MSO202A COMPLEX ANALYSIS
Assignment 4

Exercise Problems:

1. Verify Cauchy's theorem for $f(z) = z^2$ over the boundary of the square with vertices $1+i$, $-1+i$, $-1-i$ and $1-i$, counterclockwise.

2. Use ML-inequality to prove the following:

   (a) $\left|\int_\gamma \frac{1}{1+z^2}\, dz\right| \le \frac{\pi}{3}$, γ is the arc of $|z| = 2$ from 2 to $2i$.

   (b) $\left|\int_\gamma (1+z^2)\, dz\right| \le \pi R(R^2+1)$, γ is the semicircular arc of $|z| = R$.

3. By parametrizing the curve or otherwise, evaluate:

   (a) $\int_C \tan z\, dz$, where C is the circle $|z| = 1$ oriented counter -clockwise.

   (b) $\int_C \mathrm{Re}\ z^2\, dz$, C is the circle $|z| = 1$ oriented counter -clockwise.

   (c) $\int_C e^{4z}\, dz$, C is the shortest path from $8-3i$ to $8-(3+\pi)i$.

4. Use Cauchy's integral formula to find all simple closed curves C for which the following holds:
   $(a)\ \displaystyle\int_C \frac{1}{z}\, dz = 0,\quad (b)\ \int_C \frac{e^{1/z}}{z^2+9}\, dz = 0.$

5. Integrate $\frac{z^2}{z^4-1}$ counter-clockwise around the circle $(a)|z+1| = 1$ (b) $|z+i| = 1$.

6. Integrate the functions counter-clockwise on the unit circle $|z| = 1$:
   $(a)\frac{z^3}{2z-i}$ (b) $\frac{\cosh 3z}{2z}$ (c) $\frac{z^3 \sin z}{3z-1}$.

7. Let Γ denote the positively (counter-clockwise) oriented boundary of the square whose sides lie on the lines $x = \pm 2$ and $y = \pm 2$. Using Cauchy's integral formula, evaluate the following integrals:

   $$(a) \int_\Gamma \frac{\cos z}{z(z^2+8)}\, dz \quad (b) \int_\Gamma \frac{z}{2z+1}\, dz.$$

Problem for Tutorial:

8. Let C be the positively oriented circle $|z| = 3$. If $f(w) = \int_C \frac{2z^2-z-2}{z-w}\, dz, |w| \ne 3$, then show that $f(2) = 8i\pi$. What is $f(w)$, if $|w| > 3$?

9. Use Cauchy's integral formula to find closed contours C in complex plane satisfying
   $(a)\ \displaystyle\int_C \mathrm{Log}\,(z)\, dz = 0\ (b)\ \int_C \frac{\cos z}{z^6-z^2}\, dz = 0\ .$

10. Using Cauchy's integral formula, integrate counterclockwise:

$$\oint_C \frac{\text{Ln }(z+1)}{z^2+1}\, dz, \quad C:\ |z-2i|=2.$$