw S perpendicular from B and C on OA produced as shown in Fig. Then OC is nearly equal to OF

$$\begin{aligned}
 OC &= OF = OA + AF = OA + AG + GF \\
 &= OA + AG + BH \\
 V_s &= V_R + IR \cos \phi_R + I X_L \sin \phi_R
 \end{aligned}$$

THREE-PHASE SHORT TRANSMISSION LINES

For reasons associated with economy, transmission of electric power is done by 3-phase system. This system may be regarded as consisting of three single phase units, each wire

transmitting one-third of the total power. As a matter of convenience, we generally analyse 3-phase system by considering one phase only. Therefore, expression for regulation, efficiency etc. derived for a single phase line can also be applied to a 3-phase system. Since only one phase is considered, phase values of 3-phase system should be taken. Thus, V_s and V_R are the phase voltages, whereas R and X_L are the resistance and inductive reactance per phase respectively.

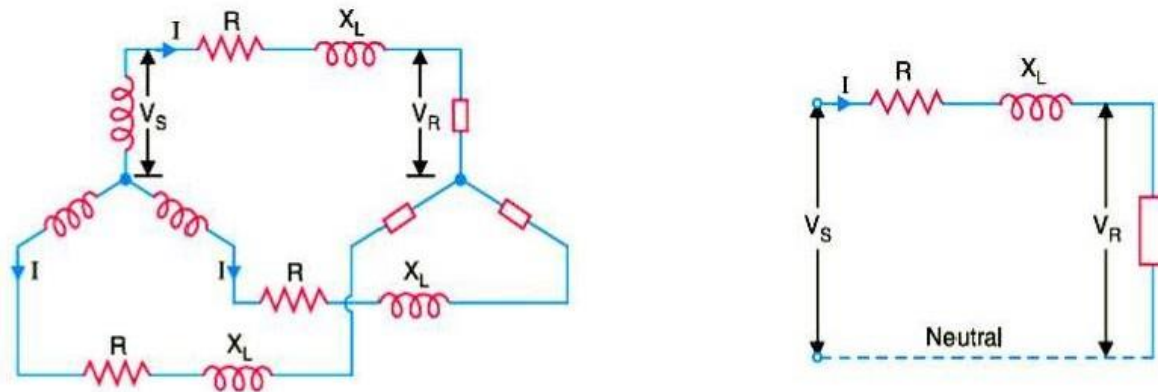


Fig (i) shows a Y-connected generator supplying a balanced Y-connected load through a transmission line. Each conductor has a resistance of $R \Omega$ and inductive reactance of $X \Omega$. Fig.(ii) shows one phase separately. The calculations can now be made in the same way as for a single phase line.

MEDIUM TRANSMISSION LINES

In short transmission line calculations, the effects of the line capacitance are neglected because such lines have smaller lengths and transmit power at relatively low voltages (< 20 kV). However, as the length and voltage of the line increase, the capacitance gradually becomes of greater importance.

Since medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, the effects of capacitance cannot be neglected. Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.

The capacitance is uniformly distributed over the entire length of the line. However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points. Such a treatment of localising the line capacitance gives reasonably accurate results. The most commonly used methods (known as localised capacitance methods) for the solution of medium transmissions

lines are :**(i) End condenser method**

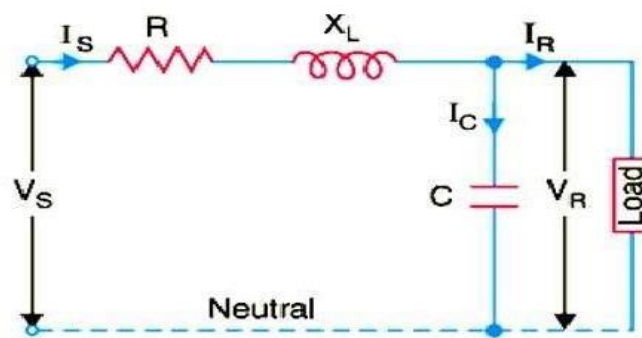
(ii) Nominal T method

(iii) Nominal π method.

Although the above methods are used for obtaining the performance calculations of medium lines, they can also be used for short lines if their line capacitance is given in a particular problem.

End Condenser Method

In this method, the capacitance of the line is lumped or concentrated at the receiving or load end as shown in Fig. This method of localising the line capacitance at the load end overestimates the effects of capacitance. In Fig, one phase of the 3-phase transmission line is shown as it is more convenient to work in phase instead of line-to-line values.



Let

I_R = load current per phase

R = resistance per phase

X_L = inductive reactance per phase

C = capacitance per phase

$\cos\phi_R$ = receiving end power factor (lagging)

V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig Taking the receiving end voltage V_R as the reference phasor,

$$\text{we have, } V_R = V_R + j0$$

$$\text{Load current, } \vec{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$$

$$\text{Capacitive current, } \vec{I}_C = j \vec{V}_R \omega C = j 2 \pi f C \vec{V}_R$$

The sending end current I_s is the phasor sum of load current I_R and capacitive current I_C i.e.

$$\begin{aligned}\vec{I}_S &= \vec{I}_R + \vec{I}_C \\ &= I_R (\cos \phi_R - j \sin \phi_R) + j 2 \pi f C V_R \\ &= I_R \cos \phi_R + j (-I_R \sin \phi_R + 2 \pi f C V_R) \\ &= \vec{I}_S \vec{Z} = \vec{I}_S (R + j X_L)\end{aligned}$$

$$\vec{V}_S = \vec{V}_R + \vec{I}_S \vec{Z} = \vec{V}_R + \vec{I}_S (R + j X_L)$$

Thus, the magnitude of sending end voltage V_s can be calculated.

$$\% \text{ Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

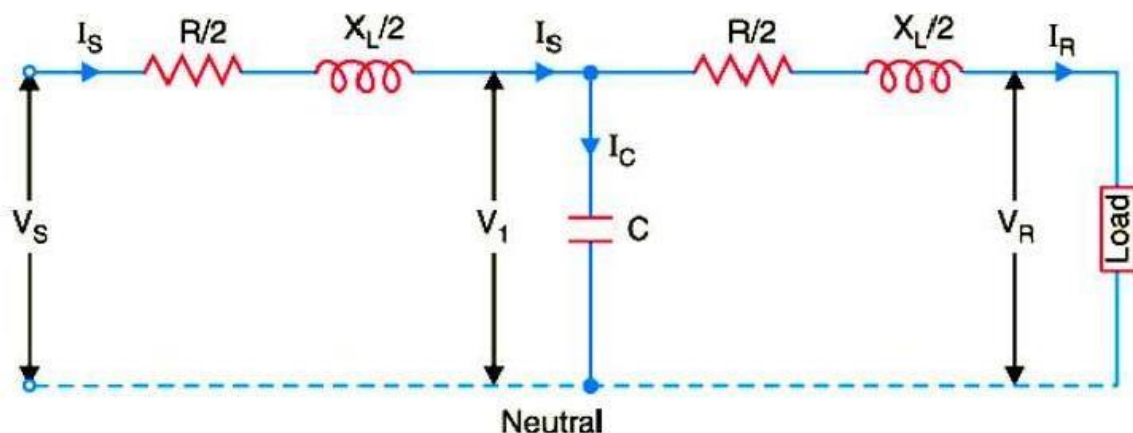
$$\begin{aligned}\% \text{ Voltage transmission efficiency} &= \frac{\text{Power delivered / phase}}{\text{Power delivered / phase} + \text{losses / phase}} \times 100 \\ &= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I_S^2 R} \times 100\end{aligned}$$

Limitations Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks :

- (i) There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.
- (ii) This method overestimates the effects of line capacitance.

ii) Nominal T Method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. Therefore, in this arrangement, full charging current flows over half the line. In Fig. one phase of 3-phase transmission line is shown as it is advantageous to work in phase instead of line-to-line values.



Let

I_R = load current per phase ;

R = resistance per phase

X_L = inductive reactance per phase ;

C = capacitance per phase

$\cos\phi_R$ = receiving end power factor (lagging) ;

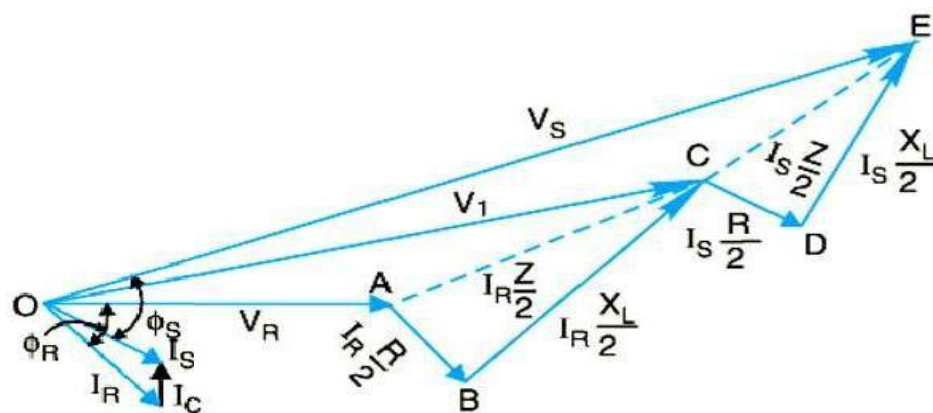
V_S = sending end voltage/phase

V_1 = voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig. Taking the receiving end voltage V_R as the reference phasor, we have,

Receiving end voltage, $\vec{V}_R = V_R + j0$

Load current, $\vec{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$



Voltage across C , $\vec{V}_1 = \vec{V}_R + \vec{I}_R \vec{Z} / 2$
 $= V_R + I_R (\cos \phi_R - j \sin \phi_R) \left(\frac{R}{2} + j \frac{X_L}{2} \right)$

Capacitive current, $\vec{I}_C = j \omega C \vec{V}_1 = j 2\pi f C \vec{V}_1$

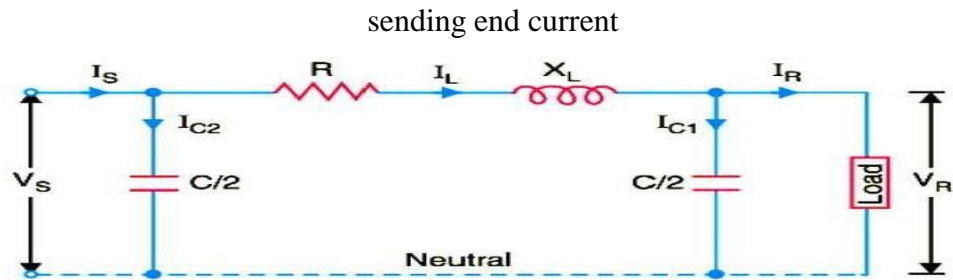
Sending end current, $\vec{I}_S = \vec{I}_R + \vec{I}_C$

Sending end voltage, $\vec{V}_S = \vec{V}_1 + \vec{I}_S \frac{\vec{Z}}{2} = \vec{V}_1 + \vec{I}_S \left(\frac{R}{2} + j \frac{X_L}{2} \right)$

iii) Nominal π Method

In this method, capacitance of each conductor (i.e., line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in Fig. It is obvious that capacitance at the sending end has no effect on the line drop.

However, its charging current must be added to line current in order to obtain the total



Let

I_R = load current per phase

R = resistance per phase

X_L = inductive reactance per phase

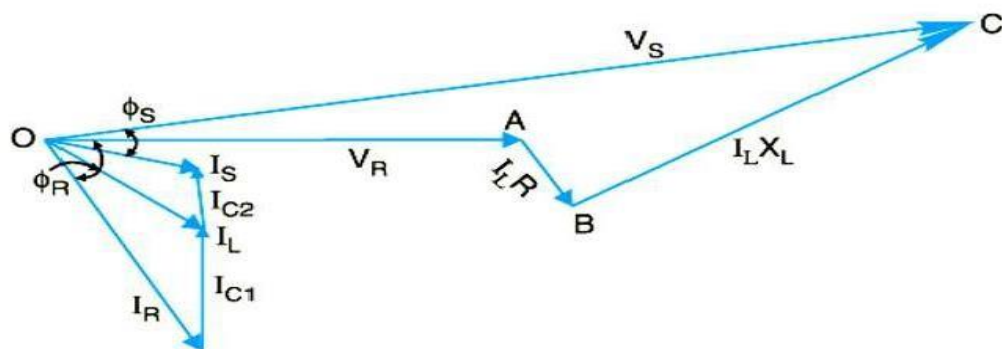
C = capacitance per phase

$\cos\phi_R$ = receiving end power factor (lagging)

V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig. Taking the receiving end voltage as the reference phasor, we have,

$$\begin{aligned}\vec{V}_R &= V_R + j0 \\ \text{Load current,} \quad \vec{I}_R &= I_R (\cos \phi_R - j \sin \phi_R) \\ \text{Charging current at load end is} \quad \vec{I}_{C1} &= j \omega (C/2) \vec{V}_R = j \pi f C \vec{V}_R\end{aligned}$$



$$\begin{aligned}\text{Line current,} \quad \vec{I}_L &= \vec{I}_R + \vec{I}_{C1} \\ \text{Sending end voltage,} \quad \vec{V}_S &= \vec{V}_R + \vec{I}_L \vec{Z} = \vec{V}_R + \vec{I}_L (R + jX_L) \\ \text{Charging current at the sending end is} \quad \vec{I}_{C2} &= j \omega (C/2) \vec{V}_S = j \pi f C \vec{V}_S \\ \therefore \text{Sending end current,} \quad \vec{I}_S &= \vec{I}_L + \vec{I}_{C2}\end{aligned}$$

LONG TRANSMISSION LINES

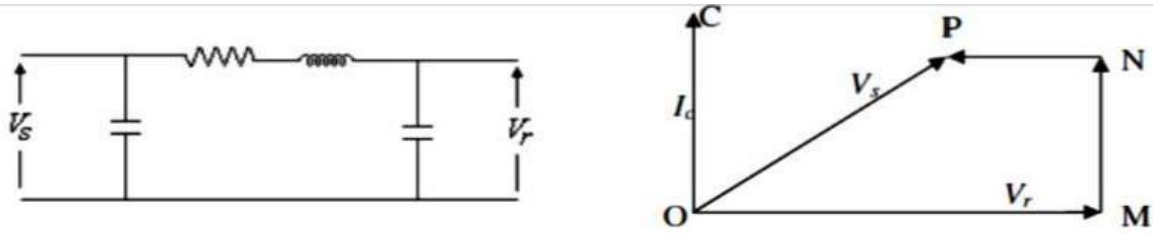
It is well known that line constants of the transmission line are uniformly distributed over the entire length of the line. However, reasonable accuracy can be obtained in line calculations for short and medium lines by considering these constants as lumped. If such an assumption of lumped constants is applied to long transmission lines (having length excess of about 150 km), it is found that serious errors are introduced in the performance calculations. Therefore, in order to obtain fair degree of accuracy in the performance calculations of long lines, the line constants are considered as uniformly distributed throughout the length of the line. Rigorous mathematical treatment is required for the solution of such lines. Fig shows the equivalent circuit of a 3-phase long transmission line on a phase-neutral basis. The whole line length is divided into n sections, each section having line constants $1/n$ th of those for the whole line. The following points may be noted :

- (i) The line constants are uniformly distributed over the entire length of line as is actually the case.
- (ii) The resistance and inductive reactance are the series elements.
- (iii) The leakage susceptance (B) and leakage conductance (G) are shunt elements. The leakage susceptance is due to the fact that capacitance exists between line and neutral. The leakage conductance takes into account the energy losses occurring through leakage over the insulators or due to corona effect between conductors. Admittance $= \sqrt{G^2 + B^2}$.
- (iv) The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached at which point its value is zero.

FERRANTI EFFECT

A long transmission line draws a substantial quantity of charging current. If such a line is open circuited or very lightly loaded at the receiving end, the voltage at receiving end may become greater than voltage at sending end. This is known as Ferranti Effect and is due to the voltage drop across the line inductance (due to charging current) being in phase with the sending end voltages. Therefore both capacitance and inductance is responsible to produce this phenomenon. The capacitance (and charging current) is negligible in short line but significant in medium line and appreciable in long line. Therefore this phenomenon occurs in medium and long lines.

Represent line by equivalent π model.



Line capacitance is assumed to be concentrated at the receiving end.

OM = receiving end voltage V_r

OC = Current drawn by capacitance = I_c

MN = Resistance drop

NP = Inductive reactance drop

Therefore;

OP = Sending end voltage at no load and is less than receiving end voltage (V_r)

Since, resistance is small compared to reactance; resistance can be neglected in calculating Ferranti effect.

From π model,

$$V_s = \left(1 + \frac{YZ}{2}\right) V_r + ZI_r$$

For open circuit line; $I_r = 0$

$$\therefore V_s = \left(1 + \frac{YZ}{2}\right) V_r$$

$$\text{or, } V_s - V_r = \left(1 + \frac{YZ}{2}\right) V_r - V_r = V_r \left(1 + \frac{YZ}{2} - 1\right)$$

$$\text{or, } V_s - V_r = \left(\frac{YZ}{2}\right) V_r = \frac{(j\omega C l)(r + j\omega L)l}{2} V_r$$

Neglecting resistance;

$$V_s - V_r = \frac{-V_r \omega^2 l^2 LC}{2}$$

Substituting the value in above equation;

$$LC = \frac{1}{(3 \times 10^5)^2}$$

$$V_s - V_r = \frac{-V_r \omega^2 l^2}{2 (3 \times 10^5)^2}$$

$$\therefore V_s - V_r = \frac{-V_r \omega^2 l^2 \times 10^{-10}}{18}$$

$$\therefore V_s = V_r \left[1 - \frac{\omega^2 l^2 \times 10^{-10}}{18} \right]$$

Now, from above expression;

$$\left[1 - \frac{\omega^2 l^2 \times 10^{-10}}{18} \right] < 1$$

$$V_s < V_r \quad \text{or;} \quad V_r > V_s$$

i.e. receiving end voltage is greater than sending end voltage and this effect is called Ferranti Effect. It is valid for open circuit condition of long line.