Contents

same as in the real case...

An infinite sequence of complex numbers indexed by natural numbers is called a *sequence*.
Eg: $\{1/n\}$, $\{z^n\}$.

A subset of a sequence indexed by a strictly increasing sequence of natural numbers is called a *subsequence*.
Eg: $\{1/2n\}$ is a subsequence of $\{1/n\}$.

A sequence $\{z_n\}$ is said to *converge to a limit* z if $z_n \to z$ as $n \to \infty$.

Given an $\epsilon > 0$ there is a $N > 0$ such that $|z_n - z| < \epsilon$ for all $n > N$.

Eg:$\{1/n\}$ converges to 0.

$\{z_n\}$ converges to z if and only if $\{Re(z_n)\}$ and $\{Im(z_n)\}$ converge to $Re(z)$ and $Im(z)$, respectively.

A sequence $\{z_n\}$ is a *Cauchy sequence* if "given an $\epsilon > 0$ there is a $N > 0$ such that $|z_n - z_m| < \epsilon$ for all $n, m > N$."

Every Cauchy sequence in $\mathbb{C}$ converges in $\mathbb{C}$ (why?).

Remark: A sequence $z_n \to z$ then the real sequence $|z_n|$ converges to $|z|$.

Determine if the following sequence converge or diverge (*i*.e., does not converge):

- $\{1/n\}$

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- $\{z^n\}$

Given a sequence of complex numbers $\{z_n\}$, an infinite sum of the form $\sum_{n=0}^{\infty} z_n$ is called a *series*.
Eg: $\sum_{n=0}^{\infty} 1/n$

Partial sums of a series: Given a series $\sum_{n=0}^{\infty} z_n$ the k-th partial sum s_k is given by $\sum_{n=0}^{k} z_n$.

A series is said to converge to a limit s if the sequence of partial sums $\{s_k\}$ converges to s.
Eg: $\sum_{n=0}^{\infty} 1/(2^n) = 2$.

- $\sum_{n=0}^{\infty} z_n$ converges if and only if $\sum_{n=0}^{\infty} Re(z_n)$ and $\sum_{n=0}^{\infty} Im(z_n)$ converges.
- $\sum_{n=0}^{k} z_n \pm \sum_{n=0}^{k} w_n = \sum_{n=0}^{k}(z_n \pm w_n)$; $c\sum_{n=0}^{k} z_n = \sum_{n=0}^{k} cz_n$.

Necessary condition for convergence of a series

If $\sum_{n=0}^{\infty} z_n$ converges then $\{z_n\}$ is a null sequence.

- Comparison test: If $\sum_{n=0}^{k} |w_n|$ converges and $|z_n| \leq |w_n|$ for all $n > N$ for some fixed N, then $\sum_{n=0}^{k} z_n$ also converges.

Ratio Test: For a series $\sum_{n=0}^{k} z_n$ if the sequence of ratios $|a_{n+1}/a_n|$ has a limit l. Then

- i $\sum_{n=0}^{k} z_n$ converges absolutely if $l < 1$;

  ii $\sum_{n=0}^{k} z_n$ diverges if $l > 1$;

  iii if $l = 1$ then the series may or may not converge.

Root Test: For a series $\sum_{n=0}^{k} z_n$ let the lim *sup* $\sqrt[n]{|a_n|}$ be l. Then

- i $\sum_{n=0}^{k} z_n$ converges absolutely if $l < 1$;

  ii $\sum_{n=0}^{k} z_n$ diverges if $l > 1$;

  iii if $l = 1$ then the series may or may not converge.