1/1

(a) $m = \frac{W}{g} = \frac{3600}{32.2} = \underline{111.8 \text{ slugs}}$

(b) $W = 3600 \text{ lb}\left[\frac{4.4482 \text{ N}}{\text{lb}}\right] = \underline{16\,010 \text{ N}}$

(c) $m = \frac{W}{g} = \frac{16\,010}{9.81} = \underline{1632 \text{ kg}}$

$\left(\text{or } m = 111.8 \text{ slugs}\left[\frac{14.594 \text{ kg}}{\text{slug}}\right] = 1632 \text{ kg}\right)$

1/2 For a 180-lb person :

$W = mg : \ 180 \text{ lb} = m\,(32.2 \text{ ft/sec}^2)$

$m = \underline{5.59 \text{ slugs}}$

$180 \text{ lb}\left(\frac{4.4482 \text{ N}}{\text{lb}}\right) = \underline{801 \text{ N}}$

$W = mg : \ 801 \text{ N} = m\,(9.81 \text{ m/s}^2)$

$m = \underline{81.6 \text{ kg}}$

1/3 $\underline{V}_1 = 15\left(\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}\right) = 12\underline{i} + 9\underline{j}$

$\underline{V}_2 = 12\,(-\cos 60°\underline{i} + \sin 60°\underline{j}) = -6\underline{i} + 10.39\underline{j}$

$V_1 + V_2 = 15 + 12 = \underline{27}$

$\underline{V}_1 + \underline{V}_2 = (12-6)\,\underline{i} + (9+10.39)\underline{j} = \underline{6\underline{i} + 19.39\underline{j}}$

$\underline{V}_1 - \underline{V}_2 = (12-(-6))\,\underline{i} + (9-10.39)\underline{j} = \underline{18\underline{i} - 1.392\underline{j}}$

$\underline{V}_1 \times \underline{V}_2 = (12\underline{i} + 9\underline{j}) \times (-6\underline{i} + 10.39\underline{j})$

$= (124.7 + 54)\underline{k} = \underline{178.7\underline{k}}$

$\underline{V}_2 \times \underline{V}_1 = -(\underline{V}_1 \times \underline{V}_2) = \underline{-178.7\underline{k}}$

$\underline{V}_1 \cdot \underline{V}_2 = (12\underline{i} + 9\underline{j}) \cdot (-6\underline{i} + 10.39\underline{j})$

$= 12(-6) + 9\,(10.39) = \underline{21.5}$

1/4 The weight of an average apple is

$W = \frac{5 \text{ lb}}{12 \text{ apples}} = 0.417 \text{ lb}$

Mass in slugs is $m = \frac{W}{g} = \frac{0.417}{32.2} = \underline{0.01294 \text{ slugs}}$

Mass in kg is $m = 0.01294 \text{ slugs}\left(\frac{14.594 \text{ kg}}{1 \text{ slug}}\right)$

$= \underline{0.1888 \text{ kg}}$

Weight in N is $W = mg = 0.1888\,(9.81) = \underline{1.853 \text{ N}}$

These apples weigh closer to 2 N each than to the rule of 1 N each !

1/5 Mass of iron sphere $m = \rho V$

$= \left(7210 \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{4}{3}\pi\,(0.050)^3\right) = 3.78 \text{ kg}$

Force of mutual attraction : $\frac{Gm^2}{d^2}$

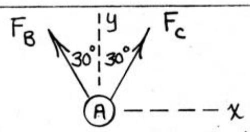
Weight of each sphere : $\frac{Gm_e m}{r^2}$

$\frac{Gm^2}{d^2} = \frac{Gm_e m}{r^2}, \quad r = d\sqrt{\frac{m_e}{m}}$

$= 0.1\sqrt{\frac{5.976 \times 10^{24}}{3.78}}\ \frac{1}{10^3}$

$= \underline{1.258\,(10^8) \text{ km}}$

1/6



$r = 0.050$ m for all spheres

$F_B = \frac{Gm_A m_B}{d_{AB}^2} = \frac{G\left(\rho_A \frac{4}{3}\pi r^3\right)\left(\rho_B \frac{4}{3}\pi r^3\right)}{d_{AB}^2}$

$= \frac{6.673(10^{-11})\left[\frac{4}{3}\pi\,(0.050)^3\right]^2(8910)(2690)}{1^2}$

$= 4.38\,(10^{-10}) \text{ N}$

$F_C = \frac{Gm_A m_C}{d_{AC}^2} = \frac{G\left[\frac{4}{3}\pi r^3\right]^2 \rho_A \rho_C}{d_{AC}^2}$

$= \frac{6.673\,(10^{-11})\left[\frac{4}{3}\pi\,(0.050)^3\right]^2(8910)(7210)}{1^2}$

$= 1.175\,(10^{-9}) \text{ N}$

$\underline{R} = \underline{F}_B + \underline{F}_C = 4.38\,(10^{-10})\left[-\sin 30°\underline{i} + \cos 30°\underline{j}\right]$

$+ 1.175\,(10^{-9})\left[\sin 30°\underline{i} + \cos 30°\underline{j}\right]$

$\underline{\underline{R} = (3.68\underline{i} + 13.98\underline{j})\,10^{-10} \text{ N}}$

1/7 $g_h = \frac{Gm_e}{(R+h)^2}$

$= \frac{(3.439 \times 10^{-8})\,(4.095 \times 10^{23})}{\left[(3959)(5280) + (150)(5280)\right]^2} = \underline{29.9 \text{ ft/sec}^2}$

Mass of man : $m = \frac{W}{g} = \frac{200}{32.174} = 6.22$ slugs

Absolute weight at h= 150 miles :

$W_h = mg_h = (6.22)(29.9) = \underline{186.0 \text{ lb}}$

The terms "zero-g" and "weightless" are definitely misnomers in this instance.

1/8 $mg = 0.1\,mg_0$

$\frac{R^2}{(R+h)^2}\,g_0 = 0.1\,g_0$

Solve for h to obtain $\underline{h = 2.16R}$

1/9 $g_{rel} = 9.780327(1 + 0.005279 \sin^2\gamma + 0.000023 \sin^4\gamma)$

At $\gamma = 45°$, $\underline{g_{rel} = 9.806 \text{ m/s}^2}$

$g_{abs} = g_{rel} + 0.03382 \cos^2\gamma$

$= 9.806198 + 0.03382 \cos^2 45°$

$\underline{= 9.823 \text{ m/s}^2}$

1/10

$g_{rel} = 9.780327(1 + 0.005279 \sin^2\gamma + 0.000023 \sin^4\gamma + \cdots)$

At $\gamma = 40°$, $g_{rel} = 9.801698 \text{ m/s}^2$

$g_{abs} = g_{rel} + 0.03382 \cos^2\gamma$

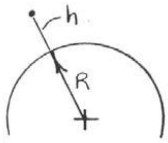
$= 9.801698 + 0.03382 \cos^2 40°$

$= 9.821544 \text{ m/s}^2$

$W_{abs} = mg_{abs} = 90(9.821544) = \underline{883.9 \text{ N}}$

$W_{rel} = mg_{rel} = 90(9.801698) = \underline{882.2 \text{ N}}$

1/11 $W = mg$, $g = g_0\left(\frac{R}{R+h}\right)^2$



From Fig. 1/1, $g_0 = 9.818 \text{ m/s}^2$

@ 28° N latitude & sea level.

At $h = 2440$ m :

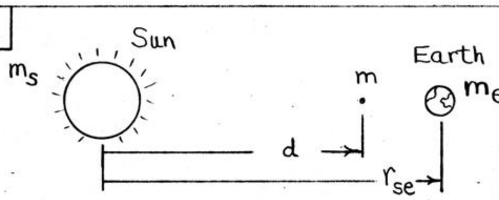
$$g = 9.818\left[\frac{6371(10^3)}{6371(10^3) + 2440}\right]^2 = 9.810 \text{ m/s}^2$$

At $h = 8848$ m :

$$g = 9.818\left[\frac{6371(10^3)}{6371(10^3) + 8848}\right]^2 = 9.791 \text{ m/s}^2$$

$\Delta W = m\Delta g = 80(9.810 - 9.791) = \underline{1.576 \text{ N}}$

1/12



Newton's Universal Gravitational Law:

$$\frac{Gmm_s}{d^2} = \frac{Gmm_e}{(r_{se} - d)^2}$$

$$d^2[m_s - m_e] - d[2m_s r_{se}] + m_s r_{se}^2 = 0$$

Substitute $m_e = 5.976(10^{24})$ kg,

$m_s = 333\,000\,[5.976(10^{24})]$ kg,

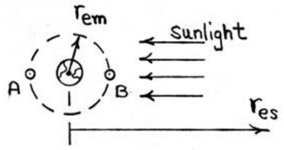
and $r_{se} = 149.6(10^9)$ m,

then solve the quadratic to obtain

$d = 149.3(10^9)$ m

or $\underline{\underline{d = 149.9(10^9) \text{ m}}}$

1/13



Force exerted by earth on moon :

$$F_e = \frac{Gm_e m_m}{r_{em}^2} = \frac{(6.673\times10^{-11})(5.976\times10^{24})^2(1)(0.0123)}{(3.84\,398\times10^8)^2}$$

$= 1.984 \times 10^{20}$ N

Forces exerted by sun on moon :

$$F_{S_A} = \frac{Gm_s m_m}{(r_{es}+r_{em})^2} = \frac{(6.673\times10^{-11})(5.976\times10^{24})^2(333{,}000)(0.0123)}{(1.496\times10^{11} + 3.84398\times10^8)^2}$$

$= 4.34 \times 10^{20}$ N

$$F_{S_B} = \frac{Gm_s m_m}{(r_{es}-r_{em})^2} = 4.38\times10^{20} \text{ N}$$

Ratios :

$R_A = 2.19$

$\underline{R_B = 2.21}$

1/14 $C_D = \dfrac{D}{\frac{1}{2}\rho v^2 S}$

$$[C_D] = \frac{MLT^{-2}}{(M/L^3)(L/T)^2 L^2} = 1$$

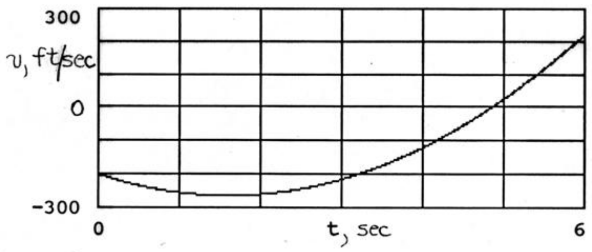
$\underline{C_D \text{ is nondimensional.}}$

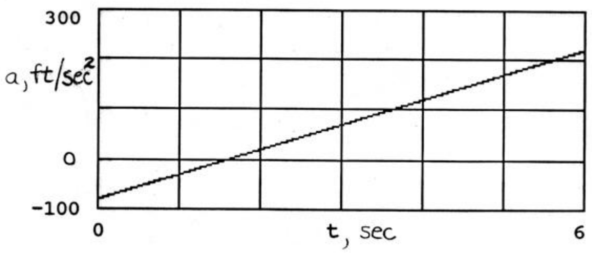
2/1 $v = 25t^2 - 80t - 200$

$a = \frac{dv}{dt} = 50t - 80$ } See plots

$a = 0: \quad 50t - 80 = 0, \quad t = 1.6$ sec

At $t = 1.6$ sec, $v = 25(1.6)^2 - 80(1.6) - 200 = \underline{-264 \ \frac{ft}{sec}}$





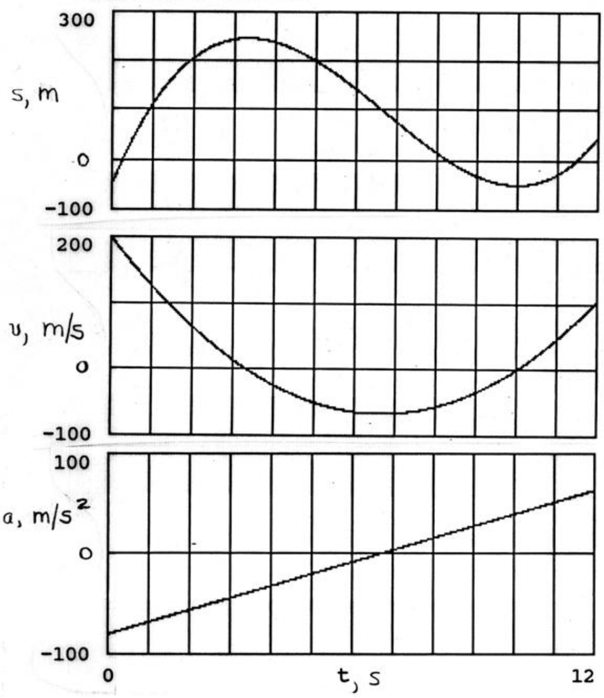
2/2 $s = 2t^3 - 40t^2 + 200t - 50$

$v = \frac{ds}{dt} = 6t^2 - 80t + 200$

$a = \frac{dv}{dt} = 12t - 80$ } See plots

$v = 0: \ 6t^2 - 80t + 200 = 0, \quad t = \frac{80 \pm \sqrt{80^2 - 4(6)(200)}}{2(6)}$

$\underline{t = 3.33 \text{ s}, \ 10 \text{ s}}$



2/3 $v = 2 - 4t + 5t^{3/2}$

$a = \frac{dv}{dt} = -4 + \frac{15}{2}t^{1/2}$

$\frac{ds}{dt} = 2 - 4t + 5t^{3/2}$

$\int_{s_0=3}^{s} ds = \int_0^t (2 - 4t + 5t^{3/2})\,dt$

$s = 3 + 2t - 2t^2 + 2t^{5/2}$

At $t = 3$ s: $\begin{cases} s = 22.2 \text{ m} \\ v = 15.98 \text{ m/s} \\ a = 8.99 \text{ m/s}^2 \end{cases}$

2/4 $s = (-2 + 3t)e^{-0.5t}$

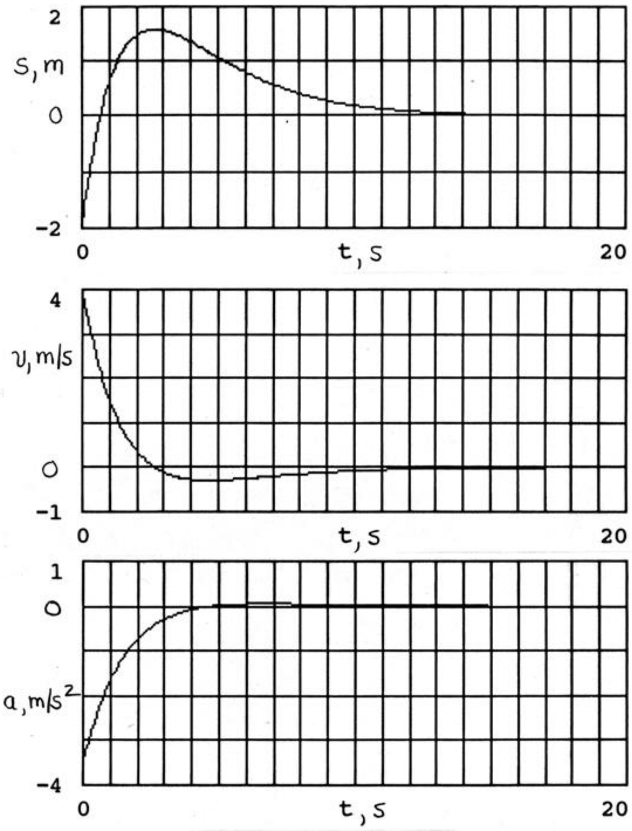
$v = \frac{ds}{dt} = 3e^{-0.5t} + (-2+3t)(-0.5)e^{-0.5t}$

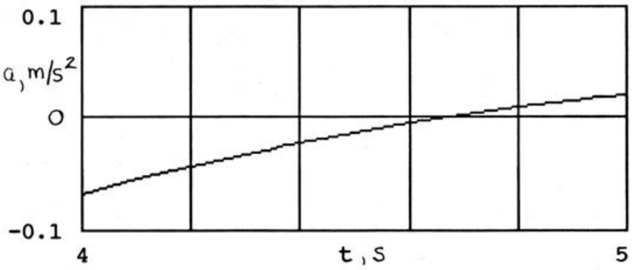
$= \underline{(4 - 1.5t)e^{-0.5t}}$

$a = \frac{dv}{dt} = -1.5e^{-0.5t} + (4 - 1.5t)(-0.5)e^{-0.5t}$

$= \underline{(-3.5 + 0.75t)e^{-0.5t}}$

$a = 0: \ (-3.5 + 0.75t)e^{-0.5t} = 0, \quad \underline{t = 4.67 \text{ s}}$





2/5

$a = \frac{dv}{dt} = 2t - 10$

$\int_{v_0=3}^{v} dv = \int_0^t (2t-10)\,dt$, $\underline{v = 3 - 10t + t^2}$ (m/s)

$\frac{ds}{dt} = 3 - 10t + t^2$

$\int_{s_0=-4}^{s} ds = \int_0^t (3 - 10t + t^2)\,dt$

$\underline{s = -4 + 3t - 5t^2 + \frac{1}{3}t^3}$ (m)

2/6

$a = v\frac{dv}{ds} = -ks^2$

$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} ks^2\,ds \Rightarrow v^2 = v_0^2 - \frac{2}{3}k(s^3 - s_0^3)$

Taking positive sign : $\underline{v = \left[v_0^2 - \frac{2}{3}k(s^3 - s_0^3)\right]^{1/2}}$

Numbers : $v = \left[10^2 - \frac{2}{3}(0.1)(5^3 - 3^3)\right]^{1/2}$

$= \underline{9.67 \text{ m/s}}$

2/7

$a = -kv^{1/2} = \frac{dv}{dt}$

$-\int_0^t k\,dt = \int_{v_0}^{v} \frac{dv}{v^{1/2}} \Rightarrow \underline{v = \left(v_0^{1/2} - \frac{1}{2}kt\right)^2}$

Numbers : $v = \left(7^{1/2} - \frac{1}{2}(0.2)(2)\right)^2 = \underline{5.98 \text{ m/s}}$

Also, $-kv^{1/2} = v\frac{dv}{ds}$

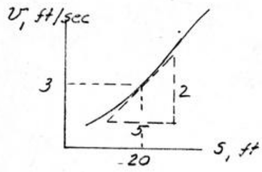
$-\int_{s_0}^{s} k\,ds = \int v^{1/2}\,dv \Rightarrow \underline{v = \left[v_0^{3/2} - \frac{3}{2}k(s - s_0)\right]^{2/3}}$

Numbers : $v = \left[7^{3/2} - \frac{3}{2}(0.2)(3-1)\right]^{2/3} = \underline{6.85 \text{ m/s}}$

2/8

$a = v\frac{dv}{ds} = 10(-3) = \underline{-30 \text{ m/s}^2}$

2/9



$a = v\frac{dv}{ds} = 3\left(\frac{2}{5}\right) = \underline{1.2 \text{ ft/sec}^2}$

2/10

+y

$y = v_0 t + \frac{1}{2}at^2$, $y = 80t - \frac{1}{2}32.2t^2$

for $y = -200$ ft,

$-200 = 80t - 16.1t^2$

or $16.1t^2 - 80t - 200 = 0$

$t = \frac{80 \pm \sqrt{(80)^2 + 4(16.1)(200)}}{2(16.1)} = \underline{6.80 \text{ sec}}$ (or -1.83 s)

For $\dot{y} = 0$, $v^2 = v_0^2 + 2ay$, $y = h = \frac{0 - 80^2}{-2(32.2)} = \underline{99.4 \text{ ft}}$

2/11

For constant acceleration,

$s = \frac{1}{2}at^2$, $t = \left(\frac{2s}{a}\right)^{1/2} = \left(\frac{2(30\,000)}{1.5(9.81)}\right)$

$= \underline{63.9 \text{ s}}$

$v = \sqrt{2as} = \sqrt{2(1.5)(9.81)(30\,000)} = \underline{940 \text{ m/s}}$

2/12

$v^2 - v_0^2 = 2a(s - s_0)$

$0 - \left[50\,\frac{5280}{3600}\right]^2 = 2a(100)$, $a = -26.9\,\frac{\text{ft}}{\text{sec}^2}$

Then $0 - \left[70\,\frac{5280}{3600}\right]^2 = 2(-26.9)\,s$

$\underline{s = 196.0 \text{ ft}}$

2/13

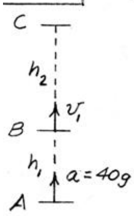
For $a =$ constant, $v^2 = v_0^2 + 2as$

$\left[\frac{180\,(5280)}{3600}\right]^2 = 0^2 + 2a\,(300)$

$a = 116.2 \text{ ft/sec}^2$

or $a = 116.2/32.2 = \underline{3.61g}$

2/14



B to C; $t = 10/2 = 5$ s

$v = v_1 + at$; $0 = v_1 - 9.81(5)$, $v_1 = 49.0$ m/s

$v^2 = v_1^2 + 2as$; $0 = (49.0)^2 + 2(-9.81)h_2$

$h_2 = 122.6$ m

A to B; $v^2 = v_0^2 + 2as$;

$(49.0)^2 = 0 + 2(40)(9.81)h_1$

$h_1 = 3.07$ m

$h = h_1 + h_2 = \underline{125.7 \text{ m}}$

2/15

$v^2 = v_0^2 + 2a(s - s_0)$

$\left(\frac{200}{3.6}\right)^2 = 0^2 + 2(0.4 \cdot 9.81)\,s$

$\underline{s = 393 \text{ m}}$

$v = v_0 + at$: $\left(\frac{200}{3.6}\right) = 0 + 0.4(9.81)\,t$

$\underline{t = 14.16 \text{ s}}$

2/16

$\int v\,dv = \int a\,ds$; $\int_{200/3.6}^{30/3.6} v\,dv = a\int_0^{600} ds$

200 km/h

= 200/3.6 m/s

$\frac{1}{2(3.6)^2}\left(\overline{30}^2 - \overline{200}^2\right) = 600a$

30 km/h

= 30/3.6 m/s

$\underline{a = -2.51 \text{ m/s}^2}$

2/16

$\int v\,dv = \int a\,ds$; $\int_{200/3.6}^{30/3.6} v\,dv = a\int_0^{600} ds$

200 km/h

= 200/3.6 m/s

$\frac{1}{2(3.6)^2}\left(\overline{30}^2 - \overline{200}^2\right) = 600a$

30 km/h

= 30/3.6 m/s

$\underline{a = -2.51 \text{ m/s}^2}$

2/17 $v = 20e^{-t/10}$

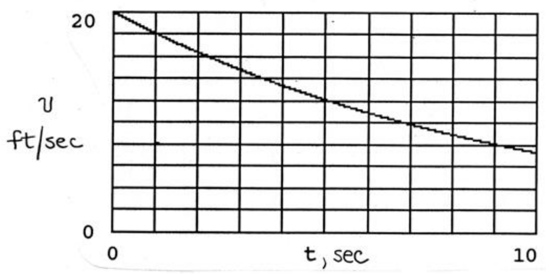
$a = \dot{v} = -2e^{-t/10}$

When $t = 10$ sec, $a = -2e^{-10/10} = \underline{-0.736 \text{ ft/sec}^2}$

From $v = \frac{ds}{dt} = 20e^{-t/10}$,

$\int_0^s ds = \int_0^{10} 20e^{-t/10}\,dt$

$s = -200e^{-t/10}\Big|_0^{10} = \underline{126.4 \text{ ft}}$



2/18 $v^2 = v_0^2 + 2as$, where $a = g/6$

$v^2 = 2^2 + 2\left(\frac{9.81}{6}\right)5$, $\underline{v = 4.51 \text{ m/s}}$

2/19

Δs = area under v-t curve

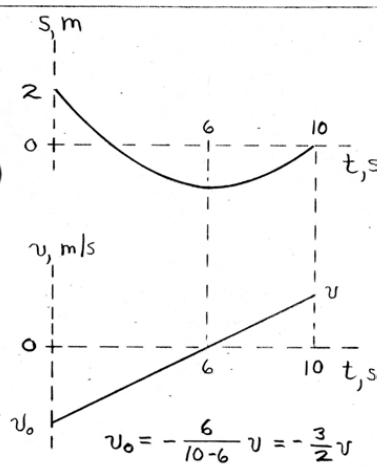
$0 - 2 = -\frac{3}{2}v\left(\frac{6}{2}\right) + \frac{1}{2}v(4)$

$\underline{v = 0.8 \text{ m/s}}$

Acceleration

$a = \frac{dv}{dt} = \frac{0.8}{4}$

$= \underline{0.2 \text{ m/s}^2}$



$v_0 = -\frac{6}{10-6}v = -\frac{3}{2}v$

2/20 Acceleration period :

$v = v_0 + at$: $\frac{22}{3.6} = 0 + \frac{9.81}{4}t_a$, $t_a = 2.49$ s

Note that the deceleration time $t_d = t_a$

$v^2 = v_0^2 + 2a\Delta s$: $\left(\frac{22}{3.6}\right)^2 = 0^2 + 2\,\frac{9.81}{4}\,\Delta s_a$

$\Delta s_a = 7.61 \text{ m} = \Delta s_d$

Cruise period : $\Delta s_c = 350 - \Delta s_a - \Delta s_d = 335$ m

$\Delta s = v_c t_c$: $335 = \frac{22}{3.6}t_c$, $t_c = 54.8$ s

Total run time $t = t_c + t_a + t_d = \underline{59.8 \text{ s}}$

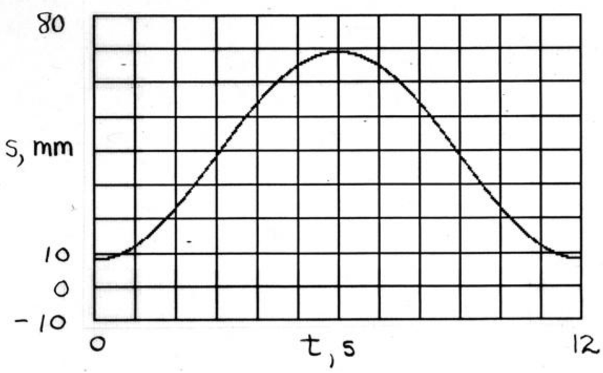
2/21 $v = \frac{ds}{dt} = 16 \sin\frac{\pi t}{6}$

$\int_8^s ds = 16\int_0^t \sin\frac{\pi t}{6}\,dt$

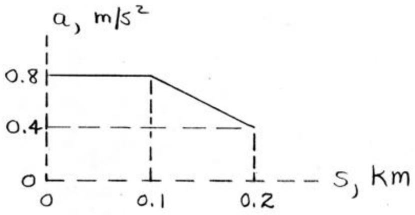
$s = 8 + 16\cdot\frac{6}{\pi}\left(-\cos\frac{\pi t}{6}\right)\Big|_0^t = 8 + \frac{96}{\pi}\left[1 - \cos\frac{\pi t}{6}\right]$

$s = s_{max}$ when $\cos\frac{\pi t}{6} = -1$ or $t = 6$ s

$s_{max} = 8 + \frac{96}{\pi}\left[1-(-1)\right] = \underline{69.1 \text{ mm}}$



2/22



From $a = v\frac{dv}{ds}$,

$\int_{v_0}^{v} v\,dv = \int_0^{200} a\,ds$ = area under a-s curve

$\frac{v^2}{2} - \frac{(40/3.6)^2}{2} = 0.8(100) + 0.6(100)$

$\underline{v = 20.1 \text{ m/s}}$ or $\underline{72.3 \text{ km/h}}$

2/23 $s = s_0 + v_0 t + \frac{1}{2}gt^2$

Sphere ①: $H = \frac{1}{2}gt_2^2$

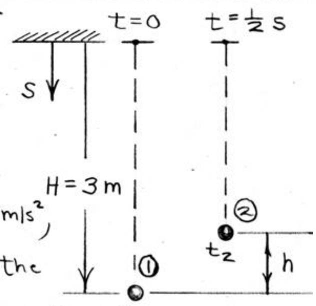
Sphere ②:

$(H-h) = \frac{1}{2}g\left(t_2 - \frac{1}{2}\right)^2$

With $H = 3$ m & $g = 9.81 \text{ m/s}^2$,

eliminate t_2 between the

2 equations & obtain $\underline{h = 2.61 \text{ m}}$.



2/24 $\Delta v = \int a\,dt$ = area under a-t curve

For $t = 4$s, $v_4 - 100 = -4(9.81)\frac{4-2}{2}$, $\underline{v_4 = 60.8 \text{ m/s}}$

For $t = 8$s, $v_8 - 60.8 = -4(9.81)(6-4)$, $\underline{v_8 = -17.72 \text{ m/s}}$

2/25

$v^2 = v_0^2 + 2a(s-s_0)$

$0 = 4^2 + 2\left(-\frac{9.81}{4}\right)(s)$, $\underline{s = 3.26\ m}$

$v = v_0 + at$: $0 = 4 + \left(-\frac{9.81}{4}\right)t_{up}$, $t_{up} = 1.631\ s$

$t = 2t_{up} = 2(1.631) = \underline{3.26\ s}$

2/26

$a = \frac{1}{2}\frac{d(v^2)}{ds} = \frac{1}{2}\frac{900-2500}{400-100} = -\frac{8}{3}\frac{ft}{sec^2}$ (constant)

$\Delta v = \int a\,dt$; $v - 50 = -\frac{8}{3}t$

At B: $30 - 50 = -\frac{8}{3}t$, $t = 7.50$ sec

$\Delta s = \int v\,dt = \int_{5.5}^{7.5}\left(50 - \frac{8}{3}t\right)dt = \underline{65.3\ ft}$

2/27

$a = 400 - kx$, where $k = \frac{400}{6/12}\ sec^{-2}$

$a = 400(1-2x)$ (x in ft)

$v\,dv = a\,dx$: $\int_0^v v\,dv = 400\int_0^x (1-2x)\,dx$

$v^2 = 800(x - x^2)$, $v = \frac{dx}{dt} = 20\sqrt{2}\sqrt{x-x^2}$ (taking + sign)

$\int_0^t dt = \int_0^x \frac{dx}{20\sqrt{2}\sqrt{x-x^2}}$

$t = -\frac{1}{20\sqrt{2}}\sin^{-1}\frac{1-2x}{\sqrt{1}}\Big|_0^x = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - \sin^{-1}(1-2x)\right]$

(a) $x = \frac{1}{4}$ ft : $t = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - \frac{\pi}{6}\right] = \underline{0.0370\ sec}$

(b) $x = \frac{1}{2}$ ft : $t = \frac{1}{20\sqrt{2}}\left[\frac{\pi}{2} - 0\right] = \underline{0.0555\ sec}$

2/28

The area under the a-s curve is

$\int_0^v v\,du = \frac{1}{2}v^2$

$\text{Area}\Big|_0^{200\ m} = \frac{3+6}{2}(100) + \frac{6+4}{2}(100) = 950\ m^2/s^2$

So $\frac{1}{2}v^2 = 950$, $\underline{v = 43.6\ m/s}$

$\frac{dv}{ds} = \frac{a}{v} = \frac{4}{43.6} = \underline{0.0918\ s^{-1}}$

2/29

$v_0 = 100/3.6 = 27.8\ m/s$

$a = -g\sin\theta = -9.81\sin\left[\tan^{-1}\frac{6}{100}\right] = -0.588\ m/s^2$

(a) $v = v_0 + at = 27.8 - 0.588(10) = \underline{21.9\ m/s}$

(b) $v^2 = v_0^2 + 2a(s-s_0) = 27.8^2 + 2(-0.588)(100)$

$v = \underline{25.6\ m/s}$

2/30

$0 < t < 4\,s$: $a = -3t/4$

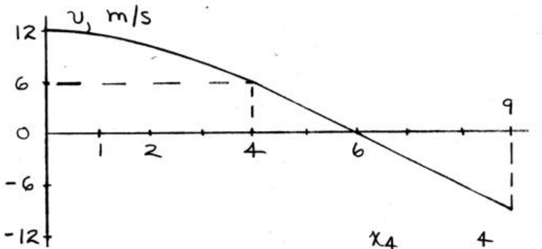
$a = \frac{dv}{dt}$: $\int_{12}^{v} dv = -\int_0^t \frac{3t}{4}dt$

$v = 12 - \frac{3}{8}t^2$, $\underline{v_4 = 6\ m/s}$

$4 < t < 9\,s$: $a = -3\ m/s^2 = \text{constant}$

$v = v_4 + a\Delta t = 6 - 3(t-4) = 18 - 3t$

$v_9 = -9\ m/s$



$0 < t < 4\,s$: $dx = v\,dt$: $\int_0^{x_4} dx = \int_0^4\left(12 - \frac{3}{8}t^2\right)dt$, $S_4 = 40\ m$

$4 < t < 9\,s$: $x_6 - x_4 = \int_4^6 v\,dt = \frac{1}{2}(6-4)6 = 6\ m$

$\therefore\ \Delta x = 40 + 6 = \underline{46\ m}$

2/31

$a = v\frac{dv}{ds} = \frac{1}{2}\frac{d(v^2)}{ds} = \frac{1}{2}\frac{\Delta(v^2)}{\Delta s} = \frac{1}{2}\frac{36-16}{80-30} = \frac{1}{5}\ m/s^2$

Counting time from A, $v = v_A + at$, $v = 4 + \frac{1}{5}t$

At B, $6 = 4 + \frac{1}{5}t_B$, $t_B = 10$ sec.

$\Delta s = \int v\,dt = \int_8^{10}\left(4 + \frac{1}{5}t\right)dt = 4(2) + \frac{1}{10}(100-64)$

$\Delta s = \underline{11.6\ m}$

2/32

$S_{car} = vt = \frac{120}{3.6}t$

$S_{cycle} = v_{av}t_1 + v_{max}t_2 = \frac{1}{2}\frac{150}{3.6}t_1 + \frac{150}{3.6}t_2$

where $t_1 = \frac{v_{max}}{a} = \frac{150}{3.6\times 6} = 6.94\ s$ & $t_2 = t - 6.94 - 2$

$S_{car} = S_{cycle}$; $\frac{120}{3.6}t = \frac{75}{3.6}6.94 + \frac{150}{3.6}(t - 8.94)$

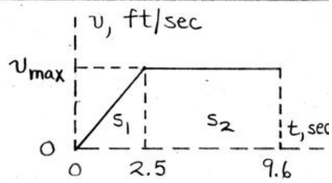
$30t = 820.8$, $t = 27.36\ s$

$S = \frac{120}{3.6}(27.36) = \underline{912\ m}$

2/33

$s_1 = \frac{1}{2}(2.5)v_{max}$

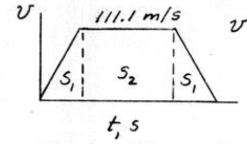
$s_2 = (9.6 - 2.5)v_{max}$



$s_1 + s_2 = (1.25 + 7.1)v_{max} = 100(3)$

$\underline{v_{max} = 35.9\ ft/sec}$

2/34 $400\ km/h \equiv \frac{400}{3.6} = 111.1\ m/s$



$v^2 = 2as,\quad s_1 = \frac{(111.1)^2}{2(0.6)(9.81)} = 1049\ m$

$s_2 = 10\,000 - 2(1049)$
$= 7903\ m$

$t_1 = \frac{v}{a} = \frac{111.1}{0.6(9.81)} = 18.88\ s$

$t_2 = \frac{s_2}{v} = \frac{7903}{111.1} = 71.13\ s$

$t = 2t_1 + t_2$
$= 2(18.88) + 71.13$
$= \underline{108.9\ s}$

or $\underline{t = 1.81\ min}$

2/35 $a = g - cy = v\frac{dv}{dy}$

$$\int_0^{y_m}(g - cy)\,dy = \int_{v_0}^{0} v\,dv$$

$$\left(gy - c\frac{y^2}{2}\right)\Big|_0^{y_m} = \frac{v^2}{2}\Big|_{v_0}^{0}$$

$$gy_m - c\frac{y_m^2}{2} = -\frac{v_0^2}{2} \Rightarrow \underline{c = \frac{v_0^2 + 2gy_m}{y_m^2}}$$

2/36 Particle 1 : $a = -kv$

$-kv = \frac{dv}{dt}$

$-k\int_0^t dt = \int_{v_0}^{v}\frac{dv}{v} \Rightarrow \underline{v = v_0 e^{-kt}}$

Then $\frac{ds}{dt} = v_0 e^{-kt}$

$\int_{s_0=0}^{s} ds = v_0\int_{t_0=0}^{t} e^{-kt}dt \Rightarrow \underline{s = \frac{v_0}{k}\left(1 - e^{-kt}\right)}$

Particle 2 : $a = -kt$

$-kt = \frac{dv}{dt}$

$-k\int_0^t t\,dt = \int_{v_0}^{v} dv \Rightarrow \underline{v = v_0 - \frac{1}{2}kt^2}$

Then $\frac{ds}{dt} = v_0 - \frac{1}{2}kt^2$

$\int_0^s ds = \int_0^t\left(v_0 - \frac{1}{2}kt^2\right)dt \Rightarrow \underline{s = v_0 t - \frac{1}{6}kt^3}$

Particle 3 : $a = -ks$

$-ks = v\frac{dv}{ds}$

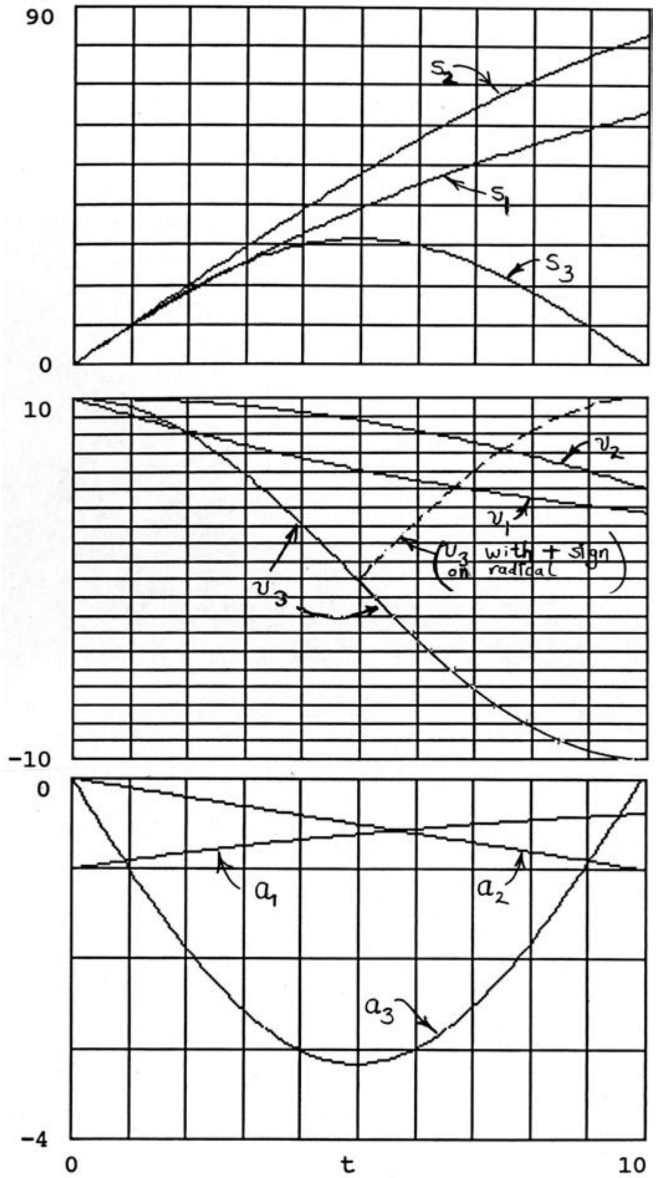
$-k\int_0^s s\,ds = \int_{v_0}^{v} v\,dv \Rightarrow \underline{v = \pm\sqrt{v_0^2 - ks^2}}$

Note : Plus sign is chosen until first reversal ($v=0$), thereafter take minus sign , etc.

Then $\frac{ds}{dt} = \pm\sqrt{v_0^2 - ks^2}$

$\int_0^s \frac{ds}{\sqrt{v_0^2 - ks^2}} = \int_0^t dt$

$\frac{1}{\sqrt{k}}\sin^{-1}\left(\frac{\sqrt{k}}{v_0}s\right) = t \Rightarrow \underline{s = \frac{v_0}{\sqrt{k}}\sin\left(\sqrt{k}\,t\right)}$



2/37 $a = P/(mv)$, P & m are constant

$v\,dv = a\,ds;\quad v\,dv = \frac{P}{mv}ds$

$\int_{v_1}^{v_2} mv^2 dv = P\int_0^s ds,\quad \underline{s = \frac{m}{3P}\left(v_2^3 - v_1^3\right)}$

$dv = a\,dt;\quad dv = \frac{P}{mv}dt$

$m\int_{v_1}^{v_2} v\,dv = \int_0^t P\,dt,\quad \underline{t = \frac{m}{2P}\left(v_2^2 - v_1^2\right)}$

2/38 $v\,dv = a\,ds;\quad \frac{v\,dv}{-Kv^2} = ds,\quad \int_{v_1}^{v_2}\frac{dv}{v} = -K\int_0^s ds$

$\ln\frac{v_2}{v_1} = -Ks,\quad K = \frac{1}{s}\ln\frac{v_1}{v_2} = \frac{1}{1500}\ln\frac{100}{20} = \underline{1.073(10^{-3})\ ft^{-1}}$

$a = \frac{dv}{dt};\quad -Kv^2 = \frac{dv}{dt},\quad \int_{v_1}^{v_2}\frac{dv}{v^2} = -Kt,\quad t = \frac{1}{K}\left(\frac{1}{v_2} - \frac{1}{v_1}\right)$

$t = \frac{10^3}{1.073}\left(\frac{1}{20} - \frac{1}{100}\right)\frac{30}{44} = \underline{25.4\ sec}$

2/39 $a = -kv = \frac{dv}{dt}$

$$-k\int_0^t dt = \int_{v_0}^{v} \frac{dv}{v} \Rightarrow v = v_0 e^{-kt}$$

Given Conditions : $1 = 4e^{-k(2)}$, $k = 0.693\ s^{-1}$

So $v = v_0 e^{-0.693t}$

When $v = \frac{v_0}{10}$: $\frac{v_0}{10} = v_0 e^{-0.693T}$, $\underline{T = 3.32\ s}$

Also, $a = -kv = v\frac{dv}{ds}$

$$-k\int_{s=0}^{s} ds = \int_{v_0}^{v} dv, \quad v = v_0 - ks$$

Given conditions : $\frac{v_0}{10} = v_0 - kD$

With $k = 0.693\ s^{-1}$ and $v_0 = 4\ m/s$:

$\frac{4}{10} = 4 - 0.693D$, $\underline{D = 5.19\ m}$

(We note that T is independent of v_0; D is not.)

2/40 $a = g - cy^2 = v\frac{dv}{dy}$

$$\int_0^{y_m}(g - cy^2)\,dy = \int_{v_0}^{0} v\,dv$$

$$\left(gy - c\frac{y^3}{3}\right)\Big|_0^{y_m} = \frac{v^2}{2}\Big|_{v_0}^{0}$$

$$gy_m - c\frac{y_m^3}{3} = -\frac{v_0^2}{2} \Rightarrow \underline{c = \frac{3v_0^2 + 6gy_m}{2y_m^3}}$$

2/41 $v\,dv = a\,dx$, $\int_0^x dx = \int_{v_0}^{v} \frac{v\,dv}{-C_1 - C_2v^2}$

$$x = \frac{-1}{2C_2}\ln(+C_1 + C_2v^2)\Big]_{v_0}^{v} = \frac{1}{2C_2}\ln\frac{C_1 + C_2v_0^2}{C_1 + C_2v^2}$$

when $v = 0$, $\underline{x = D = \frac{1}{2C_2}\ln\left(1 + \frac{C_2}{C_1}v_0^2\right)}$

2/42 (a) $g_0 = 32.2\ ft/sec^2 = $ constant

$v^2 = v_0^2 + 2a(s - s_0)$: $v^2 = 0^2 + 2(32.2)(500 \cdot 5280)$

$v = \underline{13{,}040\ ft/sec}$

(b) $a = -g_0\frac{R^2}{r^2} = v\frac{dv}{dr}$

$$-g_0R^2\int_{R+h}^{R}\frac{dr}{r^2} = \int_{v_0=0}^{v} v\,dv$$

$$-g_0R^2\left(-\frac{1}{r}\right)\Big|_{R+h}^{R} = \frac{1}{2}v^2\Big|_0^{v}$$

$$\Rightarrow v = \sqrt{\frac{2g_0Rh}{R+h}} = \sqrt{\frac{2(32.2)(3959)(500)(5280)^2}{(3959+500)(5280)}}$$

$= \underline{12{,}290\ ft/sec}$

2/43 (a) $(g_m)_0 = 5.32\ ft/sec^2 = $ constant

$v^2 = v_0^2 + 2(g_m)_0(s - s_0)$: $v^2 = 0^2 + 2(5.32)(750 \cdot 5280)$

$\underline{v = 6490\ ft/sec}$

(b) $a = -(g_m)_0\frac{R_m^2}{r^2} = v\frac{dv}{dr}$ (R_m = moon radius)

$$-(g_m)_0R_m^2\int_{R_m+h}^{R_m}\frac{dr}{r^2} = \int_{v_0=0}^{v} v\,dv$$

$$-(g_m)_0R_m^2\left(-\frac{1}{r}\right)\Big|_{R_m+h}^{R_m} = \frac{1}{2}v^2\Big|_0^{v}$$

$$v = \sqrt{\frac{2(g_m)_0R_mh}{R_m+h}} = \sqrt{\frac{2(5.32)\left(\frac{2160}{2}\right)(750)(5280)^2}{\left(\frac{2160}{2}+750\right)(5280)}}$$

$= \underline{4990\ ft/sec}$

2/44 $a = \frac{dv}{dt} = -kv$, $\int_{v_0}^{v}\frac{dv}{v} = -k\int_0^t dt$

$\ln\frac{v}{v_0} = -kt$, $\underline{v = v_0e^{-kt}}$

$v = \frac{dx}{dt} = v_0e^{-kt}$, $\int_0^x dx = \int_0^t v_0e^{-kt}dt$

$$\underline{x = \frac{v_0}{k}\left[1 - e^{-kt}\right]}$$

$v\,dv = a\,dx$, $\frac{v\,dv}{v} = -k\,dx$

$\int_{v_0}^{v} dv = -k\int_0^x dx$, $\underline{v = v_0 - kx}$

2/45 $a = \frac{dv}{dt}$; $\int_0^v \frac{dv}{g - kv} = \int_0^t dt$, $-\frac{1}{k}\ln(g - kv)\Big]_0^v = t$

$kt = \ln\frac{g}{g - kv}$, $\frac{g}{g - kv} = e^{kt}$, $\underline{v = \frac{g}{k}(1 - e^{-kt})}$

$v = \frac{dy}{dt}$; $\int_0^y dy = \frac{g}{k}\int_0^t (1 - e^{-kt})\,dt$

$y = \frac{g}{k}\left(t + \frac{1}{k}e^{-kt}\right)\Big|_0^t$, $\underline{y = \frac{g}{k}\left[t - \frac{1}{k}(1 - e^{-kt})\right]}$

2/46 (a) $a = 2 \text{ m/s}^2 = \text{constant}$

With $v = 250/3.6 = 69.4$ m/s, we have

$v^2 - v_0^2 = 2a(s - s_0)$: $69.4^2 - 0^2 = 2(2)s$

$$\underline{s = 1206 \text{ m}}$$

(b) $a = a_0 - kv^2 = v\dfrac{dv}{ds}$

$$\int_0^s ds = \int_0^v \frac{v\,dv}{a_0 - kv^2}$$

$$s = -\frac{1}{2k}\ln\left(a_0 - kv^2\right)\Big|_0^v$$

$$= -\frac{1}{2k}\ln\left[\frac{a_0 - kv^2}{a_0}\right]$$

$$s = -\frac{1}{2(4)(10^{-5})}\ln\left[\frac{2 - 4(10^{-5})(69.4)^2}{2}\right]$$

$$= \underline{1268 \text{ m}}$$

2/46 (a) $a = 2 \text{ m/s}^2 = \text{constant}$

With $v = 250/3.6 = 69.4$ m/s, we have

$v^2 - v_0^2 = 2a(s - s_0)$: $69.4^2 - 0^2 = 2(2)s$

$$\underline{s = 1206 \text{ m}}$$

(b) $a = a_0 - kv^2 = v\dfrac{dv}{ds}$

$$\int_0^s ds = \int_0^v \frac{v\,dv}{a_0 - kv^2}$$

$$s = -\frac{1}{2k}\ln\left(a_0 - kv^2\right)\Big|_0^v$$

$$= -\frac{1}{2k}\ln\left[\frac{a_0 - kv^2}{a_0}\right]$$

$$s = -\frac{1}{2(4)(10^{-5})}\ln\left[\frac{2 - 4(10^{-5})(69.4)^2}{2}\right]$$

$$= \underline{1268 \text{ m}}$$

2/47 $a = -kv^2$,

$v\,dv = a\,dx$, $\displaystyle\int_{v_0}^{v}\frac{v\,dv}{-kv^2} = \int_0^x dx$, $x = \frac{-1}{k}\ln v\Big]_{v_0}^{v}$

$$x = \frac{1}{k}\ln v_0/v$$

when $v = v_0/2$, $x = D = \frac{1}{k}\ln 2 = \underline{0.693/k}$

$v = \dfrac{dx}{dt}$ where $kx = \ln v_0/v$, $v = v_0 e^{-kx}$

so $\dfrac{dx}{v_0 e^{-kx}} = dt$ or $\displaystyle\int_0^t dt = \frac{1}{v_0}\int_0^x e^{kx}\,dx$

& $t = \dfrac{1}{v_0}\dfrac{1}{k}e^{kx}\Big]_0^x = \dfrac{1}{kv_0}\left[e^{kx} - 1\right]$

For $x = D$, $e^{kx} = 2$ so $t = \dfrac{1}{kv_0}[2-1]$, $\underline{t = \dfrac{1}{kv_0}}$

2/48 0 – 60 mi/hr: $v^2 = v_0^2 + 2as$

$0 < t < t_1$ $\quad (88)^2 = 0 + 2a(200)$, $a = 19.36$ ft/sec^2

$v = v_0 + at$, $v = 0 + 19.36\,t$

60 – 0; $v\,dv = a\,ds$; $a = -kv$ so $\dfrac{v\,dv}{-kv} = ds$

$t_1 < t < t_2$ $\quad$ or $dv = -k\,ds$ & $\displaystyle\int_{88}^{44} dv = -k\int_0^{400} ds$

$44 - 88 = -400k$, $k = 0.11$ 1/sec

& $a = -0.11v$

$a = dv/dt$, $\displaystyle\int_{88}^{v}\frac{dv}{-kv} = \int_{t_1}^{t} dt$, $\dfrac{1}{0.11}\ln\dfrac{88}{v} = t - t_1$

$t_1 = \dfrac{88}{19.36} = 4.55$ sec, $t = \dfrac{1}{0.11}\ln\dfrac{88}{44} + 4.55 = \underline{10.85 \text{ sec}}$

2/49 $a = k/x$, $v\,dv = \dfrac{k}{x}dx$

$\displaystyle\int_0^v v\,dv = k\int_{x_1}^{x}\frac{dx}{x}$; $\dfrac{v^2}{2} = k\ln\dfrac{x}{x_1}$

Thus $\dfrac{(600)^2}{2} = k\ln\dfrac{750}{7.5}$, $k = \dfrac{0.36}{2(4.605)} = 0.0391\ (\text{km/s})^2$

at $x = 375$ mm, $a = \dfrac{0.0391}{375(10^{-6})} = \underline{104.2 \text{ km/s}^2}$

2/50 $a = v\dfrac{dv}{ds} = 3.22 - 0.004v^2$

$$\int_0^{v_B}\frac{v\,dv}{3.22 - 0.004v^2} = \int_0^{600} ds$$

$$\frac{1}{2(-0.004)}\ln\left(3.22 - 0.004v^2\right)\Big|_0^{v_B} = 600$$

$$\ln\left[\frac{3.22 - 0.004v_B^2}{3.22}\right] = 600(2)(-0.004)$$

$$\frac{3.22 - 0.004v_B^2}{3.22} = 0.00823$$

$$\underline{v_B = 28.3 \text{ ft/sec}}$$

2/51 Up: $a_u = -g - kv^2 = v\dfrac{dv}{dy}$ $\quad \uparrow y$

$$\int_0^h dy = -\int_{v_0}^0 \frac{v\,dv}{g + kv^2}$$

$$h = -\frac{1}{2k}\ln\left[g + kv^2\right]_{v_0}^{0} = \frac{1}{2k}\ln\left[\frac{g + kv_0^2}{g}\right]$$

$$h = \frac{1}{2(0.002)}\ln\left[\frac{32.2 + 0.002(100)^2}{32.2}\right] = \underline{120.8 \text{ ft}}$$

Down: $a_d = -g + kv^2 = v\dfrac{dv}{dy}$

$$\int_h^0 dy = \int_0^{v_f}\frac{v\,dv}{-g + kv^2}$$

$$-h = \frac{1}{2k}\ln\left[-g + kv^2\right]\Big|_0^{v_f} = \frac{1}{2k}\ln\left[\frac{g - kv_f^2}{g}\right]$$

$$\Rightarrow v_f = \sqrt{\frac{g}{k}\left(1 - e^{-2kh}\right)}$$

$$= \sqrt{\frac{32.2}{0.002}\left(1 - e^{-2(0.002)(120.8)}\right)} = \underline{78.5 \frac{\text{ft}}{\text{sec}}}$$

2/52 Up : $a_u = -g - kv^2 = \frac{dv}{dt}$

$$\int_0^{t_u} dt = -\int_{v_0}^{0} \frac{dv}{g+kv^2}$$

$$t_u = \frac{1}{\sqrt{gk}} \tan^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Bigg|_0^{v_0} = \frac{1}{\sqrt{gk}} \tan^{-1}\left(v_0\sqrt{\frac{k}{g}}\right)$$

$$t_u = \frac{1}{\sqrt{32.2(0.002)}} \tan^{-1}\left(100\sqrt{\frac{0.002}{32.2}}\right) = \underline{2.63 \text{ sec}}$$

(Down) : $a_d = -g + kv^2 = \frac{dv}{dt}$

$$\int_0^{t_d} dt = \int_0^{v_f} \frac{dv}{-g+kv^2}$$

$$t_d = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(\frac{v\sqrt{gk}}{g}\right)\Bigg|_0^{v_f} = \frac{1}{\sqrt{gk}} \tanh^{-1}\left(v_f\sqrt{\frac{k}{g}}\right)$$

$$= \frac{1}{\sqrt{32.2(0.002)}} \tanh^{-1}\left(78.5\sqrt{\frac{0.002}{32.2}}\right)$$

$$= \underline{2.85 \text{ sec}}$$

(Refer to solution of Prob. 2/51)

2/53 $a = c_1 - c_2 v^2 = v\frac{dv}{ds}$

$$\int_0^s ds = \int_0^v \frac{v\,dv}{c_1 - c_2 v^2} = -\frac{1}{2c_2}\int_0^v \frac{-2c_2 v\,dv}{c_1 - c_2 v^2}$$

$$s = -\frac{1}{2c_2} \ln(c_1 - c_2 v^2)\Big|_0^v = \frac{1}{2c_2} \ln\left(\frac{c_1}{c_1 - c_2 v^2}\right)$$

When $s = 1320$ ft, $v = 190\left(\frac{5280}{3600}\right) = 279$ ft/sec :

$$1320 = \frac{1}{2(5)(10^{-5})} \ln\left(\frac{c_1}{c_1 - 5(10^{-5})(279)^2}\right)$$

Solve to obtain $\underline{c_1 = 31.4 \text{ ft/sec}^2}$

2/54 $a = 31.4 - 5(10^{-5})v^2 = c_1 - c_2 v^2 = \frac{dv}{dt}$

$$\int_0^t dt = \int_0^v \frac{dv}{c_1 - c_2 v^2} = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1}\sqrt{\frac{c_2}{c_1}}\,v\Bigg|_0^v$$

$$t = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1}\sqrt{\frac{c_2}{c_1}}\,v$$

For $v = 190\left(\frac{5280}{3600}\right) = 279$ ft/sec,

$$t = \frac{1}{\sqrt{31.4(5)(10^{-5})}} \tanh^{-1}\sqrt{\frac{5(10^{-5})}{31.4}}\,(279)$$

$$= \underline{9.27 \text{ sec}}$$

2/55 For an acceleration of form $a = -g - kv^2$, we cite the results from Probs. 2/51 & 2/52

$$\begin{cases} t_u = \dfrac{1}{\sqrt{gk}} \tan^{-1}\left(v_0\sqrt{\dfrac{k}{g}}\right) \\ h = \dfrac{1}{2k} \ln\left[\dfrac{g+kv_0^2}{g}\right] \end{cases}$$

For the numbers at hand :

$$t_u = \frac{1}{\sqrt{9.81(0.0005)}} \tan^{-1}\left(120\sqrt{\frac{0.0005}{9.81}}\right) = 10.11 \text{ s}$$

$$h = \frac{1}{2(0.0005)} \ln\left[\frac{9.81 + 0.0005(120)^2}{9.81}\right] = 550 \text{ m}$$

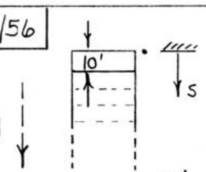
Down (v = constant) : $y = y_0 + v_{y_0} t$

$$0 = 550 - 4t_d$$

$$t_d = 137.6 \text{ s}$$

Flight time $t = t_u + t_d = 10.11 + 137.6 = \underline{147.7 \text{ s}}$

2/56



$s = \cancel{s_0} + \cancel{v_0} t + \frac{1}{2}gt^2$

When $s = 10$ ft,

$10 = \frac{1}{2}(32.2)t_{10'}^2$, $t_{10'} = 0.788$ sec

Time to pass first story from the top is $t_1 = t_{10'} - t_{0'} = 0.788 - 0 = \underline{0.788 \text{ sec}}$

10th story : $90 = \frac{1}{2}(32.2)t_{90'}^2$, $t_{90'} = 2.36$ sec

$100 = \frac{1}{2}(32.2)t_{100'}^2$, $t_{100'} = 2.49$ sec

$t_{10} = t_{100'} - t_{90'} = 2.49 - 2.36 = \underline{0.1279 \text{ sec}}$

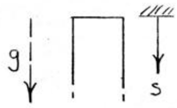
100th story : $990 = \frac{1}{2}(32.2)t_{990'}^2$,

$1000 = \frac{1}{2}(32.2)t_{1000'}^2$

$t_{100} = t_{1000'} - t_{990'} = 7.88 - 7.84 = \underline{0.0395 \text{ sec}}$

2/57

$$a = g - kv^2 = \frac{dv}{dt}$$



$$\int_0^t dt = \int_0^v \frac{dv}{g-kv^2}$$

(see Art. C/10): $t = \frac{1}{\sqrt{gk}} \tanh^{-1}\sqrt{\frac{k}{g}}\,v \Big|_0^v$

$$= \frac{1}{\sqrt{gk}} \tanh^{-1}\sqrt{\frac{k}{g}}\,v$$

$$\Rightarrow v = \frac{ds}{dt} = \sqrt{\frac{g}{k}} \tanh\left(\sqrt{gk}\,t\right)$$

$$\int_0^s ds = \sqrt{\frac{g}{k}} \int_0^t \tanh\left(\sqrt{gk}\,t\right)$$

$$s = \frac{1}{k} \ln \cosh \sqrt{gk}\,t$$

or $t = \dfrac{\cosh^{-1}(e^{sk})}{\sqrt{gk}} = \dfrac{\cosh^{-1}(e^{0.005s})}{0.401}$

s, ft	t, sec
0	0
10	0.795
90	2.54
100	2.70
990	14.06
1000	14.19

The time t_1 to pass first story is $t_1 = t_{10'} - t_0 = 0.795 - 0 = \underline{0.795\text{ sec}}$

Similarly,

$\underline{t_{10} = 0.1592\text{ sec}}$

$\underline{t_{100} = 0.1246\text{ sec}}$

2/58

$$a = \frac{d^2x}{dt^2} = Kt - k^2x$$

or $\dfrac{d^2x}{dt^2} + k^2x = Kt$, a second-order, linear differential equation whose solution is

$$x = x_h + x_p = A\sin kt + B\cos kt + \frac{K}{k^2}t$$

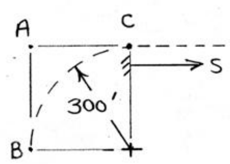
Initial conditions:

$x(0) = B = 0$

$\dot{x}(0) = kA + \dfrac{K}{k^2} = 0$, $\quad A = -\dfrac{K}{k^3}$

So $\underline{x = \dfrac{K}{k^3}(kt - \sin kt)}$

2/59



First, determine B's acceleration time:

$v = v_0 + at$

$65\left(\frac{44}{30}\right) = 25\left(\frac{44}{30}\right) + 3.22t$

$t = 18.22$ sec

Distance traveled by A in that time:

$d_A = 65\left(\frac{44}{30}\right)(18.22) = 1737$ ft

Displacement beyond C: $S_A = 1737 - 300 = 1437$ ft

Distance traveled by B in 18.22 sec:

$d_B = v_0t + \frac{1}{2}at^2 = 25\left(\frac{44}{30}\right)(18.22) + \frac{1}{2}(3.22)(18.22)^2$

$= 1202$ ft

Displacement beyond C: $S_B = 1202 - \frac{\pi(300)}{2} = 731$ ft

So A is ahead of B by $S_A - S_B = 1437 - 731$

$= \underline{706\text{ ft}}$ (in the steady-state)

2/60 For B, $v_f - v_0 =$ area under a-t curve

$(65-25)\frac{44}{30} = 3.22t_1 + \frac{1}{2}(3.22)(5)$

$t_1 = 15.72$ sec

B reaches 65 mi/hr @ $15.72 + 5 = 20.7$ sec $= t_2$

Speed of B @ t_1: $v_1 = 25\left(\frac{44}{30}\right) + 3.22(15.72)$

$= 87.3$ ft/sec

The acceleration history $(t_1 < t < t_2)$ is $a = 13.34 - 0.644t$

$\int_{v_1=87.3}^{v} dv = \int_{t_1=15.72}^{t} (13.34 - 0.644t)\,dt$ yields

$v = -42.9 + 13.34t - 0.322t^2$

Then $\int_{s_1}^{s_2} ds = \int_{t_1=15.72}^{t_2=20.7} (-42.9 + 13.34t - 0.322t^2)\,dt$

yields $(s_2 - s_1) = 463$ ft

Distance traveled by B in 20.7 sec

$d_B = 25\left(\frac{44}{30}\right)(15.72) + \frac{1}{2}(3.22)(15.72)^2 + 463$

$= 1437$ ft

Displacement beyond C: $S_B = 1437 - \frac{\pi(300)}{2} = 966$ ft

Distance traveled by A in 20.7 sec:

$d_A = 65\left(\frac{44}{30}\right)(20.7) = 1975$ ft

Displacement beyond C: $S_A = 1975 - 300 = 1675$ ft

So A is ahead of B by $S_A - S_B = 1675 - 966$

$= \underline{709\text{ ft}}$ (in the steady-state)

(Not much more than the 706 ft of Prob. 2/59)

2/61 $\underline{a}_{av} = \dfrac{\Delta \underline{v}}{\Delta t} = \dfrac{(-0.1\underline{i} + 1.8\underline{j}) - (0.1\underline{i} + 2\underline{j})}{0.1}$

$= -2\underline{i} - 2\underline{j}$ m/s

$a_{av} = \sqrt{2^2 + 2^2} = \underline{2.83\text{ m/s}^2}$

$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-2}{-2}\right) = \underline{225°}$

2/62 $x = 3t^2 - 4t$, $\dot{x} = 6t - 4$, $\ddot{x} = 6$ mm/s^2

$y = 4t^2 - \frac{1}{3}t^3$, $\dot{y} = 8t - t^2$, $\ddot{y} = 8 - 2t$ mm/s^2

when $t = 2$ s, $\dot{x} = 12 - 4 = 8$ mm/s, $\dot{y} = 16 - 4 = 12$ mm/s

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{8^2 + 12^2} = \underline{14.42\ \frac{mm}{s}}$

$\theta_x = \tan^{-1}\frac{\dot{y}}{\dot{x}} = \tan^{-1}\frac{12}{8} = \underline{56.3°}$

$\ddot{x} = 6$ mm/s^2, $\ddot{y} = 8 - 4 = 4$ mm/s^2

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{6^2 + 4^2} = \underline{7.21\text{ mm/s}^2}$

$\theta_x = \tan^{-1}\frac{\ddot{y}}{\ddot{x}} = \tan^{-1}\frac{4}{6} = \underline{33.7°}$

2/63

$x = t^2 - 4t + 20$ $y = 3\sin 2t$

$\dot{x} = 2t - 4$ $\dot{y} = 6\cos 2t$

$\ddot{x} = 2$ $\ddot{y} = -12\sin 2t$

At time $t = 3$ sec :

$\dot{x} = 2$ in./sec $\dot{y} = 5.76$ in./sec

$\ddot{x} = 2$ in./sec^2 $\ddot{y} = 3.35$ in./sec^2

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{2^2 + 5.76^2} = \underline{6.10 \text{ in./sec}}$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{2^2 + 3.35^2} = \underline{3.90 \text{ in./sec}^2}$

$\underline{v} = 2\underline{i} + 5.76\underline{j}$ in./sec , $\underline{a} = 2\underline{i} + 3.35\underline{j}$ in./sec^2

$\theta = \cos^{-1}\dfrac{\underline{v}\cdot\underline{a}}{va} = \cos^{-1}\left(\dfrac{2(2) + 5.76(3.35)}{(6.10)(3.90)}\right)$

$= \underline{11.67°}$

2/64 $x = y^2/6$ & $\dot{y} = 3$ in./sec

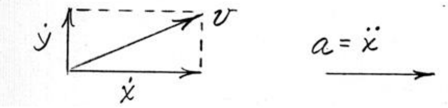
$\dot{x} = \frac{y}{3}\dot{y}$, $\ddot{x} = \frac{\dot{y}^2}{3} + \frac{y}{3}\ddot{y}$ but $\ddot{y} = 0$ & $\dot{y} = 3$ in/sec

Also when $x = 6$ in., $y = \sqrt{36} = 6$ in.

So $\dot{x} = \frac{6}{3}(3) = 6$ in/sec

Hence $v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{6^2 + 3^2} = \underline{3\sqrt{5} \text{ in/sec}}$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(3^2/3)^2 + 0} = \underline{3 \text{ in./sec}^2}$



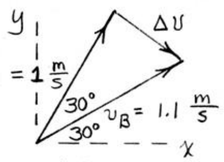
2/65 $v = \dot{s} = \frac{t}{2}$, $v_A = \frac{2}{2} = 1$ m/s, $v_B = \frac{2.2}{2} = 1.1\frac{m}{s}$

$\Delta v_x = v_{B_x} - v_{A_x} = 1.1\cos 30° - 1.0\cos 60° = 0.453\ \frac{m}{s}$

$\Delta v_y = v_{B_y} - v_{A_y} = 1.1\sin 30° - 1.0\sin 60° = -0.316\ \frac{m}{s}$

$\Delta v = \sqrt{0.453^2 + 0.316^2}$

$= 0.552$ m/s



The average acceleration is

$a_{av} = \dfrac{\Delta v}{\Delta t} = \dfrac{0.552}{0.20} = \underline{2.76 \text{ m/s}^2}$

$\underline{a}_{av} = \dfrac{\Delta \underline{v}}{\Delta t} = \dfrac{0.453\underline{i} - 0.316\underline{j}}{0.20}$

$= \underline{2.26\underline{i} - 1.580\underline{j} \text{ m/s}^2}$

2/66 $x = 20 + \frac{1}{4}t^2$, $\dot{x} = \frac{1}{2}t$, $\ddot{x} = \frac{1}{2}$ mm/s^2

$y = 15 - \frac{1}{6}t^3$, $\dot{y} = -\frac{1}{2}t^2$, $\ddot{y} = -t$ mm/s^2

For $t = 2$ s , $\dot{x} = 1$ mm/s

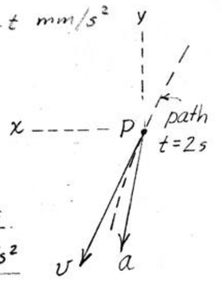
$\dot{y} = -2$ mm/s

$\ddot{x} = \frac{1}{2}$ mm/s^2

$\ddot{y} = -2$ mm/s^2

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{1^2 + (-2)^2} = \underline{2.24 \text{ mm/s}}$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(1/2)^2 + (-2)^2} = \underline{2.06 \text{ mm/s}^2}$



2/67 $\underline{r} = \left(\frac{2}{3}t^3 - \frac{3}{2}t^2\right)\underline{i} + \left(\frac{t^4}{12}\right)\underline{j}$

$\underline{v} = \dot{\underline{r}} = (2t^2 - 3t)\underline{i} + \left(\frac{1}{3}t^3\right)\underline{j}$

$\underline{a} = \dot{\underline{v}} = (4t - 3)\underline{i} + (t^2)\underline{j}$

At $t = 2$s $\begin{cases} \underline{v} = (2\cdot 2^2 - 3\cdot 2)\underline{i} + \frac{1}{3}2^3\underline{j} = 2\underline{i} + \frac{8}{3}\underline{j} \text{ mm/s} \\ \underline{a} = (4\cdot 2 - 3)\underline{i} + 2^2\underline{j} = 5\underline{i} + 4\underline{j} \text{ mm/s}^2 \end{cases}$

$\cos\theta = \dfrac{\underline{v}\cdot\underline{a}}{va} = \dfrac{(2\underline{i} + \frac{8}{3}\underline{j})\cdot(5\underline{i} + 4\underline{j})}{\sqrt{2^2 + (\frac{8}{3})^2}\sqrt{5^2 + 4^2}}$

$\underline{\theta = 14.47°}$

At $t = 3$s $\begin{cases} \underline{v} = (2\cdot 3^2 - 3\cdot 3)\underline{i} + (\frac{1}{3}3^3)\underline{j} = 9\underline{i} + 9\underline{j}\ \frac{mm}{s} \\ \underline{a} = (4\cdot 3 - 3)\underline{i} + (3^2)\underline{j} = 9\underline{i} + 9\underline{j}\ \frac{mm}{s^2} \end{cases}$

$\underline{v} \parallel \underline{a} \Rightarrow \underline{\theta = 0}$

2/68 $\begin{cases} x = 3\cos 4t; & \dot{x} = -12\sin 4t; & \ddot{x} = -48\cos 4t \\ y = 2\sin 4t; & \dot{y} = 8\cos 4t; & \ddot{y} = -32\sin 4t \end{cases}$

At time $t = 1.4$ sec :

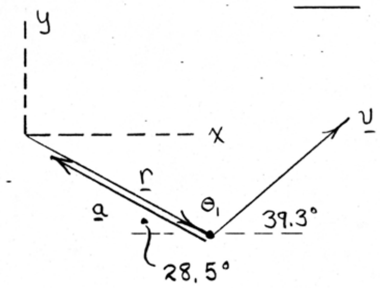
$\begin{cases} x = 2.33 \text{ ft}; & \dot{x} = 7.58 \text{ ft/sec}; & \ddot{x} = -37.2\ \frac{ft}{sec^2} \\ y = -1.263 \text{ ft}; & \dot{y} = 6.20 \text{ ft/sec}; & \ddot{y} = 20.2\ \frac{ft}{sec^2} \end{cases}$

$r = 2.65$ ft ; $v = 9.79$ ft/sec ; $a = 42.4\ \frac{ft}{sec^2}$

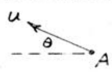
$\theta_1 = \cos^{-1}\left[\dfrac{\underline{a}\cdot\underline{v}}{av}\right] = \cos^{-1}\left[\dfrac{-37.2(7.58) + 20.2(6.20)}{42.4(9.79)}\right]$

$\underline{\theta_1 = 112.2°}$ Similarly, $\theta_2 = \cos^{-1}\left[\dfrac{\underline{a}\cdot\underline{r}}{ar}\right]$

Sketch (not to scale): $= \underline{180°}$



2/69 From Sample Prob. 2/6



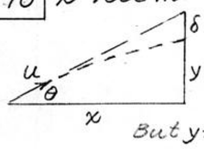
$2s = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{2(u\cos\theta)(u\sin\theta)}{g}$

But $2s = 22$ ft, $u\cos\theta = 30$ ft/sec, $u\sin\theta = v_y$

So $v_y = \dfrac{2sg}{2u\cos\theta} = \dfrac{22(32.2)}{2(30)} = \underline{11.81 \text{ ft/sec}}$

Also, $h = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{v_y^2}{2g} = \dfrac{(11.81)^2}{2(32.2)} = \underline{2.16 \text{ ft}}$

2/70 $x = 1000$ m $u = 600$ m/s $\theta = 20°$



From Sample Problem 2/6

$y = x\tan\theta - \dfrac{gx^2}{2u^2}\sec^2\theta$

But $y + \delta = x\tan\theta$, so $\delta = \dfrac{9.81(1000)^2}{2(600)^2}\dfrac{1}{(0.9397)^2}$

$\underline{\delta = 15.43 \text{ m}}$

2/71 Set up x-y axes at the initial location of G.

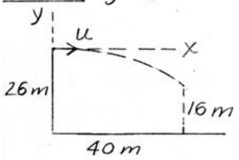
$x = x_0 + v_{x_0} t \quad : \quad 3 = (v_0 \cos\theta) t$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 : 3.5 = (v_0 \sin\theta) t - 16.1 t^2$

$v_y = v_{y_0} - g t \quad : \quad 0 = v_0 \sin\theta - 32.2 t$

Solve simultaneously : $t = 0.466$ sec, $v_0 = 16.33$ ft/sec, $\theta = 66.8°$

2/72 $a_y = -g$ so $y = 0 - \frac{1}{2} g t^2$, $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(26-16)}{9.81}}$

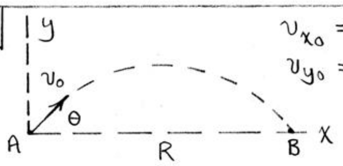


$= 1.428$ s

$x = ut$; $u = 40/1.428$

$= 28.0$ m/s

2/73



$v_{x_0} = v_0 \cos\theta$

$v_{y_0} = v_0 \sin\theta$

$x = x_0 + v_{x_0} t$ @ B : $R = 0 + (v_0 \cos\theta) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B : $0 = 0 + (v_0 \sin\theta) t_f - \frac{g}{2} t_f^2$ (2)

(2): $t_f = 0$, $\frac{2 v_0 \sin\theta}{g}$ ($t = 0$ is launch time)

(1): $R = (v_0 \cos\theta)\left(\frac{2 v_0 \sin\theta}{g}\right) = \frac{v_0^2 \sin 2\theta}{g}$

$\frac{dR}{d\theta} = 0 : \frac{v_0^2}{g} 2\cos 2\theta = 0 \Rightarrow \theta = 45°$

$R_{max} = \frac{v_0^2 \sin(2 \cdot 45°)}{g} = \frac{v_0^2}{g}$

2/74 Use x-y coordinates of the figure.

(a) $v_0 = 45$ ft/sec

$x = x_0 + v_{x_0} t$ @ left wall : $30 = 0 + 45 \cos 60° t$

$t = 1.333$ sec

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$: $y = 5 + 45 \sin 60° (1.333) - 16.1 (1.333)^2$

$= 28.3$ ft (hits wall)

Ans. : $(x, y) = (30', 28.3')$

(b) $v_0 = 60$ ft/sec

Repeat above procedure to find $y = 40.9'$ when $x = 30'$, so water clears left wall.

$x = x_0 + v_{x_0} t$ @ right wall : $50 = 0 + 60 \cos 60° t$

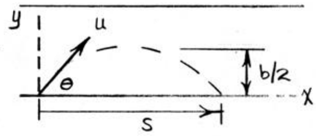
$t = 1.667$ sec

y eq. yields $y = 46.9$ ft @ $t = 1.667$ sec, so water clears building! For horizontal range: From $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ $y = 0$, $y_0 = 5$ ft, we find $t = -0.0935$ s & $t = 3.32$ s. From $x = x_0 + v_{x_0} t$: $x = 0 + 60 \cos 60° (3.32) = 99.6$ ft

2/75

$a_y = -\frac{eE}{m}$, constant

$a_x = 0$



$v_y^2 - v_{y_0}^2 = 2 a_y$: At top, $0 - (u \sin\theta)^2 = 2\left(-\frac{eE}{m}\right)\frac{b}{2}$

$E = \frac{m u^2 \sin^2\theta}{eb}$

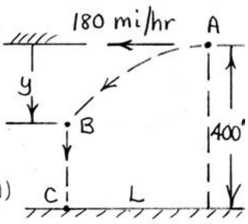
$v_y = v_{y_0} + a_y t$: At top, $0 = u \sin\theta - \frac{eE}{m} t$

$t = \frac{m u \sin\theta}{eE}$

$x = v_{x_0} t$: $s = (u \cos\theta)(2t) = u \cos\theta \left(\frac{2 m u \sin\theta}{eE}\right)$

$= u \cos\theta \left(\frac{2 m u \sin\theta}{e \frac{m u^2 \sin^2\theta}{eb}}\right) = 2b \cot\theta$

2/76



$t = 0$: package dropped at A

t_B, t_C : times package at point B, C

From A to B : $y = \frac{1}{2} g t_B^2$ (1)

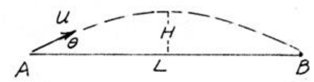
From B to C : $(400 - y) = 6(t_C - t_B)$ (2)

Also, $t_C = 37$ sec. Solve (1) & (2) to obtain $t_B = 3.52$ sec, $y = 199.1$ ft

$L = 180\left(\frac{5280}{3600}\right)(3.52) = 928$ ft

2/77 From Sample Prob. 2/6, $H = \frac{u^2 \sin^2\theta}{2g}$ ---- (a)

$L = 2s = \frac{u^2 \sin 2\theta}{g}$ ---- (b)



So $\frac{H}{L} = \frac{\sin^2\theta}{2(2\sin\theta\cos\theta)} = \frac{1}{4}\tan\theta$

Thus $\theta = \tan^{-1}(4H/L)$

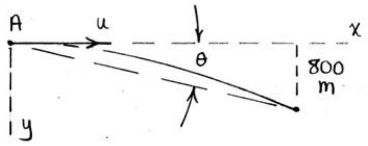
From (a) $\sin\theta = \sqrt{2gH}/u$

" (b) $Lg/u^2 = 2\sin\theta\cos\theta = 2\sin\theta\sqrt{1-\sin^2\theta}$

$= 2\frac{\sqrt{2gH}}{u}\sqrt{1-\frac{2gH}{u^2}}$

Simplify, solve for u & get $u = \sqrt{2gH}\sqrt{1+\left(\frac{L}{4H}\right)^2}$

2/78 $u = \frac{1000}{3.6} = 278 \frac{m}{s}$



y-dir. : $y = v_{y_0}t + \frac{1}{2}gt^2$

$800 = 0 + \frac{1}{2}(9.81)t^2, \quad t = 12.77\ s$

x-dir. : $x = v_{x_0}t + \frac{1}{2}a_x t^2$

$= 278(12.77) + \frac{1}{2}\left(\frac{9.81}{2}\right)(12.77)^2$

$= 3950\ m$

$\theta = \tan^{-1}\frac{800}{3950} = 11.46°$

2/79 Set up x-y coordinates with origin at A.

$x = x_0 + v_{x_0}t$ @ B : $800 + s\cos 20° = (120\cos 40°)t$ (1)

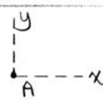
$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ @ B :

$-s\sin 20° = (120\sin 40°)t - \frac{9.81}{2}t^2$ (2)

Solve (1) & (2) simultaneously to obtain

$s = 1057\ m, \quad t = 19.50\ s$

2/80 (a) $v_0 = 140$ ft/sec and $\theta = 8°$:



$x = x_0 + v_{x_0}t$ @ B : $200 = 0 + (140\cos 8°)t$

$t = 1.443$ sec

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ @ B :

$-(7.5 - h) = 0 + 140\sin 8°(1.443) - \frac{1}{2}(32.2)(1.443)^2$

$h = 2.10$ ft

(b) $v_0 = 120$ ft/sec and $\theta = 12°$:

$x = x_0 + v_{x_0}t$ @ B : $200 = 0 + (120\cos 12°)t$

$t = 1.704$ sec

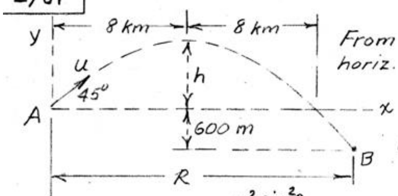
$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ @ B :

$-(7.5 - h) = 0 + (120\sin 12°)(1.704) - \frac{1}{2}(32.2)(1.704)^2$

$h = 3.27$ ft

(In baseball, the time of flight is critical; low trajectories, even with one hop, are better.)

2/81



From Sample Prob. 2/6 horiz. range is $2s = \frac{u^2 \sin 2\theta}{g}$

so $16(10^3) = \frac{u^2 \sin 90°}{9.81}$

$u = \sqrt{157000} = 396\ m/s$

Max. altitude $h = \frac{u^2\sin^2\theta}{2g}$, $H = h + 600 = \frac{(396)^2\sin^2 45°}{2(9.81)} + 600$

$= 4000 + 600 = 4600\ m$

$y = ut\sin\theta - \frac{1}{2}gt^2, \quad -600 = 396(0.7071)t - \frac{1}{2}(9.81)t^2$

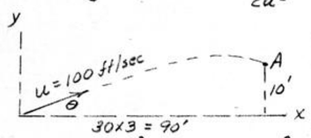
$t^2 - 57.11t - 122.3 = 0, \quad t = \frac{57.11}{2} \pm \frac{1}{2}\sqrt{3262 + 489}$

$= 59.18\ s \quad (\text{or } -2.07\ s)$

$x = ut\cos\theta, \quad R = 396(59.18)\cos 45° = 16\,579\ m$

or $R = 16.58\ km$

2/82 $y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$



Let $m = \tan\theta$

$\sec^2\theta = 1 + \tan^2\theta = 1 + m^2$

$y = xm - \frac{gx^2}{2u^2}(1+m^2), \quad m^2 - \frac{2u^2}{gx}m + \left(\frac{2u^2y}{gx^2} + 1\right) = 0$

At A, $m^2 - \frac{2(10^2)^2}{32.2(90)}m + \left(\frac{2(10^2)^2 10}{32.2(90)^2} + 1\right) = 0$

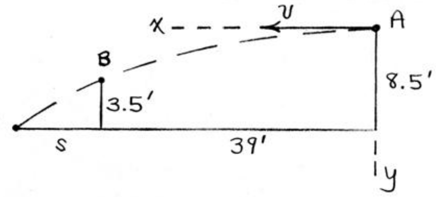
$m^2 - 6.901m + 1.7668 = 0$

$m = \frac{6.901}{2} \pm \frac{1}{2}\sqrt{(6.901)^2 - 4(1.7668)}$

$= \frac{6.901 \pm \sqrt{40.56}}{2} = 0.266$ or 6.635

$\theta = \tan^{-1}m = 14.91°$ (or $81.4°$)

2/83



$a_x = 0$: $x = v_{x_0}t, \quad 39 = vt_B$

$a_y = g$: $y = v_{y_0}t + \frac{1}{2}gt^2$

At B : $8.5 - 3.5 = 0 + \frac{1}{2}32.2t_B^2, \quad t_B = 0.557$ sec

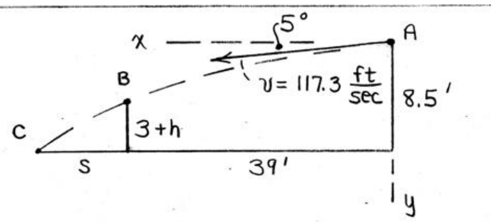
Then $v = \frac{39}{t_B} = \frac{39}{0.557} = 70.0$ ft/sec

(47.7 mi/hr)

At C : $8.5 = \frac{1}{2}(32.2)t_c^2, \quad t_c = 0.727$ sec

$s + 39 = 70.0(0.727), \quad s = 11.85$ ft

2/84



$a_x = 0$, $x = v_{x_0} t$: $39 = 117.3 \cos 5° \, t_B$, $t_B = 0.334$ sec

$a_y = g$, $y = v_{y_0} t + \frac{1}{2} g t^2$: At B,

$5.5 - h = 117.3 \sin 5° (0.334) + \frac{1}{2}(32.2)(0.334)^2$

$\underline{h = 0.296 \text{ ft}}$ or $\underline{h = 3.55 \text{ in.}}$

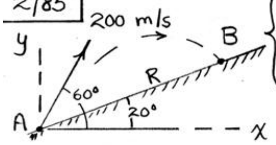
At C: $8.5 = 117.3 \sin 5° \, t_C + 16.1 t_C^2$

$t_C = 0.475$ sec

x-equation at C: $39 + s = 117.3 \cos 5° (0.475)$

$\underline{s = 16.57 \text{ ft}}$

2/85



$\begin{cases} v_{x_0} = 200 \cos 60° = 100 \text{ m/s} \\ v_{y_0} = 200 \sin 60° = 173.2 \text{ m/s} \end{cases}$

t_f = flight time

$x = x_0 + v_{x_0} t$ @ B: $R \cos 20° = 100 t_f$ (1)

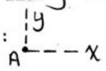
$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $R \sin 20° = 173.2 t_f - \frac{9.81}{2} t_f^2$ (2)

(1): $t_f = 0.00940 R$

(2): $R \sin 20° = 173.2(0.00940 R) - \frac{9.81}{2}(0.00940 R)^2$

$\underline{R = 2970 \text{ m}}$

2/86 x-y coordinates with origin at release point :



$v_{x_0} = v_0 \sin \theta = 12 \sin \theta$

$v_{y_0} = v_0 \cos \theta = 12 \cos \theta$

$v_y = v_{y_0} - g t$ applied at end of flight :

$-12 \cos \theta = 12 \cos \theta - 9.81 t_f$, $t_f = 2.45 \cos \theta$

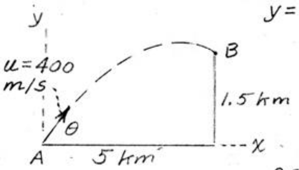
$v_x = v_{x_0} - 0.4 t$ applied at end of flight :

$-12 \sin \theta = 12 \sin \theta - 0.4(2.45 \cos \theta)$

$24 \sin \theta = 0.979 \cos \theta$, $\tan \theta = 0.041$

$\underline{\theta = 2.33°}$

2/87 Eq. of trajectory (Sample Problem 2/6)



$y = x \tan \theta - \frac{g x^2}{2u^2} \sec^2 \theta$

$= x \tan \theta - \frac{g x^2}{2u^2}(1 + \tan^2 \theta)$

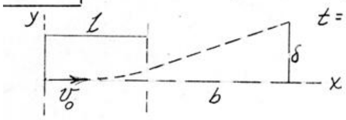
Substitute values & get

$1500 = 5000 \tan \theta - \frac{9.81(5000)^2}{2(400)^2}(1 + \tan^2 \theta)$

or $\tan^2 \theta - 6.524 \tan \theta + 2.957 = 0$

Solution gives roots $\underline{\theta_1 = 26.1°}$ & $\underline{\theta_2 = 80.6°}$

2/88 Time to travel through plates is



$t = l/v_0$ since $v_x = v_0$ = constant

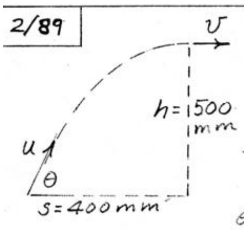
$v_y = a_y t = \frac{eE}{m} \frac{l}{v_0}$ upon emergence from plates & deflection at this point is

$y = \frac{1}{2} a_y t^2 = \frac{eE}{2m} \frac{l^2}{v_0^2}$

Also $\delta - y/b = v_y/v_x$ so $\delta = \frac{eE l^2}{2m v_0^2} + b \frac{eE l}{m v_0} / v_0$

$\underline{\delta = \frac{eEl}{m v_0^2}\left(\frac{l}{2} + b\right)}$

2/89



From results of Sample Problem 2/6

$h = \frac{u^2 \sin^2 \theta}{2g}$, $s = \frac{u^2 \sin 2\theta}{2g}$

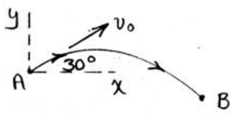
so $\frac{h}{s} = \frac{\sin^2 \theta}{\sin 2\theta} = \frac{1}{2} \tan \theta$

$\theta = \tan^{-1} \frac{2h}{s} = \tan^{-1} \frac{2(1500)}{400} = \underline{68.2°}$

$u^2 = \frac{2gh}{\sin^2 \theta} = \frac{2(9.81)(0.5)}{(0.9285)^2} = 11.38 \text{ m}^2/\text{s}^2$, $u = 3.37$ m/s

$v = u \cos \theta$ = const., $v = 3.37(0.3714) = \underline{1.253 \text{ m/s}}$

2/90 Set up x-y axes at A, target at B:



$\left.\begin{array}{l} x\text{-eq.}: \; x_B = (v_0 \cos 30°) t \\ y\text{-eq.}: \; y_B = (v_0 \sin 30°) t - \frac{1}{2} g t^2 \end{array}\right\}$

For $x_B = 12'$, $y_B = -0.333'$: $\begin{cases} v_0 = 20.6 \text{ ft/sec} \\ t = 0.672 \text{ sec} \end{cases}$

For $x_B = 14'$, $y_B = -0.333'$: $\begin{cases} v_0 = 22.4 \text{ ft/sec} \\ t = 0.723 \text{ sec} \end{cases}$

So the range is $\underline{20.6 \le v_0 \le 22.4 \text{ ft/sec}}$

2/91 Set up x-y Coordinates at A

x-eq. : $x_B = (36 \cos \theta) t$

y-eq. : $y_B = (36 \sin \theta) t - 16.1 t^2$

Solutions :

For $x_B = 40'$, $y_B = -\frac{22}{12}'$ (top of stake) :

$\theta = 34.3°$ or $\theta = 53.1°$

For $x_B = 40'$, $y_B = -3'$ (bottom of stake):

$\theta = 31.0°$ or $\theta = 54.7°$

Ranges : $31.0° \le \theta \le 34.3°$

or $\underline{53.1° \le \theta \le 54.7°}$

2/92 Set up x-y coordinates with origin at release point : $\overset{|y}{\llcorner}$ $- \theta - x$, $v_0 = 40$ m/s

$x = x_0 + v_{x_0} t$ at mitt : $20 = (40 \cos\theta) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ at mitt :

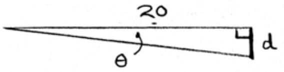
$-1.8 = 0 + (-40 \sin\theta) t_f - \frac{9.81}{2} t_f^{\,2}$ (2)

(1) : $t_f = \frac{1}{2\cos\theta}$

(2) : $-1.8 = -40 \sin\theta \left(\frac{1}{2\cos\theta}\right) - \frac{9.81}{2}\left(\frac{1}{2\cos\theta}\right)^2$

Use $\frac{1}{\cos^2\theta} = \tan^2\theta + 1$: $1.226 \tan^2\theta + 20 \tan\theta - 0.574 = 0$

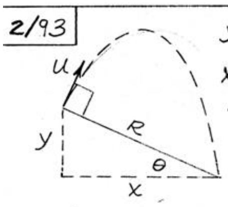
$\Rightarrow \theta = 1.640°$



$d = 20 \tan 1.640 = 0.573'$

$h = (2.2 + 0.6) - (0.573 + 1) = \underline{1.227 \text{ m}}$

2/93



$y = R \sin\theta = -ut\cos\theta + \frac{1}{2} g t^2$

$x = R\cos\theta = ut \sin\theta$

Eliminate t and get

$R \sin\theta = -R \frac{\cos\theta}{\sin\theta}\cos\theta + \frac{1}{2} g \left(\frac{R\cos\theta}{u \sin\theta}\right)^2$

$\frac{1}{\sin\theta} = \frac{g}{2} \frac{R}{u^2 \tan^2\theta}$

$R = \frac{2u^2 \tan^2\theta}{g \sin\theta} = \underline{\frac{2u^2}{g} \tan\theta \sec\theta}$

2/94 Use x-y coordinates with origin at the release point : $\overset{|y}{\llcorner}$ $- - x$

$x = x_0 + v_{x_0} t$ @ hoop : $13.75 = 0 + (v_0 \cos 50°) t_f$

$t_f = 21.4 / v_0$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ hoop :

$3 = 0 + v_0 \sin 50° \left(\frac{21.4}{v_0}\right) - 16.1 \left(\frac{21.4}{v_0}\right)^2$

$\underline{v_0 = 23.5 \text{ ft/sec}}$

2/95 Set up x-y coordinates at A : $\overset{|y}{\underset{A}{\llcorner}}$ $- x$

From $y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ evaluated at D, we have $0 = 0 + v_0 \sin\alpha\, t_D - \frac{1}{2} g t_D^{\,2}$ or $t_D = \frac{2 v_0 \sin\alpha}{g}$

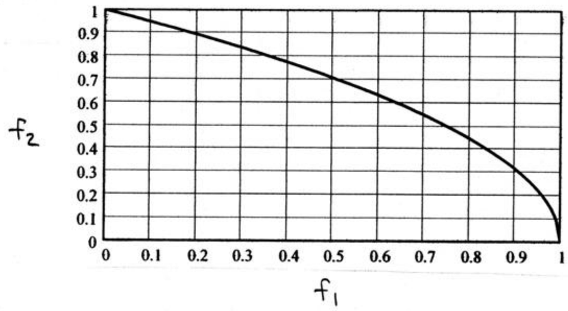
From $v_y^{\,2} = v_{y_0}^{\,2} - 2g(y - y_0)$ evaluated at apex, we have $0 = v_0^{\,2}\sin^2\alpha - 2gh$ or $h = \frac{v_0^{\,2}\sin^2\alpha}{2g}$

Time to $f_1 h$: $f_1 h = f_1 \frac{v_0^{\,2}\sin^2\alpha}{2g} = (v_0 \sin\alpha) t - \frac{1}{2} g t^2$

Solve quadratic to obtain $\begin{cases} t_B = \frac{v_0 \sin\alpha (1 - \sqrt{1 - f_1})}{g} \\ t_C = \frac{v_0 \sin\alpha (1 + \sqrt{1 - f_1})}{g} \end{cases}$

So $t_{BC} = t_C - t_B = \frac{2\sqrt{1-f_1}\, v_0 \sin\alpha}{g}$

$f_2 = \frac{t_{BC}}{t_D} = \sqrt{1 - f_1}$; $f_2 = \frac{1}{2}$ for $f_1 = \frac{3}{4}$.



2/96 With x-y coordinates, origin at A : $\overset{|y}{\underset{A}{\llcorner}}$ $- - x$

$x = x_0 + v_{x_0} t$ @ B : $360 = 0 + (100 \cos\alpha) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B : $-80 = 0 + (100 \sin\alpha) t_f - \frac{1}{2}(32.2) t_f^{\,2}$ (2)

Simultaneous solutions of (1) & (2) :

$\begin{cases} t_f = 4.03 \text{ sec}, & \alpha = 26.8° \quad (a) \\ t_f = 5.68 \text{ sec}, & \alpha = 50.7° \quad (b) \end{cases}$

Check at corner $[(x, y) = (280, 0)]$:

(a) $t_c = \frac{280}{100 \cos 26.8°} = 3.14$ sec

$y_c = 100 \sin 26.8° (3.14) - \frac{32.2}{2} (3.14)^2 = -16.94$ ft

So conditions (a) are not possible.

(b) $t_c = \frac{280}{100 \cos 50.7°} = 4.42$ sec

$y_c = 100 \sin 50.7° (4.42) - \frac{32.2}{2} (4.42)^2 = 27.5$ ft

Conditions (b) result in clearance at corner

Ans. $\underline{\alpha = 50.7°}$

▸2/97

$$\mathbf{a} = -k\mathbf{v} - g\mathbf{j}$$
$$a_x\mathbf{i} + a_y\mathbf{j} = -k(v_x\mathbf{i} + v_y\mathbf{j}) - g\mathbf{j} \quad \therefore \; a_x = -kv_x, \quad a_y = -kv_y - g$$

$\underline{x}$: $a_x = \dfrac{dv_x}{dt} = -kv_x$

$$\int_{v_{x_0}}^{v_x} \frac{dv_x}{v_x} = -\int_0^t k\,dt \Rightarrow v_x = v_{x_0}e^{-kt}$$

$$\text{or } \underline{v_x = (v_0\cos\theta)e^{-kt}}$$

$$v_x = \frac{dx}{dt} = v_{x_0}e^{-kt}$$
$$\int_0^x dx = \int_0^t v_{x_0}e^{-kt}\,dt$$
$$x = \frac{v_{x_0}}{k}\left[1 - e^{-kt}\right] = \underline{\frac{v_0\cos\theta}{k}\left[1 - e^{-kt}\right]}$$

$\underline{y}$: $a_y = \dfrac{dv_y}{dt} = -kv_y - g$

$$\int_{v_{y_0}}^{v_y} \frac{dv_y}{kv_y + g} = -\int_0^t dt$$

$$\Rightarrow v_y = \left[v_{y_0} + \frac{g}{k}\right]e^{-kt} - \frac{g}{k} = \underline{\left[v_0\sin\theta + \frac{g}{k}\right]e^{-kt} - \frac{g}{k}}$$

$$v_y = \frac{dy}{dt} = \left[v_{y_0} + \frac{g}{k}\right]e^{-kt} - \frac{g}{k}$$
$$\int_0^y dy = \int_0^t \left\{\left[v_{y_0} + \frac{g}{k}\right]e^{-kt} - \frac{g}{k}\right\}dt$$
$$\underline{y = \frac{1}{k}\left[v_0\sin\theta + \frac{g}{k}\right]\left[1 - e^{-kt}\right] - \frac{g}{k}t}$$

Terminal velocity $(t \to \infty)$: $v_x \to 0$, $\underline{v_y \to -\dfrac{g}{k}}$

▸2/98 Use the x-y coordinates of the figure. In the absence of the circular surface B-C, the range over a horizontal surface would be $\dfrac{v_0^2}{g}\sin 2\theta = \dfrac{225^2}{32.2}\sin(2\cdot 30°) = 1362$ ft > 1000 ft, so the impact point is beyond B.

$x = x_0 + v_{x_0}t$: $x = (225\cos 30°)t$ (1)

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$: $y = (225\sin 30°)t - 16.1t^2$ (2)

Surface constraint:

$$(x - 1000)^2 + (y - 500)^2 = 500^2 \qquad (3)$$

Computer solution of Eqs. (1), (2), & (3):

$\underline{x = 1242 \text{ ft}}$, $\underline{y = 62.7 \text{ ft}}$, $t = 6.38$ sec

(The nature of the solution indicates impact on the circular surface and not above C.)

▸2/99

$a_y = g$, $v_y = gt$, $y = \frac{1}{2}gt^2$

$a_x = 2v_y\omega\cos\gamma = 2gt\omega\cos\gamma$

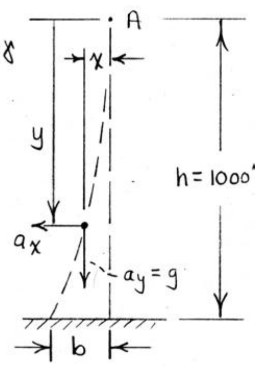
$v_x = gt^2\omega\cos\gamma$

$x = \frac{1}{3}gt^3\omega\cos\gamma$

When $y = h$, $t = \sqrt{\dfrac{2h}{g}}$

Then

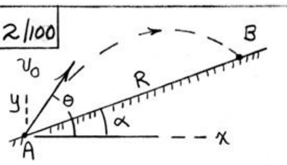
$$x = b = \frac{1}{3}g\left(\frac{2h}{g}\right)^{3/2}\omega\cos\gamma$$
$$b = \frac{2\sqrt{2}}{3}h\sqrt{\frac{h}{g}}\,\omega\cos\gamma$$



For $h = 1000$ ft, $\omega = 0.7292(10^{-4})$ rad/sec, and $g = 32.13$ ft/sec^2,

$$b = \frac{2\sqrt{2}}{3}(1000)\sqrt{\frac{1000}{32.13}}\,(0.7292\cdot 10^{-4})\cos 30°$$
$$= 0.332 \text{ ft or } \underline{3.99 \text{ in.}}$$

▸2/100



$x = x_0 + v_{x_0}t$ @ B: $R\cos\alpha = (v_0\cos\theta)t_f$

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ @ B: $R\sin\alpha = (v_0\sin\theta)t_f - \frac{1}{2}gt_f^2$

x-eq: $t_f = \dfrac{R\cos\alpha}{v_0\cos\theta}$

y-eq: $R\sin\alpha = (v_0\sin\theta)\left(\dfrac{R\cos\alpha}{v_0\cos\theta}\right) - \dfrac{1}{2}g\left(\dfrac{R\cos\alpha}{v_0\cos\theta}\right)^2$

$$\Rightarrow R = \frac{2v_0^2\cos^2\theta}{g\cos\alpha}(\tan\theta - \tan\alpha)$$

$$\frac{dR}{d\theta} = 0: \frac{4v_0^2\cos\theta(-\sin\theta)}{g\cos\alpha}(\tan\theta - \tan\alpha) + \frac{2v_0^2\cos^2\theta}{g\cos\alpha}\frac{1}{\cos^2\theta} = 0$$

$$\frac{2v_0^2}{g\cos\alpha}\left[2\cos\theta\sin\theta(\tan\alpha - \tan\theta) + 1\right] = 0$$

$$\Rightarrow 2\cos\theta\sin\theta\left(\tan\alpha - \frac{\sin\theta}{\cos\theta}\right) + 1 = 0$$
$$(2\cos\theta\sin\theta)\tan\alpha - 2\sin^2\theta + 1 = 0$$
$$\sin 2\theta\tan\alpha - 2\left(\tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta\right) + 1 = 0$$
$$\sin 2\theta\tan\alpha + \cos 2\theta = 0$$
$$\tan 2\theta = -\frac{1}{\tan\alpha}$$

$$2\theta = \tan^{-1}\left(-\frac{1}{\tan\alpha}\right) = 180° - \tan^{-1}\left(\frac{1}{\tan\alpha}\right)$$
$$= 180° - (90° - \alpha) = 90° + \alpha$$
$$\therefore \theta = \underline{\frac{90° + \alpha}{2}}$$

Specific results:
- $\alpha = 0$, $\theta = 45°$
- $\alpha = 30°$, $\theta = 60°$
- $\alpha = 45°$, $\theta = 67.5°$

2/101 For a_t = constant, $v = \cancel{v_0}^{\,0} + a_t t$

So $a_t = \dfrac{v}{t} = \dfrac{100\,(1000)}{3600}\Big/10 = 2.78\ \text{m/s}^2$

$v_8 = 2.78(8) = 22.2\ \text{m/s}$

$a_n = \dfrac{v^2}{\rho} = \dfrac{22.2^2}{80} = 6.17\ \text{m/s}^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2.78^2 + 6.17^2} = \underline{6.77\ \text{m/s}^2}$

2/102 a_1: speed v is increasing, no path curvature.

a_2: speed v increasing, car turning right.

a_3: speed v stationary, car turning right.

a_4: speed v decreasing, car turning right.

a_5: speed v decreasing, no path curvature.

a_6: speed v decreasing, car turning left.

a_7: speed v stationary, car turning left.

a_8: speed v increasing, car turning left.

2/103 $a_n = v^2/r = (0.6)^2/0.3 = 1.2\ m/s^2$

(a) $a_t = \dot{v} = 0$ so $\underline{a = a_n = 1.2\ m/s^2}$

(b) $a_t = \dot{v} = 0.9\ m/s^2$ so $a = \sqrt{a_n^2 + a_t^2} = \sqrt{(1.2)^2 + (0.9)^2}$

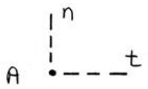
$\underline{a = 1.5\ m/s^2}$

2/104 $a = a_n = v^2/\rho$, $v = \sqrt{\rho a_n} = \sqrt{(100-0.6)0.5(9.81)}$

$= 22.08\ m/s$

or $v = 22.08(3.6) = \underline{79.5\ km/h}$

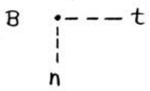
2/105



At A: $a_n = \dfrac{v_A^2}{\rho_A}$

$0.4(9.81) = \dfrac{v_A^2}{120-0.6}$

$\underline{v_A = v = 21.6\ m/s}$



At B: $a_n = \dfrac{v_B^2}{\rho_B}$

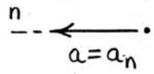
$0.25(9.81) = \dfrac{21.7^2}{\rho_B + 0.6}$

$\underline{\rho_B = 190.4\ m}$

2/106 (a) $a_n = \dfrac{v^2}{\rho} = \dfrac{1.2^2}{0.6} = 2.4\ \text{m/s}^2$

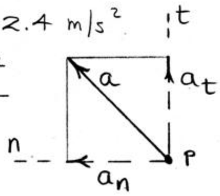
$a_t = 0$

$a = \sqrt{a_n^2 + a_t^2} = \underline{2.4\ \text{m/s}^2}$



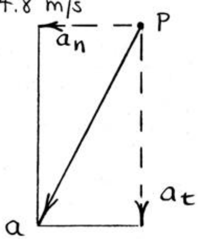
(b) $a_n = 2.4\ \text{m/s}^2$, $a_t = 2.4\ \text{m/s}^2$

$a = \sqrt{2.4^2 + 2.4^2} = \underline{3.39\ \text{m/s}^2}$

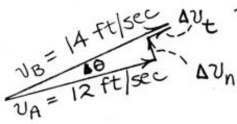


(c) $a_n = 2.4\ \text{m/s}^2$, $a_t = -4.8\ \text{m/s}^2$

$a = \sqrt{2.4^2 + 4.8^2} = \underline{5.37\ \text{m/s}^2}$



2/107



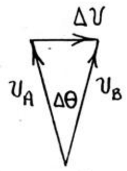
$\Delta\theta = (25-15)\dfrac{\pi}{180} = 0.1745$ rad

$v_{av} = \dfrac{12+14}{2} = 13$ ft/sec

$a_n = \dfrac{\Delta v_n}{\Delta t} = \dfrac{v_{av}(\Delta\theta)}{\Delta t} = \dfrac{13(0.1745)}{2.62-2.4} = \underline{10.31\ \dfrac{\text{ft}}{\text{sec}^2}}$

$a_t = \dfrac{\Delta v_t}{\Delta t} = \dfrac{14-12}{0.22} = \underline{9.09\ \dfrac{\text{ft}}{\text{sec}^2}}$

2/108 $v_A = v_B = v = 2$ m/s



$\Delta v = 2v\sin\dfrac{\Delta\theta}{2} = 4\sin\dfrac{\Delta\theta}{2}$ m/s

$\Delta t = \dfrac{r\,\Delta\theta}{v} = \dfrac{0.8\,\Delta\theta}{2} = 0.4\,\Delta\theta$ s

$a_{av} = \dfrac{\Delta v}{\Delta t} = \dfrac{4\sin\frac{\Delta\theta}{2}}{0.4\,\Delta\theta} = 5\,\dfrac{\sin\frac{\Delta\theta}{2}}{\Delta\theta/2}$

	$\Delta\theta°$	$\frac{\Delta\theta}{2}°$	$\frac{\Delta\theta}{2}$ rad	$\sin\frac{\Delta\theta}{2}$	a_{av}, m/s²	% diff.
(a)	30°	15°	0.262	0.259	4.94	1.1
(b)	15°	7.5°	0.1309	0.1305	4.99	0.3
(c)	5°	2.5°	0.0436	0.0436	4.998	0.03

$a_n = \dfrac{v^2}{\rho} = \dfrac{2^2}{0.8} = \underline{5\ \text{m/s}^2}$

2/109 From $a_n = \dfrac{v^2}{\rho}$, $v = \sqrt{a_n\rho} = \sqrt{0.8g\rho}$

$v_A = \sqrt{0.8g\rho_A} = \sqrt{0.8(9.81)(85)} = \underline{25.8\ \text{m/s}}$

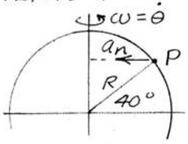
$v_B = \sqrt{0.8g\rho_B} = \sqrt{0.8(9.81)(200)} = \underline{39.6\ \text{m/s}}$

Path BB offers a considerable advantage.

2/110 $a = a_n = r\dot{\theta}^2 = R\cos\gamma\,\dot{\theta}^2$

$= \dfrac{12.742(10^6)}{2}\cos 40°\,(0.729\times10^{-4})^2$

$= \underline{0.0259\ m/s^2}$



2/111 $v = v_0 + a_t t = 0 + 1.8(5) = 9\ m/s$

$a_n = \dfrac{v^2}{\rho} = \dfrac{9^2}{40} = 2.025\ m/s^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(1.8)^2 + (2.025)^2} = \underline{2.71\ \frac{m}{s^2}}$

2/112 $a_n = v^2/\rho$; $\rho = s/\theta = \dfrac{300}{\pi/4} = 382\ ft$

$v = 45\,\frac{44}{30} = 66\ ft/sec$

$a = a_n = 66^2/382 = \underline{11.40\ ft/sec^2}$

2/113 $a_n = g = \dfrac{v^2}{\rho} = \dfrac{[17{,}369\,(5280/3600)]^2}{(3959+150)(5280)}$

$= \underline{29.91\ ft/sec^2}$

Check : $g = g_0\left(\dfrac{R}{R+h}\right)^2 = 32.22\left(\dfrac{3959}{3959+150}\right)^2$

$= \underline{29.91\ ft/sec^2}$ ✓

2/114 The radius of Jupiter is

$R = \dfrac{142\,984}{2}(10^3) = 7.15(10^7)\ m$

So $\rho = R+h = 7.15(10^7) + 10^6\ m = 7.25(10^7)\,m$

From the gravitational law

$a_n = g = g_0\dfrac{R^2}{(R+h)^2} = g_0\dfrac{R^2}{\rho^2} = 24.85\dfrac{[7.15(10^7)]^2}{[7.25(10^7)]^2}$

$= 24.2\ m/s^2$

From $a_n = \dfrac{v^2}{\rho}$, $v^2 = a_n\rho = (24.2)(7.25\cdot10^7)$

$\underline{v = 41\,900\ m/s}$ or $\underline{v = 150\,700\ km/h}$

2/115 $a_t = 20/3.6 = 5.56\ m/s^2$

$a^2 = a_n^2 + a_t^2$, $a_n^2 = \overline{3(9.81)}^2 - \overline{5.56}^2 = 835.2$

$a_n = 28.90\ m/s^2$

$a_n = v^2/\rho$, $\rho = \dfrac{(800/3.6)^2}{28.90} = \underline{1709\ m}$

2/116 $a_t = -0.6\ \frac{m}{s^2}$, constant

$v_B^2 = v_A^2 + 2a_t s = 16^2 - 2(0.6)(120)$

$v_B = 10.58\ m/s$

$a_n = v_B^2/\rho = 10.58^2/60 = 1.867\ m/s^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.6^2 + 1.867^2} = \underline{1.961\ \frac{m}{s^2}}$

2/117 $a_n = v\dot{\beta} = g$, $\dot{\beta} = \dfrac{9.79}{800(10^3)/3600} = 0.04406\ rad/s$

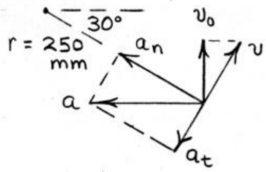
or $\dot{\beta} = 0.04406\,\dfrac{180}{\pi} = \underline{2.52\ deg/s}$

2/118 $v = v_0/\cos 30° = 2/\cos 30° = 2.31\ m/s$

$a_n = v^2/r = 2.31^2/0.250$

$= \underline{21.3\ m/s^2}$

$a_t = -a_n\tan 30° = \underline{-12.32\ \frac{m}{s^2}}$

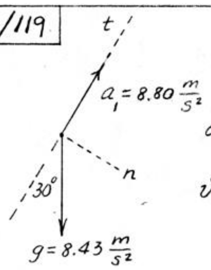


2/119 $a_n = g\sin 30° = 8.43(0.5) = 4.22\ m/s^2$

or $a_n = \dfrac{4.22}{1000}(3600)^2 = 54\,630\ km/h^2$

$a_n = \dfrac{v^2}{\rho}$, $\rho = \dfrac{v^2}{a_n} = \dfrac{(30000)^2}{54630} = \underline{16\,480\ km}$

$\dot{v} = a_t = 8.80 - 8.43\cos 30° = \underline{1.499\ m/s^2}$



2/120 $v = r\dot{\theta}$, $v_B = \frac{8}{4}2 = 4\ ft/sec$

$a_n = v^2/r$, $a_n = 4^2/\tfrac{8}{12} = 24\ ft/sec^2$

$a_t = r\ddot{\theta}$; $a_t = \frac{8}{4}6 = 12\ ft/sec^2$

$a_B = \sqrt{24^2 + 12^2} = \underline{26.8\ ft/sec^2}$

2/121 Relative to space station, $a_n = r\dot{\theta}^2$

where $a_n = 32.17\ ft/sec^2$.

Thus $32.17 = (240+20)\dot{\theta}^2$, $\dot{\theta} = 0.352\ \frac{rad}{sec}$

$N = 0.352\left(\dfrac{60}{2\pi}\right) = \underline{3.36\ rev/min}$

2/122 $a_t = \dfrac{v_f - v_i}{\Delta t} = \dfrac{6-3}{2} = 1.5\ m/s^2$

Halfway through time interval, $v = 4.5\ m/s$

$a_{P_1} = \sqrt{a_t^2 + a_n^2} = \sqrt{1.5^2 + \left(\dfrac{4.5^2}{0.060}\right)^2}$

$= \underline{338\ m/s^2}$ (34.4g!)

$a_{P_2} = a_t = \underline{1.5\ m/s^2}$

2/123 $v^2 = v_0^2 + 2a_t\Delta s$, $18^2 = 2^2 + 2a_t(8)$

$a_t = 20\ m/s^2$

$a_n = v^2/r = 3^2/0.150 = 60\ m/s^2$

$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20^2 + 60^2} = \underline{63.2\ m/s^2}$

2/124 $v_{x=50'} = \sqrt{2g\left(\frac{50}{20}\right)^2} = \frac{5}{2}\sqrt{2g}\ \frac{\text{ft}}{\text{sec}}$

$a_n = \frac{v^2}{\rho}$, where $\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left[1+\left(\frac{x}{200}\right)^2\right]^{\frac{3}{2}}}{1/200}$

$\rho_{x=50'} = 219$ ft

$a_n = \frac{25}{4}\,2(32.2)/219 = \underline{1.838\ \text{ft/sec}^2}$

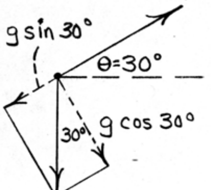
2/125 $a_n = v^2/r = 4^2/0.120 = 133.3\ \text{m/s}^2$

$a_t = -a_n \text{ ctn } 4° = -133.3 \text{ ctn } 4° = -1907\ \text{m/s}^2$

With a_t const, $v_f = v_i + a_t t$, $0 = 4 - 1907t$

$t = \frac{4}{1907} = \underline{2.10(10^{-3})\ \text{s}}$

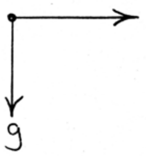
2/126 $v_0 = 100$ ft/sec



(a) $a_n = g\cos 30° = \frac{v^2}{\rho}$

$\rho = \frac{100^2}{g\cos 30°} = \underline{359\ \text{ft}}$

$\dot{v} = -g\sin 30° = \underline{-16.1\ \text{ft/sec}^2}$



(b) $a_n = g = \frac{v^2}{\rho}$

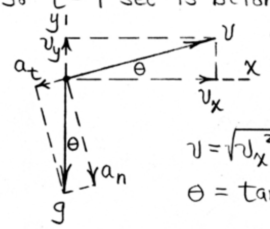
$\rho = \frac{(100\cos 30°)^2}{32.2} = \underline{233\ \text{ft}}$

$\underline{\dot{v} = 0}$

2/127 The time t_{up} to apex is found from

$v_y = v_{y_0} - gt$: $0 = 100\sin 30° - 32.2\,t_{up}$, $t_{up} = 1.553$ sec

So $t = 1$ sec is before apex and $t = 2.5$ sec is after.



(a) $t = 1$ sec

$v_x = 100\cos 30° = 86.6$ ft/sec

$v_y = 100\sin 30° - 32.2(1) = 17.80$ ft/sec

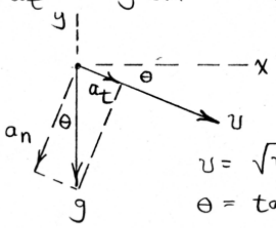
$v = \sqrt{v_x^2 + v_y^2} = 88.4$ ft/sec

$\theta = \tan^{-1}\frac{v_y}{v_x} = 11.61°$

$a_n = g\cos\theta = 32.2\cos 11.61° = 31.5\ \text{ft/sec}^2$

$\rho = \frac{v^2}{a_n} = \frac{88.4^2}{31.5} = \underline{248\ \text{ft}}$

$a_t = -g\sin\theta = -32.2\sin 11.61° = \underline{-6.48\ \text{ft/sec}^2}$



(b) $t = 2.5$ sec

$v_x = 86.6$ ft/sec

$v_y = 100\sin 30° - 32.2(2.5) = -30.5\ \frac{\text{ft}}{\text{sec}}$

$v = \sqrt{v_x^2 + v_y^2} = 91.8$ ft/sec

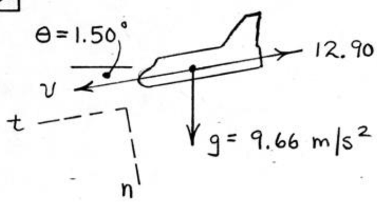
$\theta = \tan^{-1}\frac{|v_y|}{v_x} = 19.40°$

$a_n = g\cos\theta = 32.2\cos 19.40° = 30.4\ \text{ft/sec}^2$

$\rho = \frac{v^2}{a_n} = \frac{91.8^2}{30.4} = \underline{278\ \text{ft}}$

$a_t = +g\sin\theta = +32.2\sin 19.40° = \underline{10.70\ \text{ft/sec}^2}$

2/128



$\dot{v} = a_t = 9.66\sin 1.50° - 12.90 = \underline{-12.65\ \text{m/s}^2}$

$a_n = g\cos\theta = 9.66\cos 1.5° = 9.657\ \text{m/s}^2$

$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{v^2}{a_n} = \frac{(15\,450/3.6)^2}{9.657}$

$\underline{\rho = 1907\ \text{km}}$

2/129 $a_n = 0.8g = \frac{v^2}{\rho} \Rightarrow v = \sqrt{0.8g\rho}$

Car A : $v_A = \sqrt{0.8(9.81)(88)} = 26.3$ m/s

Car B : $v_B = \sqrt{(0.8)(9.81)72} = 23.8$ m/s

$t_A = \frac{s_A}{v_A} = \frac{\pi(88)}{26.3} = \underline{10.52\ \text{s}}$

$t_B = \frac{s_B}{v_A} = \frac{\pi(72) + 2(16)}{23.8} = \underline{10.86\ \text{s}}$

Car A will win the race!

2/130 Evaluate $y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ at C:

$0 = 150 + 0 - \frac{1}{2}(9.81)t^2$, $t = 3.05$ sec

Evaluate $x = x_0 + v_{x_0}t + \frac{1}{2}a_x t^2$ at C :

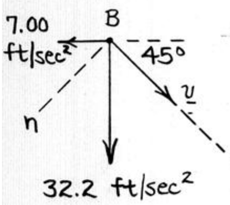
$120 = 0 + 50(3.08) + \frac{1}{2}a_x(3.08)^2$, $a_x = -7.00\ \text{ft/sec}^2$

At B: $\begin{cases} v_x = v_{x_0} + a_x t : & v_x = 50 - 7.00t \\ v_y = v_{y_0} - gt : & |v_y| = 32.2t \end{cases}$

Set $v_x = |v_y|$ & obtain $t = 1.275$ sec

So at B: $v_x = |v_y| = 32.2(1.276) = 41.1$ ft/sec

The speed at B is $v = 41.1\sqrt{2} = 58.1$ ft/sec



$a_n = 32.2\cos 45° + 7.00\cos 45°$

$= 27.7\ \text{ft/sec}^2$

From $a_n = \frac{v^2}{\rho}$,

$\rho = \frac{v^2}{a_n} = \frac{(58.1)^2}{27.7} = \underline{121.7\ \text{ft}}$

2/131 For a_t = const, $v_C^2 = v_A^2 + 2a_t\Delta s_{A-C}$

$v_A = \frac{250}{3.6}$ m/s, $v_C = \frac{200}{3.6}$ m/s, $a_t = \frac{(200)^2 - (250)^2}{(3.6)^2\,2(300)} = -2.89\ \text{m/s}^2$

$v_B^2 = v_A^2 + 2a_t\Delta s_{A-B} = \left(\frac{250}{3.6}\right)^2 + 2(-2.89)(150) = 3954\ (\text{m/s})^2$

$v_B = 62.9$ m/s

at B, $a_n = v_B^2/\rho = \frac{3954}{500} = 7.91\ \text{m/s}^2$

$a = \sqrt{a_n^2 + a_t^2} = \sqrt{(7.91)^2 + (2.89)^2} = \underline{8.42\ \text{m/s}^2}$

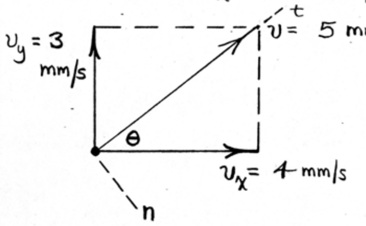
2/132 $x = 16 - 12t + 4t^2$ $\qquad y = 2 + 15t - 3t^2$

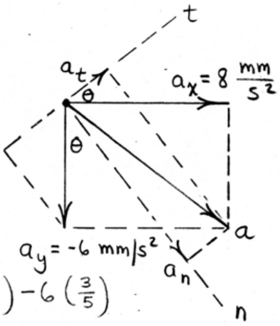
$\dot{x} = 8t - 12 \qquad \dot{y} = 15 - 6t$

$\ddot{x} = 8 \qquad \ddot{y} = -6$

At $t = 2\text{ s}$: $\dot{x} = 4 \text{ mm/s}$ $\qquad \dot{y} = 15 - 12 = 3 \frac{\text{mm}}{\text{s}}$

$\ddot{x} = 8 \text{ mm/s}^2 \qquad \ddot{y} = -6 \text{ mm/s}^2$





$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$a_t = 8\cos\theta - 6\sin\theta = 8\left(\frac{4}{5}\right) - 6\left(\frac{3}{5}\right)$

$= 2.8 \text{ mm/s}^2$

$a_n = 6\cos\theta + 8\sin\theta = 6\left(\frac{4}{5}\right) + 8\left(\frac{3}{5}\right)$

$= 9.6 \text{ mm/s}^2$

$a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{5^2}{9.6} = \underline{2.60 \text{ mm}}$

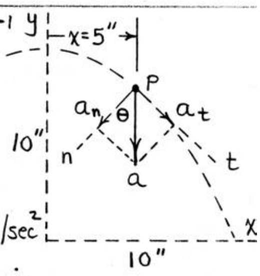
2/133 $y - 10 = kx^2$, $k = -\frac{1}{10}\text{ in.}^{-1}$

So $y = 10\left(1 - \frac{x^2}{100}\right)$ in.

$\dot{x} = 15 \text{ in./sec}$, $\ddot{x} = 0$

$\dot{y} = -x\dot{x}/5$, $\ddot{y} = -\dot{x}^2/5$

So $a = a_y = \ddot{y} = -\frac{15^2}{5} = -45 \text{ in./sec}^2$

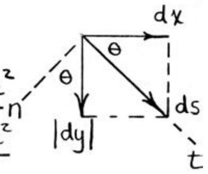


$\frac{dy}{dx} = -\frac{x}{5} = -\frac{6}{5}$ @ $x = 6$ in.

$\theta = \tan^{-1}\frac{6}{5} = 50.2°$

$a_n = a\cos\theta = 45\cos 50.2° = 28.8 \text{ in./sec}^2$

$a_t = a\sin\theta = 45\sin 50.2° = 34.6 \text{ in./sec}^2$



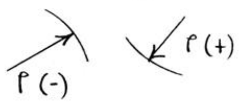
For $x = 6$ in., $\dot{y} = -6(15)/5 = -18$ in./sec

$v = \sqrt{15^2 + 18^2} = 23.4$ in./sec

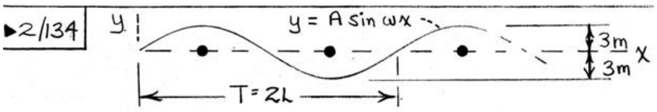
$a_n = \frac{v^2}{\rho}$, $\rho = \frac{23.4^2}{28.8} = \underline{19.06 \text{ in.}}$

Check : $\rho_{xy} = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{d^2y/dx^2} = \frac{\left[1 + (-6/5)^2\right]^{3/2}}{-1/5}$

$= -19.06$ in. ✓



▸2/134



$y = A\sin\omega x$, where $A = 3\text{ m}$ & $\omega = \frac{2\pi}{T}$

Radius of curvature $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$\frac{dy}{dx} = A\omega\cos\omega x$, $\quad \frac{d^2y}{dx^2} = -A\omega^2\sin\omega x$

Set $\frac{d\rho}{dx} = 0$ to show that $|\rho|$ is a min @ $x = \frac{T}{4}$

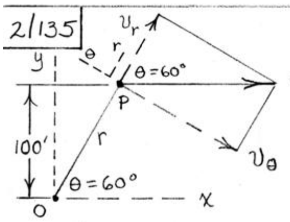
& $x = \frac{3T}{4}$.

$\rho_{min} = \frac{\left[1 + \left\{A\frac{2\pi}{T}\cos\left(\frac{2\pi}{T}\cdot\frac{T}{4}\right)\right\}^2\right]}{+A\left(\frac{2\pi}{T}\right)^2\sin\left(\frac{2\pi}{T}\cdot\frac{T}{4}\right)} = \frac{T^2}{4\pi^2 A}$

$a_n = \frac{v^2}{\rho}$: $0.7\,(9.81) = \frac{(80/3.6)^2}{T^2/(4\pi^2\cdot 3)}$

$T = 92.3 \text{ m} = 2L$, $\quad \underline{L = 46.1 \text{ m}}$

2/135



$v = 65$ mi/hr

$v = 65\left(\frac{5280}{3600}\right) = 95.3$ ft/sec

$r = \frac{100}{\sin 60°} = 115.5$ ft

$v_r = \dot{r} = v\cos\theta = 95.3\cos 60° = \underline{47.7 \text{ ft/sec}}$

$v_\theta = r\dot{\theta} = -v\sin\theta$

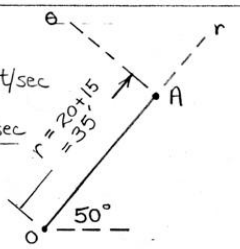
$\dot{\theta} = -\frac{v\sin\theta}{r} = -\frac{95.3\sin 60°}{115.5} = -0.715$ rad/sec

or $\dot{\theta} = -0.715\left(\frac{180}{\pi}\right) = \underline{-41.0 \text{ deg/sec}}$

2/136 $v_r = \dot{r} = 0.5$ ft/sec

$v_\theta = r\dot{\theta} = 35\left(2\frac{\pi}{180}\right) = 1.222$ ft/sec

$v = \sqrt{0.5^2 + 1.222^2} = \underline{1.320 \text{ ft/sec}}$



$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 35\left(2\frac{\pi}{180}\right)^2$

$= -0.0426 \text{ ft/sec}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.5)\left(2\frac{\pi}{180}\right) = 0.0349 \frac{\text{ft}}{\text{sec}^2}$

$a = \sqrt{0.0426^2 + 0.0349^2} = \underline{0.0551 \text{ ft/sec}^2}$

2/137 $v_r = \dot{r} = 40$ mm/s

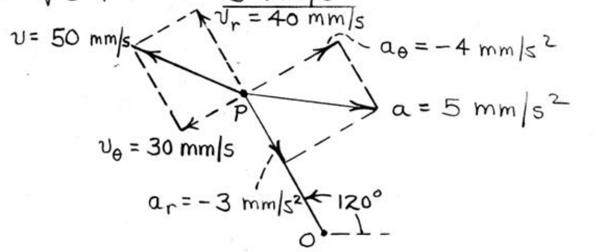
$v_\theta = r\dot{\theta} = 300(0.1) = 30$ mm/s

$v = \sqrt{40^2 + 30^2} = \underline{50 \text{ mm/s}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300(0.1)^2 = -3$ mm/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(-0.04) + 2(40)(0.1) = -4$ mm/s^2

$a = \sqrt{3^2 + 4^2} = \underline{5 \text{ mm/s}^2}$



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Position	r	$\dot{r}$	$\ddot{r}$	θ	$\dot{\theta}$	$\ddot{\theta}$
A	+	−	+	+	+	+
B	+	0	+	+	+	0
C	+	+	+	+	+	−

Notes : (1) $r \geq 0$, always, by definition

(2) $\dot{r}$ determined by inspection

(3) $\ddot{r}$ found from $a_r = \ddot{r} - r\dot{\theta}^2 = 0$

(4) $\theta \geq 0$, by definition in figure

(5) $\dot{\theta} > 0$ here, by inspection

(6) $\ddot{\theta}$ found from $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

2/139 $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = 1.5\underline{e}_r + (24+7)(5\frac{\pi}{180})\underline{e}_\theta$

$= \underline{1.5\underline{e}_r + 2.71\underline{e}_\theta \quad \text{ft/sec}}$

$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

$= \left[-4 - 31(5\frac{\pi}{180})^2\right]\underline{e}_r + \left[31(2\frac{\pi}{180}) + 2(1.5)(5\frac{\pi}{180})\right]\underline{e}_\theta$

$= \underline{-4.24\underline{e}_r + 1.344\underline{e}_\theta \quad \text{ft/sec}^2}$

2/140 $\dot{r} \neq v$, $\dot{r} \neq \underline{v}$, $\ddot{r} \neq a$, and $\ddot{r} \neq \underline{a}$ because scalars are NEVER equal to vectors.

$\dot{r} \neq v$ because $v \neq (v_r = \dot{r})$.

$\ddot{r} \neq a$ because $\ddot{r}$ is only one part of the magnitude of a_r (and therefore of a).

$\dot{\underline{r}} \neq \dot{r}\underline{e}_r$ because $\underline{v} = \dot{\underline{r}}$ also contains an $\underline{e}_\theta$-component.

$\ddot{\underline{r}} \neq \ddot{r}\underline{e}_r$ because $\underline{a} = \ddot{\underline{r}}$ contains another $\underline{e}_r$-component and also an $\underline{e}_\theta$-component.

$\dot{\underline{r}} \neq r\dot{\theta}\underline{e}_\theta$ because $\dot{\underline{r}} = \underline{v}$ also contains an $\underline{e}_r$-component.

2/141

(A) $\underline{v}_A = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = \underline{v\underline{e}_r + l\Omega\underline{e}_\theta}$

$\underline{a}_A = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

$= \underline{-l\Omega^2\underline{e}_r + 2v\Omega\underline{e}_\theta}$

(B) $\underline{v}_B = \underline{4v\underline{e}_r + 2l\Omega\underline{e}_\theta}$

$\underline{a}_B = \underline{-2l\Omega^2\underline{e}_r + 8v\Omega\underline{e}_\theta}$

2/142 $r = 375 + 125 = 500$ mm, $\dot{r} = \dot{l} = -150 \frac{\text{mm}}{\text{s}}$

$\ddot{r} = 0$, $\dot{\theta} = 60(\frac{\pi}{180}) = \frac{\pi}{3}$ rad/s, $\ddot{\theta} = 0$

$v_r = \dot{r} = -150 \frac{\text{mm}}{\text{s}}$, $v_\theta = r\dot{\theta} = 500(\frac{\pi}{3}) = 524 \frac{\text{mm}}{\text{s}}$

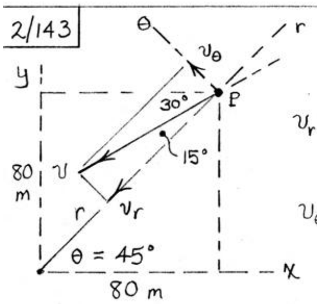
$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-150)^2 + (524)^2} = \underline{545 \frac{\text{mm}}{\text{s}}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 500(\frac{\pi}{3})^2 = -548$ mm/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-150)(\pi/3) = -314$ mm/s^2

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-548)^2 + (-314)^2} = \underline{632 \text{ mm/s}^2}$

2/143



$\begin{cases} v = 100 \text{ m/s} \\ r = 80\sqrt{2} = 113.1 \text{ m} \end{cases}$

$v_r = \dot{r} = -v\cos 15° = -100\cos 15°$

$= \underline{-96.6 \text{ m/s}}$

$v_\theta = r\dot{\theta} = v\sin 15°$

$\dot{\theta} = \frac{100\sin 15°}{113.1}$

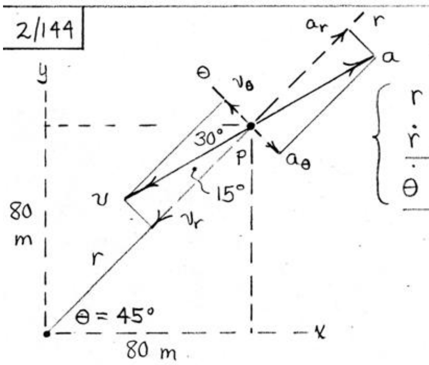
$= \underline{0.229 \text{ rad/s}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0$, $\ddot{r} = r\dot{\theta}^2 = 113.1(0.229)^2 = \underline{5.92 \frac{\text{m}}{\text{s}^2}}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$, $\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r}$

$= -\frac{2(-96.6)(0.229)}{113.1} = \underline{-0.391 \frac{\text{rad}}{\text{s}^2}}$

2/144



From Prob. 2/143,

$\begin{cases} r = 113.1 \text{ m} \\ \dot{r} = -96.6 \text{ m/s} \\ \dot{\theta} = 0.229 \text{ rad/s} \end{cases}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 20\cos 15°$

$\ddot{r} = 113.1(0.229)^2 + 20\cos 15° = \underline{25.2 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -20\sin 15°$

$\ddot{\theta} = \frac{-2(-96.6)(0.229) - 20\sin 15°}{113.1} = \underline{0.345 \text{ rad/s}^2}$

2/145 From $\mathbf{a} = [\ddot r - r\dot\theta^2]\mathbf{e}_r + [r\ddot\theta + 2\dot r\dot\theta]\mathbf{e}_\theta$

we have, for $\ddot r = \ddot\theta = 0$, $\dot\theta = \Omega$, and $r = l$:

$$a = \sqrt{(l\Omega^2)^2 + (2\dot l\Omega)^2} = \Omega\sqrt{l^2\Omega^2 + 4\dot l^2}$$

$$0.011 = 0.05\sqrt{[4.2(0.05)]^2 + 4\dot l^2}$$

Solve for $\dot l$: $\underline{\dot l = 0.0328 \text{ m/s}}$

or $\underline{\dot l = 32.8 \text{ mm/s}}$

2/146

$$\begin{cases} r = r_0\cosh Kt \\ \dot r = r_0 K\sinh Kt \\ \ddot r = r_0 K^2\cosh Kt \end{cases}$$

$a_r = \ddot r - r\dot\theta^2 = r_0K^2\cosh Kt - (r_0\cosh Kt)K^2 = 0$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2r_0K\sinh Kt\,(K)$

$= 2r_0K^2\sinh Kt$

But $\cosh^2 Kt - \sinh^2 Kt = 1$, $\sinh Kt = \sqrt{\cosh^2 Kt - 1}$

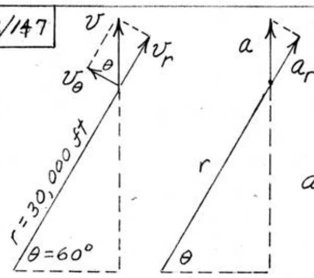
$= \sqrt{\left(\frac{r}{r_0}\right)^2 - 1}$

When $r = R$, $\sinh Kt = \sqrt{\left(\frac{R}{r_0}\right)^2 - 1}$

So at the instant of leaving the vane,

$a = a_\theta = 2r_0K^2\sqrt{\left(\frac{R}{r_0}\right)^2 - 1} = \underline{2K^2\sqrt{R^2 - r_0^2}}$

2/147



$v_\theta = r\dot\theta = 30(10^3)(0.020)$

$= 600$ ft/sec

$v = v_\theta/\cos 60°$

$= 600/0.5 = \underline{1200 \text{ ft/sec}}$

$a_r = \ddot r - r\dot\theta^2$

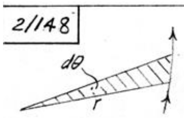
$= 70 - 30(10^3)(0.020)^2$

$= 58$ ft/sec^2

$a = a_r/\sin 60°$

$= 58/0.866 = \underline{67.0 \text{ ft/sec}^2}$

2/148

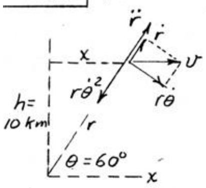


$dA = \frac{1}{2}r\,d\theta\,(r) = \frac{1}{2}r^2 d\theta$

$\dot A = \frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\dot\theta =$ constant

since $a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot\theta) = 0$

2/149 Acceleration in all directions is zero, so



$\dot\theta = -0.020$ rad/s

$a_r = \ddot r - r\dot\theta^2 = 0$, $\ddot r = r\dot\theta^2$

$r = h/\sin\theta$, $r = \frac{10}{\sqrt3/2} = 11.55$ km

$\ddot r = 11.55(-0.020)^2 = 0.00462$ km/s^2

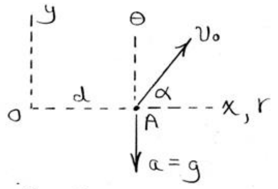
$= \underline{4.62 \text{ m/s}^2}$

$v = |r\dot\theta|/\sin\theta = h\dot\theta/\sin^2\theta$

$= \frac{|10(-0.020)|}{(\sqrt3/2)^2} = 0.267$ km/s

or $v = 0.267(3600) = \underline{960 \text{ km/h}}$

2/150



$\underline{r = d}$, $\underline{\theta = 0}$

$v_r = \dot r = v_0\cos\alpha$

$v_\theta = r\dot\theta = v_0\sin\alpha$, $\quad \dot\theta = \frac{v_0\sin\alpha}{d}$

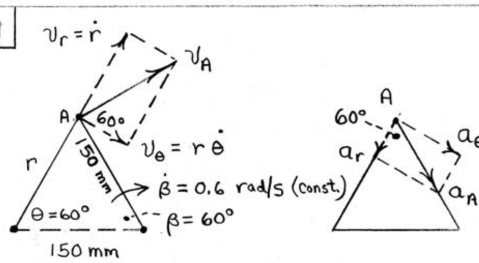
$a_r = \ddot r - r\dot\theta^2 = 0$, $\quad \ddot r = r\dot\theta^2 = \frac{v_0^2\sin^2\alpha}{d}$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = -g$, $\quad \ddot\theta = -\frac{2\dot r\dot\theta + g}{r}$

or $\ddot\theta = -\frac{2(v_0\cos\alpha)\left(\frac{v_0\sin\alpha}{r}\right) + g}{r}$

$= \underline{-\frac{1}{d}\left(\frac{2v_0^2}{d}\cos\alpha\sin\alpha + g\right)}$

2/151



For $\beta = 60°$, $\theta = 60°$, $r = 150$ mm

$v_A = 150(0.6) = 90$ mm/s

$v_\theta = r\dot\theta = -v_A\cos 60°$, $\quad \dot\theta = \frac{-90\cos 60°}{150} = -0.3\ \frac{\text{rad}}{\text{s}}$

$v_r = \dot r = v_A\sin 60° = 90\sin 60° = \underline{77.9 \text{ mm/s}}$

$a_A = a_n = 150(0.6)^2 = 54$ mm/s^2

$a_r = \ddot r - r\dot\theta^2 : -54\cos 60° = \ddot r - 150(-0.3)^2$

$\underline{\ddot r = -13.5 \text{ mm/s}^2}$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta : -54\sin 60° = 150\ddot\theta + 2(77.9)(-0.3)$

$\underline{\ddot\theta = 0}$

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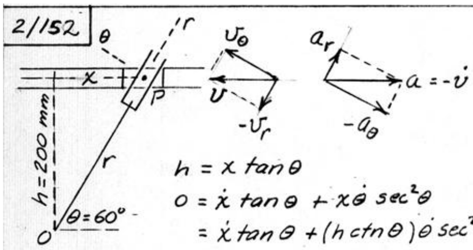


$h = x\tan\theta$

$0 = \dot x\tan\theta + x\dot\theta\sec^2\theta$

$= \dot x\tan\theta + (h\,\text{ctn}\,\theta)\dot\theta\sec^2\theta$

$v = -\dot x = h\dot\theta\,\text{ctn}\,\theta\sec^2\theta/\tan\theta$

$= h\dot\theta\csc^2\theta = 200(2)\left(\frac{2}{\sqrt3}\right)^2 = \underline{533 \text{ mm/s}}$

$v_r = -v\cos\theta = -533(1/2) = \underline{-267 \text{ mm/s}}$

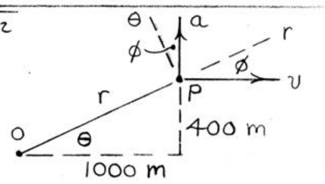
$-a = \dot v = h\dot\theta\,2\csc\theta(-\text{ctn}\,\theta\csc\theta)\dot\theta = -2h\dot\theta^2\text{ctn}\,\theta\csc^2\theta$

$a = 2(200)2^2\frac{1}{\sqrt3}\left(\frac{2}{\sqrt3}\right)^2 = \underline{1232 \text{ mm/s}^2}$

$a_r = a\cos\theta = 1232(1/2) = \underline{616 \text{ mm/s}^2}$

(Alternatively obtain $\dot r = v_r$ & $\ddot r$ from $r = h\csc\theta$)

2/153



$r = \sqrt{1000^2 + 400^2}$

$= 1077 \text{ m}$

$\theta = \tan^{-1}\frac{400}{1200} = 21.8°$

$v = \frac{600}{3.6} = 166.7 \text{ m/s}$

$a = a_n = \frac{v^2}{\rho} = \frac{166.7^2}{1200} = 23.1 \text{ m/s}^2$

$v_r = \dot{r} = v\cos\theta = 166.7\cos 21.8° = 154.7 \text{ m/s}$

$v_\theta = r\dot{\theta}$: $-166.7\sin 21.8° = 1077\dot{\theta}$, $\dot{\theta} = -0.0575 \frac{\text{rad}}{\text{s}}$

$a_r = \ddot{r} - r\dot{\theta}^2$: $23.1\sin 21.8° = \ddot{r} - 1077(-0.0575)^2$

$\underline{\ddot{r} = 12.15 \text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$: $23.1\cos 21.8° = 1077\ddot{\theta} + 2(154.7)(-0.0575)$

$\underline{\ddot{\theta} = 0.0365 \text{ rad/s}^2}$

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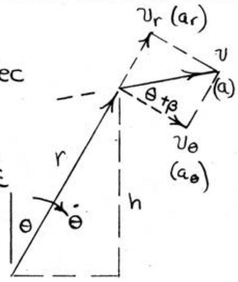


$\dot{\theta} = 2.20\left(\frac{\pi}{180}\right) = 0.0384 \text{ rad/sec}$

$v_r = \dot{r} = 360 \text{ ft/sec}$

$v_\theta = r\dot{\theta} = 12{,}000(0.0384) = 461 \frac{\text{ft}}{\text{sec}}$

$v = \sqrt{v_r^2 + v_\theta^2} = 585 \text{ ft/sec}$

or $\underline{v = 399 \text{ mi/hr}}$

$30 + \beta = \tan^{-1}\frac{360}{461} = 38.0°$, $\underline{\beta = 8.00°}$

$h = r\cos 30° = 12{,}000\cos 30° = \underline{10{,}390 \text{ ft}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 19.60 - 12{,}000(0.0384)^2 = 1.908 \frac{\text{ft}}{\text{sec}^2}$

$a = \frac{a_r}{\sin(\theta+\beta)} = \frac{1.908}{\sin 38.0°} = \underline{3.10 \text{ ft/sec}^2}$

$a_\theta = \frac{a_r}{\tan(\theta+\beta)} = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$\frac{1.908}{\tan 38.0°} = 12{,}000\ddot{\theta} + 2(360)(0.0384)$, $\underline{\ddot{\theta} = -0.00210 \frac{\text{rad}}{\text{sec}^2}}$

2/155

$\theta = 0.4 + 0.12t + 0.06t^3$	$r = 0.8 - 0.1t - 0.05t^2$
$\dot{\theta} = 0.12 + 0.18t^2$	$\dot{r} = -0.1 - 0.1t$
$\ddot{\theta} = 0.36t$	$\ddot{r} = -0.1$

At $t = 2$ s :

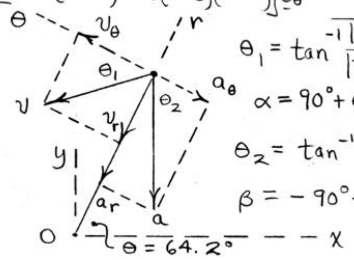
$\theta = 1.12$ rad	$r = 0.4$ m
$\dot{\theta} = 0.84$ rad/s	$\dot{r} = -0.3$ m/s
$\ddot{\theta} = 0.72$ rad/s²	$\ddot{r} = -0.1$ m/s²

$\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = -0.3\underline{e}_r + 0.4(0.84)\underline{e}_\theta$

$= \underline{-0.3\underline{e}_r + 0.336\underline{e}_\theta \text{ m/s}}$

$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta = [-0.1 - 0.4(0.84)^2]\underline{e}_r$

$+ [0.4(0.72) + 2(-0.3)(0.84)]\underline{e}_\theta = \underline{-0.382\underline{e}_r - 0.216\underline{e}_\theta \text{ m/s}^2}$



$\theta_1 = \tan^{-1}\frac{|v_r|}{|v_\theta|} = 41.8°$

$\alpha = 90° + \theta + \theta_1 = \underline{195.9°}$

$\theta_2 = \tan^{-1}\frac{|a_r|}{|a_\theta|} = 60.5°$

$\beta = -90° + \theta - \theta_2 = \underline{-86.4°}$

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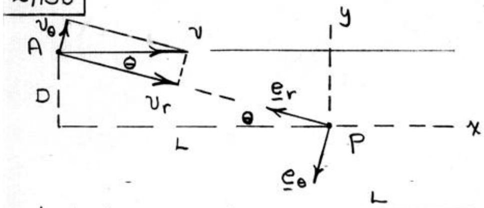

$v' = |v_r| = v\cos\theta = v\frac{L}{\sqrt{L^2 + D^2}}$

Numbers : $v' = 70\frac{500}{\sqrt{500^2 + 20^2}} = \underline{69.9 \text{ mi/hr}}$

The factor of $\cos\theta$ is the basis for the statement that, kinematically, radar can yield an accurate or low, but not high, speed measurement. As can be seen, however, adherence to the speed limit (not reliance upon $\cos\theta$) is the best policy!

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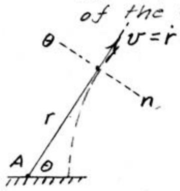
Radial line r must be tangent to trajectory for $\dot{\theta} = 0$. Thus $+\theta$ direction is in the opposite sense to the normal n-direction of the curve.



$r = 35{,}000$ ft, $\dot{r} = 1600$ ft/sec

$\dot{\theta} = 0$, $\ddot{\theta} = -7.20(10^{-3})$ rad/sec²

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 35{,}000(-7.20)(10^{-3}) + 0$

$= -252 \text{ ft/sec}^2$

$-a_\theta = a_n = v^2/\rho$, $\rho = v^2/-a_\theta$

$= \frac{(1600)^2}{252} = \underline{10.16(10^3) \text{ ft}}$

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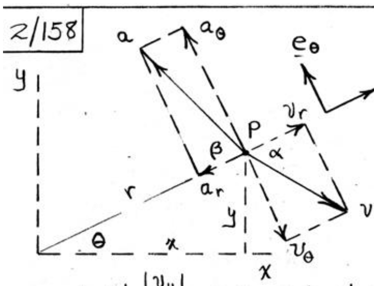

$-x = 4\text{ m},\ y = 2\text{ m}$

$\dot{x} = 2\sqrt{3}\text{ m/s},\ \dot{y} = -2\ \frac{\text{m}}{\text{s}}$

$\ddot{x} = -5\text{ m/s}^2,\ \ddot{y} = 5\ \frac{\text{m}}{\text{s}^2}$

$r = \sqrt{x^2+y^2} = \underline{2\sqrt{5}\text{ m}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \underline{26.6°}$

$\alpha = \tan^{-1}\frac{|v_y|}{|v_x|} + \theta = \tan^{-1}\frac{2}{2\sqrt{3}} + 26.6° = 56.6°$

$\beta = \tan^{-1}\left|\frac{a_y}{a_x}\right| + \theta = \tan^{-1}\frac{5}{5} + 26.6° = 71.6°$

$v = \sqrt{v_y^2+v_x^2} = \sqrt{2^2+(2\sqrt{3})^2} = 4\text{ m/s}$

$a = \sqrt{a_y^2+a_x^2} = \sqrt{5^2+5^2} = 7.07\text{ m/s}^2$

$\dot{r} = v_r = v\cos\alpha = 4\cos 56.6° = \underline{2.20\text{ m/s}}$

$v_\theta = -v\sin\alpha = -4\sin 56.6° = -3.34\text{ m/s}$

$v_\theta = r\dot{\theta}\ :\ -3.34 = 2\sqrt{5}\,\dot{\theta},\quad \underline{\dot{\theta} = -0.746\text{ rad/s}}$

$a_r = -a\cos\beta = -7.07\cos 71.6° = -2.24\text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2:\ -2.24 = \ddot{r} - 2\sqrt{5}(0.746)^2,\quad \underline{\ddot{r} = 0.255\ \frac{\text{m}}{\text{s}^2}}$

$a_\theta = a\sin\beta = 7.07\sin 71.6° = 6.71\text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}:\ 6.71 = 2\sqrt{5}\,\ddot{\theta} + 2(2.20)(-0.746),\quad \underline{\ddot{\theta} = 2.24\ \frac{\text{rad}}{\text{s}^2}}$

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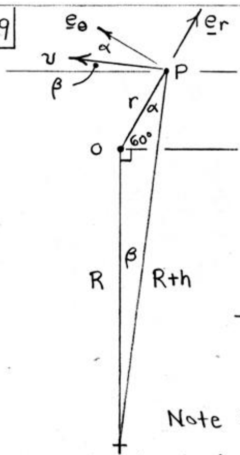

$\frac{\sin 150°}{R+h} = \frac{\sin\alpha}{R}$

$\frac{\sin 150°}{3959+150} = \frac{\sin\alpha}{3959}$

$\alpha = 28.8°$

$\alpha + \beta + 150° = 180° \Rightarrow \beta = 1.200°$

$v_r = \dot{r} = -12{,}272 = -v\sin\alpha$

$-12{,}272 = -v\sin 28.8°$

$v = \underline{25{,}474\text{ ft/sec}}$

Note : Because v is nearly parallel to the horizontal at O ($\beta = 1.200°$), one obtains a close approximation ($v = 24{,}544\ \frac{\text{ft}}{\text{sec}}$) to the correct answer by neglecting β (assuming a flat earth).

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$r = r_0 + b_0\sin 2\pi nt,\quad \dot{r} = 2\pi n b_0\cos 2\pi nt$

$\ddot{r} = -4\pi^2 b_0 n^2\sin 2\pi nt$

$\dot{\theta} = \omega,\quad \ddot{\theta} = 0$

$a_r = \ddot{r} - r\dot{\theta}^2,\quad a_r = -4\pi^2 b_0 n^2\sin 2\pi nt - (r_0 + b_0\sin 2\pi nt)\omega^2$

$= -(4\pi^2 n^2 + \omega^2)b_0\sin 2\pi nt - r_0\omega^2$

$|a_r|_{max} = \underline{(4\pi^2 n^2 + \omega^2)b_0 + r_0\omega^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta},\quad a_\theta = 0 + 4\pi b_0 n\omega\cos 2\pi nt$

$|a_\theta|_{max} = \underline{4\pi b_0 n\omega}$

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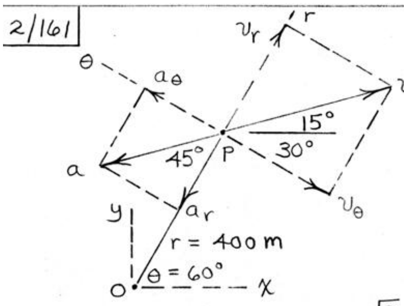

$v = \frac{90}{3.6} = 25\text{ m/s}$

$a = 0.5\text{ m/s}^2$

$v_r = \dot{r} = v\sin 45° = 25\frac{\sqrt{2}}{2} = \underline{17.68\text{ m/s}}$

$v_\theta = r\dot{\theta}\ :\ -25\cos 45° = 400\dot{\theta},\quad \underline{\dot{\theta} = -0.0442\text{ rad/s}}$

$a_r = \ddot{r} - r\dot{\theta}^2:\ -0.5\cos 45° = \ddot{r} - 400(-0.0442)^2$

$\underline{\ddot{r} = 0.428\text{ m/s}^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}\ :\ 0.5\sin 45° = 400\ddot{\theta} + 2(17.68)(-0.0442)$

$\underline{\ddot{\theta} = 0.00479\text{ rad/s}^2}$

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$$\begin{cases} r = 0.75 + 0.5 = 1.25\text{ m} & \theta = 30° \\ \dot{r} = 0.2\text{ m/s} & \dot{\theta} = 0.1745\ \frac{\text{rad}}{\text{s}} \\ \ddot{r} = -0.3\text{ m/s}^2 & \ddot{\theta} = 0 \end{cases}$$

$\underline{v} = v_r\underline{e}_r + v_\theta\underline{e}_\theta = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$

$= 0.2\underline{e}_r + 1.25(0.1745)\underline{e}_\theta = 0.2\,\underline{e}_r + 0.218\,\underline{e}_\theta\ \frac{\text{m}}{\text{s}}$

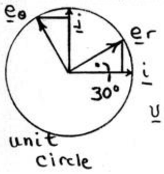
$v = \sqrt{v_r^2 + v_\theta^2} = \underline{0.296\text{ m/s}}$

$\underline{a} = a_r\underline{e}_r + a_\theta\underline{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

$= [-0.3 - 1.25(0.1745)^2]\underline{e}_r + [1.25(0) + 2(0.2)(0.1745)]\underline{e}_\theta$

$= -0.338\underline{e}_r + 0.0698\,\underline{e}_\theta\ \text{m/s}^2$

$a = \sqrt{a_r^2 + a_\theta^2} = \underline{0.345\text{ m/s}^2}$



$\underline{e}_r = \underline{i}\cos 30° + \underline{j}\sin 30°$

$\underline{e}_\theta = -\underline{i}\sin 30° + \underline{j}\cos 30°$

$\underline{v} = 0.2[\underline{i}\,c30° + \underline{j}\,s30°] + 0.218[-\underline{i}\,s30° + \underline{j}\,c30°]$

$= \underline{0.064\underline{i} + 0.289\underline{j}\ \text{m/s}}$

$\underline{a} = -0.338[\underline{i}\,c30° + \underline{j}\,s30°] + 0.0698[-\underline{i}\,s30° + \underline{j}\,c30°]$

$= \underline{-0.328\,\underline{i} - 0.1086\underline{j}\ \text{m/s}^2}$

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$r = \overline{BD} = 2R\sin\frac{\theta}{2},\quad \dot{r} = R\dot{\theta}\cos\frac{\theta}{2}$

$\ddot{r} = -\frac{R}{2}\dot{\theta}^2\sin\frac{\theta}{2}$

For $\theta = 30°$: $\begin{cases} r = 2(15)\sin 15° = 7.76\text{ in.} \\ \dot{r} = 15(4)\cos 15° = 58.0\text{ in./sec} \\ \ddot{r} = -\frac{15}{2}4^2\sin 15° = -31.1\text{ in./sec}^2 \end{cases}$

$a_r = \ddot{r} - r\dot{\theta}^2 = -31.1 - 7.76(4)^2 = -155.3\text{ in./sec}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(58.0)(4) = 464\text{ in./sec}^2$

$a = \sqrt{a_r^2 + a_\theta^2} = \underline{489\text{ in./sec}^2}$

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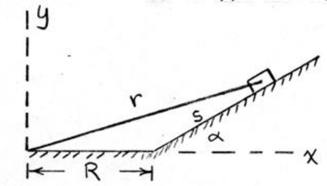


$x = R + s\cos\alpha = R + (s_0 + v_0 t + \frac{1}{2}at^2)\cos\alpha$

$= R + \frac{1}{2}at^2\cos\alpha$

$y = s\sin\alpha$

$= \frac{1}{2}at^2\sin\alpha$

$r = \sqrt{x^2+y^2} = \sqrt{(R+\frac{1}{2}at^2\cos\alpha)^2 + (\frac{1}{2}at^2\sin\alpha)^2}$

$= \sqrt{R^2 + Rat^2\cos\alpha + \frac{1}{4}a^2t^4}$

$\dot{r} = \frac{1}{2}(R^2 + Rat^2\cos\alpha + \frac{1}{4}a^2t^4)^{-1/2}[2Rat\cos\alpha + a^2t^3]$

$$= \frac{\frac{1}{2}at(2R\cos\alpha + at^2)}{\sqrt{R^2 + Rat^2\cos\alpha + \frac{1}{4}a^2t^4}}$$

2/165 From the solution to Prob. 2/164 :

$$\begin{cases} x = R + \frac{1}{2}at^2\cos\alpha \\ y = \frac{1}{2}at^2\sin\alpha \end{cases}$$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left[\frac{\frac{1}{2}at^2\sin\alpha}{R + \frac{1}{2}at^2\cos\alpha}\right]$

$$\dot{\theta} = \frac{\dfrac{(R+\frac{1}{2}at^2\cos\alpha)(at\sin\alpha) - (\frac{1}{2}at^2\sin\alpha)(at\cos\alpha)}{(R+\frac{1}{2}at^2\cos\alpha)^2}}{1 + \left[\dfrac{\frac{1}{2}at^2\sin\alpha}{R+\frac{1}{2}at^2\cos\alpha}\right]^2}$$

Simplify to

$$\dot{\theta} = \frac{Rat\sin\alpha}{R^2 + Rat^2\cos\alpha + \frac{1}{4}a^2t^4}$$

2/166 $r = 1.6 + 0.3\sin\frac{\pi t}{2}$ m ; $\theta = \frac{\pi}{4} + \frac{\pi}{8}\sin\frac{\pi t}{2}$

$\dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}$ m/s ; $\dot{\theta} = \frac{\pi^2}{16}\cos\frac{\pi t}{2}$ rad/s

$\ddot{r} = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2}$ m/s^2 ; $\ddot{\theta} = -\frac{\pi^3}{32}\sin\frac{\pi t}{2}$ rad/s^2

$v_r = \dot{r} = \frac{0.3\pi}{2}\cos\frac{\pi t}{2}$

$v_\theta = r\dot{\theta} = (1.6 + 0.3\sin\frac{\pi t}{2})(\frac{\pi^2}{16}\cos\frac{\pi t}{2})$

$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{0.3\pi^2}{4}\sin\frac{\pi t}{2} - (1.6 + 0.3\sin\frac{\pi t}{2})(\frac{\pi^2}{16}\cos\frac{\pi t}{2})^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1.6 + 0.3\sin\frac{\pi t}{2})(-\frac{\pi^3}{32}\sin\frac{\pi t}{2})$
$+ 2(\frac{0.3\pi}{2}\cos\frac{\pi t}{2})(\frac{\pi^2}{16}\cos\frac{\pi t}{2})$

At $t = 1$ s : $v_r = 0$, $v_\theta = 0$; $v = \sqrt{v_r^2 + v_\theta^2} = 0$

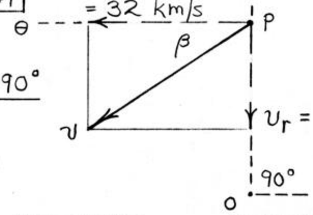
$a_r = -0.740$ m/s^2, $a_\theta = -1.841$ m/s^2 ; $a = \sqrt{0.740^2 + 1.841^2} = 1.984$ m/s^2

At $t = 2$ s : $v_r = -0.471$ m/s, $v_\theta = 0.987$ m/s ; $v = 1.094$ m/s

$a_r = -0.609$ m/s^2, $a_\theta = 0.581$ m/s^2 ; $a = 0.842$ m/s^2

2/167 $v_\theta = r\dot{\theta} = 80(0.4) = 32$ km/s $\qquad r = \overline{OP} = 80$ km

$\theta = 90°$

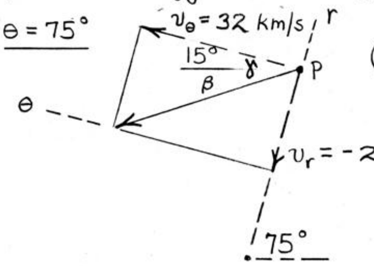


$v_r = \dot{r} = -20$ km/s

$v = \sqrt{v_\theta^2 + v_r^2} = \sqrt{32^2 + 20^2} = 37.7$ km/s

$\beta = \tan^{-1}\frac{|v_r|}{v_\theta} = \tan^{-1}\frac{20}{32} = 32.0°$

$\theta = 75°$



(v_θ & v_r unchanged)

$\gamma = 32.0°$

$v = 37.7$ km/s (unchanged)

$\beta = \gamma - 15° = 32.0 - 15° = 17.01°$

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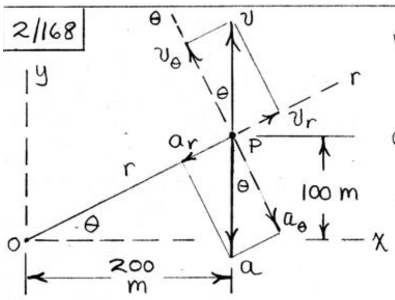

$r = \sqrt{200^2 + 100^2} = 224$ m

$\theta = \tan^{-1}\frac{100}{200} = 26.6°$

$v_r = \dot{r} = v\sin\theta = 15\sin 26.6° = 6.71$ m/s

$v_\theta = r\dot{\theta}$: $15\cos 26.6° = 224\dot{\theta}$, $\dot{\theta} = 0.06$ rad/s

$a = -g - kv^2 = -9.81 - 0.01(15)^2 = -12.06$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2$: $-12.06\sin 26.6° = \ddot{r} - 224(0.06)^2$

$\ddot{r} = -4.59$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$: $-12.06\cos 26.6° = 224\ddot{\theta} + 2(6.71)(0.06)$

$\ddot{\theta} = -0.0518$ rad/s^2

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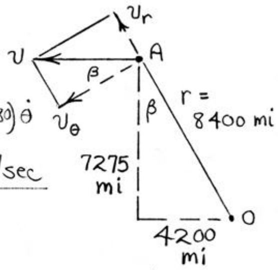

$v = 12,149\left(\frac{5280}{3600}\right) = 17,819 \frac{ft}{sec}$

$v_\theta = r\dot\theta$: $17,819 \cos 30° = 8400(5280)\dot\theta$

$\dot\theta = 3.48(10^{-4})$ rad/sec

$v_r = \dot r$: $17,819 \sin 30° = \dot r$

$\dot r = 8910 \frac{ft}{sec}$

$\beta = \tan^{-1}\left(\frac{4200}{7275}\right) = 30°$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta$: $0 = 8400(5280)\ddot\theta + 2(8910)(3.48)(10^{-4})$

$\ddot\theta = -1.398(10^{-7})$ rad/sec²

$a_r = \ddot r - r\dot\theta^2$: $-7.159 = \ddot r - 8400(5280)(3.48\times10^{-4})^2$

$\ddot r = -1.790$ ft/sec²

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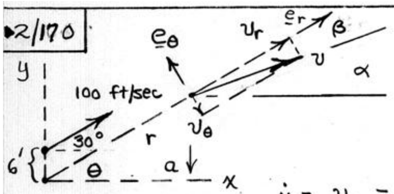

$x = x_0 + v_{x_0} t$

$= 0 + 100 \cos 30° (0.5)$

$= 43.3$ ft

$\dot x = v_{x_0} = 100 \cos 30° = 86.6 \frac{ft}{sec}$

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = 6 + 100 \sin 30°(0.5) - 16.1(0.5)^2 = 27.0$ ft

$\dot y = v_{y_0} - gt = 100 \sin 30° - 32.2(0.5) = 33.9$ ft/sec

$r = \sqrt{x^2+y^2} = \sqrt{43.3^2 + 27.0^2} = 51.0$ ft

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(27.0/43.3) = 31.9°$

$\alpha = \tan^{-1}(v_y/v_x) = \tan^{-1}(33.9/86.6) = 21.4°$

$\beta = \theta - \alpha = 10.54°$, $v = \sqrt{\dot x^2 + \dot y^2} = 93.0$ ft/sec

$v_r = v \cos\beta = 93.0 \cos 10.54° = 91.4$ ft/sec $= \dot r$

$v_\theta = -v \sin\beta = -93.0 \sin 10.54° = -17.02$ ft/sec

$-17.02 = r\dot\theta = 51.0\,\dot\theta$, $\dot\theta = -0.334$ rad/sec

$a = \ddot y = -32.2$ ft/sec²

$a_r = -g \sin\theta = -32.2 \sin 31.9° = -17.03$ ft/sec²

$-17.03 = \ddot r - r\dot\theta^2 = \ddot r - 51.0(-0.334)^2$, $\ddot r = -11.35$ ft/sec²

$a_\theta = -g\cos\theta = -32.2 \cos 31.9° = -27.3$ ft/sec²

$-27.3 = r\ddot\theta + 2\dot r\dot\theta = 51.0\ddot\theta + 2(91.4)(-0.334)$

$\ddot\theta = 0.660$ rad/sec²

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$$\begin{cases} x = 30 \cos 2t; & y = 40 \sin 2t; & z = 20t + 3t^2 \\ \dot x = -60 \sin 2t; & \dot y = 80 \cos 2t; & \dot z = 20 + 6t \\ \ddot x = -120 \cos 2t; & \ddot y = -160 \sin 2t; & \ddot z = 6 \end{cases}$$

At t = 2 s :

$$\begin{cases} x = -19.61 \text{ mm}; & y = -30.3 \text{ mm}; & z = 52 \text{ mm} \\ \dot x = 45.4 \text{ mm/s}; & \dot y = -52.3 \text{ mm/s}; & \dot z = 32 \text{ mm/s} \\ \ddot x = 78.4 \text{ mm/s}^2; & \ddot y = 121.1 \text{ mm/s}^2; & \ddot z = 6 \text{ mm/s}^2 \end{cases}$$

$r = (x^2+y^2+z^2)^{1/2} = 63.3$ mm $\quad a = (\ddot x^2 + \ddot y^2 + \ddot z^2)^{1/2}$

$v = (\dot x^2 + \dot y^2 + \dot z^2)^{1/2} = 76.3$ mm/s $\quad = 144.4$ mm/s²

$\theta_1 = \cos^{-1}\left[\frac{\mathbf{r}\cdot\mathbf{v}}{rv}\right] = \cos^{-1}\left[\frac{-19.61(45.4) - 30.3(-52.3) + 52(32)}{(63.3)(76.3)}\right]$

$= 60.8°$

$\theta_2 = \cos^{-1}\left[\frac{\mathbf{r}\cdot\mathbf{a}}{ra}\right] = \cos^{-1}\left[\frac{-19.61(78.4) - 30.3(121.1) - 52(6)}{(63.3)(144.4)}\right]$

$= 122.4°$

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$v_{z_0} = 600 \sin 60° = 520$ ft/sec

$v_{xy_0} = 600 \cos 60° = 300$ ft/sec

$v_z = v_{z_0} - gt = 520 - 32.2(20) = -124.4$ ft/sec

$v_{xy} = v_{xy_0} = 300$ ft/sec = constant

$v_x = -v_{xy} \sin 20° = -300 \sin 20° = -102.6$ ft/sec

$v_y = v_{xy} \cos 20° = 300 \cos 20° = 282$ ft/sec

$d_{xy} = v_{xy} t = 300(20) = 6000$ ft

$x = -d_{xy} \sin 20° = -6000 \sin 20° = -2050$ ft

$y = d_{xy} \cos 20° = 6000 \cos 20° = 5640$ ft

$z = v_{z_0} t - \frac{1}{2} g t^2 = 520(20) - 16.1(20)^2 = 3950$ ft

$a_x = a_y = 0$, $a_z = -g = -32.2$ ft/sec²

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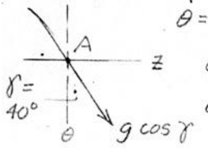
$\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ m/s, $v = \sqrt{4^2 + 2^2 + 1^2} = 4.58 \frac{m}{s}$

$a_n = a \sin 20° = 8 \sin 20° = 2.74$ m/s²

From $a_n = \frac{v^2}{\rho}$, $\rho = \frac{v^2}{a_n} = \frac{4.58^2}{2.74} = 7.67$ m

$\dot v = a_t = a \cos 20° = 8 \cos 20° = 7.52$ m/s²

2/174 $v_\theta = r\dot\theta$ & $v_\theta = v \cos\gamma$ so $\dot\theta = \frac{v}{r}\cos\gamma$



$\dot\theta = \frac{15}{5}(0.7660) = 2.298$ rad/s

$a_\theta = g \cos^2\gamma = 9.81(0.7660)^2 = 5.76$ m/s²

$a_z = g \cos\gamma \sin\gamma = 9.81(0.7660)(0.6428) = 4.83$ m/s²

$a_r = \ddot r - r\dot\theta^2 = 0 - 5(2.298)^2 = -26.41$ m/s²

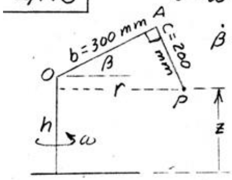
$a = \sqrt{26.41^2 + 5.76^2 + 4.83^2} = 27.5$ m/s²

2/175

$\dot\theta = \omega = \frac{40}{180}\pi = 0.698 \text{ rad/s}$

$\dot\beta = \frac{10}{180}\pi = 0.1745 \text{ rad/s}$

$\ddot\beta = \frac{20}{180}\pi = 0.349 \text{ rad/s}^2$



Use cylindrical coordinates

$r = b\cos\beta + c\sin\beta$, $\dot r = (-b\sin\beta + c\cos\beta)\dot\beta$

$\ddot r = (-b\cos\beta - c\sin\beta)\dot\beta^2 + (-b\sin\beta + c\cos\beta)\ddot\beta$

$z = h + b\sin\beta - c\cos\beta$, $\dot z = (b\cos\beta + c\sin\beta)\dot\beta$

$\ddot z = (-b\sin\beta + c\cos\beta)\dot\beta^2 + (b\cos\beta + c\sin\beta)\ddot\beta$

For $\beta = 30°$, $\dot r = (-300\times0.5 + 200\times0.866)(0.1745) = 4.050$ mm/s

$\ddot r = (-300\times0.866 - 200\times0.5)(0.1745)^2 + (-300\times0.5 + 200\times0.866)(0.349) = -2.860 \text{ mm/s}^2$

$\ddot z = (-300\times0.5 + 200\times0.866)(0.1745)^2 + (300\times0.866 + 200\times0.5)(0.349) = 126.30 \text{ mm/s}^2$

$a_r = \ddot r - r\dot\theta^2 = -2.860 - 359.8(0.698)^2 = -178.23 \text{ mm/s}^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(4.049)(0.698) = 5.65 \text{ mm/s}^2$

$a_z = \ddot z = 126.30 \text{ mm/s}^2$

$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2} = \underline{219 \text{ mm/s}^2}$

2/176

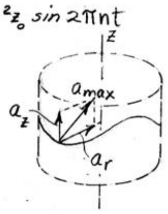
$a_r = \ddot r - r\dot\theta^2 = 0 - r\omega^2$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 0 = 0$

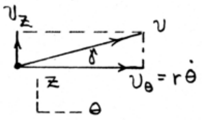
$a_z = \frac{d^2}{dt^2}(z_0 \sin 2\pi nt) = -4n^2\pi^2 z_0 \sin 2\pi nt$

$a = \sqrt{(-r\omega^2)^2 + (-4n^2\pi^2 z_0 \sin 2\pi nt)^2}$

$\underline{a_{max} = \sqrt{r^2\omega^4 + 16n^4\pi^4 z_0^2}}$



2/177 Helix angle is $\gamma = \tan^{-1}\frac{10}{24\pi} = 7.55°$

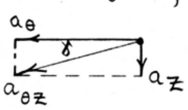


$v = 15\left(\frac{5280}{3600}\right) = 22$ ft/sec

$v_z = v\sin\gamma = 22\sin 7.55° = 2.89$ ft/sec

$v_\theta = v\cos\gamma = 22\cos 7.55° = 21.8$ ft/sec

From $v_\theta = r\dot\theta$, $\dot\theta = \frac{v_\theta}{r} = \frac{21.8}{24} = 0.909$ rad/sec



$a_{\theta z} = 2\left(\frac{5280}{3600}\right) = 2.93 \text{ ft/sec}^2$

$a_\theta = -2.93\cos 7.55° = \underline{-2.91 \text{ ft/sec}^2}$

$a_z = -2.93\sin 7.55° = \underline{-0.386 \text{ ft/sec}^2}$

$a_r = \ddot r - r\dot\theta^2 = 0 - 24(0.909)^2 = \underline{-19.82 \text{ ft/sec}^2}$

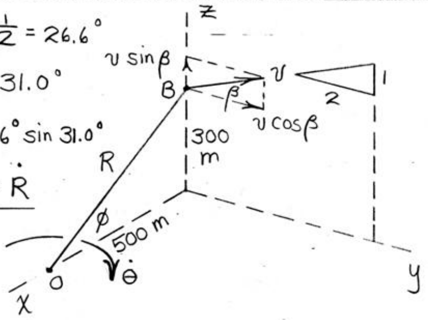
2/178

$\beta = \tan^{-1}\frac{1}{2} = 26.6°$

$\phi = \tan^{-1}\frac{3}{5} = 31.0°$

$v_R = 400\sin 26.6°\sin 31.0°$

$= \underline{92.0 \text{ km/h} = \dot R}$



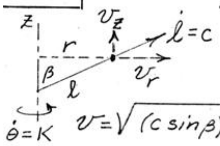
$v_\theta = R\dot\theta\cos\phi$: $\frac{400}{3.6}\cos 26.6° = 500\,\dot\theta$

$\underline{\dot\theta = 0.1988 \text{ rad/s}}$

$v_\phi = R\dot\phi$: $\frac{400}{3.6}\sin 26.6°\cos 31° = \frac{500}{\cos 31.0°}\,\dot\phi$

$\underline{\dot\phi = 0.0731 \text{ rad/s}}$

2/179



$v_r = \dot l\sin\beta = c\sin\beta$

$v_\theta = r\dot\theta = (l\sin\beta)K = Kl\sin\beta$

$v_z = \dot l\cos\beta = c\cos\beta$

$v = \sqrt{(c\sin\beta)^2 + (Kl\sin\beta)^2 + (c\cos\beta)^2}$

$= \underline{\sqrt{c^2 + K^2l^2\sin^2\beta}}$

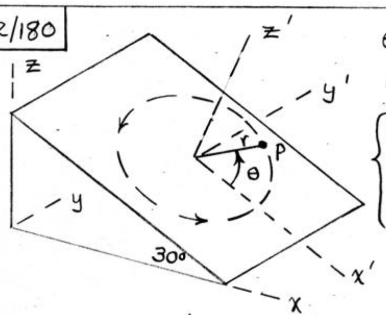
$a_r = \ddot r - r\dot\theta^2 = 0 - l\sin\beta(K^2) = -K^2 l\sin\beta$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2c\sin\beta(K) = 2Kc\sin\beta$

$a_z = \dot v_z = 0$

$a = \sqrt{(-K^2 l\sin\beta)^2 + (2Kc\sin\beta)^2} = \underline{K\sin\beta\sqrt{K^2l^2 + 4c^2}}$

2/180



$\theta = \omega t = \frac{v}{r}t$

$\dot\theta = \frac{v}{r}$

$\begin{cases} x' = r\cos\theta \\ y' = r\sin\theta \\ z' = 0 \end{cases}$

$\begin{cases} \dot x' = -r\dot\theta\sin\theta \\ \dot y' = r\dot\theta\cos\theta \\ \dot z' = 0 \end{cases}$

$\ddot x' = -r\dot\theta^2\cos\theta$, $\ddot y' = -r\dot\theta^2\sin\theta$, $\ddot z' = 0$

Component relationships : (Similar for $\dot x$, $\ddot x$, etc.) $\begin{cases} x = x'\cos 30° + z'\sin 30° \\ y = y' \\ z = -x'\sin 30° + z'\cos 30° \end{cases}$

Thus

$x = \frac{\sqrt3}{2}r\cos\frac{v}{r}t$, $y = r\sin\frac{v}{r}t$, $z = -\frac12 r\cos\frac{v}{r}t$

$\dot x = -\frac{\sqrt3}{2}v\sin\frac{v}{r}t$, $\dot y = v\cos\frac{v}{r}t$, $\dot z = \frac12 v\sin\frac{v}{r}t$

$\ddot x = -\frac{\sqrt3}{2}\frac{v^2}{r}\cos\frac{v}{r}t$, $\ddot y = -\frac{v^2}{r}\sin\frac{v}{r}t$, $\ddot z = \frac12\frac{v^2}{r}\cos\frac{v}{r}t$

Note that position relationships do not include the constants associated with the origin positions.

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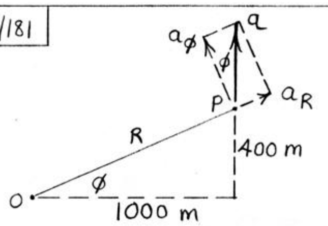
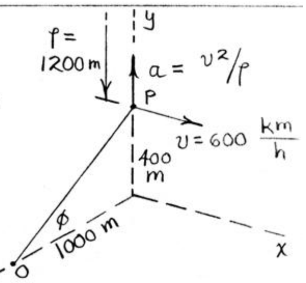




$\phi = \tan^{-1}\frac{400}{1000} = 21.8°$

$a = \frac{v^2}{\rho} = \frac{(600/3.6)^2}{1200} = 23.1 \text{ m/s}^2$

$\dot R = 0$, $\dot\phi = 0$, $\dot\theta = \frac{v_\theta}{R\cos\phi} = \frac{600(1000)/3.6}{1000} = 0.1667 \frac{\text{rad}}{\text{s}}$

Eqs. 2/19 : $a_R = \ddot R - R\dot\phi^2 - R\dot\theta^2\cos^2\phi$

$23.1\sin 21.8° = \ddot R - 0 - \frac{1000}{\cos 21.8°}(0.1667)^2\cos^2 21.8°$

$\underline{\ddot R = 34.4 \text{ m/s}^2}$

$a_\phi = \frac{1}{R}\frac{d}{dt}(R^2\dot\phi) + R\dot\theta^2\sin\phi\cos\phi = 2\dot R\dot\phi + R\ddot\phi + R\dot\theta^2\sin\phi\cos\phi$

$23.1\cos 21.8° = 0 + \frac{1000}{\cos 21.8°}\ddot\phi + \frac{1000}{\cos 21.8°}0.1667^2\sin 21.8°\cos 21.8°$

$\underline{\ddot\phi = 0.01038 \text{ rad/s}^2}$

2/182 $R = 0.75 + 0.5 = 1.25$ m, $\dot{R} = 0.2$ m/s, $\ddot{R} = -0.3\ \frac{m}{s^2}$

$\phi = 30°$, $\dot{\phi} = 10\left(\frac{\pi}{180}\right)$ rad/s, $\ddot{\phi} = 0$, $\dot{\theta} = 20\left(\frac{\pi}{180}\right)$ rad/s, $\ddot{\theta} = 0$

$$\begin{cases} v_R = \dot{R} = 0.2 \text{ m/s} \\ v_\theta = R\dot{\theta}\cos\phi = 1.25\left(20\frac{\pi}{180}\right)\cos 30° = 0.378 \frac{m}{s} \\ v_\phi = R\dot{\phi} = 1.25\left(10\frac{\pi}{180}\right) = 0.218 \text{ m/s} \end{cases}$$

$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{0.480 \text{ m/s}}$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$
$$= -0.3 - 1.25\left(10\frac{\pi}{180}\right)^2 - 1.25\left(20\frac{\pi}{180}\right)^2\cos^2 30°$$
$$= -0.4523 \text{ m/s}^2$$

$$a_\theta = \cos\phi\left[2\dot{R}\dot{\theta} + R\ddot{\theta}\right] - 2R\dot{\theta}\dot{\phi}\sin\phi$$
$$= \cos 30°\left[2(0.2)\left(20\frac{\pi}{180}\right) + 1.25(0)\right]$$
$$- 2(1.25)\left(10\frac{\pi}{180}\right)\left(20\frac{\pi}{180}\right)\sin 30° = 0.0448 \frac{m}{s^2}$$

$$a_\phi = 2\dot{R}\dot{\phi} + R\ddot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi$$
$$= 2(0.2)\left(10\frac{\pi}{180}\right) + 1.25(0) + 1.25\left(20\frac{\pi}{180}\right)^2 0.5\ \frac{\sqrt{3}}{2}$$
$$= 0.1358 \text{ m/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.474 \text{ m/s}^2}$$

2/183 Spherical coordinates

$$v_R = \dot{R} = 0.5 \text{ m/s}$$
$$v_\theta = R\dot{\theta}\cos\phi = 15\left(10\frac{\pi}{180}\right)\cos 30° = 2.27 \text{ m/s}$$
$$v_\phi = R\dot{\phi} = 15\left(7\frac{\pi}{180}\right) = 1.833 \text{ m/s}$$
$$v = \sqrt{v_R^2 + v_\theta^2 + v_\phi^2} = \underline{2.96 \text{ m/s}}$$

$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$
$$= 0 - 15\left(7\frac{\pi}{180}\right)^2 - 15\left(10\frac{\pi}{180}\right)^2\cos^2 30° = -0.567 \text{ m/s}^2$$

$$a_\theta = \frac{\cos\phi}{R}\left[R^2\ddot{\theta} + 2R\dot{R}\dot{\theta}\right] - 2R\dot{\theta}\dot{\phi}\sin\phi$$
$$= \frac{\cos 30°}{15}\left[0 + 2(15)(0.5)\left(10\frac{\pi}{180}\right)\right] - 2(15)\left(10\frac{\pi}{180}\right)\left(7\frac{\pi}{180}\right)\sin 30°$$
$$= -0.1687 \text{ m/s}^2$$

$$a_\phi = \frac{1}{R}\left[R^2\ddot{\phi} + 2R\dot{R}\dot{\phi}\right] + R\dot{\theta}^2\sin\phi\cos\phi$$
$$= \frac{1}{15}\left[0 + 2(15)(0.5)\left(7\frac{\pi}{180}\right)\right] + 15\left(10\frac{\pi}{180}\right)^2\sin 30°\cos 30°$$
$$= 0.320 \text{ m/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = \underline{0.672 \text{ m/s}^2}$$

2/184 Use Eq. 2/19 where $\varphi = -\beta$, $R = L$, $\dot{\theta} = \omega$

$$a_R = 0 - 1.2\left(-\frac{3}{2}\right)^2 - 1.2(2)^2\frac{1}{2} = \underline{-5.10 \text{ m/s}^2}$$

$$a_\theta = \frac{\sin\beta}{L}\left(2L\dot{L}\omega + 0\right) + 2L\omega\dot{\beta}\cos\beta = 2\omega\left(\dot{L}\sin\beta + L\dot{\beta}\cos\beta\right)$$
$$= 2(2)\left(0.9\frac{1}{\sqrt{2}} + 1.2\left(\frac{3}{2}\right)\frac{1}{\sqrt{2}}\right) = \frac{10.8}{\sqrt{2}} = \underline{7.64 \text{ m/s}^2}$$

$$a_\varphi = -2\dot{L}\dot{\beta} + L\omega^2\cos\beta\sin\beta = -2(0.9)\frac{3}{2} + 1.2(2^2)\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}$$
$$= -2.7 + 2.4 = \underline{-0.3 \text{ m/s}^2}$$

2/185

$R = 200 + 50\sin 4\pi t$ $\qquad$ $\dot{\theta} = 120\left(\frac{2\pi}{60}\right) = 4\pi$ rad/s

$\dot{R} = 200\pi\cos 4\pi t$ $\qquad$ $\ddot{\theta} = 0$

$\ddot{R} = -800\pi^2\sin 4\pi t$ $\qquad$ $\phi = \frac{\pi}{2} - \beta = 60°$, $\dot{\phi} = \ddot{\phi} = 0$

For $\dot{R}$ maximum, $\cos 4\pi t = 1$ and $\sin 4\pi t = 0$

Eq. 2/19 : $a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$
$$= 0 + (200+0)0 - (200+0)(4\pi)^2\cos^2 60° = -800\pi^2\ \frac{mm}{s^2}$$

$$a_\theta = \frac{\cos\phi}{R}\frac{d}{dt}\left(R^2\dot{\theta}\right) - 2R\dot{\theta}\dot{\phi}\sin\phi$$
$$= 2\dot{R}\dot{\theta}\cos\phi - 2R\dot{\theta}\dot{\phi}\sin\phi$$
$$= 2(200\pi\cdot 1)(4\pi)\cos 60° - 2(200+0)(4\pi)(0)\sin 60°$$
$$= 800\pi^2 \text{ mm/s}^2$$

$$a_\phi = \frac{1}{R}\frac{d}{dt}\left(R^2\dot{\phi}\right) + R\dot{\theta}^2\sin\phi\cos\phi$$
$$= 0 + (200+0)(4\pi)^2\sin 60°\cos 60° = 800\sqrt{3}\,\pi^2 \text{ mm/s}^2$$

$$a = \sqrt{a_R^2 + a_\theta^2 + a_\phi^2} = 17\,660 \text{ mm/s}^2 \text{ or } \underline{17.66 \text{ m/s}^2}$$

▶2/186 $R = \text{const}$ $\qquad$ $\theta = \omega t$ $\qquad$ $\sin\phi = z/R$

$z = \frac{h}{2}(1 - \cos 2\theta)$, $\dot{z} = \omega h\sin 2\theta$ where $\dot{\theta} = \omega$

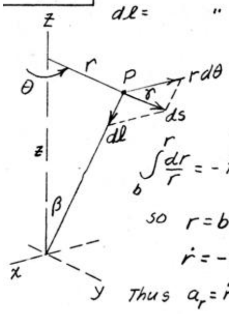
$(\cos\phi)\dot{\phi} = \frac{1}{R}\dot{z}$, $\dot{\phi} = \frac{\omega h\sin 2\theta}{R\cos\phi}$

$$v_R = \dot{R} = 0$$
$$v_\theta = R\dot{\theta}\cos\phi = R\omega\sqrt{1-\sin^2\phi} = R\omega\sqrt{1 - \left(\frac{h}{2R}[1-\cos 2\theta]\right)^2}$$
$$v_\phi = R\dot{\phi} = \frac{\omega h\sin 2\theta}{\cos\phi} = h\omega\frac{\sin 2\theta}{\sqrt{1 - \left(\frac{h}{2R}[1-\cos 2\theta]\right)^2}}$$

When $\theta = \omega t = \pi/4$, $1 - \cos 2\theta = 1$ so that

$$\underline{v_\theta = R\omega\sqrt{1-(h/2R)^2}}, \quad \underline{v_\phi = \frac{h\omega}{\sqrt{1-(h/2R)^2}}}, \quad \underline{v_R = 0}$$

▶2/187 ds = differential distance along curve
dl = " " in direction of cone element



$$r = \frac{b}{h}z, \quad \tan\beta = b/h$$
$$dl = r\,d\theta\tan\gamma$$
$$dr = -dl\sin\beta = -r\,d\theta\tan\gamma\sin\beta$$
$$\int_b^r\frac{dr}{r} = -\tan\gamma\sin\beta\int_0^\theta d\theta, \quad \ln r\Big]_b^r = -\tan\gamma\sin\beta\ \theta$$

so $r = be^{-\tan\gamma\sin\beta\,\theta}$, $r = be^{-K\theta}$ where $K = \tan\gamma\sin\beta$

$$\dot{r} = -bK\dot{\theta}e^{-K\theta}, \quad \ddot{r} = bK^2\dot{\theta}^2e^{-K\theta}, \quad \ddot{\theta} = 0$$

Thus $a_r = \ddot{r} - r\dot{\theta}^2 = b\dot{\theta}^2e^{-K\theta}(K^2 - 1)$

or $\underline{a_r = b\dot{\theta}^2\left(\tan^2\gamma\sin^2\beta - 1\right)e^{-\theta\tan\gamma\sin\beta}}$

$\underline{\text{where } \beta = \tan^{-1}\frac{b}{h}}$

2/188 The terms appearing in Eq. 2/19 are

$R = 50 + 200(1/2)^2 = 100$ mm, $\dot{R} = 400t = 400(1/2) = 200$ mm/s

$\ddot{R} = 400$ mm/s^2

$\theta = \omega t = \frac{\pi}{3}(\frac{1}{2}) = \pi/6$ rad, $\dot{\theta} = \pi/3$ rad/s, $\ddot{\theta} = 0$

$\phi = \dot{\phi} t = \frac{2\pi}{3}(\frac{1}{2}) = \pi/3$ rad, $\dot{\phi} = 2\pi/3$ rad/s, $\ddot{\phi} = 0$

$\sin\theta = 1/2$, $\cos\theta = \sqrt{3}/2$, $\sin\phi = \sqrt{3}/2$, $\cos\phi = 1/2$

$\frac{d}{dt}(R^2\dot{\theta}) = 2R\dot{R}\dot{\theta} + R^2\ddot{\theta} = 2(0.1)(0.2)\pi/3 + 0 = \frac{0.04\pi}{3}$ (m/s)^2

$\frac{d}{dt}(R^2\dot{\phi}) = 2R\dot{R}\dot{\phi} + R^2\ddot{\phi} = 2(0.1)(0.2)2\pi/3 + 0 = \frac{0.08\pi}{3}$ (m/s)^2

Thus the components of $\underline{a}$ from Eq. 2/19 become

$a_R = 0.40 - 0.10(2\pi/3)^2 - 0.10(\pi/3)^2(1/2)^2 = -0.0661$ m/s^2

$a_\theta = \frac{1/2}{0.10}\,\frac{0.04\pi}{3} - 2(0.10)(\pi/3)(2\pi/3)(\sqrt{3}/2) = -0.1704$ m/s^2

$a_\phi = \frac{1}{0.10}\,\frac{0.08\pi}{3} + 0.10(\pi/3)^2(\sqrt{3}/2)(1/2) = 0.885$ m/s^2

The magnitude of the acceleration is, then,

$a = \sqrt{(-0.0661)^2 + (-0.1704)^2 + (0.885)^2} = 0.904$ m/s^2

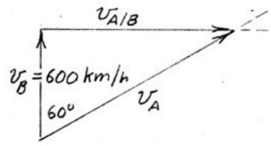
2/189 $\underline{v}_{B/A} = \underline{v}_B - \underline{v}_A = 40\underline{i} - (-80\underline{i}) = 120\underline{i}$ $\frac{km}{h}$

$\underline{a}_{B/A} = \underline{a}_B - \underline{a}_A = \underline{0} - 2\underline{i} = -2\underline{i}$ m/s^2

2/190 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$v_A = \frac{600}{\cos 60°} = 1200$ km/h

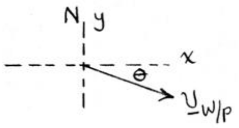
$v_{A/B} = 600 \tan 60°$
$= 1039$ km/h



2/191 (a) $\underline{v}_{W/P} = \underline{v}_W - \underline{v}_P$
$= 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - (-4\underline{i}) = 6.12\underline{i} - 2.12\underline{j}$ $\frac{mi}{hr}$

or $v_{W/P} = (6.12^2 + 2.12^2)^{1/2} = 6.48$ mi/hr

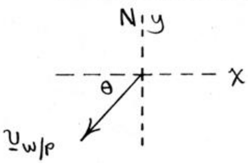
at $\theta = \tan^{-1}\frac{2.12}{6.12} = 19.11°$ south of east



(b) $\underline{v}_{W/P} = \underline{v}_W - \underline{v}_P = 3\left(\frac{\sqrt{2}}{2}\underline{i} - \frac{\sqrt{2}}{2}\underline{j}\right) - 4\underline{i} = -1.879\underline{i} - 2.12\underline{j}$ $\frac{mi}{hr}$

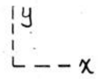
or $v_{W/P} = (1.879^2 + 2.12^2)^{1/2} = 2.83$ mi/hr

at $\theta = \tan^{-1}\frac{2.12}{1.879} = 48.5°$ South of west



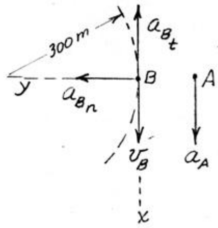
2/192 $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B$
$= 120[\cos 15°\underline{i} + \sin 15°\underline{j}] - 90[\cos 60°\underline{i} + \sin 60°\underline{j}]$
$= 70.9\underline{i} - 46.9\underline{j}$ km/h

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \underline{0} - 3(-\cos 60°\underline{i} - \sin 60°\underline{j})$
$= 1.5\underline{i} + 2.60\underline{j}$ m/s^2



2/193 $a_{B_t} = \frac{8}{3.6} = 2.22$ m/s^2, $a_A = \frac{8}{3.6} = 2.22$ m/s^2

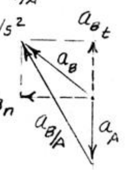
$a_{B_n} = \frac{v_B^2}{\rho} = \frac{(100/3.6)^2}{300} = 2.57$ m/s^2



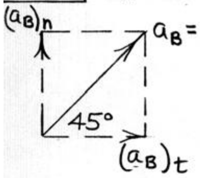
$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$-2.22\underline{i} + 2.57\underline{j} = 2.22\underline{i} + \underline{a}_{B/A}$

$\underline{a}_{B/A} = -4.44\underline{i} + 2.57\underline{j}$ m/s^2



2/194 With $\underline{a}_{B/A} = \underline{0}$, $\underline{a}_A = \underline{a}_B$



$(a_B)_n = \frac{v_B^2}{\rho} = \frac{\left(45\,\frac{44}{30}\right)^2}{600}$
$= 7.26$ ft/sec^2

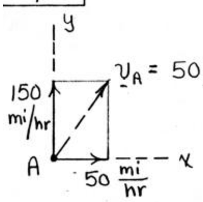
$\dot{v}_B = (a_B)_t = 7.26$ ft/sec^2

$a_A = a_B = 7.26\sqrt{2} = 10.27$ $\frac{ft}{sec^2}$

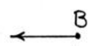
2/195 $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B$, $\Omega = 3(2\pi/60) = 0.314$ $\frac{rad}{s}$
$= \frac{18}{3.6}\underline{i} - 9(0.314)(\cos 45°\underline{i} - \sin 45°\underline{j})$
$= 3.00\underline{i} + 1.999\underline{j}$ m/s

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = 3\underline{i} - 9(0.314)^2(-\cos 45°\underline{i} - \sin 45°\underline{j})$
$= 3.63\underline{i} + 0.628\underline{j}$ m/s^2

2/196



$\underline{v}_A = 50\underline{i} + 150\underline{j}$ mi/hr



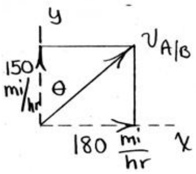
$\underline{v}_B = (180 - 50)(-\underline{i}) = -130\underline{i}$ $\frac{mi}{hr}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$: $50\underline{i} + 150\underline{j} = -130\underline{i} + \underline{v}_{A/B}$

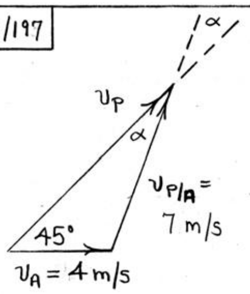
$\underline{v}_{A/B} = 180\underline{i} + 150\underline{j}$ mi/hr

$v_{A/B} = \sqrt{180^2 + 150^2} = 234$ mi/hr

$\theta = \tan^{-1}\left(\frac{180}{150}\right) = 50.2°$
(east of north)



2/197



With P being the puck:

$\underline{v}_P = \underline{v}_A + \underline{v}_{P/A}$

Law of sines :

$$\frac{\sin 45^\circ}{7} = \frac{\sin\alpha}{4}$$

$\underline{\alpha = 23.8^\circ}$

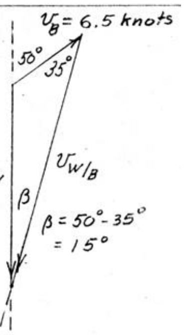
2/198 w = wind, B = boat

$\underline{v}_W = \underline{v}_B + \underline{v}_{W/B}$

Law of sines,

$$\frac{v_W}{\sin 35^\circ} = \frac{6.5}{\sin 15^\circ}$$

$$v_W = \frac{6.5(0.5736)}{0.2588} = \underline{14.40 \text{ knots}}$$



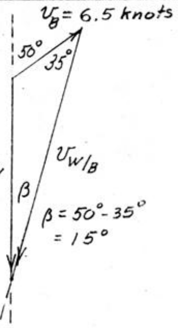
2/198 w = wind, B = boat

$\underline{v}_W = \underline{v}_B + \underline{v}_{W/B}$

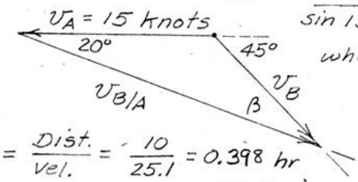
Law of sines,

$$\frac{v_W}{\sin 35^\circ} = \frac{6.5}{\sin 15^\circ}$$

$$v_W = \frac{6.5(0.5736)}{0.2588} = \underline{14.40 \text{ knots}}$$



2/199 $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$



$$\frac{v_{B/A}}{\sin 135^\circ} = \frac{15}{\sin\beta}$$

where $\beta = 180 - 20 - 135 = 25^\circ$

$$v_{B/A} = \frac{0.7071}{0.4226}15 = 25.1 \text{ knots}$$

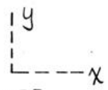
$$t = \frac{\text{Dist.}}{\text{vel.}} = \frac{10}{25.1} = 0.398 \text{ hr}$$

or 23.9 min

So collision would occur at $\underline{2{:}24 \text{ pm}}$

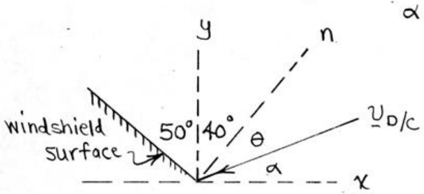
2/200 Drop : $v_D = \sqrt{2gh} = \sqrt{2(9.81)(6)} = 10.85$ m/s

Car : $v_c = 100/3.6 = 27.8$ m/s



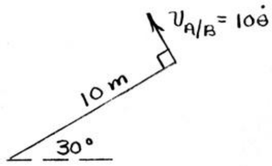
$\underline{v}_{D/c} = \underline{v}_D - \underline{v}_c = -10.85\underline{j} - 27.8\underline{i}$ m/s

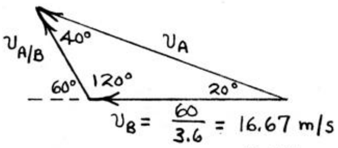
$$\alpha = \tan^{-1}\frac{10.85}{27.8} = 21.3^\circ$$



$40^\circ + \theta + \alpha = 90^\circ \Rightarrow \underline{\theta = 28.7^\circ \text{ below normal}}$

2/201 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$





$$v_B = \frac{60}{3.6} = 16.67 \text{ m/s}$$

Law of sines :

$$\frac{16.67}{\sin 40^\circ} = \frac{v_A}{\sin 120^\circ}$$

$v_A = 22.5$ m/s or $\underline{v_A = 80.8 \text{ km/h}}$

$$v_{A/B} = 16.67\frac{\sin 20^\circ}{\sin 40^\circ} = 10\dot\theta, \quad \underline{\dot\theta = 0.887 \frac{\text{rad}}{\text{s}}}$$

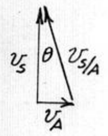
2/202 Let s = satellite , A = observer.

$\underline{v}_s = \underline{v}_A + \underline{v}_{s/A}$, $v_A = R\omega$

$= 6378(0.729)(10^{-4})(3600)$

$= 1674$ km/h (East)

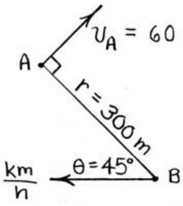
$v_s = 27\,940$ km/h (North)



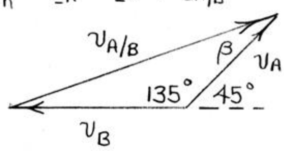
$$\theta = \tan^{-1}\frac{1674}{27\,940} = 3.43^\circ$$

Satellite appears to travel $\underline{3.43^\circ}$ $\underline{\text{west of north}}$

2/203



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$



$v_{A/B}^2 = 60^2 + 80^2 - 2(60)(80)\cos 135^\circ$

$v_{A/B} = 129.6$ km/h or $\underline{36.0 \text{ m/s}}$

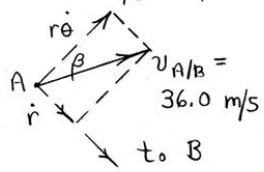
$$\frac{36.0}{\sin 135^\circ} = \frac{80/3.6}{\sin\beta}, \quad \beta = 25.9^\circ$$

$r\dot\theta = v_{A/B}\cos\beta$: $300\dot\theta = 36.0\cos 25.9^\circ$

$\underline{\dot\theta = 0.1079 \text{ rad/s}}$

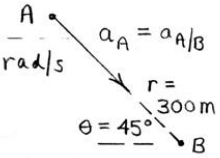
$\dot r = -v_{A/B}\sin\beta$: $\dot r = -36.0\sin 25.9^\circ$

$\underline{\dot r = -15.71 \text{ m/s}}$



2/204 From Prob. 2/203,

$\dot{r} = -15.71$ m/s , $\dot{\theta} = 0.1079$ rad/s



$\underline{a}_A = \underline{a}_B^{\,0} + \underline{a}_{A/B}$

$a_A = \dfrac{v_A^2}{\rho} = \dfrac{(60/3.6)^2}{300}$

$= 0.926$ m/s^2 $= a_{A/B}$

$(a_{A/B})_r = \ddot{r} - r\dot{\theta}^2 : \; -0.926 = \ddot{r} - 300(0.1079)^2$

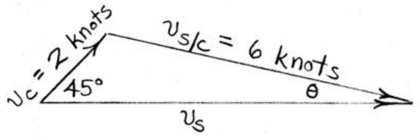
$\underline{\ddot{r} = 2.57 \text{ m/s}^2}$

$(a_{A/B})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} : \; 0 = 300\ddot{\theta} + 2(-15.71)(0.1079)$

$\underline{\ddot{\theta} = 0.01130 \text{ rad/s}^2}$

2/205
- v_s : true velocity of ship (A to B)
- v_c : true velocity of current (2 knots NE)
- $v_{s/c}$: velocity of ship relative to current (magnitude 6 knots)

$\underline{v}_s = \underline{v}_c + \underline{v}_{s/c}$



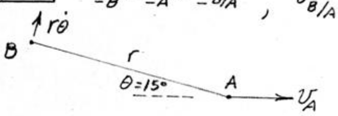
Law of sines : $\dfrac{2}{\sin\theta} = \dfrac{6}{\sin 45°}$, $\theta = 13.63°$

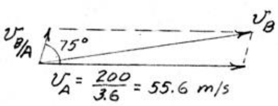
So heading $H = 90° + 13.63° \cong \underline{104°}$

$v_s = 2\cos 45° + 6\cos 13.63° = 7.25$ knots

Time $t = \dfrac{10}{7.25} = 1.380$ hr or $\underline{t = 1 \text{ hr } 23 \text{ min}}$

2/206 $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$, $v_{B/A} = r\dot{\theta} = 60(\frac{5}{180}\pi) = 5.24$ m/s





$v_B^2 = (5.24)^2 + (55.6)^2 + 2(5.24)(55.6)\cos 75° = 3264$ (m/s)^2

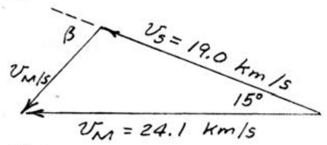
$v_B = 57.1$ m/s or $v_B = 57.1(3.6) = \underline{206 \text{ km/h}}$

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$, $a_A = 0$, $a_{B/A} = r\dot{\theta}^2 = 60(\frac{5\pi}{180})^2 = 0.457$ m/s^2

Thus $a_B = a_{B/A} = \underline{0.457 \text{ m/s}^2}$ from B to A

2/207 Mars will appear to be approaching the spacecraft head on when $v_{M/S}$ is along the line of sight M-S.

$\underline{v}_M = \underline{v}_S + \underline{v}_{M/S}$

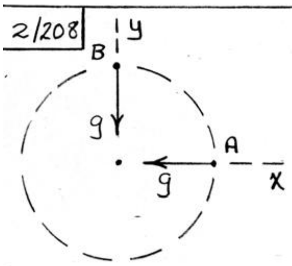


$(v_{M/S})^2 = (19.0)^2 + (24.1)^2 - 2(19.0)(24.1)\cos 15° = 57.2$ (km/s)^2

$v_{M/S} = 7.56$ km/s, $\dfrac{24.1}{\sin(\pi-\beta)} = \dfrac{7.56}{\sin 15°}$

$\sin(\pi-\beta) = \sin\beta = 0.8246$, $\underline{\beta = 55.6°}$

2/208



$g = g_0\left(\dfrac{R}{R+h}\right)^2$

$= 9.825\left(\dfrac{6371}{6371+1500}\right)^2$

$= 6.44$ m/s^2

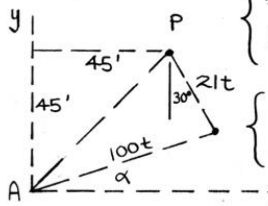
$\underline{a}_{B/A} = \underline{a}_B - \underline{a}_A$

$= -6.44\underline{j} - (-6.44\underline{i})$

$= 6.44\underline{i} - 6.44\underline{j}$ m/s^2

$a_{B/A} = 6.44\sqrt{2} = \underline{9.10 \text{ m/s}^2}$

2/209



$\begin{cases} 100t\cos\alpha = 45 + 21t\sin 30° \\ 100t\sin\alpha = 45 - 21t\cos 30° \end{cases}$

or

$\begin{cases} 100\cos\alpha = \frac{45}{t} + 21\sin 30° \\ 100\sin\alpha = \frac{45}{t} - 21\cos 30° \end{cases}$

Subtract 2nd from 1st:

$100(\cos\alpha - \sin\alpha) = 21\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$ or $\cos\alpha - \sin\alpha = 0.287$

$\cos\alpha = \sqrt{1-\sin^2\alpha} = 0.287 + \sin\alpha$

SQ BS & rearrange : $2\sin^2\alpha + 0.574\sin\alpha - 0.918 = 0$

Positive solution : $\alpha = 33.3°$; Then $t = 0.616$ sec

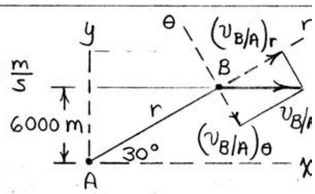
$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 100[\cos\alpha\,\underline{i} + \sin\alpha\,\underline{j}]$

$- 21[\sin 30°\,\underline{i} - \cos 30°\,\underline{j}]$

$= \underline{73.1\underline{i} + 73.1\underline{j} \text{ ft/sec}}$

►2/210 $v_{B/A} = v_B - v_A$

$= (1500-1000)/3.6 = 138.9 \frac{m}{s}$



$(v_{B/A})_r = \dot{r} = 138.9\cos 30°$

$= 120.3$ m/s

$(v_{B/A})_\theta = r\dot{\theta} : -138.9\sin 30° = \dfrac{6000}{\sin 30°}\dot{\theta}$

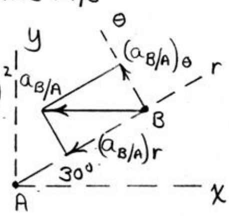
$\dot{\theta} = -0.00579$ rad/s

$a_{B/A} = a_B - a_A = 0 - 1.2 = -1.2$ m/s^2

$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2$

$-1.2\cos 30° = \ddot{r} - 12000(-0.00579)^2$

$\underline{\ddot{r} = -0.637 \text{ m/s}^2}$



$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$1.2\sin 30° = 12000\ddot{\theta} + 2(120.3)(-0.00579)$

$\underline{\ddot{\theta} = 0.1660(10^{-3}) \text{ rad/s}^2}$

▶ 2/211 Find flight time t :

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$: $7 = 3 + 100 \sin 30° t - 16.1 t^2$

Solve to obtain 0.0822 sec (discard) & $t = 3.02$ sec

Range $R = x_0 + v_{x_0} t = 0 + 100 \cos 30° (3.02)$
$= 262$ ft

Fielder must run $262 - 220 = 41.8$ ft
in $(3.02 - 0.25)$ sec $\Rightarrow v_B = \frac{41.8}{2.77} = 15.08$ ft/sec

Velocity components of ball when caught:

$v_x = v_{x_0} = 100 \cos 30° = 86.6$ ft/sec

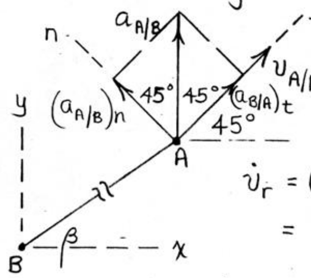
$v_y = v_{y_0} - gt = 100 \sin 30° - 32.2(3.02) = -47.4 \frac{\text{ft}}{\text{sec}}$

$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = (86.6\underline{i} - 47.4\underline{j}) - 15.08\underline{i}$
$= \underline{71.5\,\underline{i} - 47.4\,\underline{j} \text{ ft/sec}}$

▶ 2/212 (a) $\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = 50\underline{i} - (-50\underline{j}) = \underline{50\underline{i} + 50\underline{j} \text{ m/s}}$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = \frac{v_A^2}{\rho_A}\underline{j} - \underline{0} = \frac{50^2}{2000}\underline{j} = \underline{1.250\underline{j} \text{ m/s}^2}$

(b) Use the results of part (a) for a normal-tangential analysis:



$v_{A/B} = \sqrt{50^2 + 50^2}$
$= 70.7$ m/s

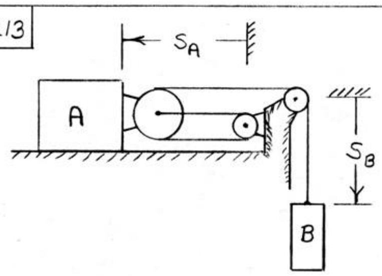
$\dot{v}_r = (a_{B/A})_t = a_{A/B} \cos 45°$
$= 1.250 \frac{\sqrt{2}}{2} = \underline{0.884 \text{ m/s}^2}$

$(a_{B/A})_n = a_{B/A} \sin 45° = \frac{v_{A/B}^2}{\rho_r}$

$1.250 \frac{\sqrt{2}}{2} = \frac{70.7^2}{\rho_r}$, $\underline{\rho_r = 5660 \text{ m}}$

(in n-direction from A)

2/213

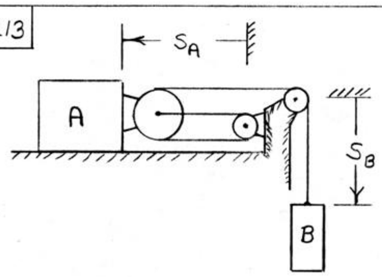


Length of cable $L = 3s_A + s_B + \text{constant}$

Differentiate : $0 = 3v_A + v_B$

$v_B = -3v_A = -3(-3.6)$
$= \underline{10.8 \text{ ft/sec (down)}}$

2/213

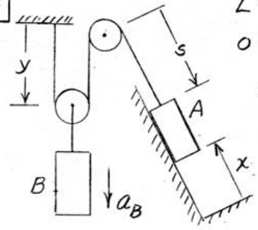


Length of cable $L = 3s_A + s_B + \text{constant}$

Differentiate : $0 = 3v_A + v_B$

$v_B = -3v_A = -3(-3.6)$
$= \underline{10.8 \text{ ft/sec (down)}}$

2/214



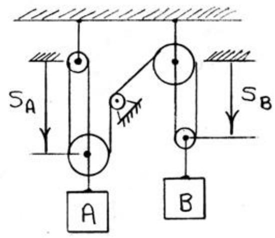
$L = 2y + s + \text{const.}$

$0 = 2\dot{y} + \dot{s}$, $0 = 2\ddot{y} + \ddot{s}$

$a_B = \ddot{y} = -\frac{\ddot{s}}{2} = +\frac{\ddot{x}}{2}$

$a_B = \frac{0.044}{2} = \underline{0.022 \text{ m/s}^2}$

2/215



The length of the main cable is

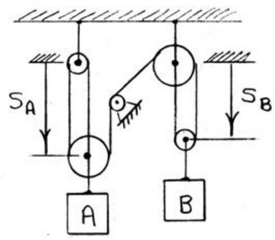
$L = 3s_A + 2s_B + \text{constants}$

$\Rightarrow 0 = 3v_A + 2v_B \ ; \quad 0 = 3a_A + 2a_B$

So $v_B = -\frac{3}{2}v_A = -\frac{3}{2}(0.8) = \underline{-1.2 \text{ m/s}}$ (up)

and $a_B = -\frac{3}{2}a_A = -\frac{3}{2}(-2) = \underline{3 \text{ m/s}^2}$ (down)

2/215



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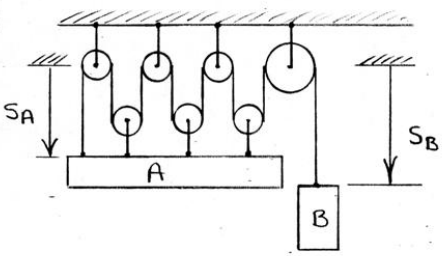
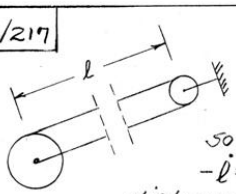
2/216



$L = 7s_A + s_B + \text{constants}$

$\underline{0 = 7a_A + a_B}$ (for coordinates shown)

2/217



Length of cable is

$L = 2\ell$

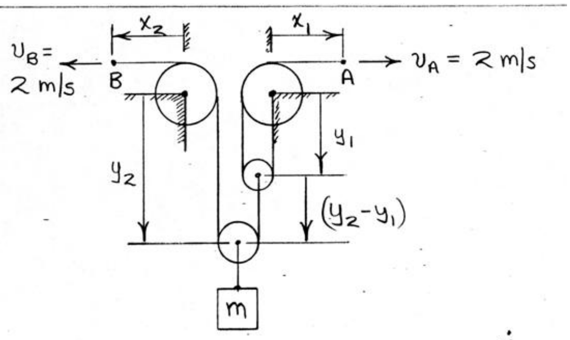
$\dot{L} = 2\dot{\ell}$

so velocity of truck is

$-\dot{\ell} = \frac{1}{2}(-\dot{L}) = \frac{1}{2}(40) = 20 \text{ mm/s}$

time $t = \frac{\text{distance}}{\text{velocity}} = \frac{4(10^3)}{20} = 200 \text{ s}$ *or* $\underline{3 \text{ min } 20 \text{ s}}$

2/218



$x_1 + 2y_1 = \text{constant} ; \quad \dot{x}_1 + 2\dot{y}_1 = 0, \quad \dot{y}_1 = -\frac{\dot{x}_1}{2}$

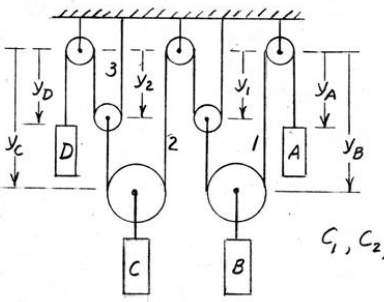
$x_2 + y_2 + (y_2 - y_1) = \text{constant} ; \quad \dot{x}_2 + 2\dot{y}_2 - \dot{y}_1 = 0$

$\dot{x}_2 + 2\dot{y}_2 - \left(-\frac{\dot{x}_1}{2}\right) = 0$

$\dot{x}_2 + 2\dot{y}_2 + \frac{\dot{x}_1}{2} = 0, \quad \dot{y}_2 = -\frac{\dot{x}_2}{2} - \frac{\dot{x}_1}{4} = -\frac{2}{2} - \frac{2}{4} = -1.5 \text{ m/s}$

or $\underline{1.5 \text{ m/s up}}$

2/219



3 degrees of freedom

C_1, C_2, C_3 *are constant lengths*

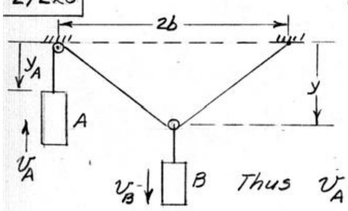
$L_1 = y_B + y_A + (y_B - y_1) + C_1; \quad 0 = 2\dot{y}_B + \dot{y}_A - \dot{y}_1$

$L_2 = y_C + 2y_1 + (y_C - y_2) + C_2 \quad\quad 0 = 2\dot{y}_C + 2\dot{y}_1 - \dot{y}_2$

$L_3 = 2y_2 + y_D + C_3, \quad\quad 0 = 2\dot{y}_2 + \dot{y}_D$

Eliminate $\dot{y}_1$ & $\dot{y}_2$ & *get* $\underline{4v_A + 8v_B + 4v_C + v_D = 0}$

2/220



Cable length is

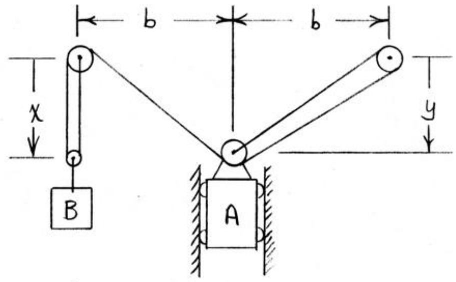
$L = y_A + 2\sqrt{y_B^2 + b^2}$

$0 = \dot{y}_A + \frac{2y\dot{y}}{\sqrt{y^2 + b^2}}$

But $v_A = -\dot{y}_A, \quad v_B = \dot{y}$

Thus $\underline{v_A = \frac{2y}{\sqrt{y^2 + b^2}} v_B}$

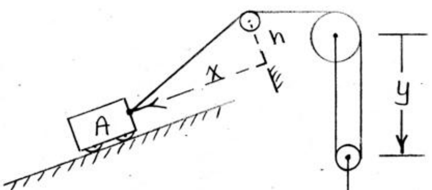
2/221



The total length of the cable is
$L = 2x + 3\sqrt{y^2+b^2}$ + constant
Differentiate to obtain
$\dot{L} = 0 = 2\dot{x} + 3\frac{y\dot{y}}{\sqrt{y^2+b^2}}$
With $\dot{x} = v_B$ & $\dot{y} = v_A$, we have

$$v_B = -\frac{3y}{2\sqrt{y^2+b^2}}v_A$$

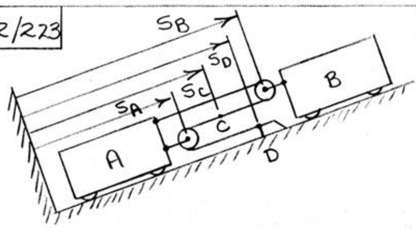
2/222



The total length of the cable is
$L = \sqrt{x^2+h^2} + 2y$ + constant
$\dot{L} = 0 = \frac{1}{2}\frac{2x\dot{x}}{\sqrt{x^2+h^2}} + 2\dot{y}$
With $v_A = \dot{x}$ & $v_B = -\dot{y}$:

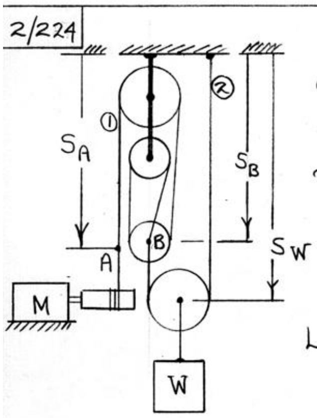
$$v_A = +\frac{2\sqrt{x^2+h^2}}{x}v_B$$

2/223



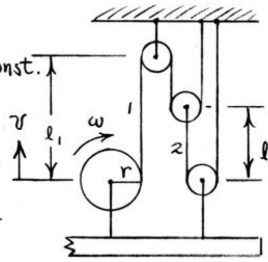
The cable length is $L = 2(s_B - s_A) + s_D - s_A$ + constants
Differentiating:
$0 = 2v_B - 3v_A$; $0 = 2a_B - 3a_A$
So $v_A = \frac{2}{3}v_B = \frac{2}{3}(3) = 2$ ft/sec
$a_A = \frac{2}{3}a_B = \frac{2}{3}(6) = 4$ ft/sec^2
$v_{B/A} = v_B - v_A = 3 - 2 =$ 1 ft/sec
$a_{B/A} = a_B - a_A = 6 - 4 =$ 2 ft/sec^2
The length of cable between A and C is
$L' = (s_B - s_A) + (s_B - s_C) = 2s_B - s_A - s_C$ + constants
$\Rightarrow 0 = 2v_B - v_A - v_C$; $v_C = 2v_B - v_A = 2(3) - 2 =$ 4 ft/sec
(All answers are quantities directed up incline.)

2/224



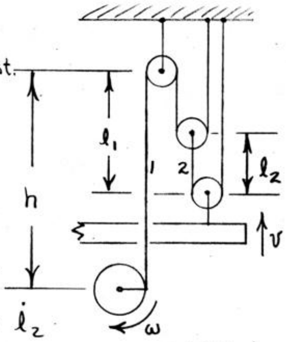
Let A be a point on cable 1, and let B be the indicated pulley.
$L_1 = s_A + 3s_B$ + constant
$0 = v_A + 3v_B$ (1)
$L_2 = s_W - s_B + s_W$
$= 2s_W - s_B$
$0 = 2v_W - v_B$ (2)
Combine (1) & (2): $v_A + 6v_W = 0$
So $v_W = -\frac{1}{6}v_A$
Hence W rises $\frac{1}{6}(180)(10) =$ 300 mm $= h$

2/225



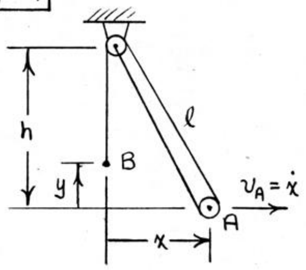
Length $L_1 = l_1 + 2(l_1 - l_2)$ + const.
$\dot{L}_1 = -r\omega = 3\dot{l}_1 - 2\dot{l}_2$
Length $L_2 = l_2 + l_1$ + const.
$\dot{L}_2 = 0 = \dot{l}_2 + \dot{l}_1$, $-\dot{l}_1 = \dot{l}_2$
But $v = -\dot{l}_1$, so
$-r\omega = 3(-v) - 2v$, $r\omega = 5v$
$v = \frac{r\omega}{5} = \frac{0.1(40)(2\pi/60)}{5} = 0.0838$ m/s
or $v =$ 83.8 mm/s

2/226



Length $L_1 = h + 2(l_1 - l_2)$ + const.
$\dot{L}_1 = -r\omega = 0 + 2\dot{l}_1 - 2\dot{l}_2$
But $v = -\dot{l}_1$
So $-r\omega = -2v - 2\dot{l}_2$
Length $L_2 = l_1 + l_2$ + const.
$\dot{L}_2 = 0 = \dot{l}_1 + \dot{l}_2$, $v = \dot{l}_2$
$\therefore r\omega = 2v + 2v$, $v = \frac{r\omega}{4} = \frac{0.1(40)(2\pi/60)}{4}$
$v = 0.1047$ m/s or $v = 104.7$ mm/s

2/227



Length of cable is

$$L = h - y + 2\ell + \text{constant} = h - y + 2\sqrt{x^2+h^2} + \text{const.}$$

$$\dot{L} = 0 = -\dot{y} + \frac{2x\dot{x}}{\sqrt{x^2+h^2}}$$

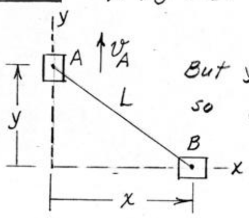
Substitute v_A for $\dot{x}$ and v_B for $\dot{y}$:

$$v_B = \frac{2x}{\sqrt{x^2+h^2}}\, v_A$$ (4 times as fast as with Sample Prob. 2/15)

2/228

$$x^2+y^2 = L^2\,; \quad x\dot{x} + y\dot{y} = 0$$

$$\& \quad \dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0$$

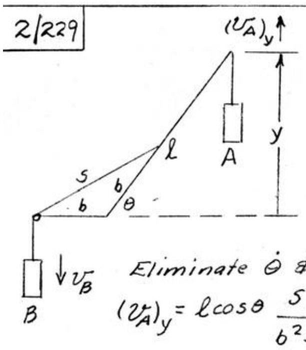


But $\dot{y} = v_A$, $\ddot{y} = 0$

so $a_x = \ddot{x} = -\dfrac{\dot{x}^2+\dot{y}^2}{x} = -\dfrac{\dfrac{\dot{y}^2 y^2}{x^2} + \dot{y}^2}{x}$

$$a_x = -\frac{L^2}{x^3}\dot{y}^2 = -\frac{L^2 v_A^2}{(L^2-y^2)^{3/2}}$$

2/229



$$s^2 = 2b^2 + 2b^2\cos\theta = 2b^2(1+\cos\theta)$$

$$2s\dot{s} = 2b^2(-\dot{\theta}\sin\theta)$$

$$v_B = -\dot{s} = \frac{b^2}{s}\dot{\theta}\sin\theta$$

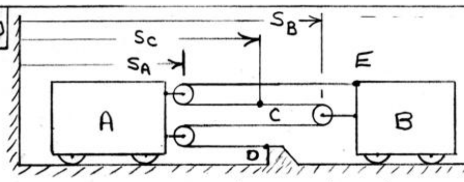
$$y = \ell\sin\theta$$

$$(v_A)_y = \dot{y} = \ell\dot{\theta}\cos\theta$$

Eliminate $\dot{\theta}$ & get

$$(v_A)_y = \ell\cos\theta\,\frac{s v_B}{b^2\sin\theta} = \frac{\ell\sqrt{2(1+\cos\theta)}}{b\tan\theta}\, v_B$$

►2/230



Cable length $L = 3(s_B - s_A) + (s_D - s_A)$

$$0 = 3v_B - 4v_A\,, \quad 0 = 3a_B - 4a_A$$

$$v_A = \tfrac{3}{4}v_B = \tfrac{3}{4}(2) = 1.5 \text{ m/s}$$

$$a_A = \tfrac{3}{4}a_B = \tfrac{3}{4}(3) = 2.25 \text{ m/s}^2$$

$$v_{B/A} = v_B - v_A = 2 - (1.5) = 0.5 \text{ m/s}$$

$$a_{B/A} = a_B - a_A = 3 - (2.25) = 0.75 \text{ m/s}^2$$

Length of cable between points E and C:

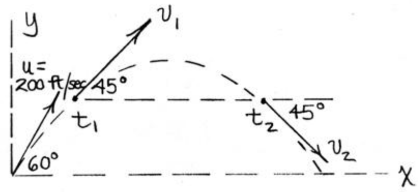
$$L' = (s_B - s_A) + (s_C - s_A) + \text{constants}$$

$$0 = v_B - 2v_A + v_C \Rightarrow v_C = 2v_A - v_B$$

or $v_C = 2(1.5) - 2 = 1$ m/s

(All answers are quantities directed to right)

2/231



$$\dot{x} = u\cos\theta = 200\cos 60° = 100 \text{ ft/sec}$$

$$\dot{y} = u\sin\theta - gt = 200\sin 60° - 32.2t = 173.2 - 32.2t$$

At t_1: $\dot{x} = \dot{y}$: $100 = 173.2 - 32.2t_1$, $t_1 = 2.27$ sec

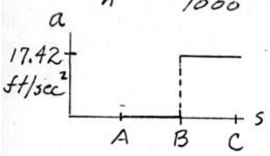
At t_2: $\dot{x} = -\dot{y}$: $100 = -173.2 + 32.2t_2$, $t_2 = 8.48$ sec

2/232

From A to B $a = 0$

From B to C $a = a_n = v^2/\rho$

$$a_n = \frac{(90 \times 44/30)^2}{1000} = 17.42 \text{ ft/sec}^2$$



Abrupt acceleration would cause abrupt forces which would be uncomfortable for passengers. A transition section to change curvature gradually over an interval of track would be required.

2/233

$$r = r_0 + b\sin\frac{2\pi t}{\tau}\,, \quad \dot{r} = \frac{2\pi}{\tau}b\cos\frac{2\pi t}{\tau}$$

$$\ddot{r} = -\frac{4\pi^2}{\tau^2}b\sin\frac{2\pi t}{\tau}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{4\pi^2}{\tau^2}b\sin\frac{2\pi t}{\tau} - r\dot{\theta}^2 = 0$$

$$\Rightarrow r = r_0\,\frac{1}{1+\left(\dfrac{\tau\dot{\theta}}{2\pi}\right)^2}$$

2/234

$\dot{x} = 20$ mm/s , $\ddot{x} = 0$

$$y = x^2/160\,, \quad \dot{y} = x\dot{x}/80\,, \quad \ddot{y} = (\dot{x}^2 + x\ddot{x})/80$$

$$v = \sqrt{\dot{x}^2+\dot{y}^2} = \sqrt{\dot{x}^2 + (x\dot{x}/80)^2} = \dot{x}\sqrt{1+(x/80)^2}$$

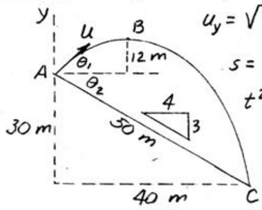
For $x = 60$ mm

$$v = 20\sqrt{1+(60/80)^2} = 25 \text{ mm/s}$$

$a = \ddot{y} = \dot{x}^2/80$ since $\ddot{x} = 0$, $a = (20)^2/80 = 5$ mm/s²

2/235

y-dir, $v^2 = v_1^2 + 2as$; $0 = u_y^2 - 2g(12)$ A to B



$$u_y = \sqrt{2(9.81)(12)} = 15.34 \text{ m/s}$$

$$s = v_1 t + \tfrac{1}{2}at^2\,; \quad -30 = 15.34t - \tfrac{9.81}{2}t^2$$

$$t^2 - 3.128t - 6.116 = 0\,,$$

$$t = 4.49 \text{ s} \quad (t = -1.36 \text{ s})$$

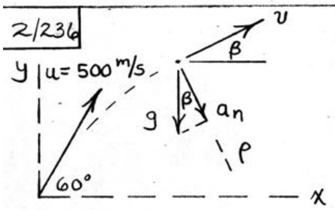
x-dir, $40 = u_x(4.49)$, $u_x = 8.91$ m/s

$$u = \sqrt{(8.91)^2 + (15.34)^2} = 17.74 \text{ m/s}$$

$$\theta_1 = \tan^{-1}\frac{15.34}{8.91} = 59.86°\,; \quad \theta_2 = \tan^{-1}\frac{3}{4} = 36.87°$$

$$\theta = \theta_1 + \theta_2 = 59.86 + 36.87 = 96.7°$$

2/236



$v_x = 500 \cos 60° = 250$ m/s

$v_y = v_{y_0} - gt = 500 \sin 60° - 9.81(30) = 138.7 \frac{m}{s}$

$v^2 = v_x^2 + v_y^2 = 250^2 + 138.7^2 = 81.7(10^3)\ m^2/s^2$

$\beta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{138.7}{250} = 29.0°$

$a_n = g\cos\beta = 9.81 \cos 29.0° = 8.58\ m/s^2$

$a_n = v^2/\rho \Rightarrow \rho = \frac{v^2}{a_n} = \frac{81.7(10^3)}{8.58} = 9529$ m

or $\underline{\rho = 9.53 \text{ km}}$

2/237 $\theta = 4[t + 30e^{-0.03t} - 30]$ (rad)

$\dot\theta = 4[1 - 0.9e^{-0.03t}]$ (rad/sec)

$\ddot\theta = 0.1080e^{-0.03t}$ (rad/sec²)

$r\dot\theta^2 = 30\left[4(1-0.9e^{-0.03t})\right]^2 = 32.2(10)$

$(1-0.9e^{-0.03t})^2 = 0.671$

$(1-0.9e^{-0.03t}) = \pm 0.819$

Take (+) as (−) will result in $t < 0$:

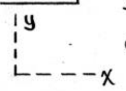
$(1-0.9e^{-0.03t}) = 0.819 \Rightarrow \underline{t = 53.5 \text{ sec}}$

$\ddot\theta = 0.1080e^{-0.03(53.5)} = 0.0217$ rad/sec²

$r\ddot\theta = 30(0.0217) = 0.651$ ft/sec² $(0.020g)$

Thus a_t can be neglected.

2/238 $\underline{v}_A = \underline{v}_W + \underline{v}_{A/W} = -48\underline{i} + 220\underline{i} = 172\underline{i}\ \frac{km}{h}$



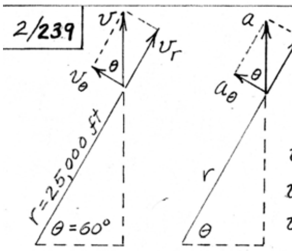
On descent: $\underline{v}_A = 172(\cos 10°\underline{i} - \sin 10°\underline{j})$ km/h

$\underline{v}_{A/c} = \underline{v}_A - \underline{v}_c = 172(\cos 10°\underline{i} - \sin 10°\underline{j}) - 30\underline{i}$

$= 139.4\underline{i} - 29.9\underline{j}$ km/h

$\beta = \tan^{-1}\left(\frac{29.9}{139.4}\right) = \underline{12.09°}$

2/239



Given $r = 25{,}000$ ft
$\dot\theta = 0.03$ rad/sec
$\theta = 60°$
$a = 64$ ft/sec²

$v_\theta = r\dot\theta = 25(10^3)(0.03) = 750$ ft/sec

$v = 750/\cos 60° = 1500$ ft/sec

$v_r = \dot r = 1500 \cos 30° = 1299$ ft/sec

$a_\theta = a\cos 60° = 64(0.5) = 32$ ft/sec²

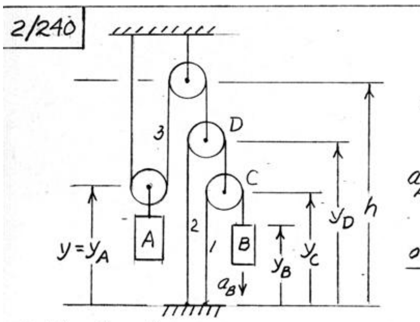
$a_\theta = r\ddot\theta + 2\dot r\dot\theta$, so $r\ddot\theta = 32 - 2(1299)(0.03) = -45.94$

$\ddot\theta = -\frac{45.94}{25{,}000} = \underline{-1.838(10^{-3})\text{ rad/sec}^2}$

$a_r = a\sin 60° = 64(\sqrt3/2) = 55.43$ ft/sec²

$a_r = \ddot r - r\dot\theta^2$, so $\ddot r = 55.43 + 25(10^3)(0.03)^2 = \underline{77.9 \text{ ft/sec}^2}$

2/240



$y = y_A = t^2/4$ m

$\dot y_A = t/2$ m/s

$a_A = \ddot y_A = 1/2$ m/s²

one degree of freedom

Cable lengths

$L_1 = y_C + (y_C - y_B) + C_1$ $\qquad 0 = 2\ddot y_C - \ddot y_B$ where $a_B = -\ddot y_B$

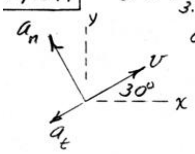
$L_2 = y_D + (y_D - y_C) + C_2$ $\qquad 0 = 2\ddot y_D - \ddot y_C$

$L_3 = 2(h - y_A) + h - y_D + C_3$, $\quad 0 = -2\ddot y_A - \ddot y_D$

Eliminate $\ddot y_C$ & $\ddot y_D$ & get $\ddot y_B = -8\ddot y_A$ or $a_B = 8\ddot y_A = \underline{4\ m/s^2}$

(By inspection of pulley displacements
$-dy_B = 8dy_A$, so $a_B = 8a_A = 8(1/2) = 4$ m/s²)

2/241 $v = \frac{1000}{3.6} = 278$ m/s, $a_t = \frac{15}{3.6} = 4.17$ m/s²



$a_n = v^2/\rho = (278)^2/1500 = 51.4$ m/s²

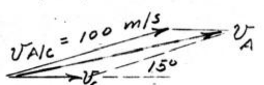
$\ddot x = -51.4 \sin 30° - 4.17 \cos 30° = \underline{-29.3\ m/s^2}$

$\ddot y = 51.4 \cos 30° - 4.17 \sin 30° = \underline{42.5\ m/s^2}$

2/242 Carrier deck has a constant velocity, so may be used as an inertial coordinate base.
Velocity of aircraft relative to carrier is

$v_{A/C}^2 = 2as = 2(50)100 = 10\,000\ (m/s)^2$, $v_{A/C} = 100$ m/s

$\underline{v}_A = \underline{v}_C + \underline{v}_{A/C}$

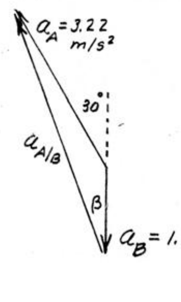


$v_C = 30(1.852)/3.6 = 15.43$ m/s

$v_A^2 = (100)^2 + (15.43)^2 + 2(100)(15.43)\cos 15° = 13\,220\ (m/s)^2$

$v_A = 115.0$ m/s or $v_A = v = 115.0(3.6) = \underline{414 \text{ km/h}}$

2/243 $\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $a_A = v_A^2/\rho = (50/3.6)^2/60 = 3.22$ m/s²



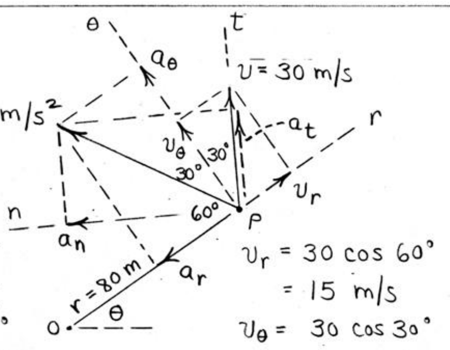
$a_{A/B} = \sqrt{(3.22\frac{\sqrt3}{2} + 1.5)^2 + (3.22[1/2])^2}$

$= \underline{4.58\ m/s^2}$

$\beta = \tan^{-1}\frac{1.608}{4.28} = \tan^{-1}0.3752$

$= \underline{20.6° \text{ west of north}}$

2/244



$a_r = -8\cos 60° = -4$ m/s²

$a_\theta = 8\cos 30° = 6.93$ m/s²

$v_r = 30\cos 60° = 15$ m/s

$v_\theta = 30\cos 30° = 26.0$ m/s

[r-θ] $v_r = \dot r = 15$ m/s

$v_\theta = r\dot\theta$: $26.0 = 80\dot\theta$, $\underline{\dot\theta = 0.325 \text{ rad/s}}$

$a_r = \ddot r - r\dot\theta^2$: $-4 = \ddot r - 80(0.325)^2$, $\underline{\ddot r = 4.44 \text{ m/s}^2}$

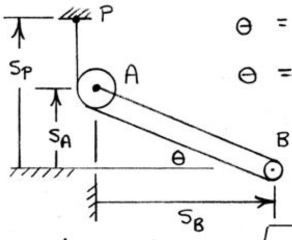
$a_\theta = r\ddot\theta + 2\dot r\dot\theta$: $6.93 = 80\ddot\theta + 2(15)(0.325)$, $\underline{\ddot\theta = -0.0352 \text{ rad/s}^2}$

[n-t] : $a_n = 8\cos 30° = \underline{6.93 \text{ m/s}^2}$

$a_t = 8\cos 60° = \underline{4 \text{ m/s}^2}$

$a_n = \dfrac{v^2}{\rho}$, $\rho = \dfrac{v^2}{a_n} = \dfrac{30^2}{6.93} = \underline{129.9 \text{ m}}$

2/245



$\theta = 45°$: $s_B/s_A = 1$

$\theta = 30°$: $s_B/s_A = \sqrt3$

$\theta = 15°$: $s_B/s_A = 3.73$

$L = (s_P - s_A) + 2\sqrt{s_A^2 + s_B^2}$

$0 = -v_A + 2(s_A v_A + s_B v_B)\Big/\sqrt{s_A^2 + s_B^2}$

$\Rightarrow v_B = \left[\sqrt{1 + (s_B/s_A)^2} - 2\right]\dfrac{v_A}{2(s_B/s_A)}$

$\theta = 45°$: $v_B = -0.293 v_A = -0.293(-1) = \underline{0.293 \frac{\text{m}}{\text{s}}}$

$\theta = 30°$: $v_B = \underline{0}$

$\theta = 15°$: $v_B = 0.250 v_A = 0.250(-1) = \underline{-0.250 \frac{\text{m}}{\text{s}}}$

2/246

$\begin{cases} x = 50 \text{ ft}, & \dot x = -10 \text{ ft/sec}, & \ddot x = -10 \text{ ft/sec}^2 \\ y = 25 \text{ ft}, & \dot y = 10 \text{ ft/sec}, & \ddot y = 5 \text{ ft/sec}^2 \end{cases}$

$v = \sqrt{\dot x^2 + \dot y^2} = \sqrt{(-10)^2 + 10^2} = \underline{10\sqrt2 \text{ ft/sec}}$

$a = \sqrt{\ddot x^2 + \ddot y^2} = \sqrt{(-10)^2 + 5^2} = \underline{11.18 \text{ ft/sec}^2}$

$\mathbf{e}_t = \mathbf{v}/v = (-10\mathbf{i} + 10\mathbf{j})/10\sqrt2 = \underline{\frac{\sqrt2}{2}(-\mathbf{i} + \mathbf{j})}$

$a_t = \mathbf{a}\cdot\mathbf{e}_t = (-10\mathbf{i} + 5\mathbf{j})\cdot\frac{\sqrt2}{2}(-\mathbf{i} + \mathbf{j}) = \underline{10.61 \text{ ft/sec}^2}$

$\mathbf{a}_t = a_t\mathbf{e}_t = 10.61\,\frac{\sqrt2}{2}(-\mathbf{i} + \mathbf{j}) = \underline{-7.5\mathbf{i} + 7.5\mathbf{j} \text{ ft/sec}^2}$

$\mathbf{a}_n = \mathbf{a} - \mathbf{a}_t = (-10\mathbf{i} + 5\mathbf{j}) - (-7.5\mathbf{i} + 7.5\mathbf{j}) = \underline{-2.5(\mathbf{i} + \mathbf{j}) \text{ ft/sec}^2}$

$a_n = \sqrt{2.5^2 + 2.5^2} = \underline{3.54 \text{ ft/sec}^2}$

$\rho = v^2/a_n = (10\sqrt2)^2/3.54 = \underline{56.6 \text{ ft}}$

$\mathbf{e}_n = \mathbf{a}_n/a_n = -2.5(\mathbf{i} + \mathbf{j})/3.54 = \underline{-\frac{\sqrt2}{2}(\mathbf{i} + \mathbf{j})}$

$\mathbf{e}_r = \mathbf{r}/r = 50\mathbf{i} + 25\mathbf{j}\Big/\sqrt{50^2 + 25^2} = \underline{0.894\mathbf{i} + 0.447\mathbf{j}}$

$\mathbf{e}_\theta = \mathbf{e}_r$ rotated CCW 90° $= \underline{-0.447\mathbf{i} + 0.894\mathbf{j}}$

$v_r = \mathbf{v}\cdot\mathbf{e}_r = (-10\mathbf{i} + 10\mathbf{j})\cdot(0.894\mathbf{i} + 0.447\mathbf{j}) = \underline{-4.47 \text{ ft/sec}}$

$\mathbf{v}_r = v_r\mathbf{e}_r = -4.47(0.894\mathbf{i} + 0.447\mathbf{j}) = \underline{-4\mathbf{i} - 2\mathbf{j} \text{ ft/sec}}$

$v_\theta = \mathbf{v}\cdot\mathbf{e}_\theta = (-10\mathbf{i} + 10\mathbf{j})\cdot(-0.447\mathbf{i} + 0.894\mathbf{j}) = \underline{13.42 \text{ ft/sec}}$

$\mathbf{v}_\theta = v_\theta\mathbf{e}_\theta = 13.42(-0.447\mathbf{i} + 0.894\mathbf{j}) = \underline{-6\mathbf{i} + 12\mathbf{j} \text{ ft/sec}}$

$a_r = \mathbf{a}\cdot\mathbf{e}_r = (-10\mathbf{i} + 5\mathbf{j})\cdot(0.894\mathbf{i} + 0.447\mathbf{j}) = \underline{-6.71 \text{ ft/sec}^2}$

$\mathbf{a}_r = a_r\mathbf{e}_r = -6.71(0.894\mathbf{i} + 0.447\mathbf{j}) = \underline{-6\mathbf{i} - 3\mathbf{j} \text{ ft/sec}^2}$

$a_\theta = \mathbf{a}\cdot\mathbf{e}_\theta = (-10\mathbf{i} + 5\mathbf{j})\cdot(-0.447\mathbf{i} + 0.894\mathbf{j}) = \underline{8.94 \text{ ft/sec}^2}$

$\mathbf{a}_\theta = a_\theta\mathbf{e}_\theta = 8.94(-0.447\mathbf{i} + 0.894\mathbf{j}) = \underline{-4\mathbf{i} + 8\mathbf{j} \text{ ft/sec}^2}$

$r = \sqrt{x^2 + y^2} = \sqrt{50^2 + 25^2} = \underline{55.9 \text{ ft}}$

$\dot r = v_r = \underline{-4.47 \text{ ft/sec}}$

$v_\theta = r\dot\theta$, $\dot\theta = v_\theta/r = 13.42/55.9 = \underline{0.240 \text{ rad/sec}}$

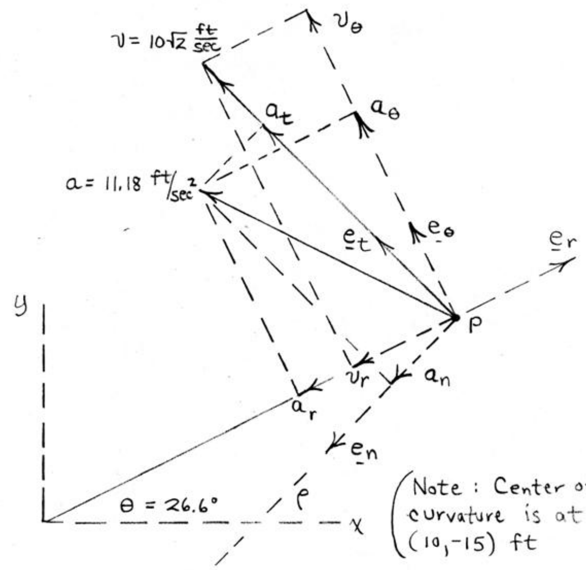
$a_r = \ddot r - r\dot\theta^2$, $\ddot r = a_r + r\dot\theta^2 = -6.71 + 55.9(0.240)^2$

$= \underline{-3.49 \text{ ft/sec}^2}$

$a_\theta = r\ddot\theta + 2\dot r\dot\theta$, $\ddot\theta = \frac{1}{r}(a_\theta - 2\dot r\dot\theta)$

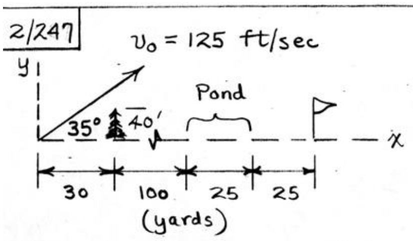
$= \frac{1}{55.9}\left[8.94 - 2(-4.47)(0.240)\right] = \underline{0.1984 \text{ rad/sec}^2}$

$\theta = \tan^{-1}(y/x) = \tan^{-1}(25/50) = \underline{26.6°}$



Note : Center of curvature is at (10, −15) ft

2/247 $v_0 = 125$ ft/sec



Time to tree: $x = x_0 + v_{x_0}t$: $90 = 0 + 125\cos 35° t$

$t = 0.879$ sec

Altitude: $y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$

$y = 0 + 125\sin 35°(0.879) - 16.1(0.879)^2 = 50.6$ ft

So ball clears (slender) tree.

Flight time (y-eq.): $0 = 0 + 125\sin 35° t_f - 16.1 t_f^2$

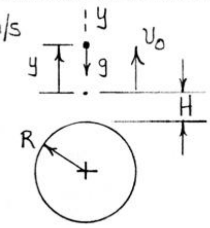
$t_f = 0$ (launch time) or $t = 4.45$ sec (impact time)

Range (x-eq.): $R = 0 + 125\cos 35°(4.45)$

$= 456$ ft or $\underline{152.0 \text{ yd}}$

Ball lands in water hazard!

2/248 $v_0 = 27000/3.6 = 7500$ m/s



$H = 35(10^4)$ m

$g_0 = 9.832$ m/s^2 (Fig. 1/1)

$R = 6371$ km or $6.371(10^6)$ m

$g = g_0\left(\frac{R}{R+H+y}\right)^2$

From $a = v\frac{dv}{dy}$, we have

$$\int_{v_0}^{0} v\,dv = \int_0^h -g_0\left(\frac{R}{R+H+y}\right)^2 dy$$

$$-\frac{1}{2}v_0^2 = g_0R^2 \frac{1}{R+H+y}\Big|_0^h$$

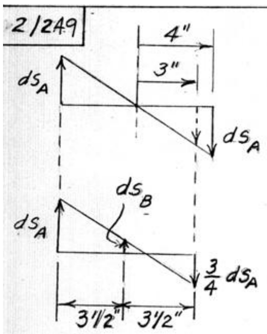
$$v_0^2 = 2g_0R^2\frac{h}{(R+H)(R+H+h)}$$

Solve for h: $h = \dfrac{(R+H)^2v_0^2}{2g_0R^2 - (R+H)v_0^2}$

Substitute numerical values and obtain

$h = 6048(10^3)$ m or $\underline{h = 6048 \text{ km}}$

2/249



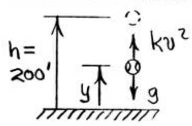
ds_B = average of ds_A and $-\frac{3}{4}ds_A$

$= \dfrac{ds_A + (-\frac{3}{4}ds_A)}{2} = \frac{1}{8}ds_A$

so $a_B = \frac{1}{8}a_A$

$= \frac{2}{8} = \underline{0.25 \text{ ft/sec}^2}$

***2/250** $a = v\dfrac{dv}{dy} = -g + kv^2$



$$\int_0^v \frac{v\,dv}{-g+kv^2} = \int_h^y dy$$

$$\frac{1}{2k}\ln\left[-g+kv^2\right]_0^v = y\Big|_h^y$$

$$\frac{1}{2k}\ln\left[\frac{-g+kv^2}{-g}\right] = y - h \Rightarrow v = \sqrt{\frac{g}{k}\left[1-e^{2k(y-h)}\right]}$$

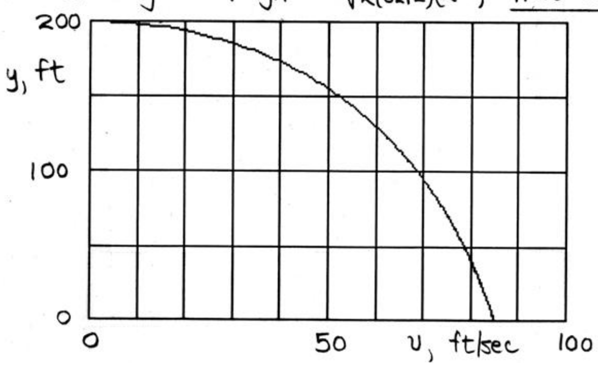
Given numbers: $85 = \sqrt{\dfrac{32.2}{k}\left[1-e^{2k(0-200)}\right]}$

Numerical solution: $\underline{k = 0.00323 \text{ ft}^{-1}}$

Terminal speed: $g = kv^2$: $32.2 = 0.00323v_t^2$

$\underline{v_t = 99.8 \text{ ft/sec}}$

Without drag: $v' = \sqrt{2gh} = \sqrt{2(32.2)(200)} = \underline{113.5 \text{ ft/sec}}$



***2/251** $a_t = \dfrac{dv}{dt} = g\cos\theta - \dfrac{k}{m}v$

With $v = r\dot{\theta}$: $\dfrac{d}{dt}(r\dot{\theta}) = g\cos\theta - \dfrac{k}{m}(r\dot{\theta})$

or $\dfrac{d^2\theta}{dt^2} + \dfrac{k}{m}\dfrac{d\theta}{dt} - \dfrac{g}{r}\cos\theta = 0$

This is a nonlinear, second-order differential equation, so a numerical integration is in order. To switch to first order form, we let

$x_1 = \theta$ & $x_2 = \dot{\theta}$

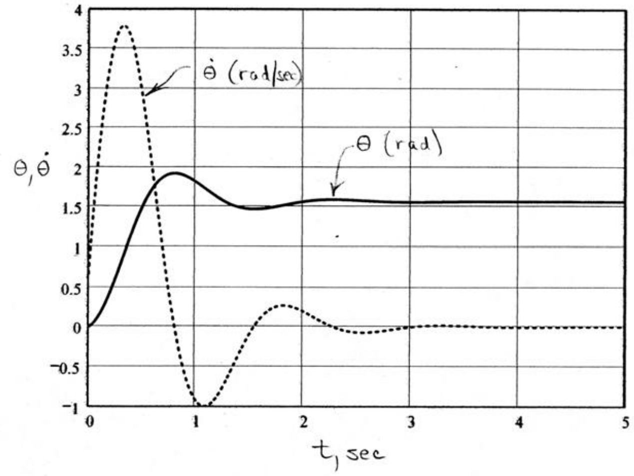
So $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{k}{m}x_2 + \frac{g}{r}\cos x_1 \end{cases}$ $\quad x_{10} = \theta_0 = 0$, $\quad x_{20} = \dot{\theta}_0 = \dfrac{v_0}{r}$

The plots below show θ and $\dot{\theta}$ as functions of t.

$\underline{\theta_{max} = 110.4° \text{ @ } t = 0.802 \text{ sec}}$

$\underline{\dot{\theta}_{max} = 3.79 \text{ rad/sec @ } t = 0.324 \text{ sec}}$

$\underline{\theta = 90° \text{ @ } t = 0.526 \text{ sec}}$



***2/252** $\ddot\theta = \dot\theta \dfrac{d\dot\theta}{d\theta} = \dfrac{g}{l}\cos\theta$

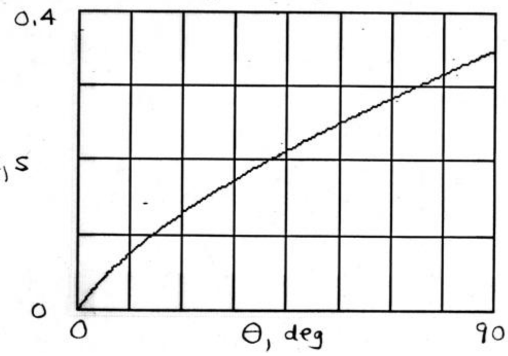
$$\int_{\dot\theta_0}^{\dot\theta} \dot\theta\, d\dot\theta = \frac{g}{l}\int_0^{\theta}\cos\theta\, d\theta$$

$$\dot\theta = \left[\dot\theta_0^{\,2} + \frac{2g}{l}\sin\theta\right]^{1/2}$$

Then $\dot\theta = \dfrac{d\theta}{dt} = \left[\dot\theta_0^{\,2} + \dfrac{2g}{l}\sin\theta\right]^{1/2}$

$$t = \int_0^{\theta} \frac{d\theta}{\sqrt{\dot\theta_0^{\,2} + \frac{2g}{l}\sin\theta}}$$

With $\dot\theta_0 = 2$ rad/s, $l = 0.6$ m, $g = 9.81$, & $\theta = \frac{\pi}{2}$, a numerical integration yields $\underline{t' = 0.349 \text{ s}}$.



***2/253** $v\,dv = a\,ds$, $a = \dfrac{T - 4.50v^2}{m}$

$$\int_0^v \frac{m\,v\,dv}{T - 4.50v^2} = \int_0^s ds\ ;\quad s = \frac{m}{9.00}\ln\frac{T}{T - 4.50v^2}$$

$$e^{9.00s/m} = \frac{T}{T - 4.50v^2},\quad v = \sqrt{\frac{T}{4.50}\left(1 - e^{-9.00s/m}\right)}$$

where $T = 250$ kN, s = distance in meters, $m = 16000$ tons (metric) (1 metric ton = 1000 kg)

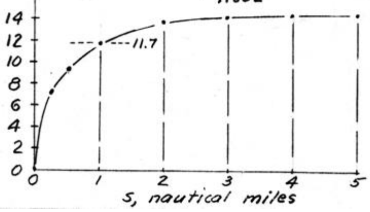
v = speed in m/s

or if v = speed in knots & s = distance in nautical miles, then

$$v = \sqrt{\frac{250}{4.50}\left(1 - e^{-9.00 \times 1852 s/16000}\right)}\ \frac{3.6}{1.852} = 14.49\sqrt{1 - e^{-1.042s}}$$

$\underline{v_{1\,mi} = 11.66 \text{ knots}}$

$\underline{v_{max} = 14.49 \text{ knots}}$



***2/254** Let $\omega_n = \sqrt{g/l}$: $\begin{cases}\theta = \theta_0 \sin\omega_n t\\ \dot\theta = \theta_0\omega_n\cos\omega_n t\\ \ddot\theta = -\theta_0\omega_n^2\sin\omega_n t\end{cases}$

$a_t = l\ddot\theta = -l\theta_0\omega_n^2\sin\omega_n t = -g\theta_0\sin\omega_n t$

$a_n = l\dot\theta^2 = l\theta_0^2\omega_n^2\cos^2\omega_n t = g\theta_0^2\cos^2\omega_n t$

$a = \sqrt{a_t^2 + a_n^2} = g\theta_0\sqrt{\sin^2\omega_n t + \theta_0^2\cos^4\omega_n t}$

a^2 (and therefore a) is an extreme when

$\dfrac{da^2}{d\theta} = 0 = g^2\theta_0^2\left[2\sin\omega_n t(\cos\omega_n t) + \theta_0^2\, 4\cos^3\omega_n t(-\sin\omega_n t)\right]$

$\Rightarrow \left[1 - 2\theta_0^2\cos^2\omega_n t\right] = 0$

$\left[1 - 2\left(\frac{\pi}{3}\right)^2\cos^2\omega_n t\right] = 0,\quad \omega_n t = 0.830$ rad

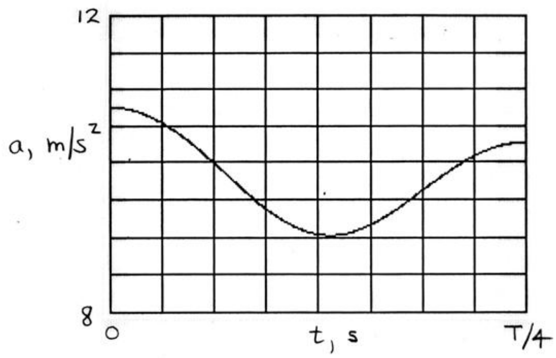
With $\omega_n = \sqrt{g/l} = \sqrt{\dfrac{9.81}{0.8}} = 3.50\ \text{s}^{-1}$, $t = 0.237$ s

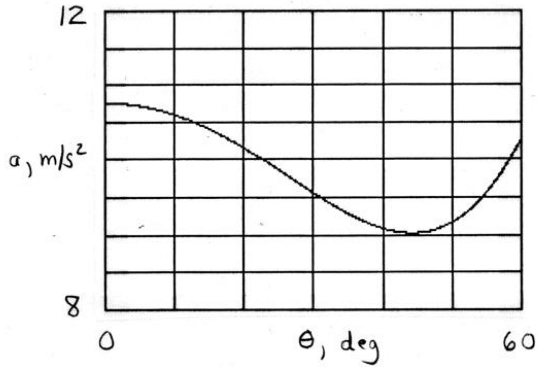
$\theta = \frac{\pi}{3}\sin(0.830) = 0.772$ rad $(44.3°)$

As can be seen from the plots below, the above represents a minimum :

$\underline{a_{min} = 9.03 \text{ m/s}^2 \text{ @ } \theta = 44.3° \text{ \& } t = 0.237 \text{ s}}$

$\underline{a_{max} = 10.76 \text{ m/s}^2 \text{ @ } \theta = 0 \text{ \& } t = 0}$





Note : Period $T = \dfrac{2\pi}{\omega_n} = \dfrac{2\pi}{3.50} = 1.794$ s

$\dfrac{T}{4} = 0.449$ s

***2/255**

$$a = \frac{dv}{dt} = c_1 - c_2 v^2$$

$$\int_0^t dt = \int_0^v \frac{dv}{c_1 - c_2 v^2} = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1}\sqrt{\frac{c_2}{c_1}}\, v \Big|_0^v$$

$$t = \frac{1}{\sqrt{c_1 c_2}} \tanh^{-1}\sqrt{\frac{c_2}{c_1}}\, v$$

Then $v = \frac{ds}{dt} = \sqrt{\frac{c_1}{c_2}} \tanh\sqrt{c_1 c_2}\, t$

$$\int_0^s ds = \sqrt{\frac{c_1}{c_2}} \int_0^t \tanh\sqrt{c_1 c_2}\, t\, dt$$

$$s = \sqrt{\frac{c_1}{c_2}} \cdot \frac{1}{\sqrt{c_1 c_2}} \ln\left(\cosh\sqrt{c_1 c_2}\, t\right)\Big|_0^t$$

$$= \frac{1}{c_2} \ln\left(\cosh\sqrt{c_1 c_2}\, t\right)$$

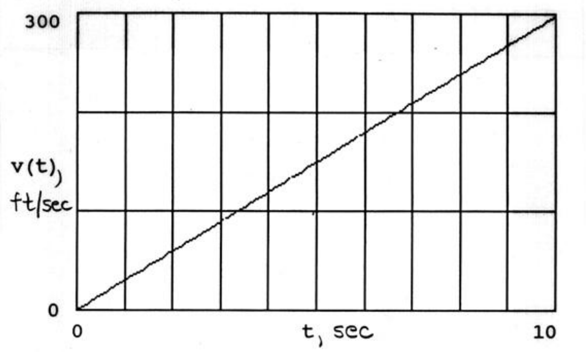
With $s = 1320$ ft, $c_1 = 30$ ft/sec^2, & $t = 9.4$ sec:

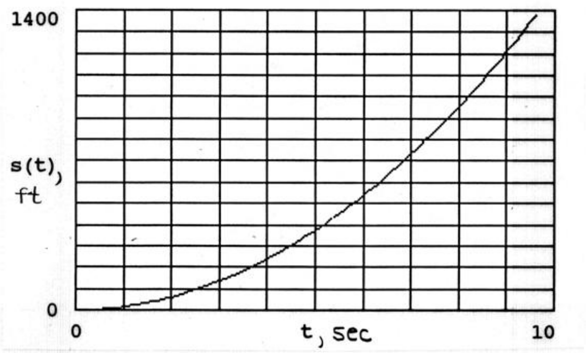
$$1320 = \frac{1}{c_2} \ln\left(\cosh\sqrt{30 c_2} \cdot 9.4\right)$$

$$= \frac{1}{c_2} \ln\left(\cosh 51.5\sqrt{c_2}\right)$$

Numerical solution: $\underline{c_2 = 9.28\,(10^{-6})\ \text{ft}^{-1}}$

(On plots below, note that v vs. t appears linear, but it is not!)





***2/256**

$$a = v\frac{dv}{dy} = -g - kv^2$$

$$\int_{v_0}^{0} \frac{v\,dv}{-g - kv^2} = \int_0^h dy$$

Let $u = g + kv^2$, $du = 2kv\,dv$: $\displaystyle\int_{v=v_0}^{v=0} -\frac{1}{2k}\frac{du}{u} = h$

$$h = -\frac{1}{2k} \ln\left(g + kv^2\right)\Big|_{v_0}^{0} = -\frac{1}{2k} \ln\left(\frac{g}{g + kv_0^2}\right)$$

$$2kh = \ln\left(1 + \frac{kv_0^2}{g}\right): \quad 2(5280)k = \ln\left(1 + \frac{2000^2 k}{32.2}\right)$$

Solve numerically to obtain $\underline{k = 3.63\,(10^{-4})\ \text{ft}^{-1}}$

***2/257** $y = x^2/4$, x & y in inches ; $x = 4\sin 2t$, t in seconds

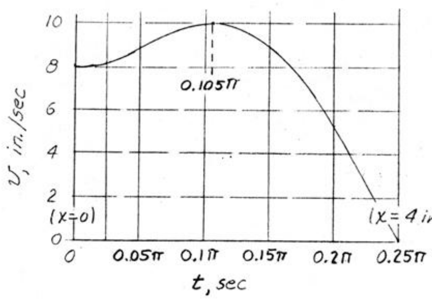
$\dot{y} = \frac{x\dot{x}}{2} = 2\sin 2t\,(8\cos 2t) = 16\sin 2t\cos 2t$ in./sec

$\dot{x} = 8\cos 2t$

$v^2 = \dot{x}^2 + \dot{y}^2 = 64\cos^2 2t + 256\sin^2 2t\cos^2 2t$

$= 64\cos^2 2t\,(1 + 4\sin^2 2t)$ (in./sec)^2

$v = 8\cos 2t\sqrt{1 + 4\sin^2 2t}$



$\frac{dv}{dt} = 0$ gives

$1 - 2\sin^2 2t = 1/4$

$\sin 2t = \sqrt{3/8}$

$\cos 2t = \sqrt{5/8}$

$v = 8\sqrt{\frac{5}{8}}\sqrt{1 + 4(3/8)}$

$= \underline{10\ \text{in./sec}}$

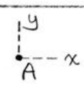
$2t = 0.659$ rad

$t = \underline{0.330\ \text{sec}}$

$(= 0.105\pi$ sec$)$

@ $\underline{x = 2.45\ \text{in.}}$

***2/258** Set up x-y coordinates @ A :



Let coordinates of B be (R, h).

$x = x_0 + v_{x_0}t$ @ B : $\quad R = 0 + (v_0\cos\alpha)t_f \quad (1)$

$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$ @ B : $\quad h = 0 + (v_0\sin\alpha)t_f - \frac{1}{2}gt_f^2 \quad (2)$

Solve (1) & (2) for R & t_f :

$$\left.\begin{aligned} R &= \frac{v_0\cos\alpha}{g}\left[v_0\sin\alpha + \sqrt{v_0^2\sin^2\alpha - 2gh}\right] \\ t_f &= \frac{1}{g}\left[v_0\sin\alpha + \sqrt{v_0^2\sin^2\alpha - 2gh}\right] \end{aligned}\right\}$$

where the + sign has been taken to maximize R.

Set $\frac{dR}{d\alpha} = 0$ (done by computer) to find

$\underline{\alpha = 48.5°}$ (for $h = 10$ m, $v_0 = 30$ m/s, and $g = 9.81$ m/s^2)

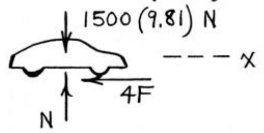
The corresponding value of R is

$\underline{R = 81.1\ \text{m}}$

So the 10-m plateau is indeed achieved, as assumed (because $R > 50$ m).

3/1 $v_2^2 - v_1^2 = 2a(x_2 - x_1)$

$0^2 - \left(\frac{100}{3.6}\right)^2 = 2a_x(50)$, $a_x = -7.72 \text{ m/s}^2$

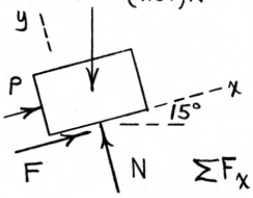


$\Sigma F_x = ma_x: \ -4F = 1500(-7.72)$

$\underline{F = 2890 \text{ N}}$

3/2 50(9.81)N $\quad \Sigma F_y = 0: \ N - 50(9.81)\cos 15° = 0$

$N = 474 \text{ N}$ throughout



(a) $P = 0$

Equilibrium check:

$\Sigma F_x = 0: \ F - 50(9.81)\sin 15° = 0$

$F = 127.0 \text{ N}$

$F_{max} = \mu_s N = 0.2(474) = 94.8 \text{ N} < F$: motion ↙

$\Sigma F_x = ma_x: \ 0.15(474) - 50(9.81)\sin 15° = 50a_x$

$\underline{a_x = -1.118 \text{ m/s}^2}$

(b) $P = 150$ N ; Equilibrium check:

$\Sigma F_x = 0: \ 150 + F - 50(9.81)\sin 15° = 0$

$F = -23.0$ N , $|F| < F_{max}$ so no motion : $\underline{a = 0}$

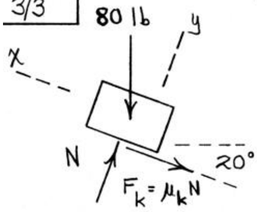
(c) $P = 300$ N ; Equilibrium check yields $F = -173.0$ N

$|F| > F_{max}$, so motion ↗ , $F = F_k$ ↙ .

$\Sigma F_x = ma_x: \ 300 - 0.15(474) - 50(9.81)\sin 15° = 50a_x$

$\underline{a_x = 2.04 \text{ m/s}^2}$

3/3 80 lb



$\Sigma F_y = 0: \ N - 80\cos 20° = 0$

$N = 75.2 \text{ lb}$

$\Sigma F_x = ma_x:$

$-0.25(75.2) - 80\sin 20° = \frac{80}{32.2}a$

$a = -18.58 \text{ ft/sec}^2$

$v = v_0 + at: \ 0 = +30 - 18.58t$, $\underline{t = 1.615 \text{ sec}}$

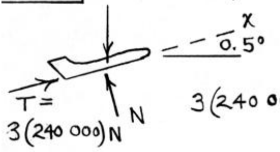
$v^2 = v_0^2 + 2a(s - s_0): \ 0^2 = 30^2 + 2(-18.58)d$

$\underline{d = 24.2 \text{ ft}}$

$v^2 = v_0^2 + 2a(s - s_0): \ 15^2 = 30^2 + 2(-18.58)d'$

$\underline{d' = 18.17 \text{ ft}}$

3/4 300 000 (9.81) N



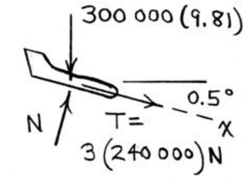
$\Sigma F_x = ma_x:$

$3(240\,000) - 300\,000(9.81)\sin\frac{1}{2}° = 300\,000\,a_x$

$a_x = 2.31 \text{ m/s}^2$

$v^2 = 2a_x s: \ (220/3.6)^2 = 2(2.31)s$, $\underline{s_u = 807 \text{ m}}$

300 000 (9.81) N



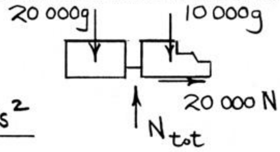
$\Sigma F_x = ma_x:$

$3(240\,000) + 300\,000(9.81)\sin\frac{1}{2}° = 300\,000\,a_x$

$a_x = 2.49 \text{ m/s}^2$

$v^2 = 2a_x s: \ (220/3.6)^2 = 2(2.49)s$, $\underline{s_d = 751 \text{ m}}$

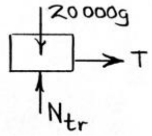
3/5 For entire unit:



$\Sigma F = ma: \ 20\,000 = 30\,000a$

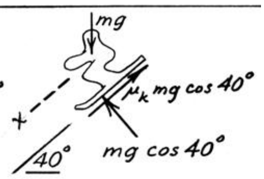
$\underline{a = 0.667 \text{ m/s}^2}$

For trailer alone, $T = 20\,000(0.667) = 13\,330$ N



or $\underline{T = 13.33 \text{ kN}}$

3/6



$\Sigma F_x = ma_x: \ mg\sin 40° - \mu_k mg\cos 40° = ma$

$a = 9.81(\sin 40° - \mu_k\cos 40°)$

$= 6.31 - 7.51\mu_k$

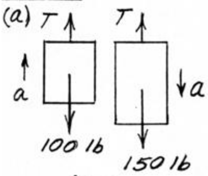
For constant accel. $s = v_0 t + \frac{1}{2}at^2$:

$20 = 0 + \frac{1}{2}(6.31 - 7.51\mu_k)2.58^2$

$\underline{\mu_k = 0.0395}$

3/7 $\Sigma F = ma; \quad T - 100 = \frac{100}{32.2}a$

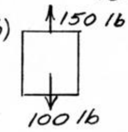
(a)



$150 - T = \frac{150}{32.2}a$

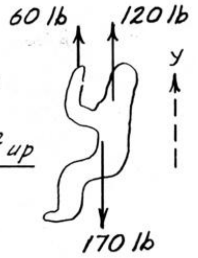
$50 = \frac{250}{32.2}a$, $\underline{a = \frac{32.2}{5} = 6.44 \frac{\text{ft}}{\text{sec}^2}}$

(b)



$150 - 100 = \frac{100}{32.2}a$, $\underline{a = \frac{32.2}{2} = 16.10 \frac{\text{ft}}{\text{sec}^2}}$

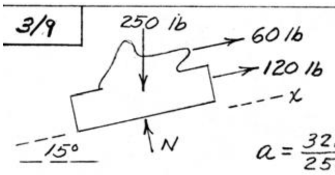
3/8



$\Sigma F_y = ma_y: \ 180 - 170 = \frac{170}{32.2} a$

$\underline{a = 1.894 \ ft/sec^2 \ up}$

3/9



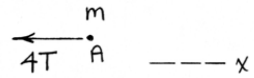
$\Sigma F_x = ma_x$

$60 + 120 - 250 \sin 15° = \frac{250}{32.2} a$

$a = \frac{32.2}{250}(180 - 64.7) = \underline{14.85 \ ft/sec^2}$

3/10

$\overset{+}{\leftarrow} \ \Sigma F = ma: \ 4(40,000) = \frac{750,000}{32.2} a$



$a = 6.87 \ ft/sec^2$

$\underline{a}_{A/B} = \underline{a}_A - \underline{a}_B = -6.87\underline{i} - \underline{0} = \underline{-6.87\underline{i} \ ft/sec^2}$

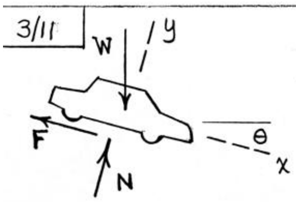
$v_A = (v_A)_0 + at = 0 + 6.87(10) = 68.7 \ ft/sec$

$v_B = 15\left(\frac{88}{60}\right) = 22 \ ft/sec$

$\underline{v}_{A/B} = \underline{v}_A - \underline{v}_B = -68.7\underline{i} - 22(\cos 30°\underline{i} + \sin 30°\underline{j})$

$= \underline{-87.7\underline{i} - 11\underline{j} \ ft/sec}$

3/11



When $\theta = \theta_1$, $a = 0$:

$\Sigma F_x = 0 = -F + W\sin\theta_1$

$F = W\sin\theta_1$

When $\theta = \theta_2$,

$\Sigma F_x = ma: \ W\sin\theta_2 - W\sin\theta_1 = \frac{W}{g} a$

$\underline{a = g(\sin\theta_2 - \sin\theta_1)}$

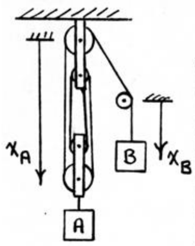
$\left.\begin{matrix}\theta_1 = 6° \\ \theta_2 = 2°\end{matrix}\right\} \ a = g(\sin 2° - \sin 6°) = \underline{-0.0696 g}$

$\underline{(-2.24 \ ft/sec^2 \ \text{or} \ -0.683 \ m/s^2)}$

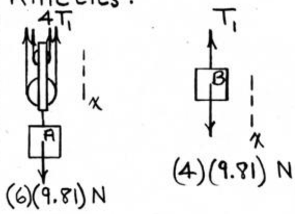
3/12

Kinematics: $4x_A + x_B = L_{rope} + \text{constant}$

$\therefore \ 4a_A + a_B = 0 \quad (1)$



Kinetics:



$A: \ \Sigma F_x = ma_x: \ 6(9.81) - 4T_1 = 6a_A \quad (2)$

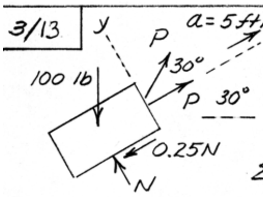
$B: \ \Sigma F_x = ma_x: \ 4(9.81) - T_1 = 4a_B \quad (3)$

Solution of Eqs. (1)-(3): $\left\{\begin{matrix} a_A = -1.401 \ m/s^2 \\ a_B = 5.61 \ m/s^2 \\ \underline{T_1 = 16.82 \ N} \end{matrix}\right.$

Tension in cable above A

$T_2 = 4T_1 = \underline{67.3 \ N}$

3/13



$\Sigma F_x = ma_x;$

$P(1 + \cos 30°) - 0.25N$

$-100\sin 30° = \frac{100}{32.2}(5)$

$\Sigma F_y = 0; \ N + P\sin 30° - 100\cos 30° = 0$

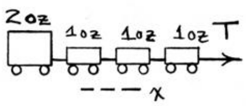
$\left.\begin{matrix} 1.866P - 0.25N = 65.53 \\ 0.5P + N = 86.6 \end{matrix}\right\}$ solve simultaneously & get $N = 64.7$ lb

$\underline{P = 43.8 \ lb}$

3/14

Coupler 1 will fail first, because it must accelerate more mass than any other coupler.

Rear part of train:

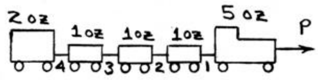


$\Sigma F_x = ma_x$

$T = 0.2 = \left(\frac{5/16}{32.2}\right) a$

$a = 20.6 \ ft/sec^2$

Whole train:

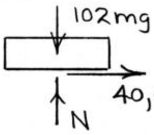


$\Sigma F_x = ma_x$

$P = \left(\frac{10/16}{32.2}\right)(20.6)$

$\underline{P = 0.4 \ lb}$

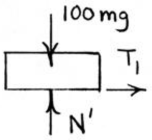
3/15 Let m be the mass of each car and $2m$ that of the locomotive.



$\xrightarrow{+} \Sigma F = ma$:

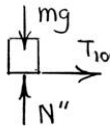
$$40{,}000 = \frac{102(200{,}000)}{32.2} a$$

$$a = 0.0631 \text{ ft/sec}^2$$



$$\xrightarrow{+} \Sigma F = ma: \; T_1 = \frac{100(200{,}000)}{32.2} 0.0631$$

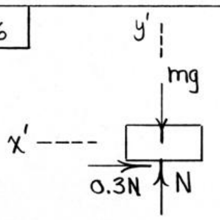
$$\underline{T_1 = 39{,}200 \text{ lb}}$$

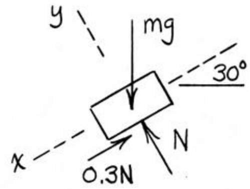


$$\xrightarrow{+} \Sigma F = ma: \; T_{100} = \frac{1(200{,}000)}{32.2} 0.0631$$

$$\underline{T_{100} = 392 \text{ lb}}$$

3/16





A to B:

$\Sigma F_y = 0 \Rightarrow \quad N = 0.866 mg$

$\Sigma F_x = ma_x: \quad mg \sin 30° - 0.3(0.866 mg) = ma$

$a_x = 2.36 \text{ m/s}^2$

$v_B^2 = v_A^2 + 2a_x d: \quad v_B^2 = 0.8^2 + 2(2.36)(2)$

$v_B = 3.17 \text{ m/s}$

B to C:

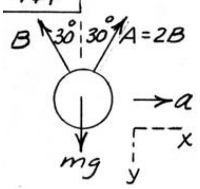
$\Sigma F_{y'} = 0 \Rightarrow \quad N = mg$

$\Sigma F_{x'} = ma_{x'}: \quad -0.3(mg) = ma_{x'}, \quad a_{x'} = -2.94 \text{ m/s}^2$

$v_C^2 = v_B^2 + 2a_{x'} s: \quad 0 = 3.17^2 - 2(2.94)s$

$\underline{s = 1.710 \text{ m}}$

3/17

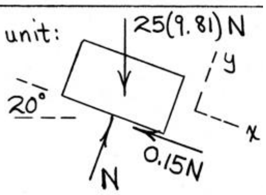


$\Sigma F_x = ma_x; \; 2B \sin 30° - B \sin 30° = ma$

$\Sigma F_y = 0; \; 2B \cos 30° + B \cos 30° - mg = 0$

Eliminate B & set $\underline{a = g/3\sqrt{3}}$

3/18 Frame & sphere as a unit:



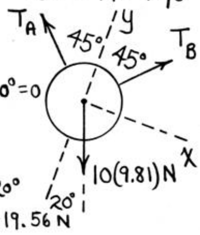
$\Sigma F_y = 0: \; N - 25(9.81)\cos 20° = 0$

$N = 230$ N

$\Sigma F_y = ma_x$:

$25(9.81)\sin 20° - 0.15(230) = 25a, \quad a = 1.973 \text{ m/s}^2$

Sphere alone:



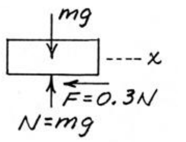
$\Sigma F_y = 0: \; (T_A + T_B)\cos 45° - 10(9.81)\cos 20° = 0$

$T_A + T_B = 130.4$ N

$\Sigma F_x = ma_x: \; (T_B - T_A)\sin 45° + 98.1 \sin 20° = 10(1.973)$, or $T_B - T_A = -19.56$ N

Solution: $\underline{T_A = 75.0 \text{ N}}$, $\underline{T_B = 55.4 \text{ N}}$

3/19 Let m = mass of crate



$\Sigma F_x = ma_x; \; -0.3mg = ma_x$

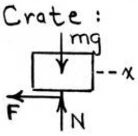
$a_x = -0.3g = -0.3(9.81) = -2.94 \text{ m/s}^2$

$\int_v^0 v\,dv = \int_0^s a_x\,dx; \; -\frac{v^2}{2} = a_x s$

$s = \frac{-(70/3.6)^2/2}{-2.94} = \underline{64.3 \text{ m}}$

3/20 Truck: $\begin{cases} v^2 - v_0^2 = 2a_T(x - x_0) \\ 0^2 - (19.44)^2 = 2a_T(50-0) \\ a_T = -3.78 \text{ m/s}^2 \end{cases}$

Crate:



$\Sigma F_x = ma_x: \; -F = m(-3.78)$

$F = 3.78 m$

$F_{max} = \mu_s N = 0.3(m\,9.81) = 2.94 m$

$F > F_{max}$, crate slips, $F = \mu_k N$

$\therefore \; \Sigma F_x = ma_x: \; -0.25 mg = ma_c, \; a_c = -2.45 \text{ m/s}^2$

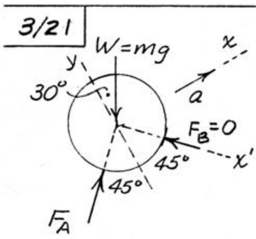
$a_{c/T} = a_c - a_T = -2.45 - (-3.78) = 1.328 \text{ m/s}^2$

$v_{c/T}^2 - v_{c/T_0}^2 = 2a_{c/T}(x_{c/T} - x_{c/T_0})$

$v_{c/T}^2 - 0^2 = 2(1.328)(3-0), \quad \underline{v_{c/T} = 2.82 \text{ m/s}}$

(Truck stopping time = 5.14 s, crate impacts at 2.13 s)

3/21



$\Sigma F_{x'} = ma_{x'};$

$mg \cos(45° + 30°) = ma \cos 45°$

$a = g \frac{\cos 75°}{\cos 45°} = 9.81 \frac{0.2588}{0.7071}$

$= \underline{0.366g}$

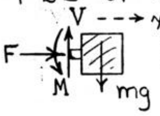
3/22

$x = X \sin \omega t$

$\dot{x} = X \omega \cos \omega t$

$\ddot{x} = -X \omega^2 \sin \omega t$, $\ddot{x}_{max} = X \omega^2$

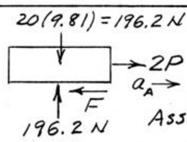
FBD of circuit board:



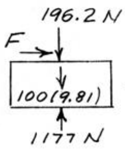
$\Sigma F_x = ma_x$: $F = m(-X\omega^2 \sin \omega t)$

$\underline{F_{MAX} = m X \omega^2}$

3/23



$\mu = 0.5$



(a) $2P = 120$ N

$F_{max} = 0.5(196.2) = 98.1 \text{ N} < 2P$

Assume slipping occurs & $F = 98.1$ N

A; $\Sigma F = ma$; $120 - 98.1 = 20\, a_A$

$\underline{a_A = 1.095 \text{ m/s}^2}$

B; $\Sigma F = ma$; $98.1 = 100\, a_B$

$a_A > a_B$ so assumption OK. $\underline{a_B = 0.981 \text{ m/s}^2}$

(b) $2P = 80 \text{ N} < F_{max}$ so no slipping occurs & for block & cart combined,

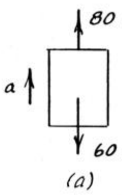
$\Sigma F = ma$; $80 = 120a$, $\underline{a_A = a_B = a = 0.667 \text{ m/s}^2}$

3/24

(a) $\Sigma F = ma$:

$80 - 60 = \frac{60}{32.2} a$,

$\underline{a = 10.73 \text{ ft/sec}^2}$



(a)

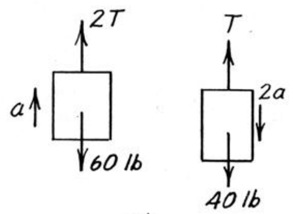
(b) $\Sigma F = ma$:

[60-lb] $2T - 60 = \frac{60}{32.2} a$

[40-lb] $40 - T = \frac{40}{32.2}(2a)$

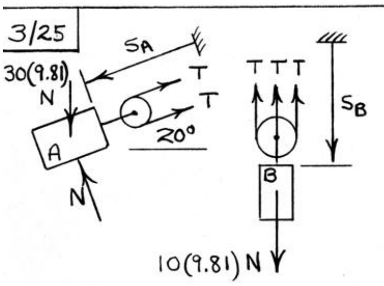
Solve & get $T = 32.7$ lb

$\underline{a = 2.93 \text{ ft/sec}^2}$



(b)

3/25



Kinematic constraint: $L = 2s_A + 3s_B$

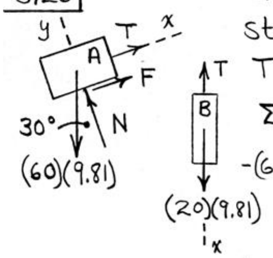
$\Rightarrow \quad 0 = 2a_A + 3a_B \quad (1)$

$\overset{+}{\swarrow}$ $\Sigma F = m_A a_A$: $30(9.81)\sin 20° - 2T = 30 a_A \quad (2)$

$+\downarrow$ $\Sigma F = m_B a_B$: $10(9.81) - 3T = 10 a_B \quad (3)$

Solution of Eqs. (1)-(3): $\begin{cases} \underline{a_A = 1.024 \text{ m/s}^2} \\ \underline{a_B = -0.682 \text{ m/s}^2} \\ T = 35.0 \text{ N} \end{cases}$

3/26



Check for motion. Assume static equilibrium. From B, $T = 196.2$ N. Mass A:

$\Sigma F_x = 0$: $196.2 + F - (60)(9.81)\sin 30° = 0$, $F = 98.1$ N

$F_{MAX} = \mu_s N = (0.25)(60)(9.81) \times \cos 30° = 127.4 \text{ N}$ (a)

No motion for (a), so $\underline{a = 0}$, $\underline{T = 196.2 \text{ N}}$

$F_{MAX} = (0.15)(60)(9.81)\cos 30° = 76.5$ N, motion for (b) ($\swarrow$)

A: $\Sigma F_x = ma_x$: $T - (60)(9.81)\sin 30° + (0.1)(60)(9.81)\cos 30° = 60\,a$

B: $\Sigma F_y = ma_x$: $(20)(9.81) - T = 20\,a$

Solution : $\underline{a = -0.589 \text{ m/s}^2}$, $\underline{T = 208 \text{ N}}$

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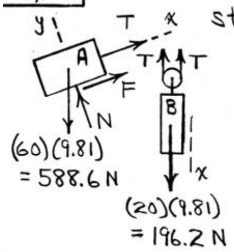


Check for motion by assuming static equilibrium.

B : $2T = 196.2$, $T = 98.1$ N

A : $\Sigma F_x = 0$: $98.1 - 588.6 \sin 30° + F = 0$, $F = 196.2$ N

$F_{max} = \mu_s N = (0.25)(588.6)\cos 30° = 127.4$ N

$F > F_{MAX} \Rightarrow$ motion ($\swarrow$)

From kinematics, $a_A = 2a_B = 2a$

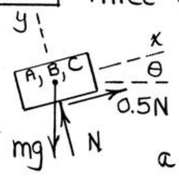
A : $\Sigma F_x = ma_x$: $T + 0.2(588.6 \cos 30°) - 588.6 \sin 30° = 60(2a)$

B : $\Sigma F_x = ma_x$: $-2T + 196.2 = 20\,a$

Solution : $\underline{a = -0.725 \text{ m/s}^2}$, $\underline{T = 105.4 \text{ N}}$

3/28

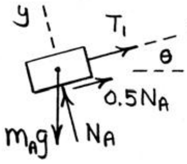
Three-car unit: $\left(\theta = \tan^{-1}\left(\frac{5}{100}\right) = 2.86°\right)$



$\begin{cases} \Sigma F_y = 0 \Rightarrow N = mg\cos\theta \\ \Sigma F_x = ma_x : 0.5mg\cos\theta - mg\sin\theta = ma \end{cases}$

$a = g(0.5\cos\theta - \sin\theta) = \underline{4.41 \text{ m/s}^2}$

Car A :



$\Sigma F_y = 0$: $N_A = m_A g \cos\theta$

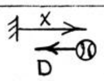
$\Sigma F_x = ma_x$: $T_1 + 0.5 m_A g \cos\theta - m_A g \sin\theta = m_A(4.41)$, $\underline{T_1 = 0}$

From FBD of Car C, $\underline{T_2 = 0}$

By similar analyses:

	(b)	(c)	(d)
a	2.78 m/s²	2.78 m/s²	2.78 m/s²
T_1	32 700 N (T)	16 330 N (C)	16 330 N (C)
T_2	16 330 N (T)	16 330 N (T)	32 700 N (C)

Chai Gr.C 92#

3/29  (Neglect weight for now)

$$\Sigma F_x = ma_x: \; -D = -C_D \tfrac{1}{2}\rho v^2 S = mv\frac{dv}{dx}$$

$$\int_0^x \left(-C_D \tfrac{1}{2}\rho S\right)dx = m\int_{v_0}^{v}\frac{dv}{v}$$

$$\Rightarrow \; v = v_0 e^{\left(-\frac{1}{2}C_D \rho S x/m\right)}$$

$$= v_0 e^{\left(-\frac{1}{2}(0.3)\left(\frac{0.07530}{32.2}\right)(\pi)\left(\frac{9.125/2\pi}{12}\right)^2 x/\frac{5.125}{16\cdot 32.2}\right)}$$

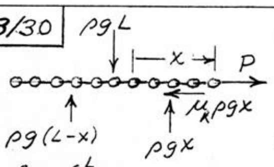
$$\underline{v = v_0 e^{-1.623(10^{-3})x}}$$

For $v_0 = 90$ mi/hr and $x = 60$ ft: $\underline{v = 81.7 \text{ mi/hr}}$

Comment on y-motion. Assume $v = 90$ mi/hr = constant. Time t to plate is

$$t = \frac{60}{90\left(5280/3600\right)} = 0.455 \text{ sec}$$

$v_y = v_{y_0} - gt = -32.2(0.455) = -14.64$ ft/sec, which would not appreciably change $v = \sqrt{v_x^2 + v_y^2}$.

3/30 

$$\Sigma F_x = ma_x; \quad P - \mu_k \rho g x = \rho L a_x$$

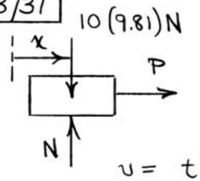
$$\int_0^v v\,dv = \int_0^L a_x\,dx$$

$$\frac{v^2}{2} = \int_0^L \left(\frac{P}{\rho L} - \frac{\mu_k g x}{L}\right)dx = \frac{P}{\rho} - \frac{\mu_k g L}{2}, \quad \underline{v = \sqrt{\frac{2P}{\rho} - \mu_k g L}}$$

From $\frac{v^2}{2} = \int_0^x \left(\frac{P}{\rho L} - \frac{\mu_k g x}{L}\right)dx$, we obtain

$$v(x) = \sqrt{2\frac{x}{L}\left(\frac{P}{\rho} - \mu_k g\frac{x}{2}\right)}$$

Note that $v(L) \geq 0$ if $\underline{P \geq \mu_k \rho g \frac{L}{2} = P_{min}}$

3/31 

$$\Sigma F_x = ma_x: \; P = 10a_x$$

$$\frac{P}{10} = \frac{dv}{dt}, \quad v = \int_0^t \frac{P}{10}\,dt$$

For $P_1 = 10t$:

$$v = t^2/2, \quad s = t^3/6$$

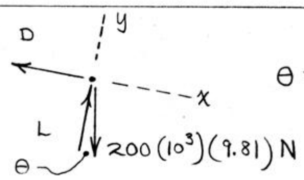
At $t = 5$ s, $\underline{v = 12.5 \text{ m/s}}$, $\underline{s = 20.8 \text{ m}}$

For $P_2 = kt^2$: $50 = k(5)^2$, $k = 2$ N/s²

So $P_2 = 2t^2$

$$v = \int_0^t \frac{2t^2}{10}dt = \frac{t^3}{15}, \quad s = t^4/60$$

At $t = 5$s, $\underline{v = 8.33 \text{ m/s}}$, $\underline{s = 10.42 \text{ m}}$

3/32 

$$\theta = \tan^{-1}\frac{250}{2000} = 7.13°$$

$$v_B^2 - v_A^2 = 2a_x(s_B - s_A):$$

$$\left(\frac{200}{3.6}\right)^2 - \left(\frac{300}{3.6}\right)^2 = 2a_x\left(\frac{2000}{\cos 7.13°}\right), \quad a_x = -0.957 \text{ m/s}^2$$

$$\Sigma F_x = ma_x: \; -D + 200(10^3)(9.81)\sin 7.13° = 200(10^3)(-0.957), \quad \underline{D = 435 \text{ kN}}$$

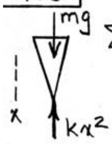
$$\Sigma F_y = 0: \; L - 200(10^3)(9.81)\cos 7.13° = 0$$

$$\underline{L = 1.947 \text{ MN}}$$

The net aerodynamic force is then

$$R = \sqrt{L^2 + D^2} = \sqrt{1.947^2 + 0.435^2} = \underline{1.995 \text{ MN}}$$

3/33 FBD of cone during penetration:

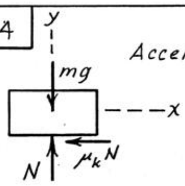


$$\Sigma F_x = ma_x: \; mg - kx^2 = mv\frac{dv}{dx}$$

$$\int_0^d \left(g - \frac{k}{m}x^2\right)dx = \int_{V_0}^0 V\,dV$$

$$gd - \frac{k}{3m}d^3 = -\frac{V_0^2}{2}, \quad \text{where } V_0 = \sqrt{2gh}$$

$$\therefore \; \underline{k = \frac{3mg}{d^3}(h+d)}$$

3/34 

Accel. down: $\Sigma F_y = ma_y: \; -mg + N = -ma$, $N = m(g-a)$

$\Sigma F_x = ma_x: \; -\mu_k m(g-a) = ma_x$, $a_x = -\mu_k(g-a)$

Accel. up: $\Sigma F_y = ma_y: \; N - mg = ma$, $N = m(g+a)$

$\Sigma F_x = ma_x: \; -\mu_k m(g+a) = ma_x$, $a_x = -\mu_k(g+a)$

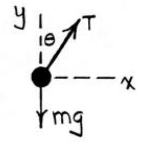
$v^2 = v_0^2 + 2as$: *Down:* $0 = v^2 - 2\mu_k(g-a)s_1$

Up: $0 = v^2 - 2\mu_k(g+a)s_2$

Eliminate v^2 *& get* $(g-a)s_1 = (g+a)s_2$,

$$\underline{a = g\frac{s_1 - s_2}{s_1 + s_2}}$$

3/35 Mass m :



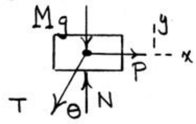
$\Sigma F_y = 0: \; T\cos\theta - mg = 0$

$T = mg/\cos\theta$

$\Sigma F_x = ma_x: \; T\sin\theta = ma$

$\left(\frac{mg}{\cos\theta}\right)\sin\theta = ma, \quad \underline{\theta = \tan^{-1}\left(\frac{a}{g}\right)}$

Cart M :



$\Sigma F_x = ma_x: \; P - T\sin\theta = Ma$

$P = ma + Ma = \underline{(m+M)a}$

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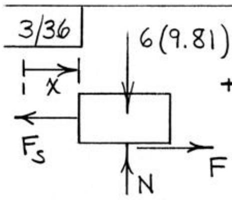

$F_s = 150x + 400x^2$ (N)

$+\uparrow \Sigma F = 0 \Rightarrow \; N = 58.9$ N

$F_{max} = \mu_s N = 0.30(58.9) = 17.66$ N

(a) $x = 50$ mm : $F_s = 150(0.050) + 400(0.050)^2 = 8.5 \text{ N} < F_{max}$

So $\underline{a = 0}$

(b) $F_s = 150(0.1) + 400(0.1)^2 = 19 \text{ N} > F_{max}$

$\Sigma F_x = ma_x: \; -19 + 0.25(58.9) = 6a$

$\underline{a = -0.714 \text{ m/s}^2}$

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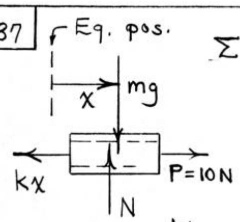

$\Sigma F_x = m\ddot{x}: \; P - kx = m\ddot{x}$

$10 - 200x = 2\ddot{x}$

$\ddot{x} = 5 - 100x$

$5 - 100x = v\frac{dv}{dx}$

$\int_0^{0.040}(5 - 100x)dx = \int_0^v v\,dv, \quad 5x - 50x^2\Big|_0^{0.040} = \frac{v^2}{2}\Big|_0^v$

$\underline{v = 0.490 \text{ m/s}}$

$5x - 50x^2 = 0$

$x = 0$ (initial condition) or $x = 0.10$ m or $\underline{100 \text{ mm}}$

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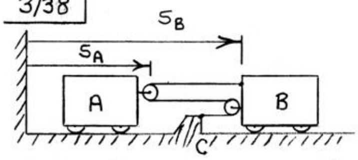
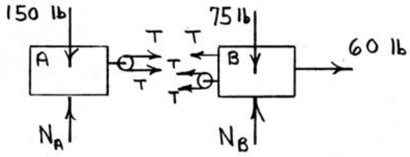


$L = 2(s_B - s_A) + (s_B - s_C) + \text{constants}$

$\Rightarrow 0 = 3a_B - 2a_A \qquad (1)$



$\xrightarrow{+} \Sigma F = ma:$ (A) $\; 2T = \frac{150}{32.2}a_A \qquad (2)$

(B) $\; 60 - 3T = \frac{75}{32.2}a_B \qquad (3)$

Solve Eqs. (1)−(3) : $\begin{cases} a_A = 7.03 \text{ ft/sec}^2 \\ a_B = 4.68 \text{ ft/sec}^2 \\ T = 16.36 \text{ lb} \end{cases}$

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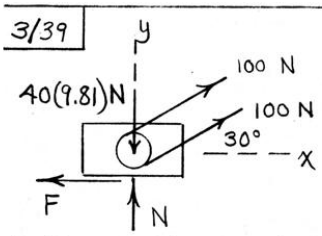


$\begin{cases} \mu_s = 0.5 \\ \mu_k = 0.4 \end{cases}$

$\Sigma F_y = 0: \; N + 200\sin 30° - 40(9.81) = 0$

$N = 292$ N

Assume static equilibrium :

$\Sigma F_x = 0: \; -F + 200\cos 30° = 0, \quad F = 173.2$ N

$F_{max} = \mu_s N = 0.5(292) = 146.2 \text{ N} < F$

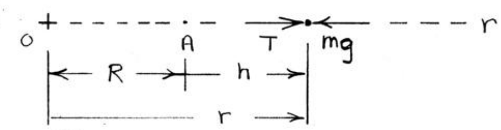
Assumption wrong, motion exists →.

$F = \mu_k N = 0.4(292) = 117.0$ N

$\Sigma F_x = ma_x: \; -117 + 200\cos 30° = 40a_x$

$\underline{a_x = a = 1.406 \text{ m/s}^2}$

3/40



$$g = g_0 \frac{R^2}{r^2} \quad \text{(all pertaining to moon)}$$

$$\Sigma F_r = ma_r: \; T - mg_0 \frac{R^2}{r^2} = mv \frac{dv}{dr}$$

$$\int_R^{2R} \left(\frac{T}{m} - g_0 \frac{R^2}{r^2} \right) dr = \int_0^v v\,dv$$

$$\Rightarrow \; v = \sqrt{\frac{2TR}{m} - g_0 R} = \sqrt{R\left(\frac{2T}{m} - g_0\right)}$$

Numbers :

$$v = \sqrt{\frac{3476(1000)}{2} \left(\frac{2(2500)}{1200} - 1.62 \right)}$$

$$= \underline{2100 \text{ m/s}}$$

3/41

$$\Sigma F_y = ma_y; \; mg - kv = ma$$

$$a = g - \frac{k}{m}v$$

$$v\,dv = a\,dy, \quad \int_0^v \frac{v\,dv}{g - \frac{k}{m}v} = \int_0^h dy$$

$$\frac{m^2}{k^2}\left[\left(g - \frac{k}{m}v\right) - g \ln\left(g - \frac{k}{m}v\right) \right]_0^v = h$$

$$h = \frac{m^2}{k^2}\left[-\frac{k}{m}v - g \ln\left(1 - \frac{kv}{mg}\right) \right]$$

$$h = \frac{m^2}{k^2}\, g \ln\left(\frac{1}{1 - \frac{kv}{mg}} \right) - \frac{mv}{k}$$

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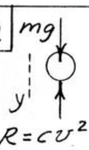

$$\Sigma F_y = ma_y; \; mg - cv^2 = ma$$

$$a = g - \frac{c}{m}v^2$$

$$v\,dv = a\,dy, \quad \int_0^v \frac{v\,dv}{g - \frac{c}{m}v^2} = \int_0^h dy$$

$$-\frac{m}{2c} \ln\left(g - \frac{c}{m}v^2\right)\Big]_0^v = h, \quad h = \frac{m}{2c} \ln\left(\frac{mg}{mg - cv^2} \right)$$

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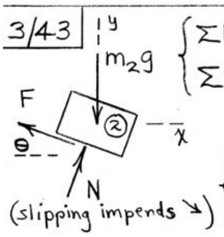


$$\begin{cases} \Sigma F_x = ma_x: \; -F\cos\theta + N\sin\theta = m_2 a \\ \Sigma F_y = 0: \; F\sin\theta + N\cos\theta - m_2 g = 0 \end{cases}$$

Solve to obtain

$$\begin{cases} F = m_2(g\sin\theta - a\cos\theta) \\ N = m_2(a\sin\theta + g\cos\theta) \end{cases}$$

For impending slip, $F = \mu_s N$, or

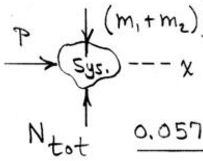
$$m_2(g\sin\theta - a\cos\theta) = \mu_s m_2(a\sin\theta + g\cos\theta)$$

Solving for a: $\quad a = g \dfrac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}$

With numbers, $a = 0.0577g$ (Note: $\tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70°$)

Let slipping impend up the inclined block (reverse F on above FBD) & obtain

$$a = g\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = 0.745g$$



$$\Sigma F_x = ma_x: \; P = (m_1 + m_2)a$$

So

$$\underline{0.0577(m_1+m_2)g \le P \le 0.745(m_1+m_2)g}$$

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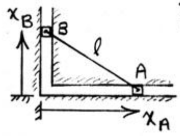
$$x_A^2 + x_B^2 = l^2$$



$$\cancel{2}x_A\dot{x}_A + \cancel{2}x_B\dot{x}_B = 0$$

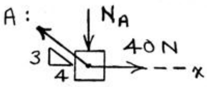
$$x_A\ddot{x}_A + \dot{x}_A^2 + x_B\ddot{x}_B + \dot{x}_B^2 = 0$$

So $\dot{x}_B = -\dfrac{x_A\dot{x}_A}{x_B} = \dfrac{-(0.4)(0.9)}{0.3} = -1.2$ m/s

$$\ddot{x}_B = \frac{-\dot{x}_B^2 - \dot{x}_A^2 - x_A\ddot{x}_A}{x_B} = \frac{-1.2^2 - 0.9^2 - 0.4\ddot{x}_A}{0.3}$$

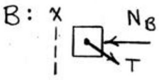
$$= -7.5 - \frac{4}{3}\ddot{x}_A \quad \text{or} \quad a_B = -7.5 - \frac{4}{3}a_A \quad (1)$$

A :



$$\Sigma F_x = ma_x: \; 40 - \frac{4}{5}T = 2a_A \quad (2)$$

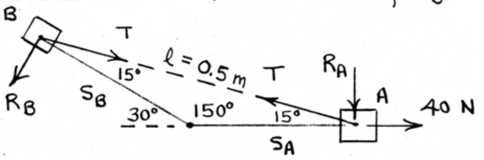
B :



$$\Sigma F_x = ma_x: \; -\frac{3}{5}T = 3a_B \quad (3)$$

Solution of Eqs. (1)-(3) : $\begin{cases} a_A = 1.364 \text{ m/s}^2 \\ a_B = -9.32 \text{ m/s}^2 \\ T = 46.6 \text{ N} \end{cases}$

3/45 $\sin 150^\circ/\ell = \sin 15^\circ/s_B$, $s_B = s_A = 0.259$ m



Law of cosines: $\ell^2 = s_A^2 + s_B^2 - 2 s_A s_B \cos 150^\circ$

$$2\ell\dot{\ell} = 0 = 2 s_A v_A + 2 s_B v_B - 2\left(-\frac{\sqrt{3}}{2}\right)(s_A v_B + s_B v_A)$$

$$s_A v_A + s_B v_B + \frac{\sqrt{3}}{2}(s_A v_B + v_A s_B) = 0^*$$

With $s_A = s_B = 0.259$ m, $v_A = 0.4$ m/s : $v_B = -0.4$ m/s

Differentiate $*$: $v_A^2 + s_A a_A + v_B^2 + s_B a_B + \frac{\sqrt{3}}{2}(s_A a_B + v_A v_B + a_A s_B + v_A v_B) = 0$

Numbers: $0.483\, a_A + 0.483\, a_B + 0.0429 = 0$ (1)

Kinetics:

$\nwarrow +$ $\Sigma F = m a_B$: $-T\cos 15^\circ = 3 a_B$ (2)

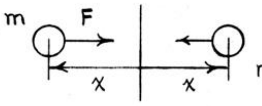
$\xrightarrow{+}$ $\Sigma F = m a_A$: $40 - T\cos 15^\circ = 2 a_A$ (3)

Solution of Eqs. (1)-(3): $T = 25.0$ N

$a_A = 7.95$ m/s^2

$a_B = -8.04$ m/s^2

▸3/46



$F = \frac{Gm^2}{x^2}$

$m = \rho V = 7210\left(\frac{4}{3}\pi\, 0.05^3\right) = 3.775$ kg

$$\Sigma F_x = m a_x : \; -\frac{Gm^2}{(2x)^2} = m v \frac{dv}{dx}$$

$$-\frac{Gm}{4}\int_{x_0=0.5}^{x}\frac{dx}{x^2} = \int_{v_0=0}^{v} v\,dv$$

$$v = \sqrt{Gm}\sqrt{\frac{1}{2x}-1} = \sqrt{(6.673\times10^{-11})(3.775)}\sqrt{\frac{1}{2(0.05)}-1}$$

$$= \underline{4.76\times10^{-5}\text{ m/s}}$$

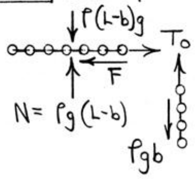
Now, $\frac{dx}{dt} = -\sqrt{Gm}\sqrt{\frac{\frac{1}{2}-x}{x}}$

$$\int_{x_0=0.5}^{x=0.05}\frac{\sqrt{x}\,dx}{\sqrt{\frac{1}{2}-x}} = -\sqrt{Gm}\int_0^t dt$$

$$\left[-\sqrt{x}\sqrt{\tfrac{1}{2}-x} + \tfrac{1}{2}\sin^{-1}\sqrt{2x}\right]_{x_0=0.5}^{x=0.05} = -\sqrt{Gm}\; t$$

Solving, $\underline{t = 48{,}800\text{ s}}$ or $\underline{t = 13\text{ hr } 33\text{ min}}$

▸3/47 Let ρ = mass/length. Length b to get started:

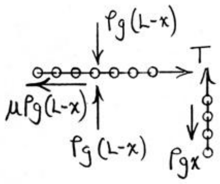


$F = \mu N = \mu g\rho(L-b)$

$\Sigma F = 0$: $T_0 - \mu g\rho(L-b) = 0$

and $T_0 = \rho g b$

Solve to obtain $b = \frac{\mu L}{1+\mu}$



$\Sigma F = ma$: $T - \mu\rho g(L-x) = \rho(L-x)a$

and $\rho g x - T = \rho x a$

Eliminate T to obtain

$$a = \ddot{x} = \frac{g}{L}\left[x(1+\mu) - \mu L\right]$$

$$v\,dv = \ddot{x}\,dx : \; \int_0^v v\,dv = \int_b^L \frac{g}{L}\left[x(1+\mu)-\mu L\right]dx$$

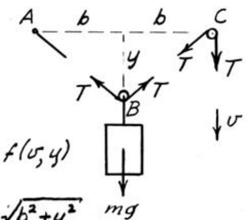
$$\frac{1}{2}v^2 = \frac{g}{L}\left[\frac{x^2}{2}(1+\mu) - \mu L x\right]_b^L$$

Substitute $b = \frac{\mu L}{1+\mu}$, Simplify, and obtain

$$\underline{v = \sqrt{\frac{gL}{1+\mu}}}$$

▸3/48

$+\uparrow \Sigma F = ma$: $2T\frac{y}{\sqrt{b^2+y^2}} - mg = ma$, $a = -\ddot{y}$



$T = \frac{m(a+g)\sqrt{b^2+y^2}}{2y}$ where $a = f(v, y)$

Let L = length of cable $ABC = 2\sqrt{b^2+y^2}$

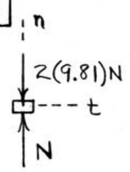
$$-v = \dot{L} = 2\frac{y\dot{y}}{\sqrt{b^2+y^2}}, \quad \ddot{L} = 2\frac{\sqrt{b^2+y^2}(\dot{y}^2+y\ddot{y})}{b^2+y^2} - 2\frac{y\dot{y}(y\dot{y})}{(b^2+y^2)\sqrt{b^2+y^2}} = 0$$

$$\text{so } \sqrt{b^2+y^2}\left(\frac{v^2(b^2+y^2)}{4y^2} + y\ddot{y}\right) = \frac{y^2}{\sqrt{b^2+y^2}}\;\frac{v^2(b^2+y^2)}{4y^2}$$

Simplify and get $\ddot{y} = -\frac{b^2v^2}{4y^3} = -a$

Thus $T = \frac{m\left(g + \frac{b^2v^2}{4y^3}\right)\sqrt{b^2+y^2}}{2y}$, $\underline{T = \frac{m}{2y}\sqrt{b^2+y^2}\left(g + \frac{b^2v^2}{4y^3}\right)}$

3/49



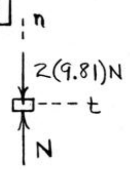
$\Sigma F_n = m a_n = m\frac{v^2}{\rho}$:

$N - 2(9.81) = 2\,\frac{4^2}{1.5}$

$\underline{N = 41.0\text{ N}}$ up

Any friction present would not enter the normal equation.

3/49



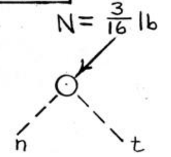
$\Sigma F_n = m a_n = m\frac{v^2}{\rho}$:

$N - 2(9.81) = 2\,\frac{4^2}{1.5}$

$\underline{N = 41.0\text{ N}}$ up

Any friction present would not enter the normal equation.

3/50

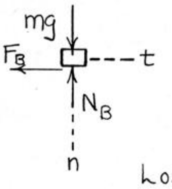


$\Sigma F_n = ma_n = m\dfrac{v^2}{\rho}$:

$\dfrac{3}{16} = \dfrac{2/16}{32.2}\left(\dfrac{5^2}{\rho}\right)$

$\underline{\rho = 0.518 \text{ ft}}$

3/51

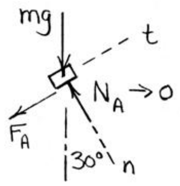


$\Sigma F_n = ma_n = m\dfrac{v^2}{\rho}$:

$2(9.81) - N = 2\,\dfrac{3.5^2}{2.4}$

$\underline{N_B = 9.41 \text{ N}}$

Loss of contact at A: $N_A \to 0$

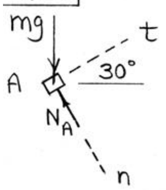


$\Sigma F_n = ma_n = m\dfrac{v^2}{\rho}$:

$\cancel{m}g\cos 30° = \cancel{m}\dfrac{v^2}{2.4}$

$\underline{v = 4.52 \text{ m/s}}$

3/52



$\Sigma F_n = ma_n : -N_A + mg\cos 30° = m\dfrac{v_A^2}{\rho}$

$N_A = m\left(g\cos 30° - \dfrac{v_A^2}{\rho}\right)$

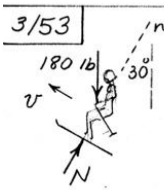
$= 2\left(9.81\cos 30° - \dfrac{4.5^2}{2.4}\right)$

$= \underline{0.1164 \text{ N}}$

$\Sigma F_t = ma_t : -mg\sin 30° = ma_t$

$a_t = -\dfrac{g}{2} = \underline{-4.90 \text{ m/s}^2}$

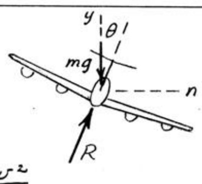
3/53



$\Sigma F_n = ma_n$; $N - 180\cos 30° = \dfrac{180}{32.2}\,\dfrac{(80)^2}{150}$

$N = 180(0.866 + 1.33) = \underline{394 \text{ lb}}$

3/54



$\Sigma F_y = 0$: $R\cos\theta - mg = 0$, $R\cos\theta = mg$

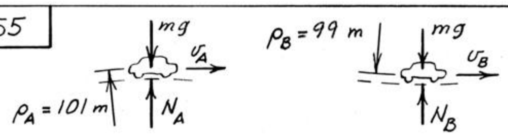
$\Sigma F_n = ma_n$: $R\sin\theta = mv^2/\rho$

Combine & get $\tan\theta = \dfrac{v^2}{\rho g}$, $\theta = \tan^{-1}\dfrac{v^2}{\rho g}$

where $v = \dfrac{400\times 5280}{3600} = 587$ ft/sec, $\rho = 2\times 5280 = 10{,}560$ ft

so $\theta = \tan^{-1}\dfrac{587^2}{10{,}560\times 32.2} = \tan^{-1}1.012$, $\underline{\theta = 45.3°}$

3/55



$\Sigma F_n = ma_n$:

A: $mg - N_A = m\dfrac{v_A^2}{\rho_A}$ B: $N_B - mg = m\dfrac{v_B^2}{\rho_B}$

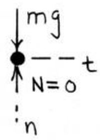
For $N_B = 2N_A$, $m\left(\dfrac{v_B^2}{\rho_B} + g\right) = 2m\left(g - \dfrac{v_A^2}{\rho_A}\right)$

$v_B^2 = \rho_B g - 2v_A^2\dfrac{\rho_B}{\rho_A} = 99(9.81) - 2\left(\dfrac{60\times 1000}{3600}\right)^2\dfrac{99}{101}$

$= 427 \text{ m}^2/\text{s}^2$

$v_B = 20.7$ m/s or $\underline{v_B = 74.4 \text{ km/h}}$

3/56 FBD of object inside airplane :



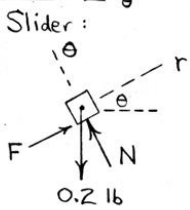
$\Sigma F_n = ma_n$: $\cancel{m}g = \cancel{m}\dfrac{v^2}{\rho}$

$\rho = \dfrac{v^2}{g} = \dfrac{\left[(600)\left(\frac{5280}{3600}\right)\right]^2}{32.2}$

$\underline{\rho = 24{,}050 \text{ ft}}$

3/57 $\Sigma F_\theta = ma_\theta = m(r\ddot\theta + 2\dot r\dot\theta)$: $N - 0.2\cos 30°$

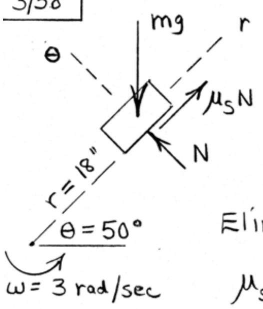
Slider :



$= \dfrac{0.2}{32.2}\left(r\overset{0}{\ddot\theta} + 2(-4)(3)\right)$

$\underline{N = 0.024 \text{ lb}}$

3/58



$\Sigma F_\theta = ma_\theta$: $N - mg\cos\theta = 0$

$N = mg\cos\theta$

$\Sigma F_r = ma_r$: $\mu_s N - mg\sin\theta$

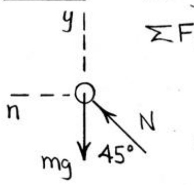
$= m(0 - r\omega^2)$

Eliminate N:

$\mu_s = \tan\theta - \dfrac{r\omega^2}{g\cos\theta}$

Numbers : $\mu_s = \tan 50° - \dfrac{(18/12)(3)^2}{32.2\cos 50°} = \underline{0.540}$

3/59



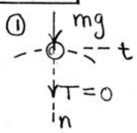
$\Sigma F_y = 0$: $N\dfrac{\sqrt2}{2} - mg = 0$, $N = \dfrac{2}{\sqrt2}mg$

$\Sigma F_n = ma_n$: $N\dfrac{\sqrt2}{2} = m\left(3R + R\dfrac{\sqrt2}{2}\right)\Omega^2$

$\dfrac{2}{\sqrt2}mg\left(\dfrac{\sqrt2}{2}\right) = mR\left(3 + \dfrac{\sqrt2}{2}\right)\Omega^2$

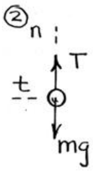
With R = 0.200 m, $\underline{\Omega = 3.64 \ \frac{\text{rad}}{\text{s}}}$

3/60

① 

$\Sigma F_n = m\frac{v^2}{\rho}$: $mg = m\frac{v^2}{1}$

$v = \sqrt{g} = 3.13$ m/s

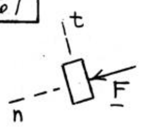
② 

$\Sigma F_n = m\frac{v^2}{\rho}$: $T - mg = m\frac{g}{1}$

$T = 2mg = 2(0.050)(9.81)$

$= 0.981$ N

3/61



$a_n = \frac{v^2}{\rho} = \frac{\left[(35)\left(\frac{5280}{3600}\right)\right]^2}{100}$

$= 26.4\ \frac{ft}{sec^2}\left(\frac{1g}{32.2\ ft/sec^2}\right)$

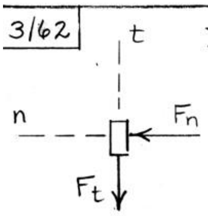
$= 0.818\ g$

$\Sigma F_n = ma_n$: $F = \frac{3000}{32.2}(26.4)$

$= 2460$ lb

(An average of 614 lb per tire!)

3/62



$\Sigma F_n = ma_n$: $F_n = \frac{3000}{32.2}\frac{\left(25\cdot\frac{5280}{3600}\right)^2}{100}$

$F_n = 1253$ lb

$\sqrt{F_n^2 + F_t^2} = F_{tot}$

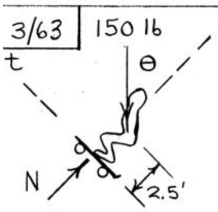
$1253^2 + F_t^2 = 2400^2$

$F_t = 2047$ lb

$\Sigma F_t = ma_t$: $-2047 = \frac{3000}{32.2}a_t$

$a_t = -22.0$ ft/sec^2

3/63



$\rho = 15 - 2.5 = 12.5'$

$\Sigma F_n = ma_n$: $N - 150\cos\theta = \frac{150}{32.2}\frac{v^2}{12.5}$

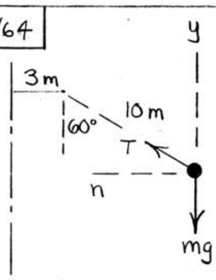
$N = 150\left(\cos\theta + \frac{v^2}{402.5}\right)$

$\theta = 0(+)$: $N_0 = 150\left(1 + \frac{28^2}{402.5}\right) = 442$ lb

$\theta = 45°$: $N_{45°} = 150\left(\frac{\sqrt{2}}{2} + \frac{20^2}{402.5}\right) = 255$ lb

$\theta = 90°$: $N_{90°} = 0$

3/64



$\Sigma F_n = ma_n$:

$T\sin 60° = m[3 + 10\sin 60°]\omega^2$

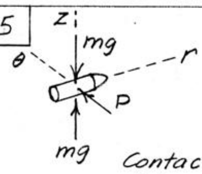
$\Sigma F_y = 0$: $T\cos 60° - mg = 0$

$\Rightarrow \tan 60° = \frac{3 + 10\sin 60°}{9.81}\omega^2$

$\omega = 1.207$ rad/s

$N = 1.207\left(\frac{60}{2\pi}\right) = 11.53$ rev/min

3/65



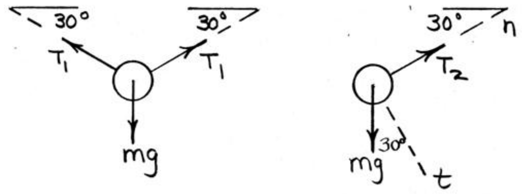
$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$P = 0.06(0 + 2[600][0.5])$

$= 36$ N

Contact is against right-hand side of barrel.

3/66

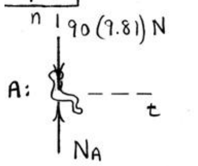


Equilibrium : $\Sigma \underline{F} = \underline{0} \Rightarrow T_1 = mg$

Motion : $\Sigma F_n = ma_n = 0$: $T_2 - mg\sin 30° = 0$

$k = \frac{T_2}{T_1} = \frac{mg\sin 30°}{mg} = 0.5$

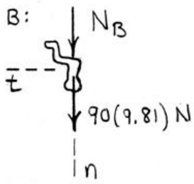
3/67

A: 

$\Sigma F_n = ma_n$:

$N_A - 90(9.81) = 90\frac{(600/3.6)^2}{1000}$

$N_A = 3380$ N

B: 

$\Sigma F_n = ma_n$:

$N_B + 90(9.81) = 90\frac{(600/3.6)^2}{1000}$

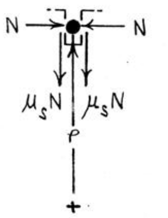
$N_B = 1617$ N

(Note static normal $mg = 90(9.81) = 883$ N)

3/68 $\omega = \left(4000\ \frac{rev}{min}\right)\left(\frac{1\ min}{60\ sec}\right)\left(\frac{2\pi\ rad}{sec}\right)$

$= 418.9$ rad/sec

FBD of pebble :

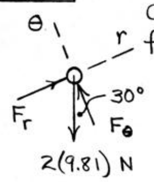


$\Sigma F_n = ma_n: \ 2\mu_s N = mr\omega^2$

$N = \frac{mr\omega^2}{2\mu_s} = \frac{(0.010)(0.350)(418.9)^2}{2(0.95)}$

$\underline{N = 323 \text{ N}}$

3/69 F_r and F_θ are the r- and θ- components of the total friction force F.



$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$

$F_r - 19.62 \sin 30° = 2[0 - 1(-0.873)^2]$

$F_r = 8.29$ N

$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$F_\theta - 19.62 \cos 30° = 2[(1)(3.49) + 2(-0.5)(-0.873)]$

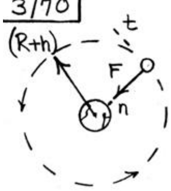
$F_\theta = 25.7$ N

$F = \sqrt{F_r^2 + F_\theta^2} = 27.0$ N

$P = \frac{F/2}{\mu_s} = \frac{27.0/2}{0.5} = \underline{27.0 \text{ N}}$

(Static gripping force $= \underline{19.62 \text{ N}}$)

3/70 $\Sigma F_n = ma_n: \ F = \frac{Gm_e m}{(R+h)^2} = m\frac{v^2}{(R+h)}$



But $v = \frac{s}{t} = \frac{2\pi(R+h)}{(23.944)(3600)}$

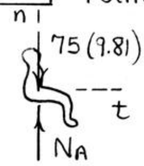
Combining the two equations :

$v = \frac{2\pi(R+h)}{(23.944)(3600)} = \sqrt{\frac{Gm_e}{(R+h)}}$

Solve for h to obtain $\underline{h = 3.580 \times 10^7 \text{ m}}$

(35,800 km)

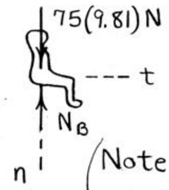
3/71 Point A : $\Sigma F_n = ma_n:$



$N_A - 75(9.81) = 75\ \frac{22^2}{40}$

$\underline{N_A = 1643 \text{ N}}$

Point B : $\Sigma F_n = ma_n:$

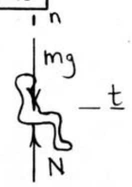


$75(9.81) - N_B = 75\ \frac{12^2}{20}$

$\underline{N_B = 195.8 \text{ N}}$

(Note static normal of magnitude $N = mg = 75(9.81) = 736$ N)

3/72 $\theta = \frac{\pi}{3}\sin 0.950t$



$\dot{\theta} = \frac{\pi}{3}(0.950)\cos 0.950t$

$\dot{\theta}_{max} = \frac{\pi}{3}(0.950) = 0.995$ rad/s when $\theta = 0$.

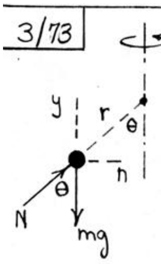
$\Sigma F_n = ma_n: \ N - mg = mr\dot{\theta}^2$

$N = mg + m(11)(0.995)^2 = \underline{20.7m}$ {N in newtons when m in kg

Riders near center experience the greatest normal force ; those at ends of unit experience the smallest normal force when θ is at a maximum (or minimum).

3/73 $\Sigma F_y = 0: \ N\cos\theta - mg = 0$

$N = mg/\cos\theta$



$\Sigma F_n = ma_n: \ N\sin\theta = m(r\sin\theta)\omega^2$

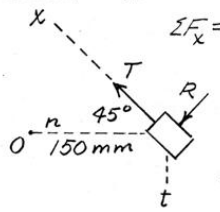
$\left(\frac{\cancel{m}g}{\cos\theta}\right)\cancel{\sin}\theta = \cancel{m}r\cancel{\sin}\theta\,\omega^2$

$\underline{\omega = \sqrt{\frac{g}{r\cos\theta}}}$

Note that $\cos\theta = \frac{g}{r\omega^2} \leq 1$

$\therefore \ \omega^2 \geq \frac{g}{r}$ is a restriction.

3/74 $a_n = r\dot{\theta}^2 = 0.15\left(300\ \frac{2\pi}{60}\right)^2 = 148.0\ m/s^2$

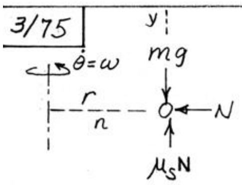


$\Sigma F_x = ma_x; \ T = 3(148.0 \cos 45°)$

$= \underline{314 \text{ N}}$

Direction of rotation does not change accel., hence has no influence on T or R.

3/75

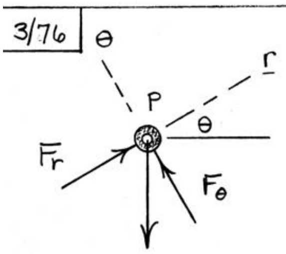


$\Sigma F_n = ma_n;\ N = mr\omega^2$

$\Sigma F_y = 0;\ \mu_s(mr\omega^2) = mg$

$\omega^2 = \dfrac{g}{\mu_s r}, \quad \omega = \sqrt{\dfrac{g}{\mu_s r}}$

3/76



$1.2(9.81) = 11.77$ N

$\theta = 30°$

$\dot\theta = 40\left(\frac{\pi}{180}\right) = 0.698$ rad/s

$\ddot\theta = 120\left(\frac{\pi}{180}\right) = 2.09$ rad/s²

$r = 1.25$ m

$\dot r = 0.4$ m/s

$\ddot r = -0.3$ m/s²

$\Sigma F_r = ma_r:\ F_r - 11.77\sin 30° = 1.2[-0.3 - 1.25(0.698)^2]$

$F_r = 4.79$ N

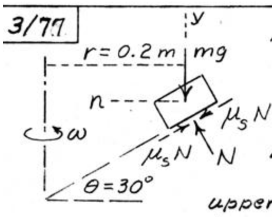
$\Sigma F_\theta = ma_\theta:\ F_\theta - 11.77\cos 30° = 1.2[1.25(2.09) + 2(0.4)(0.698)]$

$F_\theta = 14.00$ N

For static case, set $a_\theta = a_r = 0$ & obtain

$(F_r)_{st} = 5.89$ N, $\quad (F_\theta)_{st} = 10.19$ N

3/77



$\mu_s N$ is down for ω_{max}

" " up " ω_{min}

$\Sigma F_y = 0;\ N\cos\theta \mp \mu_s N\sin\theta = mg$

$\Sigma F_n = ma_n;\ N\sin\theta \pm \mu_s N\cos\theta = mr\omega^2$

upper sign for ω_{max}

lower sign for ω_{min}

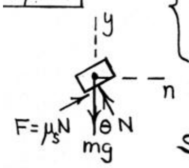
Combine & get $\dfrac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s\sin\theta} = \dfrac{r\omega^2}{g}$

$\omega = \sqrt{\dfrac{g}{r}}\sqrt{\dfrac{\sin\theta \pm \mu_s\cos\theta}{\cos\theta \mp \mu_s\sin\theta}} = \sqrt{\dfrac{9.81}{0.2}}\sqrt{\dfrac{0.5 \pm 0.3(0.866)}{0.866 \mp 0.3(0.5)}}$

Upper sign $\omega_{max} = 7.21$ rad/s

Lower sign $\omega_{min} = 3.41$ rad/s

3/78



$\Sigma F_y = 0:\ N\cos\theta - mg + \mu_s N\sin\theta = 0$

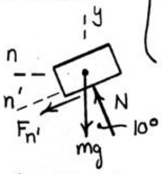
$\Sigma F_n = ma_n:\ -N\sin\theta + \mu_s N\cos\theta = mr\omega^2$

Solving for ω:

$\omega = \sqrt{\dfrac{g}{r}\dfrac{(\mu_s\cos\theta - \sin\theta)}{(\cos\theta + \mu_s\sin\theta)}} = 2.73$ rad/s

3/79

Crate:



$\Sigma F_y = 0:\ N\cos 10° - F_{n'}\sin 10° - mg = 0$

$\Sigma F_n = ma_n:\ F_{n'}\cos 10° + N\sin 10° = m\dfrac{(2t)^2}{30}$

$\Sigma F_t = ma_t:\quad F_t = m(2)$

Solve first two equations for N and $F_{n'}$ to obtain

(t, F_t into paper) $\quad F_{n'} = m\left[\dfrac{4t^2\cos 10°}{30} - g\sin 10°\right]$

$N = m\left[\dfrac{4t^2\sin 10°}{30} + g\cos 10°\right]$

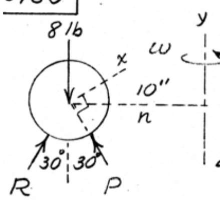
Condition for slipping: $\sqrt{F_t^2 + F_{n'}^2} = \mu_s N$

$\sqrt{2^2 + \left(\dfrac{4t^2\cos 10°}{30} - g\sin 10°\right)^2} = 0.3\left[\dfrac{4t^2\sin 10°}{30} + g\cos 10°\right]$

Square both sides and solve for t:

$t = 5.58$ s

3/80



$\omega = 30 \times 2\pi/60 = \pi$ rad/sec

Sol. I

$\Sigma F_y = 0;\ (R+P)\cos 30° = 8$

$\Sigma F_n = ma_n;\ (R-P)\sin 30° = \dfrac{8}{32.2}\dfrac{10}{12}(\pi)^2$

$2R = 8\left[\dfrac{1}{0.866} + \dfrac{10\pi^2}{32.2(12)}\dfrac{1}{0.5}\right]$

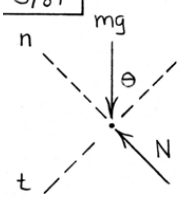
$R = 4(1.155 + 0.511) = 6.66$ lb

Sol II (one force equation)

$\Sigma F_x = ma_x;\ R\cos 30° - 8\sin 30° = \dfrac{8}{32.2}\dfrac{10}{12}(\pi)^2\cos 30°$

$R = 8\left(\tan 30° + \dfrac{10\pi^2}{32.2 \times 12}\right) = 6.66$ lb

3/81



Treat the child as a particle.

$\Sigma F_t = ma_t:\ mg\cos\theta = ma_t \quad (1)$

$\Sigma F_n = ma_n:\ N - mg\sin\theta = m\dfrac{v^2}{R} \quad (2)$

From (1): $g\cos\theta = v\dfrac{dv}{ds} = v\dfrac{dv}{Rd\theta}$

$\displaystyle\int_{\theta_0 = 20°}^{\theta} Rg\cos\theta\, d\theta = \int_{v_0 = 0}^{v} v\, dv$

$v = [2Rg(\sin\theta - \sin 20°)]^{1/2}$

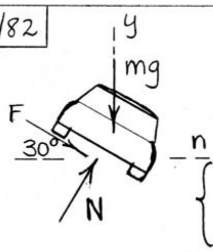
(2): $N = m\left(g\sin\theta + \dfrac{v^2}{R}\right)$

Numbers ($R = 2.5$ m, $g = 9.81$ m/s²)

$\theta = 30°$: $a_t = 8.50$ m/s², $v = 2.78$ m/s, $N = 280$ N

$\theta = 90°$: $a_t = 0$, $v = 5.68$ m/s, $N = 795$ N

3/82



For no slipping tendency, set F to zero on FBD.

$$\begin{cases} \Sigma F_y = 0: & N\cos 30° - mg = 0 \\ \Sigma F_n = m\frac{v^2}{\rho}: & N\sin 30° = m\frac{v^2}{1200} \end{cases}$$

Solve: $N = 1.155mg$, $v = 149.4$ ft/sec

or $\underline{v = 101.8 \text{ mi/hr}}$

$\underline{v_{min} = 0}$, as $\theta_{max} = \tan^{-1}\mu_s = \tan^{-1}(0.9) = 42.0° > 30°$

For v_{max}, set $F = F_{max} = \mu_s N$:

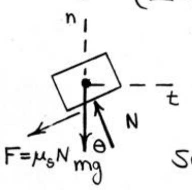
$$\begin{cases} \Sigma F_y = 0: & N\cos 30° - mg - \mu_s N \sin 30° = 0 \\ \Sigma F_n = m\frac{v^2}{\rho}: & \mu_s N\cos 30° + N\sin 30° = m\frac{v_{max}^2}{\rho} \end{cases}$$

With $\mu_s = 0.9$: $N = 2.40mg$

$v_{max} = 345$ ft/sec $\underline{(235 \text{ mi/hr})}$

3/83

Package:
$$\begin{cases} \Sigma F_t = ma_t: & -\mu_s N\cos\theta - N\sin\theta = -m\frac{g}{2} \\ \Sigma F_n = ma_n: & N\cos\theta - \mu_s N\sin\theta - mg = m\left(\frac{19.44^2}{80}\right) \end{cases}$$



First eq.: $N = \dfrac{mg/2}{\sin\theta + \mu_s\cos\theta}$

Second eq.:

$$\left(\frac{\cancel{m}g/2}{\sin\theta + \mu_s\cos\theta}\right)(\cos\theta - \mu_s\sin\theta) - \cancel{m}g = \cancel{m}(4.726)$$

$$\tan\theta = \left(\frac{1 - 2.9635\mu_s}{\mu_s + 2.9635}\right)$$

For $\mu_s = 0.2$, $\underline{\theta = 7.34°}$

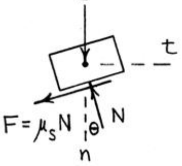
For $\mu_s = 0.4$, $\underline{\theta = -3.16°}$!!

(Note: $N > 0$ for $\theta = -3.16°$)

3/84

Package: $\Sigma F_t = ma_t: -\mu_s N\cos\theta - N\sin\theta = -mg/2$

$$N = \frac{mg/2}{\sin\theta + \mu_s\cos\theta}$$



$\Sigma F_n = ma_n: \; -N\cos\theta + \mu_s N\sin\theta + mg = m\left(\frac{19.44^2}{80}\right)$

$$\left(\frac{mg/2}{\sin\theta + \mu_s\cos\theta}\right)(\mu_s\sin\theta - \cos\theta) + mg = m(4.726)$$

$$\tan\theta = \left[\frac{1 - 1.036\mu_s}{\mu_s + 1.036}\right]$$

For $\mu_s = 0.2$, $\theta = 32.7°$ and $N > 0$.

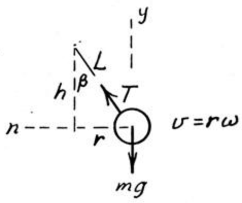
For $\mu_s = 0.4$, $\underline{\theta = 22.2° \text{ and } N > 0.}$

3/85

$\Sigma F_y = 0$: $T\cos\beta - mg = 0$, $T\cos\beta = mg$

$\Sigma F_n = ma_n$: $T\sin\beta = mv^2/r$

Divide & get $\tan\beta = \dfrac{v^2}{gr} = \dfrac{r\omega^2}{g}$



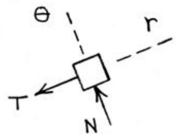
But $r = L\sin\beta$ so $\tan\beta = L\omega^2\sin\beta/g$

or $L\cos\beta = g/\omega^2$

And $h = L\cos\beta$ so $\underline{h = g/\omega^2}$ (depends only on ω & g)

Then $\underline{T} = \dfrac{mg}{\cos\beta} = \dfrac{mg}{h/L} = \dfrac{mgL}{g/\omega^2} = \underline{mL\omega^2}$

3/86



$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$:

$-T = \dfrac{3}{32.2}\left(0 - \dfrac{9}{12}6^2\right)$

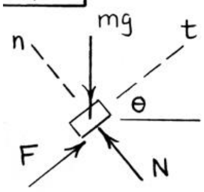
$\underline{T = 2.52 \text{ lb}}$

$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$:

$N = \dfrac{3}{32.2}\left[\frac{9}{12}(-2) + 2\left(-\frac{2}{12}\right)(6)\right]$

$\underline{N = -0.326 \text{ lb}}$ (Contact on side B)

3/87



$a_n = r\Omega^2 = \frac{13}{12}(7.5)^2 = 60.9$ ft/sec²

$\Sigma F_n = ma_n$:

$N - mg\cos\theta = 60.9m$ (1)

$\Sigma F_t = ma_t$:

$F - mg\sin\theta = 0$ (2)

Slip impends when $F = F_{max} = \mu_s N$. From (1) & (2): $\mu_s = \dfrac{32.2\sin\theta}{60.9 + 32.2\cos\theta}$

(a) $\theta = 50°$: $\underline{\mu_s = 0.302}$

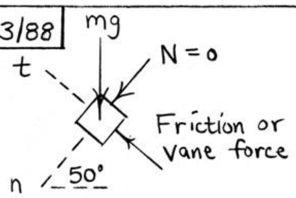
(b) $\theta = 100°$: $\underline{\mu_s = 0.573}$

From (1) $N = m(60.9 + g\cos\theta) > 0$ for all θ

So contact is maintained.

(Look ahead to solution of Prob. *3/365)

3/88

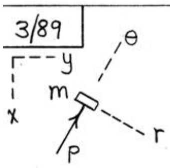


$\Sigma F_n = ma_n$: $mg\sin 50° = mr\Omega^2$

$\Omega = \sqrt{\dfrac{g\sin 50°}{r}} = \sqrt{\dfrac{9.81\sin 50°}{0.330}} = \underline{4.77 \text{ rad/s}}$

(45.6 rev/min)

3/89



(Note : mg & static normal ⊥ to paper)

$\Sigma F_r = ma_r: \quad 0 = m(\ddot{r} - r\Omega^2)$ (1)

$\Sigma F_\theta = ma_\theta: \quad P = m(r\ddot{\theta} + 2\dot{r}\Omega)$ (2) $\quad \ddot{\theta} = 0$

(1): $\ddot{r} = \dot{r}\dfrac{d\dot{r}}{dr} = r\Omega^2$

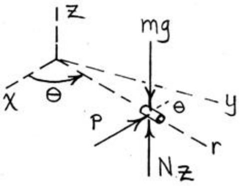
$\displaystyle\int_{\dot{r}_0}^{\dot{r}} \dot{r}\,d\dot{r} = \int_{r_0}^{r} \Omega^2 r\,dr \Rightarrow \dot{r}^2 = \dot{r}_0^2 + \Omega^2(r^2 - r_0^2)$

Numbers : (at end of tube) $\dot{r} = \left[60^2 + 7^2\left(3^2 - \left(\frac{6}{12}\right)^2\right)\right]^{1/2} = 63.5\ \frac{\text{ft}}{\text{sec}}$

(2): $P = m(2\dot{r}\Omega) = \dfrac{5/16}{32.2}(2)(63.5)(7)$

$= \underline{8.62\ \text{lb}}$

3/90



$\Sigma F_z = 0 \Rightarrow N_z = mg$

From the solution to Prob. 3/89,

$\dot{r} = \left\{\dot{r}_0^2 + \Omega^2(r^2 - r_0^2)\right\}^{1/2}$

So $\displaystyle\int_{r_0}^{r} \frac{dr}{\sqrt{\dot{r}_0^2 + \Omega^2(r^2 - r_0^2)}} = \int_0^t dt$

$\dfrac{1}{\Omega}\ln\left[r + \sqrt{r^2 + \dfrac{\dot{r}_0^2}{\Omega^2} - r_0^2}\right]_{r_0}^{r} = t$

$\dfrac{1}{\Omega}\ln\left[\dfrac{r + \sqrt{r^2 + \frac{\dot{r}_0^2}{\Omega^2} - r_0^2}}{r_0 + \frac{\dot{r}_0}{\Omega}}\right] = t$

With numbers ($r = 3$ ft, $r_0 = 0.5$ ft, $\Omega = 7\ \frac{\text{rad}}{\text{sec}}$, $\dot{r}_0 = 60$ ft/sec),

$t = 0.0408$ sec ; Then $\theta = \Omega t = 0.285$ rad

From solution to Prob. 3/89, $P = 8.62$ lb

$P_x = -P\sin\theta = -8.62 \sin 0.285^R = \underline{-2.43\ \text{lb}}$

$P_y = P\cos\theta = 8.62\cos 0.285^R = \underline{8.28\ \text{lb}}$

3/91 The distance traveled from A to C is

$(s_C - s_A) = 100 + 250\left(30\frac{\pi}{180}\right) = 231$ ft

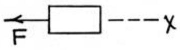
Uniform tangential acceleration : $v_C^2 = v_A^2 + 2a_t(s_C - s_A)$

$0^2 = \left[60\ \frac{5280}{3600}\right]^2 + 2a_t(231)$, $\quad a_t = -16.77\ \text{ft/sec}^2$

Speed at B: $v_B^2 = v_A^2 + 2a_t(s_B - s_A)$

$v_B^2 = \left[60\ \frac{5280}{3600}\right]^2 + 2(-16.77)(100)$, $\quad v_B = 66.3$ ft/sec

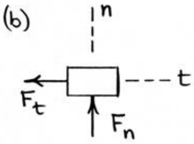
(a)



$\Sigma F_x = ma_x: \ -F = \dfrac{3000}{32.2}(-16.77)$

$\underline{F = 1562\ \text{lb}}$

(b)



$\Sigma F_t = ma_t: \ -F_t = \dfrac{3000}{32.2}(-16.77)$

$F_t = 1562$ lb

$\Sigma F_n = m\dfrac{v^2}{\rho}: \quad F_n = \dfrac{3000}{32.2}\ \dfrac{66.3^2}{250}$

$F_n = 1636$ lb

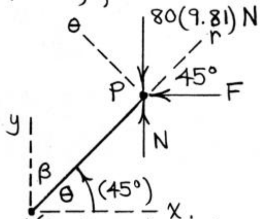
$F = \sqrt{F_t^2 + F_n^2} = \underline{2260\ \text{lb}}$

(c) v and therefore F_n go to zero;

$\underline{F = F_t = 1562\ \text{lb}}$

(In all FBDs, there is a weight into the paper and a static normal force out of the paper.)

3/92 FBD of rider at P (could be any rider!), treated as a particle :



O′ (Center of circle traced by P)

$O'P = 6$ m

$\theta = 45°$

$\dot{\theta} = -0.8$ rad/s

$\ddot{\theta} = -0.4$ rad/s²

$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2): \ -80(9.81)\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} - F\frac{\sqrt{2}}{2}$

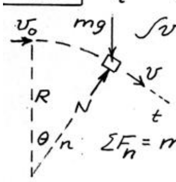
$= 80\left[0 - 6(-0.8)^2\right]$ (1)

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}): \ -80(9.81)\frac{\sqrt{2}}{2} + N\frac{\sqrt{2}}{2} + F\frac{\sqrt{2}}{2}$

$= 80\left[6(-0.4) + 0\right]$ (2)

Solve (1) & (2) : $\begin{cases} N = 432\ \text{N} \\ \underline{F = 81.5\ \text{N}} \end{cases}$ ‖ Static: $\begin{cases} N_S = 785\ \text{N} \\ \underline{F_S = 0} \end{cases}$

3/93 $\Sigma F_t = ma_t$; $mg\sin\theta = ma_t$, $a_t = g\sin\theta$



$\int v\,dv = \int a_t\,ds$; $\displaystyle\int_{v_0}^{v} v\,dv = \int_0^\theta g\sin\theta\,(R\,d\theta)$

$v^2 = v_0^2 + 2gR(1 - \cos\theta)$

$\Sigma F_n = ma_n$; $mg\cos\theta - N = m\dfrac{v^2}{R}$

$N = mg\cos\theta - \dfrac{m}{R}v_0^2 - 2mg(1 - \cos\theta)$

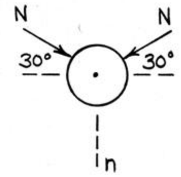
$= mg\left(3\cos\theta - 2 - \dfrac{v_0^2}{gR}\right)$

when $N = 0$, $\theta = \beta$ so $\quad 3\cos\beta = 2 + \dfrac{v_0^2}{gR}$

$\underline{\beta = \cos^{-1}\left(\frac{2}{3} + \frac{v_0^2}{3gR}\right)}$

For $v_0 = 0$, $\beta = \cos^{-1}\left(\frac{2}{3}\right) = \underline{48.2°}$

3/94 $\Sigma F_n = ma_n = mr\omega^2$:

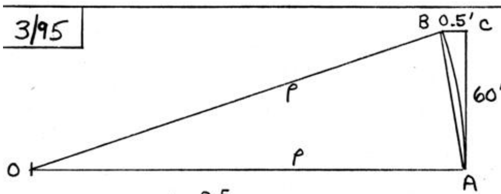


$2N \sin 30° = 2.5(0.150)\left[\frac{600(2\pi)}{60}\right]^2$

$N = 1480$ N

$F = 4N\cos 30° = \underline{5130 \text{ N}}$

3/95



$\measuredangle BAC = \tan^{-1}\frac{0.5}{60} = 0.477°$

$\measuredangle OBA = \measuredangle OAB = (90 - 0.477) = 89.5°$

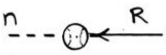
$\measuredangle BOA = 180 - 2(89.5) = 0.955° = 2\measuredangle BAC$

$\overline{AB} = \sqrt{60^2 + 0.5^2} = 60.002'$

$\frac{\sin 0.955°}{60.002'} = \frac{\sin 89.5°}{\rho}$, $\quad \underline{\rho = 3600 \text{ ft}}$

FBD: (horizontal forces)

$\Sigma F_n = ma_n$: $R = \frac{5.125/16}{32.2}\,\frac{120^2}{3600}$



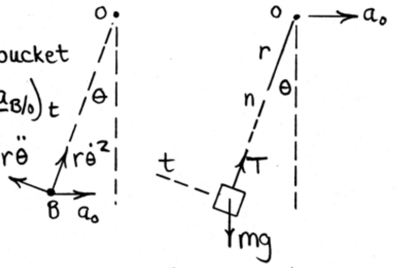
$\underline{R = 0.0398 \text{ lb}}$

(Note: $R = 0.637$ oz represents 12.4% of the weight of the baseball)

3/96

Acceleration of bucket

$a_B = a_o + (a_{B/O})_n + (a_{B/O})_t$



$\Sigma F_t = ma_t$: $-mg\sin\theta = m(r\ddot\theta - a_o\cos\theta)$

$\ddot\theta = +\frac{1}{r}(a_o\cos\theta - g\sin\theta)$

$\dot\theta$ is a maximum when $\ddot\theta = 0$: $a_o\cos\theta = g\sin\theta$

$\underline{\theta = \tan^{-1}\left(\frac{a_o}{g}\right)}$

With $\ddot\theta = \dot\theta\frac{d\dot\theta}{d\theta}$:

$\int_0^{\dot\theta}\dot\theta\, d\dot\theta = \int_0^{\theta}\frac{1}{r}(a_o\cos\theta - g\sin\theta)\,d\theta$

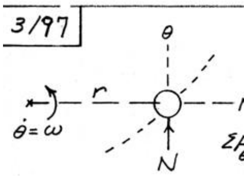
$\dot\theta^2 = \frac{2}{r}(a_o\sin\theta + g\cos\theta - g)$

$\Sigma F_n = ma_n$: $T - mg\cos\theta = m(r\dot\theta^2 + a_o\sin\theta)$

Substitute expression for $\dot\theta^2$:

$\underline{T = m(3a_o\sin\theta + 3g\cos\theta - 2g)}$

3/97



$\Sigma F_r = ma_r$; $0 = m(\ddot r - r\omega^2)$

sol. is $r = r_o\cosh\omega t$

$\dot r = r_o\omega\sinh\omega t$

$\Sigma F_\theta = ma_\theta$; $N = m(0 + 2\dot r\omega)$

$= 2mr_o\omega^2\sinh\omega t$

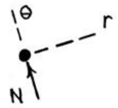
But $\cosh^2\omega t - \sinh^2\omega t = 1$, $\sinh^2\omega t = \cosh^2\omega t - 1$

$\sinh\omega t = \sqrt{\left(\frac{r}{r_o}\right)^2 - 1}$

So $N = 2mr_o\omega^2\sqrt{\left(\frac{r}{r_o}\right)^2 - 1} = \underline{2m\omega^2\sqrt{r^2 - r_o^2}}$

3/98 $\Sigma F_r = ma_r$: $0 = m(\ddot r - r\dot\theta^2)$

Particle:



$\ddot r = r\dot\theta^2 = r\omega_o^2$

$\dot r\frac{d\dot r}{dr} = r\omega_o^2$

$\int_0^{\dot r}\dot r\, d\dot r = \omega_o^2\int_{r_o}^{r} r\,dr$

$\Rightarrow \underline{\dot r = \omega_o\sqrt{r^2 - r_o^2} = v_r}$

$\frac{dr}{dt} = \omega_o\sqrt{r^2 - r_o^2}$

$\int_{r_o}^{r}\frac{dr}{\sqrt{r^2 - r_o^2}} = \omega_o\int_0^t dt$

$\ln\left[r + \sqrt{r^2 - r_o^2}\right]\Big|_{r_o}^{r} = \omega_o t \Rightarrow \underline{r = \frac{r_o}{2}\left[e^{-\omega_o t} + e^{\omega_o t}\right]}$

$v_\theta = r\dot\theta = r\omega_o = \underline{\frac{r_o\omega_o}{2}\left[e^{-\omega_o t} + e^{\omega_o t}\right]}$

As a function of t, $\underline{v_r = \frac{r_o\omega_o}{2}\left(e^{\omega_o t} - e^{-\omega_o t}\right)}$

In terms of hyperbolic functions,

$v_r = r_o\omega_o\sinh\omega_o t$

$r = r_o\cosh\omega_o t$

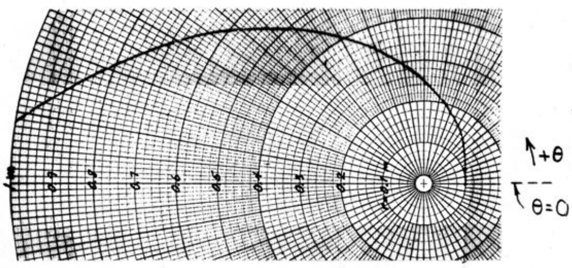
$\underline{v_\theta = r_o\omega_o\cosh\omega_o t}$

With numbers,

$v_r = 0.1\sinh t$

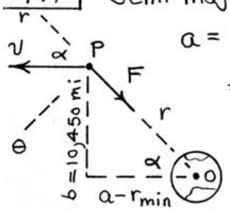
$r = 0.1\cosh t$

$v_\theta = 0.1\cosh t$



($r = 1.0$ m @ $\theta = 171.5°$)

3/99 Semi-major axis of ellipse is



$$a = \frac{r_{max} + r_{min}}{2} = \frac{26,259 + 4159}{2} = 15,209 \text{ mi}$$

$$\alpha = \tan^{-1}\frac{b}{a - r_{min}} = \tan^{-1}\frac{10,450}{15,209 - 4159} = 43.4°$$

$$\dot r = v_r = v\cos\alpha = 13,244\cos 43.4° = \underline{9620 \text{ ft/sec}}$$

$$r\dot\theta = v_\theta = v\sin\alpha, \quad \dot\theta = \frac{v\sin\alpha}{r}, \text{ where}$$

$$r = \sqrt{b^2 + (a - r_{min})^2} = \sqrt{10,450^2 + (15,209 - 4159)^2}$$

$$= 15,208 \text{ mi}; \text{ so } \dot\theta = \frac{13,244\sin 43.4°}{15,208(5280)} = \underline{1.133(10^{-4})\ \frac{\text{rad}}{\text{sec}}}$$

$$\Sigma F_r = ma_r: \; -\cancel{m}\frac{gR^2}{r^2} = \cancel{m}(\ddot r - r\dot\theta^2)$$

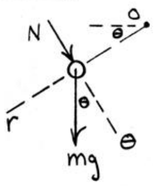
$$\ddot r = r\dot\theta^2 - \frac{gR^2}{r^2} = 15,208(5280)\left[(1.133)(10^{-4})\right]^2 - \frac{32.23(3959)^2}{15,208^2}$$

$$= \underline{-1.153 \text{ ft/sec}^2}$$

$$\Sigma F_\theta = ma_\theta: \; 0 = m(r\ddot\theta + 2\dot r\dot\theta)$$

$$\ddot\theta = -\frac{2\dot r\dot\theta}{r} = -\frac{2(9620)(1.133)(10^{-4})}{15,208(5280)} = \underline{-2.72(10^{-8})\ \frac{\text{rad}}{\text{sec}^2}}$$

▸3/100



$$\Sigma F_r = ma_r = m(\ddot r - r\dot\theta^2):$$

$$\cancel{m}g\sin\theta = \cancel{m}(\ddot r - r\omega_0^2)$$

$$\ddot r - \omega_0^2 r = g\sin\omega_0 t$$

Assume $r_h = Ce^{st}$ and substitute into equation to obtain $s_1 = -\omega_0$, $s_2 = \omega_0$. Also, assume a particular solution of form $r_p = D\sin\omega_0 t$, substitute, and obtain $D = -g/2\omega_0^2$.

$$\text{So} \quad r = r_h + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \frac{g}{2\omega_0^2}\sin\omega_0 t$$

Initial conditions:

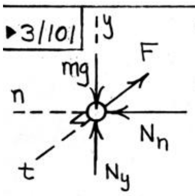
$$\begin{cases} r(0) = C_1 + C_2 = 0 \\ \dot r(0) = -\omega_0 C_1 + \omega_0 C_2 - \frac{g}{2\omega_0} = 0 \end{cases}$$

Solve for C_1 and C_2 to obtain

$$r = -\frac{g}{4\omega_0^2}e^{-\theta} + \frac{g}{4\omega_0^2}e^{\theta} - \frac{g}{2\omega_0^2}\sin\theta$$

$$\text{or} \quad \underline{r = \frac{g}{2\omega_0^2}\left[\sinh\theta - \sin\theta\right]}$$

▸3/101



$$\Sigma F_y = 0: \; N_y = mg$$

$$\Sigma F_n = ma_n: \; N_n = m\frac{v^2}{r}$$

$$F = \mu_k N_{tot} = \mu_k\sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$= \frac{\mu_k m}{r}\sqrt{r^2g^2 + v^4}$$

$$\Sigma F_t = ma_t: \; -\frac{\mu_k \cancel{m}}{r}\sqrt{r^2g^2 + v^4} = \cancel{m}v\frac{dv}{ds}$$

$$-\frac{\mu_k}{r}\int_0^s ds = \int_{v_0}^0 \frac{v\,dv}{\sqrt{v^4 + r^2g^2}} = \int_{v_0^2}^0 \frac{\frac{1}{2}dx}{\sqrt{x^2 + r^2g^2}}$$

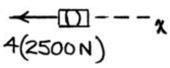
where $x = v^2$, $dx = 2v\,dv$

Integrating,

$$-\frac{\mu_k}{r}s = \frac{1}{2}\ln\left[x + \sqrt{x^2 + r^2g^2}\right]_{v_0^2}^0$$

$$\text{or} \quad \underline{s = \frac{r}{2\mu_k}\ln\left[\frac{v_0^2 + \sqrt{v_0^4 + r^2g^2}}{rg}\right]}$$

▸3/102 Motion from A to B:



$$\Sigma F_x = ma_x: \; -4(2500) = 1350a$$

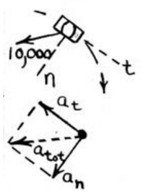
$$a = -7.407 \text{ m/s}^2, \quad v_B^2 - v_A^2 = 2a(x_B - x_A)$$

$$v_B^2 - 25^2 = 2(-7.407)(10)$$

$$v_B = 21.84 \text{ m/s}$$

$$\text{Beyond B:} \quad F = ma_{tot}, \quad a_{tot} = \frac{10,000}{1350}$$

$$= 7.407 \text{ m/s}^2$$



$$a_{tot} = \sqrt{a_n^2 + a_t^2} = \sqrt{\frac{v^4}{\rho^2} + a_t^2}$$

$$a_t = -\sqrt{a_{tot}^2 - \frac{v^4}{\rho^2}} = v\frac{dv}{ds}$$

$$\int_{10}^s ds = -\rho\int_{v_B}^0 \frac{v\,dv}{\sqrt{\rho^2 a_{tot}^2 - v^4}}$$

$$\text{Let } x = v^2: \; s - 10 = -\rho\int_{v_B^2}^0 \frac{dx/2}{\sqrt{\rho^2 a_{tot}^2 - x^2}}$$

$$s = 10 + \frac{\rho}{2}\sin^{-1}\left(\frac{v_B^2}{\rho a_{tot}}\right) = \underline{47.4 \text{ m}}$$

3/103 State ① : launch; State ② : apex

$$T_1 + U_{1-2} = T_2: \; \frac{1}{2}mv_0^2 - mgh = 0$$

$$\Rightarrow \underline{h = \frac{v_0^2}{2g}}$$

$$\text{For } v_0 = 50 \text{ m/s}: \; h = \frac{50^2}{2(9.81)} = \underline{127.4 \text{ m}}$$

3/104

$$\text{(a)} \quad U_{1-2} = \frac{1}{2}k(x_1^2 - x_2^2)$$

$$= \frac{1}{2}(3)(12)\left[\left(\frac{6}{12}\right)^2 - \left(\frac{3}{12}\right)^2\right] = \underline{3.38 \text{ ft-lb}}$$

$$\text{(b)} \quad U_{1-2} = -mgh = -14\left(\frac{9}{12}\right)\sin 15°$$

$$= \underline{-2.72 \text{ ft-lb}}$$

3/105 $T_A + U_{A\text{-}B} = T_B$

$\frac{1}{2}\cancel{m}v_A^2 + \cancel{m}gh = \frac{1}{2}\cancel{m}v_B^2$

$v_B^2 = v_A^2 + 2gh = 4^2 + 2(9.81)(1.8)$

$\underline{v_B = 7.16 \text{ m/s}}$

Knowledge of the shape of the track is unnecessary, as long as it is known that the cart passes the highest point.

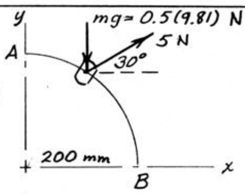
3/106 $T_A + U_{A\text{-}B} = T_B$

$\frac{1}{2}mv_A^2 + U_f + mgh = \frac{1}{2}mv_B^2$

$U_f = m\left(\frac{v_B^2}{2} - \frac{v_A^2}{2} - gh\right)$

$= 3\left(\frac{6^2}{2} - \frac{4^2}{2} - 9.81(1.8)\right) = \underline{-23.0 \text{ J}}$

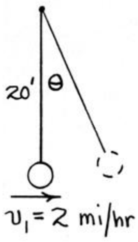
3/107



$U = \Delta T$: $5\cos 30°(0.2) - 5\sin 30°(0.2)$
$+ 0.5(9.81)(0.2)$
$= \frac{1}{2}0.5(v^2 - 0)$

$v^2 = 5.39 \text{ (m/s)}^2$, $\underline{v = 2.32 \text{ m/s}}$

3/108

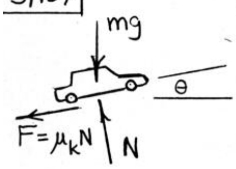


$T_1 + U_{1\text{-}2} = T_2$: $\frac{1}{2}\cancel{m}v_1^2 - \cancel{m}gh = 0$

$\frac{1}{2}\left[2\,\frac{5280}{3600}\right]^2 - 32.2\left[20(1-\cos\theta)\right] = 0$

$\underline{\theta = 6.63°}$

3/109



$\theta = \tan^{-1}\frac{6}{100} = 3.43°$

$T_A + U_{A\text{-}B} = T_B$

$\frac{1}{2}\cancel{m}v_0^2 - \mu_k\cancel{m}g\cos\theta\, s - \cancel{m}g\, s\sin\theta = 0$

$\frac{1}{2}\left(65\,\frac{5280}{3600}\right)^2 - 32.2s\left[+0.6\cos 3.43° + \sin 3.43°\right] = 0$

$\underline{s = 214 \text{ ft}}$

Going downhill $(B \to A)$: $T_B + U_{B\text{-}A} = T_A$

$\frac{1}{2}\cancel{m}v_0^2 - \mu_k\cancel{m}g\cos\theta\, s + \cancel{m}g\, s\sin\theta = 0$

$\frac{1}{2}\left(65\,\frac{5280}{3600}\right)^2 + 32.2s\left[-0.6\cos 3.43° + \sin 3.43°\right] = 0$

$\underline{s = 262 \text{ ft}}$

3/110 For collar, $U_{1\text{-}2} = \Delta T = 0$

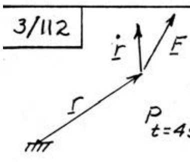
$U_{1\text{-}2} = 50\left(\frac{50-30}{12}\right) - 30\,\frac{40}{12}\sin 30° - \frac{1}{2}k\left(\frac{6}{12}\right)^2 = 0$

$\underline{k = 267 \text{ lb/ft}}$

3/111 $U_{1\text{-}2} = \Delta T$; $2\left(\frac{1}{2}kx^2\right) = \frac{1}{2}mv^2 - 0$

$k = \frac{1}{2}\,\frac{mv^2}{x^2} = \frac{1}{2}\,\frac{3500}{32.2}\left(\frac{5}{30}44\right)^2\frac{1}{(6/12)^2}\,\frac{1}{12} = \underline{974 \text{ lb/in.}}$

3/112



Power $P = \mathbf{F}\cdot\dot{\mathbf{r}}$

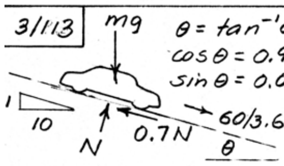
$P = (40\mathbf{i} - 20\mathbf{j} - 36\mathbf{k})\cdot(8\mathbf{i} + 2.4t\mathbf{j} - 1.5t^2\mathbf{k})$

$P_{t=4s} = (40\mathbf{i} - 20\mathbf{j} - 36\mathbf{k})\cdot(8\mathbf{i} + 9.6\mathbf{j} - 24\mathbf{k})$

$= 320 - 192 + 864 = 992 \text{ W}$

or $\underline{P = 0.992 \text{ kW}}$

3/113



$\theta = \tan^{-1}0.1 = 5.71°$

$\cos\theta = 0.9950$

$\sin\theta = 0.0995$

$N = mg\cos\theta$
$= 0.9950(9.81)m$
$= 9.76m$

$U = \Delta T$; $-0.7(9.76m)s + 9.81m(0.0995)s = -\frac{m}{2}(16.67)^2$

$5.86s = 138.9$, $\underline{s = 23.7 \text{ m}}$

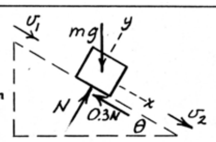
3/114 $P = W\dot{y}$ where $\dot{y} = v\sin\theta$

$\theta = \tan^{-1}0.05 = 2.86°$, $\sin\theta = 0.0499$

$P = 200\left(\frac{15}{30}44\right)0.0499 = 219.7 \text{ ft-lb/sec}$

or $P = \frac{219.7}{550} = \underline{0.400 \text{ hp}}$

3/115 $\Sigma F_y = 0$: $N - mg\cos\theta = 0$



$U = \Delta T$: $(mg\sin\theta - 0.3mg\cos\theta)\frac{1.5}{\sin\theta}$
$= \frac{1}{2}m(0.14^2 - 0.40^2)$

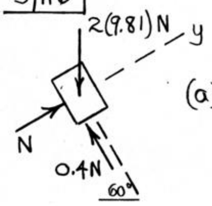
$v_1 = 0.40 \text{ m/s}$

$v_2 = 0.14 \text{ m/s}$

$1.5(9.81)\left(1 - \frac{0.3}{\tan\theta}\right) = -0.0702$

$\tan\theta = 0.299$, $\underline{\theta = 16.62°}$

3/116



$\Sigma F_y = 0$: $N - 2(9.81)\cos 60° = 0$

$N = 9.81 \text{ N}$

(a) $U_{1\text{-}2} = \Delta T$: $2(9.81)(0.5\sin 60°)$
$- 0.4(9.81)(0.5) = \frac{1}{2}2v^2$

$\underline{v = 2.56 \text{ m/s}}$

(b) $U_{1\text{-}3} = \Delta T$: $2(9.81)(0.5+x)\sin 60° - 0.4(9.81)(0.5+x)$
$-\frac{1}{2}(1600)x^2 = 0$

$800x^2 - 13.07x - 6.53 = 0$

$x = 0.0989 \text{ m}$ or $\underline{x = 98.9 \text{ mm}}$

3/117 $P = Wh/\Delta t$

or $P = \frac{120(9)}{5} / 550 = \underline{0.393 \text{ hp}}$

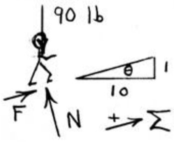
Conversions : $h = 9 \text{ ft}\left(\frac{0.3048 \text{ m}}{\text{ft}}\right) = 2.74 \text{ m}$

$W = 120 \text{ lb}\left(\frac{4.4482 \text{ N}}{\text{lb}}\right) = 534 \text{ N}$

$P = \frac{Wh}{\Delta t} = \frac{534(2.74)}{5} = \underline{293 \text{ watts}}$

Check : $0.393 \text{ hp}\left(\frac{745.7 \text{ watts}}{\text{hp}}\right) = 293 \text{ watts} \checkmark$

3/118



$v_B = 5\frac{5280}{3600} = 7.33 \text{ ft/sec}$

$v_B^2 = 2as$, $a = \frac{7.33^2}{2(50)} = 0.538 \frac{\text{ft}}{\text{sec}^2}$

$\theta = \tan^{-1}(0.1) = 5.71°$

$\xrightarrow{+} \Sigma F = ma$: $F - 90\sin 5.71° = \frac{90}{32.2}(0.538)$

$F = 10.46 \text{ lb}$

$P = Fv = 10.46(7.33) = 76.7 \frac{\text{ft-lb}}{\text{sec}}$

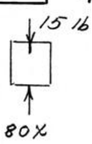
or $P = 76.7/550 = \underline{0.1394 \text{ hp}}$

3/119 Net power required $= 30(140)(24)/33{,}000$

$= 3.05 \text{ hp}$

Mechanical efficiency $= \frac{\text{Power required}}{\text{Power supplied}} = \frac{3.05}{4.00} = \underline{0.764}$

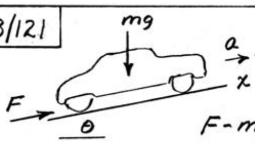
3/120 $U_{1-2} = \Delta T$; $15(18+2) - \frac{1}{2}80(2^2) = \frac{1}{2}\frac{15}{32.2}v^2(12)$



$300 - 160 = 2.795v^2$, v in ft/sec

$v^2 = 50.09$, $\underline{v = 7.08 \text{ ft/sec}}$

3/121



$\theta = \tan^{-1}0.1$, $\sin\theta = 0.0995$

$\Sigma F_x = ma_x$ where $v^2 = 2ax$

$F - mg\sin\theta = m\frac{v^2}{2x}$

$P = Fv = mgv\sin\theta + \frac{mv^3}{2x}$

$= 1500(9.81)\frac{50000}{3600}0.0995 + \frac{1500(50000/3600)^3}{2(100)}$

$= 20\,336 + 20\,094 = 40\,430 \text{ W}$

or $\underline{P = 40.4 \text{ kW}}$

3/122 For $x = 75$ mm, $U = \Delta T$

$\frac{1}{2}(0.075)R_{max} = \frac{1}{2}(0.25)(600)^2$, $R_{max} = 1.2 \text{ MN}$

For $x = 25$ mm, $R = \frac{25}{75}(1.2) = 0.4 \text{ MN}$ or $0.4(10^6)$ N

$U = \Delta T$; $\frac{1}{2}(0.025)(0.4)10^6 = \frac{1}{2}(0.25)(\overline{600}^2 - v^2)$

$v^2 = 320(10^3)\ (\text{m/s})^2$, $\underline{v = 566 \text{ m/s}}$

3/123 Power output = rate of doing work

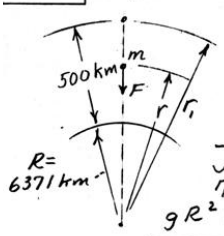
$= 300(9.81)(2) - 100(9.81)(4)$

$= 1962 \text{ J/s (W)}$

$= 1.962 \text{ kW}$

Efficiency $e = \frac{\text{Power output}}{\text{Power input}} = \frac{1.962}{2.20} = \underline{0.892}$

3/124 F = gravitational force $= Gmm_e/r^2 = gR^2m/r^2$



$U = \Delta T$

$\int_{r_1}^{r} F(-dr) = \frac{1}{2}mv^2 - 0$

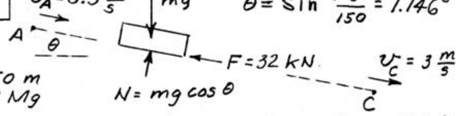
$-\int_{r_1}^{r}\frac{gR^2m}{r^2}dr = \frac{1}{2}mv^2$, $-gR^2\left(-\frac{1}{r}\right)_{r_1}^{r} = \frac{v^2}{2}$

$gR^2\left(\frac{1}{r} - \frac{1}{r_1}\right) = \frac{v^2}{2}$, $v = R\sqrt{2g\left(\frac{1}{r} - \frac{1}{r_1}\right)}$

$v = 6371\sqrt{\frac{2(9.825)}{1000}\left(\frac{1}{6371+500-100} - \frac{1}{6371+500}\right)} \ \frac{\text{km}}{\text{s}}$

$= 6371\sqrt{4.2237(10^{-8})} = \underline{1.309 \text{ km/s}}$

3/125



$\theta = \sin^{-1}\frac{3}{150} = 1.146°$

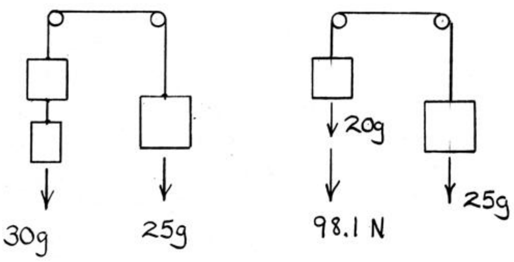
$\overline{AC} = 150$ m

$m = 68$ Mg

$U_{1-2} = \Delta T$; $68(10^3)(9.81)(3) - 32(10^3)x = \frac{1}{2}68(10^3)(3^2 - 0.5^2)$

$2001 - 32x = 297.5$, $\underline{x = 53.2 \text{ m}}$

3/126 Active-force diagrams for system :



$U = \Delta T$ for system :

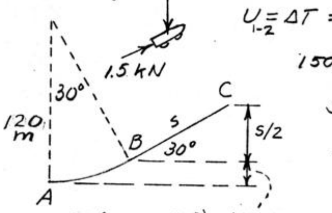
(a) $(30-25)(9.81)2 = \frac{1}{2}(30+25)v^2$

$\underline{v = 1.889 \text{ m/s}}$

(b) $[(20-25)(9.81) + 98.1]2 = \frac{1}{2}(20+25)v^2$

$\underline{v = 2.09 \text{ m/s}}$

3/127 $mg = 981$ N



$120(1-\cos 30°) = 16.08$ m

$\overline{AB} = r\theta = 120\frac{\pi}{6} = 62.8$ m

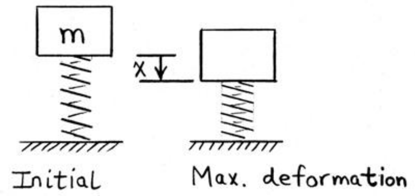
$U_{1-2} = \Delta T = 0$ since $T_C = T_A = 0$

$1500(62.8) - 981(16.08 + \frac{s}{2}) = 0$

$s = 2(94248 - 15771)/981$

$= \underline{160.0 \text{ m}}$

3/128



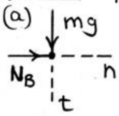
The maximum force $F = kx$ occurs when x is a maximum with $\dot{x} = 0$.

$U_{1-2} = \Delta T$: $mgx - \frac{1}{2}kx^2 = 0$, $x = \frac{2mg}{k}$

So $F = kx = \underline{2mg}$

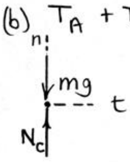
3/129 $T_A + U_{A-B} = T_B$: $0 + 2mgR = \frac{1}{2}mv_0^2$, $v_B^2 = 4gR$

(a)



$\Sigma F_n = ma_n$: $N_B = m\,\frac{4gR}{R} = \underline{4mg}$

(b) $T_A + U_{A-C} = T_C$: $0 + 3mgR = \frac{1}{2}mv_C^2$, $v_C^2 = 6gR$



$\Sigma F_n = ma_n$: $N_C - mg = m\,\frac{6gR}{R}$

$\underline{N_C = 7mg}$

(c) Call stopping point E :

$T_A + U_{A-E} = T_E$

$0 + 2mgR - mg\left(\frac{1}{2}s\right) - \mu_k\frac{\sqrt{3}}{2}mgs = 0$

$s = \underline{\frac{4R}{1+\mu_k\sqrt{3}}}$

(Note: Normal force on incline is $N = mg\cos 30° = \frac{\sqrt{3}}{2}mg$)

3/130 Let s = distance down incline before reversal of direction.

$U_{1-2} = 110(2)(10+s-s) - 300\,(10+s-s)\frac{5}{13} = 1046$ ft-lb

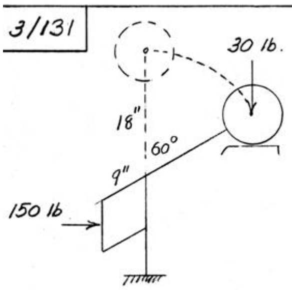
$\Delta T = \frac{1}{2}\frac{300}{32.2}\left[v^2 - (\pm 9)^2\right] = 4.66v^2 - 377$ ft-lb

$U_{1-2} = \Delta T$: $1046 = 4.66v^2 - 377$

$\underline{v = 17.48 \text{ ft/sec}}$

The initial kinetic energy is positive regardless of the velocity direction.

3/131



$U_{1-2} = \Delta T$

$150\left(\frac{9}{12}\sin 60°\right) - 30\,\frac{18}{12}(1-\cos 60°)$

$= \frac{1}{2}\,\frac{30}{32.2}(v^2 - 0^2)$

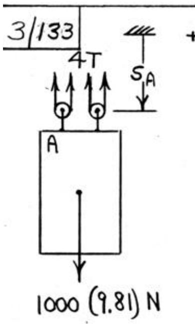
$v^2 = 160.8$, $\underline{v = 12.68 \text{ ft/sec}}$

3/132 $U_{1-2} = \Delta T$; $mg(0.8 - 1.2\cos 60°)$

$= \frac{1}{2}m(v_C^2 - 3^2)$

$9.81(0.20) = \frac{1}{2}(v_C^2 - 9)$, $v_C^2 = 12.92$, $\underline{v_C = 3.59 \text{ m/s}}$

3/133



$+\downarrow \Sigma F = 0$: $9810 - 4T = 0$, $T = 2450$ N

Length of cable $L = 4s_A$ + constants

$\dot{L} = 4v_A = 4(-3) = -12$ m/s

$P_{out} = -T\dot{L} = -2450(-12) = 29\,400$ watts

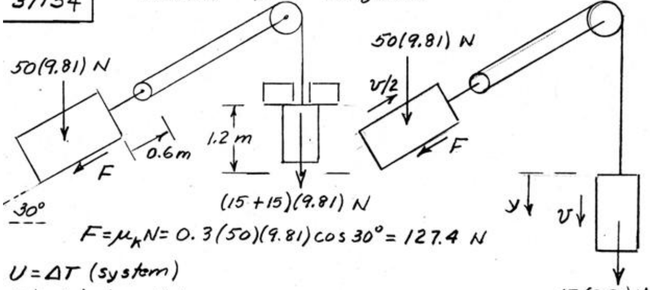
or $P_{out} = 29.4$ kW

$e = \frac{P_{out}}{P_{in}}$, $P_{in} = \frac{P_{out}}{e} = \frac{29.4}{0.8}$

$\underline{P_{in} = 36.8 \text{ kW}}$

3/134

Active - force diagrams



$F = \mu_k N = 0.3(50)(9.81)\cos 30° = 127.4$ N

$U = \Delta T$ (system)

First interval:

$30(9.81)1.2 - [50(9.81)0.5 + 127.4]\frac{1.2}{2} = \frac{1}{2}30v^2 + \frac{1}{2}50(v/2)^2$

$v^2 = 6.096$ (m/s)^2, $v = 2.469$ m/s

Second interval

$15(9.81)(y) - [50(9.81)0.5 + 127.4]\frac{y}{2} = 0 - \frac{1}{2}15(6.097)$

$-\frac{1}{2}50\left(\frac{6.097}{4}\right)$

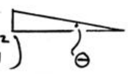
$y = 2.14$ m, $s = \frac{1}{2}(1.2 + 2.14) = \underline{1.67 \text{ m}}$

3/135 $U = \Delta T$; $-\int_0^4 (3x^2 + 60x)\,dx = \frac{1}{2}\frac{48}{32.2}(0 - v^2)/12$

$x^3 + 30x^2\Big]_0^4 = \frac{288}{32.2}v^2$, v in ft/sec.

$v^2 = \frac{32.2}{288}(64 + 480) = 60.82$ (ft/sec)^2, $\underline{v = 7.80 \text{ ft/sec}}$

3/136 $\theta = \tan^{-1}\frac{6}{100} = 3.43°$



$U_{1-2} = \Delta T$: $U_f + mgh = \frac{1}{2}m(v_2^2 - v_1^2)$

$U_f = -1400(9.81)(200\sin 3.43°)$

$+\frac{1}{2}1400\left[\left(\frac{20}{3.6}\right)^2 - \left(\frac{100}{3.6}\right)^2\right]$

$= -683\,000$ J or -683 kJ

Energy lost $\underline{Q = 683 \text{ kJ}}$

3/137 The power output of the drivetrain is

$P_{out} = Fv = 560\left(\frac{90}{3.6}\right) = 14\,000$ W

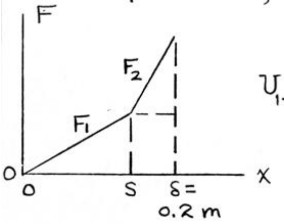
The power input to the drivetrain:

$P_{in} = \frac{P_{out}}{e} = \frac{14\,000}{0.70} = 20\,000$ W

So the motor output $\underline{P = 20 \text{ kW}}$

3/138 $F_1 = 300x$, $F_2 = 300x + 150(x-s)$

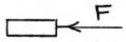
$= 450x - 150s$



$U_{1-2} = -\frac{1}{2}300s^2 - \int_s^{0.2m}(450x-150s)dx$

$= -75s^2 + 30s - 9$ J

$U_{1-2} = \Delta T$:



$-75s^2 + 30s - 9 = \frac{1}{2}(0.5)(0-s^2)$

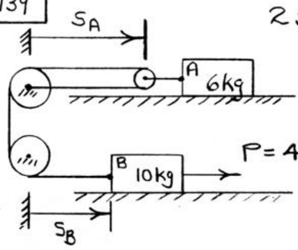
$75s^2 - 30s + 2.75 = 0$

$s = 0.1423$ m or 0.257 m

0.257 m > 200 mm, impossible

So $\underline{s = 142.3 \text{ mm}}$

3/139



$2s_A + s_B$ = constant

$2v_A + v_B = 0$

(velocities)

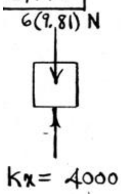
$T_1 + U_{1-2} = T_2$

$0 + 40(0.8) = \frac{1}{2}6v_A^2 + \frac{1}{2}10(2v_A)^2$

$\underline{v_A = 1.180 \text{ m/s}}$

$\underline{v_B = 2v_A = 2.36 \text{ m/s}}$ } Speeds

3/140



$U_{1-2} = \Delta T = 0$:

$6(9.81)(0.1+\delta) - \int_{0.05}^{0.05+\delta} 4000x\,dx = 0$

$2000\delta^2 + 141.1\delta - 5.89 = 0$

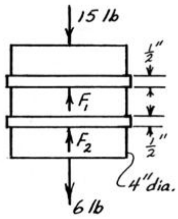
$\delta = 0.0294$ m or $\underline{\delta = 29.4 \text{ mm}}$

(Positive result taken from quadratic formula)

3/141 $F_1 + F_2 = \mu_k(\text{Area})p$

$= 0.15 \times 2 \times 4\pi \times \frac{1}{2}p$

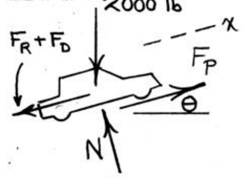
$= 1.885p$ lb



$U = \Delta T$: $(15 + 6 - 1.885p)\frac{10}{12} = \frac{1}{2}\frac{6}{32.2}(8^2 - 0^2)$

Solve & get $p = \underline{7.34 \text{ lb/in.}^2}$

3/142 mg = 2000 lb



$\theta = 0$ or $\theta = \tan^{-1}\frac{6}{100} = 3.43°$

$F_D = kv^2$: $50 = k(60)^2$

$\Rightarrow k = 0.01389 \frac{\text{lb-hr}^2}{\text{mi}^2}$

& $F_D = 0.01389v^2$

$\Sigma F_x = 0$: $F_P - F_R - F_D - mg\sin\theta = 0$

$F_P = F_R + F_D + mg\sin\theta$

(a) $\underline{\theta = 0}$ $v = 30$ mi/hr : $F_D = 0.01389(30^2) = 12.50$ lb

$F_P = F_R + F_D = 50 + 12.50 = 62.5$ lb

$P_{30} = Fv = 62.5\left(30\frac{5280}{3600}\right)/550 = \underline{5 \text{ hp}}$

$v = 60$ mi/hr : $F_D = 50$ lb, $F_P = F_R + F_D = 100$ lb

$P_{60} = Fv = 100\left(60\frac{5280}{3600}\right)/550 = \underline{16 \text{ hp}}$

(b) $\underline{\theta = 3.43°}$ $F_P = 50 + 50 + 2000\sin 3.43° = 220$ lb

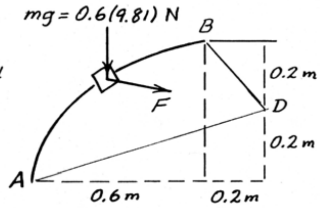
$P_{up} = 220\left(60\frac{5280}{3600}\right)/550 = \underline{35.2 \text{ hp}}$

Down: $F_P = 50 + 50 - 2000\sin 3.43° = -19.78$ lb

$P_{down} = -19.78\left(60\frac{5280}{3600}\right)/550 = \underline{-3.17 \text{ hp}}$ (brakes!)

(c) $\Sigma F_x = 0$: $50 + kv^2 - 2000\sin 3.43° = 0$, $\underline{v = 70.9 \frac{\text{mi}}{\text{hr}}}$

3/143 $U = \Delta T$



$U = F(\overline{AD} - \overline{BD}) - 0.6(9.81)0.4$

$= F(0.825 - 0.283) - 2.35$

$= 0.542F - 2.35$ J

$\Delta T = \frac{1}{2}m(v_B^2 - v_A^2)$

$= \frac{1}{2}0.6(4^2 - 0) = 4.8$ J

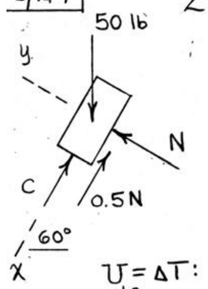
$\overline{AD} = \sqrt{0.8^2 + 0.2^2} = 0.825$ m

$\overline{BD} = \sqrt{0.2^2 + 0.2^2} = 0.283$ m

Thus $0.542F - 2.35 = 4.8$,

$\underline{F = 13.21 \text{ N}}$

3/144



$\Sigma F_y = 0$: $N - 50\cos 60° = 0$, $N = 25$ lb

Displacement is $3 + \frac{4}{12} = 3.33$ ft

$U_{1-2} = (50\sin 60° - 0.5 \cdot 25)\,3.33$

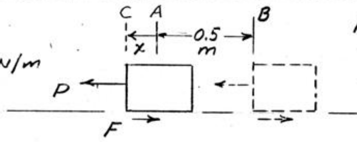
$- \frac{1}{12}\int_0^{4''}(100x + 9x^2)\,dx$

$= 20.0$ ft-lb

$U_{1-2} = \Delta T$: $20.0 = \frac{1}{2}\frac{50}{32.2}(v^2 - 2^2)$

$\underline{v = 5.46 \text{ ft/sec}}$

3/145



$k = 300$ N/m

$F = \mu_k N$
$= 0.30(10)(9.81)$
$= 29.43$ N

(a)
From B to A: $U_{1-2} = \Delta T$

$\frac{1}{2}(300)(0.5)^2 - 29.43(0.5) = \frac{1}{2}(10)v^2$

$v^2 = 4.557\ (m/s)^2$, $\underline{v = 2.13\ m/s}$

(b) From A to C; $U_{1-2} = \Delta T$

$-\frac{1}{2}(300)x^2 - 29.43x = 0 - \frac{1}{2}(10)(4.557)$

$x^2 + 0.1962x - 0.1519 = 0$

$x = \frac{-0.1962}{2} \pm \frac{1}{2}\sqrt{(0.1962)^2 + 4(0.1519)}$

$= -0.0981 \pm 0.4019$, $\underline{x = 0.304\ m}$ $(x = -0.48)$

3/146 $P = Fv$; $F = ma$, so $P = mav$
& $a = \frac{P}{mv}$.

But $v\,dv = a\,ds$, so $mv^2 dv = P\,ds$

$\int_{v_1}^{v_2} mv^2 dv = \int_0^s P\,ds$; $\frac{m}{3}(v_2^3 - v_1^3) = Ps$

$\underline{v_2 = \left(\frac{3Ps}{m} + v_1^3\right)^{1/3}}$

3/147 $U'_{1-2} = 0 = \Delta T + \Delta V_g + \Delta V_e$

$\Delta T = \frac{1}{2}3(v^2 - 0) = \frac{3}{2}v^2$

$\Delta V_g = -3(9.81)(0.8) = -23.5$ J

$\Delta V_e = \frac{1}{2}200\left[\left(\sqrt{0.8^2 + 0.6^2} - 0.4\right)^2 - (0.8 - 0.4)^2\right]$

$= 20$ J

So $0 = \frac{3}{2}v^2 - 23.5 + 20$, $\underline{v = 1.537\ m/s}$

3/148 For the system, $U'_{1-2} = 0$, so $\Delta V_g + \Delta T = 0$

$-mg\left(\frac{18}{12}\right) + \frac{1}{2}(2m)(v^2 - 0) = 0$

$\underline{v = 6.95\ ft/sec}$

3/149 Establish datum @ A.

(a) $T_A + V_A = T_B + V_B$

$0 + 0 = \frac{1}{2}mv_B^2 - mgh_B$

$v_B = \sqrt{2gh_B} = \sqrt{2(9.81)(4.5)} = \underline{9.40\ m/s}$

(b) State F : spring fully compressed

$T_A + V_A = T_F + V_F$

$0 + 0 = 0 - mgh_f + \frac{1}{2}k\delta^2$

$\delta = \sqrt{\frac{2mgh_f}{k}} = \sqrt{\frac{2(1.2)(9.81)(3)}{24000}} = 0.0542$ m

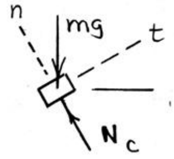
or $\underline{\delta = 54.2\ mm}$

3/150 Establish datum @ A.

$T_A + V_A = T_C + V_C$: $0 + 0 = \frac{1}{2}mv_C^2 - mgh_C$

$v_C = \sqrt{2gh_C} = \sqrt{2(9.81)(3 + 1.5\cos 30°)}$

$= 9.18$ m/s



(a) $\Sigma F_n = m\frac{v^2}{\rho}$: $N_C - 1.2(9.81)\cos 30° = 1.2\frac{9.18^2}{1.5}$

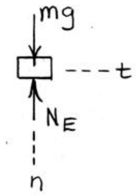
$\underline{N_C = 77.7\ N}$

(b) $\Sigma F_n = 0$: $N_C - 1.2(9.81)\cos 30° = 0$

$\underline{N_C = 10.19\ N}$

$T_A + V_A = T_E + V_E$: $0 + 0 = \frac{1}{2}mv_E^2 - mgh_E$

$v_E = \sqrt{2gh_E} = \sqrt{2(9.81)(3)} = 7.67$ m/s



$\Sigma F_n = m\frac{v^2}{\rho}$: $-N_E + 1.2(9.81) = 1.2\frac{7.67^2}{1.5}$

$\underline{N_E = -35.3\ N}$ (down)

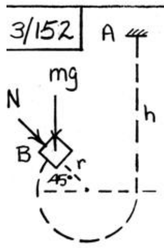
3/151 $\Delta T + \Delta V_e + \Delta V_g = 0$, $\Delta T = 0$

$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}500(0.050^2 - 0.100^2) = -1.875$ J

$\Delta V_g = mg\Delta h = 2(9.81)h = 19.62h$

Thus $0 - 1.875 + 19.62h = 0$, $h = 0.0956$ m or $\underline{h = 95.6\ mm}$

3/152



$U'_{1-2} = \Delta T + \Delta V_g = 0$

$\frac{1}{2}mv^2 - mg\left(h - \frac{r}{\sqrt{2}}\right) = 0$

$v^2 = 2g\left(h - \frac{r}{\sqrt{2}}\right)$

$\Sigma F_n = ma_n$: $N + \frac{mg}{\sqrt{2}} = m\frac{v^2}{r}$

$\Rightarrow N = mg\left[\left(\frac{h}{r} - \frac{1}{\sqrt{2}}\right)2 - \frac{1}{\sqrt{2}}\right]$

$= mg\left[2\frac{h}{r} - \frac{3}{\sqrt{2}}\right]$

With $m = 0.25$ kg, $r = 0.15$ m, & $h = 0.6$ m,

$\underline{N = 14.42\ N}$

3/153 $T_A + V_A = T_B + V_B$, datum @ B

$0 + mgR + \frac{1}{2}k\left[R\sqrt{2} - R\right]^2 = \frac{1}{2}mv_B^2 + 0$

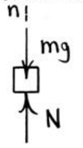
$$v_B = \sqrt{2gR + \frac{kR^2}{m}(3-2\sqrt{2})}$$

$T_A + V_A = T_C + V_C$, datum @ C

$0 + 2mgR + \frac{1}{2}k\left[R\sqrt{2} - R\right]^2 = \frac{1}{2}mv_C^2 + 0$

$$v_C = \sqrt{4gR + \frac{kR^2}{m}(3-2\sqrt{2})}$$

Kinetics at C:



$\Sigma F_n = ma_n$: $N - mg = m\frac{v_C^2}{R}$

$$\Rightarrow N = m\left[5g + \frac{kR}{m}(3-2\sqrt{2})\right]$$

3/154 For the system, $T_1 + V_1 + U'_{1-2} = T_2 + V_2$

$\frac{1}{2}m\cancel{v_1^2}^{0} + \frac{1}{2}kx_1^2 + 0 = \frac{1}{2}mv^2 + \frac{1}{2}kx_2^2 - mgh$,

where the datum is the initial position and h is the drop distance. Note that the spring deflection runs at twice that of the cylinder. Numbers:

$\frac{1}{2}6(12)\left[\frac{3}{12}\right]^2 = \frac{1}{2}\frac{100}{32.2}v^2 + \frac{1}{2}6(12)\left[\frac{3+2(\frac{1}{2})}{12}\right]^2 - 100\left(\frac{1/2}{12}\right)$

$$v = 1.248 \text{ ft/sec}$$

3/155 (a) $\Delta T + \Delta V_g = 0$

$\frac{1}{2}\frac{5}{32.2}v^2 + \frac{1}{2}\frac{10}{32.2}\left(\frac{12}{18}v\right)^2 + 5\frac{18}{12}\sin 60° - 10\frac{12}{12}\sin 60° = 0$

$0.1467v^2 = 2.165$, $v^2 = 14.76$ (ft/sec)^2

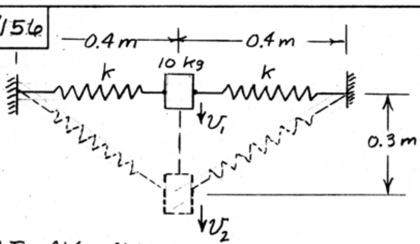
$$v = 3.84 \text{ ft/sec}$$

(b) For entire interval $\Delta T = 0$, $\Delta V_g + \Delta V_e = 0$

$-2.165(12) + \frac{1}{2}(200)x^2 = 0$, $x^2 = 0.2598$ (in)^2

$$x = 0.510 \text{ in.}$$

3/156



$k = 800$ N/m
$v_1 = 2$ m/s
Initial stretch
$\delta_1 = 0.1$ m
$\delta_2 = 0.5 - 0.3 = 0.2$ m

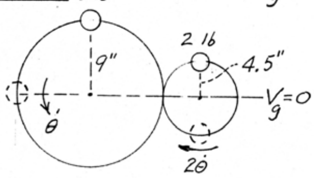
$\Delta T + \Delta V_g + \Delta V_e = 0$

$\frac{1}{2}10(v_2^2 - 2^2) - 10(9.81)(0.3) + \frac{2}{2}800(0.2^2 - 0.1^2) = 0$

$5v_2^2 = 20 + 29.43 - 24$, $v_2^2 = 5.086$ (m/s)^2

$$v_2 = 2.26 \text{ m/s}$$

3/157



$\Delta T + \Delta V_g = 0$; $\Delta T = \frac{1}{2}\frac{3}{32.2}\left(\frac{9}{12}\dot\theta\right)^2$

$+ \frac{1}{2}\frac{2}{32.2}\left(\frac{4.5}{12}[2\dot\theta]\right)^2$

$= 0.04367\dot\theta^2$ ft-lb

$\Delta V_g = -3\left(\frac{9}{12}\right) - 2\left(\frac{4.5+4.5}{12}\right)$

$= -\frac{15}{4} = -3.75$ ft-lb

Thus $0.04367\dot\theta^2 - 3.75 = 0$, $\dot\theta^2 = 85.87$ (rad/sec)^2

$$\dot\theta = 9.27 \text{ rad/sec}$$

3/158 Let m be the mass of the car

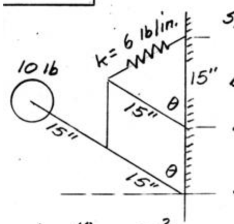
$U'_{1-2} = \Delta T + \Delta V_g$: $0 = \frac{1}{2}m(v^2 - v_0^2) + mgy$

$v^2 = v_0^2 - 2gy$

$a_n = \frac{v^2}{\rho}$: $\frac{v_0^2}{\rho_0} = \frac{v_0^2 - 2gy}{\rho}$, $\rho = \rho_0\left(1 - \frac{2gy}{v_0^2}\right)$

For car to remain in contact with the track at the top, $a_n > g$, so for constant a_n, $v_0^2/\rho_0 > g$ so $v_{0_{min}} = \sqrt{\rho_0 g}$

3/159 For the interval from $\theta = 60°$ to $\theta = 90°$;



Spring stretch is

$\delta = 15\sqrt{2} - 15 = 6.21$ in.

$\Delta V_e = \frac{1}{2}6(6.21)^2 = 115.8$ in.-lb

$\Delta V_g = -mg\Delta h = -10(30)\cos 60°$

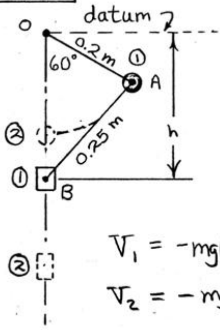
$= -150$ in.-lb.

$\Delta T + \Delta V_g + \Delta V_e = 0$

$\frac{1}{2}\frac{10}{32.2(12)}v^2 - 150 + 115.8 = 0$, $v^2 = 2642$, $v = 51.4$ $\frac{\text{in.}}{\text{sec}}$

or $v = 4.28$ ft/sec

3/160



$T_1 + V_1 = T_2 + V_2$

$T_1 = 0$

$T_2 = \frac{1}{2}mv_A^2$

Note that

$h = 0.2\cos 60° + \sqrt{0.25^2 - (0.2\sin 60°)^2}$

$= 0.280$ m

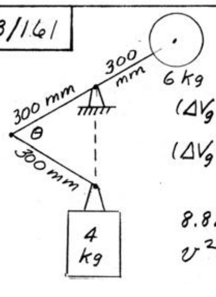
$V_1 = -mg(0.2\cos 60) - mg(0.280) = -0.380mg$

$V_2 = -mg(0.2) - mg(0.45) = -0.650mg$

So $-0.380mg = \frac{1}{2}mv_A^2 - 0.650mg$

$$v_A = 2.30 \text{ m/s}$$

3/161



For motion from $\theta = 60°$ to $\theta = 180°$

$\Delta V_g + \Delta T = 0$

$(\Delta V_g)_{6\,kg} = 6(9.81)(0.3)(1 - \sin 30°) = 8.829$ J

$(\Delta V_g)_{4\,kg} = -4(9.81)(2)(0.3)(1 - \sin 30°)$

$= -11.772$ J

$8.829 - 11.772 + \frac{1}{2}6v^2 + 0 = 0$

$v^2 = 0.981$ (m/s)^2, $v = 0.990$ m/s

3/162 Establish datum at release point.

$T_A + V_A = T_B + V_B$

$0 + \frac{1}{2}k_A x_A^2 = 0 + mg(x_A + d + x_B) + \frac{1}{2}k_B x_B^2$

$\frac{1}{2}(48)(12)\left(\frac{5}{12}\right)^2 = 14\left(\frac{5+14+x_B}{12}\right) + \frac{1}{2}(10)(12)\left(\frac{x_B}{12}\right)^2$

$\underline{x_B = 6.89 \text{ in.}}$

The fact that $x_B > x_A$ is due to the difference in spring stiffnesses (along with the particular value $d = 20 - 6 = 14''$). Note that $d = 14''$ is the distance which the collar moves when out of contact with the springs.

3/163 A force analysis reveals that A will move down & B will move up.

Kinematics : $3v_A = 2v_B$ (speeds)

$T_1 + V_1 = T_2 + V_2$, datum @ initial position

$0 + 0 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B\left(\frac{3}{2}v_A\right)^2 + m_B g h_B - m_A g h_A$

$0 = \frac{1}{2}(40)v_A^2 + \frac{1}{2}8\,\frac{9}{4}v_A^2 + 8(9.81)(1) - 40(9.81)\left(\frac{2}{3}(1)\sin 20°\right)$

$\underline{v_A = 0.616 \text{ m/s}}$, $\underline{v_B = \frac{3}{2}v_A = 0.924 \text{ m/s}}$

3/164 *1st interval of motion (0.4 m)* $\Delta T + \Delta V_g = 0$ *for system*

$\frac{1}{2}(4+6+8)v^2 + 9.81 \times 0.4(8-4-6) = 0,\ v^2 = 0.872\ (\text{m/s})^2$

$v = 0.934$ m/s

2nd interval for 6- & 8-kg cylinders $\Delta T + \Delta V_g = 0$

$0 - \frac{1}{2}(6+8)(0.872) + 9.81(h-0.4)(8-6) = 0,\ h = 0.711$ m

or $\underline{h = 711 \text{ mm}}$

Kinetic energy of collar is dissipated into heat & sound during impact with bracket.

3/165 *Constant total energy is* $E = T_A + V_{g_A} = T_P + V_{g_A}$

Thus $\frac{1}{2}mv_A^2 - \frac{mgR^2}{r_A} = \frac{1}{2}mv_P^2 - \frac{mgR^2}{r_P}$

$v_A^2 = v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)$, $\underline{v_A = \sqrt{v_P^2 - 2gR^2\left(\frac{1}{r_P} - \frac{1}{r_A}\right)}}$

3/166 $U'_{1\text{-}2} = \Delta T + \Delta V_e + \Delta V_g$ *for system*

$U'_{1\text{-}2} = 50(1.5)\cos 30° = 64.95$ J

$\Delta T = \frac{1}{2}\,2\,v^2 = v^2$

$\Delta V_e = \frac{1}{2}30\left[\left(\sqrt{2^2+1.5^2} - 1.5\right)^2 - (2-1.5)^2\right] = 11.25$ J

$\Delta V_g = 2(9.81)1.5 = 29.43$ J

So $64.95 = v^2 + 11.25 + 29.43$, $v^2 = 24.27$, $\underline{v = 4.93 \frac{\text{m}}{\text{s}}}$

3/167 $\Delta T + \Delta V_g = 0$, $V_g = -\frac{mgR^2}{r}$

Mean radius of earth is $R = 6371$ km

$g = 9.825(3600)^2/1000 = 127.3(10^3)$ km/h^2

Thus

$\frac{1}{2}m\left(v_B^2 - [24\,000]^2\right) + 127.3(10^3)(6371)^2 m\left(-\frac{1}{6500} + \frac{1}{7000}\right) = 0$

$\frac{1}{2}v_B^2 - 288(10^6) + 5167(10^9)(-0.01099)(10^{-3}) = 0$

$v_B^2 = 2[288 + 56.8]10^6 = 690(10^6)$, $\underline{v_B = 26\,300 \text{ km/h}}$

3/168 $\Delta T + \Delta V_g + \Delta V_e + U_f = 0$

Spring elongation at A is $\delta_A = \sqrt{3^2+4^2} - 2 = 3$ ft

" " " B " $\delta_B = \sqrt{3^2+3^2} - 2 = 2.24$ ft

$\Delta V_e = \frac{1}{2}k(\delta_B^2 - \delta_A^2) = \frac{1}{2}2(2.24^2 - 3^2) = -3.97$ ft-lb

$\Delta T = \frac{1}{2}m(v_B^2 - v_A^2) = \frac{1}{2}\frac{5}{32.2}(10^2 - 6^2) = 4.97$ ft-lb

$\Delta V_g = W\Delta z = 5(0-4) = -20$ ft-lb

Thus $4.97 - 20 - 3.97 + U_f = 0$, $\underline{U_f = 19.00 \text{ ft-lb (loss)}}$

$U_f = F_{av}s$, $F = \frac{19.00}{5} = \underline{3.80 \text{ lb}}$

3/169 Ellipse eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}}$

$= \sqrt{1 - \frac{0.6^2}{0.8^2}} = 0.661$

$r_{min} = a(1-e) = 0.8(1-0.661) = 0.271$ m

$r_{max} = a(1+e) = 0.8(1+0.661) = 1.329$ m

$T_A + V_A = T_C + V_C$

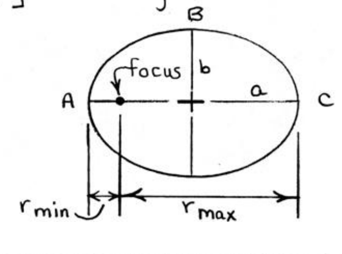
$\frac{1}{2}mv_A^2 + 0 = 0 + \frac{1}{2}kx_C^2$

$\frac{1}{2}(0.4)v_A^2 = \frac{1}{2}(3)[1.329 - 0.271]^2$, $v_A = 2.90$ m/s

Then $T_A + V_A = T_B + V_B$

$\frac{1}{2}(0.4)(2.90)^2 + 0 = \frac{1}{2}(0.4)v_B^2 + \frac{3}{2}\left\{\left[(0.8-0.271)^2 + (0.6)^2\right]^{1/2} - 0.271\right\}^2$

$\underline{v_B = 2.51 \text{ m/s}}$



3/170 $U'_{1\text{-}2} = \Delta T + \Delta V_g = 0$

$\Delta T = \frac{1}{2}m\left[v^2 - \left(2000\,\frac{44}{30}\right)^2\right]$

$\Delta V_g = -mgR^2\left(\frac{1}{R} - \frac{1}{2R}\right) = -\frac{mgR}{2}$

$= -\frac{1}{2}m\ 5.32\ (1080)(5280)$

So $v^2 - \left(2000\,\frac{44}{30}\right)^2 = 5.32\,(1080)(5280)$

$\underline{v = 6240 \text{ ft/sec}}$ or $\underline{4250 \text{ mi/hr}}$

3/171 $U'_{1-2} = 0$ so $T_1 + V_{g_1} = T_2 + V_{g_2}$

Take datum $V_g = 0$ at ground level.

$T_1 = \frac{1}{2}\frac{175+10}{32.2} v^2 = 2.87 v^2$, $T_2 = 0$

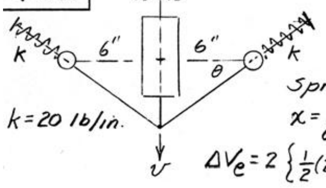
$V_{g_1} = (175+10)\frac{42}{12} = 648$ ft-lb

$V_{g_2} = 175(18) + 10(8) = 3230$ ft-lb

So $2.87 v^2 + 648 = 0 + 3230$

$\underline{v = 30.0 \text{ ft/sec}}$ or $\underline{20.4 \text{ mi/hr}}$

3/172



For interval $\theta = 0$ to $\theta = 30°$

$\Delta V_g = -10(6\tan 30°)$
$= -34.64$ in.-lb

Spring compression is

$x = \frac{6}{\cos 30°} - 6 = 0.928$ in.

$\Delta V_e = 2\left\{\frac{1}{2}(20)(0.928)^2\right\} = 17.23$ in.-lb

$\Delta T = \frac{1}{2}\frac{10}{32.2}\frac{1}{12}v^2$, ($v$ in in./sec)

$= 0.01294 v^2$

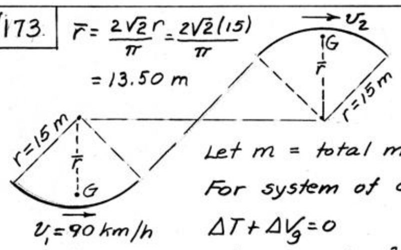
$\Delta T + \Delta V_g + \Delta V_e = 0$; $0.01294 v^2 - 34.64 + 17.24 = 0$

$v^2 = 1345$ (in./sec)^2, $v = 36.7$ in/sec

or $\underline{v = 3.06 \text{ ft/sec}}$

3/173 $\bar{r} = \frac{2\sqrt{2}r}{\pi} = \frac{2\sqrt{2}(15)}{\pi}$

$= 13.50$ m



Let m = total mass of train

For system of cars

$\Delta T + \Delta V_g = 0$

$\frac{1}{2}m(v_2^2 - v_1^2) + mg(2\bar{r}) = 0$, $v_2^2 = v_1^2 - 4g\bar{r}$

$v_2^2 = \left[\frac{90(1000)}{3600}\right]^2 - 4(9.81)(13.50) = 625 - 529.9 = 95.07\ (\text{m/s})^2$

$v_2 = 9.75$ m/s or $\underline{v_2 = 35.1 \text{ km/h}}$

3/174 $U' = \Delta T + \Delta V_g = 0$ where $V_g = -\frac{mgR^2}{r}$

$\Delta V_g = -9.825\, m[6371(10^3)]^2\left(\frac{1}{(2500+6371)10^3} - \frac{1}{(2200+6371)10^3}\right)$

$= 1.573(10^6)m$

$\Delta T = \frac{1}{2}m\left(v_B^2 - \left[\frac{25\,000\times10^3}{3600}\right]^2\right)$

Thus $\frac{1}{2}v_B^2 - \frac{1}{2}\left[\frac{25\,000}{3.6}\right]^2 + 1.573(10^6) = 0$

$v_B^2 = 45.08(10^6)\left(\frac{m}{s}\right)^2$, $v_B = 6714$ m/s

or $\underline{v_B = 24\,170 \text{ km/h}}$

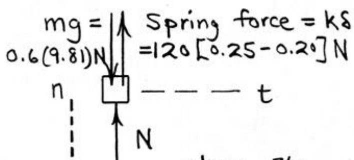
3/175 $T_A + V_A = T_B + V_B$, datum @ B.

$0 + 0.6(9.81)(0.5) + \frac{1}{2}120\left[\sqrt{0.25^2+0.5^2} - 0.2\right]^2$

$= \frac{1}{2}(0.6)v_B^2 + \frac{1}{2}120[0.25 - 0.20]^2$

$\underline{v_B = 5.92 \text{ m/s}}$

Kinetics at B:



Radius of curvature $\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$y = kx^2$; $0.5 = k(0.5)^2 \Rightarrow k = 2$

$y = 2x^2$, $\frac{dy}{dx} = 4x$, $\frac{d^2y}{dx^2} = 4$

When $x = 0$, $\rho = \frac{[1+0^2]^{3/2}}{4} = 0.25$ m

$\Sigma F_n = ma_n$: $N + 120(0.05) - 0.6(9.81)$

$= 0.6\,\frac{5.92^2}{0.25}$

$\underline{N = 84.1 \text{ N}}$

3/176 $\Delta T + \Delta V_g + \Delta V_e = 0$

$\Delta T = \frac{1}{2}m\dot{y}^2$; $\Delta V_g = -mgy$

$\Delta V_e = 2\left\{\frac{1}{2}kx^2\right\} = k(y\sin\theta)^2 = ky^2(1-\cos^2\theta)$

$= ky^2(1 - c^2/b^2)$

$\frac{1}{2}m\dot{y}^2 - mgy + ky^2(1 - c^2/b^2) = 0$

$\underline{\dot{y} = \sqrt{2y\left(g - \frac{k}{m}y\,\frac{b^2-c^2}{b^2}\right)}}$

$y_{max} = y$ for $\dot{y} = 0$, so $2gy - \frac{2k}{m}y^2(1 - c^2/b^2) = 0$

Hence ($y_{min} = 0$), $\underline{y_{max} = \frac{mg}{k}\,\frac{b^2}{b^2-c^2}}$

3/177 $x^2 + y^2 = 0.9^2$, $x\dot{x} + y\dot{y} = 0$, $v_A = -\dot{y} = \frac{x}{y}\dot{x} = \frac{x}{y}v_B$

$\Delta T + \Delta V_g = 0$; $\frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mg\left(y - \frac{0.9}{\sqrt{2}}\right) = 0$

$\dot{x}^2\left(1 + \frac{x^2}{y^2}\right) = 2(9.81)\left(\frac{0.9}{\sqrt{2}} - y\right)$, $\dot{x}^2\,\frac{x^2+y^2}{y^2} = 19.62\left(\frac{0.9}{\sqrt{2}} - y\right)$

$0.9^2\dot{x}^2 = 19.62\left(\frac{0.9}{\sqrt{2}}y^2 - y^3\right)$

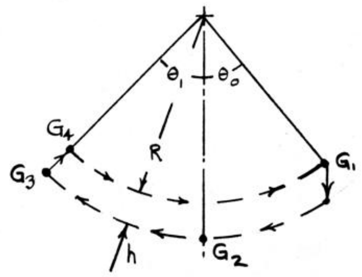
For max. $\dot{x}$, $\frac{d(\dot{x}^2)}{dy} = \frac{19.62}{0.81}\left(\frac{1.8}{\sqrt{2}}y - 3y^2\right) = 0$

so $y\left(\frac{1.8}{\sqrt{2}} - 3y\right) = 0$, $y = 0.6/\sqrt{2}$ m

& $\dot{x}^2 = \frac{19.62}{0.81}\left(\frac{0.9}{\sqrt{2}}\,\frac{0.36}{2} - \frac{0.108}{\sqrt{2}}\right) = \frac{19.62\sqrt{2}}{30}$

$v_{B_{max}} = \dot{x} = \sqrt{\frac{19.62\sqrt{2}}{30}} = \underline{0.962 \text{ m/s}}$

3/178



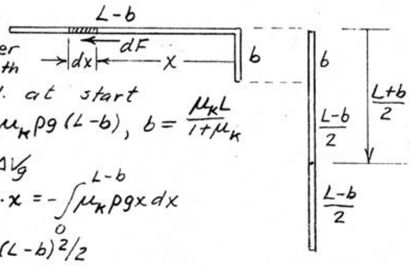
$U'_{1-3} = \Delta T + \Delta V_g$

$U'_{1-2} = U'_{2-3} = 0$, $\Delta T = 0$, so $V_{g_1} = V_{g_3}$

Thus $R\cos\theta_0 = (R+h)\cos\theta_1$

$$\underline{\underline{\theta_1 = \cos^{-1}\left(\frac{R}{R+h}\cos\theta_0\right)}}$$

▸3/179



ρ = mass per unit length

For equil. at start

$\rho g b = \mu_k \rho g (L-b)$, $b = \frac{\mu_k L}{1+\mu_k}$

$U = \Delta T + \Delta V_g$

$U = -\int dF\cdot x = -\int_0^{L-b} \mu_k \rho g x\, dx$

$= -\mu_k \rho g (L-b)^2/2$

$\Delta T = \frac{1}{2}\rho L v^2$

$\Delta V_g = -\rho g (L-b)\left(\frac{L+b}{2}\right)$

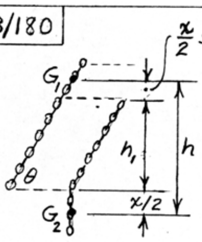
Thus $-\mu_k \rho g (L-b)^2/2 = \frac{1}{2}\rho L v^2 - \rho g \frac{L^2-b^2}{2}$

$v^2 = g(1-b/L)(L+b-\mu_k[L-b])$; Now substitute b

so $v^2 = g\left(1-\frac{\mu_k}{1+\mu_k}\right)\left(L\left[1+\frac{\mu_k}{1+\mu_k}\right] - \mu_k\left[L - \frac{\mu_k L}{1+\mu_k}\right]\right)$

$= \frac{gL}{1+\mu_k}$, $\underline{v = \sqrt{\frac{gL}{1+\mu_k}}}$

3/180



$h_1 = (L-x)\sin\theta$

$h = (L-x)\sin\theta + \frac{x}{2}\sin\theta + \frac{x}{2}$

$= L\sin\theta + \frac{x}{2}(1-\sin\theta)$

Let ρ = mass per unit length

$\Delta V_g + \Delta T = 0$

ΔV_g is that of the length x dropping a distance h

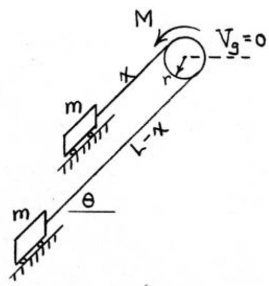
$\Delta V_g = -\rho g x h = -\rho g\left[Lx\sin\theta + \frac{x^2}{2}(1-\sin\theta)\right]$

$\Delta T = \frac{1}{2}\rho L v^2$

Thus $-\rho g\left[Lx\sin\theta + \frac{x^2}{2}(1-\sin\theta)\right] + \frac{1}{2}\rho L v^2 = 0$

$$v = \sqrt{2gx\left[\sin\theta + \frac{x}{2L}(1-\sin\theta)\right]}$$

▸3/181



$U'_{1-2} = \Delta T + \Delta V$

$U'_{1-2} = M\frac{x}{r}$

$\Delta V_e = 0$

$V_{g_2} = -g\left[m(L-x) + mx + \rho(L-x)\frac{L-x}{2} + \rho x\frac{x}{2}\right]\sin\theta$

$= -g\sin\theta\left\{mL + \frac{\rho}{2}\left[(L-x)^2 + x^2\right]\right\}$

$V_{g_1} = -g\sin\theta\left\{mL + \rho L\frac{L}{2}\right\}$

$\Delta V_g = -g\sin\theta\left\{mL + \frac{\rho}{2}\left[(L-x)^2+x^2\right] - mL - \frac{\rho L^2}{2}\right\}$

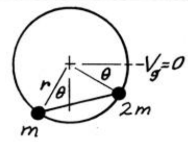
$= -g\sin\theta\left\{\frac{\rho}{2}\left[2x^2 - 2Lx\right]\right\}$

$\Delta T = \frac{1}{2}(2m+\rho L)v^2$

$\therefore M\frac{x}{r} = \frac{1}{2}(2m+\rho L)v^2 - g\sin\theta\left\{\frac{\rho}{2}\left[2x^2-2Lx\right]\right\}$

Solving, $\underline{v = \sqrt{\frac{2}{2m+\rho L}}\sqrt{\frac{Mx}{r} - \rho g x(L-x)\sin\theta}}$

▸3/182 For the unit $U' = \Delta T + \Delta V_g = 0$



$\Delta V_g = (-2mgr\sin\theta - mgr\cos\theta) - (-mgr + 0)$

$= mgr(-2\sin\theta - \cos\theta + 1)$

so $\frac{1}{2}3mv^2 - 0 + mgr(-2\sin\theta - \cos\theta + 1) = 0$

or $v^2/gr = \frac{2}{3}(2\sin\theta + \cos\theta - 1)$

(a) Rod is horiz. when $\theta = 45°$

$v^2/gr = \frac{2}{3}(2\sin 45° + \cos 45° - 1) = 0.748$, $\underline{v_{45°} = 0.865\sqrt{gr}}$

(b) $\frac{d}{d\theta}\left(\frac{v^2}{gr}\right) = \frac{2}{3}(2\cos\theta - \sin\theta) = 0$ for max v^2 & hence max v

$\tan\theta = 2$, $\theta = \tan^{-1}2 = 63.4°$

so $v^2_{max}/gr = \frac{2}{3}(2\sin 63.4° + \cos 63.4° - 1) = 0.824$

$\underline{v_{max} = 0.908\sqrt{gr}}$

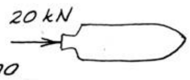
(c) $\theta = \theta_{max}$ when $T = \Delta T = 0$ so $2\sin\theta + \cos\theta - 1 = 0$

$2\sqrt{1-\cos^2\theta} = 1 - \cos\theta$, $5\cos^2\theta - 2\cos\theta - 3 = 0$

$\cos\theta = 0.2 \pm 0.8 = 1$ or -0.6, $\theta = 0$ or $\underline{\theta_{max} = 126.9°}$

3/183

$\int \Sigma F\, dt = \Delta G$

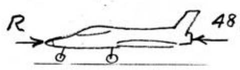


$(20\,000)(3\times 60) = 30\,000(v - 24\,000)\frac{1000}{3600}$

$\underline{v = 24\,400 \text{ km/h}}$

3/184

$\int \Sigma F dt = m\Delta v$



$[48(10^3) - R]10 = 6450\left(\frac{250\times 1000}{3600} - 0\right)$

$R = 3208$ N or $\underline{R = 3.21 \text{ kN}}$

3/185 $\int F\,dt = m\Delta v$

$2(26)10^3 t = 90(10^3)[28\,100 - 28\,000]/3.6$

$\underline{t = 48.1\ \text{s}}$

3/186 $\begin{cases} \underline{v} = 1.5t^3\underline{i} + (2.4-3t^2)\underline{j} + 5\underline{k} & (\text{m/s}) \\ \dot{\underline{v}} = 4.5t^2\underline{i} - 6t\underline{j} & (\text{m/s}^2) \end{cases}$

At $t = 2$s : $\begin{cases} \underline{v} = 12\underline{i} - 9.6\underline{j} + 5\underline{k} \ \text{m/s} \\ \dot{\underline{v}} = 18\underline{i} - 12\underline{j} \ \text{m/s}^2 \end{cases}$

Then $\underline{G} = m\underline{v} = 1.2(12\underline{i} - 9.6\underline{j} + 5\underline{k})$

$= \underline{14.40\underline{i} - 11.52\underline{j} + 6\underline{k}\ \text{kg·m/s}}$

$G = \sqrt{14.40^2 + 11.52^2 + 6^2} = \underline{19.39\ \text{kg·m/s}}$

$\Sigma\underline{F} = \dot{\underline{G}}$: $\underline{R} = m\dot{\underline{v}} = 1.2(18\underline{i} - 12\underline{j})$

$= \underline{21.6\underline{i} - 14.4\underline{j}\ \text{N}}$

3/187 Conservation of system linear momentum:

$\xrightarrow{+}$ $0.075(600) = 50.075 v_f$, $v_f = 0.899$ m/s

Initial energy $T_1 = \frac{1}{2}(0.075)(600)^2 = 13\,500$ J

Final energy $T_2 = \frac{1}{2}(50.075)(0.899)^2 = 20.2$ J

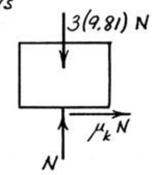
Absolute energy loss $|\Delta E| = T_1 - T_2 = \underline{13\,480\ \text{J}}$

Percent lost : $n = \frac{|\Delta E|}{T_1}(100\%) = \underline{99.9\%}$

3/188. For system of bullet and block $\Delta G = 0$, $G_1 = G_2$:

$(0.060)(600) = (0.060)(400) + 3v$

Initial velocity of block is $v = 4$ m/s



For block $U = \Delta T$: $-\mu_k(3\times 9.81)(2.70) = \frac{1}{2}3(0-4^2)$

$\underline{\mu_k = 0.302}$

3/189 $\Delta G = 0$; $150{,}000\times 2 + 120{,}000\times 3$

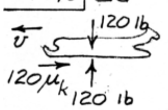
$= (150{,}000 + 120{,}000)v$, $\underline{v = 2.44\ \text{mi/hr}}$

$|\Delta E| = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 - \frac{1}{2}(m_A+m_B)v^2$

$= \frac{1}{2(32.2)}\left(\frac{44}{30}\right)^2[150{,}000\times 2^2 + 120{,}000\times 3^2 - 270{,}000\times \overline{2.44}^2]$

$\underline{= 2230\ \text{ft-lb loss}}$

3/190 $\Delta G = 0$; $100(15) = 120v$, $v = 12.5$ ft/sec



$v^2 = 2as$; $a = F/m = \frac{120\mu_k}{120/g} = \mu_k g$

$32.2\mu_k = \frac{\overline{12.5}^2}{2(80)}$, $\underline{\mu_k = 0.030}$

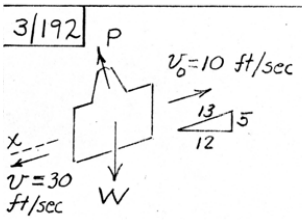
3/191 No difference between cases (a) & (b).

$G_1 = G_2$: $mv = (3m)v'$, $v' = \frac{v}{3}$

$T = \frac{1}{2}mv^2$, $T' = \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{1}{6}mv^2$

$n = \frac{T-T'}{T} = \frac{\frac{1}{2}mv^2 - \frac{1}{6}mv^2}{\frac{1}{2}mv^2} = \underline{\frac{2}{3}}$

3/192

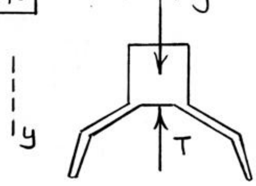


$\int \Sigma F_x\,dt = m\Delta v_x$

$W\left(\frac{5}{13}\right)t = \frac{W}{32.2}(30 - [-10])$

$\underline{t = 3.23\ \text{sec}}$

3/193 $mg = 200(1.62)$ N



$\int \Sigma F_y\,dt = m\Delta v_y$:

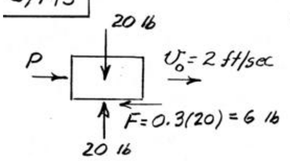
$200(1.62)(5) - [\frac{1}{2}2(800) + 2(800)] = 200(v-6)$

$\underline{v = 2.10\ \text{m/s}}$

3/194 $\Delta G = 0$; $600(18\,000) - \{400v_3 + 200(18\,060)\} = 0$

$\underline{v_3 = 17\,970\ \text{km/h}}$

3/195

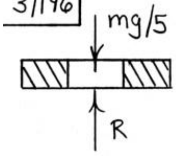


$\int_0^t \Sigma F\,dt = m\Delta v$

$16(0.2) + 8(0.2) - 6(0.4) = \frac{20}{32.2}(v-2)$

$\underline{v = 5.86\ \text{ft/sec}}$

3/196



Washer :

$+\downarrow$ $\frac{m}{5}v + \left[\frac{mg}{5} - R\right]\Delta t = 0$

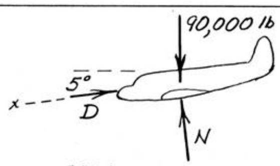
$\underline{R = \frac{m}{5}\left(\frac{v}{\Delta t} + g\right)}$

Initial energy : $\frac{1}{2}\frac{6m}{5}v^2 = \frac{3}{5}mv^2$

Final energy : $\frac{1}{2}mv^2$

$n = \frac{\frac{3}{5} - \frac{1}{2}}{3/5}(100\%) = \underline{16.67\%}$

3/197



$\int \Sigma F_x\,dt = \Delta G_x$:

$(90{,}000 \sin 5° - D)120 = \frac{90{,}000}{32.2}(360 - 400)\frac{5280}{3600}$

$\underline{D = 9210\ \text{lb}}$

3/198 $\Delta G = 0;\ (0.140)(600) - [0.140 + 3\times 0.100]v = 0$

$v = 190.9\ m/s$

$|\Delta E| = \frac{1}{2}(0.140)(600)^2 - \frac{1}{2}(0.140 + 0.300)(190.9)^2$

$= 25.2(10^3) - 8.018(10^3) = \underline{17.18(10^3)\ J}$ loss

3/199 $\int F\,dt = m\Delta v$

$(50{,}000 \cos 20°)\,t = \frac{150{,}000 \times 2240}{32.2}\ \frac{1 \times 1.151}{1}\ \frac{44}{30}$

$46{,}985\,t = 17.62 \times 10^6$

$t = 375$ sec or $\underline{t = 6.25\ min}$

3/199 $\int F\,dt = m\Delta v$

$(50{,}000 \cos 20°)\,t = \frac{150{,}000 \times 2240}{32.2}\ \frac{1 \times 1.151}{1}\ \frac{44}{30}$

$46{,}985\,t = 17.62 \times 10^6$

$t = 375$ sec or $\underline{t = 6.25\ min}$

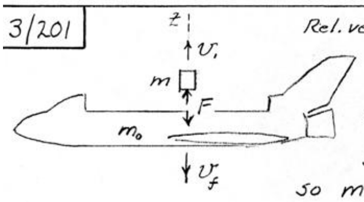
3/200 $\Delta G = 0;\ 320(28) - (320 + 20\times 18)v = 0$

Initial velocity of chain is $\underline{v = 13.18\ m/s}$

$\int \Sigma F\,dt = m\Delta v;\ (20\times 18)9.81(0.7)t = (320 + 20\times 18)(13.18)$

$\underline{t = 3.62\ s}$

3/201



Rel. velocity is

$v_1 + v_f = 0.3\ m/s$ --(1)

$\int F\,dt = m v_1$

$\int -F\,dt = m_f(-v_f)$

so $m v_1 = m_o v_f$

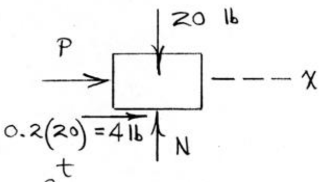
$800\,v_1 = 90\,000\,v_f$ ----(2)

Solve (1) & (2) & get $v_f = 0.3 - \frac{90000}{800} v_f$

$\underline{v_f = 0.00264\ m/s}$

so $F_{av}\int_0^4 dt = 90\,000\,(0.00264)$, $F_{av} = \frac{90(2.64)}{4} = \underline{59.5\ N}$

3/202

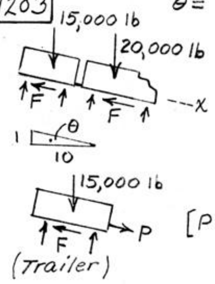


$\xrightarrow{+}\ m v_1 + \int_0^t \Sigma F\,dt = 0:$

$-\frac{20}{32.2}(4) + 5(0.2) + 2.5(t - 0.2) + 4t = 0$

$\underline{t = 0.305\ sec}$

3/203



$\theta = \tan^{-1} 0.1 = 5.71°,\ \sin\theta = 0.0995$

$\int \Sigma F_x\,dt = m\Delta v_x$

$[35{,}000 \times 0.0995 - 2F]5$

$= \frac{35{,}000}{32.2}\left(0 - 20\frac{44}{30}\right)$

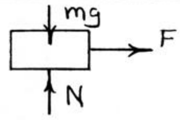
$F = 4930$ lb

$[P - 4930 + 15{,}000 \times 0.0995]5$

$= \frac{15{,}000}{32.2}\left(0 - 20\frac{44}{30}\right)$

$\underline{P = 704\ lb\ (tension)}$

3/204



$\xrightarrow{+}\ m v_1 + \int \Sigma F\,dt = m v_2:$

$0 + \int_0^t F_0 e^{-bt}\,dt = m v$

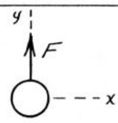
$\underline{v = \frac{F_0}{mb}\left(1 - e^{-bt}\right)}$, $\underline{v \rightarrow \frac{F_0}{mb}\ \text{as}\ t \rightarrow \infty}$

$\frac{ds}{dt} = \frac{F_0}{mb}\left(1 - e^{-bt}\right)$

$\int_{s_0=0}^{s} ds = \int \frac{F_0}{mb}\left(1 - e^{-bt}\right)dt$

$\underline{s = \frac{F_0}{mb}\left[t + \frac{1}{b}\left(e^{-bt} - 1\right)\right]}$

3/205



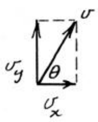
$\int \Sigma F_y\,dt = \Delta G_y:$

$\int_0^4 \left(2 + \frac{3t^2}{4}\right)dt = 2.4\left(v_y - \left[-\frac{3}{5}5\right]\right)$

$2t + \frac{t^3}{4}\Big|_0^4 = 2.4(v_y + 3),\ v_y = 7\ m/s$

$\int \Sigma F_x\,dt = \Delta G_x:\ 0 = 2.4\left(v_x - \frac{4}{5}5\right),\ v_x = 4\ m/s$ constant

$v = \sqrt{4^2 + 7^2} = \underline{8.06\ m/s}$, $\theta = \tan^{-1}\frac{7}{4} = \underline{60.3°}$



3/206 Impact velocity $v_0 = \sqrt{2gh} = \sqrt{2(9.81)(1.4)}$

$= 5.24\ m/s$

$\Delta G = 0;\ 450(5.24) + 0 = (450 + 240)v$

$\underline{v = 3.42\ m/s}$

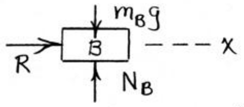
Impulse of weights is negligible compared with impulse of impact forces.

3/207 (a) $m_A v_A = (m_A + m_B) v'$

$v' = \frac{m_A}{m_A + m_B} v_A = \frac{4000/g}{(4000+2000)/g} 20$

$= 13.33$ mi/hr (19.56 ft/sec)

(c) Car B :



$m_B v_B + R\Delta t = m_B v' : 0 + R(0.1) = \frac{2000}{32.2}(19.56)$

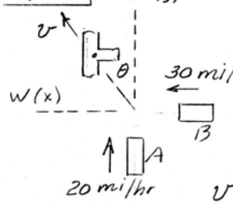
$R = 12,150$ lb

(The force which car B exerts on car A is 12,150 lb to the left, by Newton's Third Law.)

(b) $a_A = \frac{\Delta v}{\Delta t} = \frac{19.56 - 20\left(\frac{5280}{3600}\right)}{0.1} = -97.8 \frac{\text{ft}}{\text{sec}^2}$

$a_B = \frac{\Delta v}{\Delta t} = \frac{19.56 - 0}{0.1} = 195.6 \frac{\text{ft}}{\text{sec}^2}$

3/208



$\Delta G_x = 0;\ \frac{3200}{g}(30) = \frac{(3200+3400)}{g} v_x$

$v_x = 14.55$ mi/hr

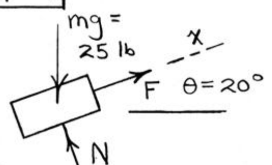
$\Delta G_y = 0;\ \frac{3400(20)}{g} = \frac{(3200+3400)}{g} v_y$

$v_y = 10.30$ mi/hr.

$v = \sqrt{(14.55)^2 + (10.30)^2} = 17.82$ mi/hr

$\theta = \tan^{-1}\frac{v_x}{v_y} = \tan^{-1}\frac{14.55}{10.30} = 54.7°$

3/209



$F = b + 10 \sin 6t$

(a) $m v_{x_1} + \int_{t_1}^{t_2} \Sigma F_x \, dt = m v_{x_2}$:

$0 - (mg \sin\theta)\Delta t + \int_0^{\Delta t} F\, dt = m v$

$-25 \sin 20°(1.5) + \left[5t - \frac{10}{6}\cos 6t\right]_0^{1.5} = \frac{25}{32.2} v$

$v = -2.76$ ft/sec

(b) b must equal $mg \sin\theta$

or $b = 25 \sin 20° = 8.55$ lb

3/210 $G_1 = G_2$: $m_s v_s + m_m v_m = (m_s + m_m) v$

$1000(2000)\underline{j} + 10(5000)\left[\frac{+5\underline{i} - 4\underline{j} - 2\underline{k}}{\sqrt{5^2 + 4^2 + 2^2}}\right] = (1000+10)\underline{v}$

$\underline{v} = 36.9\underline{i} + 1951\underline{j} - 14.76\underline{k}$ m/s

The angle between v_s and v is

$\beta = \cos^{-1}\frac{\underline{v}\cdot\underline{v}_s}{v v_s}$

$= \cos^{-1}\left[\frac{(36.9\underline{i} + 1951\underline{j} - 14.76\underline{k})\cdot 2000\underline{j}}{\sqrt{36.9^2 + 1951^2 + 14.76^2}\ 2000}\right]$

$= 1.167°$

3/211 $\int F dt = F t = m\Delta v$

$\underline{F} = \frac{0.20}{0.04}\left([18\cos 20°]\underline{i} + [18 \sin 20°]\underline{j} - [-12\underline{i}]\right)$

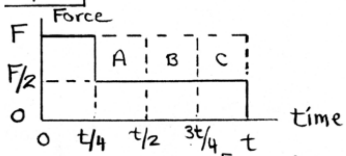
$= 5(18 \times 0.9397\underline{i} + 18 \times 0.3420\underline{j} + 12\underline{i})$

$= 30(4.819\underline{i} + 1.026\underline{j})$ N

$F = 30\sqrt{4.819^2 + 1.026^2} = 147.8$ N

$\beta = \tan^{-1} v_y/v_x = \tan^{-1}\frac{1.026}{4.819} = 12.02°$

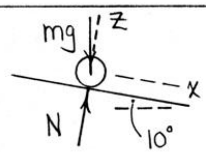
3/212



Solid area is 5/8 of nominal area, so $n = 62.5\%$

In order to compensate, areas A, B, & C must be added after time t, so the extra time $t' = \frac{3}{4} t$.

3/213 $\int F_x dt = m\Delta v_x$:



$(mg \sin 10°)2 = m[v_x - (-3 \sin 15°)]$

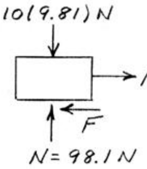
$v_x = 2.63$ m/s

$\int F_y dt = m\Delta v_y$: $0 = m[v_y - 3\cos 15°]$

$v_y = 2.90$ m/s

$v = \sqrt{v_x^2 + v_y^2} = 3.91$ m/s

3/214



$F_s = \mu_s N = 0.6(98.1) = 58.9$ N

$F_k = \mu_k N = 0.4(98.1) = 39.2$ N

Block does not move until $P = F_s$ or $25t = 58.9$, $t = 2.35$ s

Then F becomes 39.2 N

$\int \Sigma F dt = m\Delta v$; $\int_{2.35}^{4}(25t - 39.2)dt = 10(v - 0)$

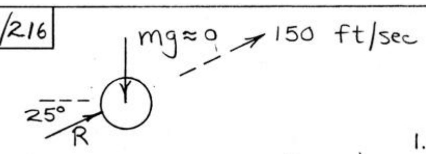
$\left.\frac{25t^2}{2} - 39.2t\right]_{2.35}^{4} = 10v$, $10v = 66.1$, $v = 6.61$ m/s

3/215 $\int F_x\,dt = m\Delta v_x$: $0.2t = \dfrac{1.2}{32.2}\left[v_x - (-10\sin 30^\circ)\right]$

$v_x = \dfrac{dx}{dt} = 5.37t - 5$

$\int_0^0 dx = \int_0^t (5.37t-5)\,dt$, $\underline{t = 1.863 \text{ sec}}$

3/216



$\overset{+}{\nearrow}$ $R\Delta t = mv$: $R(0.001) = \dfrac{1.62/16}{32.2}(150)$

$\underline{R = 472 \text{ lb}}$

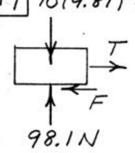
$\overset{+}{\nearrow}$ $R = ma$: $472 = \dfrac{1.62/16}{32.2}\,a$

$\underline{a = 150{,}000 \text{ ft/sec}^2 \ (4660g)}$

$v^2 - v_0^2 = 2ad$: $150^2 - 0^2 = 2(150{,}000)\,d$

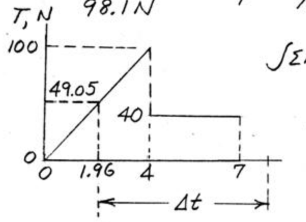
$\underline{d = 0.075 \text{ ft or } 0.900 \text{ in.}}$

3/217



Block begins to move when $T = F = \mu W = 0.5(98.1) = 49.05\text{ N}$

which occurs at $t_1 = \dfrac{49.05}{100}\,4 = 1.96 \text{ s}$



$\int \Sigma F\,dt = m\Delta v$

Max. velocity reached by block occurs at $t = 4$ s

$\dfrac{(100-49.05)}{2}(4-1.96) = 10(v-0)$

$\underline{v_{max} = 5.19 \text{ m/s}}$

For total motion $\Delta v = 0$, so

$\dfrac{100+49.05}{2}(4-1.96) + 40(7-4) - 49.05\,\Delta t = 0$

$\underline{\Delta t = 5.54 \text{ s}}$

3/218 $\Delta G = 0$, $G_1 = G_2$

$\left(\dfrac{2/16}{32.2} + 0\right)2000 = \dfrac{2/16+50}{32.2}\,v_2$, $v_2 = 4.99$ ft/sec

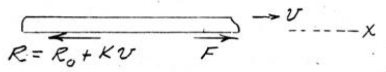
$U = \Delta T$: $v_2 = \sqrt{2gh}$, $4.99^2 = 2(32.2)(6)(1-\cos\theta)$ where $h = 6(1-\cos\theta)$

$\cos\theta = 0.936$, $\underline{\theta = 20.7^\circ}$

% energy loss $= \dfrac{\frac{1}{2}m_1 v_1^2 - (m_1+m_2)gh}{\frac{1}{2}m_1 v_1^2} \times 100\% = \left(1 - \dfrac{m_1+m_2}{m_1}\dfrac{2gh}{v_1^2}\right)100\%$

$= \left[1 - \dfrac{2/16+50}{2/16}\,\dfrac{2(32.2)6(1-0.936)}{2000^2}\right]100\% = \underline{99.8\%}$

3/219



$\Sigma F\,dt = m\,dv$, $(F - R_0 - Kv)\,dt = m\,dv$

$\int_0^t dt = \int_0^v \dfrac{m\,dv}{F-R_0-Kv}$; $t = -\dfrac{m}{K}\ln(F-R_0-Kv)\Big]_0^v$

$t = -\dfrac{m}{K}\ln\dfrac{F-R_0-Kv}{F-R_0}$

$\underline{t = \dfrac{m}{K}\ln\dfrac{F-R_0}{F-R_0-Kv}}$

3/220 For plug: $\Delta T + \Delta V_g = 0$; $\frac{1}{2}m_A v^2 - m_A g r = 0$

$v = \sqrt{2gr}$

Plug & block: $\Delta G = 0$; $m_A\sqrt{2gr} = (m_A + m_C)v'$

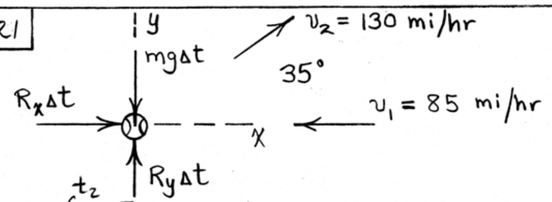
where v' = velocity of block & plug after impact

Friction force $F = \mu_k(m_A + m_C)g$

Deceleration $a = F/(m_A+m_C) = \mu_k g$

$v'^2 = 2as$, $s = \left(\dfrac{m_A}{m_A+m_C}\right)^2 2gr\,\dfrac{1}{2\mu_k g} = \underline{\dfrac{r}{\mu_k}\left(\dfrac{m_A}{m_A+m_C}\right)^2}$

3/221



$mv_{x_1} + \int_{t_1}^{t_2}\Sigma F_x\,dt = mv_{x_2}$:

$-\dfrac{5.125/16}{32.2}\left(85\,\dfrac{5280}{3600}\right) + R_x(0.005) = \dfrac{5.125/16}{32.2}\left(130\,\dfrac{5280}{3600}\cos 35^\circ\right)$

$\underline{R_x = 559 \text{ lb}}$

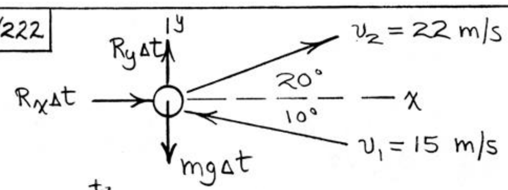
$mv_{y_1} + \int_{t_1}^{t_2}\Sigma F_y\,dt = mv_{y_2}$:

$0 + R_y(0.005) - \dfrac{5.125}{16}(0.005) = \dfrac{5.125/16}{32.2}\left(130\,\dfrac{5280}{3600}\sin 35^\circ\right)$

$\underline{R_y = 218 \text{ lb}}$

If mg is neglected during impact, $R_y = 218$ lb – a good assumption! The quantity mg may not be neglected thereafter – otherwise we obtain a record home-run distance!

3/222



$mv_{x_1} + \int_{t_1}^{t_2}\Sigma F_x\,dt = mv_{x_2}$:

$0.060(-15\cos 10^\circ) + R_x(0.05) = 0.060(22\cos 20^\circ)$

$R_x = 42.5$ N

$mv_{y_1} + \int_{t_1}^{t_2}\Sigma F_y\,dt = mv_{y_2}$:

$0.060(15\sin 10^\circ) + R_y(0.05) - 0.060(9.81)(0.05) = 0.060(22\sin 20^\circ)$

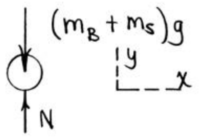
$R_y = 6.49$ N

Weight $mg = 0.060(9.81) = 0.589$ N is about 9% of R_y – no need to omit mg from analysis!

$R = \sqrt{R_x^2 + R_y^2} = \underline{43.0 \text{ N}}$

$\beta = \tan^{-1}\dfrac{R_y}{R_x} = \underline{8.68^\circ}$

3/223 System :



$$m_B v_{B_x} + m_s \cancel{v_{s_x}}^{0} = (m_B + m_s) v$$

$$v = \frac{m_B v_{Bx}}{(m_B + m_s)} = \frac{80/32.2}{90(32.2)}(16 \cos 30°)$$

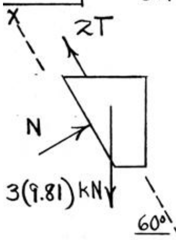
$$= \underline{12.32 \text{ ft/sec}}$$

$$m_B v_{B_y} + m_s \cancel{v_{s_y}}^{0} + \int_0^{\Delta t} [N - (m_B + m_s) g]\, dt = 0$$

$$-\frac{80}{32.2}(16 \sin 30°) + N(0.05) - 90(0.05) = 0$$

$$\underline{N = 488 \text{ lb}}$$

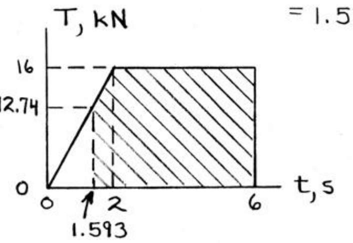
3/224 Skip begins to move when



$2T = 3(9.81)\frac{\sqrt{3}}{2}$, $T = 12.74$ kN

which occurs at $t = \frac{12.74}{16}$

$= 1.593$ s



$\int \Sigma F_x\, dt = m \Delta v_x$:

$$2\left[\frac{16 + 12.74}{2}(2 - 1.593) + 16(6-2)\right] - 3(9.81)\frac{\sqrt{3}}{2}(6 - 1.593) = 3(v - 0)$$

$$\underline{v = 9.13 \text{ m/s}}$$

3/225 $T_y = 600 \cos\theta$; $\dot{\theta} = \pi/10$ rad/s, so $dt = \frac{10}{\pi} d\theta$

$$\int \Sigma F_y\, dt = m \Delta v_y ; \int_0^{\pi/2} 600 \cos\theta \left(\frac{10}{\pi} d\theta\right) = 260 (v_y - 0)$$

$$\frac{6000}{\pi} \sin\theta \Big]_0^{\pi/2} = 260\, v_y , \quad v_y = \frac{6000}{260\pi} = \underline{7.35 \text{ m/s}}$$

3/225 $T_y = 600 \cos\theta$; $\dot{\theta} = \pi/10$ rad/s, so $dt = \frac{10}{\pi} d\theta$

$$\int \Sigma F_y\, dt = m \Delta v_y ; \int_0^{\pi/2} 600 \cos\theta \left(\frac{10}{\pi} d\theta\right) = 260 (v_y - 0)$$

$$\frac{6000}{\pi} \sin\theta \Big]_0^{\pi/2} = 260\, v_y , \quad v_y = \frac{6000}{260\pi} = \underline{7.35 \text{ m/s}}$$

3/226 (a) $\Delta G = 0$; $m(4) + 0 = m v_A + m v_B$

$$v_A + v_B = 4$$

$$\Delta T = 0.4T ; \tfrac{1}{2} m (4^2) - \left[\tfrac{1}{2} m v_A^2 + \tfrac{1}{2} m v_B^2\right] = 0.4\left[\tfrac{1}{2} m (4^2)\right]$$

$$v_A^2 + v_B^2 = 9.6$$

Solve simultaneously & get $(4 - v_B)^2 + v_B^2 = 9.6$

or $v_B^2 - 4v_B + 3.2 = 0$, $v_B = \frac{4}{2} \pm \frac{1}{2}\sqrt{16 - 4(3.2)}$

$= 2 \pm 0.894$

(sol. I) $v_B = 2.894$ ft/sec, $v_A = 4 - 2.894 = 1.106$ ft/sec

(sol. II) $v_B = 1.106$ ft/sec, $v_A = 4 - 1.106 = 2.894$ ft/sec

sol. II is ruled out since distance between A & B would be decreasing so that $v_B > v_A$

Thus $\underline{v_B = 2.89 \text{ ft/sec}}$

(b) For initial to final condition

$\Delta G = 0$; $m(4) + 0 = 2m v_c$, $\underline{v_c = 2 \text{ ft/sec}}$

3/227 (a) $\underline{H}_o = \underline{r} \times m\underline{v}$

$$\underline{H}_o = (-6\underline{i} + 8\underline{j}) \times 2(7)(-\sin 30°\underline{i} - \cos 30°\underline{j})$$

$$= 128.7\underline{k} \text{ kg}\cdot\text{m}^2/\text{s}$$

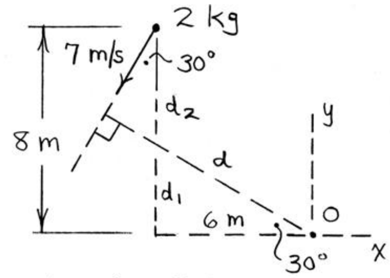
So $\underline{H_o = 128.7 \text{ kg}\cdot\text{m}^2/\text{s}}$

$d_1 = 6 \tan 30°$

$= 3.46$ m

$d_2 = 8 - d_1$

$= 4.54$ m



$$d = \frac{6}{\cos 30°} + 4.54 \sin 30° = 9.20 \text{ m}$$

$$H_o = mvd = 2(7)(9.20) = \underline{128.7 \text{ kg}\cdot\text{m}^2/\text{s}}$$

3/228 (a) $\underline{G} = m\underline{v} = 3 \cdot 5(-\cos 30°\underline{i} - \sin 30°\underline{j})$

$$= \underline{-12.99\underline{i} - 7.5\underline{j} \text{ kg}\cdot\text{m/s}}$$

(b) $\underline{H}_o = \underline{r} \times m\underline{v} = \underline{r} \times \underline{G}$

$$= 2(\cos 15°\underline{i} - \sin 15°\underline{j}) \times (-12.99\underline{i} - 7.5\underline{j})$$

$$= \underline{-21.2\underline{k} \text{ kg}\cdot\text{m}^2/\text{s}}$$

(c) $T = \frac{1}{2} m v^2 = \frac{1}{2}(3)(5)^2 = \underline{37.5 \text{ J}}$

3/229 $\underline{H}_o = \underline{r} \times m\underline{v}$

$$= (a\underline{i} + b\underline{j} + c\underline{k}) \times mv\underline{k}$$

$$= \underline{mv(b\underline{i} - a\underline{j})}$$

$$\dot{\underline{H}}_o = \underline{M}_o = (a\underline{i} + b\underline{j} + c\underline{k}) \times F\underline{j}$$

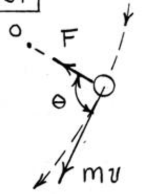
$$= \underline{F(-c\underline{i} + a\underline{k})}$$

3/230 Angular momentum about O is conserved:

$H_{O_1} = H_{O_2}$: $3mv(L) + 2mv(L) = 3mL^2\omega$

$\omega = \dfrac{5}{3}\dfrac{v}{L}$

3/231



$\Sigma M_O = \dot{H}_O = 0$, so $H_O = \text{const.}$

$H_{O_A} = H_{O_B}$

$m(4)(0.350 \sin 54°) = m v_B (0.230 \sin 65°)$

$v_B = 5.43$ m/s

3/232 (a) $v_B = \sqrt{2gr}$

$H_O = mrv_B = mr\sqrt{2gr}$, $\dot{H}_O = mgr$

(b) $v_C = \sqrt{2g(2r)} = 2\sqrt{gr}$

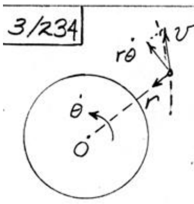
$H_O = mrv_C = 2mr\sqrt{gr}$, $\dot{H}_O = 0$

3/233 $H_1 + \int_{t_1}^{t_2} M\,dt = H_2$

$0 + 20(0.1)t = 4(3)(0.4)^2\left[150\left(\frac{1}{60}\right)(2\pi)\right]$

$t = 15.08$ s

3/234

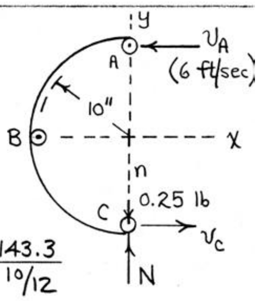


$\Sigma M_O = \dot{H}_O$; $0 = \dfrac{d}{dt}(m r\dot{\theta} \times r)$

or $\dfrac{d}{dt}(r^2\dot{\theta}) = 0$

so $r^2\dot{\theta} = \text{const.}$

3/235 $T_A + U_{A\text{-}C} = T_C$



$\frac{1}{2}mv_A^2 + mgh_{A\text{-}C} = \frac{1}{2}mv_C^2$

$v_C^2 = v_A^2 + 2g\,h_{A\text{-}C}$

$= 6^2 + 32.2\left(\frac{20}{12}\right)(2)$

$= 143.3 \text{ ft}^2/\text{sec}^2$

$\Sigma F_y = ma_y$: $N - 0.25 = \dfrac{0.25}{32.2}\,\dfrac{143.3}{10/12}$

$N = 1.585$ lb

$\dot{\underline{H}}_B = \underline{M}_B = (1.585 - 0.25)\frac{10}{12}\underline{k} = 1.113\underline{k}$ lb-ft

3/236 Velocity of plug upon impact is

$v = \sqrt{2gh} = \sqrt{2(9.81)(0.6)} = 3.43$ m/s

For system, $\Delta H_O = 0$. Take C.W. positive

Initial $H_O = -4(0.5)^2(2) - 6(0.3)^2(2) + 2(3.43)(0.5)$

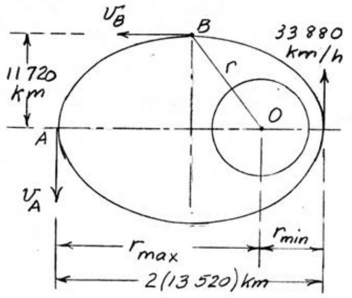
$= -2 - 1.08 + 3.43 = 0.351$ N·m·s

Final $H_O = [(4+2)(0.5)^2 + 6(0.3)^2]\omega$

$= 2.04\omega$

So $0.351 = 2.04\omega$, $\omega = 0.1721$ rad/s CW

3/237 $\Sigma M_O = \dot{H}_O = 0$ so $H_O = \text{constant}$



$r_{min} = 6371 + 390 = 6761$ km

$r_{max} = 2(13\,520) - 6761 = 20\,279$ km

For H_O constant

$6761(33\,880) = 11\,720\,v_B = 20\,279\,v_A$

$v_A = 11\,300$ km/h

$v_B = 19\,540$ km/h

3/238 For the entire system, $\Sigma M_O = \dot{H}_O = 0$, so angular momentum is conserved.

$H_{O_1} = H_{O_2}$: $2mr^2\omega_0 + 0 = 2mr^2\omega + 2m(2r)^2\omega$

$\omega = \omega_0/5$

Kinetic energy loss $\Delta Q = T_1 - T_2$

$\Delta Q = 2\left(\frac{1}{2}mr^2\omega_0^2\right) - \left\{2\left(\frac{1}{2}mr^2\omega^2\right) + 2\left(\frac{1}{2}m(2r)^2\omega^2\right)\right\}$

$= mr^2\omega_0^2 - mr^2\left(5\left(\frac{\omega_0}{5}\right)^2\right) = \frac{4}{5}mr^2\omega_0^2$

So $n = \dfrac{\Delta Q}{T_1}(100\%) = \dfrac{\frac{4}{5}mr^2\omega_0^2}{2\left(\frac{1}{2}mr^2\omega_0^2\right)}(100\%) = 80\%$

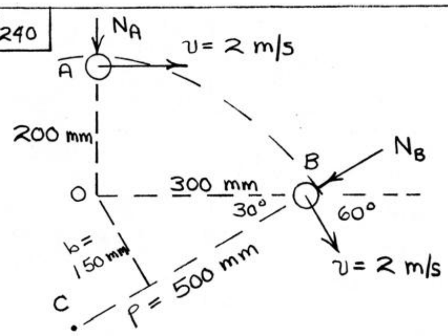
3/239 $\Delta H = 0$; $2m\,r\omega_0(r) - 2m(2r)\omega(2r) = 0$

$\omega = \omega_0/4$

$\Delta T = 2\left(\frac{1}{2}m[r\omega_0]^2\right) - 2\left(\frac{1}{2}m\left[2r\frac{\omega_0}{4}\right]^2\right) = mr^2\omega_0^2\,(3/4)$

$n = \Delta T/T = \frac{3}{4}mr^2\omega_0^2 / mr^2\omega_0^2 = 3/4$

3/240



$\Sigma M_{O_z} = \dot{H}_{O_z}$:

At A, $\Sigma M_{O_z} = 0$, so $\dot{H}_{O_z} = 0$

At B, $\Sigma M_{O_z} = -N_B b$, where $N_B = m\dfrac{v^2}{\rho} = 0.1\,\dfrac{2^2}{0.5} = 0.8$ N

So $\dot{H}_{O_z} = -N_B b = -0.8(0.150) = -0.120$ N·m (or -0.120 kg·m²/s²)

3/241 $\Sigma M_O = \dot{H}_O = 0$, so angular momentum is conserved : $H_{O_1} = H_{O_2}$ (O: any point on axis)

$$0.2\,(0.3\cos 30°)^2 4 = 0.2(0.2\cos 30°)^2\,\omega$$

$$\underline{\omega = 9 \text{ rad/s}}$$

$$U'_{1\text{-}2} = \Delta T + \Delta V_g$$

$$\Delta T = \tfrac{1}{2}(0.2)\left[(0.2\cos 30°\cdot 9)^2 - (0.3\cos 30° \times 4)^2\right]$$

$$= 0.1350 \text{ J}$$

$$\Delta V_g = 0.2\,(9.81)\,(0.1\sin 30°) = 0.0981 \text{ J}$$

So $U'_{1\text{-}2} = 0.1350 + 0.0981 = \underline{0.233 \text{ J}}$

3/242 $\int \Sigma M_O\,dt = \Delta H_O = H_{O_B} - H_{O_A}$

$$H_{O_A} = 0.02(4)(0.090)\sin 30° = 0.0036 \text{ kg·m}^2/\text{s}$$

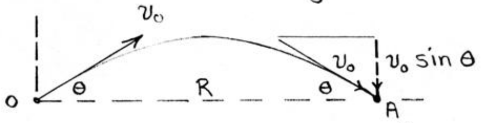
$$H_{O_B} = 0.02(6)(0.180)\sin 60° = 0.01871 \text{ kg·m}^2/\text{s}$$

$$\Delta H_O = 0.01871 - 0.0036 = 0.01511 \text{ kg·m}^2/\text{s}$$

$M_{O_{av}} \times 0.5 = 0.01511$, $\underline{M_{O_{av}} = 0.0302 \text{ N·m}}$

3/243 (a) $\underline{H_O = 0}$ when projectile is at O.

(b) Range $R = \dfrac{2v_0^2\cos\theta\sin\theta}{g}$

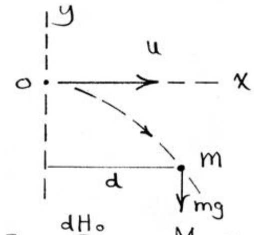


$$H_O = m v_y R = m v_0 \sin\theta \,\frac{2v_0^2\cos\theta\sin\theta}{g}$$

$$= \underline{\frac{2 m v_0^3 \sin^2\theta\cos\theta}{g}}$$

The moment of the projectile weight about point O is always increasing the angular momentum about O.

3/244



(Note : $d = ut$)

$$\underline{H}_O = \frac{d\underline{H}_O}{dt} = \underline{M}_O = -mgd\,\underline{k}$$

$$\int_0^{\underline{H}_O} d\underline{H}_O = -\int_0^t mgd\,\underline{k}\,dt = -\int_0^t mgut\,\underline{k}\,dt$$

$$\Rightarrow \underline{\underline{H}_O = -\tfrac{1}{2} mgut^2\,\underline{k}}$$

3/245 Conservation of angular momentum :

$$(\cancel{m} r v_\theta)_A = (\cancel{m} r v_\theta)_B$$

$$50(10^6)(188{,}500) = 75(10^6)\,v_\theta$$

$$\underline{v_\theta = 125{,}700 \text{ ft/sec}} \text{ @ B}$$

Energy conservation $T_A + V_A = T_B + V_B$

$$\tfrac{1}{2}\cancel{m}v_A^2 - \frac{Gm_s\cancel{m}}{r_A} = \tfrac{1}{2}\cancel{m}v_B^2 - \frac{Gm_s\cancel{m}}{r_B}$$

$$\tfrac{1}{2}(188{,}500)^2 - \tfrac{1}{2}v_B^2 = 3.439(10^{-8})(333{,}000)(4.095)10^{23}\left[\frac{1}{50(10^6)} - \frac{1}{75(10^6)}\right]\frac{1}{5280}$$

$$v_B = 153{,}900 \text{ ft/sec}$$

$$v_r = \sqrt{v_B^2 - v_\theta^2} = \sqrt{153{,}900^2 - 125{,}700^2} = \underline{88{,}870 \ \tfrac{\text{ft}}{\text{sec}}}$$

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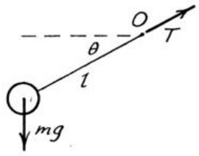


$$\Sigma M_O = \dot{H}_O : mgl\cos\theta = \frac{d}{dt}(ml^2\dot\theta) = ml^2\ddot\theta$$

$$\underline{\ddot\theta = \frac{g}{l}\cos\theta}$$

From $\int \dot\theta\,d\dot\theta = \int\ddot\theta\,d\theta$, $\left.\dfrac{\dot\theta^2}{2}\right|_0^{\dot\theta} = \int_0^\theta \dfrac{g}{l}\cos\theta\,d\theta$,

$$\dot\theta^2 = \frac{2g}{l}\sin\theta,\quad \dot\theta_{\theta=90°} = \sqrt{\frac{2g}{l}}$$

so at $\theta = 90°$, $v = l\dot\theta = \underline{\sqrt{2gl}}$

By work-energy $U = \Delta T$, $mgl = \frac{1}{2}mv^2$, $v = \sqrt{2gl}$

3/247 Forces on particle exert no moment about the central axis, so angular momentum is conserved about this axis. Thus $\Delta H_z = 0$ &

$$m v_0\cos\beta\,(r) = m v\cos\theta\,(r),\quad v_0\cos\beta = v\cos\theta$$

Also energy is conserved so that

$$\Delta T + \Delta V_g = 0;\ \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 - mgh = 0$$

Eliminate v & get $\cos\theta = \dfrac{v_0\cos\beta}{\sqrt{v_0^2 + 2gh}}$

or $\underline{\theta = \cos^{-1}\dfrac{\cos\beta}{\sqrt{1 + \dfrac{2gh}{v_0^2}}}}$

3/248 System angular momentum conserved during impact: $\circlearrowleft{+}\ H_{O_1} = H_{O_2}$:

$$0.050(300)(0.4\cos 20°) - 3.2(0.2)^2 6 - 3.2(0.4)^2 6$$

$$= (0.050 + 3.2)(0.4)^2\omega' + 3.2(0.2)^2\omega'$$

$$\underline{\omega' = 2.77 \text{ rad/s} \quad (\text{CCW})}$$

Energy considerations after impact :

$T' + V' = \overset{\nearrow 0}{T} + V$, choose datum @ O :

$$\tfrac{1}{2}(0.05 + 3.2)[0.4(2.77)]^2 + \tfrac{1}{2}(3.2)[0.2(2.77)]^2$$

$$+[3.2(0.2) - (3.2 + 0.05)(0.4)]9.81 = 0 +$$

$$[3.2(0.2) - (3.2 + 0.05)(0.4)]9.81\cos\theta$$

$$\underline{\theta = 52.1°}$$

3/249 Path form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\left(\begin{array}{l} a = 5 \text{ ft} \\ b = 4 \text{ ft} \end{array}\right.$

Angular momentum about O is conserved:

$$m r_A v_A = m r_B v_B : \quad v_B = \frac{r_A}{r_B} v_A = \frac{a}{b} v_A$$

$$= \frac{5}{4}(8) = 10 \text{ ft/sec}$$

$$y = b\left[1 - \left(\frac{x}{a}\right)^2\right]^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} b \left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}} \cdot \left(-\frac{2x}{a^2}\right) = -\frac{bx}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}}$$

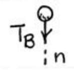
$$\frac{d^2y}{dx^2} = -\frac{b}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}} - \frac{bx}{a^2}\left(-\frac{1}{2}\right)\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{3}{2}}\left(-\frac{2x}{a^2}\right)$$

$$= -\frac{b}{a^2}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{1}{2}} - \frac{bx^2}{a^4}\left[1 - \left(\frac{x}{a}\right)^2\right]^{-\frac{3}{2}}$$

Now, $\left.\frac{dy}{dx}\right|_{x=0} = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{x=0} = -\frac{b}{a^2} = -\frac{4}{5^2} = -\frac{4}{25}$

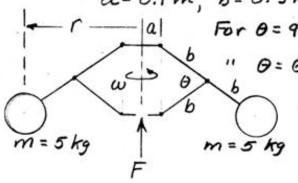
$$\rho_{xy} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2} = \frac{[1+0]^{3/2}}{-4/25} = -6.25'$$

So $\rho = 6.25'$; $\Sigma F_n = m\frac{v^2}{\rho}$: $T_B = \frac{1.5}{32.2} \frac{10^2}{6.25} = \underline{0.745 \text{ lb}}$



▶3/250 $\omega_0 = 40(2\pi)/60 = 4.19$ rad/s

$a = 0.1$ m, $b = 0.3$ m



For $\theta = 90°$, $r_0 = 0.1 + 2(0.3)\cos 45° = 0.524$ m

" $\theta = 60°$, $r = 0.1 + 2(0.3)\cos 30° = 0.620$ m

$\Delta H = 0$; $2 m r_0^2 \omega_0 - 2 m r^2 \omega = 0$

$$\omega = \frac{r_0^2}{r^2}\omega_0 = \left(\frac{0.524}{0.620}\right)^2 (4.19)$$

$$= \underline{3.00 \text{ rad/s}}$$

$\left(\text{or } \frac{3.00}{2\pi} 60 = 28.6 \text{ rev/min}\right)$

$U = \Delta T + \Delta V_g = 2\left(\frac{1}{2}m\right)(r^2\omega^2 - r_0^2\omega_0^2) + 2mg\,\Delta h$

where $\Delta h = 2b(\sin 45° - \sin 30°)$
$= 2(0.3)(0.7071 - 0.5) = 0.1243$ m

$U = 5\left([0.620 \times 3.00]^2 - [0.524 \times 4.19]^2\right) + 2(5)(9.81)(0.1243)$

$= -6.850 + 12.190 = \underline{5.34 \text{ J}}$

3/251 $v = \sqrt{2gh}$, $v' = \sqrt{2gh'}$

$$e = \frac{v'}{v} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{1100}{2100}} = \underline{0.724}$$

$$n = \frac{mgh - mgh'}{mgh}(100\%) = \frac{2100 - 1100}{2100}(100\%)$$

$= \underline{47.6\ \%}$

3/252 System linear momentum:

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1.5}{32.2}(0.8) + \frac{2}{32.2}(-2.4) = \frac{1.5}{32.2} v_1' + \frac{2}{32.2} v_2'$$

Restitution: $e = \frac{v_2' - v_1'}{v_1 - v_2}$: $0.5 = \frac{v_2' - v_1'}{0.8 - (-2.4)}$

Solve the two equations to obtain

$v_1' = -1.943$ ft/sec

$\underline{v_2' = -0.343 \text{ ft/sec}}$

Original energy: $T_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$$T_1 = \frac{1}{2}\frac{1.5}{32.2}(0.8)^2 + \frac{1}{2}\frac{2}{32.2}(2.4)^2$$

$= 0.1938$ ft-lb

$$T_2 = \frac{1}{2}\frac{1.5}{32.2}(1.943)^2 + \frac{1}{2}\frac{2}{32.2}(0.343)^2$$

$= 0.0916$ ft-lb

$$n = \frac{T_1 - T_2}{T_1}(100\%) = \frac{0.1938 - 0.0916}{0.1938}(100\%)$$

$= \underline{52.7\ \%}$

3/253 System momentum:

$$\frac{1.5}{32.2}(0.8) + \frac{2}{32.2} v_2 = \frac{1.5}{32.2} v_1' + 0$$

Restitution: $\frac{-v_1'}{0.8 - v_2} = 0.5$

Solve to obtain $\left\{\begin{array}{l} v_1' = -1.120 \text{ ft/sec} \\ \underline{v_2 = -1.440 \text{ ft/sec}} \end{array}\right.$

(Note: v_2 assumed totally unknown above – no leftward direction assumed.)

3/254 Consider the case $v_2' = v_1$. Conservation of system linear momentum:

$$m_1 v_1 + m_2 \overset{\nearrow 0}{v_2} = m_1 v_1' + m_2 v_2' = m_1 v_1' + m_2 v_1$$

$$v_1' = \frac{(m_1 - m_2)}{m_1} v_1$$

Restitution: $e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{v_1 - \left(\frac{m_1 - m_2}{m_1}\right) v_1}{v_1}$

$$\frac{m_1}{m_2} = \frac{1}{e}$$

So for $v_2' > v_1$, $\underline{\frac{m_1}{m_2} > \frac{1}{e}}$

3/255 System linear momentum :

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \quad \xrightarrow{+}$$

$$\cancel{m} v + 0 = \cancel{m} v_A' + p\cancel{m} v_B' \quad (1)$$

Restitution : $e = \dfrac{v_B' - v_A'}{v_A - v_B}$: $0.1 = \dfrac{v_B' - v_A'}{v - 0}$ (2)

Solve (1) & (2) to obtain

$$v_A' = \left(\frac{1-0.1p}{1+p}\right) v, \quad v_B' = \frac{1.1}{1+p} v$$

For $p = \frac{1}{2}$: $v_A' = 0.633 v$, $v_B' = 0.733 v$

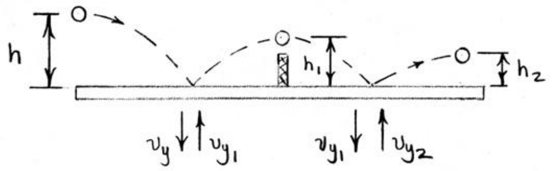
3/256 Impact velocity $v = \sqrt{2gh} = \sqrt{2(32.2)(4)}$
$= 16.05$ ft/sec

$\Delta G = 0$; $500(16.05) + 0 = 0 + 800 v'$

$v' = 10.03$ ft/sec

$e = \dfrac{v'}{v} = \dfrac{10.03}{16.05} = 0.625$

3/257



(Note : Drop distances reduced by $r = 0.75''$)

$v_y = \sqrt{2g(h-r)}$, $v_{y_1} = e v_y = e\sqrt{2g(h-r)}$
$= \sqrt{2g(h_1 - r)}$

$\therefore \; 0.90^2(2g)(h - 0.75) = (2g)(9 - 0.75)$, $h = 10.94$ in.

$v_{y_2} = e v_{y_1}$: $\sqrt{2g(h_2 - r)} = 0.9\sqrt{2g(h_1 - r)}$

With $r = 0.75$ in. & $h_1 = 9$ in. : $h_2 = 7.43$ in.

3/258 $\Delta G = 0$; $m_A v_A + 0 = m_A v_A' + m_B v_B'$

$e = 0$; $v_A' = v_B'$

Thus $m_A v_A = (m_A + m_B) v_A'$

$$|\Delta T| = -\tfrac{1}{2} m_A v_A'^2 - \tfrac{1}{2} m_B v_B'^2 + \tfrac{1}{2} m_A v_A^2$$

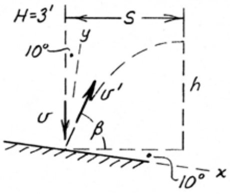
$$= -\tfrac{1}{2} m_A \left(\frac{m_A}{m_A + m_B} v_A\right)^2 - \tfrac{1}{2} m_B \left(\frac{m_A}{m_A + m_B} v_A\right)^2 + \tfrac{1}{2} m_A v_A^2$$

$$= -\tfrac{1}{2}\left(\frac{m_A}{m_A + m_B} v_A\right)^2 (m_A + m_B) + \tfrac{1}{2} m_A v_A^2$$

$$= \frac{1}{2} \frac{m_A m_B}{m_A + m_B} v_A^2 \quad (\text{loss})$$

$$\frac{|\Delta T|}{T} = \frac{1}{2} \frac{m_A m_B}{m_A + m_B} v_A^2 \frac{1}{\frac{1}{2} m_A v_A^2} = \frac{m_B}{m_A + m_B}$$

3/259 $v = \sqrt{2gH} = \sqrt{2 \times 32.2 \times 3}$
$= 13.90$ ft/sec



At impact $\Sigma F_x = 0$ so $\Delta G_x = 0$ so

$v' \cos(\beta + 10°) - 13.90 \sin 10° = 0$ -- (a)

$e = 0.7 = \dfrac{v' \sin(\beta + 10°)}{13.90 \cos 10°}$ ----- (b)

Combine & get $\tan(\beta + 10°) = 3.97$

$\beta + 10° = 75.9°$, $\beta = 65.9°$

From Eq. (a) $v' = \dfrac{13.90 \sin 10°}{\cos 75.9°} = 9.88$ ft/sec

From Sample Prob. 2/6, $h = \dfrac{v'^2 \sin^2\beta}{2g} = \dfrac{9.88^2 \sin^2 65.9°}{2 \times 32.2} = 1.263$ ft

$s = \dfrac{v'^2 \sin 2\beta}{2g} = \dfrac{9.88^2 \sin 131.7°}{2 \times 32.2} = 1.132$ ft

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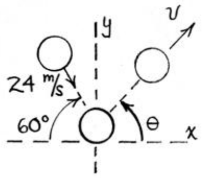

During impact $\Sigma F_x = 0$ so no change in x velocity component.

$v \cos\theta = 24(0.5) = 12$ m/s

In y-dir., $e = \dfrac{v \sin\theta}{24 \cos 30°} = 0.8$

$\tan\theta = \dfrac{16.63}{12} = 1.386$

$\theta = 54.2°$

$v = \dfrac{12}{\cos 54.2°} = 20.5$ m/s

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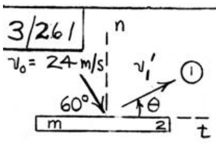

t- momentum conserved :

Ball : $m_1 v_{1_t} = m_1 v_{1_t}'$

$v_{1t}' = v_{1t} = 24 \cos 60° = 12$ m/s

Plate : $m_2 v_{2_t} = m_2 v_{2_t}'$, $v_{2t}' = v_{2t} = 0$

n- momentum :

$$\cancel{m_1} v_{1n} + \cancel{m_2} v_{2n} = \cancel{m_1} v_{1n}' + \cancel{m_2} v_{2n}'$$

$$-24 \sin 60° = v_{1n}' + v_{2n}'$$

Restitution : $e = \dfrac{v_{2n}' - v_{1n}'}{v_{1n} - v_{2n}}$, $0.8 = \dfrac{v_{2n}' - v_{1n}'}{-24 \sin 60° - 0}$

Solve to find $v_{1n}' = -2.08$ m/s, $v_{2n}' = -18.71$ m/s

$v_1' = \sqrt{v_{1t}'^2 + v_{1n}'^2} = 12.20$ m/s, $\theta = \tan^{-1}\left(\dfrac{v_{1n}'}{v_{1t}'}\right) = -9.83°$

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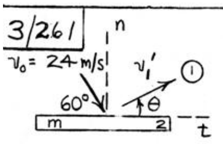

t-momentum conserved:

Ball: $m_1 v_{1_t} = m_1 v_{1_t}'$

$v_{1t}' = v_{1t} = 24\cos 60° = 12$ m/s

Plate: $m_2 v_{2_t} = m_2 v_{2_t}'$, $v_{2_t}' = v_{2_t} = 0$

n-momentum:

$\cancel{m}_1 v_{1_n} + \cancel{m}_2 v_{2_n} = \cancel{m}_1 v_{1_n}' + \cancel{m}_2 v_{2_n}'$

$-24\sin 60° = v_{1n}' + v_{2n}'$

Restitution: $e = \dfrac{v_{2n}' - v_{1n}'}{v_{1n} - v_{2n}}$, $0.8 = \dfrac{v_{2n}' - v_{1n}'}{-24\sin 60° - 0}$

Solve to find $v_{1n}' = -2.08$ m/s, $v_{2n}' = -18.71$ m/s

$v_1' = \sqrt{v_{1t}'^2 + v_{1n}'^2} = \underline{12.20 \text{ m/s}}$, $\theta = \tan^{-1}\left(\dfrac{v_{1n}'}{v_{1t}'}\right) = \underline{-9.83°}$

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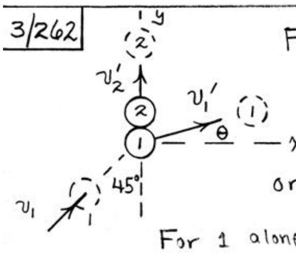

For 1 & 2 together:

$m v_1 \cos 45° = m v_1' \sin\theta + m v_2'$

or $v_1'\sin\theta + v_2' = v_1/\sqrt{2}$ (1)

For 1 alone: $m v_1 \sin 45° = m v_1' \cos\theta$

or $v_1'\cos\theta = v_1/\sqrt{2}$ (2)

Restitution: $v_2' - v_1'\sin\theta = e v_1 \cos 45°$

or $v_2' - v_1'\sin\theta = 0.9 v_1/\sqrt{2}$ (3)

(1) & (3): $v_1'\sin\theta = 0.0354 v_1$; Divide by (2): $\underline{\theta = 2.86°}$

$n = \dfrac{T_1 - T_2}{T_1} = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{\frac{1}{2}mv_2'^2 + \frac{1}{2}mv_1'^2}{\frac{1}{2}mv_1^2}$

$= 1 - \dfrac{v_2'^2 + v_1'^2}{v_1^2}$, where $v_1' = 0.708 v_1$, $v_2' = 0.672 v_1$

So $n = 1 - \dfrac{0.672^2 + 0.708^2}{1} = \underline{0.0475}$

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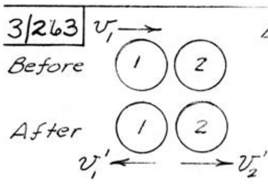

$\Delta G = 0$; $m v_1 = -m v_1' + m v_2'$

$v_2' = v_1 + v_1'$

$e = \dfrac{v_2' + v_1'}{v_1}$, $v_1' = e v_1 - v_2'$

Combine & get

$v_2' = v_1 + e v_1 - v_2'$

or $v_2' = \dfrac{1+e}{2} v_1$

It follows that $v_3' = \dfrac{1+e}{2} v_2' = \left(\dfrac{1+e}{2}\right)^2 v_1$

$v_4' = \dfrac{1+e}{2} v_3' = \left(\dfrac{1+e}{2}\right)^3 v_1$

⋮

so $\underline{v_n = \left(\dfrac{1+e}{2}\right)^{n-1} v_1}$

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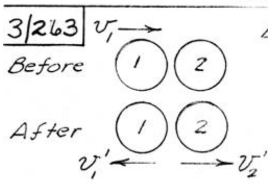


$\Delta G = 0$; $m v_1 = -m v_1' + m v_2'$

$v_2' = v_1 + v_1'$

$e = \dfrac{v_2' + v_1'}{v_1}$, $v_1' = e v_1 - v_2'$

Combine & get

$v_2' = v_1 + e v_1 - v_2'$

or $v_2' = \dfrac{1+e}{2} v_1$

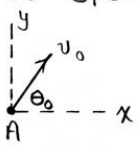
It follows that $v_3' = \dfrac{1+e}{2} v_2' = \left(\dfrac{1+e}{2}\right)^2 v_1$

$v_4' = \dfrac{1+e}{2} v_3' = \left(\dfrac{1+e}{2}\right)^3 v_1$

⋮

so $\underline{v_n = \left(\dfrac{1+e}{2}\right)^{n-1} v_1}$

3/264 Let the launch conditions at A be speed v_0, launch angle θ_0:



The range L_1 is

$L_1 = \dfrac{2v_0^2 \sin\theta_0 \cos\theta_0}{g}$

and the velocity components coming into B are $\begin{cases} v_x = v_0\cos\theta_0 \\ v_y = -v_0\sin\theta_0 \end{cases}$

The velocity components after impact at B are $v_x = v_0\cos\theta_0$, $v_y = e v_0 \sin\theta_0$, which result in the range $L_2 = \dfrac{2e v_0^2 \sin\theta_0\cos\theta_0}{g}$

So $\underline{L_2 = e L_1}$.

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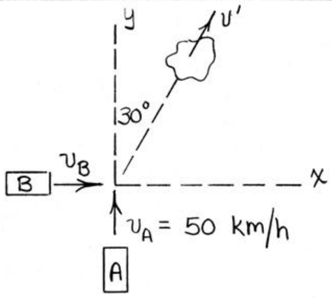

$$G_{1x} = G_{2x}: \quad m_B v_B + 0 = (m_A + m_B) v' \sin 30°$$
$$1600 v_B = 2800 v' \left(\tfrac{1}{2}\right) \qquad (1)$$
$$G_{1y} = G_{2y}: \quad m_A v_A + 0 = (m_A + m_B) v' \cos 30°$$
$$1200\,(50) = 2800 v' (0.866) \qquad (2)$$

From (2): $v' = 24.7$ km/h

From (1): $\underline{v_B = 21.7 \text{ km/h}}$

3/266 System linear momentum is conserved:

$$m_s \underline{v}_s + m_m \underline{v}_m = (m_s + m_m)\underline{v}'$$
$$(1000)(2000\,\underline{i}) + 100\,(-v_m \underline{j}) = (1000+100)[v'(\cos 20°\,\underline{i} - \sin 20°\,\underline{j})]$$

Equating coefficients:

$\underline{i}$: $(1000)(2000) = 1100 \cos 20°\, v'$
$$v' = 1935 \text{ m/s}$$

$\underline{j}$: $-100\, v_m = -(1100)(1935) \sin 20°$
$$\underline{v_m = 7280 \text{ m/s}}$$

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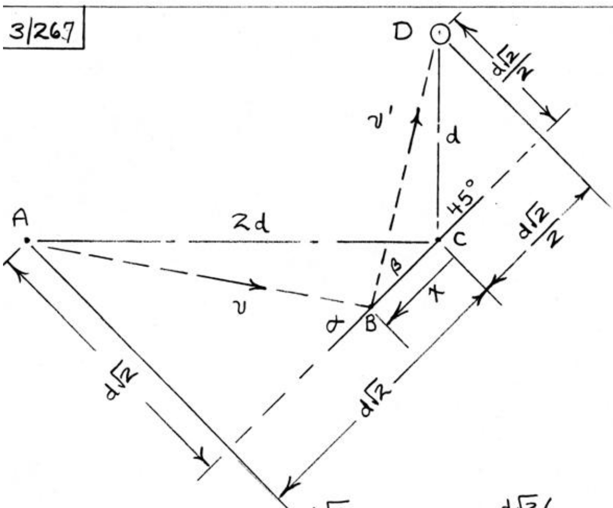

$$\tan\alpha = \frac{1}{e}\tan\beta: \quad \frac{d\sqrt{2}}{d\sqrt{2} - x} = \frac{1}{0.8}\,\frac{d\sqrt{2}/2}{\left(\frac{d\sqrt{2}}{2} + x\right)}$$

Solve for $\underline{x = 0.1088\,d}$

3/268 Let v_s and v_b stand for rebound velocities from steel and brass plates.

Impact speed $= \sqrt{2gh} = \sqrt{2(9.81)(0.15)} = 1.716$ m/s

$0.6 = \dfrac{v_s}{1.716}$, $v_s = 1.029$ m/s

$0.4 = \dfrac{v_s}{1.716}$, $v_b = 0.686$ m/s

$$\omega = \frac{1.029 - 0.686}{0.60} = \underline{0.572 \text{ rad/s CCW}}$$

3/268 Let v_s and v_b stand for rebound velocities from steel and brass plates.

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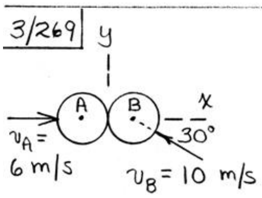
3/269



$$v_{A_y}' = v_{A_y} = 0$$
$$v_{B_y}' = v_{B_y} = 10 \sin 30° = 5 \text{ m/s}$$

$$m_A v_{A_x} + m_B v_{B_x} = m_A v_{A_x}' + m_B v_{B_x}'$$
$$6 - 10\cos 30° = v_{A_x}' + v_{B_x}' \qquad (1)$$
$$e = \frac{v_{B_x}' - v_{A_x}'}{v_{A_x} - v_{B_x}}: \quad 0.75 = \frac{v_{B_x}' - v_{A_x}'}{6 - (-10\cos 30°)} \qquad (2)$$

Solve Eqs. (1) & (2): $\begin{cases} v_{A_x}' = -6.83 \text{ m/s} \\ v_{B_x}' = 4.17 \text{ m/s} \end{cases}$

Magnitudes and directions (θ measured from x):
$$v_A' = 6.83 \tfrac{m}{s} @ \theta_A = 180°$$
$$\underline{v_B' = 6.51 \tfrac{m}{s} @ \theta_B = 50.2°}$$

Initial: $T_1 = \frac{1}{2} m (6^2 + 10^2) = 68\,m$

Final: $T_2 = \frac{1}{2} m (6.83^2 + 6.51^2) = 44.5\,m$

$$n = \frac{68 - 44.5}{68}(100\%) = \underline{34.6\%}$$

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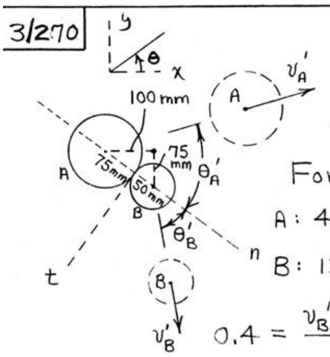

For system, $\Delta G_n = 0$

$23 v_A' \cos\theta_A' + 4 v_B' \cos\theta_B'$
$= 23(4)(\frac{4}{5}) - 4(12)(\frac{4}{5})$ --- (a)

For each sphere, $\Delta G_t = 0$

A: $4(\frac{3}{5}) = v_A' \sin\theta_A'$ --- (b)

B: $12(\frac{3}{5}) = v_B' \sin\theta_B'$ --- (c)

$0.4 = \dfrac{v_B' \cos\theta_B' - v_A' \cos\theta_A'}{12(\frac{4}{5}) + 4(\frac{4}{5})}$ --- (d)

Solve (a), (b), (c), (d) : $v_A' = 2.46$ m/s, $\theta_A' = 77.2°$

$v_B' = 9.16$ m/s, $\theta_B' = 51.8°$

Relative to the $+x$-axis, the directions of the final velocities are

$\begin{cases} \theta_A = 77.2 - 36.9 = 40.3° \\ \theta_B = -51.8 - 36.9 = -88.7° \end{cases}$

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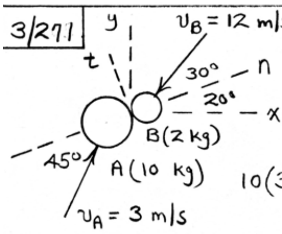

$v_{A_t}' = v_{A_t} = 3 \sin 45° = 2.12 \frac{m}{s}$

$v_{B_t}' = v_{B_t} = -12 \sin 30° = -6 \frac{m}{s}$

$m_A v_{A_n} + m_B v_{B_n} = m_A v_{A_n}' + m_B v_{B_n}'$:

$10(3 \cos 45°) + 2(-12 \cos 30°) =$
$10 v_{A_n}' + 2 v_{B_n}'$ (1)

$e = \dfrac{v_{B_n}' - v_{A_n}'}{v_{A_n} - v_{B_n}}$: $0.5 = \dfrac{v_{B_n}' - v_{A_n}'}{3 \cos 45° - (-12 \cos 30°)}$ (2)

Solve (1) & (2) : $v_{A_n}' = -1.007$ m/s, $v_{B_n}' = 5.25$ m/s

Then
$\begin{cases} v_{A_x}' = -2.12 \sin 20° - 1.007 \cos 20° = -1.672 \text{ m/s} \\ v_{A_y}' = 2.12 \cos 20° - 1.007 \sin 20° = 1.649 \text{ m/s} \\ v_{B_x}' = -(-6 \sin 20°) + 5.25 \cos 20° = 6.99 \text{ m/s} \\ v_{B_y}' = -6 \cos 20° + 5.25 \sin 20° = -3.84 \text{ m/s} \end{cases}$

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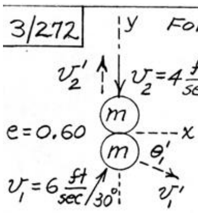


For system, $\Delta G_y = 0$ so

$[m(6 \cos 30°) - m(4)]$
$-[m(-v_1' \sin\theta_1') + m v_2'] = 0$

or $v_2' - v_1' \sin\theta_1' = 1.196$ (1)

For each ball $\Delta v_x = 0$ so

$v_1' \cos\theta_1' = 6(\frac{1}{2})$, $v_{2_x}' = 0$

Also $e = 0.60 = \dfrac{v_2' + v_1' \sin\theta_1'}{4 + 6 \cos 30°}$, $v_2' + v_1' \sin\theta_1' = 5.518$ (2)

Combine (1) & (2) & get $2v_2' = 1.196 + 5.518$, $v_2' = 3.36 \frac{ft}{sec}$

& $v_1' \sin\theta_1' = 2.16$; *Divide by* $v_1' \cos\theta_1' = 3$

& *get* $\theta_1' = \tan^{-1} 0.7203 = 35.8°$ & $v_1' = \dfrac{3}{\cos 35.8°} = 3.70 \frac{ft}{sec}$

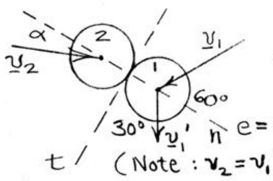
Initial kinetic energy $= \frac{1}{2} m (6^2 + 4^2) = \frac{1}{2} m (52)$

Final " " $= \frac{1}{2} m (3.70^2 + 3.36^2) = \frac{1}{2} m (24.9)$

% loss $= \dfrac{52 - 24.9}{52} = 0.520$ *or* 52.0%

3/273

Conservation of n-momentum :
$m(-v_1 \cos 60°) + m(v_2 \cos\alpha) =$
$m v_{1n}' + m v_{2n}'$ (a)



(Note : $v_2 = v_1$)

Restitution :

$e = 0.8 = \dfrac{v_{2n}' - v_{1n}'}{-v_1 \cos 60° - v_2 \cos\alpha}$ (b)

Simultaneous solution of Eqs. (a) and (b) :

$v_{1n}' = v_1 [0.9 \cos\alpha - 0.05]$

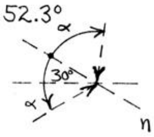
$v_{1t}' = v_{1t} = v_1 \sin 60° = \frac{\sqrt{3}}{2} v_1$

$\tan 30° = \dfrac{v_{1n}'}{v_{1t}'} = \dfrac{v_1 [0.9 \cos\alpha - 0.05]}{\frac{\sqrt{3}}{2} v_1}$

Solving, $\cos\alpha = 0.611$, $\alpha = \pm 52.3°$

So $\theta = 30° + 52.3° = 82.3°$

or $\theta = 30° - 52.3° = -22.3°$



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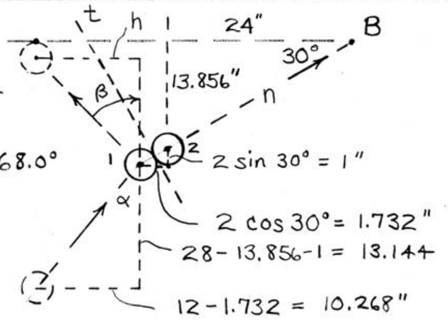
$\alpha = \tan^{-1} \dfrac{10.268}{13.144}$
$= 38.0°$

$\theta_1 = \alpha + 30° = 68.0°$



Mom. : $\cancel{m_1}(v_1)_n + \cancel{m_2}(v_2)_n = \cancel{m_1}(v_1')_n + \cancel{m_2}(v_2')_n$

$v_1 \sin 68.0° = (v_1')_n + (v_2')_n$

Restitution : $e = \dfrac{(v_2')_n - (v_1')_n}{(v_1)_n - (v_2)_n}$

$0.9 = \dfrac{(v_2')_n - (v_1')_n}{v_1 \sin 68.0° - 0}$

Solving, $(v_1')_n = 0.0464 v_1$

Also, $(v_1')_t = (v_1)_t = v_1 \cos 68.0° = 0.375 v_1$

$\tan\theta_1' = \dfrac{(v_1')_n}{(v_1')_t} = \dfrac{0.0464 v_1}{0.375 v_1}$, $\theta_1' = 7.05°$

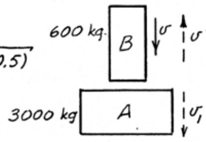
$\beta = 30° - \theta_1' = 22.95°$, $\tan\beta = \dfrac{h}{13.856 - 1 + 1} = 0.423$

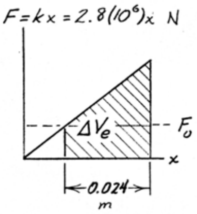
$h = 5.87$". Then $x = 24 - 1.732 - 5.87 = 16.40$ in.

3/275

$F = kx = 2.8(10^6)x$ N

Hammer: $v = \sqrt{2gh}$
$= \sqrt{2(9.81)(0.5)}$
$= 3.13$ m/s





For anvil: $\Delta T + \Delta V_e + \Delta V_g = 0$

$\Delta T = 0 - \frac{1}{2}3000 v_1^2$ J

$\Delta V_e = \frac{1}{2}(2.8\times10^6)(0.024)^2 + 29.4(10^3)(0.024)$ J

$F_0 = mg = 3000(9.81) = 29.4(10^3)$ N

$\Delta V_g = -29.4(10^3)(0.024)$ J

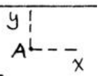
Substitute & get $-\frac{1}{2}(3000)v_1^2 + \frac{1}{2}(2.8\times10^6)(0.024)^2 = 0$,
$v_1 = 0.733$ m/s

Hammer & anvil impact: $\Delta G_x = 0$: $3000(0.733) - 600v' - 600(3.13) = 0$
$v' = 0.534$ m/s

Hammer after impact: $v' = \sqrt{2gh}$, $h = \frac{0.534^2}{2\times9.81} = 0.01453$ m
or $\underline{h = 14.53 \text{ mm}}$

$e = \frac{0.534 - (-0.733)}{3.13} = \underline{0.405}$

▸3/276 $v_{x_A} = 50\cos\alpha$, $v_{y_A} = 50\sin\alpha$



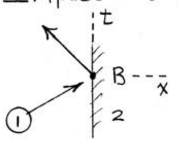
$t_{AB} = \frac{10}{v_{x_A}} = \frac{10}{50\cos\alpha} = \frac{1}{5\cos\alpha}$

$v_{x_B} = v_{x_A} = 50\cos\alpha$

$v_{y_B} = v_{y_A} - gt = 50\sin\alpha - \frac{g}{5\cos\alpha}$

$y_B = y_A + v_{y_A}t - \frac{1}{2}gt^2 = 0 + (50\sin\alpha)\left(\frac{1}{5\cos\alpha}\right) - \frac{g}{2}\left(\frac{1}{25\cos^2\alpha}\right) = 10\tan\alpha - \frac{g}{50\cos^2\alpha}$

Impact at B:



$v_{y_B}' = v_{y_B} = 50\sin\alpha - \frac{g}{5\cos\alpha}$

$e = \frac{v_{2x}' - v_{1x}'}{v_{1x} - v_{2x}} = \frac{0 - v_{1x}'}{50\cos\alpha - 0} = 0.5$

$v_{1x}' = -25\cos\alpha$

$t_{BA} = \frac{10}{25\cos\alpha} = \frac{2}{5\cos\alpha}$

$y_A = y_B + v_{y_B}'t - \frac{1}{2}gt^2$

$0 = \left(10\tan\alpha - \frac{g}{50\cos^2\alpha}\right) + \left(50\sin\alpha - \frac{g}{5\cos\alpha}\right)\left(\frac{2}{5\cos\alpha}\right) - \frac{g}{2}\left(\frac{2}{5\cos\alpha}\right)^2$

Collect terms:

$30\tan\alpha - \frac{9g}{50}\frac{1}{\cos^2\alpha} = 0$

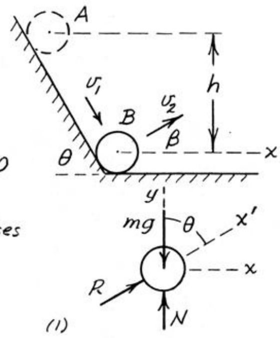
Use $\frac{1}{\cos^2\alpha} = (\tan^2\alpha + 1)$ to obtain

$5.796\tan^2\alpha - 30\tan\alpha + 5.796 = 0$

Quadratic solution : $\tan\alpha = 0.201,\ 4.97$

$\Rightarrow \underline{\alpha = 11.37° \text{ or } 78.6°}$

▸3/277



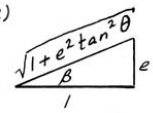
From A to B
$\Delta T + \Delta V_g = 0$: $\frac{1}{2}mv_1^2 - mgh = 0$,
$v_1 = \sqrt{2gh}$

During impact
$\Sigma F_{x'} \geq 0$: $R - mg\cos\theta + N\cos\theta \geq 0$
$R < mg\cos\theta$

During small time of impact, impulses of R & mg are negligible
so $\int \Sigma F_x\, dt \approx 0$ & $\Delta G_x \approx 0$:
$mv_1\cos\theta = mv_2\cos\beta$ (1)

$\int \Sigma F_y\, dt \neq 0$ & $v_2\sin\beta = ev_1\sin\theta$ (2)

Divide (2) by (1) & get $e\tan\theta = \tan\beta$



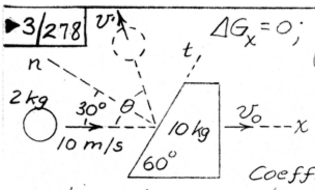
$v_x = v_2\cos\beta = v_1\cos\theta$, $\underline{v_x = \sqrt{2gh}\cos\theta}$

$n = \frac{|\Delta T|}{T} = \frac{\frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = 1 - \frac{v_2^2}{v_1^2} = 1 - \frac{(ev_1\sin\theta/\sin\beta)^2}{v_1^2}$

$n = 1 - \frac{e^2\sin^2\theta}{\sin^2\beta} = \underline{1 - (\cos^2\theta + e^2\sin^2\theta)}$ where $\sin^2\beta = \frac{e^2\tan^2\theta}{1 + e^2\tan^2\theta}$

For a rounded corner of radius greater than that of the sphere, there would be no discontinuity in the magnitude of the velocity and, hence, no impact.

▸3/278



$\Delta G_x = 0$; $2(10) + 0 = -2v'\cos\theta + 10v_0$
(For system) (1)

$\Delta G_t = 0$ (for sphere)
$2(10\sin 30°) = 2v'\sin(\theta - 30°)$ (2)

Coefficient of restitution applies to velocity components in n-dir.

$0.6 = \frac{v_0\sin 60° + v'\cos(\theta - 30°)}{10\cos 30° + 0}$ (3)

Eq.(3) is $5.196 = 0.866v_0 + 0.866v'\cos\theta + 0.5v'\sin\theta$

Sub. Eq.(1) to eliminate v_0 & get

$5.196 = 0.866(2 + 0.2v'\cos\theta) + 0.866v'\cos\theta + 0.5v'\sin\theta$

or $1.039v'\cos\theta + 0.5v'\sin\theta = 3.464$ (4)

Eq.(2) becomes $0.866v'\sin\theta - 0.5v'\cos\theta = 5$ (5)

Solve (4) & (5) & get $\underline{v' = 6.04 \text{ m/s}}$, $\underline{\theta = 85.9°}$

From Eq. (1) $v_0 = 2.087$ m/s

For carriage $\Delta T + \Delta V_e = 0$; $-\frac{1}{2}10(2.087)^2 + \frac{1}{2}1600\delta^2 = 0$

$\delta^2 = 0.02722$, $\delta = 0.1650$ m or $\underline{\delta = 165.0 \text{ mm}}$

3/279

$v = \sqrt{\frac{Gm_s}{r}} = \sqrt{\frac{(3.439\times10^{-8})(333{,}000)(4.095\times10^{23})}{(93\times10^6)(5280)}}$

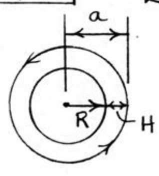
$= 97{,}725$ ft/sec $= \underline{18.51 \text{ mi/sec}}$

3/280 For a circular orbit, $r_{min} = r_{max}$ and $a = R + h$, so Eq. 3/48 becomes

$v = R\sqrt{\frac{g}{R+h}} = 6371(10^3)\sqrt{\frac{9.825}{(6371+590)10^3}}$

$= \underline{7569 \text{ m/s}}$ or $\underline{27\,250 \text{ km/h}}$

3/281 Eq. 3/47 with $r = a = R + H$:



$$v^2 = 2gR^2\left(\frac{1}{a} - \frac{1}{2a}\right) = \frac{gR^2}{R+H}$$

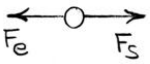
$$= \frac{1.62\,(3476/2)^2}{\frac{3476}{2} + 80}(1000)$$

$\underline{v = 1641 \text{ m/s}}$ or $\underline{5910 \text{ km/h}}$

3/282

$$\frac{F_s}{F_e} = \frac{G m_s m / d_{m-s}^2}{G m_e m / d_{m-e}^2}$$

Moon m



$$= \left(\frac{d_{m-e}}{d_{m-s}}\right)^2 \frac{m_s}{m_e}$$

$$= \left(\frac{384\ 398}{149.6(10^6) - 384\ 398}\right)^2 333\ 000 = 2.21$$

Therefore, the acceleration of the moon is toward the sun, and thus the path is concave toward the sun!

3/283 From Eq. 3/47 for a circular orbit of altitude H & with $a = r = R + H$

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2r}\right) = gR^2/(R+H), \quad v = R\sqrt{g/(R+H)}$$

$$v^2_{escape} = 2gR^2\left(\frac{1}{r} - \frac{1}{\infty}\right) = 2gR^2/(R+H), \quad v = R\sqrt{2g/(R+H)}$$

$$\Delta v = v_{escape} - v = R\sqrt{\frac{g}{R+H}}(\sqrt{2} - 1) = (\sqrt{2} - 1)v = 0.414\,v$$

For $R = 3959$ mi, $g = 32.23$ ft/sec^2, $H = 200$ mi,

$$v = 3959\sqrt{\frac{32.23/5280}{3959 + 200}} = 4.80 \text{ mi/sec}, \ \Delta v = 0.414(4.80) = \underline{1.987 \tfrac{mi}{sec}}$$

3/284 $r_{min} = 6371 + 240 = 6611$ km

$r_{max} = 6371 + 400 = 6771$ km

From Eq. 3/43 $\dfrac{r_{min}}{r_{max}} = \dfrac{1-e}{1+e}$

So $(1+e)6611 = (1-e)6771$, $\underline{e = 0.01196}$

From Eq. 3/44 with $a = \frac{1}{2}(r_{max} + r_{min}) = 6691$ km

$$\tau = 2\pi\frac{(6691 \times 10^3)^{3/2}}{(6371 \times 10^3)\sqrt{9.824}} = \underline{5446 \text{ s}} \text{ or}$$

$\underline{\tau = 1 \text{ h } 30 \text{ min } 46 \text{ s}}$

3/285 $F = G\dfrac{m_1 m_2}{r^2}$

$$= 6.673(10^{-11})\frac{1.490(10^{23})\,(1.900)(10^{27})}{(1.070 \times 10^9)^2} = \underline{16.50(10^{21})\text{N}}$$

$$F = mr\omega^2, \quad \omega = \sqrt{\frac{F}{mr}} = \sqrt{\frac{16.50(10^{21})}{1.490(10^{23})\,1.070(10^9)}}$$

$$= 1.017(10^{-5}) \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega} = \frac{2\pi}{1.017(10^{-5})} = \underline{6.18(10^5)\text{s}} \text{ or } \underline{7.17 \text{ days}}$$

$$a_n = \frac{F}{m_1} = \frac{16.50(10^{21})}{1.490(10^{23})} = \underline{110.7(10^{-3}) \text{ m/s}^2}$$

3/286 $r_{min} = 2R$, $r_{max} = 3R$

$$a = \frac{r_{min} + r_{max}}{2} = 2.5R$$

$$v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}} = R\sqrt{\frac{g}{2.5R}}\sqrt{\frac{3R}{2R}} = \sqrt{\frac{3gR}{5}}$$

The velocity in the original circular orbit is

$$v_c = R\sqrt{\frac{g}{a}} = R\sqrt{\frac{g}{2R}} = \sqrt{\tfrac{1}{2}gR}$$

$$\Delta v = v_p - v_c = \sqrt{gR}\left(\sqrt{\tfrac{3}{5}} - \sqrt{\tfrac{1}{2}}\right) = 0.0675\sqrt{gR}$$

Numbers: $\Delta v = 0.0675\sqrt{9.825(6371)(1000)}$

$= \underline{534 \text{ m/s}}$

(Δv to occur opposite B)

3/287 $r = a = 6371 + 300 = 6671 \text{ km} = 6.671(10^6)\text{ m}$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi\,(6.671 \times 10^6)^{3/2}}{6.371 \times 10^6\sqrt{9.825}}$$

$= 5421$ s

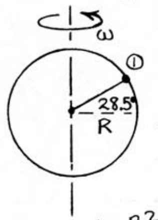
Speed of ground point on equator

$v_e = R_e\omega_e = (6378)(7.292 \times 10^{-5}) = 0.4651$ km/s

Required distance $d = v_e\tau = (0.4651)(5421)$

$= \underline{2520 \text{ km}}$

3/288



① On ground, speed $v_1 = (R\cos 28.5)\omega$

$= 6371(1000)\cos 28.5°\,(0.7292 \cdot 10^{-4})$

$= 408$ m/s

$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(80\,000)\,408^2$

$= 6.67(10^9)$ J

$$V_1 = -\frac{mgR^2}{R} = -80\,000(9.825)(6371 \cdot 1000) = -5.01(10^{12})\text{J}$$

② In circular orbit: $v_2 = R\sqrt{\frac{g}{r}}$

$$= 6371(10^3)\sqrt{\frac{9.825}{(6371 + 300)(1000)}} = 7.73(10^3) \text{ m/s}$$

$$T_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}\,80\,000\,[7.73(10^3)]^2 = 2.39(10^{12})\text{ J}$$

$$V_2 = -\frac{mgR^2}{r} = -\frac{80\,000(9.825)(6371 \cdot 1000)^2}{(6371 + 300) \cdot 1000} = -4.78(10^{12})\text{ J}$$

$$\Delta E = T_2 + V_2 - (T_1 + V_1) = \underline{2.61(10^{12})\text{ J}}$$

3/289 (a) $v = R\sqrt{\frac{g}{r}} = 6371(1000)\sqrt{\frac{9.825}{(6371+637)(1000)}}$

$= \underline{7544 \text{ m/s}}$

(b) From $r_{min} = a(1-e)$, $a = \frac{r_{min}}{1-e} = \frac{1.1(6371)}{1-0.1}$

$= 7787 \text{ km}$

$v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{1+e}{1-e}} = 6371(1000)\sqrt{\frac{9.825}{7787(1000)}}\sqrt{\frac{1+0.1}{1-0.1}}$

$= \underline{7912 \text{ m/s} = v}$

(c) $a = \frac{r_{min}}{1-e} = \frac{1.1(6371)}{1-0.9} = 70\,081 \text{ km}$

$v_p = 6371(1000)\sqrt{\frac{9.825}{70\,081(1000)}}\sqrt{\frac{1+0.9}{1-0.9}}$

$= \underline{10\,398 \text{ m/s} = v}$

(d) Eq. 3/47 with $a \to \infty$: $v = R\sqrt{\frac{2g}{r}}$

This is $\sqrt{2}$ times answer for part (a), so

$v = \sqrt{2}\,(7544) = \underline{10\,668 \text{ m/s}}$

3/290 $v_A = R\sqrt{\frac{g}{r}} = (3959)(5280)\sqrt{\frac{32.23}{(4759)(5280)}}$

$= 23{,}676 \text{ ft/sec}$

$v_B = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}}$

$= (3959)(5280)\sqrt{\frac{32.23}{\frac{(2(3959)+1800)}{2}(5280)}}\sqrt{\frac{4959}{4759}}$

$= 23{,}917 \text{ ft/sec}$

Momentum conservation during impact:

$m_A v_A + m_B v_B = (m_A + m_B) v_C$. But $m_A = m_B$, so

$v_C = \frac{1}{2}(v_A + v_B) = 23{,}796 \text{ ft/sec}$

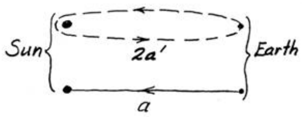
From $v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}}$,

$r_{max} = \frac{r_{min}}{\left(\frac{2gR^2}{v_p^2 r_{min}} - 1\right)} = 2.5652 \times 10^7 \text{ ft}$ (4858 mi)

$h_{max} = r_{max} - R = 4858 - 3959 = \underline{899 \text{ mi}}$

3/291

Radius of actual orbit around the sun is a, which is the major axis 2a′ of the degenerate ellipse.



R = radius of sun

g = gravitational accel. on surface of sun

Orbital period Eq. 3/44

For actual orbit $\tau = 2\pi \frac{a^{3/2}}{R\sqrt{g}}$

For degenerate ellipse $\tau' = 2\pi \frac{(a/2)^{3/2}}{R\sqrt{g}}$

so $\frac{\tau'}{\tau} = \frac{(\frac{1}{2})^{3/2}}{1}$

But time t to fall is $t = \frac{1}{2}\tau' = \frac{1}{2}\left(\frac{1}{2}\right)^{3/2}\tau = \frac{1}{4\sqrt{2}} 365.26$

$= \underline{64.6 \text{ days}}$

3/292 The apogee speed at C is

$v_a = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}}$

$= 6371(10^3)\sqrt{\frac{9.825}{(2\cdot 6371+240+320)1000/2}}\sqrt{\frac{6371+240}{6371+320}}$

$= 7697 \text{ m/s}$

The circular orbit speed at $h = 320$ km is

$v_{circ} = R\sqrt{g/r_{max}} = 7720 \text{ m/s}$

$\Delta v = v_{circ} - v_a = 7720 - 7697 = 23.25 \text{ m/s}$

$F\Delta t = m\Delta v$: $2(30\,000)(\Delta t) = 85\,000\,(23.25)$

$\underline{\Delta t = 32.9 \text{ s}}$

The burn to increase speed is at C.

3/293 The linear impulses from drag and from the thruster must be equal in magnitude, or

$Dt = \sum T t_{burn}$

$t = 10\tau$, where $\tau = 2\pi \frac{a^{3/2}}{R\sqrt{g}}$

or $\tau = 2\pi \frac{(6.571 \times 10^6)^{3/2}}{6.371 \times 10^6 \sqrt{9.825}} = 5300 \text{ s}$

$t = 10\tau = 53{,}000 \text{ s}$

$D = \frac{\sum T t_{burn}}{t} = \frac{2(300)}{53{,}000} = \underline{0.01132 \text{ N}}$

3/294 From $r_{min} = a(1-e)$:

$7959 = [4000 + 3959 + 16{,}000](1-e)$, $e = 0.668$

$b = a\sqrt{1-e^2} = 23{,}959\sqrt{1-0.668^2} = \underline{17{,}833 \text{ mi}}$

At B, $r = \sqrt{16{,}000^2 + 17{,}833^2} = 23{,}959 \text{ mi}$

$v_B^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

$= 2(32.23)3959^2(5280)\left[\frac{1}{23{,}959} - \frac{1}{2(23{,}959)}\right]$

$\underline{v_B = 10{,}551 \text{ ft/sec}}$

3/295 Eq. 3/44: $\tau_f = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi a^{3/2}}{\sqrt{Gm_A}}$

$= 2\pi \frac{[200(10^9)]^{3/2}}{\sqrt{6.673(10^{-11})\,10^{31}}} = \underline{21{,}760{,}000 \text{ s}}$

Eq. 3/49b: $\tau_{nf} = \frac{2\pi a^{3/2}}{\sqrt{G(m_A + m_B)}}$

$= 2\pi \frac{[200(10^9)]^{3/2}}{\sqrt{6.673(10^{-11})(10^{31} + 10^{30})}} = \underline{20{,}740{,}000 \text{ s}}$

(−4.7 percent difference)

3/296 Speed in circular orbit is

$$v = R\sqrt{\frac{g}{a}} = (3959)(5280)\sqrt{\frac{32.23}{(4459)(5280)}}$$

$= 24{,}458$ ft/sec

Time required for B to return to C's burn position : $t = \frac{2\pi r - 1000(5280)}{v}$

$= 5832$ s

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}}, \quad a = \left(\frac{\tau R\sqrt{g}}{2\pi}\right)^{2/3}$$

$$a = \left(\frac{(5832)(3959)(5280)\sqrt{32.23}}{2\pi}\right)^{2/3} = 2.29799\,(10)^7 \text{ ft}$$

At apogee, $v_C = \sqrt{2gR^2\left[\frac{1}{r} - \frac{1}{2a}\right]} = 24{,}156$ ft/sec

$\Delta v = v - v_C = 24{,}458 - 24{,}156 = \underline{302 \text{ ft/sec}}$

(Can check to ensure that C does not strike the earth by finding $r_{min} = 2.242 \times 10^7$ ft $> R = 2.090 \times 10^7$ ft !)

3/297 From previous solution, the circular orbit speed is $v = 24{,}458$ ft/sec.

Time required for B to return to C's burn position over almost two circular orbits:

$$t = \frac{4\pi r - (1000)(5280)}{v} = 11{,}881 \text{ s}$$

$$a = \left(\frac{\tau R\sqrt{g}}{2\pi}\right)^{2/3} = \left[\frac{\left(\frac{11{,}881}{2}\right)(3959)(5280)\sqrt{32.23}}{2\pi}\right]^{2/3}$$

$= 2.32626\,(10^7)$ ft

At apogee, $v_C = \sqrt{2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)} = 24{,}309$ ft/sec

$\Delta v = v - v_C = 24{,}458 - 24{,}309 = \underline{148 \text{ ft/sec}}$

3/298 Circular orbit speed

$$v_0 = R\sqrt{\frac{g}{a}} = R\sqrt{\frac{g}{3R}} = \sqrt{\tfrac{1}{3}gR}$$

Speed at A (apogee) in elliptical orbit:

$$v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}} = R\sqrt{\frac{g}{2R}}\sqrt{\frac{R}{3R}} = \sqrt{\tfrac{1}{6}gR}$$

$$v_r = v_0 - v_A = \sqrt{gR}\left[\sqrt{1/3} - \sqrt{1/6}\right] = 0.1691\sqrt{gR}$$

Numbers : $v_r = 0.1691\sqrt{1.62\,\frac{3476}{2}\,(1000)}$

$= \underline{284 \text{ m/s}}$ (directed rearward)

Call the circular orbit period τ_0 and the elliptical orbit period τ_{AB}

$$\tau_0 = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \frac{2\pi(3R)^{3/2}}{R\sqrt{g}}; \quad \tau_{AB} = \frac{2\pi(2R)^{3/2}}{R\sqrt{g}}$$

$$\theta = \left(\frac{\tau_{AB}/2}{\tau_0/2}\right)\pi = \left(\frac{2}{3}\right)^{3/2}\pi = 1.710 \text{ rad or } \underline{98.0°}$$

3/299 Circular orbit : $v = R\sqrt{\frac{g}{r}}$

$$v = (3959)(5280)\sqrt{\frac{32.23}{(4159)(5280)}} = 25{,}324 \text{ ft/sec}$$

During burn : $a_t = \frac{F}{m} = \frac{2(6000)}{(175{,}000)/32.2} = 2.208 \frac{\text{ft}}{\text{sec}^2}$

$v_a = v - a_t t = 25{,}324 - 2.208(150) = 24{,}993 \frac{\text{ft}}{\text{sec}}$

$v^2 = 2gR^2\left[\frac{1}{r} - \frac{1}{2a}\right]$

Substitute conditions at B to find $a = 2.1403(10^7)$ ft

Use $v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{1-e}{1+e}}$ to obtain $e = 0.02599$

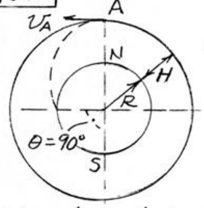
$r = \frac{a(1-e^2)}{1+e\cos\theta}$ utilized at point C :

$$(3959)(5280) = \frac{(2.1403 \times 10^7)(1 - 0.02599^2)}{1 + 0.02599\cos\theta}$$

$\theta = 26.7°$

$\beta = 180 - \theta = \underline{153.3°}$

3/300 From Eq. 3/43 $\frac{1}{r} = \frac{1 + e\cos\theta}{a(1-e^2)}$



when $\theta = 90°$, $r = R$

" $\theta = 180°$, $r = R + H$

Thus $\frac{1}{R} = \frac{1}{a(1-e^2)}$

& $\frac{1}{R+H} = \frac{1-e}{a(1-e^2)} = \frac{1}{a(1+e)}$

Solve & get $1 - e = \frac{R}{R+H}$ & $a(1+e) = R + H$

From Eq. 3/48

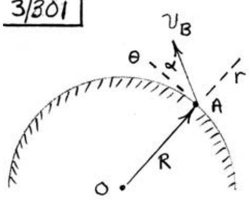
$$v_A = R\sqrt{\frac{g(1-e)}{a(1+e)}} = R\sqrt{g}\sqrt{\frac{R}{R+H}\,\frac{1}{R+H}} = \frac{R\sqrt{gR}}{R+H}$$

For circular orbit

$$v = R\sqrt{\frac{g}{R+H}} \quad \text{so } \Delta v = R\sqrt{\frac{g}{R+H}} - \frac{R\sqrt{gR}}{R+H}$$

$$= \underline{R\sqrt{\frac{g}{R+H}}\left(1 - \sqrt{\frac{R}{R+H}}\right)}$$

3/301



$v_\theta = v\cos\alpha = 2000\cos 30°$
$= 1732$ m/s

$v_r = v\sin\alpha = 2000\sin 30°$
$= 1000$ m/s

$$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$$

Using conditions at B :
$a = 3.2906 \times 10^6$ m

$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(2000)^2 = 2 \times 10^6$ m

$V_B = -\frac{mgR^2}{r} = \frac{-m(9.825)(6.371 \times 10^6)^2}{6.371 \times 10^6}$

$= -6.2595 \times 10^7$ m

$E = T_B + V_B = -6.0595 \times 10^7$ m

$h = rv_\theta = 6.371(10^6)(1732) = 1.1035 \times 10^{10}$

Now use $e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}}$ to get $e = 0.9525$

Finally, $r_{max} = a(1+e) = 3.2906(10^6)(1 + 0.9525)$
$= 6.4249 \times 10^6$ m

$h_{max} = r_{max} - R = 53\,900$ m or $\underline{53.9 \text{ km}}$

3/302 Point A is the apogee, so we have $r_{max} = \frac{3R}{2} = a(1+e)$.

$r = \frac{a(1-e^2)}{1+e\cos\theta}$; At B : $R = \frac{a(1-e^2)}{1+e\cos(135°)}$

Solving, $e = 0.6306$, $a = 0.9199R$

Now, $v_B^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

At A : $v_B^2 = 2(9.825)(6.371\times10^6)^2 \times \left(\frac{1}{6.371\times10^6} - \frac{1}{2(0.9199)(6.371)(10^6)}\right)$

$\underline{v_B = 7560 \text{ m/s}}$

3/303

$v_A = R\sqrt{\frac{g}{a}}\sqrt{\frac{1-e}{1+e}} = R\sqrt{\frac{9.825}{0.9199R}}\sqrt{\frac{1-0.6306}{1+0.6306}}$

$= 1.555\sqrt{R}$

$h = r_A v_A = \frac{3}{2}R(1.555\sqrt{R}) = 2.3332R^{3/2}$

Conservation of angular momentum requires

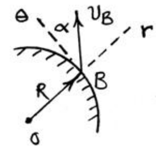
$h = r_B v_{B\theta} = R v_{B\theta} = 2.3332R^{3/2}$

$v_{B\theta} = 2.3332R^{1/2}$

$= 2.3332(6.371\times10^6)^{1/2} = 5889 \text{ m/s}$

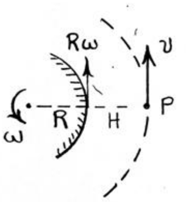
$v_{B\theta} = v_B\cos\alpha$

$\alpha = \cos^{-1}\left(\frac{v_{B\theta}}{v_B}\right) = \cos^{-1}\left(\frac{5889}{7560}\right) = \underline{38.8°}$



(Value of v_B from previous solution)

3/304 $a = \frac{1}{2}[2(6371) + 150 + 1500] = 7196$ km



Eq. 3/47 at perigee P :

$v^2 = 2(9.825)[6371(10^3)]^2 \times \left[\frac{1}{(6371+150)10^3} - \frac{1}{2(7196)10^3}\right]$

$v = 8179$ m/s

$R\omega = 6371(10^3)(0.7292\cdot10^{-4}) = 465$ m/s

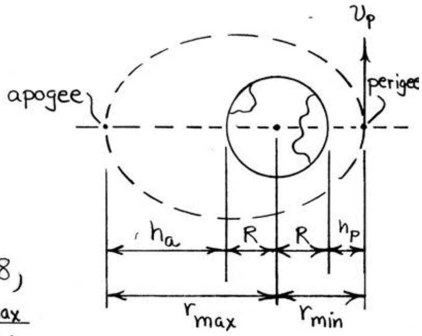
Absolute dish angular velocity $p_a = \frac{v - R\omega}{H}$

Relative dish angular velocity $p = p_a - \omega$

$p = \frac{v - R\omega}{H} - \omega = \frac{8179 - 465}{150(10^3)} - 0.7292(10^{-4})$

$= \underline{0.0514 \text{ rad/s}}$

3/305



At perigee, $a = a_n = \frac{v_p^2}{\rho_p}$,

so $\rho_p = \frac{v_p^2}{a_n}$

From Eq. 3/48,

$v_p^2 = R^2\frac{g}{a}\frac{r_{max}}{r_{min}}$

But from Eqs. 3/43 : $r_{min} + r_{max} = 2a$, so

$v_p^2 = gR^2\frac{r_{max}}{r_{min}}\frac{2}{r_{min}+r_{max}}$

Also, $a_n = g_{perigee} = g\left(\frac{R}{r_{min}}\right)^2$ (from Chapter 1)

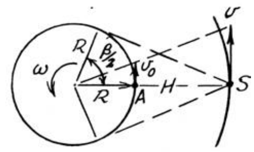
Thus $\rho_p = 2gR^2\frac{r_{max}}{r_{min}}\frac{1}{r_{min}+r_{max}} \Big/ g\frac{R^2}{r_{min}^2}$

$= 2\frac{r_{max}r_{min}}{r_{min}+r_{max}}$

or $\underline{\rho_p = 2\frac{(R+h_a)(R+h_p)}{2R+h_a+h_p}}$

3/306

Path is limited to an equatorial orbit in order to remain above a point A on the equator.



$\frac{v}{R+H} = \omega$ & for circular orbit Eq. 3/47 with $a = r = R+H$ gives $v = R\sqrt{\frac{g}{R+H}}$

$\omega = 0.7292\times10^{-4}$ rad/s

$R = 6371$ km

Combine & get $R+H = \sqrt[3]{\frac{gR^2}{\omega^2}}$, $H = \sqrt[3]{\frac{9.825(6371\times10^3)^2}{(0.7292\times10^{-4})^2}} - 6371\times10^3$

$= (42\,170 - 6371)10^3 = 35.8\times10^6$ m

or $\underline{H = 35\,800 \text{ km}}$

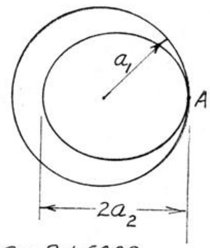
$\frac{\beta}{2} = \cos^{-1}\frac{R}{R+H} = \cos^{-1}\frac{6371}{42\,170} = 81.3°$, $\underline{\beta = 162.6° \text{ of longitude}}$

3/307 $Ft = m\Delta v$

For circular orbit, $v_1 = R\sqrt{g/a_1}$

$= 6371(10^3)\sqrt{\frac{9.825}{12\,371(10^3)}}$

$= 5678$ m/s



For elliptical orbit at apogee A

$v_A = R\sqrt{\frac{g}{a_2}}\sqrt{\frac{r_{min}}{r_{max}}}$

where $r_{min} = 6371 + 3000 = 9371$ km

$r_{max} = 6371 + 6000 = 12\,371$ km

So $v_A = 6371(10^3)\sqrt{\frac{9.825}{10\,871(10^3)}}\sqrt{\frac{9371}{12\,371}}$

$= 5271$ m/s

Thus $\Delta v = 5678 - 5271 = 406$ m/s

So $2000t = 800(406)$

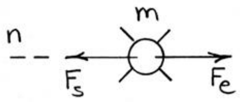
$\underline{t = 162 \text{ s}}$

$a_1 = R + 6000$
$= 6371 + 6000$
$= 12\,371$ km

$2a_2 = 2a_1 - 3000$
$= 21\,742$ km

$a_2 = 10\,871$ km

*3/308



F_s : force exerted on spacecraft by sun

F_e : force exerted on spacecraft by earth

$\Sigma F_n = ma_n : \quad F_s - F_e = m\frac{v^2}{\rho} = m\rho\omega^2 = m(D-h)\left(\frac{2\pi}{T}\right)^2$

where D is the earth-sun distance and T is the earth orbital period.

$$\frac{Gm_s \cancel{m}}{(D-h)^2} - \frac{Gm_e\cancel{m}}{h^2} = \cancel{m}(D-h)\left(\frac{2\pi}{T}\right)^2$$

With $G = 3.439(10^{-8})\frac{ft^4}{lb\text{-}sec^4}$, $m_s = 333{,}000\, m_e$, $m_e = 4.095(10^{23})$ slugs, $D = 92.96(10^6)(5280)$ ft, and $T = 365.26(24)(3600)$ sec, solve numerically for h as $h = 4.87(10^9)$ ft or $\underline{922{,}000 \text{ mi}}$

▶3/309 For 1, $v_1 = R\sqrt{g/r_1}$; for 2, $v_2 = R\sqrt{g/r_2}$

For transfer ellipse at A, $v_1' = R\sqrt{g/a}\sqrt{r_2/r_1}$ $\Big\}$ $a = \frac{r_1+r_2}{2}$

For transfer ellipse at B, $v_2' = R\sqrt{g/a}\sqrt{r_1/r_2}$ $\Big\}$ (Eq. 3/48)

At A, $\Delta v_A = v_1' - v_1 = R\sqrt{g/a}\sqrt{r_2/r_1} - R\sqrt{g/r_1} = \underline{R\sqrt{g/r_1}\left(\sqrt{\frac{2r_2}{r_1+r_2}}-1\right)}$

At B, $\Delta v_B = v_2 - v_2' = R\sqrt{g/r_2} - R\sqrt{g/a}\sqrt{r_1/r_2} = \underline{R\sqrt{g/r_2}\left(1-\sqrt{\frac{2r_1}{r_1+r_2}}\right)}$

$$\Delta v_A = 6371(10^3)\sqrt{\frac{9.825(10^3)}{6871}}\left(\sqrt{\frac{2(42171)}{6871+42171}}-1\right) = \underline{2370 \text{ m/s}}$$

$$\Delta v_B = 6371(10^3)\sqrt{\frac{9.825(10^3)}{42171}}\left(1-\sqrt{\frac{2(6871)}{6871+42171}}\right) = \underline{1447 \text{ m/s}}$$

▶3/310 Eq. 3/47 : $v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

$$7400^2 = 2(9.825)(6371\cdot 1000)^2\left[\frac{1}{7371\cdot 1000} - \frac{1}{2a}\right]$$

$\underline{a = 7462 \text{ km}}$

$T = \frac{1}{2}mv^2 = \frac{1}{2}m(7400)^2 = 27.38(10^6)m$

$V = -\frac{mgR^2}{r} = -\frac{m(9.825)(6371\cdot 1000)^2}{7371(1000)} = -54.1(10^6)m$

$E = T + V = -26.7(10^6)m$ (in Joules)

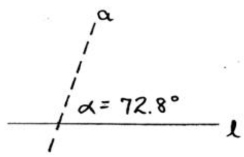
$h = rv_\theta = 7371(10^3)(7400 \cos 2°) = 5.45(10^{10})\frac{m^2}{s}$

$$e = \sqrt{1+\frac{2Eh^2}{mg^2R^4}} = \sqrt{1+\frac{2(-26.7)10^6 m\ 5.45^2\ 10^{20}}{m(9.825)^2(6371\cdot 1000)^4}}$$

$= 0.0369$

From $r = \frac{a(1-e^2)}{1+e\cos\theta}$: $7371 = \frac{7462(1-0.0369^2)}{1+0.0369\cos\theta}$

$\theta = \pm 72.8°$ So $\underline{\alpha' = 72.8°}$

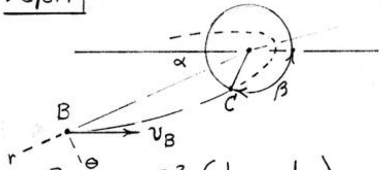


$r_{min} = a(1-e) = 7462(1-0.0369)$

$= 7186 > R = 6371$ m

$\therefore$ $\underline{\text{Does not strike earth}}$

▶3/311



At B, $r = \sqrt{29}R$

$\alpha = \tan^{-1}\left(\frac{2R}{5R}\right) = 21.8°$

$v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

At B : $3200^2 = 2(9.825)(6.371\times 10^6)^2\left[\frac{1}{\sqrt{29}\,6.371(10^6)} - \frac{1}{2a}\right]$

$a = 3.066\times 10^7$ m

$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}m(3200)^2 = 5.120\times 10^6$ m

$V_B = -\frac{mgR^2}{r_B} = -m\frac{(9.825)(6.371\times 10^6)^2}{\sqrt{29}(6.371\times 10^6)} = -1.162\times 10^7$ m

$E = T_B + V_B = -6.504\times 10^6$ m

$v_\theta = 3200 \sin\alpha = 1188.5$ m/s

$h = rv_\theta = \sqrt{29}(6.371\times 10^6)(1188.5) = 4.077\times 10^{10}$ kg-m²/s

$$e = \sqrt{1+\frac{2Eh^2}{mg^2R^4}}$$

$$e = \sqrt{1+\frac{2(-6.504m)(4.077\times 10^{10})^2}{m(9.825)^2(6.371\times 10^6)^4}}$$

$= 0.9295$

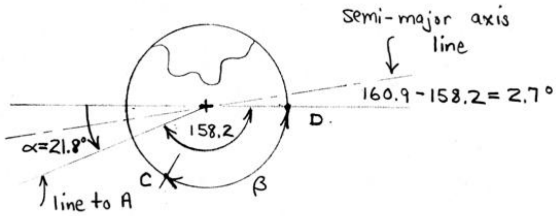
$r = \frac{a(1-e^2)}{1+e\cos\theta}$

At B : $\sqrt{29}(6.371\times 10^6) = \frac{(3.066\times 10^7)(1-0.9295^2)}{1+0.9295\cos\theta}$

$\theta = 160.9°$

At C : $6371(10^6) = \frac{(3.066\times 10^7)(1-0.9295^2)}{1+e\cos\theta}$

$\theta = 111.8°$



$\beta = 111.8 - 2.7 = \underline{109.1°}$

3/312 The speed of the orbiter is

$$v_0 = \sqrt{\frac{Gm_e}{r}} = \sqrt{\frac{6.673(10^{-11})(5.976)(10^{24})}{(6371+200)(1000)}} = 7790 \text{ m/s}$$

The speed of the satellite is

$$v = \sqrt{v_0^2 + v_{s/o}^2} = 7791 \text{ m/s}$$

Eq. 3/47: $v^2 = 2gR^2\left(\frac{1}{r} - \frac{1}{2a}\right)$

$$(7791)^2 = 2(9.825)(6371 \cdot 1000)^2\left[\frac{1}{6571(1000)} - \frac{1}{2a}\right]$$

$$\underline{a = 6572 \text{ km}}$$

$$\tau = \frac{2\pi a^{3/2}}{R\sqrt{g}} = \underline{5301 \text{ s}}$$

Energy $E = \frac{1}{2}mv^2 - \frac{Gm_e m}{r} = -30.3m(10^6)$ J

$h = rv_\theta = 6571(1000)7790 = 5.12(10^{10})$ m^2/s

Eq. 3/45: $e = \sqrt{1 + \frac{2Eh^2}{mg^2R^4}} = \underline{0.01284}$

From $\frac{1}{r} = \frac{1 + e\cos\theta}{a(1-e^2)}$, $\underline{\theta = 90° \text{ exactly}}$

(semimajor axis is parallel to x-axis)

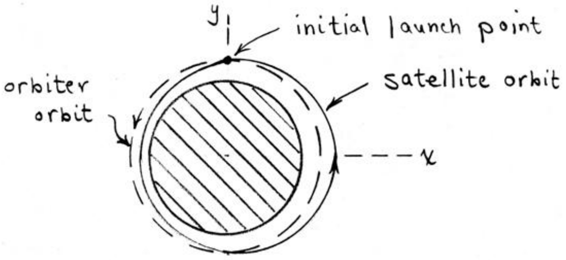
$r_{min} = a(1-e) = \underline{6.49(10^6) \text{ m}}$

$r_{max} = a(1+e) = \underline{6.66(10^6) \text{ m}}$

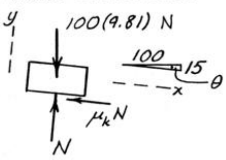
$v_p = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{max}}{r_{min}}} = \underline{7890 \text{ m/s}}$

$v_a = R\sqrt{\frac{g}{a}}\sqrt{\frac{r_{min}}{r_{max}}} = \underline{7690 \text{ m/s}}$

Sketch (not to scale):



3/313 *Truck bed is a constant-velocity frame of reference so that $U_{rel} = \Delta T_{rel}$ holds.*



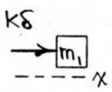
$\Sigma F_y = 0$: $N - 981\cos 8.53° = 0$

$N = 970$ N

$U_{rel} = \Delta T_{rel}$: $(981\sin 8.53° - 970\mu_k)2 = \frac{1}{2}100(0 - 3^2)$ $\qquad \theta = \tan^{-1}0.15 = 8.53°$

$$\underline{\mu_k = 0.382}$$

3/314 $\Sigma F_x = ma_x$: $k\delta = m_1 a_{x_1}$



$-k\delta = m_2 a_{x_2}$

$a_{1/2} = a_{x_1} - a_{x_2} = \frac{k\delta}{m_1} - \left(-\frac{k\delta}{m_2}\right)$

or $\underline{a_{1/2} = a_{rel} = k\delta\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}$

3/315 $v_{rel} = l\dot{\theta} = 0.5(2) = 1$ m/s $\rightarrow$

$v = v + v_{rel} = 2 + 1 = 3$ m/s $\rightarrow$

$\underset{\sim}{G} = m\underset{\sim}{v} = 3(3\underset{\sim}{i}) = \underline{9\underset{\sim}{i} \text{ kg}\cdot\text{m/s}}$

$\underset{\sim}{G}_{rel} = m\underset{\sim}{v}_{rel} = 3(1\underset{\sim}{i}) = \underline{3\underset{\sim}{i} \text{ kg}\cdot\text{m/s}}$

$T = \frac{1}{2}mv^2 = \frac{1}{2}(3)(3)^2 = \underline{13.5 \text{ J}}$

$T_{rel} = \frac{1}{2}mv_{rel}^2 = \frac{1}{2}(3)(1)^2 = \underline{1.5 \text{ J}}$

$\underset{\sim}{H}_O = -lmv\underset{\sim}{k} = -(0.5)(3)(3)\underset{\sim}{k} = \underline{-4.5\underset{\sim}{k} \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$

$\underset{\sim}{H}_{B_{rel}} = -lmv_{rel}\underset{\sim}{k} = -(0.5)(3)(1)\underset{\sim}{k} = \underline{-1.5\underset{\sim}{k} \ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$

3/316 *Rel. to carrier* $U_{rel} = \Delta T_{rel}$

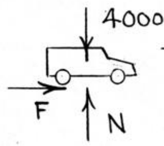
$(22 + P)(10^3)75 = \frac{1}{2}(3)(10^3)\left[(240/3.6)^2 - 0\right]$

$22 + P = 88.9$ kN, $\underline{P = 66.9 \text{ kN}}$

3/317 Barge-fixed frame is Newtonian.

$v^2 = v_0^2 + 2a(s - s_0)$: $\left(15\,\frac{5280}{3600}\right)^2 = 2a(80)$

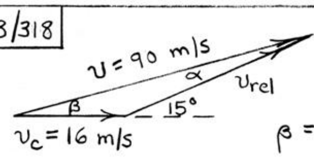
$a = 3.03$ ft/sec$^2 = a_{rel}$



$\xrightarrow{+} \Sigma F = ma$: $F = \frac{4000}{32.2}(3.03)$

$$\underline{F = 376 \text{ lb}}$$

3/318



$\frac{\sin 165°}{90} = \frac{\sin\alpha}{16}$

$\alpha = 2.64°$

$\beta = 180 - 165 - \alpha = 12.36°$

$\frac{\sin\beta}{v_{rel}} = \frac{\sin 165°}{90}$, $\qquad v_{rel} = 74.4$ m/s

$U_{rel} = \Delta T_{rel}$: $Fd = \frac{1}{2}m(v_{rel}^2 - 0)$

$F(100) = \frac{1}{2}7000(74.4^2)$

$\underline{F = 194\,000 \text{ N}}$ or $\underline{194.0 \text{ kN}}$

3/319 $- - - x$

For truck, $a_T = -0.9g$, $t_{stop} = \frac{15 \text{ m/s}}{0.9(9.81 \text{ m/s}^2)} = 1.699$ s

For crate, $a_c = -0.7g$

$a_{c/T} = a_c - a_T = -0.7g - (-0.9g) = 0.2g$

(As long as truck is moving)

At $t = t_{stop}$,

$x_{c/T} = (x_{c/T})_0 + (v_{c/T})_0 t_{stop} + \frac{1}{2}a_{c/T}t_{stop}^2$

$= 0 + 0 + \frac{1}{2}(0.2g)(1.699)^2 = 2.83$ m

$v_{c/T} = (v_{c/T})_0 + a_{c/T}t_{stop} = 0 + (0.2g)(1.699) = 3.33 \ \frac{\text{m}}{\text{s}}$

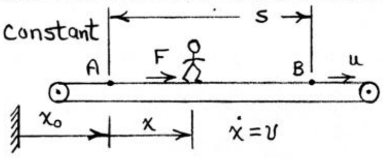
Then : $v_c^2 = v_{c_0}^2 + 2a_c(x - x_0)$

$= (3.33)^2 + 2(-0.7g)(3.2 - 2.83)$

$$\underline{v_c = 2.46 \text{ m/s}}$$

Chai Gr.C 92#

3/320 F = constant



Absolute: $U = \Delta T$: $F(s + \Delta x_o) = \frac{1}{2}m(u+v)^2 - \frac{1}{2}mu^2$

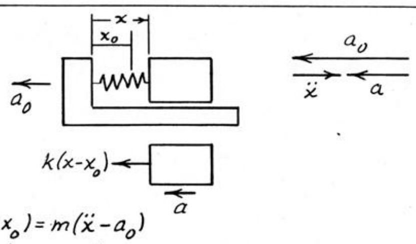
$$Fs + F\Delta x_o = \frac{1}{2}mv^2 + muv \quad (1)$$

Relative to walkway: $U_{rel} = \Delta T_{rel}$: $Fs = \frac{1}{2}mv^2 - 0$ (2)

Subtract (2) from (1): $F\Delta x_o = muv$

The term muv represents the work done by force F due only to the movement of the walkway.

3/321



$\Sigma F_x = ma_x$: $-k(x-x_o) = m(\ddot{x} - a_o)$

$\dot{x}\,d\dot{x} = \ddot{x}\,dx$ so $\int_0^{\dot{x}} \dot{x}\,d\dot{x} = \int_{x_o}^{x} \left[a_o - \frac{k}{m}(x - x_o)\right] dx$

$$\frac{1}{2}\dot{x}^2 = \left(a_o - \frac{kx_o}{m}\right)(x - x_o) - \frac{k}{2m}(x^2 - x_o^2)$$

$\frac{d}{dx}\left(\frac{\dot{x}^2}{2}\right) = a_o + \frac{kx_o}{m} - \frac{kx}{m} = 0$ for max. $\frac{\dot{x}^2}{2}$ & hence max $\dot{x}$

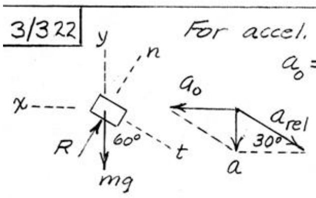
so $\frac{kx}{m} = a_o + \frac{kx_o}{m}$, $x = x_o + \frac{ma_o}{k}$

Thus

$$(v_{rel})^2_{max} = \dot{x}^2_{max} = 2\left(a_o + \frac{kx_o}{m}\right)\left(x_o + \frac{ma_o}{k} - x_o\right) - \frac{k}{m}\left(x_o^2 + \frac{2ma_o}{k}x_o + \frac{m^2a_o^2}{k^2} - x_o^2\right) = \frac{ma_o^2}{k}$$

$$\underline{(v_{rel})_{max} = a_o\sqrt{m/k}}$$

3/322



For accel. a vertically down,

$a_o = a_{rel} \cos 30°$

$a_x = 0$ so $\Sigma F_x = 0$

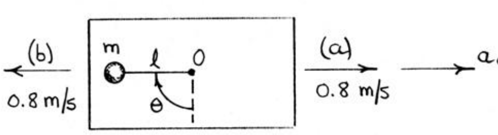
$-R \sin 30° = 0$, $\underline{R = 0}$

$\Sigma F_y = ma_y$, $mg = ma$

$a = g$ & $a_o = a\sqrt{3} = g\sqrt{3}$

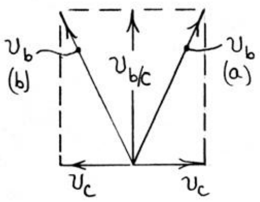
$= 9.81\sqrt{3} = \underline{16.99 \frac{m}{s^2}}$

3/323



$m_b = 10$ kg $\quad l = 0.8$ m $\quad \theta = 90°$

$m_c = 250$ kg $\quad \dot{\theta} = 3$ rad/s



$v_{b/c} = l\dot{\theta} = 0.8(3) = 2.4$ m/s

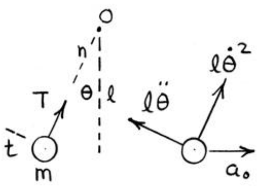
$T_b = \frac{1}{2}m_bv_b^2 = \frac{1}{2}(10)[0.8^2 + 2.4^2] = 32$ J

(same for cases (a) and (b))

$T_c = \frac{1}{2}m_cv_c^2 = \frac{1}{2}(250)(0.8)^2 = 80$ J

$T = T_b + T_c = 32 + 80 = \underline{112 \text{ J}}$ for both cases

3/324 $\Sigma \underline{F} = m(\underline{a}_o + \underline{a}_{rel})$. In t-dir., $\Sigma F_t = 0$,



so $a_t = l\ddot{\theta} - a_o\cos\theta = 0$

$\ddot{\theta} = \frac{a_o}{l}\cos\theta \quad (1)$

In n-dir., $\Sigma F_n = ma_n$

$T = m(l\dot{\theta}^2 + a_o\sin\theta) \quad (2)$

Integrate (1): $\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{a_o}{l}\cos\theta$

$$\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \int_0^{\theta} \frac{a_o}{l}\cos\theta\,d\theta$$

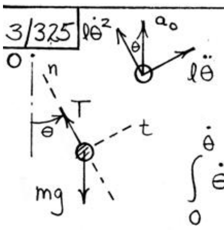
$$\frac{1}{2}\dot{\theta}^2 = \frac{a_o}{l}\sin\theta$$

From (2): $T = m[2a_o\sin\theta + a_o\sin\theta]$

or $\underline{T = 3ma_o\sin\theta}$

For $\theta = \frac{\pi}{2}$, $T_{\pi/2} = 3ma_o = 3(10)(3) = \underline{90 \text{ N}}$

3/325



$\Sigma F_t = ma_t$:

$-mg\sin\theta = m(l\ddot{\theta} + a_o\sin\theta)$

$\ddot{\theta} = -\left(\frac{a_o + g}{l}\right)\sin\theta$

$$\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = -\int_{\theta_o}^{\theta}\left(\frac{a_o+g}{l}\right)\sin\theta\,d\theta$$

$$\dot{\theta}^2 = 2\left(\frac{a_o+g}{l}\right)(\cos\theta - \cos\theta_o)$$

$\Sigma F_n = ma_n$: $T - mg\cos\theta = m(l\dot{\theta}^2 + a_o\cos\theta)$

$T = m[g(3\cos\theta - 2\cos\theta_o) + a_o(3\cos\theta - 2\cos\theta_o)]$

$= m(g + a_o)(3\cos\theta - 2\cos\theta_o)$

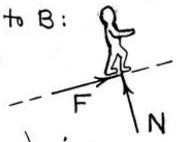
When $\theta = 0$,

$\underline{T_o = m(g + a_o)(3 - 2\cos\theta_o)}$

If $\theta_o = \pi/2$,

$\underline{T_o = 3m(g + a_o)}$

3/326 For motion from A to B:



Absolute: $U'_{abs} = \Delta T + \Delta V_g$:

$F(\Delta x_0 + s) = \frac{1}{2} m (v_r + u)^2 - \frac{1}{2} m u^2 + mg(\Delta x_0 + s)\sin\theta$

$= \frac{1}{2} m v_r^2 + m v_r u + mg(\Delta x_0 + s)\sin\theta$

Relative: $U'_{rel} = \Delta T_{rel} + \Delta V_{g_{rel}}$: $Fs = \frac{1}{2} m v_r^2 + mgs\sin\theta$

Work done by walkway: $U'_{abs} - U'_{rel} = m v_r u + mg\Delta x_0 \sin\theta$

$m v_r u$ represents the work done by the belt due only to the motion of the walkway.

For $m = \frac{150}{32.2}$ slugs, $v_r = 2.5$ ft/sec, $u = 2$ ft/sec, $\theta = 10°$, $s = 30$ ft:

$\Sigma F_x = m a_{x_{rel}}$: $a_{x_{rel}} = \frac{v_r^2}{2s} = \frac{2.5^2}{2(30)} = 0.1042 \frac{\text{ft}}{\text{sec}^2}$

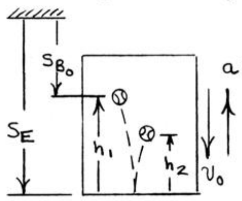
$F - 150\sin 10° = \frac{150}{32.2}(0.1042)$, $F = 26.5$ lb

Power by boy: $P_{rel} = F v_r = 26.5(2.5) = 66.3 \frac{\text{ft-lb}}{\text{sec}}$

or $P_{rel} = 66.3/550 = \underline{0.1206 \text{ hp}}$

3/327 (a) $a = 0$, elevator is Newtonian frame

$v' = ev = e\sqrt{2gh_1} = \sqrt{2gh_2}$



$a = \frac{g}{4}$ $\quad \underline{h_2 = e^2 h_1}$

(b) $a = \frac{g}{4}$ up

Let B = ball , E = elevator, $\downarrow +$

At impact, $s_B = s_E$: $s_{B_0} + v_{B_0}t + \frac{1}{2}gt^2 = s_{E_0} + v_{E_0}t - \frac{1}{2}\frac{g}{4}t^2$

$s_{B_0} + v_0 t + \frac{1}{2}gt^2 = (s_{B_0} + h_1) + v_0 t - \frac{1}{8}gt^2$, $t = 2\sqrt{\frac{2h_1}{5g}}$

$v_{B/E} = v_B - v_E = \left(v_0 + g\,2\sqrt{\frac{2h_1}{5g}}\right) - \left(v_0 - \frac{g}{4}2\sqrt{\frac{2h_1}{5g}}\right)$

$= \sqrt{\frac{5h_1 g}{2}}$

After collision, $v'_{B/E_0} = -e\sqrt{\frac{5h_1 g}{2}}$ (up)

$v'_{B/E} = v'_{B/E_0} + a_{B/E}t = -e\sqrt{\frac{5h_1 g}{2}} + \frac{5}{4}gt$

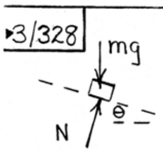
When $v'_{B/E} = 0$, $t = 2e\sqrt{\frac{2h_1}{5g}}$

$s'_{B/E} = s'_{B/E_0} + v'_{B/E_0}t + \frac{1}{2}\frac{5}{4}gt^2$

$= 0 - e\sqrt{\frac{5h_1 g}{2}}\,2e\sqrt{\frac{2h_1}{5g}} + \frac{5}{8}g\,4e^2\frac{2h_1}{5g}$

$= -e^2 h_1 \Rightarrow \underline{h_2 = e^2 h_1}$

▸3/328



$U_{rel} = \Delta T_{rel}$

$mg\,l\sin\theta = \frac{1}{2} m v_{rel}^2 - 0$

$v_{rel}^2 = 2gl\sin\theta$

$U = \Delta T$: $mg\,l\sin\theta + (N\sin\theta)d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

where d is the horizontal distance traveled by the block.

Time to slide from B to C : $l = \frac{1}{2}at^2 = \frac{1}{2}g\sin\theta t^2$

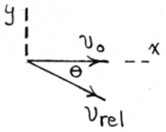
$t = \left(\frac{2l}{g\sin\theta}\right)^{1/2}$. So $d = v_0 t = v_0\sqrt{\frac{2l}{g\sin\theta}}$

Also, $N = mg\cos\theta$

Solving the work-energy equation for v^2:

$\underline{v_A = \left(v_0^2 + 2gl\sin\theta + 2v_0\cos\theta\sqrt{2lg\sin\theta}\right)^{1/2}}$

Check :



$\underline{v}_A = \underline{v}_0 + \underline{v}_{rel}$

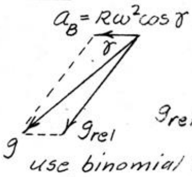
$= v_0\underline{i} + \sqrt{2gl\sin\theta}(\cos\theta\,\underline{i} - \sin\theta\,\underline{j})$

$= (v_0 + \sqrt{2gl\sin\theta}\cos\theta)\underline{i} - \sqrt{2gl\sin^3\theta}\,\underline{j}$

$v_A^2 = (v_0 + \sqrt{2gl\sin\theta}\cos\theta)^2 + (2gl\sin^3\theta)$

✓ $v_A^2 = v_0^2 + 2gl\sin\theta + 2v_0\cos\theta\sqrt{2gl\sin\theta}$

▸3/329 From law of cosines



$g_{rel}^2 = g^2 + a_B^2 - 2g\,a_B\cos\gamma$

$= g^2\left(1 + \left[\frac{a_B}{g}\right]^2 - 2\frac{a_B}{g}\cos\gamma\right)$

$g_{rel} = g\left[1 + \frac{a_B}{g}\left(\frac{a_B}{g} - 2\cos\gamma\right)\right]^{1/2}$

use binomial expansion for 1st two terms $(1+x)^n = 1 + nx + \cdots$ & get

$g_{rel} = g\left[1 + \frac{a_B}{g}\left(\frac{a_B}{2g} - \cos\gamma\right) + \cdots\right]$

$= g + a_B\left(\frac{a_B}{2g} - \cos\gamma\right) + \cdots$

$g_{rel} = g - R\omega^2\cos^2\gamma\left(1 - \frac{R\omega^2}{2g}\right) + \cdots$

$R\omega^2 = 6.371(10^6)(0.7292\times10^{-4})^2 = 0.03388 \text{ m/s}^2$

$g_{rel} = 9.825 - 0.03388\left(1 - \frac{0.03388}{2\times9.825}\right)\cos^2\gamma + \cdots$

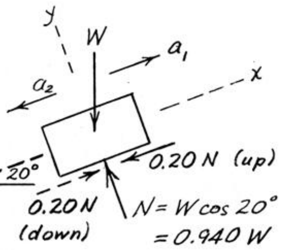
$\underline{= 9.825 - 0.03382\cos^2\gamma \text{ m/s}^2}$

▸3/330

Case (a): Orbital speed is constant so that $\ddot{x}$ is both the absolute and relative acceleration in the x-direction. Hence $F = m\ddot{x}$ holds.

Case (b): Orbital speed is decreasing in the position shown so that a component of acceleration in the negative x-direction exists so that the true (absolute) acceleration in the x-direction is $\ddot{x}$ minus the tangential orbital deceleration. Consequently $F \neq m\ddot{x}$. Only at the perigee and apogee positions where $\dot{r} = 0$ would $F = m\ddot{x}$ be true.

3/331



$(Up)\ \Sigma F_x = ma_x:\ -0.20(0.940\,W) - W\sin 20° = \frac{W}{g}a_1$

$a_1 = -17.06$ ft/sec^2

$v^2 = v_1^2 + 2a_1 s:$

$0 = 20^2 + 2(-17.06)s$

$s = 11.72$ ft

$(Down)\ \Sigma F_x = ma_x:\ -W\sin 20° + 0.20(0.940\,W) = \frac{W}{g}(-a_2)$

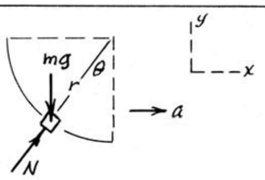
$a_2 = -4.96$ ft/sec^2

$v^2 = v_0^2 + 2a_2 s:\ v_2^2 = 0^2 + 2(4.96)(11.72)$

$= 116.3$ (ft/sec)^2

$\underline{v_2 = 10.78 \text{ ft/sec}}$

3/332



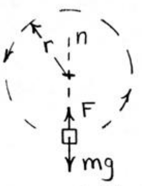
$\Sigma F_y = 0:\ N\cos\theta - mg = 0,\ N\cos\theta = mg$

$\Sigma F_x = ma_x:\ N\sin\theta = ma$

Divide & get $\tan\theta = a/g$

$\underline{\theta = \tan^{-1}\frac{a}{g}}$

3/333 Critical condition will occur when weight is at bottom position.



$\Sigma F_n = ma_n:\ F - mg = mr\omega^2$

$80 - 0.030(9.81) = 0.030(0.175)\omega^2$

$\omega = 123.2\ \frac{\text{rad}}{\text{s}}$

$N = \omega\left(\frac{60}{2\pi}\right) = \underline{1177 \text{ rev/min}}$

3/334 $v^2 = 2gh = 2(9.81)(0.4 + 0.4\cos 30°)$

$= 14.64$ m^2/s^2

$\Sigma F_n = ma_n:\ T - 2(9.81)\cos 30° = 2\frac{14.64}{0.4}$

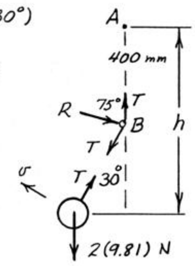
$T = 90.2$ N

Equil. of forces at B:

$R = 2T\cos 75°$

$= 2(90.2)(0.259)$

$= \underline{46.7 \text{ N}}$



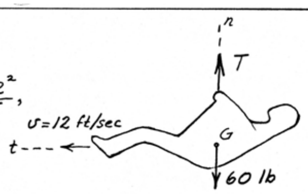
3/335

System:

$\Sigma F_n = m\frac{v^2}{r}:\ T - 60 = \frac{60}{32.2}\frac{12^2}{15},$

$T = 60(1 + 0.298)$

$= \underline{77.9 \text{ lb}}$

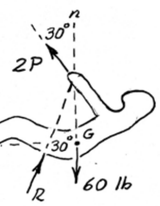


Girl:

$\Sigma F_n = m\frac{v^2}{r}:\ 2P\cos 30° + R\cos 30° - 60$

$= \frac{60}{32.2}\frac{12^2}{15} = 17.89$ lb

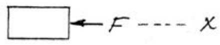
$\Sigma F_t = 0:\ 2P\sin 30° - R\sin 30° = 0$



Solve & get

$\underline{P = 22.5 \text{ lb}},\ \underline{R = 45.0 \text{ lb}}$

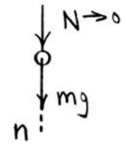
3/336



$\int \Sigma F_x\,dt = m\Delta v_x$

$-\frac{1}{2}(0.4)(8) - \frac{1}{2}(0.4)(10) = 2(v - 4),\ \underline{v = 2.2 \text{ m/s}}$

3/337 Dynamics at B (top of loop)



$\Sigma F_n = ma_n:\ mg = m\frac{v_B^2}{R}$

$v_B^2 = gR$

Work-kinetic energy from A to B:

$T_A + U_{A-B} = T_B:\ 0 + \frac{1}{2}k\delta^2 - mg\mu_k R - mg(2R) = \frac{1}{2}m(gR)$

$$\underline{\delta = \sqrt{\frac{mgR(5 + 2\mu_k)}{k}}}$$

3/338 Possibilities considered $\begin{cases} \text{(a) 2 masses with speed } v_1 \\ \text{(b) 1 mass with speed } 2v_1 \end{cases}$

Both (a) and (b) conserve system momentum since $2(1mv_1) = 1(2m)v_1$.

But with $e = 1$, kinetic energy must also be conserved.

Initial: $T = 2(\frac{1}{2}mv_1^2) = mv_1^2$

Final: $\begin{cases} T'_a = 2(\frac{1}{2}mv_1^2) = mv_1^2 \\ T'_b = 1(\frac{1}{2}m(2v_1)^2) = 2mv_1^2 \end{cases}$

So choice (b) is ruled out.

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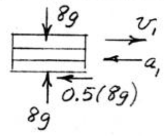
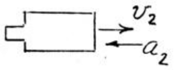


$F = ma;\ a_1 = 0.5g$

$v_1 = \sqrt{2a_1 s_1} = \sqrt{2(0.5\times 9.81)\times 0.8}$

$= 2.80$ m/s



$a_2 = 0.5g$

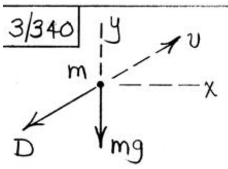
$v_2 = \sqrt{2a_2 s_2} = \sqrt{2(0.5\times 9.81)\times 1.2}$

$= 3.43$ m/s

$\Delta G = 0;\ 0.060v + 0 = 8(2.80) + (6 + 0.06)3.43$

$\underline{v = 720 \text{ m/s}}$

3/340



$\Sigma \underline{F} = m\underline{a}: \ -C_D \frac{1}{2}\rho v^2 S \frac{\underline{v}}{v} - mg\underline{j} = m(a_x\underline{i} + a_y\underline{j})$

$-C_D \frac{1}{2}\rho v S(v_x\underline{i} + v_y\underline{j}) - mg\underline{j} = m(a_x\underline{i} + a_y\underline{j})$

So $\begin{cases} a_x = -C_D \frac{1}{2}\rho S v v_x / m \\ a_y = -C_D \frac{1}{2}\rho S v v_y / m - g \end{cases}$

where $v = \sqrt{v_x^2 + v_y^2}$

The two acceleration expressions are coupled through the speed term. And The expressions are nonlinear.

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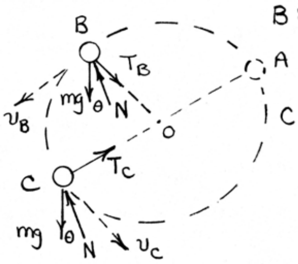
$U'_{1-2} = 0$, so $\Delta T + \Delta V_g = 0$



$B: \frac{1}{2}m(v_B^2 - u^2) - mgr\sin\theta = 0$

$a_n = \frac{v_B^2}{r} = \frac{u^2}{r} + 2g\sin\theta$

$C: \frac{1}{2}m(v_C^2 - u^2) - mg(2r\sin\theta) = 0$

$a_n = \frac{v_C^2}{r} = \frac{u^2}{r} + 4g\sin\theta$

$\Sigma F = ma_n:$

$B: \ \underline{T_B = m\left(\frac{u^2}{r} + 2g\sin\theta\right)}$

$C: \ T_C - mg\sin\theta = m\left(\frac{u^2}{r} + 4g\sin\theta\right)$

$\underline{T_C = m\left(\frac{u^2}{r} + 5g\sin\theta\right)}$

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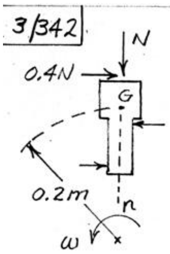
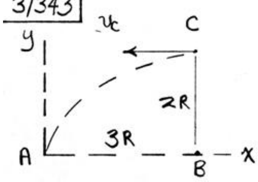


$\omega = \frac{3000(2\pi)}{60} = 314.2 \text{ rad/s}$

$\Sigma F_n = ma_n; \quad N = 2(0.2)(314.2)^2$
$= 39.5(10^3) \text{ N}$

$M = 4\mu_k N r = 4(0.4)(39.5)(10^3)(0.3)$
$= \underline{18.96 \text{ kN}\cdot\text{m}}$

3/343



$y_A = y_C + v_{y_C} t - \frac{1}{2}gt^2:$

$0 = 2R + 0(t) - \frac{1}{2}gt^2$

$t = 2\sqrt{R/g}$

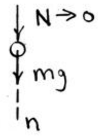
$x_A = x_C + v_{x_C} t:$

$0 = 3R - v_C 2\sqrt{R/g}$

$v_C = \frac{3}{2}\sqrt{gR}$

$T_B + U_{B-C} = T_C: \ \frac{1}{2}mu^2 - mg(2R) = \frac{1}{2}m\left[\frac{3}{2}\sqrt{gR}\right]^2$

$\underline{u = \frac{5}{2}\sqrt{gR}}$



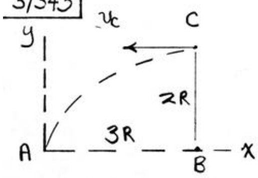
$\Sigma F_n = ma_n: \ mg = m\frac{v_C^2}{R}, \ v_C = \sqrt{gR}$

$x_A = x_C + v_{x_C} t:$

$0 = x_{min} - \sqrt{gR}\, 2\sqrt{R/g}$

$\underline{x_{min} = 2R}$

3/343



$y_A = y_C + v_{y_C} t - \frac{1}{2}gt^2:$

$0 = 2R + 0(t) - \frac{1}{2}gt^2$

$t = 2\sqrt{R/g}$

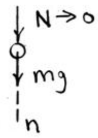
$x_A = x_C + v_{x_C} t:$

$0 = 3R - v_C 2\sqrt{R/g}$

$v_C = \frac{3}{2}\sqrt{gR}$

$T_B + U_{B-C} = T_C: \ \frac{1}{2}mu^2 - mg(2R) = \frac{1}{2}m\left[\frac{3}{2}\sqrt{gR}\right]^2$

$\underline{u = \frac{5}{2}\sqrt{gR}}$



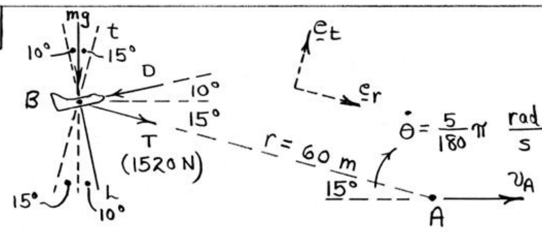
$\Sigma F_n = ma_n: \ mg = m\frac{v_C^2}{R}, \ v_C = \sqrt{gR}$

$x_A = x_C + v_{x_C} t:$

$0 = x_{min} - \sqrt{gR}\, 2\sqrt{R/g}$

$\underline{x_{min} = 2R}$

3/344



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = \underline{0} + r\dot{\theta}^2\underline{e}_r + 0\underline{e}_t$

$a_B = 60\left[\frac{5\pi}{180}\right]^2 = 0.457 \text{ m/s}^2$

$\Sigma F_t = ma_t: \ L\cos 25° - D\sin 25° - 200(9.81)\cos 15° = 0$

$\Sigma F_r = ma_r: \ 1520 + 200(9.81)\sin 15° - L\sin 25°$
$- D\cos 25° = 200(0.457)$

Solve the above two equations to obtain

$\underline{D = 954 \text{ N}}$

$\underline{L = 2540 \text{ N}}$

3/345 Velocity of plug at bottom is

$\sqrt{2gh} = \sqrt{2(32.2)6} = 19.66$ ft/sec

$\Delta G = 0; \quad \frac{2(19.66)}{g} - \frac{(2+4)v}{g} = 0, \quad v = 6.55$ ft/sec

$\Delta T + \Delta V_e = 0; \quad \frac{1}{2}\frac{6}{32.2}(0 - 6.55^2) + \frac{1}{2}80(x^2 - 0) = 0$

$x^2 = 0.100$ ft^2, $x = 0.316$ ft

$n = \frac{\Delta T}{T}; \quad n = \left[\frac{1}{2}\frac{2}{g}(19.66)^2 - \frac{1}{2}\frac{6}{g}(6.55)^2\right] \Big/ \frac{1}{2}\frac{2}{g}(19.66)^2$

$= 1 - \frac{6}{2}\left(\frac{6.55}{19.66}\right)^2 = 1 - 3(0.1111) = 0.667$

3/346 The method of work-energy cannot handle forces which are functions of time; the impulse-momentum method cannot accept forces which vary with displacement. Newton's Second Law gives the acceleration as

$$a = -\frac{k}{m}x + \frac{F(t)}{m}$$

which is not easily integrated by standard (non-numerical) methods.

3/347 Final Skidding : $U_{1-2} = \Delta T$

(Prime denotes speed after impact) $\quad -\mu_k mgd = 0 - \frac{1}{2}mv'^2$

$v' = \sqrt{2\mu_k g d}$

A : $v_A' = \sqrt{2(0.9)(32.2)(50)} = 53.8$ ft/sec

B : $v_B' = \sqrt{2(0.9)(32.2)(100)} = 76.1$ ft/sec

Collision : $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$\frac{4000}{g}v_A + 0 = \frac{4000}{g}(53.8) + \frac{2000}{g}(76.1)$

$v_A = 91.9$ ft/sec

Initial skidding : $U_{1-2} = \Delta T$

$-\mu_k \cancel{m} g d = \frac{1}{2}\cancel{m}(v_A^2 - v_{A_0}^2)$

$-(0.9)(32.2)(50) = \frac{1}{2}(91.9^2 - v_{A_0}^2)$, $v_{A_0} = 106.5 \frac{\text{ft}}{\text{sec}}$

(Speed limit was exceeded!) $= 72.6$ mi/hr

3/348 For the system of man and cord for full fall

(a) $U'_{1-2} = 0 = \Delta V_g + \Delta V_e : 0 = 80(9.81)(-44) + \frac{1}{2}k(44-20)^2$,

$k = 119.9$ N/m

(b) $U'_{1-2} = 0 = \Delta T + \Delta V_g + \Delta V_e : \; 0 = \frac{1}{2}80v^2 - 80(9.81)(20+y) + \frac{1}{2}119.9y^2$

where y = elongation of bungee cord.

$40\frac{d(v^2)}{dy} = 80(9.81) - 119.9y = 0$ for max v^2, $y = 6.55$ m

& $v^2_{max} = \frac{1}{40}\left\{80(9.81)(20+6.55) - \frac{1}{2}119.9(6.55)^2\right\} = 457$ m^2/s^2

$v_{max} = 21.4$ m/s

(c) Max. acceleration occurs at bottom where tension is greatest

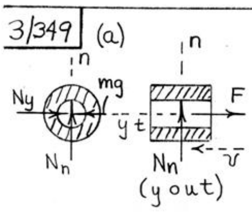
$T_{max} = Ky = 119.9(44-20) = 2880$ N

$\uparrow \Sigma F_y = ma_{max} : \; 2880 - 80(9.81) = 80\, a_{max}$

$a_{max} = 26.2$ m/s^2 or $\frac{8}{3}g$

80(9.81) N

3/349 (a)



$\Sigma F_y = 0 : \quad N_y = mg$

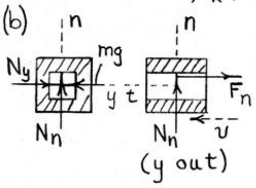
$\Sigma F_n = m\frac{v^2}{\rho} : \quad N_n = m\frac{v^2}{\rho}$

$N_{tot} = \sqrt{N_y^2 + N_n^2} = m\sqrt{g^2 + \frac{v^4}{\rho^2}}$

$F = \mu_k N_{tot} = \mu_k m\sqrt{g^2 + \frac{v^4}{\rho^2}}$

$\Sigma F_t = ma_t : \; -\mu_k \cancel{m}\sqrt{g^2 + \frac{v^4}{\rho^2}} = \cancel{m}a_t, \quad a_t = -10.75 \frac{m}{s^2}$

(b)



As in part (a), $N_y = mg$ and $N_n = m\frac{v^2}{\rho}$.

But $F_y = \mu_k N_y = \mu_k mg$

and $F_n = \mu_k N_n = \mu_k m\frac{v^2}{\rho}$.

$a_t = -\frac{F_y + F_n}{m} = -\mu_k g - \mu_k\frac{v^2}{\rho} = -14.89$ m/s^2

3/350 (Roman numeral : process; Arabic number : state)

I. Engine moves 1 ft : $U = \Delta T$: $Fd = \frac{1}{2}m(v_2^2 - v_1^2)$

(State ① → State ②) $\quad 40{,}000(1) = \frac{1}{2}\frac{400{,}000}{32.2}(v_2^2 - 0^2)$

$v_2 = 2.54$ ft/sec

II. "Collision" with A : $m_L v_2 = (m_L + m_A)v_3$

(② → ③) $\quad 400{,}000(2.54) = 600{,}000 v_3$, $v_3 = 1.692 \frac{\text{ft}}{\text{sec}}$

III. L & A move 1 ft : $40{,}000(1) = \frac{1}{2}\frac{600{,}000}{32.2}(v_4^2 - 1.692^2)$

(③ → ④) $\quad v_4 = 2.67$ ft/sec

IV. "Collision" with B : $(m_L + m_A)v_4 = (m_L + m_A + m_B)v_5$

(④ → ⑤) $\quad 600{,}000(2.67) = 800{,}000 v_5$

$v_5 = 2.01$ ft/sec

V. L, A, & B move 1 ft : $40{,}000(1) = \frac{800{,}000}{32.2}(v_6^2 - 2.01^2)$

(⑤ → ⑥) $\quad v_6 = 2.69$ ft/sec

VI. "Collision" with C : (⑥ → ⑦)

$(m_L + m_A + m_B)v_6 = (m_L + m_A + m_B + m_C)v_7$

$800{,}000(2.69) = 1{,}000{,}000 v_7$

(a) $\quad v_7 = 2.15$ ft/sec $= v$

With no slack,

$U = \Delta T : \; 40{,}000(3) = \frac{1}{2}\frac{10^6}{32.2}(v'^2 - 0^2)$

(b) $\quad v' = 2.78$ ft/sec

3/351 D to E : $y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$

$-\rho = -\frac{1}{2}gt^2, \quad t = \sqrt{\frac{2\rho}{g}}$

$x = x_0 + v_{x_0}t$: $d = v_D\sqrt{\frac{2\rho}{g}}, \quad v_D = d\sqrt{\frac{g}{2\rho}}$

A to D : $U = \Delta T$

$\frac{1}{2}k\delta^2 - \mu_k mg\rho - mg\rho = \frac{1}{2}m\left(d^2\frac{g}{2\rho}\right) - 0$

$\delta = \sqrt{\frac{mg}{k}}\sqrt{\frac{d^2}{2\rho} + 2\rho(1+\mu_k)}$

But speed at top of hill must be ≥ 0:

$U = \Delta T$: $\frac{1}{2}k\delta^2 - \mu_k mg\rho - 3mg\rho = \frac{1}{2}mv^2 - 0 \geq 0$

or $\delta \geq \sqrt{\frac{2mg\rho}{k}(3+\mu_k)}$

$\therefore \frac{mg}{k}\left(\frac{d^2}{2\rho} + 2\rho[1+\mu_k]\right) \geq \frac{2mg\rho}{k}(3+\mu_k)$

or $\underline{d \geq 2\sqrt{2}\,\rho}$

3/352 For a minimum escape orbit (parabolic) $e = 1$ & $a \to \infty$ so from Eq. 3/47

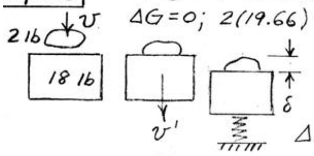
$v_{esc} = R\sqrt{\frac{2g}{R+H}} = 6371\sqrt{\frac{2 \times 9.825 \times 10^{-3}}{6371+2000}} \times 3600$

$= 35\,140$ km/h

Thus $\Delta v = 35140 - 26140 = \underline{9000 \text{ km/h}}$

3/353 $v = \sqrt{2gh} = \sqrt{2(32.2)(6)} = 19.66$ ft/sec

$\Delta G = 0;\ 2(19.66) + 0 = (18+2)v',\ v' = 1.966\ \frac{ft}{sec}$



Initial spring deflection

$\delta_0 = W/(2k) = \frac{18}{2(3)} = 3$ in.

$\Delta T + \Delta V_g + \Delta V_e = 0$

$\Delta T = 0 - \frac{1}{2}\frac{20}{32.2}(1.966)^2 = -1.200$ ft-lb

$\Delta V_g = -20\ \delta/12 = -1.667\delta$ ft-lb where δ is in inches

$\Delta V_e = \frac{1}{2}(2)(3)\left[(3+\delta)^2 - 3^2\right]\frac{1}{12} = \frac{(3+\delta)^2 - 9}{4}$ ft-lb

Thus $-1.200 - 1.667\delta + \frac{(3+\delta)^2 - 9}{4} = 0$

or $\delta^2 - 0.667\delta - 4.800 = 0$

$\delta = \frac{0.667}{2} \pm \frac{1}{2}\sqrt{0.444 + 19.20} = 0.333 \pm 2.216$

$\underline{\delta = 2.55 \text{ in.}}$ (or $\delta = -1.88$ in.)

3/354 $\overset{\rightarrow}{+}$ $G_1 = G_2$

$\cancel{m}v_A + \cancel{m}v_B = 2\cancel{m}v'$

$3 - 5 = 2v',\ \underline{v' = -1 \text{ m/s} \quad (\text{left})}$

$H_{G_1} = H_{G_2}$: $\cancel{m}\cancel{r}v_A + \cancel{m}\cancel{r}v_B = 2\cancel{m}r^{\cancel{2}}\dot\theta'$

$\dot\theta' = \frac{v_A + v_B}{2r} = \frac{3+5}{2(0.4)}$

$= \underline{10 \text{ rad/s (CCW)}}$

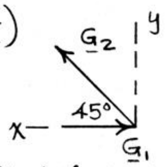
3/355 From S.P. 2/6, $2s = \frac{v^2 \sin 2\theta}{g}$

$350 = \frac{v^2 \sin 90°}{32.2}, \quad v = 106.2$ ft/sec

$\underline{G}_1 = m\underline{v}_1 = \frac{5/16}{32.2}\left(90\ \frac{5280}{3600}\right)(-\underline{i}) = -1.281\underline{i}$ lb-sec

$\underline{G}_2 = m\underline{v} = \frac{5/16}{32.2}\ 106.2\left(\frac{\underline{i}}{\sqrt{2}} + \frac{\underline{j}}{\sqrt{2}}\right)$

$= 0.729(\underline{i} + \underline{j})$ lb-sec



$\underline{F}_{av}\Delta t = \underline{G}_2 - \underline{G}_1$: $\underline{F}_{av}(0.005) = 0.729(\underline{i}+\underline{j}) - (-1.281\underline{i})$

$\underline{F}_{av} = 402\underline{i} + 145.7\underline{j}$ lb

$F_{av} = \sqrt{402^2 + 145.7^2} = \underline{428 \text{ lb}}$

(Note: The weight of the baseball is ignored during its impact with the bat. With the weight included, F_{av} still rounds to 428 lb!)

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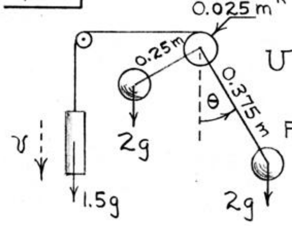


Work done during angular displacement θ is

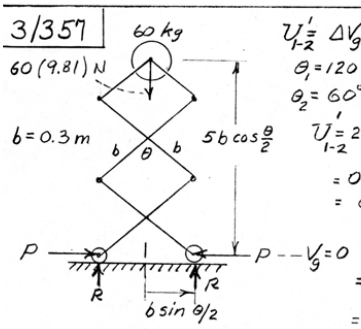
$U = 1.5g(0.025\theta) + 2g(0.25\sin\theta) - 2g(0.375)(1-\cos\theta)$

For $\theta = 30°$, $U = 1.659$ J

$\Delta T = \frac{1}{2}\left[1.5v^2 + 2\left(\frac{0.25}{0.025}v\right)^2 + 2\left(\frac{0.375}{0.025}v\right)^2\right]$

$= 325.8v^2$. $U_{1-2} = \Delta T$ yields $\underline{0.0714 \text{ m/s } (71.4\ \frac{mm}{s})}$

3/357



$U'_{1-2} = \Delta V_g + \Delta T$

$\theta_1 = 120°,\ v_1 = 0$

$\theta_2 = 60°,\ v_2 = 3$ m/s

$U'_{1-2} = 2Pb\left(\sin\frac{\theta_1}{2} - \sin\frac{\theta_2}{2}\right)$

$= 0.6P(\sin 60° - \sin 30°)$

$= 0.2196P$ J

$\Delta V_g = mg\Delta h$

$= mg(5b)\left(\cos\frac{\theta_2}{2} - \cos\frac{\theta_1}{2}\right)$

$= 60(9.81)(1.5)(\cos 30° - \cos 60°)$

$= 323.2$ J

$\Delta T = \frac{1}{2}60(3)^2 - 0 = 270$ J

Thus $0.2196P = 323.2 + 270$, $P = \frac{323.2 + 270}{0.2196} = 2701$ N

or $\underline{P = 2.70 \text{ kN}}$

For complete system

$\Sigma F = ma;\ 2R - 60(9.81) = 60(20)$

$\underline{R = 894 \text{ N}}$

3/358 Drop of A (state ① → state ②):

$T_1 + U_{1-2} = T_2: \quad 0 + m_A g\, 1.8(1-\cos 60°) = \frac{1}{2} m_A v_{A_2}^2$

$v_{A_2} = 4.20 \text{ m/s}$

Collision (② → ③):

$$\begin{cases} m_A v_{A_2} + m_B v_{B_2} = m_A v_{A_3} + m_B v_{B_3} & (1) \\ v_{B_3} - v_{A_3} = 0.7(v_{A_2} - v_{B_2}) & (2) \end{cases} \qquad (v_{B_2} = 0)$$

Solution: $v_{A_3} = 2.42$ m/s, $v_{B_3} = 5.36$ m/s

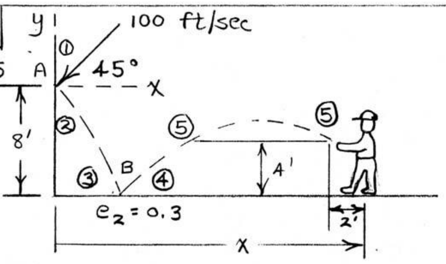
Rise of B (③ → ④):

$T_3 + U_{3-4} = T_4$:

$\frac{1}{2} m_B (5.36)^2 - m_B (9.81)[2.4(1-\cos 30°) + s \sin 30°] = 0$

$\underline{s = 2.28 \text{ m}}$

3/359



Use coordinates & states ① - ⑤ shown.

$v_{1x} = -100 \cos 45° = -70.7$ ft/sec

$v_{1y} = -100 \sin 45° = -70.7$ ft/sec

$v_{2x} = -e_1 v_{1x} = -0.5(-70.7) = 35.4$ ft/sec

$v_{2y} = v_{1y} = -70.7$ ft/sec

$v_{3x} = v_{2x} = 35.4$ ft/sec

$v_{3y} = -\sqrt{v_{2y}^2 + 2g(8)} = -\sqrt{70.7^2 + 2(32.2)(8)} = -74.3 \frac{\text{ft}}{\text{sec}}$

$v_{3y} = v_{2y} - g t_3: \; -74.3 = -70.7 - 32.2\, t_3, \quad t_3 = 0.1104$ sec

$v_{4x} = v_{3x} = 35.4$ ft/sec

$v_{4y} = -e_2 v_{3y} = -0.3(-74.3) = 22.3$ ft/sec

$y_5 = y_4 + v_{y_4} t_5 - \frac{1}{2} g t_5^2$:

$-4 = -8 + 22.3\, t_5 - 16.1\, t_5^2: \quad t_5 = 0.212,\ 1.172$ sec

Then $x = x_3 + v_{4x} t_5 + 2$, where $x_3 = v_{2x} t_3$

$= 35.4(0.1104) = 3.73$ ft

Thus $x = 3.73 + 35.4(0.212) + 2 = \underline{13.40 \text{ ft}}$

or $x = 3.73 + 35.4(1.172) + 2 = \underline{47.3 \text{ ft}}$

3/360 Results of Prob. 3/309:

$\Delta v_A = R\sqrt{\frac{g}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right)$

Nominally,

$(\Delta v_A)_n = (3959)(5280)\sqrt{\frac{32.23}{(3959+170)(5280)}} \times$

$\left(\sqrt{\frac{2(3959+22{,}300)}{(3959+170)+(3959+22{,}300)}} - 1\right) = 7997 \frac{\text{ft}}{\text{sec}}$

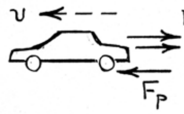
Actually,

$(\Delta v_A)_a = (3959)(5280)\sqrt{\frac{32.23}{(3959+170)(5280)}} \times$

$\left(\sqrt{\frac{2(3959+700)}{(3959+170)+(3959+700)}} - 1\right) = 755 \frac{\text{ft}}{\text{sec}}$

$\frac{(\Delta v_A)_a}{(\Delta v_A)_n} = \frac{t'}{t}, \quad t' = \frac{(\Delta v_A)_a}{(\Delta v_A)_n} t = \frac{755}{7997}(90) = \underline{8.50 \text{ sec}}$

▶3/361



$F_D = C_D\left(\frac{1}{2}\rho v^2\right)S$

$F_R = 200$ lb

$\overset{+}{\leftarrow}\Sigma F = 0: \quad F_P = F_D + F_R$

Undamaged: $F_P = 0.3\left[\frac{1}{2}\,\frac{0.07530}{32.2}\left(200 \cdot \frac{5280}{3600}\right)^2\right]30 + 200 = 1105$ lb

Power required: $P = F_P \cdot v = 1105 \cdot \left(200\, \frac{5280}{3600}\right) = 324(10^3)$ ft-lb/sec

Damaged (power available is unchanged)

$P = F_P' \cdot v': \; 324(10^3) = \left[0.4\left(\frac{1}{2}\,\frac{0.07530}{32.2} v'^2\right)30 + 200\right] v'$

Solve cubic: $v' = 268$ ft/sec or $\underline{182.9 \text{ mi/hr}}$

►3/362

$$\begin{cases} F_R = -k_1 v, & k_1 = 0.833 \ \frac{lb\text{-}hr}{mi} = 0.5682 \ \frac{lb\text{-}sec}{ft} \\ F_D = -k_2 v^2, & k_2 = 0.0139 \ \frac{lb\text{-}hr^2}{mi^2} = 0.006457 \ \frac{lb\text{-}sec^2}{ft^2} \end{cases}$$

(a) $P_{30} = Fv = \left[0.833(30) + 0.0139(30)^2\right]\left[30\left(\frac{5280}{3600}\right)\right]$

$= 1650 \ \frac{ft\text{-}lb}{sec} = \underline{3 \ hp}$

$P_{60} = Fv = \left[0.833(60) + 0.0139(60)^2\right]\left[60\left(\frac{5280}{3600}\right)\right]$

$= 8800 \ \frac{ft\text{-}lb}{sec} = \underline{16 \ hp}$

(b) $-k_1 v - k_2 v^2 = m \frac{dv}{dt}$

$$\int_0^t dt = -m \int_{v_1}^{v_2} \frac{dv}{v(k_1 + k_2 v)}$$

$$t = -\frac{m}{k_1} \ln\left[\frac{v_2(k_1 + k_2 v_1)}{v_1(k_1 + k_2 v_2)}\right]$$

$$t = -\frac{2000/32.2}{0.5682} \ln\left[\frac{7.33(0.5682 + 0.006457(88))}{88(0.5682 + 0.006457(7.33))}\right]$$

$= \underline{205 \ sec}$

$-k_1 v - k_2 v^2 = m v \frac{dv}{ds}$

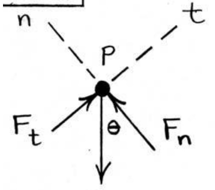
$$\int_0^s ds = -m \int_{v_1}^{v_2} \frac{dv}{k_1 + k_2 v}$$

$$s = -\frac{m}{k_2} \ln(k_1 + k_2 v)\Big|_{v_1}^{v_2}$$

$$= -\frac{m}{k_2} \ln\left[\frac{k_1 + k_2 v_2}{k_1 + k_2 v_1}\right] = \underline{5898 \ ft}$$

***3/363**

$\theta = t^2, \ \dot\theta = 2t, \ \ddot\theta = 2 \ rad/s^2$



$0.4(9.81) = 3.924$ N

$\Sigma F_n = ma_n$: $F_n - 3.924 \cos t^2 = 0.4(1.5)(2t)^2$

$F_n = 3.924 \cos t^2 + 2.4t^2$ (N)

$\Sigma F_t = ma_t$: $F_t - 3.924 \sin t^2 = 0.4(1.5)(2)$

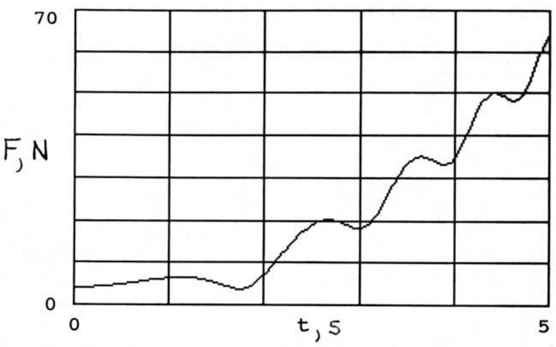
$F_t = 3.924 \sin t^2 + 1.2$ (N)

$F = \sqrt{F_n^2 + F_t^2}$; simplify to

$F = \sqrt{16.84 + 18.84t^2 \cos t^2 + 9.42 \sin t^2 + 5.76t^4}$

When $F = 30$ N, $\underline{t = 3.40 \ s}$ (numerically)

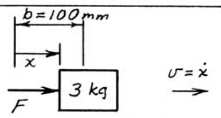
$\theta = 3.40^2 = \underline{11.57 \ rad}$ or $\theta = \underline{663°}$



***3/364**

Power $P = Fv$ where $F = k(b - x)$

$F = 1.8(10^3)(0.1 - x)$ N



$U = \Delta T$: $U = \int_0^x F dx = \int_0^x 1.8(10^3)(0.1 - x) dx$

$= 1.8(10^3)(0.1x - \frac{x^2}{2})$ J

$\Delta T = \frac{1}{2}mv^2 - 0 = \frac{1}{2}3v^2$ J

Thus $v^2 = \frac{2}{3} 1.8(10^3)(0.1x - \frac{x^2}{2})$

$P^2 = F^2 v^2 = (1.8)^2(10^6)(0.1 - x)^2 \times \frac{2}{3} 1.8(10^3)(0.1x - \frac{x^2}{2})$

$= 3.89(10^9)(0.1 - x)^2(0.1x - \frac{x^2}{2})$ watts^2 with x in meters

$\frac{d(P^2)}{dx} = 3.89(10^9)\{2(0.1 - x)(-1)(0.1x - \frac{x^2}{2}) + (0.1 - x)^2(0.1 - x)\}$

$= 3.89(10^9)(-0.2x + x^2 + 0.01 - 0.2x + x^2)(0.1 - x)$

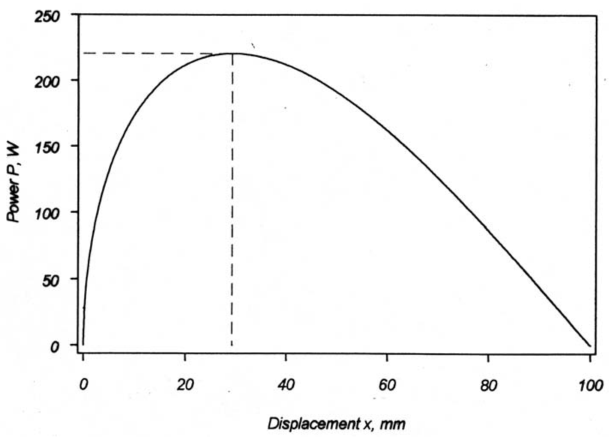
$= 3.89(10^9)(0.1 - x)(x^2 - 0.2x + 0.005) \times 2 = 0$ for max or min

so $x = 0.1$ or $x = 0.1 \pm 0.0707 = 0.1707$ m or $\underline{x = 0.0293 \ m}$

$P = \sqrt{38.9(10^8)}(0.1 - x)\sqrt{0.1x - 0.5x^2}$ W

Substitute x from above & get

$\underline{P_{max} = 220 \ W}$



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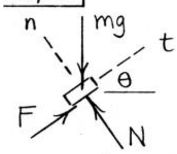
$a_n = r\Omega^2 = \frac{13}{12}(7.5)^2 = 60.9 \frac{ft}{sec^2}$



$$\begin{cases} \Sigma F_n = ma_n: N - mg\cos\theta = 60.9m \\ \Sigma F_t = ma_t: F - mg\sin\theta = 0 \end{cases}$$

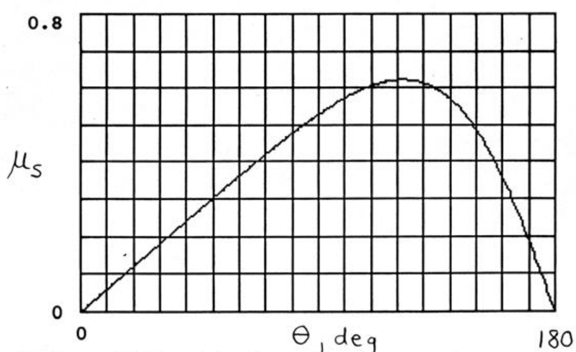
Slipping impends when $F = F_{max} = \mu_s N$

Simultaneous solution : $\mu_s = \frac{32.2 \sin\theta}{60.9 + 32.2\cos\theta}$

See plot of μ_s vs. θ below. Set $\frac{d\mu_s}{d\theta} = 0$

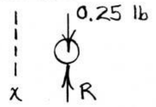
or numerically determine that

$\underline{\mu_{min} = 0.622}$ @ $\underline{\theta = 121.9°}$



Note that it is impossible to see slippage first occur at any angle greater than θ = 121.9° !

***3/366**



$\Sigma F_x = ma_x: \quad 0.25 - R = \dfrac{0.25}{32.2}\, a$

$(0.25 - R)\,dx = \dfrac{0.25}{32.2}\, a\,dx$

But $a\,dx = v\,dv$, so

$(0.25 - R)\,dx = \dfrac{0.25}{32.2}\, v\,dv = 0.00388\, d(v^2)$

For small intervals : $(0.25 - R)\,\Delta x = 0.00388\,\Delta(v^2)$

or $\Delta(v^2) = (64.4 - 258R)\,\Delta x$

Set up program to produce the following table :

x ft	Δx ft	R lb	64.4-258 R ft/sec²	Δv² (ft/sec)²	v² (ft/sec)²	v ft/sec
0		0	64.4		0	0
	1			64.4		
1		0.04	54.1		64.4	8.02
	1			54.1		
2		0.08	43.7		118.5	10.9
	1			43.7		
3		0.13	30.9		162.2	12.7
	1			30.9		
4		0.16	23.1		193.1	13.9
	1			23.1		
5		0.19	15.4		216.2	14.7
	1			15.4		
6		0.21	10.2		231.6	15.2
	1			10.2		
7		0.22	7.6		241.8	15.6
	1			7.6		
8		0.23	5.1		249.4	15.8
	1			5.1		
9		0.25	0		254.5	16.0
	1			0		
10					254.5	16.0

So $v = 16.0$ ft/sec.

For $R = kv^2 \qquad W - kv^2 = \dfrac{W}{g}\, a$

$\displaystyle\int_0^x \frac{g}{W}\,dx = \int_0^v \frac{v\,dv}{W - kv^2}$

$\Rightarrow v = \sqrt{\dfrac{W}{k}\left(1 - e^{-2gkx/W}\right)}$

With numbers, $\underline{v = 16.3 \text{ ft/sec}}$

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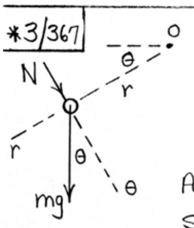


$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$

$mg\sin\theta = m(\ddot{r} - r\omega_0^2)$

$\ddot{r} - \omega_0^2 r = g\sin\omega_0 t$

Assume $r_h = Ce^{st}$ to obtain $s_1 = -\omega_0$, $s_2 = \omega_0$. Assume particular solution of form $r_p = D\sin\omega_0 t$ and find $D = -\dfrac{g}{2\omega_0^2}$. So

$r = r_h + r_p = C_1 e^{-\omega_0 t} + C_2 e^{\omega_0 t} - \dfrac{g}{2\omega_0^2}\sin\omega_0 t$

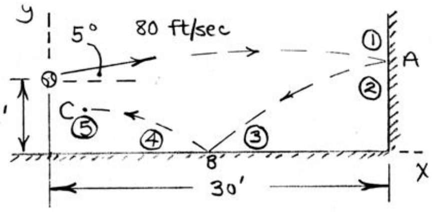
Use the initial conditions $r(0) = \dot{r}(0) = 0$ to find C_1 and C_2, allowing us to write the solution as

$r = \dfrac{g}{4\omega_0^2}\left(-e^{-\theta} + e^{\theta} - 2\sin\theta\right)$

Now, set $r = 1$ m and $\omega_0 = 0.5$ rad/s and use Newton's method to solve for θ as $\theta = 0.535$ rad, or $\underline{30.6°}$. From

$\theta = \omega_0 t, \quad t = \dfrac{0.535}{0.5} = \underline{1.069 \text{ s}}$.

***3/368**



Define states ① - ⑤ as shown.

$\begin{cases} v_0 = 80 \text{ ft/sec} \\ \theta = 5° \end{cases}$

$v_{0x} = v_0\cos\theta = 80\cos 5° = 79.7$ ft/sec

$v_{0y} = v_0\sin\theta = 80\sin 5° = 6.97$ ft/sec

$t_{01} = \dfrac{30}{79.7} = 0.376$ sec

$y_1 = 3 + 6.97(0.376) - 16.1(0.376)^2 = 3.34$ ft

$v_{1x} = v_{0x} = 79.7$ ft/sec

$v_{1y} = v_{0y} - gt_{01} = 6.97 - 32.2(0.376) = -5.15$ ft/sec

Now program the following numbered equations :

$v_{2x} = -ev_{1x}$ (1)

$v_{2y} = v_{1y}$ (2)

$v_{3x} = v_{2x}$ (3)

$v_{3y} = -\sqrt{v_{2y}^2 + 2gy_1}$ (4)

$t_{23} = (v_{2y} - v_{3y})/g$ (5)

$x_3 = 30 + v_{2x}t_{23}$ (6)

$v_{4x} = v_{3x}$ (7)

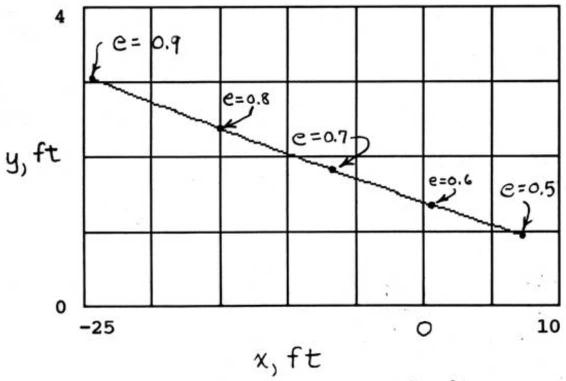
$v_{4y} = -ev_{3y}$ (8)

$t_{45} = v_{4y}/g$ (9)

$x_5 = x_3 + v_{4x}t_{45} = x$ (10)

$y_5 = v_{4y}t_{45} - \frac{1}{2}gt_{45}^2 = y$ (11)

Solve Eqs. (1)-(11) for $0.5 \le e \le 0.9$ to obtain the following plot.



For $x = 0$, $\underline{e = 0.610,\ y = 1.396 \text{ ft}}$

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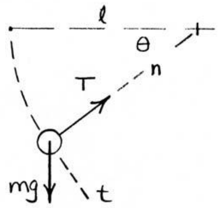


$v_0 = l\dot{\theta}_0$
$= 0.5\,(0.2)$
$= 0.1$ m/s

$\Sigma F_t = ma_t: \quad mg\cos\theta = ml\ddot{\theta}, \quad \ddot{\theta} = \frac{g}{l}\cos\theta$

$v\,dv = a_t\,ds: \quad v\,dv = l\ddot{\theta}\,(l\,d\theta) = gl\cos\theta\,d\theta$

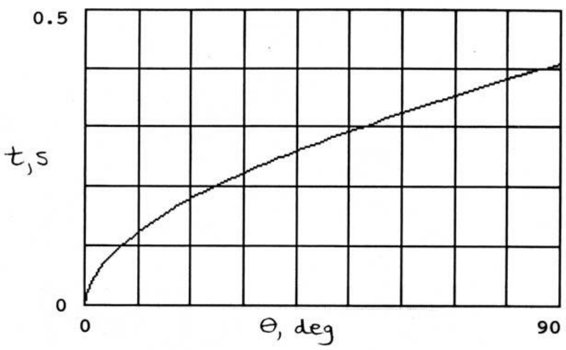
$\int_{v_0=0.1}^{v} v\,dv = \int_{\theta_0=0}^{\theta} gl\cos\theta\,d\theta, \quad v^2 = 2gl\sin\theta + v_0^2$

$v = \frac{ds}{dt} = \frac{l\,d\theta}{dt}: \quad \sqrt{2gl\sin\theta + v_0^2} = l\frac{d\theta}{dt}$

Rearrange to $\int_{t_0=0}^{t} dt \quad \int_{\theta_0=0}^{\theta} l\frac{d\theta}{\sqrt{2gl\sin\theta + v_0^2}}$

So $t = 0.5\int_0^{\theta}\frac{d\theta}{\sqrt{9.81\sin\theta + 0.01}}$

Set up a numerical integration scheme (see Appendix C/12) and integrate the above for various upper limits $(0 \le \theta \le \pi/2)$



When $\theta = 90°$, $\underline{t = 0.409\text{ s}}$.

*3/370 For $\theta \neq 0$,

$V_g = \rho g\left(\frac{\pi r}{2} - r\theta\right)\bar{r}\sin\alpha - \rho g r\theta\,\frac{r\theta}{2}$

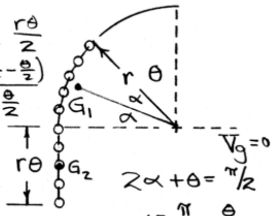
where $\bar{r} = \frac{r\sin\alpha}{\alpha} = r\,\frac{\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\frac{\pi}{4}-\frac{\theta}{2}}$

$V_g = \rho g r^2\left\{\left(\frac{\pi}{2}-\theta\right)\frac{\sin^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\frac{\pi}{4}-\frac{\theta}{2}} - \frac{\theta^2}{2}\right\}$

For $\theta = 0$, $V_g = \rho g\frac{\pi r}{2}\frac{2r}{\pi} = \rho g r^2$

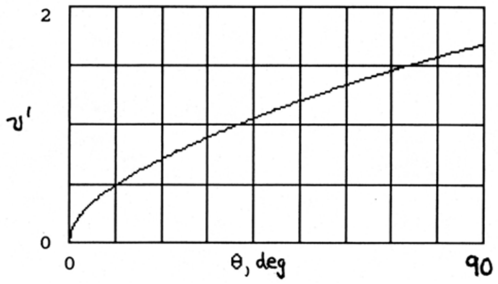
Thus $\Delta V_g = (V_g)_\theta - (V_g)_{\theta=0}$

$= 3\rho g r^2\left\{1 + \frac{\theta^2}{2} - \left(\frac{\pi}{2}-\theta\right)\frac{\sin^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\frac{\pi}{4}-\frac{\theta}{2}}\right\}$

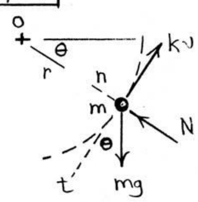


$U'_{1-2} = 0 = \Delta T + \Delta V_g$; with $\Delta T = \frac{1}{2}\rho\frac{\pi r}{2}v^2$, we get

$\underline{v' = \frac{v}{\sqrt{gr}} = \sqrt{2}\sqrt{\frac{1}{\pi}(2+\theta^2) - \left(1-\frac{2\theta}{\pi}\right)\frac{\sin^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\frac{\pi}{4}-\frac{\theta}{2}}}}$



*3/371



$\Sigma F_t = ma_t:$

$mg\cos\theta - kv = mv\frac{dv}{ds}$

But $ds = r\,d\theta$, so

$mg\cos\theta - kv = mv\frac{dv}{r\,d\theta}$

$\frac{dv}{d\theta} = \frac{gr\cos\theta}{v} - \frac{kr}{m}$

It is not possible to separate variables, so we numerically integrate to obtain v as a function of θ.

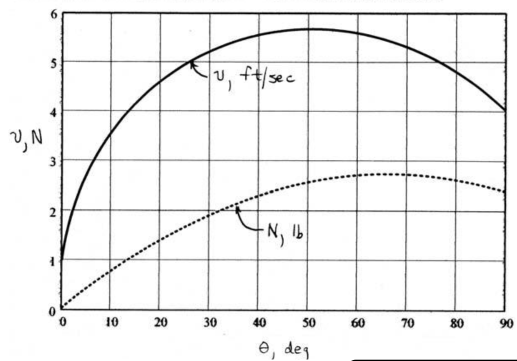
$\Sigma F_n = m\frac{v^2}{\rho}: \quad N - mg\sin\theta = m\frac{v^2}{r}$

$N = m\left[g\sin\theta + \frac{v^2}{r}\right]$

where v is available from the previously mentioned numerical integration. Plots of both v and N as functions of θ are shown below. The maxima are

$\underline{v_{max} = 5.69\text{ ft/sec} \;@\; \theta = 50.8°}$

$\underline{N_{max} = 2.75\text{ lb} \;@\; \theta = 66.2°}$



4/1

$$\bar{\underline{r}} = \frac{\Sigma m_i \underline{r}_i}{\Sigma m_i} = \frac{m(d\underline{i}) + 2m(2d\underline{j}) + 4m(1.5d\underline{k})}{m+2m+4m}$$

$$= \frac{d}{7}(\underline{i} + 4\underline{j} + 6\underline{k})$$

$$\dot{\bar{\underline{r}}} = \frac{\Sigma m_i \dot{\underline{r}}_i}{\Sigma m_i} = \frac{m(2v\underline{j}) + 2m(3v\underline{k}) + 4m(v\underline{i})}{7m}$$

$$= \frac{v}{7}(4\underline{i} + 2\underline{j} + 6\underline{k})$$

$$\ddot{\bar{\underline{r}}} = \frac{\Sigma \underline{F}}{\Sigma m_i} = \frac{F\underline{k}}{7m}$$

$$T = \Sigma \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2}\left[m(2v)^2 + 2m(3v)^2 + 4m(v)^2\right]$$

$$= 13mv^2$$

$$\underline{H}_O = \Sigma \underline{r}_i \times m_i \underline{v}_i = 2mvd\underline{k} + 12mvd\underline{i} + 6mvd\underline{j}$$

$$= mvd(12\underline{i} + 6\underline{j} + 2\underline{k})$$

$$\dot{\underline{H}}_O = \Sigma \underline{M}_O = -Fd\underline{j}$$

4/2 From Eq. 4/10 with P replaced by O:

$$\underline{H}_O = \underline{H}_G + \bar{\underline{r}} \times \Sigma m_i \bar{\underline{v}}$$

or

$$\underline{H}_G = \underline{H}_O - \bar{\underline{r}} \times \Sigma m_i \bar{\underline{v}}$$

$$\underline{H}_G = mvd(12\underline{i} + 6\underline{j} + 2\underline{k}) - \frac{d}{7}(\underline{i} + 4\underline{j} + 6\underline{k}) \times 7m \cdot \frac{v}{7}(4\underline{i} + 2\underline{j} + 6\underline{k})$$

$$= \frac{mvd}{7}(72\underline{i} + 24\underline{j} + 28\underline{k})$$

($\underline{H}_O$, $\bar{\underline{r}}$, and $\bar{\underline{v}} = \dot{\bar{\underline{r}}}$ from Prob. 4/1)

From Eq. 4/11:

$$\Sigma \underline{M}_O = \dot{\underline{H}}_G + \bar{\underline{r}} \times \Sigma m_i \bar{\underline{a}}$$

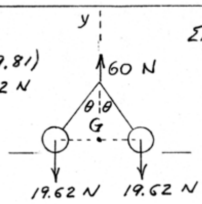
$$\dot{\underline{H}}_G = \Sigma \underline{M}_O - \bar{\underline{r}} \times \Sigma m_i \bar{\underline{a}}$$

$$= -Fd\underline{j} - \frac{d}{7}(\underline{i} + 4\underline{j} + 6\underline{k}) \times 7m\left(\frac{F\underline{k}}{7m}\right)$$

$$= -\frac{2Fd}{7}(2\underline{i} + 3\underline{j})$$

4/3

$mg = 2(9.81)$
$= 19.62$ N

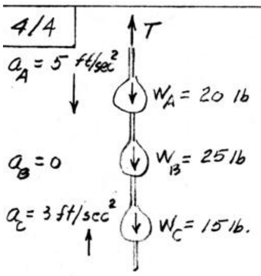


$\Sigma F_y = m\bar{a}_y$; $60 - 2(19.62) = 4\bar{a}_y$

$a_y = \bar{a}_y = 5.19$ m/s^2

(independent of θ)

4/4



$\Sigma F = \Sigma m_i a_i$

$T - (20 + 25 + 15)$

$= \frac{1}{32.2}(20[-5] + 25[0] + 15[3])$

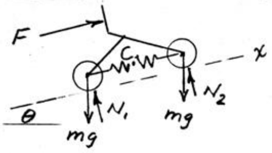
$T - 60 = \frac{1}{32.2}(-55)$

$T = 60 - 1.708 = 58.3$ lb

4/5 $\Sigma F = m\bar{a}$: $6.4 = (0.8 + 0.5 + 0.3)\bar{a}$

$\bar{a} = 4$ m/s^2

4/6

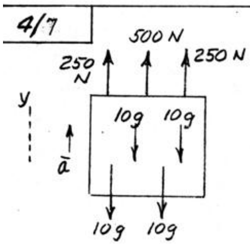


$\Sigma F_x = m\bar{a}_x$ where $a_C = \bar{a}_x$

$F - 2mg\sin\theta = 2ma_C$

$a_C = \frac{F}{2m} - g\sin\theta$

4/7



$\Sigma F_y = m\bar{a}_y$

$500 + 250 + 250 - 40(9.81) = 40\bar{a}$

$40\bar{a} = 1000 - 392$

$\bar{a} = 15.19$ m/s^2

4/8 The principle of the motion of the mass center gives $F = m\bar{a}$ for each case, so the mass-center accelerations are identical. In the two cases of hinged members, however, the mass center is not a point attached to a member, and for these two cases the accelerations of the members will differ.

4/9

$$\underline{F}_{av} = \frac{\Delta \underline{G}}{\Delta t} = \left[(3.7 - 3.4)\underline{i} + (-2.2 + 2.6)\underline{j} + (4.9 - 4.6)\underline{k}\right]/0.2$$

$$= 1.5\underline{i} + 2.0\underline{j} + 1.5\underline{k} \text{ N}$$

$$F = |\underline{F}_{av}| = \sqrt{1.5^2 + 2.0^2 + 1.5^2} = 2.92 \text{ N}$$

4/10 For system, $\Delta T + \Delta V_g = 0$

$\Delta T = 3(\frac{1}{2}mv^2) - 0 = \frac{3}{2}mv^2$

$\Delta V_g = 0 - mg\frac{b}{\sqrt{2}} - mg\frac{2b}{\sqrt{2}} = -\frac{3b}{\sqrt{2}}mg$

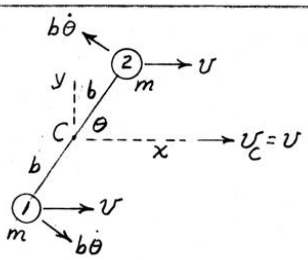
Thus $\frac{3}{2}mv^2 - \frac{3b}{\sqrt{2}}mg = 0$, $v^2 = bg\sqrt{2}$

$v = \sqrt{bg\sqrt{2}}$

4/11

For sphere 1,

$\underline{G}_1 = m[(v + b\dot{\theta}\sin\theta)\underline{i} - (b\dot{\theta}\cos\theta)\underline{j}]$



For sphere 2

$\underline{G}_2 = m[(v - b\dot{\theta}\sin\theta)\underline{i} + (b\dot{\theta}\cos\theta)\underline{j}]$

$\underline{G} = \underline{G}_1 + \underline{G}_2 = m[v + v]\underline{i} = 2mv\underline{i}$

4/12

$\underline{H}_O = \underline{H}_G + \bar{\underline{r}} \times \underline{G}$, $\underline{G} = 3(3\underline{i} + 4\underline{j})$ kg·m/s

$= 1.20\underline{k} + (0.4\underline{i} + 0.3\underline{j}) \times 3(3\underline{i} + 4\underline{j})$

$= 1.20\underline{k} + 3(1.6\underline{k} - 0.9\underline{k})$

$= 1.20\underline{k} + 3(0.7\underline{k}) = 3.3\underline{k}$ kg·m^2/s

4/13 Mass center is center of middle bar, so Eq. 4/1 for the entire system gives

$\Sigma F = m\bar{a}: \quad 10 = 3\frac{8}{32.2}a, \quad \underline{a = 13.42\ ft/sec^2}$

4/14

$\Sigma M_O = \dot{H}_O$ where O-O is the axis of rotation

$M = \frac{dH_O}{dt}, \quad \int_0^t M\,dt = \int_0^{H_O} dH_O = H_O$

$Mt = 4m(r\omega)r, \quad \underline{t = \frac{4mr^2\omega}{M}}$

5/15 $\Sigma M_O = \dot{H}_O = \frac{dH_O}{dt}, \quad \int \Sigma M_O\,dt = \Delta H_O$

$M_O t = \Delta\left|\Sigma m_i r_i (r_i\dot{\theta})\right| = \Sigma m_i r^2 \Delta\dot{\theta}$

$30\times 5 = [3(0.5)^2 + 4(0.4)^2 + 3(0.6)^2](\dot{\theta}' - 20)$

$150 = 2.47(\dot{\theta}' - 20), \quad \dot{\theta}' = 60.7 + 20 = \underline{80.7\ \frac{rad}{s}}$

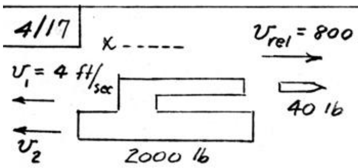
4/16 $\int_0^t M_z\,dt = H_{z_2} - H_{z_1}, \quad H_z = \Sigma m_i r_i (r_i\dot{\theta})$

$H_z = 2(3)(0.3)^2\dot{\theta} + 2(3)(0.5)^2\dot{\theta} = 2.04\dot{\theta}$

so $30t = 2.04(20 - [-20]) = 81.6$

$\underline{t = 2.72\ s}$

4/17



$\Delta G_x = 0, \quad G_1 = G_2$

$\frac{1}{g}(2000 + 40)4 = \frac{1}{g}(2000v_2 - 40[800 - v_2])$

$8160 = 2040v_2 - 32000$

$\underline{v_2 = 19.69\ ft/sec}$

4/18 For entire system $\Delta G_x = 0$, x horiz.

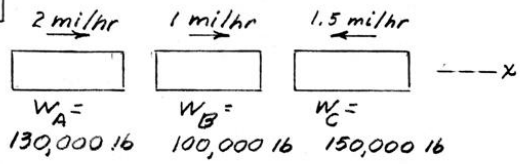
$(300 + 400 + 100)v$

$- (300\times 0.6 - 400\times 0.3 + 100\times 1.2\cos 30°) = 0$

$800v = 163.9, \quad \underline{v = 0.205\ m/s}$

Momentum is conserved regardless of sequence of events, so final velocity would be the same.

4/19



$\Sigma F_x = 0$ for system so $\Delta G_x = 0$

$(130\times 2 + 100\times 1 - 150\times 1.5)\frac{44}{30}\frac{10^3}{32.2}$

$- (130 + 100 + 150)v\frac{44}{30}\frac{10^3}{32.2} = 0$

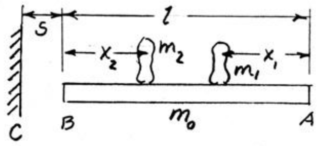
$v = \frac{260 + 100 - 225}{130 + 100 + 150} = \underline{0.355\ mi/hr}$

% loss of energy $= \frac{T_i - T_f}{T_i}100 = 100\left(1 - \frac{T_f}{T_i}\right) = n$

$n = 100\left\{1 - \frac{\frac{1}{2g}(130 + 100 + 150)(0.355)^2}{\frac{1}{2g}(130\times 2^2 + 100\times 1^2 + 150\times 1.5^2)}\right\} = 100\left(1 - \frac{47.96}{957.5}\right)$

$\underline{n = 95.0\ \%}$

4/20 With respect to C, $\Sigma m_i x_i$ = constant



$m_1 l + m_2(0) + m_0\frac{l}{2} =$
$m_1(s + l - x_1) + m_2(s + x_2)$
$+ m_0\left(s + \frac{l}{2}\right)$

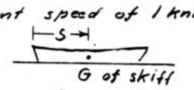
Simplify & get $s = \frac{m_1 x_1 - m_2 x_2}{m_0 + m_1 + m_2}$

But they meet when $x_2 + x_1 = l$ so

$\underline{s = \frac{(m_1 + m_2)x_1 - m_2 l}{m_0 + m_1 + m_2}}$

4/21 With neglect of hydraulic forces linear momentum is conserved & velocity $v_2 = v_1 = 1$ knot. Center of mass does not change position with respect to reference axes moving with constant speed of 1 knot.

Thus $(\Sigma m_i x_i)_1 = (\Sigma m_i x_i)_2$



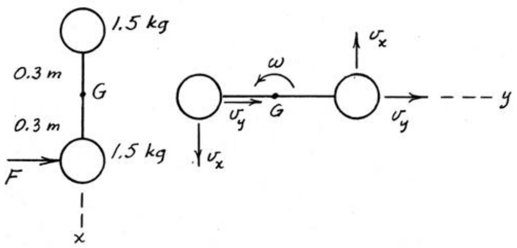
$\frac{1}{32.2}[120(2) + 180(8) + 160(16) + 300(s)]$

$= \frac{1}{32.2}[120(14 + x) + 180(4 + x) + 160(10 + x) + 300(s + x)]$

$4240 = 4000 + 760x, \quad \underline{x = \frac{240}{760} = 0.316\ ft}$

Timing & sequence of changed positions does not affect final result because all forces are internal.

4/22



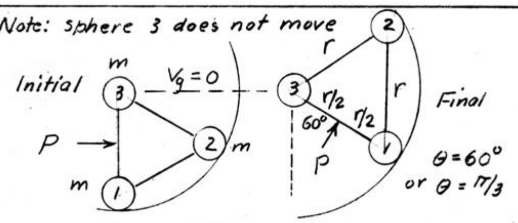
$\int \Sigma F_x\, dt = 0$ so $\Delta G_x = 0$

$\int \Sigma F_y\, dt = \Delta G_y$: $10 = 2(1.5)v_y$, $v_y = 3.33$ m/s

$\int \Sigma M_G\, dt = \Delta H_G$: $10(0.3) = 2(1.5)v_x(0.3)$, $v_x = 3.33$ m/s

$v = 3.33\sqrt{2} = \underline{4.71 \text{ m/s both spheres}}$

4/23 Note: sphere 3 does not move



(a) $U' = \Delta V_g$: $U' = P_{min} \frac{r}{2}\frac{\pi}{3}$

$\Delta V_g = mg\left(r + \frac{r}{2}\right) = \frac{3}{2}mgr$

Thus $\frac{\pi r}{6} P_{min} = \frac{3}{2}mgr$, $\underline{P_{min} = \frac{9}{\pi}mg}$

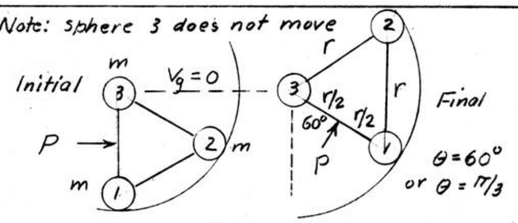
(b) $U' = \Delta T + \Delta V_g$ with $P = 2P_{min} = \frac{18}{\pi}mg$

$U' = \frac{18}{\pi}mg\frac{r}{2}\frac{\pi}{3} = 3mgr$, $\Delta V_g = \frac{3}{2}mgr$

$\Delta T = 2\left(\frac{1}{2}mv^2\right) = mv^2$

Thus $3mgr = mv^2 + \frac{3}{2}mgr$, $\underline{v = \sqrt{3gr/2}}$

4/23 Note: sphere 3 does not move



(a) $U' = \Delta V_g$: $U' = P_{min} \frac{r}{2}\frac{\pi}{3}$

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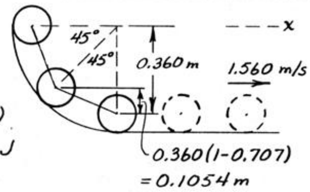
Thus $3mgr = mv^2 + \frac{3}{2}mgr$, $\underline{v = \sqrt{3gr/2}}$

4/24



$U'_{1-2} = \Delta T + \Delta V_g$

$= 3\left(\frac{1}{2}\times 2.75 \times 1.560^2\right) - 0$

$- 2.75\times 9.81(0.360 + 0.1054)$

$= 10.04 - 12.56 = -2.52$ J

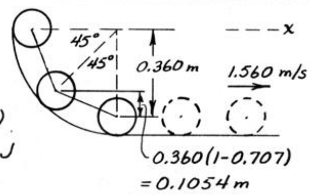
so loss is $\underline{\Delta Q = 2.52 \text{ J}}$

$I_x = \int \Sigma F_x\, dt = \Delta G_x = G_2 - G_1$, $G_2 = 3mv = 3(2.75)(1.560)$

$= 12.87$ N·s, $G_1 = 0$

$\underline{I_x = 12.87 \text{ N·s}}$

4/24



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$= 3\left(\frac{1}{2}\times 2.75 \times 1.560^2\right) - 0$

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$= 12.87$ N·s, $G_1 = 0$

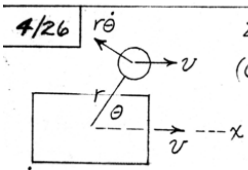
$\underline{I_x = 12.87 \text{ N·s}}$

4/25 (a) $\Sigma F_x = m\bar{a}_x$; $F = 2m\bar{a}$, $\underline{\bar{a} = F/2m}$

(b) $H_G = 2m\left(\frac{L}{2}\right)^2\dot{\theta}$, $\dot{H}_G = mL^2\ddot{\theta}/2$

$\Sigma M_G = \dot{H}_G$; $Fb = mL^2\ddot{\theta}/2$, $\underline{\ddot{\theta} = \frac{2Fb}{mL^2}}$

4/26



$\Sigma F_x = 0$ for system so $\Delta G_x = 0$

$(G_x)_{\theta=0} = (20+5)(0.6) = 15.0$ N·s

$(G_x)_{\theta=60°} = (20+5)v - 5(1.6)\sin 60°$

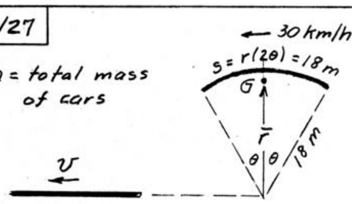
$= 25v - 6.93$ N·s

$r\dot{\theta} = 0.4(4) = 1.6$ m/s

Thus $15.0 = 25v - 6.93$, $v = 21.9/25 = \underline{0.877 \text{ m/s}}$

4/27

m = total mass of cars



$\theta = \frac{s}{2r} = \frac{18}{2(18)} = \frac{1}{2}$ rad

$= \frac{1}{2}\frac{180}{\pi} = 28.65°$

$\bar{r} = \frac{r\sin\theta}{\theta} = \frac{18\sin 28.65°}{1/2}$

$= 17.26$ m

For system $\Delta T + \Delta V_g = 0$

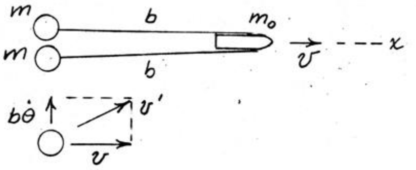
$\frac{1}{2}m\left(v^2 - \left[\frac{30}{3.6}\right]^2\right) - mg(17.26) = 0$

$v^2 = (30/3.6)^2 + 2(9.81)(17.26) = 69.4 + 338.6$

$= 408$ $(m/s)^2$

$v = 20.2$ m/s or $v = 20.2(3.6) = \underline{72.7 \text{ km/h}}$

4/28



For system $\Delta G_x = 0$: $(m_0 v + 2mv) - m_0 v_0 = 0$

$$v = \frac{m_0}{m_0 + 2m} v_0$$

$U = \Delta T$: $0 = \frac{1}{2} m_0 v^2 + 2\left[\frac{1}{2} m (v^2 + b^2\dot\theta^2)\right] - \frac{1}{2} m_0 v_0^2$

$$(m_0 + 2m) v^2 + 2mb^2\dot\theta^2 = m_0 v_0^2$$

Substitute v & get $\dfrac{m_0^2 v_0^2}{m_0 + 2m} + 2mb^2\dot\theta^2 = m_0 v_0^2$

Solve for $\dot\theta$ & get $\quad \underline{\dot\theta = \dfrac{v_0}{b}\sqrt{\dfrac{m_0}{m_0 + 2m}}}$

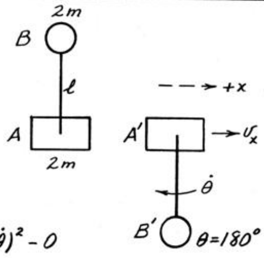
4/29

For entire system,

$\int \Sigma F_x \, dt = \Delta G_x = G_{A'} + G_{B'} - 0$

$0 = 2mv_x + 2m(v_x - l\dot\theta)$

$2v_x = l\dot\theta \quad$ - - - - (1)



For entire system,

$U'_{1-2} = \Delta T + \Delta V_g$

$\Delta T = \frac{1}{2}(2m)v_x^2 + \frac{1}{2}(2m)(v_x - l\dot\theta)^2 - 0$

$\quad = m(2v_x^2 - 2l\dot\theta v_x + l^2\dot\theta^2)$

$\Delta V_g = -2mg(2l) = -4mgl$

$U'_{1-2} = 0$ so $0 = m(2v_x^2 - 2l\dot\theta v_x + l^2\dot\theta^2) - 4mgl$

or $2v_x^2 - 2l\dot\theta v_x + l^2\dot\theta^2 = 4gl \quad$ - - - (2)

Combine (1) & (2): $2v_x^2 - 4v_x^2 + 4v_x^2 = 4gl$, $v_x^2 = 2gl$, $\underline{v_x = \sqrt{2gl}}$

$\dot\theta = 2v_x/l = 2\sqrt{2gl}/l = \underline{2\sqrt{\dfrac{2g}{l}}}$

4/30 System is conservative so $\Delta T + \Delta V_g = 0$

Flatcar; $\Delta T = \frac{1}{2} m v^2 - 0$

$\quad = \frac{1}{2} \frac{50{,}000}{32.2} v^2$

$\Delta V_g = 0$

Vehicle; $\Delta T = \frac{1}{2} m v^2 - 0 = \frac{1}{2} \frac{15000}{32.2}\left[(\dot s \cos 5° - v)^2 + (\dot s \sin 5°)^2\right] - 0$

$\Delta V_g = -W\Delta h = -15000\,(40 \sin 5°)$

Thus $776.4 v^2 + 232.9\left[(\dot s \cos 5° - v)^2 + (\dot s \sin 5°)^2\right] - 52290 = 0 \quad$ - - - - (1)

Also for system, $\Sigma F_x = 0$ so $\Delta G_x = 0$

$\frac{15{,}000}{32.2}(\dot s \cos 5° - v) - \frac{50{,}000}{32.2} v = 0$

$\dot s \cos 5° - v = 3.33 v \quad$ & $\quad \dot s \sin 5° = 4.33 v \tan 5° = 0.379 v$

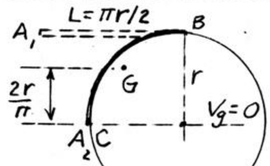
Substitute into (1) & get

$776.4 v^2 + 232.9\left[(3.33v)^2 + (0.379v)^2\right] - 52290 = 0$

$v^2(776.4 + 2588 + 33.5) = 52290$

$v^2 = 15.39\,(\text{ft/sec})^2$, $\quad \underline{v = 3.92 \text{ ft/sec}}$

4/31



Initial energy

$V_{g_1} = \rho g \frac{\pi r}{2} r = \rho g \frac{\pi r^2}{2}$

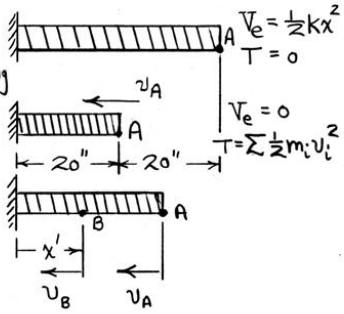
Final energy

$V_{g_2} = \rho g \frac{\pi r}{2} \frac{2r}{\pi} = \rho g r^2$

Energy loss $\Delta Q = V_{g_1} - V_{g_2} = \rho g r^2 (\frac{\pi}{2} - 1) = \underline{0.571 \rho g r^2}$

With $\Delta T = 0$ ($T_2 = T_1 = 0$) the loss of potential is dissipated into heat energy due to impact of rope against the drum.

4/32



Potential energy of spring

$V_e = \frac{1}{2} k x^2$

$\quad = \frac{1}{2} \frac{200}{20/12} \left(\frac{20}{12}\right)^2$

$\quad = 166.7$ ft-lb

During collapse of spring as $x \to 0$:

$v_B = \dfrac{x'}{20/12} v_A$, x' in feet.

Let ρ = mass per unit length; $dm = \rho\, dx'$

$T = \int \frac{1}{2} v_B^2 \, dm = \frac{1}{2}\int_0^{20/12} \frac{x'^2}{(20/12)^2} v_A^2 \rho \, dx'$

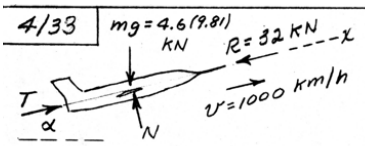
$\quad = \frac{1}{6} \rho \frac{20}{12} v_A^2$, where $\rho \frac{20}{12} = m$

So $T = \frac{1}{6} m v_A^2 = \frac{1}{6} \frac{3}{32.2} v_A^2 = \dfrac{v_A^2}{64.4}$

From $V_e = T$: $166.7 = \dfrac{v_A^2}{64.4}$, $\quad \underline{v_A = 103.6 \text{ ft/sec}}$

(Kinetic energy goes into heat, sound, & deformation.)

4/33



$T = m'_a(u - v) + m'_f u$

$\quad = 106(680 - 1000/3.6)$

$\quad + 4\,(680)$

$\quad = 45400$ N

$\Sigma F_x = ma_x = 0$; $\quad 45.4 - 32 - 4.6(9.81)\sin\alpha = 0$

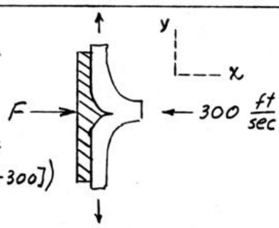
$\sin\alpha = 0.2960$, $\quad \underline{\alpha = 17.22°}$

4/34

$m' = \frac{\mu}{g} Q$

$\quad = \frac{0.0753}{32.2}(6.50)$

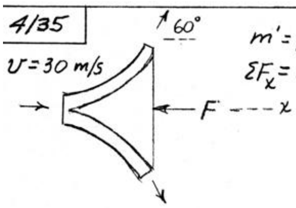
$\quad = 0.0152$ lb×sec/ft



$\Sigma F_x = m'\Delta v_x$: $F = 0.0152(0 - [-300])$

$\quad = \underline{4.56 \text{ lb}}$

4/35



$m' = \rho Q = 1000\,(0.05) = 50$ kg/s

$\Sigma F_x = m'\Delta v_x$; $-F = 50(30 \cos 60° - 30)$

$\underline{F = 750 \text{ N}}$

4/36 Resistance R equals net thrust T

where $T = m'(u - v)$

Nozzle velocity $u = Q/A = \dfrac{0.082}{\frac{\pi(0.050)^2}{4}} = 41.8$ m/s

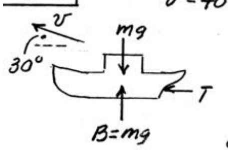
Density of salt water, Table D-1, $\rho = 1030$ kg/m^3

$m' = \rho Q = 1030(0.082) = 84.5$ kg/s

$v = 70\,\dfrac{1000}{3600} = 19.44$ m/s

$R = T = 84.5(41.8 - 19.44) = \underline{1885\text{ N}}$

4/37



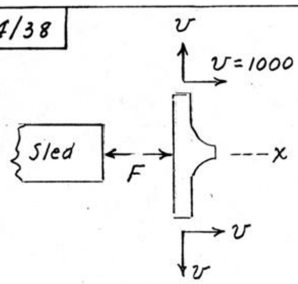
$v = 40$ m/s , $m' = \rho Q = 1030(0.080) = 82.4$ kg/s

$\Sigma F = m'\Delta v$:

$T = 82.4(40\cos 30° - 0) = 2850$ N

or $\underline{T = 2.85\text{ kN}}$

4/38



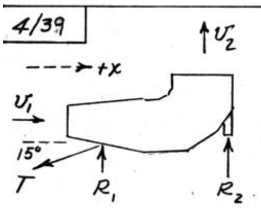
$m' = \rho A v = \dfrac{62.4}{32.2}\dfrac{15}{144}\,1000 = 202$ lb×sec/ft

$\Sigma F_x = m'\Delta v_x$

$F = 202(1000 - 0)$

$\underline{F = 202{,}000\text{ lb}}$

4/39



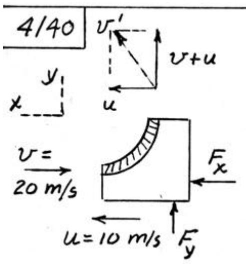
$\Sigma F_x = m'\Delta v_x$

$-T\cos 15° = (43 + 0.8)(0 - 720)$

$T = 32\,600$ N

or $\underline{T = 32.6\text{ kN}}$

4/40



Final velocity of diverted water is $\underline{v}' = \underline{u} + \underline{v}_{rel}$

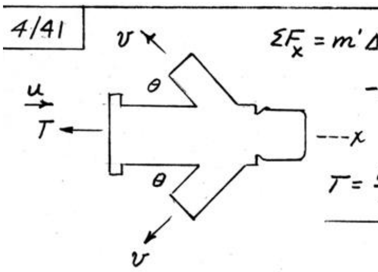
where $v_{rel} = v + u$

$m' = \rho A v_{rel} = 1000\frac{\pi}{4}(0.025)^2(20 + 10) = 14.73$ kg/s

$\Sigma F_x = m'\Delta v_x$: $F_x = 14.73(10 - [-20]) = \underline{442\text{ N}}$

$\Sigma F_y = m'\Delta v_y$: $F_y = 14.73(20 + 10 - 0) = \underline{442\text{ N}}$

4/41

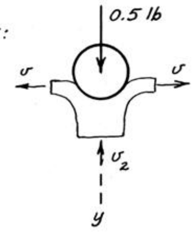


$\Sigma F_x = m'\Delta v_x$,

$-T = m'(-v\cos\theta - u)$

$\underline{T = \dfrac{\pi d^2}{4}\rho u(v\cos\theta + u)}$

4/42



Ball & stream just under it:

$\Sigma F_y = m'\Delta v_y$:

m' at ball $= m'$ at nozzle

$= \rho A v = \dfrac{62.4}{32.2}\dfrac{\pi(0.5)^2}{4}\left(\dfrac{1}{12}\right)^2 \cdot 35$

$= 0.925$ lb-sec/ft

so $0.5 = 0.925(0 - [-v_2])$

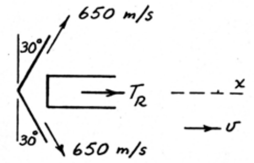
$v_2 = 5.41$ ft/sec

For water stream $\Delta V_g + \Delta T = 0$:

$mgh + \frac{1}{2}m(v_2^2 - v_1^2) = 0$,

$h = \dfrac{1}{2\times 32.2}(35^2 - 5.41^2) = \underline{18.57\text{ ft}}$

4/43 $\Sigma F = \Sigma m'u$



With reversers in place,

$T_R = m_g' u\sin 30° + m_a' v$

$T_R = (50 + 0.65)(650)\sin 30° + 50(55.6 - 0)$

$= 16\,460 + 2780$

$= 19\,240$ N

$v = 200/3.6 = 55.6$ m/s

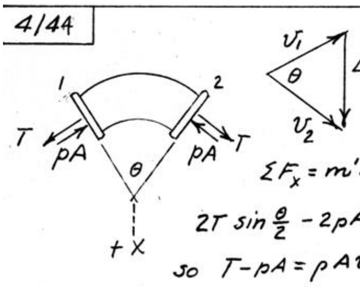
Without reversers $T = m_g' u - m_a' v$

$T = (50 + 0.65)650 - 50(55.6)$

$= 32\,900 - 2780 = 30\,100$ N

so $n = \dfrac{19\,240}{30\,100} = \underline{0.638}$

4/44



$\Delta v = 2v\sin\frac{\theta}{2}$

where $v = |\underline{v}_1| = |\underline{v}_2|$

$\Sigma F_x = m'\Delta v_x$ entire system

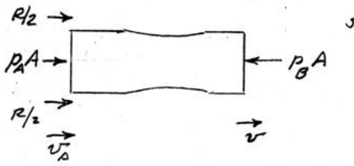
$2T\sin\frac{\theta}{2} - 2pA\sin\frac{\theta}{2} = \rho A v\left(2v\sin\frac{\theta}{2}\right)$

so $T - pA = \rho A v^2$, $\underline{T = A(p + \rho v^2)}$

independent of θ

4/45 Continuity requires $\rho_A A v_A = \rho_B A v$

so $v_A = v\,\rho_B/\rho_A$



$m' = \rho A v = \rho_B A v$ at B

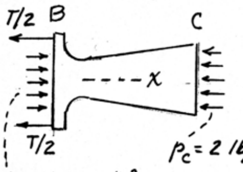
$\Sigma F = m'\Delta v$; $R + p_A A - p_B A = \rho_B A v(v - v_A)$

$= \rho_B A v^2(1 - \rho_B/\rho_A)$

Also, $A = \pi d^2/4$, so $R = \rho_B\dfrac{\pi d^2}{4}v^2\left(1 - \dfrac{\rho_B}{\rho_A}\right) + (p_B - p_A)\dfrac{\pi d^2}{4}$

$\underline{R = \dfrac{\pi d^2}{4}\left[\rho_B\left(1 - \dfrac{\rho_B}{\rho_A}\right)v^2 + (p_B - p_A)\right]}$

4/46 $A_C = \frac{\pi 4^2}{4(144)} = 0.0873\ ft^2$, $A_B = 4A_C = 0.349\ ft^2$



$m' = \rho A v$

$m'_B = \frac{0.840}{32.2}(0.349)50 = 0.455\ \frac{lb\text{-}sec}{ft}$

$m'_C = \frac{0.0760}{32.2}(0.0873)v_C = 2.06(10^{-4})v_C$

$m'_B = m'_C$ so $v_C = \frac{0.455}{2.06(10^{-4})}$

$p_B = 150\ lb/in.^2$

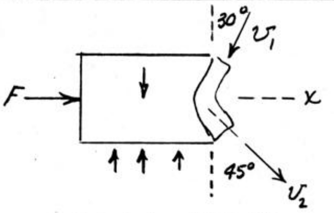
$\rightarrow v_B = 50\ ft/sec$ $\qquad = 2210\ ft/sec$

$\Sigma F_x = m'\Delta v_x$: $150(0.349)144 - 2(0.0873)(144) - T$

$= 0.455(2210 - 50)$

$\underline{T = 6530\ lb}$

4/47



$Q = Av$:

$\frac{1}{2}(231) = \frac{0.01^2\pi}{4}v_1$

$v_1 = 1.471(10^6)\ in./min$

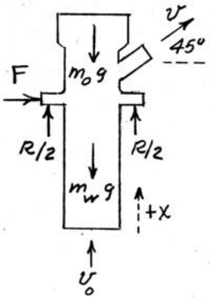
or $v_1 = 2042\ ft/sec$ & $v_2 = 0.60(2042) = 1225\ ft/sec$

$\Sigma F_x = m'\Delta v_x$: $m' = \frac{231}{2}\frac{1}{60}\frac{1}{1728}\frac{68}{32.2} = 2.35(10^{-3})\ lb\text{-}sec/ft$ (slugs/sec)

$F = 2.35(10^{-3})(1225 \sin 45° - [-2042 \sin 30°])$

$\underline{F = 4.44\ lb}$

4/48



$\Sigma F_x = m'\Delta v_x$:

$R - m_0 g - m_w g = \rho Q(v \cos 45° - v_0)$

$m_0 = 310\ kg$

Mass of water $m_w = \rho V$

$= 1000\frac{\pi}{4}(0.2)^2(6)$

$= 188.5\ kg$

$Q = 0.125\ m^3/s$

$A = \frac{\pi}{4}(0.1)^2 = 0.00785\ m^2$

$A_0 = \frac{\pi}{4}(0.25)^2 = 0.0491\ m^2$

$v = Q/A = 0.125/0.00785 = 15.92\ m/s$

$v_0 = Q/A_0 = 0.125/0.0491 = 2.55\ m/s$

Thus $R - (310 + 188.5)9.81 = 1000(0.125)(15.92 \cos 45° - 2.55)$

$= 1088\ N$

$\underline{R = 5980\ N}$

4/49 $kx = m'\Delta v$, $m' = \rho A v = 1000\frac{\pi}{4}(0.030)^2 v$

$= 0.7069 v$

$15000(0.150) = 0.7069 v(v - 0)$

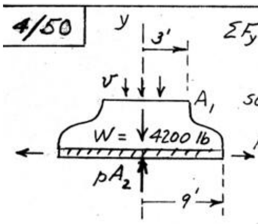
$v^2 = 3183$, $\underline{v = 56.4\ m/s}$

$\Sigma M_A = m'vd$; $M = 15(150)(15 \sin 75° - 4.8 \cos 75°)$

$= 2250(13.25) = 29\ 800\ N{\cdot}m$

or $\underline{M = 29.8\ kN{\cdot}m}$

4/50



$\Sigma F_y = m'\Delta v_y$; $m' = \rho A_1 v$

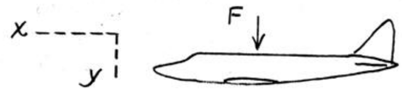
$\Delta v_y = 0 - (-v) = v$

so $pA_2 - W = \rho A_1 v^2$

$p(\pi \times 9^2)(144) - 4200 = \frac{0.076}{32.2}\pi \times 3^2(150^2)$

$\underline{p = 0.1556\ lb/in.^2}$

4/51 Consider motion as measured from the inertial reference of the aircraft.



$F = m'\Delta v_y$ where $\Delta v_y = 0 - 20 = -20\ ft/sec$

Volume of water striking horizontal surface per second is $Q = A v_y = (2960\ ft^2)(\frac{1}{12}\frac{ft}{hr})(\frac{1}{3600}\frac{1}{sec/hr})$

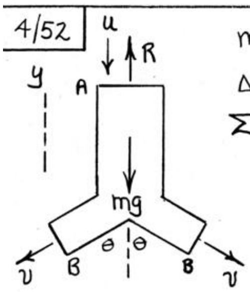
$= 0.0685\ ft^3/sec$

$m' = \rho Q = \frac{62.4}{32.2}(0.0685) = 0.1328\ lb\text{-}sec/ft$ (slugs/sec)

$F = m'|\Delta v_y|$ $\quad F = 0.1328(20)$

$\underline{= 2.66\ lb}$

4/52



$m' = \rho A v_A = \rho\frac{\pi d^2}{4}u$

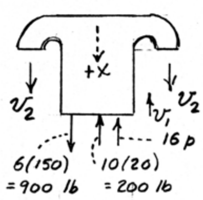
$\Delta v_y = -v\cos\theta - (-u) = u - v\cos\theta$

$\Sigma F_y = m'\Delta v_y$:

$R - mg = \rho\frac{\pi d^2}{4}u(u - v\cos\theta)$

$\underline{R = mg + \rho\frac{\pi d^2}{4}u(u - v\cos\theta)}$

4/53 $Q = 1.6\ ft^3/sec$ $\quad v_1 = \frac{Q}{A_1} = \frac{1.6}{20/144} = 11.52\ ft/sec$



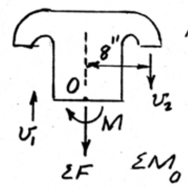
$v_2 = \frac{Q}{A_2} = \frac{1.6/2}{3.2/144} = 36\ ft/sec$

$m' = \rho Q = \frac{64.4}{32.2}1.6 = 3.2\ lb\text{-}sec/ft$

$\Sigma F_x = m'\Delta v_x$:

$900 - 200 - 16p = 3.2(36 - [-11.52])$

$\underline{p = 34.2\ lb/in.^2}$



For left side blocked off & with

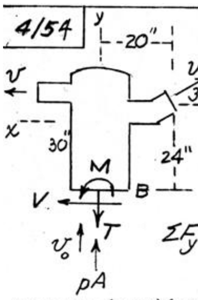
$Q = 1.6/2 = 0.8\ ft^3/sec$,

$v_2 = Q/A_2 = \frac{0.8}{3.2/144} = 36\ \frac{ft}{sec}$, $m' = \frac{3.2}{2} = 1.6\ lb\text{-}sec/ft$

$\Sigma M_O = m'(v_2 d_2 - v_1 d_1)$

$M = 1.6(36 \times 8 - 0) = \underline{461\ lb\text{-}in.}$

4/54



$v = \dfrac{\text{Volume rate}}{\text{area}} = \dfrac{10/2}{0.040} = 125$ ft/sec

For each outlet $m' = \dfrac{5 \times 62.4}{32.2}$

$= 9.69$ lb-ft^{-1}-sec

$v_o = \dfrac{10}{0.75} = 13.33$ ft/sec

$\Sigma F_y = m'\Delta v_y;\ -T + 12{,}960 = 9.69(0 - 13.33) + 9.69(+125 \sin 30° - 13.33)$

$pA = 120(0.75)(144) = 12{,}960$ lb

$= -129.2 + 476.5 - 12{,}960$

$\underline{T = 12{,}610 \text{ lb}}$

$\Sigma F_x = m'\Delta v_x;\ V = 9.69(125 - 0) + 9.69(-125 \cos 30° - 0)$

$= 1211 - 1049 = \underline{162.3 \text{ lb}}$

$\Sigma M_B = \Sigma m' v d;\ M = 9.69(125)\frac{30}{12} - 9.69(125 \cos 30°)\frac{24}{12} + 9.69(125 \sin 30°)\frac{20}{12}]$

$M = 3028 - 2098 + 1009 = \underline{1939 \text{ lb-ft}}$

4/55 For the truck and plow as a system:

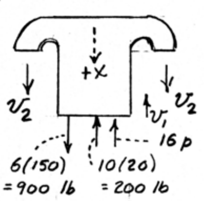
$\Sigma F_x = m'\Delta v_x:\ P = \dfrac{60\,000}{60}\left[\dfrac{20}{3.6} - 0\right] = 5560$ N

or $\underline{P = 5.56 \text{ kN}}$

$\Sigma F_y = m'\Delta v_y:\ R = \dfrac{60\,000}{60}\left[12 \cos 45° - 0\right] = 8490$ N

or $\underline{R = 8.49 \text{ kN}}$

4/53 $Q = 1.6$ ft^3/sec $\quad v_1 = \dfrac{Q}{A_1} = \dfrac{1.6}{20/144} = 11.52$ ft/sec



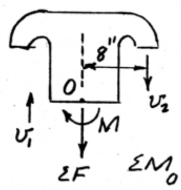
$v_2 = \dfrac{Q}{A_2} = \dfrac{1.6/2}{3.2/144} = 36$ ft/sec

$m' = \rho Q = \dfrac{64.4}{32.2} 1.6 = 3.2$ lb-sec/ft

$\Sigma F_x = m'\Delta v_x:$

$900 - 200 - 16p = 3.2(36 - [-11.52])$

$\underline{p = 34.2 \text{ lb/in.}^2}$



For left side blocked off & with

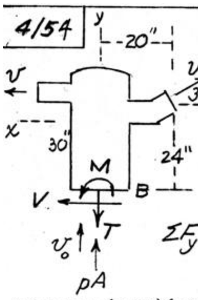
$Q = 1.6/2 = 0.8$ ft^3/sec,

$v_2 = Q/A_2 = \dfrac{0.8}{3.2/144} = 36 \dfrac{\text{ft}}{\text{sec}}$, $m' = \dfrac{3.2}{2} = 1.6$ lb-sec/ft

$\Sigma M_O = m'(v_2 d_2 - v_1 d_1)$

$M = 1.6(36 \times 8 - 0) = \underline{461 \text{ lb-in.}}$

4/54



$v = \dfrac{\text{Volume rate}}{\text{area}} = \dfrac{10/2}{0.040} = 125$ ft/sec

For each outlet $m' = \dfrac{5 \times 62.4}{32.2}$

$= 9.69$ lb-ft^{-1}-sec

$v_o = \dfrac{10}{0.75} = 13.33$ ft/sec

$\Sigma F_y = m'\Delta v_y;\ -T + 12{,}960 = 9.69(0 - 13.33) + 9.69(+125 \sin 30° - 13.33)$

$pA = 120(0.75)(144) = 12{,}960$ lb

$= -129.2 + 476.5 - 12{,}960$

$\underline{T = 12{,}610 \text{ lb}}$

$\Sigma F_x = m'\Delta v_x;\ V = 9.69(125 - 0) + 9.69(-125 \cos 30° - 0)$

$= 1211 - 1049 = \underline{162.3 \text{ lb}}$

$\Sigma M_B = \Sigma m' v d;\ M = 9.69(125)\frac{30}{12} - 9.69(125 \cos 30°)\frac{24}{12} + 9.69(125 \sin 30°)\frac{20}{12}]$

$M = 3028 - 2098 + 1009 = \underline{1939 \text{ lb-ft}}$

4/55 For the truck and plow as a system:

$\Sigma F_x = m'\Delta v_x:\ P = \dfrac{60\,000}{60}\left[\dfrac{20}{3.6} - 0\right] = 5560$ N

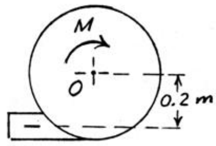
or $\underline{P = 5.56 \text{ kN}}$

$\Sigma F_y = m'\Delta v_y:\ R = \dfrac{60\,000}{60}\left[12 \cos 45° - 0\right] = 8490$ N

or $\underline{R = 8.49 \text{ kN}}$

4/56 $M = M_O = m'(v_2 d_2 - 0)$

$v_2 = \dfrac{Q}{A} = \dfrac{16}{\pi(0.150)^2/4} \dfrac{1}{60} = 15.09 \dfrac{\text{m}}{\text{s}}$



From Table D-1, air density is 1.206 kg/m^3

so $m' = \rho Q = 1.206(16)/60 = 0.322$ kg/s

$M_O = 0.322(15.09 \times 0.2 - 0) = 0.971$ N·m

$P = 0.32 + M_O\omega/1000 = 0.32 + \dfrac{0.971(3450 \times 2\pi/60)}{1000}$

$P = 0.32 + 0.351 = \underline{0.671 \text{ kW}}$

4/57 $v = 40$ m/s, $u = 420$ m/s

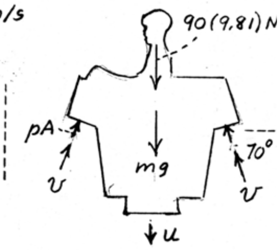
$pA = -1.8(10^3)(0.1320)$

$= -238$ N

$m' = \rho A v$

$= 1.206(0.1320)(40)$

$= 6.37$ kg/s



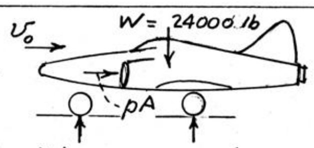
For steady flow

$\Sigma F_y = m'\Delta v_y$

$(90 + m)9.81 - (-238 \sin 70°) = 6.37(420 - [-40 \sin 70°])$

$9.81m + 1106 = 2914$, $\underline{m = 184.3 \text{ kg}}$

4/58



$m'_{air} = \dfrac{106}{32.2} \dfrac{\text{lb/sec}}{\text{ft/sec}^2} = 3.29$ lb×sec/ft

$m'_{fuel} = 3.29/18 = 0.1829$ lb×sec/ft

air intake velocity $v_o = \dfrac{m'_{air}}{\rho A} = \dfrac{3.29}{(0.0753/32.2)(1800/144)}$

$= 112.6$ ft/sec

$m'_{exhaust} = 3.29 + 0.1829$

$= 3.47$ lb×sec/ft

$pA = -0.30(1800) = -540$ lb

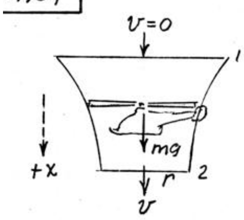
Net thrust $T = m'_{ex} u - m'_{air} v_o - pA$

$= 3.47(3100) - 3.29(112.6) - (-540)$

$= 10{,}940$ lb

$\Sigma F_x = ma_x:\ 10940 = \dfrac{24000}{32.2} a$, $\underline{a = 14.68 \text{ ft/sec}^2}$

4/59



mg = weight of helicopter = force on system of air stream & helicopter

For system between sections 1 & 2

$\Sigma F_x = m'\Delta v_x$

$mg = \rho \pi r^2 v(v-0)$

$v = \frac{1}{r}\sqrt{\frac{mg}{\pi\rho}}$

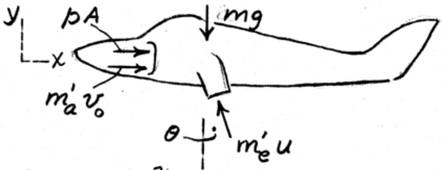
Power = rate of increase of kinetic energy

$P = \frac{1}{2}m'(v_2^2 - v_1^2) = \frac{1}{2}m'v^2 = m'v\frac{v}{2} = mg\frac{v}{2}$

$P = \frac{mg}{2r}\sqrt{\frac{mg}{\pi\rho}}$

4/60

Simulated FBD



$mg = 8600(9.81) = 84.4(10^3)$ N

mass rate of air $= m'_a = 90$ kg/s

" " " fuel $= 90/18 = 5$ kg/s

" " " exhaust $= m'_e = 95$ kg/s

$pA = -2(10^3)(1.10) = -2200$ N

$v_o = m'_a/\rho A = \frac{90}{1.206(1.10)} = 67.8$ m/s, $m'_a v_o = 90(67.8) = 6110$ N

$m'_e u = 95(1020) = 96\,900$ N

For vertical take off $\Sigma F_x = 0$: $6110 - 2200 - 96\,900 \sin\theta = 0$

$\sin\theta = 0.0403$, $\theta = 2.31°$

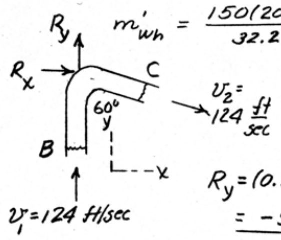
$\Sigma F_y = ma_y$: $96\,900\cos 2.31° - 84\,400 = 8600\,a_y$

$a_y = 1.448$ m/s²

4/61

$m'_{air} = \frac{18(2000)}{32.2}\frac{1}{3600} = 0.3106$ slugs/sec

$m'_{wh} = \frac{150(2000)}{32.2}\frac{1}{3600} = 2.588$ slugs/sec



$\Sigma F = m'\Delta v$

$R_x = (0.3106 + 2.588)(124\sin 60° - 0)$

$= 311$ lb

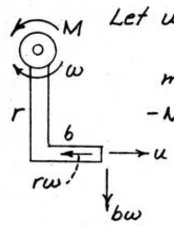
$R_y = (0.3106 + 2.588)(-124\cos 60° - 124)$

$= -539$ lb

Forces acting on pipe bend & mass within it

1) tension $pA = 4.42\frac{\pi(14)^2}{4} = 680$ lb due to vacuum
2) tension in pipe at B
3) " " " " C
4) weight of bend
5) balance of external support forces from crane
6) shear force and bending moment at C

4/62



components of absolute velocity of water at exit

For entire system $\Sigma M = m'(v_2 d_2 - v_1 d_1)$

Let u = velocity of water relative to nozzle $= Q/4A$

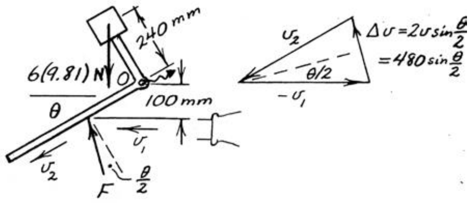
$m' = \rho Q$

$-M = \rho Q(r^2\omega + b^2\omega - \frac{Q}{4A}r - 0)$

$M = \rho Q(\frac{Qr}{4A} - [r^2 + b^2]\omega)$

For $M = 0$, $\omega = \omega_o = \frac{Qr}{4A(r^2+b^2)}$

4/63



$F = m'\Delta v$: $m' = \rho A v = 1.206\frac{\pi \times 0.040^2}{4}240 = 0.364$ kg/s

$F = 0.364 \times 480\sin\frac{\theta}{2} = 174.6\sin\frac{\theta}{2}$ N

For vane:

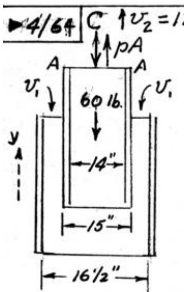
$\Sigma M_O = 0$: $174.6\sin\frac{\theta}{2}\cos\frac{\theta}{2}\left(\frac{0.100}{\sin\theta}\right) - 6(9.81)(0.240\sin\theta) = 0$

$87.3 \times 0.100 = 6(9.81)(0.240\sin\theta)$

$\sin\theta = 0.618$, $\theta = 38.2°$

Assumption: Entire air stream is diverted downward along the vane, with no flow toward O.

►4/64



$v_2 = 124$ ft/sec

Air inlet area $= \frac{\pi}{4}([16.5]^2 - [15]^2) = 37.1$ in.²

$= 37.1/144 = 0.258$ ft²

Exit area $= \frac{\pi}{4}(14)^2 = 153.9$ in.²

$= 153.9/144 = 1.069$ ft²

$pA = -(-4.42)(153.9) = 680$ lb

$m'_{air} = \frac{18(2000)}{32.2}\frac{1}{3600} = 0.3106$ slugs/sec

$m'_{wh} = \frac{150(2000)}{32.2}\frac{1}{3600} = 2.588$ slugs/sec

$v_1 = m'_{air}/\rho A = \frac{0.3106}{(\frac{0.075}{32.2})(0.258)} = 517$ ft/sec

$\Sigma F_y = m'\Delta v_y$; $-C + 680 - 60 = 0.3106(124 - [-517]) + 2.588(124 - 0)$

$= 199.2 + 320.9$

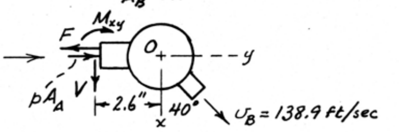
$C = 100.3$ lb

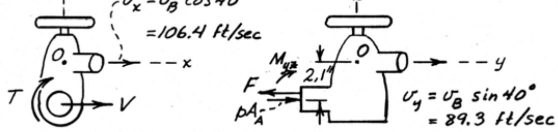
►4/65

Flow rate $Q = \frac{340 \times 231}{1728 \times 60} = 0.758\frac{\text{ft}^3}{\text{sec}}$, $m' = \rho Q = \frac{62.4}{32.2}0.758 = 1.468\frac{\text{lb-sec}}{\text{ft}}$

Flow area $A_A = \frac{\pi 2^2}{4}/144 = 0.0218$ ft², $A_B = \frac{\pi \times 1^2}{4}/144 = 0.00545$ ft²

Velocity $v_A = \frac{Q}{A_A} = \frac{0.758}{0.0218} = 34.7\frac{\text{ft}}{\text{sec}}$, $v_B = \frac{Q}{A_B} = \frac{0.758}{0.00545} = 138.9\frac{\text{ft}}{\text{sec}}$





$(x\text{-}y)\ \Sigma F_x = m'\Delta v_x$: $150(\frac{\pi 2^2}{4}) - F = 1.468(89.3 - 34.7)$, $F = 391$ lb

$\Sigma F_y = m'\Delta v_y$: $V = 1.468(106.4 - 0)$, $V = 156.2$ lb

$\Sigma M_{A-A} = m'\Delta(vd)$: $M_{xy} = 1.468(106.4 \times \frac{2.6}{12}) = 33.8$ lb-ft

$(y\text{-}z)\ \Sigma M_{A-A} = m'\Delta(vd)$: $M_{yz} = 1.468(89.3 \times \frac{2.1}{12}) = 22.9$ lb-ft

$M = \sqrt{M_{xy}^2 + M_{yz}^2} = (33.8^2 + 22.9^2)^{1/2} = 40.9$ lb-ft

$(x\text{-}z)\ \Sigma M_O = 0$: $T - Vd = 0$, $T = 156.2(\frac{2.1}{12}) = 27.3$ lb-ft

▸4/66 From Part (b) of Sample Problem $m' = \rho A(v-u)$

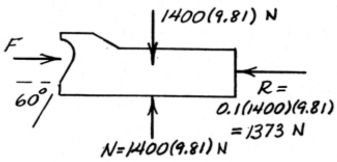
$= (1000)\frac{\pi \times 0.140^2}{4}(150-u)$

$= 15.39(150-u)$ kg/s

& $F = \rho A(v-u)^2(1-\cos 120°)$, $\theta = 90° + 30° = 120°$

$= 15.39(150-u)^2(1-(-0.5))$

$= 23.1(150-u)^2$



$\Sigma F = m\dot{u}$: $23.1(150-u)^2 - 1373 = 1400\,\dot{u}$

$\int_0^u \frac{du}{0.01649(150-u)^2 - 0.981} = \int_0^3 dt$

To integrate let $w = 150-u$, $\int_{u=0}^{u=u} \frac{dw}{0.981 - 0.01649w^2} = 3$

$\frac{1}{2\sqrt{0.01649}\sqrt{0.981}} \ln \left| \frac{0.990 + 0.1284(150-u)}{0.990 - 0.1284(150-u)} \right|_0^u = 3$

$3.93 \ln \frac{1-0.00634u}{1-0.00703u} = 3$, $\frac{1-0.00634u}{1-0.00703u} = 2.145$

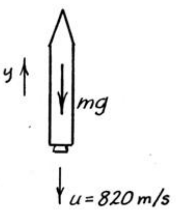
Solve for u & get $\underline{u = 131.0 \text{ m/s}}$

4/67

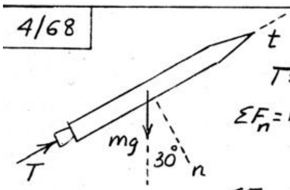
$\Sigma F_y = ma + \dot{m}u$: $-9.81m = 6.80m - 220(820)$

$m = 10.86(10^3)$ kg

or $\underline{m = 10.86 \text{ Mg}}$



4/68



$mg = 3(10^3)(9.60) = 28.8(10^3)$ N

$T = m'\Delta v = 130(600) = 78(10^3)$ N

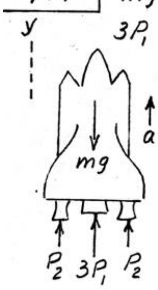
$\Sigma F_n = ma_n$: $28.8(10^3)\cos 30° = 3(10^3)a_n$

$\underline{a_n = 8.31 \text{ m/s}^2}$

$\Sigma F_t = ma_t$: $78(10^3) - 28.8(10^3)\sin 30° = 3(10^3)a_t$

$\underline{a_t = 21.2 \text{ m/s}^2}$

4/69



$mg = 2.04(10^6)(9.81) = 20.0(10^6)$ N

$3P_1 = 3(2.00)(10^6) = 6.00(10^6)$ N

$2P_2 = 2(11.80)(10^6) = 23.6(10^6)$ N

Specific impulse $I = \frac{u}{g} = 455$ s

So $u = 455(9.81) = 4460$ m/s

$\Sigma F_y = ma_y$: $(6.00)10^6 + (23.6)10^6 - 20.0(10^6) = 2.04(10^6)a$

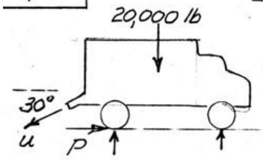
$\underline{a = 4.70 \text{ m/s}^2}$

$P_1 = m'u$, $2.00(10^6) = m'(4460)$

$\underline{m' = 448 \text{ kg/s}}$

4/70

$\Sigma F_x = m\dot{v} + \dot{m}u$



$u = 60$ ft/sec

$\dot{m} = -\frac{80}{32.2} = -2.48 \frac{\text{slugs}}{\text{sec}}$

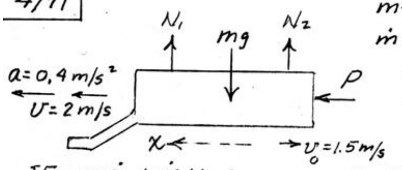
$m = \frac{20,000}{32.2} = 621$ slugs

(a) Water on; $P = 621(2) - 2.48(60)\cos 30°$

$= 1242 - 129 = \underline{1113 \text{ lb}}$

(b) Water off; $\dot{m} = 0$, $\underline{P = 1242 \text{ lb}}$

4/71



$m = 50 + 80 = 130$ kg

$\dot{m} = \rho A u = \rho A(v + v_0)$

$= 1000(2000)10^{-6}(2 + 1.5)$

$= 7.0$ kg/s

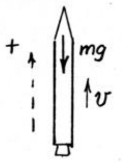
$u = v + v_0 = 3.5$ m/s

$\Sigma F_x = m\dot{v} + \dot{m}u$; $P = 130(0.4) + 7.0(3.5)$

$= 52 + 24.5 = \underline{76.5 \text{ N}}$

4/72 $\Sigma F = m\dot{v} + \dot{m}u$; $-mg = ma + \dot{m}u$

$-m(a+g) = \dot{m}u$

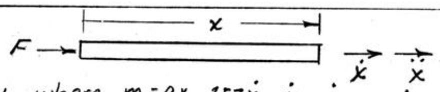


$u\frac{dm}{dt} = -m(a+g)$

$\int_{m_0}^{m} \frac{dm}{m} = -\frac{a+g}{u}\int_0^t dt$, $\ln\frac{m}{m_0} = -\frac{a+g}{u}t$

$\underline{m = m_0 e^{-\frac{a+g}{u}t}}$

4/73



$\Sigma F = m\dot{v} + \dot{m}u$ where $m = \rho x$, $v = \dot{x}$, $\dot{m} = \rho\dot{x}$, $u = \dot{x}$

Thus $F = \rho x\ddot{x} + \rho\dot{x}\dot{x}$, $\underline{F = \rho(x\ddot{x} + \dot{x}^2)}$

4/74

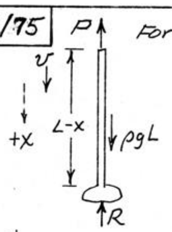
With added moisture particles initially at rest, relative velocity of attachment of mass is $u = v$

Thus with $\Sigma F = m\dot{v} + \dot{m}u$

we have $\Sigma F = m\dot{v} + \dot{m}v = \underline{\frac{d}{dt}(mv)}$

where $\Sigma F = mg - R$

4/75



For $\dot{x} = v = $ const, $P =$ weight of descending links $= \rho g(L-x)$

$\Sigma F_x = \frac{dG_x}{dt}$; $\rho gL - R - \rho g(L-x) = \frac{d}{dt}(\rho[L-x]v)$

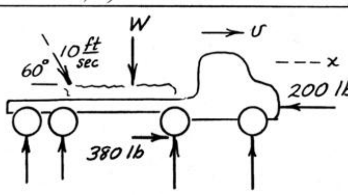
$\rho gx - R = -\rho v\dot{x} = -\rho v^2$

So $\underline{R = \rho gx + \rho v^2}$

4/76 $\Sigma F_x = m\dot{v} + \dot{m}u$:

$\Sigma F_x = 380 - 200 = 180$ lb

$m = \frac{12000 + 4(220)}{32.2}$



$= 400$ lb-sec²/ft

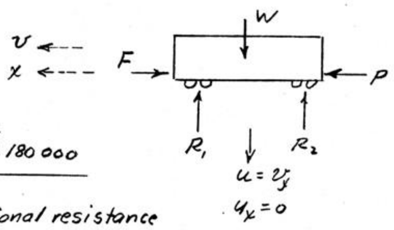
at $t = 4$ sec.

$\dot{m} = 220/32.2 = 6.83$ lb-sec/ft 1.5 mi/hr = 2.20 ft/sec

$u = 2.20 - 10\cos 60° = -2.80$ ft/sec

So $180 = 400\dot{v} + 6.83(-2.80)$, $a = \dot{v} = \underline{0.498 \text{ ft/sec}^2}$

4/77



$\Sigma F_x = m\dot{v}_x + \dot{m}u_x$

$F = 4\,\dfrac{54{,}600 + \frac{1}{2}\,180\,000}{2000}$

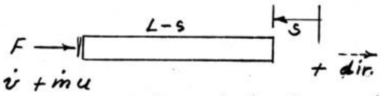
$= 289$ lb frictional resistance

u_x = rel. velocity with respect to the car in x-dir. $= 0$

Thus $P - 289 = \dfrac{54{,}600 + \frac{1}{2}\,180{,}000}{32.2}\,0.15 + 0$

So $P = 674 + 289$, $\underline{P = 963\ \text{lb}}$

4/78



Eq. 4/20 $\Sigma F = m\dot{v} + \dot{m}u$

where $\Sigma F = F$, $m = \rho(L-s)$, $\dot{m} = -\rho\dot{s}$, $\dot{v} = -\ddot{s}$, $u = -\dot{s}$

Thus $F = \rho(L-s)(-\ddot{s}) + (-\rho\dot{s})(-\dot{s})$

$\underline{F = \rho\dot{s}^2 - \rho(L-s)\ddot{s}}$

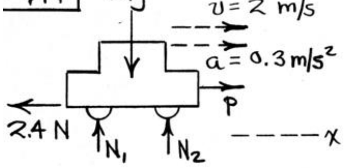
Eq. 4/6 for entire system $\Sigma F = \dot{G}$

$G = \rho(L-s)(-\dot{s}) = -\rho(L-s)\dot{s}$

$\dot{G} = -\rho(L-s)\ddot{s} - \rho(-\dot{s})\dot{s} = \rho\dot{s}^2 - \rho(L-s)\ddot{s}$

Thus $\underline{F = \rho\dot{s}^2 - \rho(L-s)\ddot{s}}$

4/79



$m = 40 + 30(1.2)$
$= 76$ kg

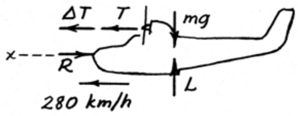
$\dot{m} = -\rho v = -1.2(2)$
$= -2.4$ kg/s

$\Sigma F_x = m\dot{v} + \dot{m}u$: $P - 2.4 = 76(0.3) - 2.4(2)$

$\underline{P = 20.4\ \text{N}}$

4/80

For constant initial speed propeller thrust T = drag R.



Added power $= \Delta T \cdot v$,

$\Delta T \times \dfrac{280 \times 1000}{3600} = 223.8\,(10^3)$ watts (joules/second)

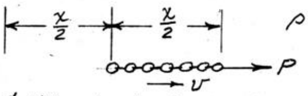
$\Delta T = 2880$ N

$\Sigma F_x = m\dot{v} + \dot{m}u$ where $\dot{m} = 4.5 \times 1000/12 = 375$ kg/s,

$u = v = \dfrac{280 \times 1000}{3600} = 77.8$ m/s

So $2880 = 16.4(10^3)\,\dot{v} + 375(77.8)$, $\dot{v} = \underline{a = -1.603\ \text{m/s}^2}$
(deceleration)

4/81



$\rho = 48/8 = 6$ kg/m

$\dot{m} = \rho\frac{d}{dt}\left(\frac{x}{2}\right) = \frac{1}{2}\rho\dot{x} = \frac{1}{2}\rho v = \frac{1}{2}(6)(1.5) = 4.5$ kg/s

$u = v = 1.5$ m/s

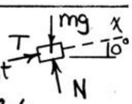
$\Sigma F = m\dot{v} + \dot{m}u$

(a) $\dot{v} = 0$; $P = 0 + 4.5(1.5) = \underline{6.75\ \text{N}}$

(b) $m = \rho\frac{x}{2} = 6\left(\frac{4}{2}\right) = 12$ kg

$20 = 12\dot{v} + 4.5(1.5)$, $a = \dot{v} = \underline{1.104\ \text{m/s}^2}$

4/82 $\Sigma F_x = ma_x$: $T - mg\sin\theta = ma_x$



$T = m'u = \dfrac{2}{32.2}(400) = 24.8$ lb constant

$m = m_0 - m't = \dfrac{1}{32.2}(125 - 2t)$ lb-sec²/ft

Propulsion time $t = \frac{20}{2} = 10$ sec

So $m'u - (m_0 - m't)g\sin\theta = (m_0 - m't)\dfrac{dv}{dt}$

$\displaystyle\int_0^t \left[\frac{m'u}{m_0 - m't} - g\sin\theta\right] dt = \int_0^v dv$

$\Rightarrow v = u\ln\left(\dfrac{m_0}{m_0 - m't}\right) - gt\sin\theta$

When $t = 10$ sec, $v = 400\ln\left(\dfrac{125}{125-20}\right) - 32.2(10)\sin 10°$

$= \underline{13.83\ \text{ft/sec}}$

4/83 With v = const., $\dot{v}$ = accel. = 0

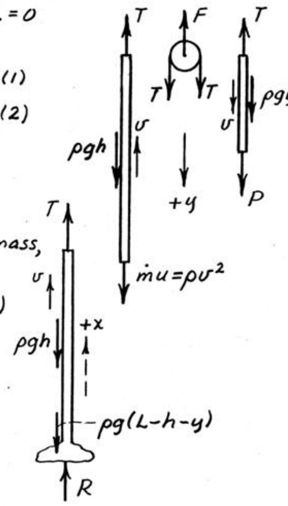
so $\Sigma F_y = 0$ for all bodies:

$P + \rho gy - T = 0$ ---(1)

$\rho v^2 + \rho gh - T = 0$ --- (2)

Eliminate T & get

$\underline{P = \rho v^2 + \rho g(h-y)}$



Left-hand portion (constant mass, upper end moving up):

$\Sigma F_x = \dot{G}_x$: $T + R - \rho gh - \rho g(L-h-y)$
$= \frac{d}{dt}(\rho hv)$

$\rho v^2 + \rho gh + R - \rho gh - \rho g(L-h-y)$
$= \frac{d}{dt}(\rho hv) = \rho v^2$
where $\dot{h} = v$

$\rho v^2 + R - \rho g(L-h-y) = \rho v^2$

$R = \underline{\rho g(L-h-y)}$ = weight of pile of chain

4/84 Let m_0 = initial mass of car $= 25(10^3)$ kg

$\dot{m} = 4(10^3)$ kg/s

The car acquires mass which has zero initial horizontal velocity, so for horizontal x-dir, $\Sigma F_x = \frac{d}{dt}(mv)$

$0 = \frac{d}{dt}(m_0 + \dot{m}t)v$, $(m_0 + \dot{m}t)a + \dot{m}v = 0$

$a = \dfrac{dv}{dt} = -\dfrac{\dot{m}v}{m_0 + \dot{m}t}$

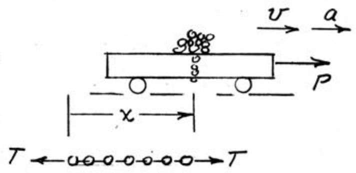
$\displaystyle\int_{v_0}^{v} \frac{dv}{v} = -\int_0^t \frac{\dot{m}}{m_0 + \dot{m}t}\,dt \Rightarrow v = \frac{dx}{dt} = \frac{m_0 v_0}{m_0 + \dot{m}t}$

Then $\displaystyle\int_0^x dx = \int_0^t \frac{m_0 v_0}{m_0 + \dot{m}t}\,dt \Rightarrow x = \frac{m_0 v_0}{\dot{m}}\ln\left(\frac{m_0 + \dot{m}t}{m_0}\right)$

With $t = \frac{32}{4} = 8$ s, $x = \dfrac{25(10^3)(1.2)}{4(10^3)}\ln\left(\dfrac{25 + 4(8)}{25}\right)$

$\underline{x = 6.18\ \text{m}}$

4/85 $m = m_0 - \rho x$



T is not transmitted to cart so $\Sigma F = ma$

$$P = (m_0 - \rho x)a$$

$$a = \frac{P}{m_0 - \rho x}$$

$v\,dv = a\,dx$, $\displaystyle\int_{v_0}^{v} v\,dv = \int_0^x \frac{P\,dx}{m_0 - \rho x}$

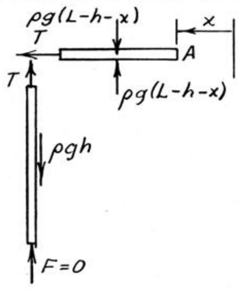
$$\frac{v^2}{2}\Big]_{v_0}^{v} = -\frac{P}{\rho}\ln(m_0 - \rho x)\Big]_0^x = \frac{P}{\rho}\ln\frac{m_0}{m_0 - \rho x}$$

Thus $v = \sqrt{v_0^2 + \dfrac{2P}{\rho}\ln\dfrac{m_0}{m_0 - \rho x}}$

$T = m'\Delta v$, $T = \rho v(v)$, $\underline{T = \rho v^2}$

There is no reaction (R in Fig. 4/6) between departing links & cart, so $\dot m u$ is zero & $\Sigma F = ma$

4/86 Let x be the displacement of the chain & T be the tension in the chain at the corner.



Horiz. part $\Sigma F_x = ma_x$:

$$T = \rho(L-h-x)\ddot x$$

Vert. part $\Sigma F_y = ma_y$:

$$\rho g h - T = \rho h \ddot x$$

Eliminate T & get

$$\ddot x = \frac{gh}{L-x}$$

$\dot x\,d\dot x = \ddot x\,dx$: $\displaystyle\int_0^{v_1^2}\frac{1}{2}d(\dot x^2) = \int_0^{L-h}\frac{gh}{L-x}dx$

$$\frac{\dot x^2}{2}\Big|_{\dot x = 0}^{v_1} = -gh\ln(L-x)\Big|_0^{L-h},\quad \frac{v_1^2}{2} = gh\ln(L-x)\Big|_{L-h}^{0} = gh\ln\frac{L}{h}$$

(a) $\underline{v_1 = \sqrt{2gh\ln(L/h)}}$

(b) Free fall of end A gives $v_2^2 = v_1^2 + 2gh = 2gh\ln\frac{L}{h} + 2gh$

$\underline{v_2 = \sqrt{2gh(1+\ln[L/h])}}$

(c) Q = loss of potential energy since $\Delta T = 0$

$Q = \rho g h\frac{h}{2} + \rho g(L-h)h$, $\underline{Q = \rho g h(L - \frac{h}{2})\ \text{loss}}$

4/87 For airplane plus moving portion of chains

$$\Sigma F = 0 = m\dot v + \dot m u = (m + 2\rho\tfrac{x}{2})\dot v + [2\tfrac{d}{dt}(\rho\tfrac{x}{2})]v$$

$$-(m+\rho x)\frac{dv}{dt} = \rho v\frac{dx}{dt}\qquad \frac{dv}{v} = -\frac{\rho\,dx}{m+\rho x}$$

$$\int_{v_0}^{v}\frac{dv}{v} = -\int_0^x\frac{\rho\,dx}{m+\rho x};\quad \ln\frac{v}{v_0} = -\ln\frac{m+\rho x}{m},\quad \frac{v}{v_0} = \frac{m}{m+\rho x}$$

or $v = \dfrac{v_0}{1+\rho x/m}$ & for $x = 2L$, $\underline{v = \dfrac{v_0}{1+\frac{2\rho L}{m}}}$

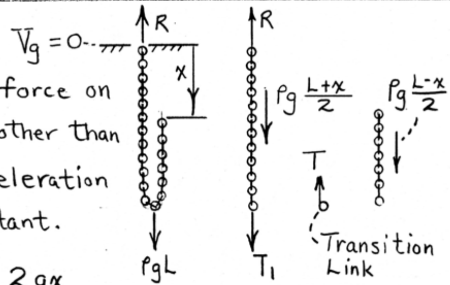
Also, $v = \dfrac{dx}{dt}$ so $\displaystyle\int_0^x(1+\frac{\rho x}{m})dx = \int_0^t v_0\,dt$

$$x + \frac{\rho x^2}{2m} = v_0 t,\quad x^2 + \frac{2m}{\rho}x - \frac{2mv_0 t}{\rho}$$

$$x = -\frac{m}{\rho} \pm \frac{1}{2}\sqrt{\frac{4m^2}{\rho^2} + \frac{8mv_0t}{\rho}},\quad \underline{x = \frac{m}{\rho}\left[\sqrt{1+\frac{2v_0t\rho}{m}} - 1\right]}$$

for + root

▸4/88



There is no force on moving part other than weight so acceleration $\ddot x = g$ = constant.

Thus, $v^2 = 2gx$

$$T_1 = m'\Delta v:\quad T_1 = \left[\frac{d}{dt}\left(\rho\frac{L-x}{2}\right)\right][0 - v]$$

$$= \frac{1}{2}\rho v^2 = \frac{1}{2}\rho(2gx) = \underline{\rho g x}$$

Equilibrium of links at rest:

$$\Sigma F_x = 0:\quad T_1 + \rho g\frac{L+x}{2} - R = 0$$

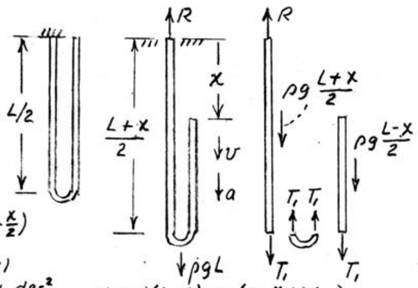
$$\Rightarrow\quad \underline{R = \frac{1}{2}\rho g(L+3x)}$$

$$\text{Loss } Q = |V_{g_1} - V_{g_2}| = \left|\rho g L\left(-\frac{L}{4}\right) - \rho g L\left(-\frac{L}{2}\right)\right|$$

$$= \underline{\frac{1}{4}\rho g L^2}$$

When $x \to L$, $v \to \infty$. The loss of potential energy equals the gain in kinetic energy, so the gain is concentrated in the last element and is lost during impact when the last element is abruptly brought to rest.

▸4/89



$$\Delta V_g = \rho g\frac{L-x}{2}(-x) + \rho g\frac{x}{2}\left(-\frac{x}{2}\right) = -\frac{1}{2}\rho g x\left(L - \frac{x}{2}\right)$$

$$\Delta T = \frac{1}{2}\rho\frac{L-x}{2}v^2$$

$$\Delta V_g + \Delta T = 0$$

$$\frac{1}{4}\rho(L-x)v^2 = \frac{1}{2}\rho g x\left(L - \frac{x}{2}\right)$$

$$v^2 = 2gx\frac{L - x/2}{L-x}\quad ---(1)$$

$$v\,dv = a\,dx\ \text{ so }\ a = \frac{1}{2}\frac{dv^2}{dx} = g\frac{(L-x)(L-x) - x(L - x/2)(-1)}{(L-x)^2}$$

$$\underline{a = g\left(1 + \frac{x(L - x/2)}{(L-x)^2}\right)}\quad ---(2)$$

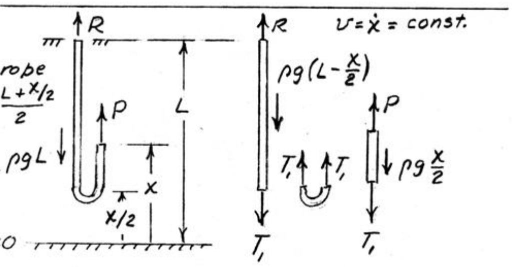
For entire rope

$$\Sigma F_x = \dot G_x:\quad \rho g L - R = \frac{d}{dt}\left(\rho\frac{L-x}{2}v\right) = \frac{\rho}{2}\left[(L-x)a - v^2\right]$$

Sub. (1) & (2) & get $\quad \underline{R = \frac{1}{2}\rho g\left[(L+x) + \frac{x(L - x/2)}{L-x}\right]}$

Equil of fixed part gives $\quad \underline{T_1 = \frac{1}{2}\rho g x(L - x/2)/(L-x)}$

▶4/90



For entire rope
$V_g = \rho g(L-\frac{x}{2})\frac{L+x/2}{2} + \rho g\frac{x}{2}\frac{3x}{4}$
$= \frac{\rho g}{2}(L^2+\frac{x^2}{2})$

Entire rope: $\Sigma F_x = \dot G_x$: $R+P-\rho gL = \frac{d}{dt}(\rho\frac{x}{2}v) = \frac{1}{2}\rho v^2$ ---(1)

Work-energy: $dU' = dT + dV_g$: $P\,dx = d(\frac{1}{2}\rho\frac{x}{2}v^2) + d\{\frac{\rho g}{2}(L^2+\frac{x^2}{2})\}$

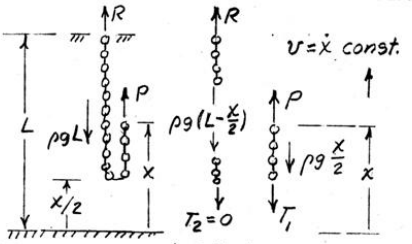
$= \frac{1}{4}\rho v^2 dx + \frac{1}{2}\rho g x\,dx$

$P = \frac{1}{4}\rho v^2 + \frac{1}{2}\rho g x$ ---- (2)

Sub. (2) into (1) $R = \frac{1}{4}\rho v^2 + \rho g(L - x/2)$

Equil. of part not moving $\Sigma F_y = 0$: $R - \rho g(L-\frac{x}{2}) - T_1 = 0$, $T_1 = \frac{1}{4}\rho v^2$

▶4/91



Entire chain: $\Sigma F_x = \dot G_x$: $R+P-\rho gL = \frac{d}{dt}(\rho\frac{x}{2}v)$

$R+P = \frac{1}{2}\rho v^2 + \rho gL$ (same as for rope)

$T_1 = m'\Delta v$: $T_1 = \frac{d}{dt}(\rho\frac{x}{2})v$, $T_1 = \frac{1}{2}\rho v^2$

Moving part: $\Sigma F_x = 0$: $P - \rho g\frac{x}{2} - \frac{1}{2}\rho v^2 = 0$, $P = \frac{1}{2}\rho(v^2+gx)$

Equil. of part not moving $\Sigma F_x = 0$: $R - \rho g(L-\frac{x}{2}) = 0$, $R = \rho g(L-\frac{x}{2})$

$U_{1-2} = \Delta T + \Delta V_g + Q$, $\Delta T = \frac{1}{2}\rho\frac{x}{2}v^2$, $\Delta V_g = \rho g\frac{x}{2}\frac{x}{2}$

$U = Px = \frac{1}{2}\rho(v^2x + gx^2)$ so $Q = \frac{1}{4}\rho x(v^2+gx)$

▶4/92 For falling part $\Sigma F = m\dot v + \dot m u$

where $\Sigma F = \rho g x$, $m = \rho x$, $\dot m = \rho v$, $u = v = \dot x$

Thus $\rho g x = \rho x\dot v + \rho v\dot x$, $gx\,dt = x\,dv + v\,dx$

or $gx\,dt = d(xv)$; $gx^2v\,dt = xv\,d(xv)$

so $gx^2dx = \frac{1}{2}d[(xv)^2]$ & $g\int_0^x x^2dx = \frac{1}{2}\int_0^{(xv)^2} d[(xv)^2]$

$\frac{gx^3}{3} = \frac{1}{2}(xv)^2$, $v = \sqrt{\frac{2gx}{3}}$

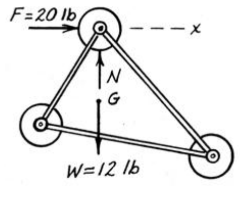
$a = \dot v = \sqrt{\frac{2g}{3}}\,\frac{1}{2}x^{-1/2}\dot x = \sqrt{\frac{2g}{3}}\frac{1}{2\sqrt{x}}\sqrt{\frac{2gx}{3}}$, $a = \frac{g}{3}$ constant

$Q = -\Delta V_g - \Delta T = +\frac{\rho gL^2}{2} - \frac{\rho L}{2}v^2_{x=L} = \frac{\rho gL^2}{2} - \frac{\rho gL^2}{3} = \frac{\rho gL^2}{6}$

4/93

$\Sigma F_x = m\bar a_x$: $20 = \frac{12}{32.2}\bar a_x$

$\bar a_x = \bar a = 53.7$ ft/sec²



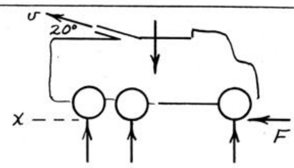
4/94 For the system, $\Sigma M_o = \dot H_o = 0$, so H_o is conserved:

$$\frac{2}{16}(1000)\frac{10}{12} = \frac{2}{16}\left(\frac{10}{12}\right)^2\omega + 3\left(\frac{20}{12}\right)^2\omega$$

$\omega = 12.37$ rad/sec

A large horizontal force is exerted on the rod by the bearing so that $\Sigma F \neq 0$ in the horizontal direction. Thus $\dot G_x \neq 0$ and the linear momentum of the bullet-pendulum system is not conserved.

4/95



$F = m'\Delta v_x$: $\Delta v_x = v\cos 20°$

$Q = Av$: $\frac{1400\times231}{1728}\frac{1}{60}\frac{ft^3}{sec}$

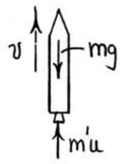
$= \frac{\pi\times2^2/4}{144}v$, $v = 143.0$ ft/sec

$\Delta v_x = 143.0\cos 20° - 0 = 134.4$ ft/sec

$m' = \rho Q = \frac{62.4}{32.2}\frac{1400\times231}{1728\times60} = 6.04$ lb-sec/ft

$F = 6.04(134.4) = 812$ lb

4/96 $m = m_0 - m't$



$\Sigma F = ma$: $m'u - (m_0-m't)g = (m_0-m't)a$

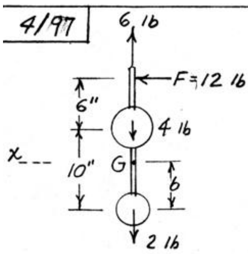
$a = \frac{dv}{dt} = \frac{m'u}{m_0-m't} - g$

$\int_0^v dv = \int_0^t \frac{m'u}{m_0-m't}dt - \int_0^t g\,dt$

$v = -u\ln(m_0-m't)\Big|_0^t - gt\Big|_0^t$

$v = u\ln\left(\frac{m_0}{m_0-m't}\right) - gt$

4/97



$\Sigma F_x = m\bar a_x$; $12 = \frac{4+2}{32.2}\bar a$

$\bar a = 64.4$ ft/sec²

$4(10-b) = 2b$, $b = 6.67$ in.

$H_G = \Sigma mr^2\dot\theta = \frac{4(3.33)^2 + 2(6.67)^2}{32.2(12)^2}\dot\theta$

$= 0.0288\dot\theta$ lb-ft-sec

$\Sigma M_G = \dot H_G$; $12\frac{(6+3.33)}{12} = 0.0288\ddot\theta$

$\ddot\theta = \frac{9.33}{0.0288} = 325$ rad/sec²

4/98

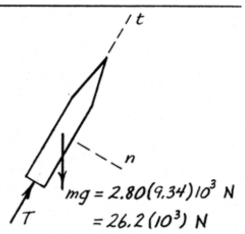
$T = m'u = 120(640) = 76.8(10^3)$ N

$\Sigma F_t = ma_t$: $76.8(10^3) - 26.2(10^3)\cos 30° = 2.80(10^3)a_t$

$\underline{a_t = 19.34 \text{ m/s}^2}$

$\Sigma F_n = ma_n$: $26.2(10^3)\sin 30° = 2.80(10^3)a_n$

$\underline{a_n = 4.67 \text{ m/s}^2}$



4/99

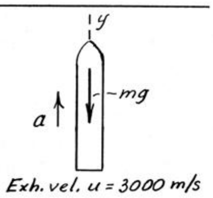
$\dot m = -m' = -5.2$ kg/s

$m = 200 + 1200 - 5.2t = 1400 - 5.2t$ kg

$\Sigma F = m\dot v + \dot m u$: $-mg = ma - 5.2(3000)$

$(1400 - 5.2t)(a + 8.70) = 15\,600$

$a = \dfrac{15\,600}{1400 - 5.2t} - 8.70$ m/s²



When $t = 60$ s, $a = \dfrac{15\,600}{1400 - 5.2(60)} - 8.70 = 14.34 - 8.70 = \underline{5.64 \text{ m/s}^2}$

Max. accel. occurs when $5.2t = 1200$, $\underline{t = 231 \text{ s}}$

$a_{max} = \dfrac{15\,600}{1400 - 5.2(231)} - 8.70 = 78.0 - 8.70 = \underline{69.3 \text{ m/s}^2}$

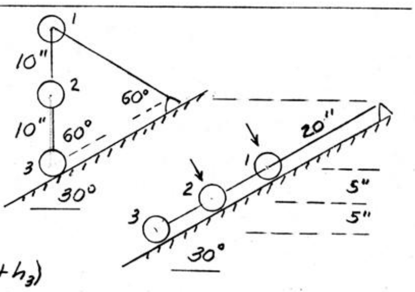
4/100

Vertical drop of spheres is

$h_1 = 20''$

$h_2 = 15''$

$h_3 = 10''$



For system

$\Delta V_g = -mg(h_1 + h_2 + h_3)$

$\Delta T = \frac{1}{2}m(v_1^2 + v_2^2 + 0) = \frac{1}{2}m(v^2 + [\frac{v}{2}]^2) = \frac{5}{8}mv^2$

$U' = \Delta T + \Delta V$: $0 = \frac{5}{8}mv^2 - mg(h_1 + h_2 + h_3)$

$v^2 = \frac{8}{5}(32.2)\dfrac{20 + 15 + 10}{12} = 193.2$ (ft/sec)²

$\underline{v = 13.90 \text{ ft/sec}}$

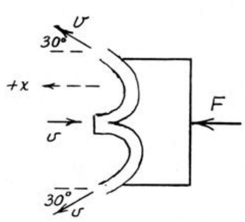
Potential energy loss goes into impact energy loss

4/101 $\Sigma F_x = m'\Delta v_x$

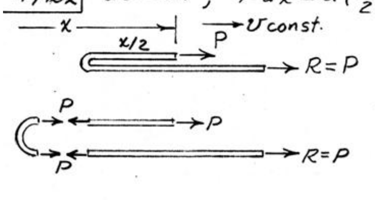
$m' = \rho A v = \dfrac{62.4}{32.2}\left(\dfrac{\pi}{4}\dfrac{(3/4)^2}{144}\right)120 = 0.713$ lb-sec/ft

$\Delta v_x = v\cos 30° - (-v) = v(1 + \cos 30°) = 120(1 + 0.866) = 224$ ft/sec

$F = 0.713 \times 224 = \underline{159.8 \text{ lb}}$



4/102 $dU = dT$; $P\,dx = d(\frac{1}{2}\rho\frac{x}{2}v^2) = \frac{1}{4}\rho v^2 dx$



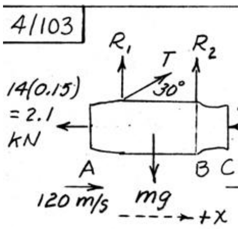
$\underline{P = \frac{1}{4}\rho v^2}$

or $\Sigma F_x = \dot G_x$

$P + R = \dfrac{d}{dt}(\rho\frac{x}{2}v)$

$R = \frac{1}{2}\rho v^2 - \frac{1}{4}\rho v^2 = \frac{1}{4}\rho v^2$

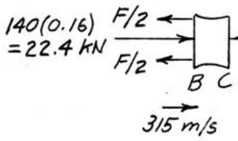
4/103



$\Sigma F_x = m'\Delta v_x$

$-2.1 - 0.84 + T\cos 30° = [31.6(600) - 30(120)]10^{-3}$

$\underline{T = 21.1 \text{ kN}}$

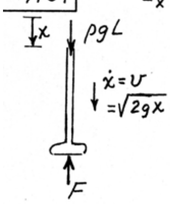


$\Sigma F_x = m'\Delta v_x$

$22.4 - 0.84 - F = 31.6(600 - 315)10^{-3}$

$\underline{F = 12.55 \text{ kN}}$

4/104 $G_x = \rho(L - x)\sqrt{2gx} = \rho\sqrt{2g}(Lx^{1/2} - x^{3/2})$



$\Sigma F_x = \dot G_x$ for entire system

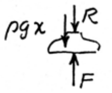
$\rho g L - F = \rho\sqrt{2g}\left(\frac{1}{2}Lx^{-1/2} - \frac{3}{2}x^{1/2}\right)\dot x$

$= \rho\sqrt{2g}\left(\frac{1}{2}L\sqrt{2g} - \frac{3}{2}\sqrt{2g}\,x\right)$

$= \rho g L - 3\rho g x$

So that $\underline{F = 3\rho g x}$

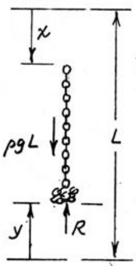
Alternatively



$R = m'\Delta v = \rho\dot x(\dot x) = \rho\dot x^2 = \rho(2gx) = 2\rho g x$

$\Sigma F_x = 0$; $2\rho g x + \rho g x - F = 0$, $\underline{F = 3\rho g x}$

4/105 Take entire chain as system (constant mass)



$\ddot x = g$, $\ddot y = a$, $G_x = \rho(L - x - y)\dot x - \rho(x + y)\dot y$

$\dot x = gt$, $\dot y = at$

$x = \frac{1}{2}gt^2$, $y = \frac{1}{2}at^2$

$\Sigma F_x = \dot G_x$; $\rho g L - R = \rho[(-\dot x - \dot y)\dot x + (L - x - y)\ddot x]$

$-\rho[(\dot x + \dot y)\dot y + (x + y)\ddot y]$

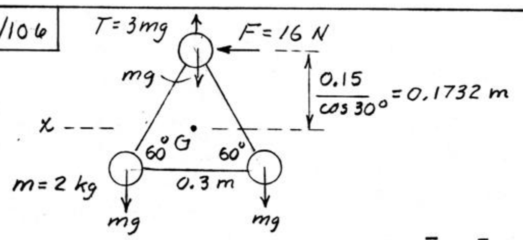
$\rho g L - R = \rho[-(\dot x + \dot y)^2 - (x + y)(\ddot x + \ddot y) + L\ddot x]$

$= \rho[-(a + g)^2t^2 - \frac{1}{2}(a + g)^2t^2 + Lg]$

$= -\frac{3}{2}\rho(a + g)^2t^2 + \rho g L$

So $\underline{R = \frac{3}{2}\rho(a + g)^2t^2}$

4/106



For system: $\Sigma F_x = m\bar a_x$: $16 = 3(2)\bar a$, $\bar a = 2.67 \frac{\text{m}}{\text{s}^2}$

$\Sigma M_G = \dot H_G$: $16(0.1732) = \dfrac{d}{dt}(3 \times 2 \times 0.1732^2\dot\theta)$

$\ddot\theta = \dfrac{16(0.1732)}{6(0.1732)^2} = \underline{15.40 \frac{\text{rad}}{\text{s}^2}}$

For top sphere

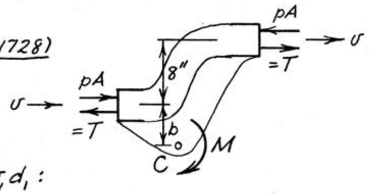
$a = \bar a + \bar r\ddot\theta = 2.67 + 0.1732(15.40) = \underline{5.33 \text{ m/s}^2}$

4/107

$Q = Av$

$v = Q/A = \dfrac{5000 \times 231/(60\times1728)}{\dfrac{\pi\times4^2}{4}\ \dfrac{1}{144}}$

$= 127.7$ ft/sec



$\Sigma M_o = \dot{H}_o = m'v_2 d_2 - m'v_1 d_1$:

$m' = \frac{\mu}{g} Q = \frac{62.4}{32.2}\ \frac{5000\times231}{60\times1728} = 21.6$ lb-sec/ft

so $M = 21.6(127.7)(8/12) = \underline{1837 \text{ lb-ft}}$

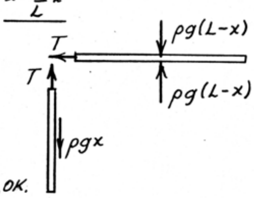
4/108 System is conservative, so $\Delta V_g + \Delta T = 0$.

$-\rho g x \frac{x}{2} + \frac{1}{2}\rho L \dot{x}^2 = 0,\ \frac{g}{L}x^2 = \dot{x}^2,\ \dot{x} = \sqrt{\frac{g}{L}}\,x$

(a) accel $a = \ddot{x} = \sqrt{\frac{g}{L}}\,\dot{x} = \sqrt{\frac{g}{L}}\sqrt{\frac{g}{L}}\,x$, so $\underline{a = \frac{g}{L}x}$

(b) $\Sigma F = ma$: $T = \rho(L-x)\frac{g}{L}x$

$\underline{T = \rho g x(1 - \frac{x}{L})}$

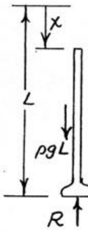


Check from vertical part

$\rho g x - T = \rho x \frac{g}{L}x,\ T = \rho g x(1 - \frac{x}{L})$, OK.

(c) $v\,dv = a_x\,dx$: $\int_0^v v\,dv = \frac{g}{L}\int_0^L x\,dx,\ \frac{v^2}{2} = \frac{g}{L}\frac{L^2}{2},\ \underline{v = \sqrt{gL}}$

▸4/109 For entire rope of constant mass



$\Sigma F_x = \dot{G}_x$; $\rho g L - R = \frac{d}{dt}(\rho[L-x]\dot{x} + 0)$ ----(1)

$\Delta T + \Delta V_g = 0$; $\frac{1}{2}\rho(L-x)\dot{x}^2 = \rho g x(L - \frac{x}{2})$

$\dot{x}^2 = \frac{x(2L-x)}{L-x}g$ or $\dot{x}(L-x) = \frac{x}{\dot{x}}(2L-x)g$

Substitute into (1) & get

$\rho g L - R = \frac{d}{dt}\left(\frac{x(2L-x)}{\dot{x}}\rho g\right) = \rho g \frac{\dot{x}[2L-2x]\dot{x} - x[2L-x]\ddot{x}}{\dot{x}^2}$

$= 2\rho g(L-x) - \rho g x(2L-x)\frac{\ddot{x}}{\dot{x}^2}$

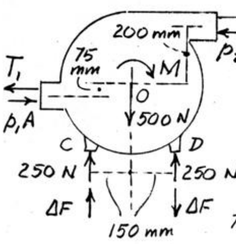
Differentiate $\dot{x}^2$ & get $\ddot{x} = \left[1 + \frac{x(L-\frac{x}{2})}{(L-x)^2}\right]g$. Substitute

& get $\rho g L - R = 2\rho g(L-x) - \rho g x(2L-x)\left[1 + \frac{x(L-\frac{x}{2})}{(L-x)^2}\right]g/\dot{x}^2$

Substitute $\dot{x}^2$, simplify, & solve for R

& get $\underline{R = \rho g x \frac{4L-3x}{2(L-x)}}$, (Less than $R_{a=0}$ of Prob. 4/105 with $x < \frac{2L}{3}$)

▸4/110 $T_1 = p_1 A,\ T_2 = p_2 A$



Power $P = M\omega$

$M = \frac{40(10^3)}{900(2\pi/60)} = 424$ N·m

$\Sigma M_o = m'(v_2 d_2 - v_1 d_1)$

$424 + 0.3\Delta F = \frac{20}{60}(1000)\left[18(0.2) - (18)(-0.075)\right]$

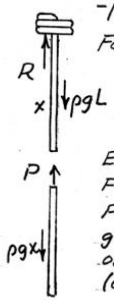
$\Delta F = \frac{1650 - 424}{0.3} = 4090$ N

Thus $C = 250 + 4090 = \underline{4340 \text{ N}}$ (up)

$D = 4090 - 250 = \underline{3840 \text{ N}}$ (down)

▸4/111 Entire system is conservative so $\Delta V_g + \Delta T = 0$

$-\rho g x \frac{x}{2} + \frac{1}{2}\rho x v^2 = 0,\ \underline{v = \sqrt{gx}},\ a = \dot{v} = \frac{1}{2}\frac{\sqrt{g}}{\sqrt{x}}\sqrt{gx} = g/2$



For entire system $\Sigma F_x = \dot{G}_x$

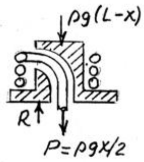
$\rho g L - R = \frac{d}{dt}(\rho x v) = \frac{d}{dt}(\rho\sqrt{g}\,x^{3/2}) = \frac{3}{2}\rho g x$

Thus $R = \rho g L - \frac{3}{2}\rho g x$, $\underline{R = \rho g(L - \frac{3}{2}x)}$

Explanation of $R = 0$ for $x = 2L/3$:

For vertical section $\Sigma F_x = m a_x$; $\rho g x - P = \rho x \frac{g}{2}$

$P = \rho g x/2$. For idealized flow, a frictionless guide must be introduced. Guide & coil of length $L-x$ is isolated. $\Sigma F_x = m'\Delta v_x$ (or $\Sigma F_x = m(0) + \dot{m}u$) gives



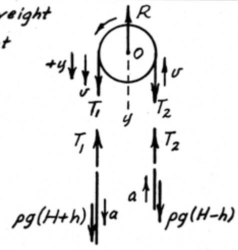
$\rho g x/2 + \rho g(L-x) - R = \rho v(v) = \rho(gx)$

$R = \rho g(\frac{x}{2} + L - x - x) = \rho g(L - 3x/2)$

& $R = 0$ when $x = 2L/3$ as before.

When $x > 2L/3$, the rate of momentum change requires more force than $P + \rho g(L-x)$ so R reverses.

▸4/112 With neglect of mass of pulley & weight of small portion of chain in contact with pulley



$\Sigma M_o \approx 0$ so $T_1 = T_2 = T$

$\Sigma F = ma$ for chains

$\rho g(H+h) - T = \rho(H+h)a$ --- (1)

$T - \rho g(H-h) = \rho(H-h)a$ --- (2)

Eliminate T from (1) & (2) & get

$a = \frac{h}{H}g$ (3)

So $T = \rho(H-h)\frac{h}{H}g + \rho g(H-h)$, $T = \rho g(H - \frac{h^2}{H})$ (4)

Pulley & chain on it: steady flow gives

$\Sigma F_y = m'\Delta v_y$: $2T - R = \rho v(v - [-v])$, $R = 2T - 2\rho v^2$ (5)

But $\int_0^v v\,dv = \int_0^h \frac{g}{H}h\,dh,\ v^2 = \frac{g}{H}h^2,\ \underline{v = \sqrt{\frac{g}{H}}\,h}$ (6)

Substitute (4) & (6) into (3) & get

$R = 2\rho g(H - \frac{h^2}{H}) - 2\rho\frac{g}{H}h^2,\ \underline{R = 2\rho g(H - \frac{2h^2}{H})}$

5/1 $\alpha = \dfrac{\omega_2 - \omega_1}{t} = \dfrac{900-300}{6/60} = 6000 \text{ rev/min}^2$

$\omega_2^2 = \omega_1^2 + 2\alpha\theta, \quad \theta = N = \dfrac{\omega_2^2 - \omega_1^2}{2\alpha}$

$= \dfrac{(900)^2 - (300)^2}{2(6000)} = \underline{60 \text{ rev}}$

5/2 (a) $\underline{v}_A = \underline{\omega} \times \underline{r}_{A/O} = -6\underline{k} \times 45\underline{j}$

$= \underline{270\underline{i} \text{ mm/s}}$

$\underline{a}_A = \underline{\alpha} \times \underline{r}_{A/O} - \omega^2 \underline{r}_{A/O} = 4\underline{k} \times 45\underline{j} - 6^2(45\underline{j})$

$= \underline{-180\underline{i} - 1620\underline{j} \text{ mm/s}^2}$

(b) $\underline{v}_B = \underline{\omega} \times \underline{r}_{B/O} = -6\underline{k} \times (-30\underline{i} + 45\underline{j})$

$= \underline{270\underline{i} + 180\underline{j} \text{ mm/s}}$

$\underline{a}_B = \underline{\alpha} \times \underline{r}_{B/O} - \omega^2 \underline{r}_{B/O}$

$= 4\underline{k} \times (-30\underline{i} + 45\underline{j}) - 6^2(-30\underline{i} + 45\underline{j})$

$= \underline{900\underline{i} - 1740\underline{j} \text{ mm/s}^2}$

5/3 $\omega = 12 - 3t^2$; when $\omega = 0$, $t^2 = 4$, $t = 2$ s

$\int_0^{\Delta\theta} d\theta = \int_0^t \omega\, dt$; $\Delta\theta = \int_0^3 (12-3t^2)\,dt = [12t - t^3]_0^3$

$= \underline{9 \text{ rad}}$

$\theta_1 = \int_0^2 (12-3t^2)\,dt = [12t - t^3]_0^2 = 16 \text{ rad (CW)}$

$\theta_2 = \int_2^3 (12-3t^2)\,dt = [12t - t^3]_2^3 = -7 \text{ rad (CCW)}$

The total number of turns is

$N = (16+7)/2\pi = \underline{3.66 \text{ rev}}$

5/4 Let $\underline{k}$ be a unit vector out of paper.

(a) $\underline{v}_A = \underline{\omega} \times \underline{r}_{A/O} = 3\underline{k} \times (-0.4\underline{e}_n) = \underline{1.2\underline{e}_t \text{ m/s}}$

$\underline{a}_A = \underline{\alpha} \times \underline{r}_{A/O} - \omega^2 \underline{r}_{A/O} = -14\underline{k} \times (-0.4\underline{e}_n) - 3^2(-0.4\underline{e}_n)$

$= \underline{-5.6\underline{e}_t + 3.6\underline{e}_n \text{ m/s}^2}$

(b) $\underline{v}_B = \underline{\omega} \times \underline{r}_{B/O} = 3\underline{k} \times (-0.4\underline{e}_n + 0.1\underline{e}_t)$

$= \underline{1.2\underline{e}_t + 0.3\underline{e}_n \text{ m/s}}$

$\underline{a}_B = \underline{\alpha} \times \underline{r}_{B/O} - \omega^2 \underline{r}_{B/O}$

$= -14\underline{k} \times (-0.4\underline{e}_n + 0.1\underline{e}_t) - 3^2(-0.4\underline{e}_n + 0.1\underline{e}_t)$

$= \underline{-6.5\underline{e}_t + 2.2\underline{e}_n \text{ m/s}^2}$

5/5 For v constant $a_t = 0$ & $a = a_n = v^2/r$

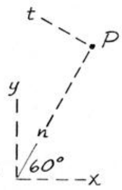
$\left(\dfrac{v^2}{r}\right)_A = \dfrac{2}{3}\left(\dfrac{v^2}{3}\right)_B$, $\underline{r = 4.5 \text{ in.}}$

5/6 For $\theta = 90°$, $\underline{a} = -a_t\underline{i} - a_n\underline{j}$ so $a_t = r\alpha = 1.8 \text{ m/s}^2$,

$\alpha = \dfrac{1.8}{0.3} = \underline{6 \text{ rad/s}^2}$

& $a_n = r\omega^2 = 4.8 \text{ m/s}^2$, $\omega = \sqrt{4.8/0.3} = \underline{4 \text{ rad/s}}$

5/7



$a_x = -3.02 \text{ m/s}^2$

$a_y = -1.624 \text{ m/s}^2$

$a_t = 3.02 \sin 60° - 1.624 \cos 60° = 1.803 \frac{\text{m}}{\text{s}^2}$

$a_n = 3.02 \cos 60° + 1.624 \sin 60° = 2.92 \frac{\text{m}}{\text{s}^2}$

$a_t = r\alpha$: $\alpha = 1.803/0.3 = \underline{6.01 \text{ rad/s}^2}$

$a_n = r\omega^2$: $\omega^2 = 2.92/0.3 = 9.72 \text{ (rad/s)}^2$, $\underline{\omega = 3.12 \text{ rad/s}}$

5/8 $\alpha = -k\omega^2 = \omega \dfrac{d\omega}{d\theta}$

$-k\int_{\theta_0}^{\theta} d\theta = \int_{\omega_0}^{\omega} \dfrac{d\omega}{\omega}$

$-k(\theta - \theta_0) = \ln\left(\dfrac{\omega}{\omega_0}\right) \Rightarrow \omega = \omega_0 e^{-k(\theta-\theta_0)}$

When $\omega = \dfrac{\omega_0}{3}$: $\dfrac{\omega_0}{3} = \omega_0 e^{-k(\theta-\theta_0)}$

With $k = 0.1$, $\underline{(\theta - \theta_0) = 10.99 \text{ rad}}$

Now set $\alpha = -k\omega^2 = \dfrac{d\omega}{dt}$

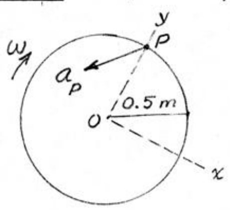
$-k\int_0^t dt = \int_{\omega_0}^{\omega} \dfrac{d\omega}{\omega^2}$

$-kt = -\left(\dfrac{1}{\omega} - \dfrac{1}{\omega_0}\right)$

When $\omega = \omega_0/3$, with $k = 0.1$: $t = \dfrac{20}{\omega_0}$

With $\omega_0 = 12$ rad/s, $\underline{t = 1.667 \text{ s}}$

5/9 For point P, $\underline{a}_P = -3\underline{i} - 4\underline{j} \text{ m/s}^2$



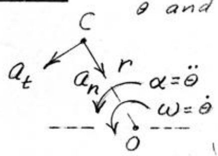
$a_n = r\omega^2$, $4 = 0.5\omega^2$, $\omega = \sqrt{8} \frac{\text{rad}}{\text{s}}$

$\underline{\underline{\omega} = -\sqrt{8}\,\underline{k} \text{ rad/s}}$

$a_t = r\alpha$, $3 = 0.5\alpha$, $\alpha = 6 \text{ rad/s}^2$

$\underline{\underline{\alpha} = 6\underline{k} \text{ rad/s}^2}$

5/10 All lines including OC have the same $\dot\theta$ and $\ddot\theta$; $r = \frac{2}{3}(0.150)\frac{\sqrt{3}}{2} = 0.0866$ m



$a_n = r\omega^2$, $\dot\theta = \omega = \sqrt{a_n/r}$

$= \sqrt{80/0.0866}$

$= \underline{30.4 \text{ rad/s}}$

$a_t = r\alpha$, $\ddot\theta = \alpha = a_t/r$

$= 30/0.0866$

$= \underline{346 \text{ rad/s}^2}$

5/11 $\omega_{av} = \dfrac{\Delta\theta}{\Delta t}$ where $\Delta\theta = \pi/2$ rad

$\Delta t = \dfrac{\Delta s}{v} = \dfrac{40}{10} = 4$ sec

so $\omega_{av} = \dfrac{\pi/2}{4} = \underline{0.393 \text{ rad/sec}}$

5/12 $v_P = \omega \times r = 2k \times [0.5i + 0.2j + 0.050k]$

$= -0.4i + j \quad m/s$

$a_P = \alpha \times r + \omega \times (\omega \times r)$

$= -3k \times [0.5i + 0.2j + 0.050k]$

$+ 2k \times [2k \times (0.5i + 0.2j + 0.050k)]$

$= -1.4i - 2.3j \quad m/s^2$

Note that r could have been taken as $0.5i + 0.2j$ m

The magnitudes of the above results are

$v_P = 1.077$ m/s and $a_P = 2.69$ m/s^2.

These magnitudes check with

$v_P = r_{xy}\,\omega = \sqrt{0.5^2+0.2^2}\,(2) = 1.077$ m/s^2 ✓

and $a_P = \sqrt{a_t^2 + a_n^2} = \sqrt{(r_{xy}\alpha)^2 + (r_{xy}\omega^2)^2}$

$= \sqrt{0.5^2+0.2^2}\,\sqrt{3^2+2^4} = 2.69$ m/s^2 ✓

5/13 $\omega = 40(\frac{3}{5}j + \frac{4}{5}k) = 8(3j + 4k)$ rad/sec

$r = 15i + 16j - 12k$ in.

$$v = \omega \times r = 8\begin{vmatrix} i & j & k \\ 0 & 3 & 4 \\ 15 & 16 & -12 \end{vmatrix} = -800i + 480j - 360k \text{ in./sec}$$

$= 40(-20i + 12j - 9k)$ in./sec

$a = \dot\omega \times r + \omega \times v$

$$= 0 + 8\begin{vmatrix} i & j & k \\ 0 & 3 & 4 \\ -800 & 480 & -360 \end{vmatrix} = 800(-30i - 32j + 24k)\ \frac{\text{in.}}{\text{sec}^2}$$

$= 1600(-15i - 16j + 12k)$ in./sec^2

$r = \sqrt{15^2+16^2+12^2} = 25$ in.

$v = r\omega = 25(40) = 1000$ in./sec ; $|v| = 40\sqrt{20^2+12^2+9^2} = 40(25) = 1000$ in./sec

$a_n = r\omega^2 = 25(40)^2 = 40(10^3)$ in./sec^2 ; $|a| = 1600\sqrt{15^2+16^2+12^2}$

$= 1600(25) = 40(10^3)$ in./sec^2

(Checks)

5/14 $\omega_{OA} = \omega_{BC} = -6k$ rad/s

$r_A = 0.3i + 0.28j$ m

$v_A = \omega \times r_A = -6k \times (0.3i + 0.28j) = -1.8j + 1.68i$ m/s

$v_A = 1.68i - 1.8j$ m/s

$a_A = \dot\omega \times r_A + \omega \times v_A = 0 + (-6k) \times (1.68i - 1.8j)$

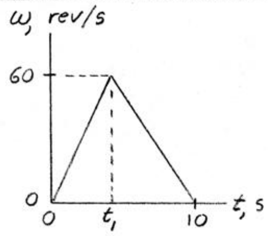
$= -10.08j - 10.8i$

$a_A = -10.8i - 10.08j$ m/s^2

5/15 $N = \Delta\theta = \int_0^{10} \omega\, dt$ = area

$= \frac{1}{2}(10)(60) = 300$ rev

(independent of t_1)

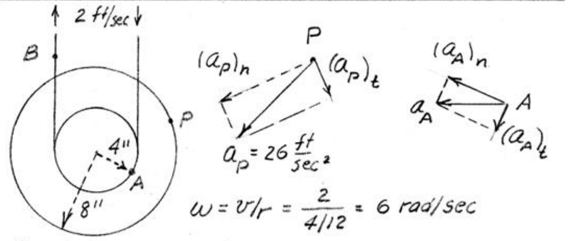


5/16 At B, $v = \frac{50}{30}44 = 73.3$ ft/sec, $r = 180 - \frac{18}{12} = 178.5$ ft

$\omega = v/r = 73.3/178.5 = 0.411$ rad/sec

Between A & B $\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{30}{180}\pi/1.52 = 0.344$ rad/sec

5/17



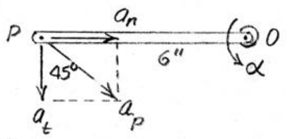
$\omega = v/r = \frac{2}{4/12} = 6$ rad/sec

$(a_P)_n = r\omega^2 = \frac{8}{12}6^2 = 24$ ft/sec^2

$(a_P)_t = \sqrt{26^2 - 24^2} = 10$ ft/sec^2, $\alpha = \frac{a_t}{r} = \frac{10}{8/12} = 15$ rad/sec^2

$a_B = (a_A)_t = \frac{4}{8}(a_P)_t = \frac{4}{8}10 = 5$ ft/sec^2

5/18



$\alpha = \frac{600(2\pi)}{60}\,\frac{1}{2} = 10\pi$ rad/sec^2

$a_t = r\alpha = 6(10\pi) = 60\pi$ in./sec^2

$a_n = r\omega^2 = 60\pi$ in./sec^2 for $45°$

so $\omega^2 = 60\pi/6 = 10\pi$, $\omega = 5.60$ rad/s

$\omega = \omega_0 + \alpha t$: $5.60 = 0 + 10\pi t$, $t = 0.1784$ sec

5/19 $\Delta\theta = (30 - 0)2\pi = 60\pi$ rad

$\alpha = 10 + k\theta$, $20 = 10 + 60\pi k$, $k = \frac{1}{6\pi}$

so $\alpha = 10 + \frac{\theta}{6\pi}$

$\int_{\omega_0}^{90} \omega\, d\omega = \int_0^{60\pi} (10 + \frac{\theta}{6\pi})\, d\theta$, $(90)^2 - \omega_0^2 = 2\left[10\theta + \frac{\theta^2}{12\pi}\right]_0^{60\pi}$

$\omega_0^2 = 8100 - 2[600\pi + 300\pi] = 2445$, $\omega_0 = 49.4$ rad/s

5/20 (a) $\alpha = -0.05\omega = \frac{d\omega}{dt}$

$-0.05\,dt = \frac{d\omega}{\omega}$

$-0.05\int_0^t dt = \int_{\omega_0}^{\omega} \frac{d\omega}{\omega}$

$-0.05t = \ln\left(\frac{\omega}{\omega_0}\right)$

$\Rightarrow \omega = \omega_0 e^{-0.05t}$, $\omega = 100e^{-0.05(10)} = 60.7$ rad/s

(b) $\alpha = -0.05\omega = \omega\frac{d\omega}{d\theta}$

$-0.05\,d\theta = d\omega$

$-0.05\int_0^\theta d\theta = \int_{\omega_0}^{\omega} d\omega$

$-0.05\theta = \omega - \omega_0$

$\omega = \omega_0 - 0.05\theta$, $\omega = 100 - 0.05(10 \cdot 2\pi)$

$= 96.9$ rad/s

5/21

$\omega\, d\omega = \alpha\, d\theta$, $\frac{d\omega}{d\theta} = k$, so $\frac{\alpha}{\omega} = k$,

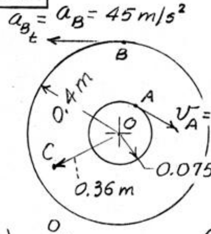
$\frac{d\omega}{dt}\frac{1}{\omega} = k$, $\int_{\omega_0}^{\omega}\frac{d\omega}{\omega} = \int_0^t k\, dt \Rightarrow \omega = \omega_0 e^{kt}$

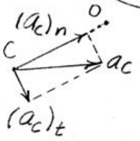
From $\frac{d\theta}{dt} = \omega_0 e^{kt}$, $\int_0^{\theta} d\theta = \int_0^t \omega_0 e^{kt} dt$

$\Rightarrow \theta = \frac{\omega_0}{k}(e^{kt}-1)$

$\alpha = \dot{\omega} = \omega_0 k e^{kt}$

5/22





$\omega = v/r = \frac{1.5}{0.075} = 20$ rad/s

$\alpha = a_t/r = \frac{45}{0.4} = 112.5$ rad/s²

$(a_C)_n = r\omega^2 = 0.36(20)^2 = 144$ m/s²

$(a_C)_t = r\alpha = 0.36(112.5) = 40.5$ m/s²

$a_C = \sqrt{(144)^2 + (40.5)^2} = 149.6$ m/s²

5/23

$\omega = v_A/r_A = \frac{10}{8/12} = 15$ rad/sec, $\underline{\omega} = 15\underline{k}$ rad/sec

$\alpha = (a_A)_t/r_A = \frac{24}{8/12} = 36$ rad/sec², $\underline{\alpha} = -36\underline{k}$ rad/sec²

$\underline{a}_B = \underline{\alpha}\times\underline{r}_B + \underline{\omega}\times(\underline{\omega}\times\underline{r}_B)$

$= -36\underline{k}\times\frac{6}{12}\underline{j} + 15\underline{k}\times(15\underline{k}\times\frac{6}{12}\underline{j}) = 18\underline{i} - 112.5\underline{j}$ ft/sec²

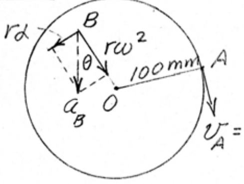
5/24

For gear A, $\Delta\omega = \int_2^6 \alpha_A\, dt$, $N_A = 2N_B$

$(N_A - 600)\frac{2\pi}{60} = \frac{4+8}{2}(6-2)$, $N_A = 600 + 229 = 829$ rev/min

so at $t = 6$ s, $N_B = \frac{829}{2} = 415$ rev/min

5/25

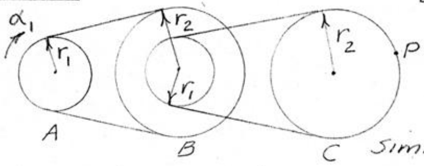


$\tan\theta = \frac{r\alpha}{r\omega^2} = \frac{\alpha}{\omega^2} = 0.6$

$v_A = r_A\omega$, $\omega = \frac{800}{100} = 8$ rad/s

Thus $\alpha = 0.6(8^2) = 38.4\ \frac{rad}{s^2}$

►5/26



$r_2\omega_C = r_1\omega_B$

$r_2\omega_B = r_1\omega_A$

so that $\omega_C = (\frac{r_1}{r_2})^2\omega_A$

Similarly $\alpha_C = (\frac{r_1}{r_2})^2\alpha_1$

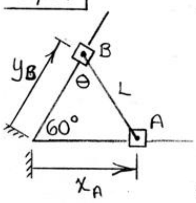
$\omega_A = \alpha_1 t$ so that $\omega_C = (\frac{r_1}{r_2})^2\alpha_1 t$

For P, $a_n = r_2\omega_C^2 = r_2[(\frac{r_1}{r_2})^2\alpha_1 t]^2$

$a_t = r_2\alpha_C = r_2(\frac{r_1}{r_2})^2\alpha_1$

$a_P = \sqrt{a_n^2 + a_t^2} = \frac{r_1^2}{r_2}\alpha_1\sqrt{1 + (\frac{r_1}{r_2})^4\alpha_1^2 t^4}$

5/27



$\frac{x_A}{\sin\theta} = \frac{L}{\sin 60°}$ (1)

$x_A = \frac{2}{\sqrt{3}}L\sin\theta$

$\dot{x}_A = v = \frac{2}{\sqrt{3}}L\cos\theta\,\dot{\theta}$ (2)

We need $\cos\theta$ in terms of x_A.

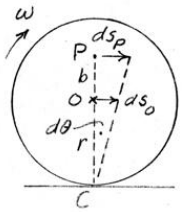
From (1): $\sin\theta = \frac{\sqrt{3}}{2}\frac{x_A}{L}$

Then $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1 - \frac{3}{4}\frac{x_A^2}{L^2}}$

(2): $\dot{\theta} = \omega = \frac{\sqrt{3}\,v}{2L\cos\theta} = \frac{\sqrt{3}\,v}{2L\sqrt{1-\frac{3}{4}\frac{x_A^2}{L^2}}}$

$(0 \le x_A \le L)$

5/28



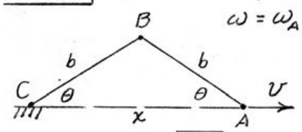
$ds_P = \overline{PC}\, d\theta$

$v_P = \frac{ds_P}{dt} = \overline{PC}\,\dot{\theta} = \overline{PC}\,\omega = (b+r)\omega$

$\frac{ds_P}{b+r} = \frac{ds_O}{r}$ so $\frac{v_P}{b+r} = \frac{v_O}{r}$

$v_P = \frac{b+r}{r}v_O$

5/29



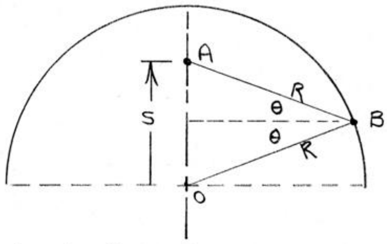
$x = 2b\cos\theta$, $\dot{x} = -2b\dot{\theta}\sin\theta$, $v = \dot{x}$

$\omega = \omega_{AB} = \dot{\theta}$ so $\omega = \frac{-v}{2b\sin\theta}$ CW

For $a = \ddot{x}$ const., $\dot{x}^2 = 2ax$, $v = \sqrt{2ax}$

so $\omega = \frac{\sqrt{2ax}}{2b\sqrt{1-\cos^2\theta}} = \frac{\sqrt{2ax}}{\sqrt{4b^2 - x^2}}$

5/30

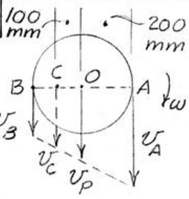


$s = 2(R\sin\theta)$, $\dot{s} = v = 2R\cos\theta\,\dot{\theta}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{s^2}{4R^2}}$

So $\dot{\theta} = \omega = \dfrac{v}{2R\sqrt{1-\frac{s^2}{4R^2}}}$

5/31



$v_A = 0.4$ m/s, $v_B = 0.2$ m/s

$\omega = \dfrac{v_A - v_B}{\overline{AB}} = \dfrac{0.4-0.2}{0.400} = 0.5 \frac{\text{rad}}{\text{s}}$ CW

$v_P = v_B + \overline{BO}\omega$

$= 0.2 + 0.200(0.5) = 0.3$ m/s

$v_C = v_B + \overline{BC}\omega$

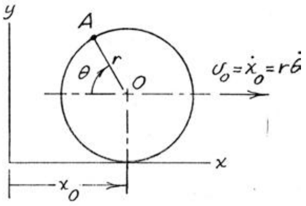
$= 0.2 + 0.100(0.5) = 0.25$ m/s

5/32

Coordinates of A are

$x = x_0 - r\cos\theta$

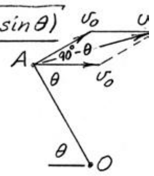
$y = r + r\sin\theta$



$\dot{x} = \dot{x}_0 + r\dot{\theta}\sin\theta = v_0(1+\sin\theta)$

$\dot{y} = r\dot{\theta}\cos\theta = v_0\cos\theta$

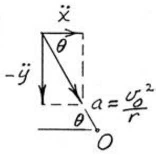
$v = \sqrt{\dot{x}^2+\dot{y}^2} = v_0\sqrt{(1+\sin\theta)^2+\cos^2\theta} = v_0\sqrt{2(1+\sin\theta)}$



$\ddot{x} = v_0\dot{\theta}\cos\theta = v_0\left(\frac{v_0}{r}\right)\cos\theta = \frac{v_0^2}{r}\cos\theta$

$\ddot{y} = -v_0\dot{\theta}\sin\theta = -v_0\left(\frac{v_0}{r}\right)\sin\theta = -\frac{v_0^2}{r}\sin\theta$

$a = \sqrt{\ddot{x}^2+\ddot{y}^2} = \frac{v_0^2}{r}\sqrt{\cos^2\theta+\sin^2\theta} = \frac{v_0^2}{r}$ toward O



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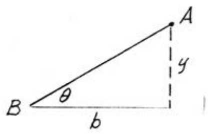


$y = b\tan\theta$

$\dot{y} = -v = b\dot{\theta}\sec^2\theta$

v = constant so that

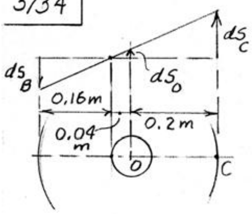
$0 = b\ddot{\theta}\sec^2\theta + 2b\dot{\theta}(\sec\theta\cdot\sec\theta\tan\theta)\dot{\theta}$

$0 = b\sec^2\theta(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)$

$\alpha = \ddot{\theta} = -2\dot{\theta}^2\tan\theta = -2\left(\frac{-v\cos^2\theta}{b}\right)^2\tan\theta = -\frac{2v^2}{b^2}\sin\theta\cos^3\theta$

or $\alpha = -\dfrac{v^2}{b^2}\sin 2\theta\cos^2\theta$

5/34



$v_B = \sqrt{2a_B s_B}$ for constant accel.

$v_B = \sqrt{2(0.2)(1.6)} = 0.8$ m/s

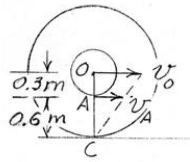
$\dfrac{ds_B}{0.16} = \dfrac{ds_O}{0.04} = \dfrac{ds_C}{0.24}$

$ds_O = \dfrac{0.04}{0.16}ds_B$, $a_O = \dfrac{0.04}{0.16}a_B$

$a_O = \dfrac{0.2}{4} = 0.05$ m/s^2 up

$v_C = \dfrac{0.24}{0.16}v_B = \frac{3}{2}(0.8) = 1.2$ m/s up

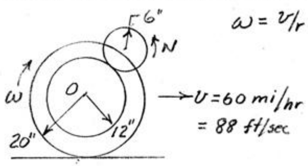
5/35



$v_O = \overline{OC}\omega = \dfrac{\overline{OC}}{\overline{AC}}v_A = \dfrac{0.9}{0.6}0.8 = 1.2$ m/s

$\omega = \dfrac{v_A}{\overline{AC}} = \dfrac{v_O}{\overline{OC}} = \dfrac{1.2}{0.9} = 1.333$ rad/s CW

5/36



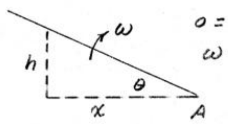
$\omega = v/r = \dfrac{88}{20/12}\dfrac{60}{2\pi} = 504$ rev/min

$N = \frac{12}{6}(504)$

$= 1008$ $\frac{\text{rev}}{\text{min}}$

5/37

$v_A = r\omega_0 = -\dot{x}$, $h = x\tan\theta$



$0 = \dot{x}\tan\theta + x\dot{\theta}\sec^2\theta$

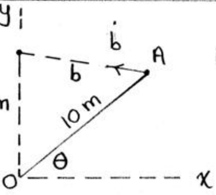
$\omega = \dot{\theta} = -\dfrac{\dot{x}}{x}\sin\theta\cos\theta$

$= -\dfrac{\dot{x}}{x}\dfrac{hx}{x^2+h^2}$

$\omega = \dfrac{rh\omega_0}{x^2+h^2}$

5/38

$\dot{b} = \frac{1}{4}v_B = \frac{1}{4}(3.2) = 0.8\ \frac{\text{ft}}{\text{sec}}$



$b^2 = 6^2 + 10^2 - 2(6)(10)\sin\theta$

$2b\dot{b} = -120\cos\theta\,\dot{\theta}$

$2\dot{b}^2 + 2b\ddot{b} = 120\dot{\theta}^2\sin\theta - 120\ddot{\theta}\cos\theta$

For $\theta = 30°$, $\dot{b} = -0.8$ m/s, $\ddot{b} = 0$,

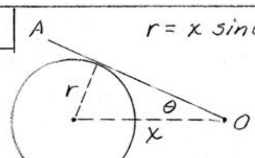
$b^2 = 36 + 100 - 120\left(\frac{1}{2}\right)$, $b = 8.72$ m

$2(8.72)(-0.8) = -120\frac{\sqrt{3}}{2}\dot{\theta}$, $\dot{\theta} = \omega = 0.1342$ rad/s

$2(-0.8)^2 + 2(8.72)(0) = 120(0.1342)^2\left(\frac{1}{2}\right) - 120\ddot{\theta}\frac{\sqrt{3}}{2}$

$\ddot{\theta} = -0.001916$ rad/s^2

5/39



$r = x\sin\theta$, $0 = \dot{x}\sin\theta + x\dot{\theta}\cos\theta$

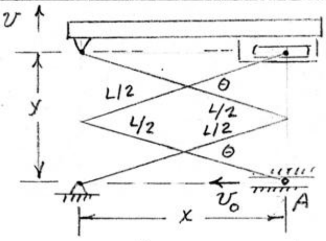
$\omega = \dot{\theta} = -\dfrac{\dot{x}}{x}\tan\theta$

But $v = -\dot{x}$

& $\tan\theta = \dfrac{r}{\sqrt{x^2-r^2}}$

so $\omega = \dfrac{v}{x}\dfrac{r}{\sqrt{x^2-r^2}} = \dfrac{v}{x\sqrt{(x/r)^2-1}}$

5/40



$y = 2L \sin\frac{\theta}{2}$

$v = \dot{y} = L\dot{\theta}\cos\frac{\theta}{2}$

$x = L\cos\frac{\theta}{2}$

$\dot{x} = -v_0 = -\frac{L}{2}\dot{\theta}\sin\frac{\theta}{2}$

so $L\dot{\theta} = 2v_0 / \sin\frac{\theta}{2}$

& $v = \frac{2v_0}{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}$, $\underline{v = 2v_0 \operatorname{ctn}\frac{\theta}{2}}$

5/41

$y = \frac{h}{2}\left(1 + \cos\frac{\pi x}{b}\right)$

$\dot{y} = -\frac{\pi h}{2b}\dot{x}\sin\frac{\pi x}{b} = -\frac{\pi h}{2b}v\sin\frac{\pi x}{b}$

$\ddot{y} = -\left(\frac{\pi v}{b}\right)^2\frac{h}{2}\cos\frac{\pi x}{b}$, $\ddot{y}_{max} = 2g = \left(\frac{\pi v}{b}\right)^2\frac{h}{2}$

where $a_G = g = \frac{1}{2}\ddot{y}_{max}$

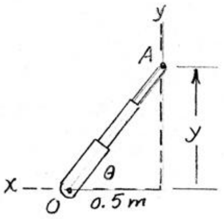
So $\underline{h = 4g\left(\frac{b}{\pi v}\right)^2}$

For $b = 1$ m, $v = \frac{20}{3.6} = 5.56$ m/s :

$h = 4(9.81)\left(\frac{1}{\pi 5.56}\right)^2 = 0.1288$ m

or $\underline{h = 128.8 \text{ mm}}$

5/42



$y = 0.5\tan\theta$

$\dot{y} = 0.5\sec^2\theta\,\dot{\theta}$

$\ddot{y} = 0 = \sec\theta(\tan\theta\sec\theta)\dot{\theta}^2 + 0.5\sec^2\theta\,\ddot{\theta}$

$\dot{\theta} = 2\dot{y}/\sec^2\theta$

$\ddot{\theta} = -2\tan\theta\,\dot{\theta}^2$

For $y = 0.6$ m, $\tan\theta = \frac{0.6}{0.5} = 1.2$, $\theta = 50.2°$

$\sec\theta = 1.562$

So for $\dot{y} = 0.2$ m/s, $\dot{\theta} = \frac{2(0.2)}{(1.562)^2} = \underline{0.1639 \text{ rad/s}}$

$\ddot{\theta} = -2(1.2)(0.1639)^2 = \underline{-0.0645 \text{ rad/s}^2}$

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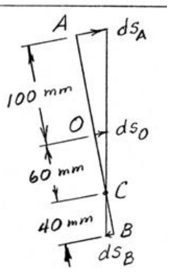


$v_B = 30(4) = 120$ mm/s

Also $v_B = \frac{ds_B}{dt} = \frac{40\,d\theta}{dt} = 40\omega$,

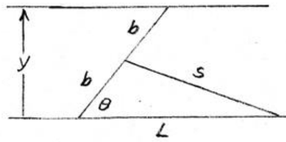
$\underline{\omega = \frac{120}{40} = 3 \text{ rad/s CW}}$

$v_A = \frac{ds_A}{dt} = 160\frac{d\theta}{dt} = 160\omega = 160(3) = \underline{480 \text{ mm/s}}$

$v_O = \frac{ds_O}{dt} = 60\frac{d\theta}{dt} = 60\omega = 60(3) = \underline{180 \text{ mm/s}}$

From Sample Problem 5/4 $a_C = r\omega^2 = 60(3^2) = \underline{540 \text{ mm/s}^2}$ toward O

5/44



$y = 2b\sin\theta$

$v = \dot{y} = 2b\dot{\theta}\cos\theta$

$s^2 = b^2 + L^2 - 2bL\cos\theta$

$2s\dot{s} = 0 + 0 + 2bL\dot{\theta}\sin\theta$

$\dot{\theta} = \frac{s\dot{s}}{bL\sin\theta}$

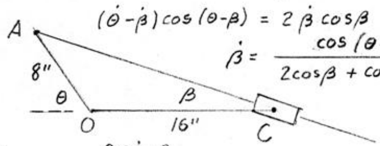
so $v = 2b\frac{s\dot{s}}{bL\sin\theta}\cos\theta = \underline{2\frac{\sqrt{b^2 + L^2 - 2bL\cos\theta}}{L\tan\theta}\dot{s}}$

5/45

Law of sines $\frac{8}{\sin\beta} = \frac{16}{\sin(\theta - \beta)}$

$(\dot{\theta} - \dot{\beta})\cos(\theta - \beta) = 2\dot{\beta}\cos\beta$

$\dot{\beta} = \frac{\cos(\theta - \beta)}{2\cos\beta + \cos(\theta - \beta)}\dot{\theta}$



$\tan\beta = \frac{8\sin\theta}{16 + 8\cos\theta}$, For $\theta = 60°$, $\tan\beta = \frac{8\sin 60°}{16 + 8\cos 60°}$

$= 0.346$, $\beta = 19.11°$

$\theta - \beta = 60 - 19.11 = 40.9°$

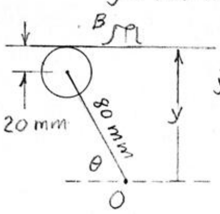
$\cos(\theta - \beta) = 0.756$, $\cos\beta = 0.945$

$\omega = \dot{\beta} = \frac{0.756}{2(0.945) + 0.756}\,\frac{600(2\pi)}{60} = \underline{17.95 \text{ rad/sec CW}}$

5/46

$y = 20 + 80\sin\theta$, $\dot{y} = 80\dot{\theta}\cos\theta$

$\ddot{y} = 80\ddot{\theta}\cos\theta - 80\dot{\theta}^2\sin\theta$



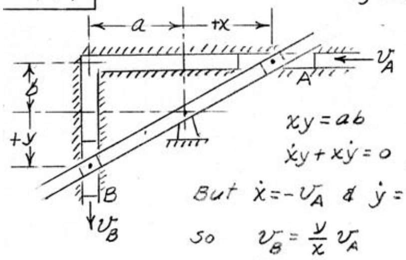
For $\theta = 60°$, $\dot{\theta} = 4\frac{\text{rad}}{\text{s}}$, $\ddot{\theta} = 8\frac{\text{rad}}{\text{s}^2}$,

$\ddot{y} = 80(8)\left(\frac{1}{2}\right) - 80(4)^2\frac{\sqrt{3}}{2}$

$= 320 - 1109 = -789$ mm/s²

Thus $a_B = \underline{789 \text{ mm/s}^2 \text{ down}}$

5/47



By similar triangles

$\frac{b}{x} = \frac{y}{a}$

so $\underline{y = \frac{ab}{x}}$

$xy = ab$

$\dot{x}y + x\dot{y} = 0$, $\dot{y} = -\frac{y}{x}\dot{x}$

But $\dot{x} = -v_A$ & $\dot{y} = +v_B$

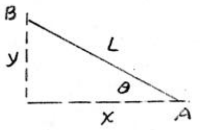
so $\underline{v_B = \frac{y}{x}v_A}$

$v_B x = v_A y$, $a_B x + v_B\dot{x} = \dot{v}_A y + v_A\dot{y}$, $\dot{v}_A = 0$

$a_B x = v_A v_B - v_B(-v_A) = 2v_A v_B$

$a_B = \frac{2v_A v_B}{x} = \underline{\frac{2v_A^2 y}{x^2}}$

5/48 $x = L\cos\theta$



$\dot{x} = -v_0 = -L\dot{\theta}\sin\theta$

$\omega = \dot{\theta} = \dfrac{v_0}{L\sin\theta}$

where $L\sin\theta = y = \sqrt{L^2 - x^2}$

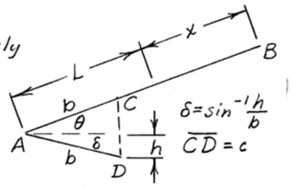
so $\omega = \dfrac{v_0}{\sqrt{L^2-x^2}}$

$\alpha = \ddot{\theta} = \dfrac{v_0}{L}\dfrac{d}{dt}\csc\theta = \dfrac{v_0}{L}(-\text{ctn}\,\theta\csc\theta)\dot{\theta}$

$= -\dfrac{v_0}{L}\dfrac{x}{y}\dfrac{L}{y}\dot{\theta} = \dfrac{-x v_0^2}{y^2\sqrt{L^2-x^2}}$

$= \dfrac{-x v_0^2}{(L^2-x^2)^{3/2}}$

5/49 For vertical motion only of B, its horizontal coordinate remains constant so



$\dfrac{d}{dt}\{(L+x)\cos\theta\} = 0$

or $-(L-x)\dot{\theta}\sin\theta + \dot{x}\cos\theta = 0,$

$\dot{x} = (L+x)\dot{\theta}\tan\theta$

$\overline{CD}^2 = c^2 = b^2 + b^2 - 2b^2\cos(\theta+\delta) = 2b^2(1-\cos(\theta+\delta))$

$2c\dot{c} = 2b^2\dot{\theta}\sin(\theta+\delta),\quad \dot{\theta} = \dfrac{c\dot{c}}{b^2\sin(\theta+\delta)} = \dfrac{\sqrt{2}\sqrt{1-\cos(\theta+\delta)}}{b\sin(\theta+\delta)}\dot{c}$

Thus $\dot{x} = (L+x)\tan\theta\dfrac{\sqrt{2}\sqrt{1-\cos(\theta+\delta)}}{b\sqrt{1-\cos^2(\theta+\delta)}}\dot{c}$

$= \dfrac{L+x}{b}\tan\theta\dfrac{\sqrt{2}}{\sqrt{1+\cos(\theta+\delta)}}\dot{c} = \dfrac{L+x}{b}\dfrac{\tan\theta}{\cos\frac{1}{2}(\theta+\delta)}\dot{c}$

where $\delta = \sin^{-1}\dfrac{h}{b}$

5/50 Belt velocity is the same for both pulleys

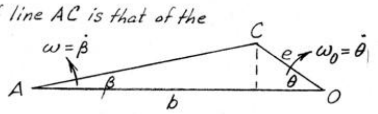
so $r_1\omega_1 = r_2\omega_2$

Thus $\dot{r}_1\omega_1 + r_1\dot{\omega}_1 = \dot{r}_2\omega_2 + r_2\dot{\omega}_2$

For $\dot{\omega}_1 = 0$ & $\alpha_2 = \dot{\omega}_2$, we have

$\alpha_2 = \dot{\omega}_2 = \dfrac{\dot{r}_1\omega_1 - \dot{r}_2\omega_2}{r_2} = \dfrac{\dot{r}_1 r_2 - r_1\dot{r}_2}{r_2^2}\omega_1$

5/51 Angular velocity of line AC is that of the fork, whose sides are parallel to AC.



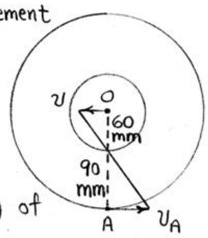
$\tan\beta = \dfrac{e\sin\theta}{b - e\cos\theta}$

$\sec^2\beta\,\dot{\beta} = \dfrac{(b-e\cos\theta)e\dot{\theta}\cos\theta - e\sin\theta(e\dot{\theta}\sin\theta)}{(b-e\cos\theta)^2} = \dfrac{b\cos\theta - e}{(b-e\cos\theta)^2}e\dot{\theta}$

Substitute $\sec^2\beta = 1 + \tan^2\beta = \dfrac{b^2 - 2be\cos\theta + e^2}{(b-e\cos\theta)^2}$ & get

$\omega = \dot{\beta} = \dfrac{b\cos\theta - e}{b^2 - 2be\cos\theta + e^2}e\omega_0$ where $\omega_0 = \dot{\theta}$

5/52 Let ds = differential movement



$\dfrac{ds_0}{60} = \dfrac{ds_A}{90}$

So $\dfrac{v_0}{60} = \dfrac{v_A}{90}$, $v_0 = \dfrac{2}{3}v_A$

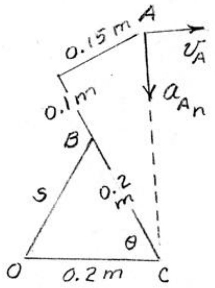
Pitch (distance between teeth) of large gear is $\pi\dfrac{300}{48} = 19.63$ mm

19.63 mm is the advancement per revolution of worm.

Thus $v_A = 19.63\left(\dfrac{120}{60}\right) = 39.3$ mm/s

So $v_0 = \dfrac{2}{3}(39.3) = 26.2$ mm/s

5/53 Given $\dot{s} = 0.260$ m/s



$s = 2(0.2)\sin\dfrac{\theta}{2}$

$\dot{s} = 0.2\dot{\theta}\cos\dfrac{\theta}{2}$

For $\theta = 60°$

$\dot{s} = 0.260 = 0.2\dot{\theta}\cos\dfrac{60°}{2}$

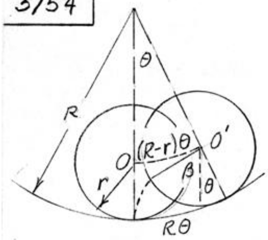
$\dot{\theta} = \omega_{AC} = \dfrac{0.260}{0.2\cos 30°} = 1.501$ rad/s

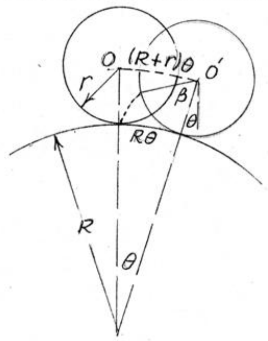
$\overline{AC} = \sqrt{0.3^2 + 0.15^2} = 0.335$ m

$a_{A_n} = \overline{AC}\,\omega_{AC}^2$

$= 0.335(1.501)^2 = 0.756$ m/s²

5/54





$v_0 = v = (R-r)\dot{\theta}$

$R\theta = r(\theta+\beta)$

so $\theta(R-r) = r\beta$

$\dot{\theta}(R-r) = r\dot{\beta}$

so $v = r\dot{\beta}$ & $\omega = \dot{\beta}$

$v = r\omega$ so $a_t = r\alpha$

(β = absolute angle)

$v_0 = v = (R+r)\dot{\theta}$

$R\theta = r\beta$ so $\theta + \beta = \left(\dfrac{r+R}{r}\right)\theta$

so $r(\dot{\theta}+\dot{\beta}) = (R+r)\dot{\theta}$

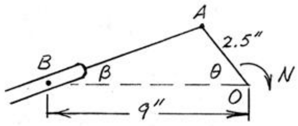
where $\omega = (\dot{\beta}+\dot{\theta})$

so $v = r\omega$ so $a_t = r\alpha$

($\beta+\theta$ = absolute angle)

5/55

$\tan\beta = \dfrac{2.5\sin\theta}{9-2.5\cos\theta}$



$\dot\theta = \dfrac{2\pi N}{60} = \dfrac{120}{30}\pi = 12.57 \text{ rad/s}$

$\sec^2\beta\,\dot\beta = \dfrac{(9-2.5\cos\theta)2.5\dot\theta\cos\theta - 2.5\sin\theta(2.5\dot\theta\sin\theta)}{(9-2.5\cos\theta)^2}$

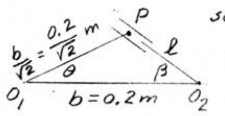
$= \dfrac{22.5\cos\theta - 6.25}{(9-2.5\cos\theta)^2}\dot\theta$

$\dot\beta = \dfrac{22.5\cos\theta-6.25}{(9-2.5\cos\theta)^2}\dot\theta\cos^2\beta$

But $\cos^2\beta = \dfrac{(9-2.5\cos\theta)^2}{9^2+2.5^2-2(9)(2.5)\cos\theta}$

so $\dot\beta = \dfrac{22.5\cos\theta-6.25}{87.2-45\cos\theta}12.57$ or $\underline{\dot\beta = \dfrac{12.57}{2}\dfrac{\cos\theta-0.278}{1.939-\cos\theta}\dfrac{\text{rad}}{\text{sec}}}$

▸5/56 $\dfrac{b/\sqrt2}{\sin\beta} = \dfrac{b}{\sin(\pi-\theta-\beta)} = \dfrac{b}{\sin(\theta+\beta)}$



so $\sqrt2\sin\beta = \sin(\theta+\beta)$ ----- (a)

$\sqrt2\dot\beta\cos\beta = (\dot\theta+\dot\beta)\cos(\theta+\beta)$

$\omega_2 = -\dot\beta = \dot\theta\dfrac{\cos(\theta+\beta)}{\cos(\theta+\beta)-\sqrt2\cos\beta}$ --- (b)

From (a) $\sin\beta(\sqrt2-\cos\theta) = \sin\theta\cos\beta$, $\tan\beta = \dfrac{\sin\theta}{\sqrt2-\cos\theta}$

For $\theta = 20°$, $\beta = \tan^{-1}\dfrac{0.3420}{\sqrt2-0.9397} = \tan^{-1}0.7208 = 35.8°$

& for $\dot\theta = -2$ rad/s, Eq. (b) gives

$\omega_2 = -2\dfrac{\cos(20°+35.8°)}{\cos(20°+35.8°)-\sqrt2\cos35.8°} = -2\dfrac{0.5623}{-0.5849}$

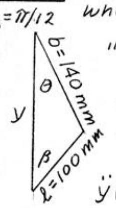
$\underline{\omega_2 = 1.923 \text{ rad/s}}$

▸5/57 $\theta = \theta_0\sin2\pi t$, $\dot\theta = 2\pi\theta_0\cos2\pi t$, $\ddot\theta = -4\pi^2\theta_0\sin2\pi t$

$\theta_0 = \pi/12$

when $\theta = 0$, $t = 1/2$ & $\dot\theta = 2\pi\theta_0 = \pi^2/6$ rad/s, $\ddot\theta = 0$

" $\theta = \frac{\pi}{12}$, $t = 1/4$ & $\dot\theta = 0$, $\underline{\ddot\theta = -4\pi^2\theta_0 = -\pi^3/3 \text{ rad/s}^2}$



$\ell^2 = y^2+b^2-2yb\cos\theta$, $0 = y\dot y + yb\dot\theta\sin\theta - \dot yb\cos\theta$

$0 = y\ddot y + \dot y^2 + \dot yb\dot\theta\sin\theta + yb\ddot\theta\sin\theta + yb\dot\theta^2\cos\theta$

$-\ddot yb\cos\theta + \dot yb\dot\theta\sin\theta$

$\ddot y(b\cos\theta - y) = \dot y^2 + 2\dot yb\dot\theta\sin\theta + yb\ddot\theta\sin\theta + yb\dot\theta^2\cos\theta$

(a) $\theta = 0$, $\dot y = 0$, $\ddot\theta = 0$: $\ddot y(b-[b+\ell]) = 0+0+0+(b+\ell)b(\pi^2/6)^2$

$\ddot y = \dfrac{\pi^4}{36}\dfrac{b(b+\ell)}{-\ell} = \dfrac{-\pi^4}{36}\dfrac{0.14(0.24)}{0.1} = -0.909\ \dfrac{\text{m}}{\text{s}^2}$ $\underline{\text{(up)}}$

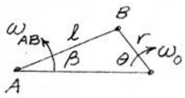
(b) $\theta = \pi/12$, $\dot y = 0$, $\dot\theta = 0$: $b/\sin\beta = \ell/\sin\frac{\pi}{12}$, $\beta = \sin^{-1}\left(\frac{0.14}{0.1}\sin\frac{\pi}{12}\right) = 21.24°$

$y = b\cos\theta + \ell\cos\beta = 0.14\cos\frac{\pi}{12} + 0.1\cos21.24° = 0.2284$ m

$\ddot y(0.14\cos\frac{\pi}{12} - 0.2284) = 0+0+0.2284(0.14)\left(-\frac{\pi^3}{3}\right)\sin\frac{\pi}{12} + 0$

$\ddot y(-0.09320) = -0.08555$, $\underline{\ddot y = 0.918 \text{ m/s}^2 \text{ (down)}}$

▸5/58 $\ell\sin\beta = r\sin\theta$, $\ell\dot\beta\cos\beta = r\dot\theta\cos\theta$

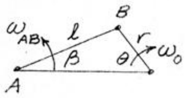


so $\omega_{AB} = \dot\beta = \dfrac{r}{\ell}\dot\theta\dfrac{\cos\theta}{\cos\beta} = \underline{\dfrac{r}{\ell}\omega_0\dfrac{\cos\theta}{\sqrt{1-\left(\frac{r}{\ell}\sin\theta\right)^2}}}$

$\ell\ddot\beta\cos\beta - \ell\dot\beta^2\sin\beta = -r\dot\theta^2\sin\theta$, $\ddot\theta = \dot\omega_0 = 0$

$\alpha_{AB} = \ddot\beta = \dfrac{\ell\dot\beta^2\sin\beta - r\dot\theta^2\sin\theta}{\ell\cos\beta} = \underline{\dfrac{r\omega_0^2}{\ell}\sin\theta\dfrac{\frac{r^2}{\ell^2}-1}{\left(1-\frac{r^2}{\ell^2}\sin^2\theta\right)^{3/2}}}$

▸5/58 $\ell\sin\beta = r\sin\theta$, $\ell\dot\beta\cos\beta = r\dot\theta\cos\theta$



so $\omega_{AB} = \dot\beta = \dfrac{r}{\ell}\dot\theta\dfrac{\cos\theta}{\cos\beta} = \underline{\dfrac{r}{\ell}\omega_0\dfrac{\cos\theta}{\sqrt{1-\left(\frac{r}{\ell}\sin\theta\right)^2}}}$

$\ell\ddot\beta\cos\beta - \ell\dot\beta^2\sin\beta = -r\dot\theta^2\sin\theta$, $\ddot\theta = \dot\omega_0 = 0$

$\alpha_{AB} = \ddot\beta = \dfrac{\ell\dot\beta^2\sin\beta - r\dot\theta^2\sin\theta}{\ell\cos\beta} = \underline{\dfrac{r\omega_0^2}{\ell}\sin\theta\dfrac{\frac{r^2}{\ell^2}-1}{\left(1-\frac{r^2}{\ell^2}\sin^2\theta\right)^{3/2}}}$

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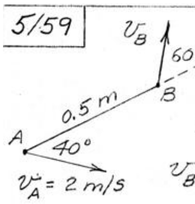
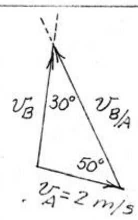


$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$\dfrac{v_B}{\sin50°} = \dfrac{2}{\sin30°}$

$v_B = \dfrac{2\sin50°}{\sin30°} = \underline{3.06 \text{ m/s}}$



$v_{B/A} = v_B\cos30° + v_A\cos50°$

$= 3.06\cos30° + 2\cos50° = 3.94$ m/s

$\omega_{AB} = \dfrac{v_{B/A}}{\overline{AB}} = \dfrac{3.94}{0.5} = \underline{7.88 \text{ rad/s ccw}}$

5/60 $\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O}$ where $v_{A/O} = \overline{AO}\,\omega = \frac{10}{12}\omega\ \frac{\text{ft}}{\text{sec}}$

$v_O = 4$ ft/sec

(a) $v_A = 4$ (←), $v_O = 4$ (→), $v_{A/O} = 8$ ft/sec (←)

$\omega = \dfrac{8}{10/12} = 9.6\dfrac{\text{rad}}{\text{sec}}$, $N = 9.6\dfrac{60}{2\pi} = 91.7\ \dfrac{\text{rev}}{\text{min}}$ $\underline{\text{CCW}}$

(b) $v_O = 4$ (→), $v_{A/O} = 4$ ft/sec (←)

$v_A = 0$, $\omega = \dfrac{4}{10/12} = 4.8\dfrac{\text{rad}}{\text{sec}}$, $N = 45.8\ \dfrac{\text{rev}}{\text{min}}$ $\underline{\text{CCW}}$

(c) $v_O = 4$ (→), $v_{A/O} = 4$ (→), $v_A = 8$ ft/sec (→)

$\omega = \dfrac{4}{10/12} = 4.8\dfrac{\text{rad}}{\text{sec}}$, $\underline{N = 45.8\ \frac{\text{rev}}{\text{min}}\ \text{CW}}$

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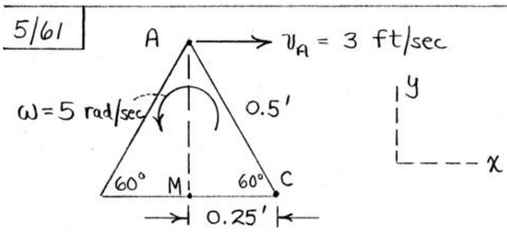


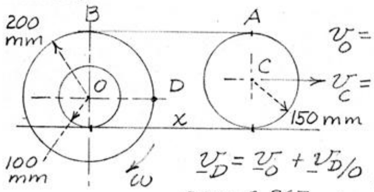
$\overline{AM} = 0.25\sqrt3 = 0.433$ ft

$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_A + \boldsymbol{\omega}\times\mathbf{r}_{C/A}$

$= 3\mathbf{i} + 5\mathbf{k}\times[0.25\mathbf{i} - 0.433\mathbf{j}]$

$= \underline{5.17\mathbf{i} + 1.25\mathbf{j} \text{ ft/sec}}$

6/62 $v_B = v_A = 2v_C = 0.8\ m/s$, $\omega = \frac{v_B}{r} = \frac{0.8}{0.3} = \frac{8}{3}\ \frac{rad}{s}$

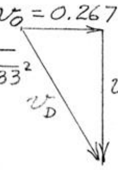


$v_O = \frac{0.1}{0.3} v_B = \frac{0.8}{3} = 0.267\ \frac{m}{s}$

$v_C = 0.4\ m/s$

$v_D = v_O + v_{D/O}$, $v_{D/O} = \overline{OD}\omega = 0.2\left(\frac{8}{3}\right) = 0.533\ \frac{m}{s}$

$v_O = 0.267 m/s$



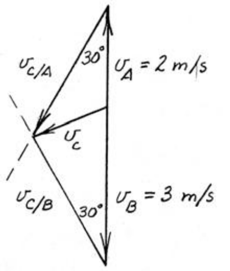
$v_D = \sqrt{0.267^2 + 0.533^2}$
$= 0.596\ m/s$

$\dot{x} = v_{C/O} = (v_C - v_O) = (0.4 - 0.267) = 0.133\ m/s$

5/63 $v_C = v_A + v_{C/A} = v_B + v_{C/B}$

From geometry of isosceles triangle

$v_C = \sqrt{(2.5\tan 30°)^2 + 0.5^2} = 1.528\ m/s$



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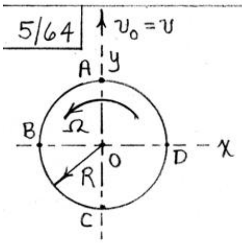


$v_O = 107\ 257 j$ km/h

$R\Omega = 6371(10^3)[7.292(10^{-5})]$
$= 465\ \frac{m}{s}\left(3.6\ \frac{km/h}{m/s}\right)$
$= 1672$ km/h

$v_A = v_O + v_{A/O} = -1672 i + 107\ 257 j$ km/h

$v_B = v_O + v_{B/O} = 107\ 257 j - 1672 j = 105\ 585 j$ km/h

$v_C = v_O + v_{C/O} = 1672 i + 107\ 257 j$ km/h

$v_D = v_O + v_{D/O} = (107\ 257 + 1672) j = 108\ 929 j$ km/h

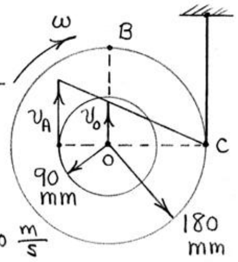
5/65

$v_O = \frac{180}{180+90} v_A = \frac{2}{3}(0.9) = 0.6\ \frac{m}{s}$

$v_B = v_O + v_{B/O}$, $\omega = \omega_{BO} = \frac{0.6}{0.180}$
$= 3.33$ rad/s

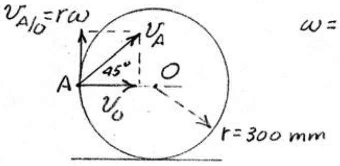
$v_{B/O} = \overline{BO}\,\omega_{OB} = 0.180(3.33) = 0.60\ \frac{m}{s}$

$v_B = 0.6\sqrt{2} = 0.849$ m/s



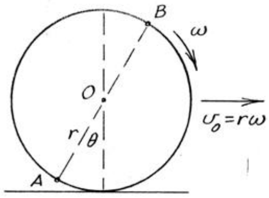
5/66 $|v_O| = |v_{A/O}| = r\omega = 12\cos 45° = 8.49\ m/s$

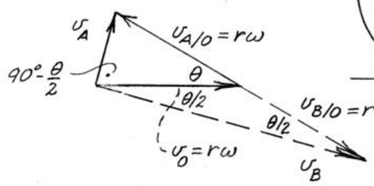
$\omega = \frac{8.49}{0.325} = 26.1$ rad/s



5/67

$v_A = v_O + v_{A/O}$
$v_B = v_O + v_{B/O}$

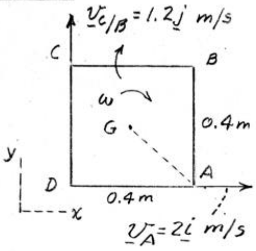




$v_A = 2r\omega\sin\frac{\theta}{2}$

Angle between v_A & v_B is 90°

5/68



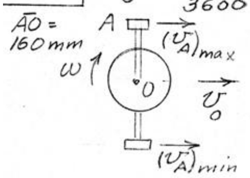
$v_{C/B} = \overline{CB}\,\omega$, $\omega = \frac{1.2}{0.4} = 3$ rad/s CW

$\omega = -3k$ rad/s

$v_G = v_A + \omega \times r_{AG}$
$= 2i - 3k \times (-0.2i + 0.2j)$
$= 2i + 0.6j + 0.6i$

$v_G = 2.6i + 0.6j$ m/s

5/69 $v_O = \frac{16000}{3600} = 4.444\ m/s$, $2r = 660$ mm



$\omega = \frac{70}{170}\frac{v_O}{r} = \frac{70}{170}\frac{4.44}{0.33} = 5.55$ rad/s

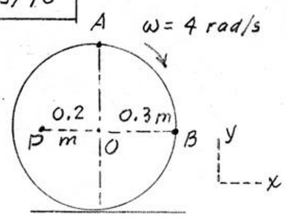
$v_A = v_O + v_{A/O}$

$|v_{A/O}| = \overline{AO}\,\omega_{AO} = 0.160(5.55)$
$= 0.887$ m/s

$(v_A)_{max} = 4.444 + 0.887 = 5.33$ m/s

$(v_A)_{min} = 4.444 - 0.887 = 3.56$ m/s

5/70



$v_{A/B} = \omega \times r_{A/B}$

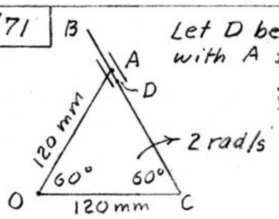
$\omega = -4k$ rad/s

$r_{A/B} = -0.3i + 0.3j$ m

$v_{A/B} = -4k \times 0.3(-i + j)$
$= 1.2(i + j)$ m/s

$v_P = v_O + v_{P/O} = r\omega i + \overline{PO}\omega j = 4(0.3i + 0.2j)$ m/s

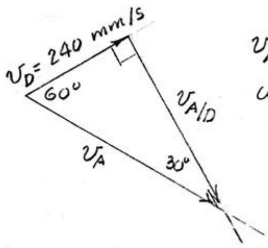
5/71



Let D be point on BC coincident with A for θ = 60°

$v_A = v_D + v_{A/D}$

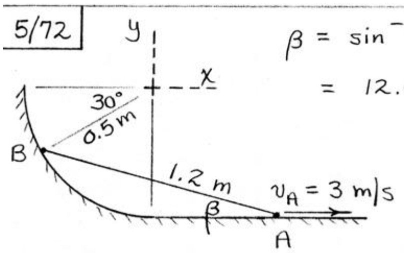
where $v_D = \overline{DC}\,\omega_{CB}$
$= 120(2)$
$= 240$ mm/s



$v_A = \frac{240}{\cos 60°} = 480$ mm/s

$\omega_{OA} = \frac{v_A}{\overline{OA}} = \frac{480}{120} = 4$ rad/s CW

5/72



$$\beta = \sin^{-1}\frac{0.5 - 0.5\sin 30°}{1.2} = 12.02°$$

$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = \underline{v}_A + \underline{\omega}\times\underline{r}_{B/A}$$

$$v_B(\sin 30°\,\underline{i} - \cos 30°\,\underline{j}) = 3\underline{i} + \omega\underline{k}\times 1.2(-\cos\beta\,\underline{i} + \sin\beta\,\underline{j})$$

$$= 3\underline{i} + \omega\underline{k}\times 1.2(-\cos 12.02°\,\underline{i} + \sin 12.02°\,\underline{j})$$

$$= 3\underline{i} - 1.174\omega\underline{j} - 0.250\omega\underline{i}$$

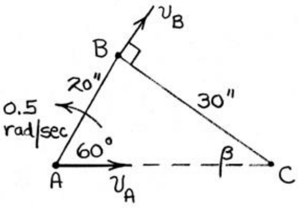
$$\left.\begin{array}{l}\underline{i}:\ \frac{1}{2}v_B = 3 - 0.250\omega \\ \underline{j}:\ -\frac{\sqrt{3}}{2}v_B = -1.174\omega\end{array}\right\}\quad \begin{array}{l}v_B = 4.38 \text{ m/s} \\ \underline{\omega = 3.23 \text{ rad/s}}\end{array}$$

5/73

$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$$

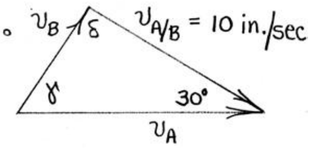
$$\frac{20}{\sin\beta} = \frac{30}{\sin 60°},\quad \beta = 35.3°$$

$$v_{A/B} = \overline{AB}\,\omega_{AB} = 20(0.5) = 10 \text{ in./sec}$$



$$\gamma = 90° - 35.3° = 54.7°$$

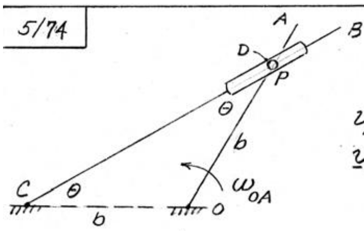
$$\delta = 180° - 54.7° - 30° = 95.3°$$



$$\frac{v_A}{\sin 95.3°} = \frac{10}{\sin 54.7°}$$

$$\underline{v_A = 12.20 \text{ in./sec}}$$

5/74

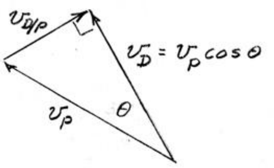


Let P = pin

D = point on CB coincident with P

$$v_P = r\omega = b\omega_{OA}$$

$$\underline{v}_D = \underline{v}_P + \underline{v}_{D/P}$$



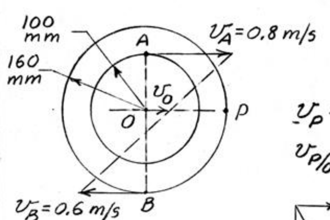
$$v_D = \overline{CD}\,\omega_{CB} = 2b\cos\theta\,\omega_{CB}$$

So $2b\cos\theta\,\omega_{CB} = b\,\omega_{OA}\cos\theta$

Thus $\underline{\omega_{CB} = \frac{1}{2}\omega_{OA}}$

5/75

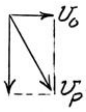
$$\omega = \frac{v_A + v_B}{\overline{AB}} = \frac{0.8 + 0.6}{0.26} = 5.38 \frac{\text{rad}}{\text{s}} \text{ CW}$$



$$v_O = v_A - \overline{AO}\,\omega = 0.8 - 0.1(5.38) = 0.262 \frac{\text{m}}{\text{s}}$$

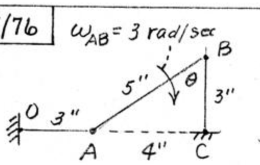
$$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O}$$

$$v_{P/O} = \overline{PO}\,\omega = 0.16(5.38) = 0.862 \frac{\text{m}}{\text{s}}$$



$$v_P = \sqrt{0.262^2 + 0.862^2} = \underline{0.900 \text{ m/s}}$$

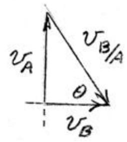
5/76



$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A},\quad \omega_{BC} = \frac{v_B}{\overline{BC}}$$

$$v_{B/A} = \overline{AB}\,\omega_{AB} = 5(3) = 15 \text{ in./sec}$$

$$\theta = \cos^{-1}\frac{3}{5}$$

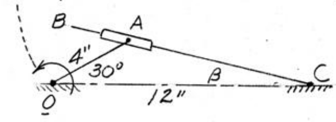


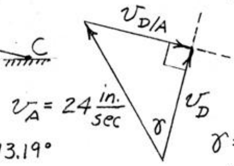
$$v_B = v_{B/A}\cos\theta = 15(3/5) = 9 \text{ in./sec}$$

$$\omega_{BC} = 9/3 = \underline{3 \text{ rad/sec CW}}$$

5/77 Let D be point on BC coincident with A

$$\underline{v}_D = \underline{v}_A + \underline{v}_{D/A},\quad v_A = r\omega = 4(6) = 24 \text{ in./sec}$$





$$\beta = \tan^{-1}\frac{4\sin 30°}{12 - 4\cos 30°} = 13.19°\qquad \gamma = 30 + \beta = 43.19°$$

$$v_D = v_A\cos\gamma = 24\cos 43.19° = 17.50 \text{ in./sec}$$

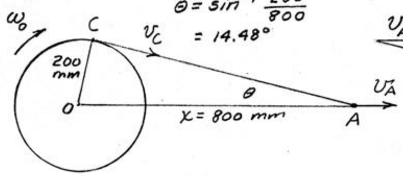
$$\omega_{CB} = v_D/\overline{DC} \text{ where } \overline{DC} = \sqrt{4^2 + 12^2 - 2(4)(12)\cos 30°} = 8.77 \text{ in.}$$

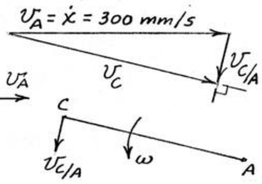
$$= \frac{17.50}{8.77} = \underline{2.00 \text{ rad/sec CW}}$$

5/78

$$\underline{v}_C = \underline{v}_A + \underline{v}_{C/A},\quad v_A = 300 \text{ mm/s}$$

$$\theta = \sin^{-1}\frac{200}{800} = 14.48°$$





$$v_C = 300\cos 14.48° = 300(0.9682) = 290 \text{ mm/s}$$

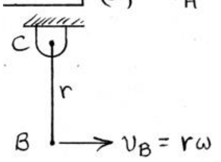
$$v_{C/A} = 300\sin 14.48° = 300/4 = 75 \text{ mm/s}$$

$$\overline{CA} = 800\cos 14.48° = 775 \text{ mm}$$

$$\omega_{AB} = v_{C/A}/\overline{CA} = 75/775 = \underline{0.0968 \text{ rad/s CCW}}$$

$$\omega_o = v_C/\overline{CO} = 290/200 = \underline{1.452 \text{ rad/s CW}}$$

5/79 (a) $v_A = v_B = r\omega$ (right)

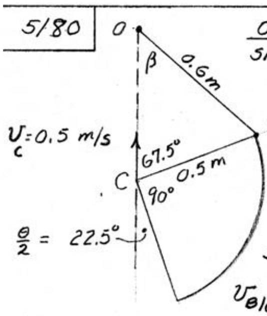


$$\omega_{BC} = \frac{v_B}{\overline{BC}} = \frac{r\omega}{r} = \underline{\omega \text{ CCW}}$$

(b) $v_A = v_B = 2r\omega$ (right)

$$\omega_{BC} = \frac{v_B}{\overline{BC}} = \frac{2r\omega}{r} = \underline{2\omega \text{ CCW}}$$

5/80



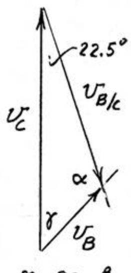
$\frac{0.5}{\sin\beta} = \frac{0.6}{\sin 67.5°}$, $\beta = \sin^{-1} 0.7699 = 50.3°$

$v_B = v_C + v_{B/C}$

$\frac{v_{B/C}}{\sin 39.7°} = \frac{v_C}{\sin 117.8°}$

$v_{B/C} = 0.361$ m/s

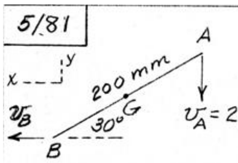
$\omega = \omega_{BC} = \frac{0.361}{0.5} = \underline{0.722 \text{ rad/s}}$



$\gamma = 90 - \beta = 39.7°$

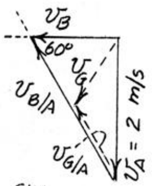
$\alpha = 180 - 39.7 - 22.5 = 117.8°$

5/81



$v_B = v_A + v_{B/A}$

$\omega = \frac{v_{B/A}}{\overline{BA}} = \frac{2/\cos 30°}{0.200} = \underline{11.55 \text{ rad/s CW}}$



$v_G = v_A + v_{G/A}$

$v_{G/A} = \overline{GA}\,\omega = \frac{1}{2} v_{B/A}$

From diagram $v_G = 2/\sqrt{3} = \underline{1.155 \text{ m/s}}$

$\mathbf{v}_A = -2\mathbf{j}$ m/s, $\mathbf{v}_B = v_B\mathbf{i}$, $\boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k}$

$v_B\mathbf{i} = -2\mathbf{j} + \omega_{AB}\mathbf{k} \times (0.2\cos 30°\,\mathbf{i} - 0.2\sin 30°\,\mathbf{j})$

$= (-2 + 0.1732\,\omega_{AB})\mathbf{j} + 0.1\,\omega_{AB}\mathbf{i}$

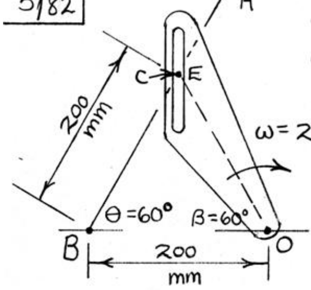
$\omega_{AB} = \frac{2}{0.1732} = \underline{11.55 \text{ rad/s}}$ CW,

$\mathbf{v}_G = -2\mathbf{j} + 11.55\mathbf{k} \times (0.1\cos 30°\,\mathbf{i} - 0.1\sin 30°\,\mathbf{j})$

$= (-2 + 1.00)\mathbf{j} + 0.577\mathbf{i} = -\mathbf{j} + 0.577\mathbf{i}$ m/s

$v_G = \sqrt{1^2 + 0.577^2} = \underline{1.155 \text{ m/s}}$

5/82



Let E be a point on D coincident with pin C.

$v_C = v_E + v_{C/E}$

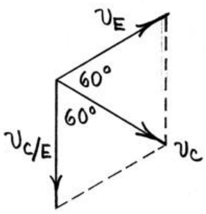
where $v_E = \overline{EO}\,\omega = 200(2) = 400$ mm/s

From vector triangles,

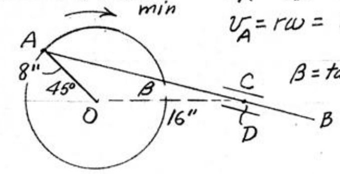
$v_C = 400$ mm/s

$v_A = \frac{200 + 130}{200}(400)$

$= \underline{660 \text{ mm/s}}$



5/83



$v_A = v_D + v_{A/D}$

$v_A = r\omega = 8\,\frac{600(2\pi)}{60} = 503$ in./sec

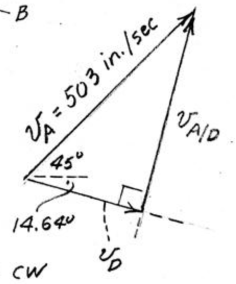
$\beta = \tan^{-1}\frac{8\sin 45°}{16 + 8\cos 45°} = 14.64°$

$v_{A/D} = 503\sin(45° + 14.64°) = 434$ in./sec

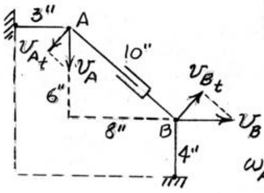
$\omega_{AB} = \omega_{AD} = \frac{v_{A/D}}{\overline{AD}}$

$\overline{AD} = \frac{8\cos 45°}{\sin 14.64°} = 22.4$ in.

$\omega_{AB} = \frac{434}{22.4} = \underline{19.38 \text{ rad/sec CW}}$



5/84



$v_A = 3(0.5) = 1.5$ in./sec

$v_{A_t} = 1.5\left(\frac{4}{5}\right) = 1.2$ in./sec

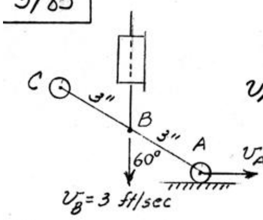
$v_B = 4(0.5) = 2.0$ in./sec

$v_{B_t} = 2.0\left(\frac{3}{5}\right) = 1.2$ in./sec

$\omega_{AB} = \omega = \frac{v_{A_t} + v_{B_t}}{\overline{AB}}$

$\omega = \frac{1.2 + 1.2}{10} = \underline{0.24 \text{ rad/sec CCW}}$

5/85



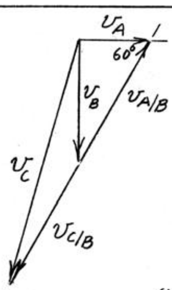
$v_A = v_B + v_{A/B}$

$v_C = v_B + v_{C/B}$

$v_{A/B} = \overline{AB}\,\omega$

$= \overline{CB}\,\omega$

$= v_{C/B}$



From geometry

$v_{A/B} = 3/\sin 60° = 3.46$ ft/sec, $v_A = 3/\tan 60° = 1.732$ ft/sec

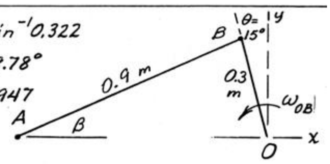
$v_{C/B} = 3.46$ ft/sec

$v_C = \sqrt{(3+3)^2 + (1.732)^2} = \sqrt{39} = \underline{6.24 \text{ ft/sec}}$

5/86

$0.9\sin\beta = 0.3\cos 15°$, $\beta = \sin^{-1} 0.322 = 18.78°$

$\cos\beta = 0.947$



$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

I (Vector algebra) $\mathbf{v}_A = v_A\mathbf{i}$, $\mathbf{v}_B = \boldsymbol{\omega}_{OB} \times \mathbf{r}_{OB}$

$= \omega_{OB}\mathbf{k} \times (-0.3 \times 0.259\mathbf{i} + 0.3 \times 0.966\mathbf{j}) = \omega_{OB}(-0.0776\mathbf{j} - 0.290\mathbf{i})$

$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{AB} = -0.086\mathbf{k} \times 0.9(-0.947\mathbf{i} - 0.322\mathbf{j}) = 0.0733\mathbf{j} - 0.0249\mathbf{i}$ m/s

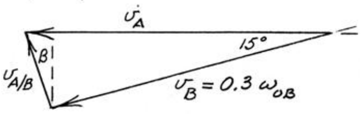
So $v_A\mathbf{i} = -0.0776\,\omega_{OB}\mathbf{j} - 0.290\,\omega_{OB}\mathbf{i} + 0.0733\mathbf{j} - 0.0249\mathbf{i}$

$\mathbf{j}$-terms: $\omega_{OB} = \frac{0.0733}{0.0776} = \underline{0.944 \text{ rad/s CCW}}$

$\mathbf{i}$-terms: $v_A = -0.290(0.944) - 0.0249 = \underline{-0.298 \text{ m/s}}$ (neg. x-dir)

II (Vector geometry)

$v_{A/B} = 0.9(0.086) = 0.0774$ m/s

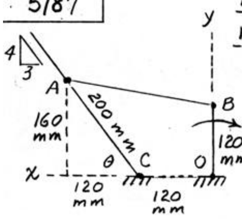


Law of sines: $\frac{0.0774}{\sin 15°} = \frac{0.3\,\omega_{OB}}{\sin(90° - 18.78°)}$, $\underline{\omega_{OB} = 0.944 \text{ rad/s CCW}}$

$v_A = 0.3(0.944)\cos 15° + 0.0774\sin 18.78°$,

$\underline{v_A = 0.298 \text{ m/s to the left}}$

5/87



$\underline{r}_{CA} = 0.12\underline{i} + 0.16\underline{j}$ m

$\underline{r}_{OB} = 0.12\underline{j}$ m, $\underline{r}_{BA} = 0.24\underline{i} + 0.04\underline{j}$ m

$\underline{v}_A = \underline{\omega}_{AC} \times \underline{r}_{CA}$

$= \omega_{AC}\underline{k} \times (0.12\underline{i} + 0.16\underline{j})$

$= 0.12\omega_{AC}\underline{j} - 0.16\omega_{AC}\underline{i}$

$\underline{v}_B = \underline{\omega}_{OB} \times \underline{r}_{OB} = 0.5\underline{k} \times 0.12\underline{j}$

$= -0.06\underline{i}$ m/s

$\underline{v}_{A/B} = \underline{\omega}_{AB} \times \underline{r}_{BA} = \omega_{AB}\underline{k} \times (0.24\underline{i} + 0.04\underline{j})$

$= 0.24\omega_{AB}\underline{j} - 0.04\omega_{AB}\underline{i}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$, so $0.12\omega_{AC}\underline{j} - 0.16\omega_{AC}\underline{i} = -0.06\underline{i} + 0.24\omega_{AB}\underline{j} - 0.04\omega_{AB}\underline{i}$

Equate coefficients & get

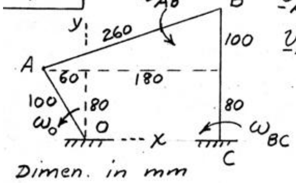
$0.16\omega_{AC} - 0.04\omega_{AB} = 0.06$

$0.12\omega_{AC} - 0.24\omega_{AB} = 0$

Solve & get

$\underline{\omega}_{AB} = 0.214\underline{k}$ rad/s, $\underline{\omega_{CA} = 0.429\underline{k}\text{ rad/s}}$

5/88



Dimen. in mm

$\omega_{AO} = \omega_O = 10$ rad/s

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$\underline{v}_A = \underline{\omega}_{AO} \times \underline{r}_{AO}$

$= 10\underline{k} \times (-0.06\underline{i} + 0.08\underline{j})$

$= -0.6\underline{j} - 0.8\underline{i}$ m/s

$\underline{v}_B = \underline{\omega}_{BC} \times \underline{r}_{BC}$

$= \omega_{BC}\underline{k} \times 0.18\underline{j} = -0.18\omega_{BC}\underline{i}$ m/s

$\underline{v}_{A/B} = \underline{\omega}_{AB} \times \underline{r}_{A/B}$

$= \omega_{AB}\underline{k} \times (-0.24\underline{i} - 0.1\underline{j})$

$= -0.24\omega_{AB}\underline{j} + 0.1\omega_{AB}\underline{i}$ m/s

Thus,

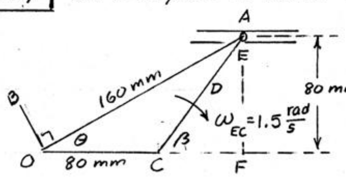
$-0.6\underline{j} - 0.8\underline{i} = -0.18\omega_{BC}\underline{i} - 0.24\omega_{AB}\underline{j} + 0.1\omega_{AB}\underline{i}$

Equate $\underline{j}$ terms & get $\omega_{AB} = \dfrac{0.6}{0.24} = 2.5$ rad/s

$\underline{\underline{\omega}_{BC} = 5.83\underline{k}\text{ rad/s}}$

$\underline{\underline{\omega}_{AB} = 2.5\underline{k}\text{ rad/s}}$

5/89 Let E be point on member D coincident with A



$\theta = \sin^{-1}\dfrac{80}{160} = 30°$

$\overline{CF} = 160\cos 30° - 80 = 58.6$ mm

$\beta = \tan^{-1}\dfrac{80}{58.6} = 53.8°$

$\overline{EC} = \dfrac{80}{\sin 53.8°} = 99.1$ mm

$\underline{v}_A = \underline{v}_E + \underline{v}_{A/E}$

$v_E = 99.1(1.5) = 148.7\ \frac{mm}{s}$

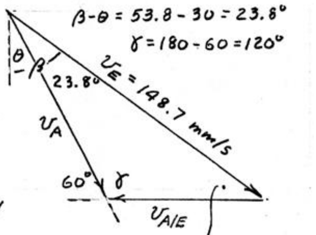
$\dfrac{v_A}{\sin 36.2°} = \dfrac{148.7}{\sin 120°}$

$v_A = 148.7\,\dfrac{0.591}{0.866} = 101.4\ \frac{mm}{s}$

$\omega_{AOB} = \dfrac{101.4}{160} = \underline{0.634\text{ rad/s CW}}$

$\beta - \theta = 53.8 - 30 = 23.8°$

$\gamma = 180 - 60 = 120°$

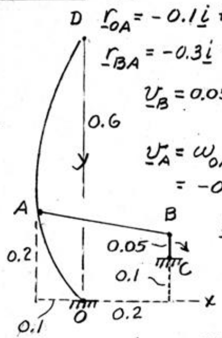


$90 - \beta = 36.2°$

Alternatively, draw vector triangle to scale & measure $v_A \approx 101$ mm/s. Etc.

5/90

$\omega_{CB} = -\frac{2\pi}{2}\frac{rad}{s}$ or $\underline{\omega}_{CB} = -\pi\underline{k}$ rad/s



$\underline{r}_{OA} = -0.1\underline{i} + 0.2\underline{j}$ m, $\underline{r}_{CB} = 0.05\underline{j}$ m

$\underline{r}_{BA} = -0.3\underline{i} + 0.05\underline{j}$ m, $\underline{r}_{OD} = 0.6\underline{j}$,

$\underline{v}_B = 0.05\pi\underline{i}$ m/s, $\underline{v}_{A/B} = \omega_{AB}\underline{k} \times (-0.3\underline{i} + 0.05\underline{j})$

$= -0.3\omega_{AB}\underline{j} - 0.05\omega_{AB}\underline{i}$

$\underline{v}_A = \omega_{OA}\underline{k} \times (-0.1\underline{i} + 0.2\underline{j})$

$= -0.1\omega_{OA}\underline{j} - 0.2\omega_{OA}\underline{i}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$ so

$-0.1\omega_{OA}\underline{j} - 0.2\omega_{OA}\underline{i} = 0.05\pi\underline{i} - 0.3\omega_{AB}\underline{j} - 0.05\omega_{AB}\underline{i}$

Dimensions in meters

Thus $-0.2\omega_{OA} + 0.05\omega_{AB} = 0.05\pi$

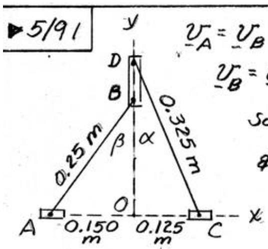
$-0.1\omega_{OA} + 0.3\omega_{AB} = 0$

Solve & get $\underline{\omega}_{AB} = -0.0909\pi\underline{k} = -0.286\underline{k}$ rad/s (CW)

$\underline{\omega}_{OA} = -0.273\pi\underline{k} = -0.857\underline{k}$ rad/s (CW)

$v_E = v_D = 0.6\,\omega_{OD} = 0.6\,\omega_{OA} = 0.6(0.857) = \underline{0.514\text{ m/s}}$

▶5/91



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$, $\underline{v}_C = \underline{v}_D + \underline{v}_{C/D}$

$\underline{v}_B = \underline{v}_D$, $\underline{v}_C - \underline{v}_A = -0.2\underline{i}$ m/s

So $\underline{v}_A = (\underline{v}_C - \underline{v}_{C/D}) + \underline{v}_{A/B}$

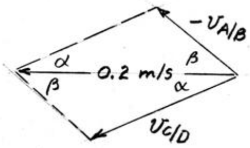
& $\underline{v}_{C/D} - \underline{v}_{A/B} = \underline{v}_C - \underline{v}_A = -0.2\underline{i}$ m/s

$\overline{OD} = \sqrt{(0.325)^2 - (0.125)^2}$

$= 0.3$ m

$\overline{OB} = \sqrt{(0.25)^2 - (0.15)^2}$

$= 0.2$ m



$\sin\alpha = 0.125/0.325 = 5/13$

$\cos\alpha = 0.3/0.325 = 12/13$

$\sin\beta = 0.150/0.250 = 3/5$

$\cos\beta = 0.2/0.25 = 4/5$

$v_{C/D}\cos\alpha + v_{A/B}\cos\beta = 0.2$

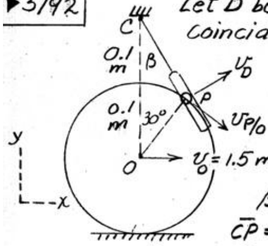
$v_{C/D}\sin\alpha - v_{A/B}\sin\beta = 0$

solve & get $v_{C/D} = \frac{39}{280} = 0.1393$ m/s

$v = v_D = \underline{v}_C - \underline{v}_{C/D}$; $\underline{v}_J = \underline{v}_C\underline{i} - \left[\frac{39}{280}\right](-\underline{i}\cos\alpha - \underline{j}\sin\alpha)$

So $v = \dfrac{39}{280}\,\dfrac{5}{13} = 3/56 = \underline{0.0536\text{ m/s}}$

▶5/92 Let D be point on slotted arm coincident with P



$\underline{v}_D = \underline{v}_P + \underline{v}_{D/P}$ - - - - (a)

$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O}$

$v_{P/O} = \overline{PO}\,\omega_{PO} = \overline{PO}\,\dfrac{v_O}{\overline{PO}} = v_O$

$\beta = \tan^{-1}\dfrac{0.1\sin 30°}{0.2 - 0.1\cos 30°} = 23.8°$

$\overline{CP} = 0.1\,\dfrac{\sin 30°}{\sin 23.8°} = 0.1239$ m

$\underline{v}_D = v_D(\underline{i}\cos\beta + \underline{j}\sin\beta) = v_D(0.915\underline{i} + 0.403\underline{j})$

$\underline{v}_P = 1.5\underline{i} + (1.5\cos 30°)\underline{i} - (1.5\sin 30°)\underline{j} = 2.799\underline{i} - 0.75\underline{j}$ m/s

$\underline{v}_{D/P} = v_{D/P}(-\underline{i}\sin\beta + \underline{j}\cos\beta) = v_{D/P}(-0.403\underline{i} + 0.915\underline{j})$

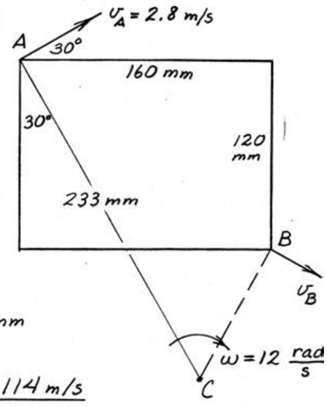
Substitute in Eq. (a) & separate $\underline{i}$ & $\underline{j}$ terms to get

$$\left.\begin{aligned}0.915v_D - 2.799 + 0.403v_{D/P} &= 0\\ 0.403v_D + 0.75 - 0.915v_{D/P} &= 0\end{aligned}\right\}\ \text{solve \& get}\ v_D = 2.26\text{ m/s},\ v_{D/P} = 1.816\ \tfrac{m}{s}$$

Thus $\omega = \omega_{CD} = \dfrac{2.26}{0.1239} = \underline{18.22\text{ rad/s CCW}}$

5/93 Instantaneous center C of zero velocity must lie on the perpendicular to v_A at a distance from A of

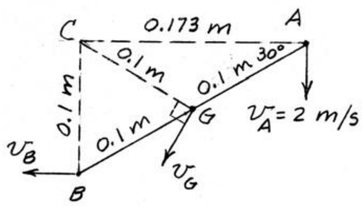
$v = r\omega$, $\overline{AC} = r = \frac{v}{\omega} = \frac{2.8}{12}$
$= 0.233$ m or 233 mm



$\overline{CB}^2 = (160 - 233\sin 30°)^2 + (233\cos 30° - 120)^2$
$= 8614\ \text{mm}^2$, $\overline{CB} = 92.8$ mm

$v_B = \overline{CB}\,\omega = 0.0928(12) = \underline{1.114\ \text{m/s}}$

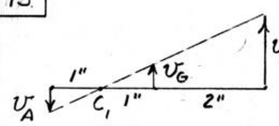
5/94



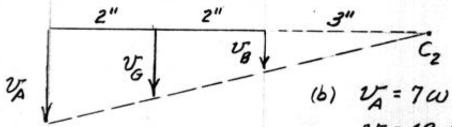
$\omega = v_A/\overline{AC}$
$= 2/0.1732 = \underline{11.55\ \text{rad/s CW}}$

$v_G = \overline{CG}\,\omega$
$= 0.1(11.55)$
$= \underline{1.155\ \text{m/s}}$

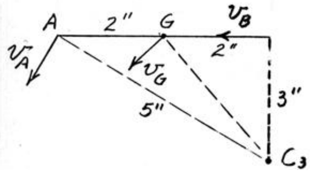
5/95



(a) $v_A = (1)\omega = 1(6)$
$\underline{v_A = 6\ \text{in./sec}}$
$\underline{v_G = 6\ \text{in./sec}}$

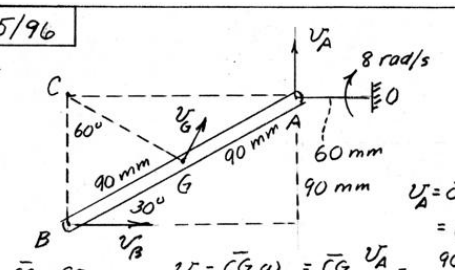


(b) $v_A = 7\omega = 7(6)$
$\underline{v_A = 42\ \text{in./sec}}$
$v_G = 5\omega = 5(6)$
$\underline{v_G = 30\ \text{in./sec}}$



(c) $v_A = 5\omega = 5(6) = \underline{30\ \text{in./sec}}$
$v_G = \overline{C_3G}\,\omega = \sqrt{3^2+2^2}\,(6)$
$= \underline{21.6\ \text{in./sec}}$

5/96

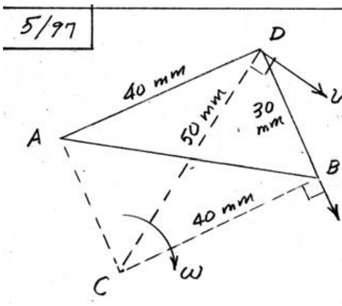


$v_A = \overline{OA}\,\omega_{OA}$
$= 60(8) = 480$ mm/s

$\overline{CG} = 90$ mm, $v_G = \overline{CG}\,\omega_{CG} = \overline{CG}\,\frac{v_A}{\overline{AC}} = \frac{90}{180\cos 30°}\,480$

$\underline{v_G = 277\ \text{mm/s}}$

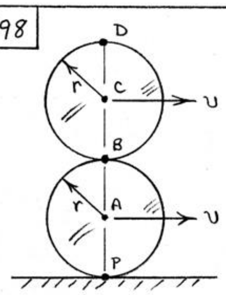
5/97



$\omega = \frac{v_B}{\overline{CB}} = \frac{200}{40} = 5$ rad/s

$v_D = \overline{CD}\,\omega = 50(5)$
$= \underline{250\ \text{mm/s}}$

5/98

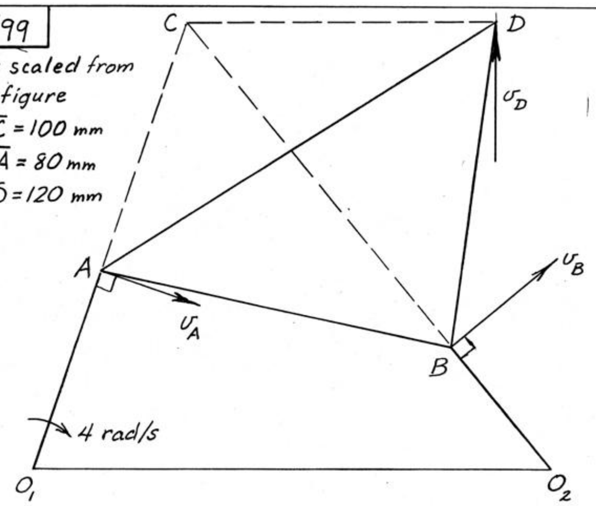


(a) $\omega_\ell = \frac{v}{r}$ CW
(b) $\omega_u = \frac{v}{r}$ CCW
(c) $v_A = v$ (right)
$v_B = 2v$ (right)
$v_C = v$ (right)
$v_D = v_P = 0$

The mechanic's hands have no absolute velocity!

5/99

As scaled from figure
$\overline{AC} = 100$ mm
$\overline{O_1A} = 80$ mm
$\overline{CD} = 120$ mm

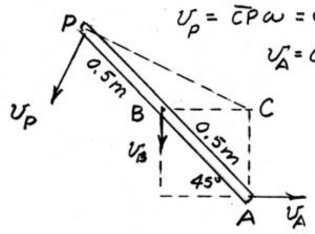


$v_A = \overline{O_1A}\,\omega = 0.80(4) = 0.32$ m/s

$v_D/\overline{CD} = v_A/\overline{AC}$, $v_D = \frac{0.120}{0.100}\,0.32 = \underline{0.38\ \text{m/s}}$

5/100 $\overline{CP} = \sqrt{(1\times\cos 45°)^2 + (0.5\sin 45°)^2} = 0.791$ m

$v_P = \overline{CP}\,\omega = 0.791(2) = \underline{1.581\ \text{m/s}}$
$v_A = \overline{CA}\,\omega = 0.5\sin 45°(2)$
$\underline{v_A = 0.707\ \text{m/s}}$



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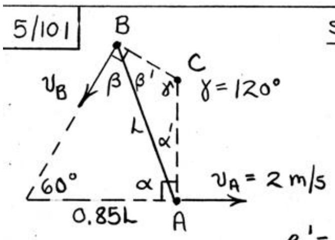

$\frac{\sin 60°}{L} = \frac{\sin\beta}{0.85L}$

$\beta = 47.4°$

$\alpha = 180° - 60° - 47.4° = 72.6°$

$\beta' = 42.6°, \quad \alpha' = 17.40°$

$\frac{\sin\gamma}{L} = \frac{\sin\beta'}{\overline{AC}}, \quad \overline{AC} = 0.782L$

$\frac{\sin\gamma}{L} = \frac{\sin\alpha'}{\overline{BC}}, \quad \overline{BC} = 0.345L$

$v_A = \overline{AC}\omega, \quad \omega = \frac{2}{0.782L} = \frac{2}{0.782(0.8)} = \underline{3.20 \text{ rad/s}}$

$v_B = \overline{BC}\omega = 0.345(0.8)(3.20) = \underline{0.884 \text{ m/s}}$

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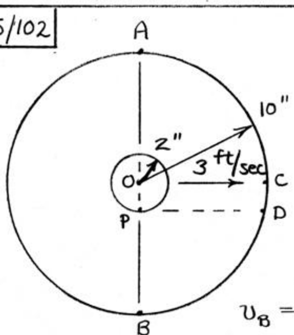


Point P is the instantaneous center

$v_0 = \overline{OP}\omega, \quad \omega = \frac{3}{2/12}$

$\omega = 18$ rad/sec CW

$v_A = \overline{AP}\omega = \frac{12}{12}(18) = \underline{18 \text{ ft/sec}} \rightarrow$

$v_B = \overline{BP}\omega = \frac{8}{12}(18) = \underline{12 \frac{ft}{sec}} \leftarrow$

$v_C = \overline{CP}\omega = \sqrt{\left(\frac{2}{12}\right)^2 + \left(\frac{10}{12}\right)^2}\,(18) = \underline{15.30 \text{ ft/sec}}$

$\alpha = \tan^{-1}\frac{2}{12} = 9.46°$

$v_D = \overline{DP}\omega = \frac{10}{12}(18) = \underline{15 \text{ ft/sec}} \downarrow$

5/103

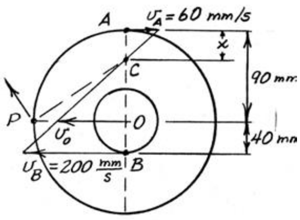
$\frac{60}{x} = \frac{60+200}{90+40}, \quad x = 30$ mm

$\frac{v_o}{\overline{CO}} = \frac{v_A}{\overline{AC}}, \quad v_o = \frac{60}{30}60 = \underline{120 \frac{mm}{s}}$

$\overline{CP} = \sqrt{60^2 + 90^2} = 108.2$ mm

$v_P = \overline{CP}\omega = \overline{CP}\frac{v_A}{\overline{AC}} = 108.2\frac{60}{30}$

$= \underline{216 \text{ mm/s}}$



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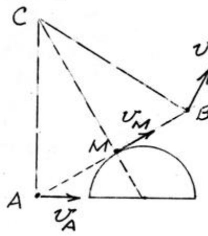
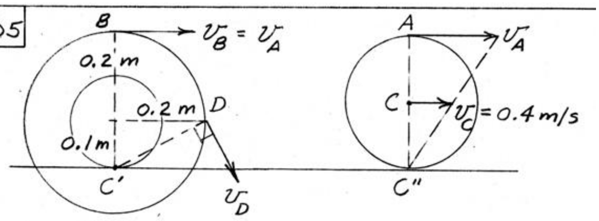


For given position $\overline{CB} = \overline{CA}$ so

$v_B = \overline{CB}\omega = \overline{CB}\frac{v_A}{\overline{AC}} = v_A$

5/105



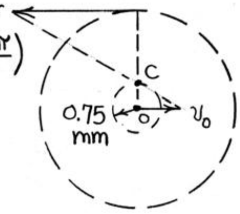
$v_B = v_A = \frac{\overline{AC''}}{\overline{CC''}}v_C = 2(0.4) = 0.8$ m/s

$v_D = \overline{C'D}\,\omega = \overline{C'D}\frac{v_B}{\overline{BC'}} = \frac{\sqrt{0.1^2 + 0.2^2}}{0.3}0.8 = \underline{0.596 \text{ m/s}}$

5/106 C = instantaneous center

$v_o = \overline{OC}\omega = 0.75(10^{-3})\left(\frac{1800 \cdot 2\pi}{60}\right)$

$= \underline{0.1414 \text{ m/s}}$



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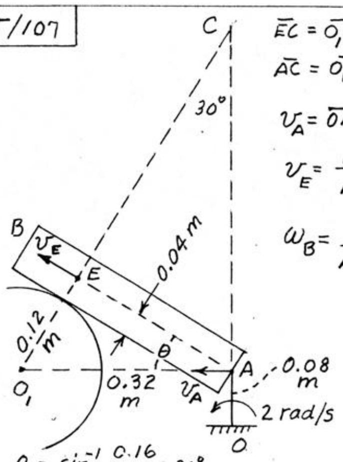


$\theta = \sin^{-1}\frac{0.16}{0.32} = 30°$

$\overline{EC} = \overline{O_1C} - 0.16 = 0.64 - 0.16 = 0.48$ m

$\overline{AC} = \overline{O_1A}\,\text{ctn}\,30° = 0.32\sqrt{3} = 0.554$ m

$v_A = \overline{OA}\,\omega_{OA} = 0.08(2) = 0.16$ m/s

$v_E = \frac{\overline{EC}}{\overline{AC}}v_A = \frac{0.48}{0.554}0.16 = \underline{0.1386 \text{ m/s}}$

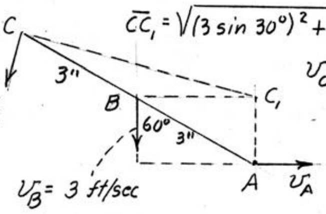
$\omega_B = \frac{v_A}{\overline{AC}} = \frac{0.16}{0.554} = \underline{0.289 \text{ rad/s CW}}$

5/108

$v_C = \overline{CC_1}\,\omega_{AC}\,; \quad \omega_{AC} = \omega_{AB} = v_B/\overline{BC_1} = \frac{3}{\frac{3}{12}\sin 60°} = 13.86$ rad/s

$\overline{CC_1} = \sqrt{(3\sin 30°)^2 + (6\cos 30°)^2} = 5.41$ in.

$v_C = \frac{5.41}{12}(13.86) = \underline{6.24 \frac{ft}{sec}}$



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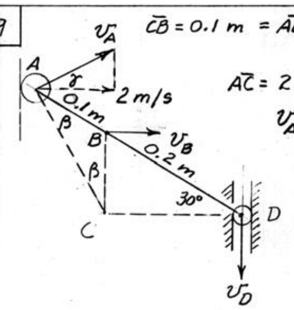


$\overline{CB} = 0.1 \text{ m} = \overline{AB}, \quad 2\beta + (180 - 60) = 180$

$\beta = 30°, \ \gamma = 30°$

$\overline{AC} = 2(0.1)\cos 30° = 0.1732$ m

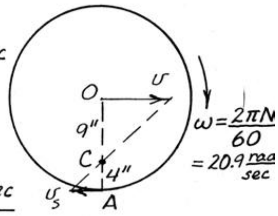
$v_A = \frac{2}{\cos 30°} = 2.31$ m/s

$\omega = \frac{v_A}{\overline{AC}} = \frac{2.31}{0.1732} = \underline{13.33 \frac{rad}{s}}$

$v_D = \overline{CD}\omega = 0.2\cos 30°(13.33)$

$= \underline{2.31 \text{ m/s}}$

5/110

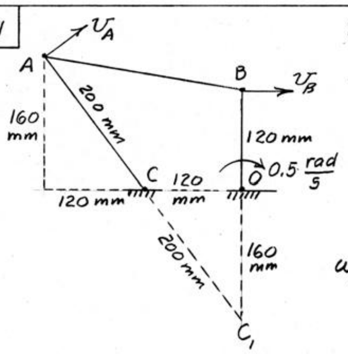


$\omega = v/\overline{OC}$, $v = \frac{9}{12} \cdot 20.9 = 15.71$ ft/sec

or $\underline{v = 10.71 \text{ mi/hr}}$

$v_s = \frac{4}{9} v = \frac{4}{9}(15.71)$, $\underline{v_s = 6.98 \text{ ft/sec}}$

5/111



$v_B = 0.120(0.5) = 0.060$ m/s

$\omega_{CA} = v_A/\overline{CA}$

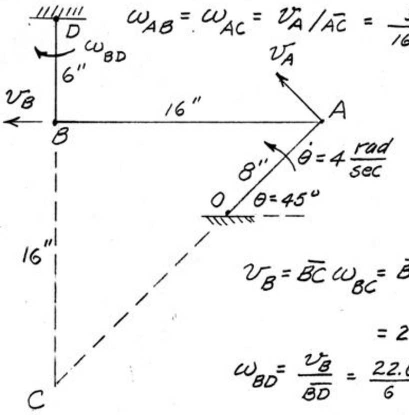
$v_A = \frac{\overline{AC_1}}{\overline{BC_1}} v_B = \frac{0.400}{0.280}(0.060) = 0.0857$ m/s

$\omega_{CA} = \frac{0.0857}{0.200} = \underline{0.429 \frac{rad}{s}}$

5/112

$v_A = \overline{AO}\,\dot\theta = 8(4) = 32$ in./sec

$\omega_{AB} = \omega_{AC} = v_A/\overline{AC} = \frac{32}{16\sqrt{2}} = \sqrt{2} = 1.414 \frac{rad}{sec}$ $\underline{\text{CCW}}$

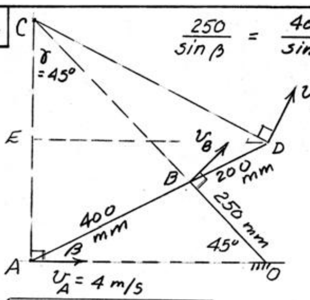


$v_B = \overline{BC}\,\omega_{BC} = \overline{BC}\frac{v_A}{\overline{AC}} = \frac{16}{16\sqrt{2}}(32)$

$= 22.6$ in./sec

$\omega_{BD} = \frac{v_B}{\overline{BD}} = \frac{22.6}{6} = \underline{3.77 \text{ rad/sec CW}}$

5/113



$\frac{250}{\sin\beta} = \frac{400}{\sin 45°}$, $\beta = 26.2°$

$\overline{AO} = 400\cos 26.2° + 250\cos 45° = 535.6$ mm

$\overline{AC} = \overline{AO}\tan 45° = 535.6$ mm

$\overline{ED} = 600\cos 26.2° = 538.2$ mm

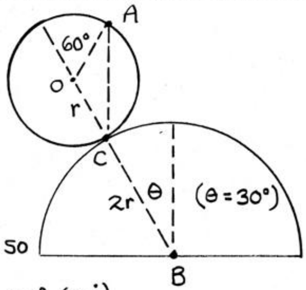
$\overline{CE} = 535.6 - 600\sin 26.2° = 270.4$ mm

$\overline{CD} = \sqrt{(270.4)^2 + (538.2)^2} = 602.4$ mm

$v_D = v_A\frac{\overline{CD}}{\overline{CA}} = 4\,\frac{602.4}{535.6} = \underline{4.50 \text{ m/s}}$

$\omega_{ABD} = \omega = v_A/\overline{CA} = \frac{4000}{535.6} = \underline{7.47 \text{ rad/s}}$

5/114



$v_O = \overline{OB}\,\dot\theta = 3r\dot\theta$

$\omega_{OC} = \omega_{AC} = \frac{v_O}{\overline{OC}} = \frac{3r\dot\theta}{r} = 3\dot\theta$

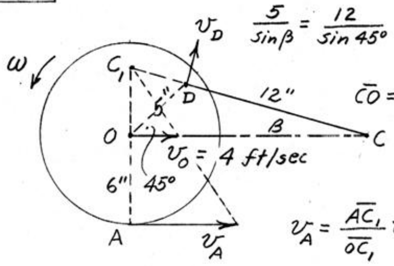
C is the instantaneous center of zero velocity, so

$v_A = \overline{AC}\,\omega_{AC} = 2r\cos 30°\,(3\dot\theta)$

$= 2r\,\frac{\sqrt{3}}{2}\,(3\dot\theta) = \underline{3\sqrt{3}\,r\dot\theta}$

5/115

C_1 = instantaneous center



$\frac{5}{\sin\beta} = \frac{12}{\sin 45°}$, $\beta = \sin^{-1}\left(\frac{5}{12}\frac{1}{\sqrt{2}}\right)$

$= 17.14°$

$\overline{CO} = 5\cos 45° + 12\cos 17.14°$

$= 15.00$ in.

$\overline{C_1O} = 15.00\tan 17.14°$

$= 4.63$ in.

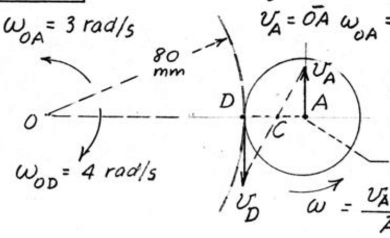
$v_A = \frac{\overline{AC_1}}{\overline{OC_1}} v_o = \frac{6 + 4.63}{4.63}\,4$

$= \underline{9.19 \text{ ft/sec}}$

5/116

$v_D = \overline{OD}\,\omega_{OD} = 80(4) = 320$ mm/s

$v_A = \overline{OA}\,\omega_{OA} = 100(3) = 300$ mm/s

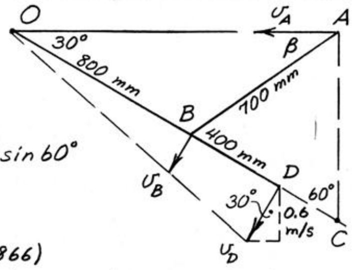


C = instant. center of small gear

$\omega = \frac{v_A + v_D}{\overline{AD}} = \frac{320 + 300}{20}$

$\underline{\omega = 31 \text{ rad/s CCW}}$

5/117



$800\sin 30° = 700\sin\beta$

$\beta = \sin^{-1}\frac{4}{7} = 34.8°$

$700\cos 34.8° = (400 + \overline{DC})\sin 60°$

$\overline{DC} = 263$ mm

$v_B = \frac{800}{1200} v_D = \frac{8}{12}(0.6/0.866)$

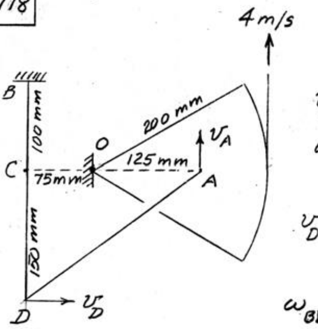
$= 0.462$ m/s

C = inst. center for AB

$\overline{AC} = 700\sin 34.8° + (400 + 263)\cos 60° = 732$ mm

$v_A/\overline{AC} = v_B/\overline{BC}$, $v_A = \frac{732}{400 + 263}\,0.462 = \underline{0.509 \text{ m/s}}$

5/118



C = instantaneous center of zero velocity for AD.

$v_A = \frac{125}{200}(4) = 2.5\ m/s$

$\omega_{AD} = \frac{v_A}{\overline{AC}} = \frac{2.5}{0.200} = 12.5\ \frac{rad}{s}$

$v_D = \overline{CD}\,\omega_{CD} = \overline{CD}\,\omega_{AD}$
$= 0.150\ (12.5)$
$= 1.875\ m/s$

$\omega_{BD} = \frac{v_D}{BD} = \frac{1.875}{0.25} = 7.5\ rad/s$

5/119 C is the instantaneous center of zero velocity for DBA

From geometry,

$\overline{AC} = \frac{5}{3}(120) = 200\ mm$

$\overline{BC} = 160\ mm$

$\overline{DC} = \sqrt{60^2 + 160^2}$

$= 170.9\ mm$

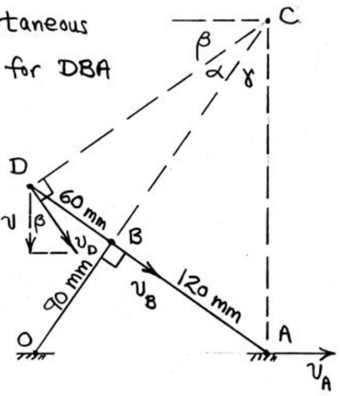
$\gamma = \sin^{-1}\frac{120}{200} = 36.9°$

$\alpha = \tan^{-1}\frac{60}{160} = 20.6°$

$\beta = 90 - 36.9 - 20.6 = 32.6°$

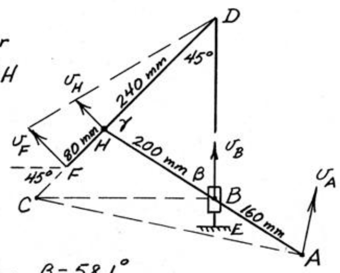
$v_D = \frac{v}{\cos\beta} = \frac{0.2}{\cos 32.6°} = 0.237\ m/s$

$\frac{v_D}{\overline{DC}} = \frac{v_A}{\overline{AC}}$; $v_A = \frac{200}{170.9}(0.237) = 0.278\ m/s$



5/120

C = instantaneous center of zero velocity of ABH



$v_F \cos 45° = v_G = 2\ m/s$

so $v_F = 2.83\ m/s$

& $v_H = \frac{240}{80+240}\,2.83$
$= 2.12\ m/s$

Law of sines, $\frac{240}{\sin\beta} = \frac{200}{\sin 45°}$, $\beta = 58.1°$

$\gamma = 180° - (58.1° + 45°) = 76.9°$

$\frac{\overline{BD}}{\sin 76.9°} = \frac{200}{\sin 45°}$, $\overline{BD} = 276\ mm$ & $\overline{DC} = \frac{276}{\cos 45°} = 390\ mm$

$\overline{CA}^2 = 276^2 + 160^2 - 2(276)(160)\cos(90° + 58.1°)$, $\overline{CA} = 420\ mm$

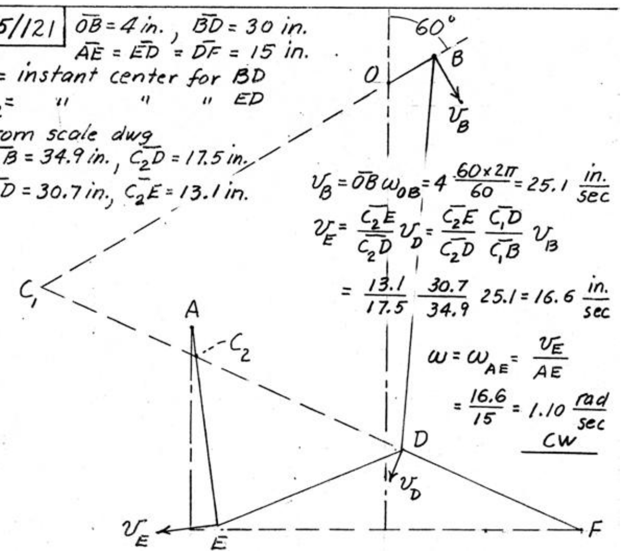
$\overline{CH} = \overline{CD} - 240 = 390 - 240 = 149.7\ mm$

$v_A/\overline{AC} = v_H/\overline{CH}$, $v_A = 2.12\,\frac{420}{149.7} = 5.95\ m/s$

►5/121 $\overline{OB} = 4\ in.$, $\overline{BD} = 30\ in.$
$\overline{AE} = \overline{ED} = \overline{DF} = 15\ in.$

C_1 = instant center for BD
C_2 = " " " ED

From scale dwg
$\overline{C_1B} = 34.9\ in.$, $\overline{C_2D} = 17.5\ in.$
$\overline{C_1D} = 30.7\ in.$, $\overline{C_2E} = 13.1\ in.$



$v_B = \overline{OB}\,\omega_{OB} = 4\,\frac{60 \times 2\pi}{60} = 25.1\ \frac{in.}{sec}$

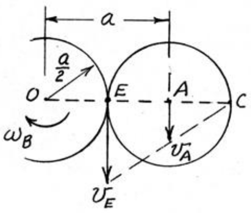
$v_E = \frac{\overline{C_2E}}{\overline{C_2D}}\,v_D = \frac{\overline{C_2E}}{\overline{C_2D}}\,\frac{\overline{C_1D}}{\overline{C_1B}}\,v_B$

$= \frac{13.1}{17.5}\,\frac{30.7}{34.9}\,25.1 = 16.6\ \frac{in.}{sec}$

$\omega = \omega_{AE} = \frac{v_E}{AE}$

$= \frac{16.6}{15} = 1.10\ \frac{rad}{sec}$ CW

►5/122

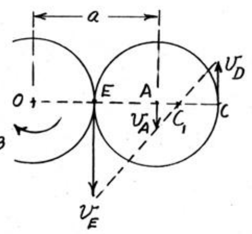


(a) $v_A = \omega_{OA}\,a$

$v_E = 2v_A = 2a\,\omega_{OA}$

$\omega_B = \frac{v_E}{a/2} = \frac{2a\,\omega_{OA}}{a/2} = 4(90)$

$= 360\ rev/min$



(b) $v_D = \overline{OC}\,\omega_D = \frac{3a}{2}\,80 = 120a$

$v_A = \overline{OA}\,\omega_{OA} = 90a$

$\frac{v_A + v_D}{a} = \frac{v_E - v_A}{a}$, $v_E = v_D + 2v_A$
$= 300a$

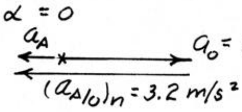
$\omega_{OE} = \omega_B = \frac{v_E}{a/2} = \frac{300a}{a/2} = 600\ \frac{rev}{min}$

5/123 $\underline{a}_A = \underline{a}_0 + (\underline{a}_{A/0})_n + (\underline{a}_{A/0})_t$ not function of v_0 or sense of ω

$(a_{A/0})_n = \overline{OA}\,\omega^2 = 0.8(2^2) = 3.2\ m/s^2$

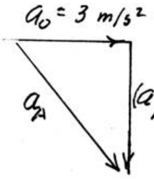
$(a_{A/0})_t = \overline{OA}\,\alpha = 0$

(a) $\theta = 0$



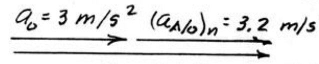
$a_A = 0.2\ m/s^2$

(b) $\theta = 90°$



$a_A = \sqrt{3^2 + 3.2^2}$
$= 4.39\ m/s^2$

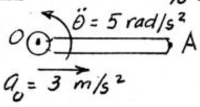
(c) $\theta = 180°$



$a_A = 3 + 3.2 = 6.2\ m/s^2$

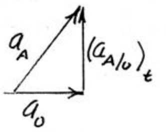
5/124 $\underline{a}_A = \underline{a}_O + (\underline{a}_{A/O})_n + (\underline{a}_{A/O})_t$

$\overline{OA} = 0.8$ m



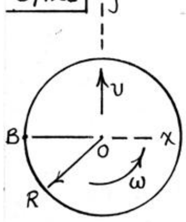
$(a_{A/O})_n = \overline{AO}\,\dot{\theta}^2 = 0$

$(a_{A/O})_t = \overline{AO}\,\ddot{\theta} = 0.8(5) = 4$ m/s²



$a_A = \sqrt{3^2 + 4^2} = \underline{5 \text{ m/s}^2}$

5/125



$$a_O = \frac{Gm_s}{r^2} = \frac{6.673(10^{-11})[5.976\cdot 10^{24}\cdot 333\,000]}{[149.6(10^9)]^2}$$

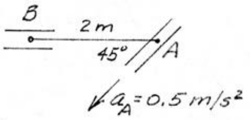
$= 0.00593$ m/s² $(\leftarrow)$

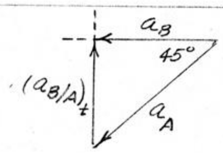
$R\omega^2 = 6371(10^3)[7.292(10^{-5})]^2 = 0.0339$ m/s² $(\rightarrow)$

$\underline{a}_B = \underline{a}_O + \underline{a}_{B/O} = -0.00593\underline{i} + 0.0339\underline{i}$

$= \underline{0.0279\underline{i} \text{ m/s}^2}$

5/126 $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$





$(a_{B/A})_t = 0.5 \sin 45° = 0.354$ m/s²

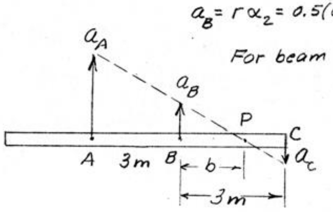
$\alpha_{AB} = \frac{0.354}{2} = \underline{0.1768 \text{ rad/s}^2}$ CW

5/127 $a_A = r\alpha_1 = 0.5(0.5) = 0.25$ m/s²

$a_B = r\alpha_2 = 0.5(0.2) = 0.1$ m/s²

For beam $\alpha = \frac{a_{A/B}}{\overline{AB}} = \frac{0.25 - 0.1}{3}$

$= \underline{0.05 \text{ rad/s}^2 \text{ CW}}$



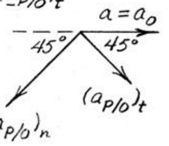
$+\downarrow\ \underline{a}_C = \underline{a}_B + \underline{a}_{C/B} = -0.1 + 3(0.05) = \underline{0.05 \text{ m/s}^2 \text{ down}}$

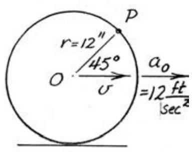
$+\downarrow\ \underline{a}_P = 0 = \underline{a}_B + \underline{a}_{P/B} = -0.1 + b(0.05)$, $\underline{b = 2\text{ m}}$

5/128 $\underline{a}_P = \underline{a}_O + (\underline{a}_{P/O})_n + (\underline{a}_{P/O})_t$

$(a_{P/O})_n = r\omega^2 = r\left(\frac{v}{r}\right)^2 = \frac{v^2}{r}$

$(a_{P/O})_t = r\alpha = r\left(\frac{a_O}{r}\right) = a_O$





For $(a_P)_{horiz} = 0$, $\frac{v^2}{1}\cos 45° = 12 + 12\cos 45°$

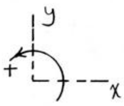
$v^2 = 29.0$ ft²/sec²

$v = 5.38$ ft/sec or $\underline{v = 3.67 \text{ mi/hr}}$

5/129 In the coordinates shown, the no-slip kinematic constraints are $v_O = -r\omega$, $a_O = -r\alpha$.

So $\omega = -\frac{v_O}{r} = -\frac{3}{0.4} = -7.5$ rad/s

$\alpha = -\frac{a_O}{r} = -\frac{-5}{0.4} = 12.5$ rad/s²



$\underline{v}_A = \underline{v}_O + \underline{v}_{A/O} = \underline{v}_O + \underline{\omega} \times \underline{r}_{A/O}$

$= 3\underline{i} + (-7.5\underline{k}) \times 0.4[-\cos 45°\underline{i} + \sin 45°\underline{j}]$

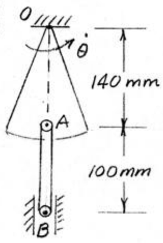
$= \underline{5.12\underline{i} + 2.12\underline{j} \text{ m/s}}$

$\underline{a}_B = \underline{a}_O + \underline{a}_{B/O} = \underline{a}_O + \underline{\alpha} \times \underline{r}_{B/O} - \omega^2 \underline{r}_{B/O}$

$= -5\underline{i} + 12.5\underline{k} \times 0.2\underline{i} - (-7.5)^2(0.2\underline{i})$

$= \underline{-16.25\underline{i} + 2.5\underline{j} \text{ m/s}^2}$

5/130



$\theta = \frac{\pi}{12}\sin 2\pi t$, $\dot{\theta} = \frac{\pi^2}{6}\cos 2\pi t$

$\ddot{\theta} = -\frac{\pi^3}{3}\sin 2\pi t$

$\theta = 0$, $\dot{\theta} = \pi^2/6$ rad/s², $\ddot{\theta} = 0$

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$, $a_A = 0.140\left(\frac{\pi^2}{6}\right)^2 = 0.379$ m/s² ↑

$v_A = 0.140\left(\frac{\pi^2}{6}\right) = 0.230$ m/s

$\omega_{AB} = \frac{0.230}{0.100} = 2.30$ rad/s

$a_{B/A} = (a_{B/A})_n = 0.100(2.30)^2 = 0.530$ m/s² ↑

$a_B = 0.379 + 0.530 = \underline{0.909 \text{ m/s}^2 \text{ (up)}}$

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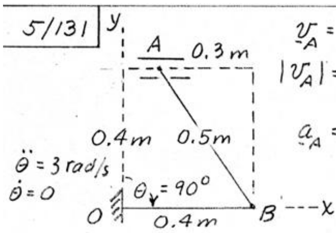


$\underline{v}_A = \underline{v}_B + \underline{\omega}_{AB} \times \underline{r}_{A/B}$

$|v_A| = |v_B| = |\omega_{AB}| = 0$ for $\dot{\theta} = 0$

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_t$, $(\underline{a}_{A/B})_n = \underline{0}$

$a_A\underline{i} = 0.4(3)(-\underline{j}) + \alpha_{AB}\underline{k} \times (-0.3\underline{i} + 0.4\underline{j})$

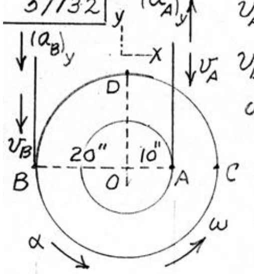
$= -1.2\underline{j} - 0.3\alpha_{AB}\underline{j} - 0.4\alpha_{AB}\underline{i}$

$a_A = -0.4\alpha_{AB}$ & $0 = -1.2 - 0.3\alpha_{AB}$

$\alpha_{AB} = -4$ rad/s², $\underline{\alpha}_{AB} = \underline{-4\underline{k} \text{ rad/s}^2}$

$a_A = -0.4(-4) = +1.6$ m/s², $\underline{a}_A = \underline{1.6\underline{i} \text{ m/s}^2}$

5/132



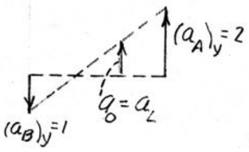
$v_A = \frac{6}{12}4 = 2$ ft/sec, $(a_A)_y = \frac{6}{12}4 = 2$ ft/sec²

$v_B = \frac{6}{12}6 = 3$ ft/sec, $(a_B)_y = \frac{6}{12}2 = 1$ ft/sec²

$\omega = \frac{3-2}{30/12} = 0.4$ rad/sec CCW

$\alpha = \frac{2+1}{30/12} = 1.2$ rad/sec² CCW

From diagram, $\underline{a}_L = \underline{a}_O = \underline{1\underline{j} \text{ ft/sec}^2}$



$\underline{a}_C = \underline{a}_O + (\underline{a}_{C/O})_n + (\underline{a}_{C/O})_t$

$= 1\underline{j} - \frac{20}{12}(0.4)^2\underline{i} + \frac{20}{12}(1.2)\underline{j}$

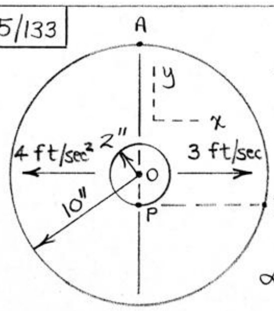
$= \underline{3\underline{j} - 0.267\underline{i} \text{ ft/sec}^2}$

$\underline{a}_D = \underline{a}_O + (\underline{a}_{D/O})_n + (\underline{a}_{D/O})_t$

$= 1\underline{j} - \frac{20}{12}(0.4)^2\underline{j} - \frac{20}{12}(1.2)\underline{i}$

$= \underline{0.733\underline{j} - 2\underline{i} \text{ ft/sec}^2}$

5/133



From the solution to Prob. 5/102 or from $v_0 = r\omega$, $\omega = \frac{v_0}{r} = \frac{3}{2/12} = 18$ rad/sec CW.

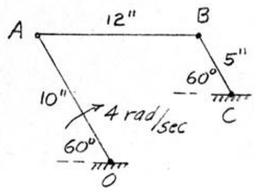
From $a_0 = r\alpha$, $\alpha = \frac{a_0}{r} = \frac{4}{2/12} = 24 \frac{\text{rad}}{\text{sec}^2}$ CCW

$$\underline{a}_A = \underline{a}_0 + \underline{a}_{A/0} = \underline{a}_0 + \underline{\alpha} \times \underline{r}_{A/0} - \omega^2 \underline{r}_{A/0}$$
$$= -4\underline{i} + 24\underline{k} \times \tfrac{10}{12}\underline{j} - 18^2\left(\tfrac{10}{12}\underline{j}\right)$$
$$= \underline{-24\underline{i} - 270\underline{j} \text{ ft/sec}^2}$$

$$\underline{a}_D = \underline{a}_0 + \underline{a}_{D/0} = \underline{a}_0 + \underline{\alpha} \times \underline{r}_{D/0} - \omega^2 \underline{r}_{D/0}$$
$$= -4\underline{i} + 24\underline{k} \times \left(\tfrac{10}{12}\cos\sin^{-1}\tfrac{2}{10}\,\underline{i} - \tfrac{2}{12}\underline{j}\right)$$
$$- 18^2\left(\tfrac{10}{12}\cos\sin^{-1}\tfrac{2}{10}\,\underline{i} - \tfrac{2}{12}\underline{j}\right)$$
$$= \underline{-265\underline{i} + 73.6\underline{j} \text{ ft/sec}^2}$$

(Could use P as a base point for $\underline{a}_D$.)

5/134 $v_A = r\omega = 10(4) = 40$ in./sec $= v_B$

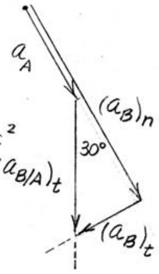


$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$\omega_{AB} = 0$ so $(a_{B/A})_n = 0$

$a_A = 10(4^2) = 160$ in./sec²

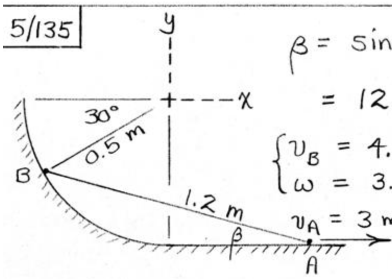
$(a_B)_n = v_B^2/\overline{BC} = 40^2/5 = 320 \frac{\text{in.}}{\text{sec}^2}$



$(a_{B/A})_t = \frac{160}{\cos 30°} = 185 \frac{\text{in.}}{\text{sec}^2}$

$\alpha_{AB} = \frac{185}{12} = \underline{15.40 \text{ rad/sec}^2}$ CW

5/135



$\beta = \sin^{-1}\frac{0.5 - 0.5\sin 30°}{1.2} = 12.02°$

$\begin{cases} v_B = 4.38 \text{ m/s} \\ \omega = 3.23 \text{ rad/s} \end{cases}$ (Prob. 5/68)

$v_A = 3$ m/s = constant

$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = \underline{a}_A + \underline{\alpha} \times \underline{r}_{B/A} - \omega^2 \underline{r}_{B/A}$$
$$a_{B_t}(\sin 30°\,\underline{i} - \cos 30°\,\underline{j}) + \frac{4.38^2}{0.5}(\cos 30°\,\underline{i} + \sin 30°\,\underline{j})$$
$$= \underline{0} + \alpha\underline{k} \times 1.2(-\cos 12.02°\,\underline{i} + \sin 12.02°\,\underline{j})$$
$$- 3.23^2(1.2)(-\cos 12.02°\,\underline{i} + \sin 12.02°\,\underline{j})$$

Carry out vector algebra & equate coefficients:

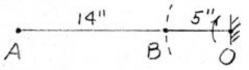
$\underline{i}$: $\frac{1}{2}a_{B_t} + 33.3 = -0.250\alpha + 12.28$

$\underline{j}$: $-\frac{\sqrt{3}}{2}a_{B_t} + 19.21 = -1.174\alpha - 2.61$

Solution: $\underline{a_{B_t} = -23.9 \text{ m/s}^2}$, $\underline{\alpha = -36.2 \text{ rad/s}^2}$

5/136 $\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$ ---→ +x

(a)



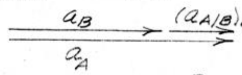
From Sample Prob. 5/15

$a_B = (a_B)_n = 10{,}280$ ft/sec²

$v_B = 65.45$ ft/sec

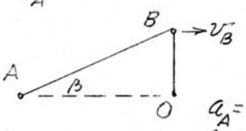
$v_{A/B} = v_B = 65.4$ ft/sec

$(a_{A/B})_n = (65.45)^2/\frac{14}{12} = 3670$ ft/sec²



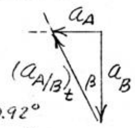
$\underline{a}_A = 13{,}950\underline{i}$ ft/sec²

(b) $\beta = \sin^{-1}\frac{5}{14} = 20.92°$



$\omega_{AB} = 0$

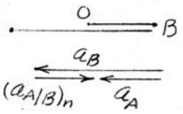
$(a_{A/B})_n = 0$



$a_A = 10{,}280\tan 20.92°$

$\underline{a}_A = \underline{-3930\underline{i} \text{ ft/sec}^2}$

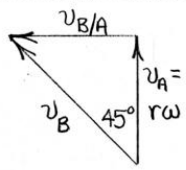
(c)



$a_A = 10{,}280 - 3670$ ft/sec²

$\underline{a}_A = \underline{-6610\underline{i} \text{ ft/sec}^2}$

5/137 $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$



$v_{B/A} = r\omega$, $\omega_{AB} = \frac{r\omega}{r} = \omega$

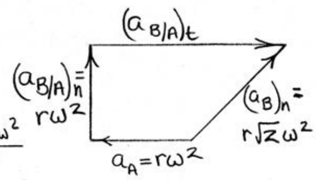
$v_B = r\omega\sqrt{2}$, $\omega_{BC} = \frac{r\omega\sqrt{2}}{r\sqrt{2}} = \omega$

$(\underline{a}_B)_n + (\underline{a}_B)_t = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$a_A = r\omega^2 \leftarrow$; $(a_{B/A})_n = r\omega^2 \uparrow$

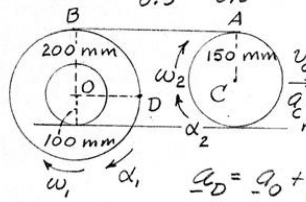
$(a_B)_n = r\sqrt{2}\,\omega^2$ ∠45°

$(a_B)_t = 0$ so $\underline{\alpha_{BC} = 0}$



$(a_{B/A})_t = 2r\omega^2$, so $\alpha_{AB} = \frac{2r\omega^2}{r} = \underline{2\omega^2}$

5/138 $\omega_1 = \frac{v_B}{0.3} = \frac{v_A}{0.3} = \omega_2 = \frac{2(0.4)}{0.3} = \frac{8}{3}$ rad/s



$\alpha_1 = \frac{a_{B_x}}{0.3} = \frac{a_{A_x}}{0.3}$
$= \frac{2(0.8)}{0.3} = \frac{16}{3}\ \frac{rad}{s^2}$

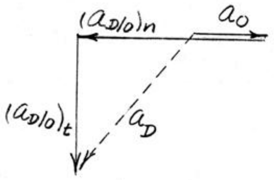
$\underline{a}_D = \underline{a}_O + (\underline{a}_{D/O})_n + (\underline{a}_{D/O})_t$

$a_O = \frac{100}{300} a_{B_x} = \frac{1}{3} a_{A_x} = \frac{1}{3}(2)(0.8) = \frac{1.6}{3} = 0.533\ m/s^2$

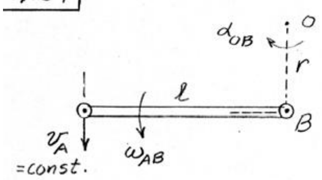
$(a_{D/O})_n = 0.2(8/3)^2 = 1.422\ m/s^2$

$(a_{D/O})_t = 0.2(16/3) = 1.067\ m/s^2$

$a_D = \sqrt{(1.422 - 0.533)^2 + (1.067)^2}$
$= \sqrt{1.928} = \underline{1.388\ m/s^2}$



5/139

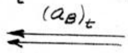


$\omega_{AB} = v_A / l$

$v_B = 0$ so $(a_B)_n = \frac{v_B^2}{r} = 0$

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$\underline{a}_{B_t} = 0 + (\underline{a}_{B/A})_n + 0$

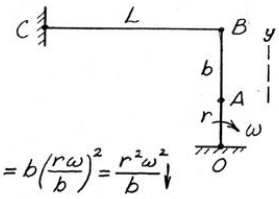


$(a_{B/A})_n = l\omega_{AB}^2 = v_A^2 / l$

Thus $\alpha_{OB} = \frac{(a_B)_t}{r} = \underline{\frac{v_A^2}{rl}}$

5/140

For this position $(v_A)_y = 0$
so $v_B = 0$, $\omega_{BC} = 0$
$\omega_{AB} = v_A / b = r\omega / b$ CCW

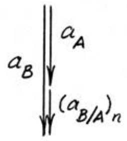


$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$, $a_A = r\omega^2 \downarrow$, $(a_{B/A})_n = b\left(\frac{r\omega}{b}\right)^2 = \frac{r^2\omega^2}{b} \downarrow$

$(a_B)_n = L\omega_{BC}^2 = 0$

$a_B = (a_B)_t = r\omega^2 + \frac{r}{b} r\omega^2 = r\omega^2\left(1 + \frac{r}{b}\right)$

$\alpha_{BC} = \frac{(a_B)_t}{L} = \underline{\frac{r\omega^2}{L}\left(1 + \frac{r}{b}\right)\ CW}$



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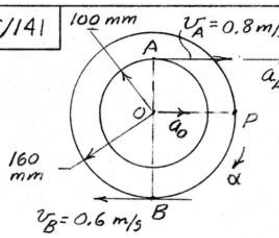


$a_A = 2\ m/s^2$

$a_O = \frac{160}{260}(2) = 1.231\ m/s$

$\alpha = \frac{a_O}{r} = \frac{1.231}{0.160} = 7.69\ rad/s^2$

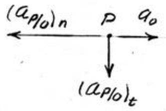
$\omega = \frac{v_A + v_B}{\overline{AB}} = \frac{0.8 + 0.6}{0.260} = 5.38\ \frac{rad}{s}$

$\underline{a}_P = \underline{a}_O + (\underline{a}_{P/O})_n + (\underline{a}_{P/O})_t$

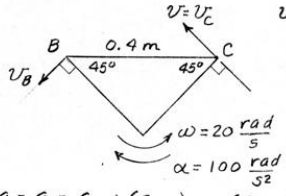
$(a_{P/O})_n = \overline{PO}\,\omega^2 = 0.160(5.38)^2 = 4.64\ m/s^2$

$(a_{P/O})_t = \overline{PO}\,\alpha = 0.16(7.69) = 1.231\ m/s^2$

$a_P = \sqrt{(4.64 - 1.231)^2 + (1.231)^2} = \underline{3.62\ m/s^2}$



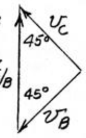
5/142



$\underline{v} = \underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$

$v_{C/B} = \overline{CB}\,\omega$
$= 0.4(20)$
$= 8$ m/s

$v = v_C = 8/\sqrt{2}$
$= \underline{5.66\ m/s}$

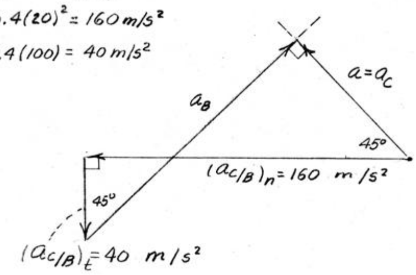


$\underline{a} = \underline{a}_C = \underline{a}_B + (\underline{a}_{C/B})_n + (\underline{a}_{C/B})_t$

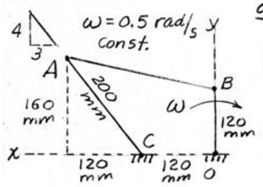
$(a_{C/B})_n = \overline{CB}\,\omega^2 = 0.4(20)^2 = 160\ m/s^2$

$(a_{C/B})_t = \overline{CB}\,\alpha = 0.4(100) = 40\ m/s^2$

From diagram
$a = 160/\sqrt{2} - 40/\sqrt{2}$
$= \underline{84.9\ m/s^2}$



5/143 From Prob. 5/87, $\underline{\omega}_{AB} = 0.214\underline{k}$, $\underline{\omega}_{CA} = 0.429\underline{k}\ \frac{rad}{s}$



$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$ ---- (a)

$\underline{a}_A = \underline{\omega}_{CA} \times (\underline{\omega}_{CA} \times \underline{r}_{CA}) + \underline{\alpha}_{CA} \times \underline{r}_{CA}$
$= (0.429)^2(\underline{k} \times [\underline{k} \times \{0.12\underline{i} + 0.16\underline{j}\}])$
$+ \alpha_{CA}\underline{k} \times (0.12\underline{i} + 0.16\underline{j})$
$= 0.1837(-0.12\underline{i} - 0.16\underline{j})$
$+ 0.12\alpha_{CA}\underline{j} - 0.16\alpha_{CA}\underline{i}$

$\underline{a}_B = \underline{\omega} \times (\underline{\omega} \times \underline{r}_{OB}) = (0.5)^2(0.12)(-\underline{j}) = -0.03\underline{j}\ m/s^2$

$(\underline{a}_{A/B})_n = \underline{\omega}_{AB} \times (\underline{\omega}_{AB} \times \underline{r}_{BA}) = (0.214)^2(\underline{k} \times [\underline{k} \times \{0.24\underline{i} + 0.04\underline{j}\}])$
$= 0.0459(-0.24\underline{i} - 0.04\underline{j})\ m/s^2$

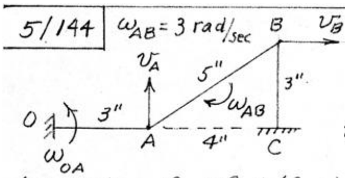
$(\underline{a}_{A/B})_t = \alpha_{AB}\underline{k} \times \underline{r}_{BA} = \alpha_{AB}\underline{k} \times (0.24\underline{i} + 0.04\underline{j}) = \alpha_{AB}(0.24\underline{j} - 0.04\underline{i})$

Substitute terms into Eq.(a) & equate separately $\underline{i}$ & $\underline{j}$ coefficients & get $\alpha_{AB} - 4\alpha_{CA} = 0.2755$
$2\alpha_{AB} - \alpha_{CA} = 0.02041$

Solve & get $\alpha_{CA} = -0.0758\ rad/s^2$, $\alpha_{AB} = -0.0277\ rad/s^2$

$\underline{\alpha}_{CA} = \underline{-0.0758\,\underline{k}\ rad/s^2}$

5/144 $\omega_{AB} = 3$ rad/sec



C = instant. center of AB

$\omega_{AB} = \frac{v_A}{\overline{AC}}$, $v_A = \frac{4}{12}3 = 1$ ft/sec

$v_B = \overline{BC}\,\omega_{AB} = \frac{3}{12}3 = 0.75$ ft/sec

$(\alpha_{OA} = 0)$ $\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$(a_B)_n = v_B^2 / \overline{BC} = (0.75)^2 / \frac{3}{12} = 2.25\ ft/sec^2$

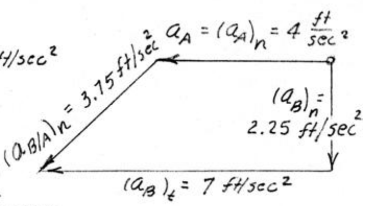
$(a_B)_t = \overline{BC}\,\alpha_{BC}$

$(a_{B/A})_n = \overline{AB}\,\omega_{AB}^2 = \frac{5}{12}3^2 = 3.75\ ft/sec^2$

$(a_{B/A})_t = \overline{AB}\,\alpha_{AB}$

$(a_A)_n = v_A^2 / \overline{AO} = 1^2 / \frac{3}{12} = 4\ ft/sec^2$

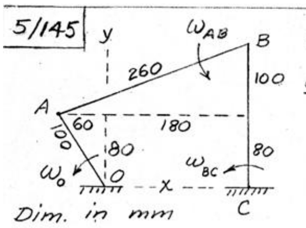
$(a_A)_t = \overline{AO}\,\alpha_{AO} = 0$



From diag,
$(a_{B/A})_t = 0$ so $\underline{\alpha}_{AB} = \alpha_{ABD} = 0$

$\alpha_{BC} = 7 / \frac{3}{12} = \underline{28\ rad/sec^2\ CCW}$

5/145



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$\underline{a}_B = \underline{\omega}_{BC}\times(\underline{\omega}_{BC}\times \underline{r}_{B/C}) + \underline{\alpha}_{BC}\times \underline{r}_{B/C}$

$= 5.83\underline{k}\times(5.83\underline{k}\times 0.18\underline{j})$

$+ \alpha_{BC}\underline{k}\times 0.18\underline{j}$ m/s^2

$= -6.125\underline{j} - 0.18\alpha_{BC}\underline{i}$ m/s^2

$\underline{a}_A = \underline{\omega}_0\times(\underline{\omega}_0\times \underline{r}_{A/O}) = 10\underline{k}\times(10\underline{k}\times[-0.06\underline{i}+0.08\underline{j}])$

$= 6\underline{i} - 8\underline{j}$ m/s^2 $\quad(\alpha_{OA}=0)$

$(a_{B/A})_n = \underline{\omega}_{AB}\times(\underline{\omega}_{AB}\times \underline{r}_{B/A}) = 2.5\underline{k}\times(2.5\underline{k}\times[0.24\underline{i}+0.1\underline{j}])$

$= -1.5\underline{i} - 0.625\underline{j}$ m/s^2

$(a_{B/A})_t = \alpha_{AB}\underline{k}\times(0.24\underline{i}+0.1\underline{j}) = -0.1\alpha_{AB}\underline{i} + 0.24\alpha_{AB}\underline{j}$

Substitute in accel. equation & equate coefficients & set

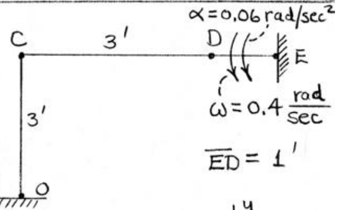
$-0.18\alpha_{BC} = 6 - 1.5 - 0.1\alpha_{AB}$

$-6.125 = -8 - 0.625 + 0.24\alpha_{AB}$

Sol. is $\underline{\alpha}_{AB} = \underline{10.42\underline{k}\ rad/s^2}$

$(\underline{\alpha}_{BC} = -19.21\underline{k}\ rad/s^2)$

5/146



$v_C = 0$

$\omega_{CD} = \frac{v_D}{\overline{CD}} = \frac{1(0.4)}{3}$

$= 0.1333$ rad/sec

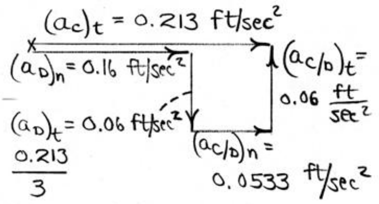
$\underline{a}_C = \underline{a}_D + (\underline{a}_{C/D})_n + (\underline{a}_{C/D})_t$

$(a_D)_n = 1(0.4)^2 = 0.16\ ft/sec^2 \rightarrow$

$(a_D)_t = 1(0.06) = 0.06\ ft/sec^2 \downarrow$

$(a_{C/D})_n = 3(0.1333)^2 = 0.0533\ ft/sec^2 \rightarrow$

$(a_C)_n = \frac{v_C^2}{\overline{CO}} = 0$



$\alpha_{AB} = \alpha_{CO} = \frac{(a_C)_t}{\overline{CO}} = \frac{0.213}{3}$

$= 0.0711\ rad/sec^2$ CW

$\omega_{AB} = \omega_{CO} = \frac{v_C}{\overline{CO}} = 0$

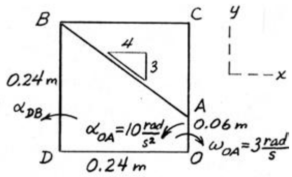
So $(\underline{a}_{A/B}) = (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t = \underline{0} + \overline{AB}\alpha_{AB}\underline{j}$

$= (4+6)(0.0711)\underline{j} = \underline{0.711\underline{j}\ ft/sec^2}$

5/147

$v_B = v_A = \overline{OA}\,\omega_{OA} = 0.06(3) = 0.18\ m/s^2$

$\omega_{DB} = v_B/\overline{DB} = 0.18/0.24 = 0.75\ rad/s$



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$\underline{a}_{B_n} + \underline{a}_{B_t} = \underline{a}_{A_n} + \underline{a}_{A_t} + \underline{a}_{B/A_n} + \underline{a}_{B/A_t}$

$\underline{a}_{B_n} = \overline{DB}\,\omega_{DB}^2(-\underline{j}) = 0.24(0.75)^2(-\underline{j})$

$= -0.135\underline{j}$ m/s^2

$\underline{a}_{B_t} = \overline{DB}\,\alpha_{DB}(-\underline{i}) = -0.24\alpha_{DB}\underline{i}$

$\underline{a}_{A_n} = \overline{OA}\,\omega_{OA}^2(-\underline{j}) = 0.06(3^2)(-\underline{j}) = -0.54\underline{j}$ m/s^2

$\underline{a}_{A_t} = \overline{OA}\,\alpha_{OA}(-\underline{i}) = 0.06(10)(-\underline{i}) = -0.6\underline{i}$ m/s^2

$(\underline{a}_{B/A})_n = \overrightarrow{BA}\,\omega_{AB}^2 = \underline{0}$ since $\omega_{AB} = 0$

$(\underline{a}_{B/A})_t = \alpha_{AB}\underline{k}\times\overrightarrow{AB} = \alpha_{AB}\underline{k}\times(-0.24\underline{i}+0.18\underline{j}) = -0.24\alpha_{AB}\underline{j} - 0.18\alpha_{AB}\underline{i}$

$-0.135\underline{j} - 0.24\alpha_{DB}\underline{i} = -0.54\underline{j} - 0.6\underline{i} + \underline{0} - 0.24\alpha_{AB}\underline{j} - 0.18\alpha_{AB}\underline{i}$

$\underline{j}$-terms: $-0.135 = -0.54 - 0.24\alpha_{AB}$, $\alpha_{AB} = -1.688\ rad/s^2$ (CW)

$\underline{i}$-terms: $-0.24\alpha_{DB} = -0.6 - 0.18(-1.688)$,

$\underline{\alpha_{DB} = 1.234\ rad/s^2\ CCW}$

5/148 Using C as the instant. center for AB gives

$v_B = 0.150(40) = 6\ m/s$, $\omega_{BC} = \frac{6}{0.15} = 40\ rad/s$

$v_A = 0.2(40) = 8\ m/s$, $\omega_{AO} = \frac{8}{0.1} = 80\ rad/s$

$(\underline{a}_A)_n + (\underline{a}_A)_t = (\underline{a}_B)_n + (\underline{a}_B)_t + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$(\underline{a}_A)_n = 0.1(80)^2(-\underline{i}) = -640\underline{i}$ m/s^2

$(\underline{a}_B)_n = 0.150(40)^2(-\underline{j}) = -240\underline{j}$ m/s^2

$(\underline{a}_{A/B})_n = \underline{\omega}_{AB}\times(\underline{\omega}_{AB}\times \underline{r}_{A/B}) = 40\underline{k}\times(40\underline{k}\times[0.2\underline{i} - 0.15\underline{j}])$

$= -320\underline{i} + 240\underline{j}$ m/s^2

$(\underline{a}_{A/B})_t = \underline{\alpha}_{AB}\times \underline{r}_{A/B} = 0$ for ω_{AB} const.

Substitute & equate $\underline{i}$ & $\underline{j}$ coefficients & get

$(\underline{a}_A)_t = -240\underline{j} + 240\underline{j} = 0$ so $\underline{\alpha_{OA} = 0}$

$(\underline{a}_B)_t = -320\underline{i}$ m/s^2, $(\underline{a}_D)_n = \frac{225}{150}(240)(-\underline{j}) = -360\underline{j}$ m/s^2

$(\underline{a}_D)_t = \frac{225}{150}(320)(-\underline{i}) = -480\underline{i}$ m/s^2, $\underline{a}_D = \underline{-120(4\underline{i}+3\underline{j})\ m/s^2}$

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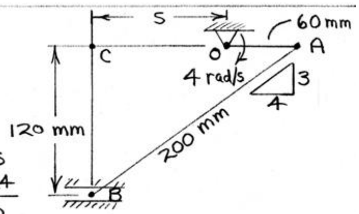


$(60+s)^2 + 120^2 = 200^2$

$s = 100$ mm

$v_A = 0.06(4) = 0.24$ m/s

$\omega_{AB} = \frac{v_A}{\overline{AC}} = \frac{0.24}{0.160}$

$= 1.5$ rad/s

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$; $a_A = (a_A)_n = 0.06(4)^2$

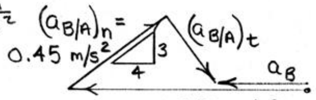
$= 0.96\ \frac{m}{s^2} \leftarrow$

$(a_{B/A})_n = 0.2(1.5)^2$

$= 0.45\ m/s^2$ $\measuredangle$ 45°

From the diagram,

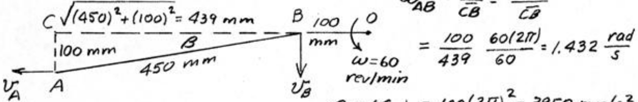
$(a_{B/A})_t = \frac{3}{4}(0.45) = 0.338\ \frac{m}{s^2}$



$\alpha_{AB} = (a_{B/A})_t/\overline{AB}$

$= \frac{0.338}{0.2} = \underline{1.688\ rad/s^2\ ccw}$

5/150 C = instantaneous center of zero velocity for AB

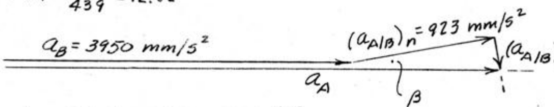


$$\omega_{AB} = \frac{v_B}{\overline{CB}} = \frac{\overline{OB}\omega}{\overline{CB}} = \frac{100}{439}\,\frac{60(2\pi)}{60} = 1.432\ \frac{rad}{s}$$

$$a_B = (a_B)_n = 100(2\pi)^2 = 3950\ mm/s^2$$

$$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t\ ; \quad (a_{A/B})_n = 450(1.432)^2 = 923\ mm/s^2$$

$$\beta = \tan^{-1}\frac{100}{439} = 12.82°$$



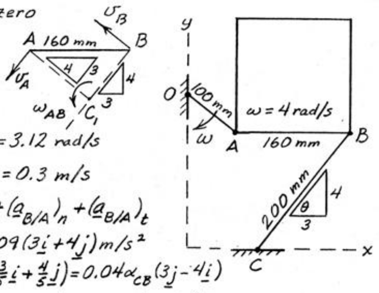
$$(a_{A/B})_t = 923\tan 12.82 = 923\,\frac{100}{439} = 210\ mm/s^2$$

$$\alpha_{AB} = \frac{(a_{A/B})_t}{\overline{AB}} = \frac{210}{450} = \underline{0.467\ rad/s^2\ CCW}$$

$$a_A = 3950 + 923\cos 12.82° + 210\sin 12.82° = 4890\ mm/s^2$$

or $\underline{a_A = 4.89\ m/s^2}$

5/151 C_1 = inst. center of zero vel. for AB



$\overline{AC}_1 = \frac{4}{5}160 = 128$ mm

$\overline{BC}_1 = \frac{3}{5}160 = 96$ mm

$\omega_{AB} = v_A/\overline{AC}_1 = 0.1(4)/0.128 = 3.12$ rad/s

$v_B = \overline{C_1B}\,\omega_{AB} = 0.096(3.12) = 0.3$ m/s

$(\underline{a}_B)_n + (\underline{a}_B)_t = (\underline{a}_A)_n + (\underline{a}_A)_t + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$

$(\underline{a}_B)_n = \frac{0.3^2}{0.2}(-\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}) = -0.09(3\underline{i}+4\underline{j})\,m/s^2$

$(\underline{a}_B)_t = \underline{\alpha}_{CB}\times\underline{r}_{CB} = \alpha_{CB}\underline{k}\times 0.2(\frac{3}{5}\underline{i}+\frac{4}{5}\underline{j}) = 0.04\alpha_{CB}(3\underline{j}-4\underline{i})$

$(\underline{a}_A)_n = 0.1(4^2)(-\frac{4}{5}\underline{i}+\frac{3}{5}\underline{j}) = 0.32(-4\underline{i}+3\underline{j})\ m/s^2$

$(\underline{a}_A)_t = \underline{0}$

$(\underline{a}_{B/A})_n = 0.160(3.12^2)(-\underline{i}) = -1.562\,\underline{i}\ m/s^2$

$(\underline{a}_{B/A})_t = \underline{\alpha}_{AB}\times\underline{r}_{AB} = \alpha_{AB}\underline{k}\times 0.16\underline{i} = 0.16\alpha_{AB}\underline{j}$

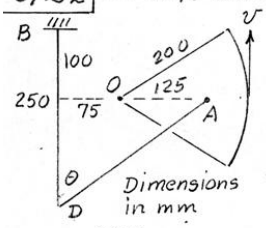
Thus $-0.09(3\underline{i}+4\underline{j}) + 0.04\alpha_{CB}(3\underline{j}-4\underline{i}) = 0.32(-4\underline{i}+3\underline{j}) - 1.562\underline{i} + 0.16\alpha_{AB}\underline{j}$

Equate $\underline{i}$-terms: $-0.27 - 0.16\alpha_{CB} = -1.28 - 1.562$, $\underline{\alpha_{CB} = 16.08\ rad/s^2\ CCW}$

" $\underline{j}$-terms: $-0.36 + 0.12(16.08) = 0.96 + 0.16\alpha_{AB}$,

$\underline{\alpha_{AB} = 3.81\ rad/s^2\ CCW}$

5/152 $v = 4$ m/s const.



From Prob. 5/118

$v_A = 2.5\ \frac{m}{s}$, $v_D = 1.875\ \frac{m}{s}$

$\omega_{AD} = 12.5$ rad/s

$\underline{a}_D = \underline{a}_A + \underline{a}_{D/A}$

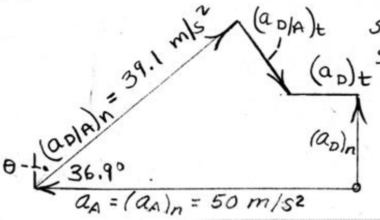
$(a_D)_n = v_D^2/\overline{BD} = \frac{1.875^2}{0.250} = 14.06\ m/s^2$

$a_A = (a_A)_n = v_A^2/\overline{OA} = \frac{2.5^2}{0.125} = 50\ m/s^2$

$(a_{D/A})_n = \overline{AD}\,\omega_{AD}^2 = 0.250(12.5)^2 = 39.1\ m/s^2$

$\theta = \tan^{-1}\frac{200}{150} = 53.1°$

$\overline{AD} = 250$ mm



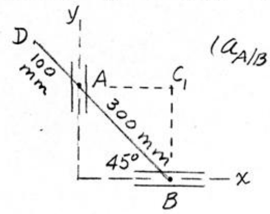
Solution of polygon gives $(a_{D/A})_t = 11.72\,m/s^2$

$(a_D)_t = 11.72\,m/s^2$

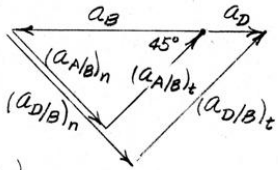
$\alpha_{BD} = (a_D)_t/\overline{BD}$

$= \frac{11.72}{0.25} = \underline{46.9\ \frac{rad}{s^2}\ CW}$

5/153 $\underline{a}_A = \underline{0} = \underline{a}_B + \underline{a}_{A/B}$; $\omega_{AB} = \frac{v_A}{\overline{AC}_1} = \frac{0.5}{0.3/\sqrt{2}} = 2.36\ \frac{rad}{sec}$



$(a_{A/B})_n = 0.3(2.36)^2 = 1.667\ m/s^2$



$\underline{a}_D = \underline{a}_B + (\underline{a}_{D/B})_n + (\underline{a}_{D/B})_t$

where $(a_{D/B})_n = \frac{400}{300}(a_{A/B})_n = \frac{4}{3}(1.667) = 2.22\ m/s^2$

$(a_{D/B})_t = \frac{400}{300}(a_{A/B})_t = 2.22\ m/s^2$

$a_D = (2.22 - 1.667)\sqrt{2} = 0.786\ m/s^2$

or $\underline{\underline{a}_D = 0.786\,\underline{i}\ m/s^2}$

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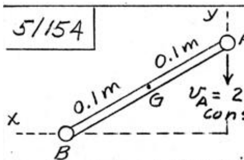


By inspection using instant center,

$\underline{\omega}_{AB} = \frac{2}{0.2}\,\frac{2}{\sqrt{3}}\underline{k} = \frac{20}{\sqrt{3}}\underline{k}$ rad/s

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_n + (\underline{a}_{B/A})_t$, $\underline{a}_A = \underline{0}$

Vector algebra: $\underline{a}_B = a_B\underline{i}$, $(\underline{a}_{B/A})_n = \underline{\omega}_{AB}\times(\underline{\omega}_{AB}\times\underline{r}_{AB})$

$= (\frac{20}{\sqrt{3}})^2\underline{k}\times[\underline{k}\times 0.2(\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j})]$

$= \frac{40}{3}(-\sqrt{3}\,\underline{i} + \underline{j})\ m/s^2$

$(\underline{a}_{B/A})_t = \underline{\alpha}_{AB}\times\underline{r}_{AB} = \alpha_{AB}\underline{k}\times 0.2(\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j})$

$= \frac{\alpha_{AB}}{10}(\sqrt{3}\,\underline{j} + \underline{i})$

Thus $a_B\underline{i} = \underline{0} + \frac{40}{3}(-\sqrt{3}\,\underline{i} + \underline{j}) + \frac{\alpha_{AB}}{10}(\sqrt{3}\,\underline{j} + \underline{i})$

so $a_B = -\frac{40}{\sqrt{3}} + \frac{\alpha_{AB}}{10}$ & $0 = \frac{40}{3} + \frac{\alpha_{AB}}{10}\sqrt{3}$

giving $\alpha_{AB} = -\frac{400}{3\sqrt{3}}$ rad/s² & $a_B = -\frac{160}{3\sqrt{3}}$ m/s²

$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = 0 + \frac{1}{2}(\underline{a}_{B/A})_n + \frac{1}{2}(\underline{a}_{B/A})_t = \frac{20}{3}(-\sqrt{3}\,\underline{i} + \underline{j}) - \frac{20}{3\sqrt{3}}(\sqrt{3}\,\underline{j} + \underline{i})$

$= -\frac{80}{3\sqrt{3}}\underline{i} = \underline{-15.40\underline{i}\ m/s^2}$

Vector geometry:

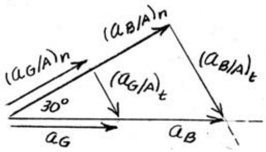
$(a_{B/A})_n = 0.2(20/\sqrt{3})^2 = 80/3\ m/s^2$

$(a_{B/A})_t = 0.2\alpha_{AB}$

$(a_{G/A})_n = \frac{1}{2}(a_{B/A})_n = 40/3\ m/s^2$

$(a_{G/A})_t = \frac{1}{2}(a_{B/A})_t = 0.1\alpha_{AB}$

$a_G = \frac{40}{3}\,\frac{2}{\sqrt{3}} = \underline{15.40\ m/s^2}$



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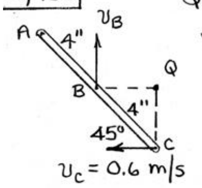


Q is instantaneous center of zero velocity for bar AC.

$$\omega_{AC} = \frac{v_C}{\overline{QC}} = \frac{0.6}{\frac{4}{12}\cos 45°} = 2.55 \text{ rad/sec CW}$$

$\underline{a}_B = \underline{a}_C + (\underline{a}_{B/C})_n + (\underline{a}_{B/C})_t$

$(a_{B/C})_n = \overline{BC}\,\omega_{AC}^2 = \frac{4}{12}(2.55)^2 = 2.16 \text{ ft/sec}^2$

$\alpha_{BC} = \frac{(a_{B/C})_t}{\overline{BC}} = \frac{2.16}{4/12} = 6.48 \text{ rad/sec}^2$ CCW

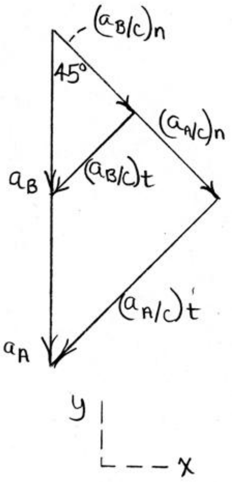
$\underline{a}_A = \underline{a}_C + (\underline{a}_{A/C})_n + (\underline{a}_{A/C})_t$

$(a_{A/C})_n = \overline{AC}\,\omega_{AC}^2 = \frac{8}{12}(2.55)^2 = 4.32 \text{ ft/sec}^2$

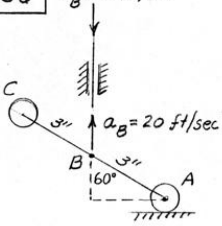
$(a_{A/C})_t = 4.32 \text{ ft/sec}^2$

$a_A = 4.32\sqrt{2} = 6.11 \text{ ft/sec}^2$

$\therefore \underline{a}_A = -6.11\underline{j} \text{ ft/sec}^2$



5/156 $v_B = 3$ ft/sec



From solution to Prob. 5/85

$v_{A/B} = 3.46$ ft/sec

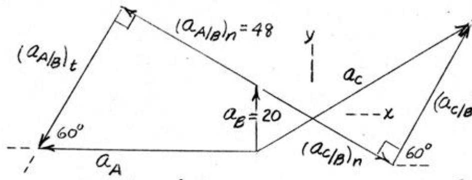
$\omega_{AB} = 3.46/\frac{3}{12} = 13.86$ rad/sec

$\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$

$(a_{A/B})_n = \frac{3}{12}(13.86)^2 = 48 \text{ ft/sec}^2$

From diagram $(a_{A/B})_t = \frac{20+24}{\sin 60°} = 50.8 \frac{ft}{sec^2} = (a_{C/B})_t$

Also $(a_{C/B})_n = (a_{A/B})_n = 48 \frac{ft}{sec^2}$



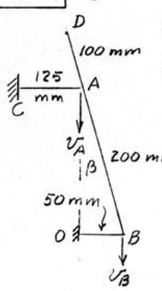
Components of a_C are

$(a_C)_x = 48\cos 30° + 50.8\cos 60° = 66.97 \frac{ft}{sec^2}$

$(a_C)_y = 20 - 48\sin 30° + 50.8\sin 60° = 40 \frac{ft}{sec^2}$

$a_C = \sqrt{(66.97)^2 + (40)^2} = 78.0 \text{ ft/sec}^2$

►5/157 $v_B = r\omega = 50\frac{(120)2\pi}{60} = 200\pi = 628$ mm/s

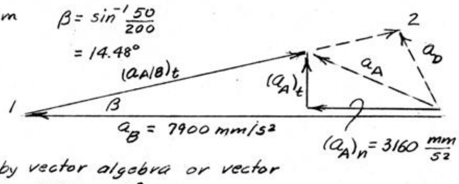


v_B parallel to v_A so $v_A = v_B$ & $\omega_{AB} = 0$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$; $a_B = (a_B)_n = \frac{v_B^2}{r} = \frac{(200\pi)^2}{50} = 7900 \text{ mm/s}^2$

$(a_A)_n = \frac{(200\pi)^2}{125} = 3160 \text{ mm/s}^2$

$\beta = \sin^{-1}\frac{50}{200} = 14.48°$

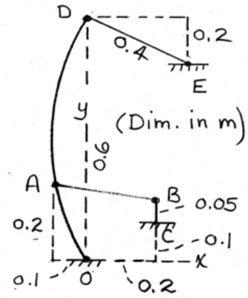


From solution by vector algebra or vector geometry, $(a_{A/B})_t = 4890 \text{ mm/s}^2$

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$; $a_{D/B} = (a_{D/B})_t = \frac{\overline{BD}}{\overline{BA}}(a_{A/B})_t = \frac{300}{200}(4890) = 7340 \frac{mm}{s^2}$ (= 1-2)

$a_D = \sqrt{(7340\sin 14.48°)^2 + (7900 - 7340\cos 14.48°)^2} = 1997 \text{ mm/s}^2$

►5/158 $\underline{\omega}_{CB} = -\pi\underline{k}$ rad/s , $\underline{r}_{OA} = -0.1\underline{i} + 0.2\underline{j}$ m



$\underline{r}_{CB} = 0.05\underline{j}$ m, $\underline{r}_{BA} = -0.3\underline{i} + 0.05\underline{j}$ m

$\underline{r}_{OD} = 0.6\underline{j}$ m , $\underline{v}_B = 0.05\pi\,\underline{i}$ m/s

$\underline{v}_{A/B} = \omega_{AB}\underline{k} \times (-0.3\underline{i} + 0.05\underline{j})$
$= -0.3\,\omega_{AB}\,\underline{j} - 0.05\,\omega_{AB}\,\underline{i}$

$\underline{v}_A = \omega_{OA}\underline{k} \times (-0.1\underline{i} + 0.2\underline{j})$
$= -0.1\,\omega_{OA}\,\underline{j} - 0.2\,\omega_{OA}\,\underline{i}$

From $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$:

$-0.1\,\omega_{OA}\,\underline{j} - 0.2\,\omega_{OA}\,\underline{i} = 0.05\pi\,\underline{i} - 0.3\,\omega_{AB}\,\underline{j} - 0.05\,\omega_{AB}\,\underline{i}$

Equate like coefficients: $\begin{cases} \underline{\omega}_{AB} = -0.286\underline{k} \text{ rad/s} \\ \underline{\omega}_{OA} = -0.857\underline{k} \text{ rad/s} \end{cases}$

Now, $\underline{a}_A = \underline{a}_B + (\underline{a}_{A/B})_n + (\underline{a}_{A/B})_t$ *

$\underline{a}_A = -\omega_{OA}^2\,\underline{r}_{OA} + \underline{\alpha}_{OA} \times \underline{r}_{OA}$
$= 0.734(0.1\underline{i} - 0.2\underline{j}) + \alpha_{OA}(-0.1\underline{j} - 0.2\underline{i})$

$(\underline{a}_{A/B})_n = -\omega_{AB}^2\,\underline{r}_{BA} = 0.0816(0.3\underline{i} - 0.05\underline{j})$ m/s²

$(\underline{a}_{A/B})_t = \underline{\alpha}_{AB} \times \underline{r}_{BA} = \alpha_{AB}(-0.3\underline{j} - 0.05\underline{i})$ m/s²

Substitute into *, equate like coefficients, & obtain

$\alpha_{OA} = -0.0519$ rad/s², $\alpha_{AB} = -1.186$ rad/s²

$\underline{a}_E = (\underline{a}_D)_n + (\underline{a}_D)_t + (\underline{a}_{E/D})_n + (\underline{a}_{E/D})_t$, $(\underline{a}_{E/D})_n = \underline{0}$ since $\omega_{DE} = 0$

$a_E\,\underline{i} = -0.6(0.857)^2\underline{j} + 0.6(0.0519)\underline{i} + \alpha_{ED}\underline{k} \times (0.12\underline{i} - 0.2\underline{j})$

Solve to obtain $\alpha_{ED} = 1.272$ rad/s², $\underline{a_E = 0.285 \text{ m/s}^2}$

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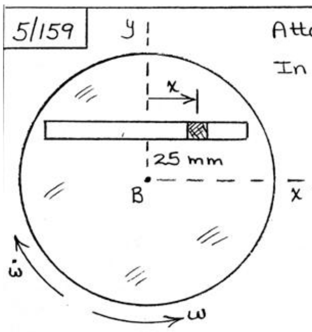

Attach Bxy to disk as shown.

In Eqs. 5/12 & 5/14 :

$\begin{cases} \underline{v}_B = \underline{a}_B = \underline{0} \\ \underline{\omega} = 5\underline{k} \frac{rad}{s}, \ \dot{\underline{\omega}} = -3\underline{k} \frac{rad}{s^2} \\ \underline{r} = 36\underline{i} + 25\underline{j} \text{ mm} \\ \underline{v}_{rel} = -100\underline{i} \text{ mm/s} \\ \underline{a}_{rel} = 150\underline{i} \text{ mm/s}^2 \end{cases}$

(5/12): $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$ gives

$\underline{v}_A = -225\underline{i} + 180\underline{j}$ mm/s

(5/14) : $\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

gives $\underline{a}_A = -675\underline{i} - 1733\underline{j}$ mm/s²

5/160 For the coordinates $\overset{y}{\underset{+\circlearrowleft}{\vdash}} x$, the no-slip constraints are $v_0 = -r\omega$ & $a_0 = -r\alpha$. So

$$\omega = -\frac{v_0}{r} = -\frac{-3}{0.30} = 10 \text{ rad/s}$$

$$\alpha = -\frac{a_0}{r} = -\frac{5}{0.30} = -16.67 \text{ rad/s}^2$$

Use the frame Oxy as disk-fixed.

(5/12): $\underline{v}_A = \underline{v}_0 + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

(5/14): $\underline{a}_A = \underline{a}_0 + \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

Ingredients: $\left\{\begin{array}{l} \underline{v}_0 = -3\underline{i} \text{ m/s} \\ \underline{a}_0 = 5\underline{i} \text{ m/s}^2 \\ \underline{\omega} = 10\underline{k} \text{ rad/s} \\ \underline{\alpha} = -16.67\underline{k} \text{ rad/s}^2 \end{array}\right.$ $\quad$ $\underline{r} = 0.24\underline{j}$ m, $\underline{v}_{rel} = 2\underline{i}$ m/s, $\underline{a}_{rel} = -7\underline{i} - \frac{2^2}{0.24}\underline{j} = -7\underline{i} - 16.67\underline{j}$ m/s²

Substitute into (5/12) & (5/14) & simplify:

$\underline{v}_A = -3.4\underline{i}$ m/s

$\underline{a}_A = 2\underline{i} - 0.667\underline{j}$ m/s²

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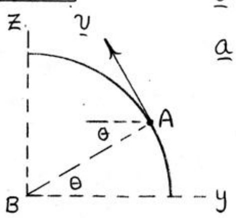


$\underline{v} = v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$

$\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}$

$= 2\Omega\underline{k} \times v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$

$= 2\Omega v \sin\theta\,\underline{i}$ (west)

For $v = 500$ km/h,

(a) Equator, $\theta = 0$: $a_{cor} = 0$

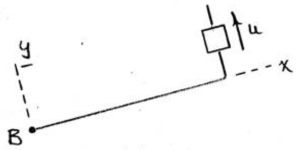
(b) North pole, $\theta = 90°$: $a_{cor} = 2(7.292 \cdot 10^{-5})\frac{500}{3.6}$

$= 0.0203$ m/s²

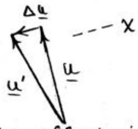
The track provides the necessary westward acceleration so that the velocity vector is properly rotated and reduced in magnitude.

5/162 $\underline{a}_{cor} = 2\underline{\omega} \times \underline{v}_{rel}$

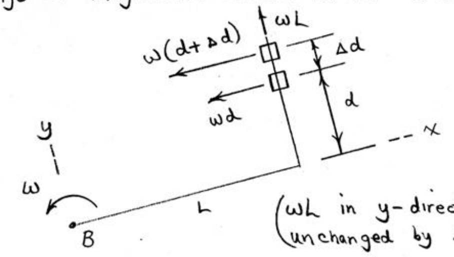
$= 2\omega\underline{k} \times u\underline{j} = -2\omega u\underline{i}$



Change-of-direction effect is in $-x$ direction:



Change-of-magnitude effect is in $-x$ direction:



(ωL in y-direction unchanged by Δd.)

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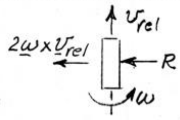
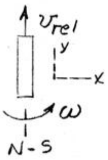
(a) North pole



Only horizontal component of acceleration is $|2\underline{\omega} \times \underline{v}_{rel}| = 2(0.7292)(10^{-4})(15) = 0.00219$ m/s²

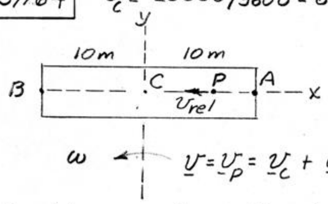
$\Sigma F = ma$; $R = 50\,000\,(0.00219) = 109.4$ N

(b) Equator



$\omega_{xy} = 0$ where x-y is horizontal plane so $2\underline{\omega} \times \underline{v}_{rel} = 0$ & there is no other horizontal acceleration so $R = 0$

5/164 $v_C = 25000/3600 = 6.94$ m/s



$\omega = v_C/\rho = 6.94/60 = 0.1157$ rad/s

$\underline{v}_{rel} = -1.5\underline{i}$ m/s

$\underline{\omega} = 0.1157\underline{k}$ rad/s

$\underline{v} = \underline{v}_P = \underline{v}_C + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

For A; $\underline{r} = 10\underline{i}$ m; $\underline{v}_A = -6.94\underline{i} + 0.1157\underline{k} \times 10\underline{i} - 1.5\underline{i}$

$= -8.44\underline{i} + 1.157\underline{j}$ m/s

For C; $r = 0$, $\underline{v}_C = -6.94\underline{i} - 1.5\underline{i} = -8.44\underline{i}$ m/s

For B; $\underline{r} = -10\underline{i}$ m; $\underline{v}_B = -6.94\underline{i} - 1.157\underline{j} - 1.5\underline{i}$

$= -8.44\underline{i} - 1.157\underline{j}$ m/s

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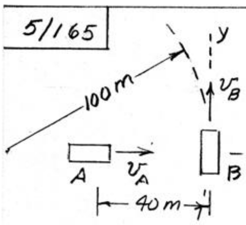


$v_A = \frac{72 \times 1000}{3600} = 20$ m/s

$v_B = \frac{54 \times 1000}{3600} = 15$ m/s

$\omega = \omega_B = \frac{15}{100} = 0.15$ rad/s

$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

$20\underline{i} = 15\underline{j} + 0.15\underline{k} \times (-40\underline{i}) + \underline{v}_{rel}$

$\underline{v}_{rel} = 20\underline{i} - 9\underline{j}$ m/s

$(v_{rel})_{\text{rotating axes}}$ differs from $(v_{rel})_{\text{translating axes}}$ by $\underline{\omega} \times \underline{r}$

5/166 From Prob. 5/165 $\underline{v}_{rel} = 20\underline{i} - 9\underline{j}$ m/s

$\underline{\omega} = 0.15\underline{k}$ rad/s

$\underline{a}_A = \underline{a}_B + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \dot{\underline{\omega}} \times \underline{r} + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_A = \underline{0}$, $\underline{a}_B = \frac{v_B^2}{R}(-\underline{i}) = -\frac{15^2}{100}\underline{i} = -2.25\underline{i}$ m/s²

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = 0.15\underline{k} \times (0.15\underline{k} \times [-40\underline{i}]) = 0.90\underline{i}$ m/s²

$\dot{\underline{\omega}} \times \underline{r} = \underline{0}$

$2\underline{\omega} \times \underline{v}_{rel} = 2(0.15\underline{k}) \times (20\underline{i} - 9\underline{j}) = 2.7\underline{i} + 6\underline{j}$ m/s²

Thus $\underline{0} = -2.25\underline{i} + 0.90\underline{i} + \underline{0} + 2.7\underline{i} + 6\underline{j} + \underline{a}_{rel}$

$\underline{a}_{rel} = -1.35\underline{i} - 6\underline{j}$ m/s²

5/167 $x = 2 \sin 4\pi t$, $\dot{x} = 8\pi \cos 4\pi t$, $\ddot{x} = -32\pi^2 \sin 4\pi t$

$\theta = 0.2 \sin 8\pi t$, $\dot{\theta} = 1.6\pi \cos 8\pi t$, $\ddot{\theta} = -12.8\pi^2 \sin 8\pi t$

(a) For $x = 0$ & $\dot{x}$ (+); $t = 0$, $v_{rel} = \dot{x} = 8\pi$ in./sec

$\underline{a}_A = 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$ $\qquad a_{rel} = \ddot{x} = 0$

$= 2(1.6\pi)(8\pi)\underline{j} + 0$ $\qquad \omega = \dot{\theta} = 1.6\pi$ rad/sec

$= 253\underline{j}$ in./sec² $\qquad \dot{\omega} = \ddot{\theta} = 0$

(b) For $x = +2$ in., $\sin 4\pi t = 1$, $\cos 4\pi t = 0$, $t = 1/8$ sec

$\theta = 0$

$v_{rel} = \dot{x} = 0$, $\ddot{x} = -32\pi^2$ in./sec²

$\omega = \dot{\theta} = -1.6\pi$ rad/sec

$\dot{\omega} = \ddot{\theta} = 0$

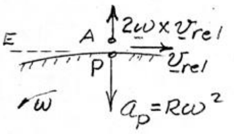
$\underline{a}_A = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\dot{\underline{\omega}} \times \underline{r} = 0$, $\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -2(1.6\pi)^2\underline{i} = -5.12\pi^2\underline{i}$ in./sec²

$2\underline{\omega} \times \underline{v}_{rel} = 0$, $\underline{a}_{rel} = \ddot{x}\underline{i} = -32\pi^2\underline{i}$ in./sec²

$\underline{a}_A = -5.12\pi^2\underline{i} - 32\pi^2\underline{i} = -366\underline{i}$ in./sec²

5/168 Let P be a point on the road coincident with A. $\underline{a}_A = \underline{a}_p + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$



For zero vertical acceleration,

$|2\underline{\omega} \times \underline{v}_{rel}| = R\omega^2$

$v_{rel} = \frac{1}{2}R\omega$

For $R = 6378$ km, $\omega = 0.7292(10^{-4})$ rad/s,

$v_{rel} = \frac{1}{2}(6378 \times 10^3)(0.7292 \times 10^{-4}) = 233$ m/s

or $v_{rel} = 233(3.6) = 837$ km/h

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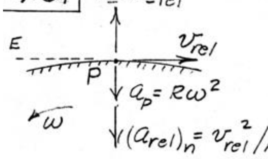

For zero vertical accel,

$|2\underline{\omega} \times \underline{v}_{rel}| = R\omega^2 + v_{rel}^2/R$

$v_{rel}^2 - 2\omega R v_{rel} + R^2\omega^2 = 0$

$(v_{rel} - R\omega)^2 = 0$, $v_{rel} = R\omega$

(zero absolute velocity)

[$v_{rel} = 6378(0.7292)(10^{-4}) = 0.4651$ km/s

or $0.4651(3600) = 1674$ km/h]

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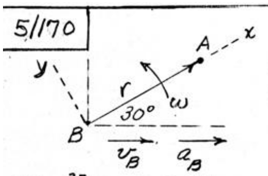

$\underline{r} = (20 + b)\underline{i} = 25\underline{i}$ ft

$\underline{v}_{rel} = \dot{r}\underline{i} = 2\underline{i}$ ft/sec

$\underline{a}_{rel} = \ddot{r}\underline{i} = -1\underline{i}$ ft/sec²

$\underline{\omega} = \frac{10}{180}\pi\underline{k} = 0.1745\underline{k}$ rad/sec

$\dot{\underline{\omega}} = \underline{0}$

$v_B = \frac{35}{30}44 = 51.3$ ft/sec, $\underline{v}_B = 51.3(\underline{i}\cos 30° - \underline{j}\sin 30°)$

$a_B = -10$ ft/sec² $\qquad \underline{a}_B = -10(\underline{i}\cos 30° - \underline{j}\sin 30°)$

Eq. 5/14, $\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

so $\underline{a}_A = -10(0.866\underline{i} - 0.5\underline{j}) + \underline{0} + (0.1745)^2\underline{k} \times (\underline{k} \times 25\underline{i})$

$+ 2(0.1745\underline{k}) \times 2\underline{i} - 1\underline{i}$

(b) $\underline{a}_A = -10.42\underline{i} + 5.70\underline{j}$ ft/sec² with respect to ground

(a) $\underline{a}_A - \underline{a}_B = -10.42\underline{i} + 5.70\underline{j} - (-10)(0.866\underline{i} - 0.5\underline{j})$

$= -1.76\underline{i} + 0.70\underline{j}$ ft/sec² with respect to truck

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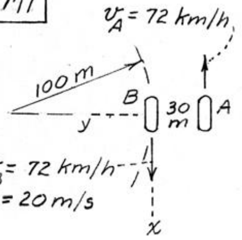

$\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r} + \underline{v}_{rel}$

Angular velocity of axes is $\omega = \frac{72/3.6}{100} = 0.2$ rad/s

so $\underline{\omega} = 0.2\underline{k}$ rad/s

& $\underline{r} = \underline{r}_{A/B} = -30\underline{j}$ m

Thus $-20\underline{i} = 20\underline{i} + 0.2\underline{k} \times (-30\underline{j}) + \underline{v}_{rel}$

$\underline{v}_{rel} = -40\underline{i} - 6\underline{i} = -46\underline{i}$ m/s

Curvature of road for A has no effect on $\underline{v}_{rel}$ & hence $\underline{v}_A$.

5/172 Refer to figure and solution for Prob. 5/171 where $\underline{v}_{rel} = -46\underline{i}$ m/s

$\underline{\omega} = 0.2\underline{k}$ rad/s const.

$\underline{a}_A = \underline{a}_B + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \dot{\underline{\omega}} \times \underline{r} + 2\underline{\omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_A = (v^2/\rho)(-\underline{j}) = -\frac{20^2}{100}\underline{j} = -4\underline{j}$ m/s², $\quad (a_A)_t = 0$

$\underline{a}_B = (v^2/\rho)(+\underline{j}) = +4\underline{j}$ m/s² $\quad (a_B)_t = 0$

$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = 0.2\underline{k} \times (0.2\underline{k} \times [-30\underline{j}]) = 1.2\underline{j}$ m/s²

$\dot{\underline{\omega}} \times \underline{r} = 0$

$2\underline{\omega} \times \underline{v}_{rel} = 2(0.2\underline{k}) \times (-46\underline{i}) = -18.4\underline{j}$ m/s²

so $\underline{a}_{rel} = -4\underline{j} - 4\underline{j} - 1.2\underline{j} + 18.4\underline{j} = 9.2\underline{j}$ m/s²

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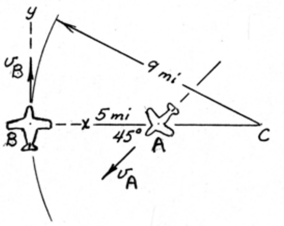


$v_B = 480\frac{44}{30} = 704$ ft/sec

$v_A = 360\frac{44}{30} = 528$ ft/sec

$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

Angular vel. of axes $= \underline{\omega} = \frac{v_B}{\rho}(-\underline{k})$

$= \frac{-704}{9\times5280}\underline{k} = -0.01481\underline{k}$ rad/sec

v_{rel} = vel. of A rel. to B

$\underline{r} = 5(5280)\underline{i} = 26{,}400\underline{i}$ ft

Thus $528(-0.707\underline{i} - 0.707\underline{j}) = 704\underline{j} - 0.01481\underline{k}\times 26{,}400\underline{i} + \underline{v}_{rel}$

$\underline{\underline{v}_{rel} = -373\underline{i} - 686\underline{j}}$ ft/sec with $v_{rel} = 781$ ft/sec

or $\underline{533 \text{ mi/hr}}$

5/174 Refer to solution for Prob. 5/173.

$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_A = \underline{0}$, $\underline{a}_B = \frac{v_B^2}{\rho}\underline{i} = \frac{704^2}{9\times5280}\underline{i} = 10.43\underline{i}$ ft/sec²

$\dot{\underline{\omega}}\times\underline{r} = \underline{0}$

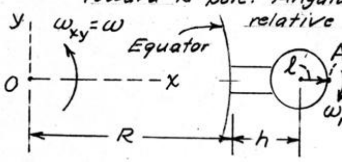
$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -0.01481\underline{k}\times(-0.01481\underline{k}\times26{,}400\underline{i}) = -5.79\underline{i}$ ft/sec²

$2\underline{\omega}\times\underline{v}_{rel} = 2(-0.1481\underline{k})\times(-373\underline{i} - 686\underline{j}) = 11.05\underline{j} - 23.0\underline{i}$ ft/sec²

$\underline{a}_{rel} = \underline{0} - 10.43\underline{i} - \underline{0} + 5.79\underline{i} - 11.05\underline{j} + 20.3\underline{i} = \underline{15.69\underline{i} - 11.05\underline{j} \text{ ft/sec}^2}$

where $\underline{a_{rel} = 19.19 \text{ ft/sec}^2}$

5/175 Attach x-y axes to earth with z-axis pointing toward N-pole. Angular velocity of hour hand relative to clock is $\omega_r = 2\pi/12$ CW



But $\omega = 2\pi/24$ CCW

so $\omega_r = 2\omega$

$\underline{v}_{rel} = \ell\omega_r(-\underline{j}) = 2\ell\omega(-\underline{j})$

$\underline{v}_A = \underline{v}_O + \underline{\omega}\times\underline{r} + \underline{v}_{rel} = \underline{0} + \omega\underline{k}\times(R+h+\ell)\underline{i} + 2\ell\omega(-\underline{j})$

$\underline{v}_A = \underline{(R+h-\ell)\omega\underline{j}}$

$\underline{a}_A = \underline{a}_O + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_O = \underline{0}$, $\dot{\underline{\omega}} = \underline{0}$, $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -(R+h+\ell)\omega^2\underline{i}$

$2\underline{\omega}\times\underline{v}_{rel} = 2\omega\underline{k}\times(-2\ell\omega\underline{j}) = 4\ell\omega^2\underline{i}$

$\underline{a}_{rel} = -\ell\omega_r^2\underline{i} = -4\ell\omega^2\underline{i}$

so $\underline{a}_A = -(R+h+\ell)\omega^2\underline{i} + 4\ell\omega^2\underline{i} - 4\ell\omega^2\underline{i}$

$\underline{a}_A = \underline{-(R+h+\ell)\omega^2\underline{i}}$

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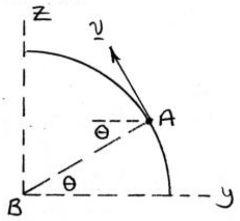


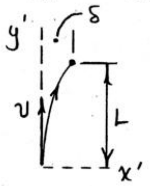
$\underline{v} = v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$

$\underline{a}_{cor} = 2\underline{\omega}\times\underline{v}$

$= 2\Omega\underline{k}\times v(-\sin\theta\,\underline{j} + \cos\theta\,\underline{k})$

$= 2\Omega v\sin\theta\,\underline{i}$ (west)

With no westward force mechanism available, the ball will drift to the east (relative to the ground) with an acceleration of magnitude a_{cor}.



$a_{x'} = 2\Omega v\sin\theta$

$\delta = \frac{1}{2}a_{x'}t^2 = \frac{1}{2}(2\Omega v\sin\theta)\left(\frac{L}{v}\right)^2$

$= \underline{\frac{\Omega L^2}{v}\sin\theta}$ (assumes $\delta \ll L$)

With $\Omega = 7.292(10^{-5})$ rad/sec, $v = 15$ ft/sec, $L = 60$ ft, & $\theta = 40°$: $\delta = 0.01125$ ft

$\underline{(0.1350 \text{ in.})}$

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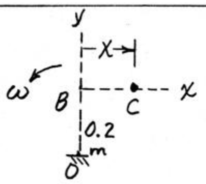


	(a) $t = 3$ s	(b) $t = 0.5$ s
$x = 0.04\sin\pi t$ =	0	0.04
$\dot{x} = 0.04\pi\cos\pi t$ =	-0.04π	0
$\ddot{x} = -0.04\pi^2\sin\pi t$ =	0	$-0.04\pi^2$
$\omega = 2\sin\frac{\pi}{2}t$ =	-2	$\sqrt{2}$
$\dot{\omega} = \pi\cos\frac{\pi}{2}t$ =	0	$\pi/\sqrt{2}$

$\underline{a}_C = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

(a) $\underline{a}_B = 0.2(-2)^2(-\underline{j}) + 0\underline{i} = -0.8\underline{j}$ m/s²

$\dot{\underline{\omega}}\times\underline{r} = 0$, $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -2\underline{k}\times(-2\underline{k}\times 0) = \underline{0}$

$2\underline{\omega}\times\underline{v}_{rel} = 2(-2\underline{k})\times(-0.04\pi\underline{i}) = 0.503\underline{j}$ m/s², $\underline{a}_{rel} = \underline{0}$

Substitute & get $\underline{\underline{a}_C = -0.297\underline{j} \text{ m/s}^2}$

(b) $\underline{a}_B = -0.2\sqrt{2}^2\underline{j} - 0.2\frac{\pi}{\sqrt{2}}\underline{i} = -0.444\underline{i} - 0.4\underline{j}$ m/s²

$\dot{\underline{\omega}}\times\underline{r} = \pi/\sqrt{2}\,\underline{k}\times 0.04\underline{i} = 0.0889\underline{j}$ m/s²

$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = \sqrt{2}\underline{k}\times(\sqrt{2}\underline{k}\times0.04\underline{i}) = -0.08\underline{i}$ m/s²

$2\underline{\omega}\times\underline{v}_{rel} = 2\sqrt{2}\underline{k}\times\underline{0} = \underline{0}$, $\underline{a}_{rel} = -0.04\pi^2\underline{i} = -0.395\underline{i}$ m/s²

Substitute & get $\underline{a}_C = \underline{-0.919\underline{i} - 0.311\underline{j} \text{ m/s}^2}$

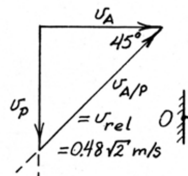
5/178 Let P = point on ODE coincident with A.

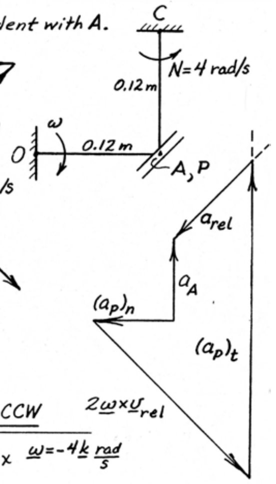
$v_A = v_P + v_{A/P}$

$v_A = 0.12(4) = 0.48$ m/s

$v_P = 0.48$ m/s,

$\omega_{OP} = \omega = \dfrac{0.48}{0.12} = \underline{4 \text{ rad/s CW}}$





$a_A = a_P + 2\omega \times v_{rel} + a_{rel}$

$a_A = (a_A)_n = 0.12(4^2) = 1.92$ m/s² ↑

$|2\omega \times v_{rel}| = 2(4)(0.48\sqrt{2}) = 5.43$ m/s² ↘

$(a_P)_n = 0.12(4^2) = 1.92$ m/s² ⟵

From diagram, $a_{rel} = 2.72$ m/s²,

$(a_P)_t = 7.68$ m/s²

$\alpha_{ODE} = \alpha = 7.68/0.12 = \underline{64.0 \text{ rad/s}^2 \text{ CCW}}$

Alternatively, with O —ω— P — — x (y up), $\omega = -4k$ rad/s

$a_A = (a_A)_n = 0.12(4^2)j = 1.92j$ m/s²

$(a_P)_n = 0.12(4^2)(-i) = -1.92i$ m/s²

$(a_P)_t = \alpha k \times 0.12 i = 0.12\alpha j$

$2\omega \times v_{rel} = 2(-4k) \times 0.48(i + j) = 3.84(i - j)$ m/s²

$a_{rel} = a_{rel}\dfrac{1}{\sqrt{2}}(i + j)$

Substitute: i-terms give $a_{rel}/\sqrt{2} = -1.92$, $a_{rel} = -2.72$ m/s² so $a_{rel} = \swarrow$

j-terms: $0.12\alpha = 1.92 + 3.84 + 1.92$, $\underline{\alpha = 64 \text{ rad/s}^2 \text{ CCW}}$

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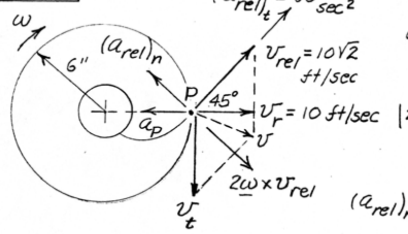


$a = a_P + 2\omega \times v_{rel} + a_{rel}$

$a_P = r\omega^2 = \dfrac{6}{12}\left[\dfrac{200 \times 2\pi}{60}\right]^2 = 219$ ft/sec²

$|2\omega \times v_{rel}| = 2\left[\dfrac{200 \times 2\pi}{60}\right]10\sqrt{2} = 592$ ft/sec²

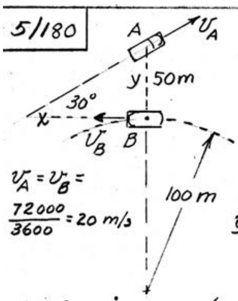
$(a_{rel})_n = \dfrac{v_{rel}^2}{\rho} = \dfrac{(10\sqrt{2})^2}{8/12} = 300 \dfrac{ft}{sec^2}$

Let n_1 = unit vector in n-dir. & t_1 = unit vector in t-dir.

$a = \dfrac{219}{\sqrt{2}} n_1 - \dfrac{219}{\sqrt{2}} t_1 - 592 n_1 + 80 t_1 + 300 n_1$

$= -137.3 n_1 - 75.1 t_1$ ft/sec², $a = \sqrt{137.3^2 + 75.1^2} = \underline{156.5 \text{ ft/sec}^2}$

5/180



$v_A = v_B = \dfrac{72000}{3600} = 20$ m/s

$v_A = v_B + \omega \times r + v_{rel}$

$v_A = 20(-0.866i + 0.5j)$ m/s

$v_B = 20i$ m/s

$\omega = \dfrac{20}{100}(-k) = -0.2k$ rad/s

$\omega \times r = -0.2k \times 50j = 10i$ m/s

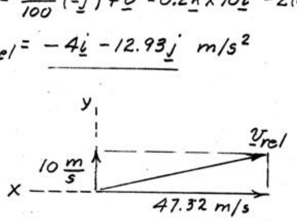
$v_{rel} = 20(-0.866i + 0.5j) - 20i - 10i$

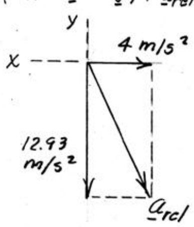
$= \underline{-47.3i + 10j \text{ m/s}}$

$a_A = a_B + \dot{\omega} \times r + \omega \times (\omega \times r) + 2\omega \times v_{rel} + a_{rel}$

$0 = \dfrac{(20)^2}{100}(-j) + 0 - 0.2k \times 10i - 2(0.2k) \times (-47.32i + 10j) + a_{rel}$

$a_{rel} = \underline{-4i - 12.93j \text{ m/s}^2}$





5/181 $a_C = \dot{\omega} \times r + \omega \times (\omega \times r) + 2\omega \times v_{rel} + a_{rel}$

where $\omega = \dot{\theta}k = 2\pi f_1 \theta_0 \cos 2\pi f_1 t\, k$

$\dot{\omega} = \ddot{\theta}k = -4\pi^2 f_1^2 \theta_0 \sin 2\pi f_1 t\, k$

$r = (8 + y)j$ ft, $v_{rel} = \dot{r} = \dot{y}j = 2\pi f_2 y_0 \cos 2\pi f_2 t\, j$

$a_{rel} = \ddot{r} = \ddot{y}j = -4\pi^2 f_2^2 y_0 \sin 2\pi f_2 t\, j$

For $t = 2$ sec, $\theta_0 = \pi/4$ rad, $f_1 = \frac{1}{4}$ cycle/sec, $f_2 = \frac{1}{2}$ cycle/sec, $y_0 = 6$ in.

$\omega = -\pi^2/8\, k$ rad/sec, $\dot{\omega} = 0$, $r = (8 + y)j = 8j$ ft

$v_{rel} = \frac{\pi}{2} j$ ft/sec, $a_{rel} = 0$

so $a_C = 0 + \left(-\frac{\pi^2}{8}k\right) \times \left(-\frac{\pi^2}{8}k \times 8j\right) + 2\left(-\frac{\pi^2}{8}k\right) \times \frac{\pi}{2}j$

$= -\frac{\pi^4}{8}j + \frac{\pi^3}{8}i$

$a_C = \underline{\frac{\pi^3}{8}(i - \pi j)}$ ft/sec²

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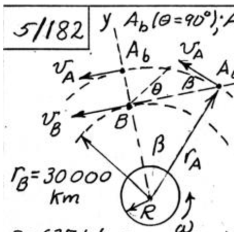


$R = 6371$ km

$g = 9.825$ m/s²

$\omega_0 = 0.7292(10^{-4})$ rad/s

$\omega = \omega_{xy} = v_B / r_B$

$= \dfrac{13125}{30000} = 0.4375 \dfrac{rad}{h}$

For circular orbit $v = R\sqrt{g/r}$

For geosyn. orbit $v = r_A \omega_0$

so $r_A \omega_0 = R\sqrt{g/r_A}$, $r_A = \left(\dfrac{R^2 g}{\omega_0^2}\right)^{1/3}$

$r_A = \left\{\dfrac{(6371)^2 (9.825/10^3)}{[0.7292(10^{-4})]^2}\right\}^{1/3} = 42\,171$ km

$\beta = \cos^{-1}\dfrac{30000}{42171} = 44.65°$

$\overline{BA} = \sqrt{(42171)^2 - (30000)^2} = 29638$ km

$v_A = 6371\sqrt{\dfrac{9.825/10^3}{42171}}(3600) = 11070$ km/h

$v_B = 6371\sqrt{\dfrac{9.825/10^3}{30000}}(3600) = 13125$ km/h

$v_A = v_B + \omega \times r + v_{rel}$, $v_{rel} = v_A - v_B - \omega \times r$

(a) $\theta = 0$; $r = 29638i$ km

$v_{rel} = 11070(-i\cos\beta + j\sin\beta) - (-13125i) - 0.4375k \times 29638i$

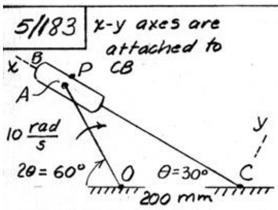
$= \underline{5250i - 5190j \text{ km/h}}$

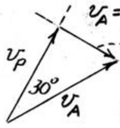
(b) $\theta = 90°$, $r = (42171 - 30000)j = 12171j$ km

$v_{rel} = -11070i - (-13125i) - 0.4375k \times 12171j$

$= \underline{7380i \text{ km/h}}$

5/183 x-y axes are attached to CB





$v_A = 200(10) = 2000$ mm/s

$v_{A/P} = v_{rel} = 2000\left(\frac{1}{2}\right) = 1000$ mm/s

$v_P = 2000\dfrac{\sqrt{3}}{2} = 1732 \dfrac{mm}{s}$

$\omega_{xy} = \omega = \dfrac{v_P}{\overline{PC}} = \dfrac{1732}{2(200)\sqrt{3}/2} = \underline{5 \text{ rad/s CW}}$

$\overline{OA} = 200$ mm

$a_A = a_C + \dot{\omega} \times r + \omega \times (\omega \times r) + 2\omega \times v_{rel} + a_{rel}$

$\dot{\omega} = \ddot{\theta} = 0$ since $\dfrac{d^2(2\theta)}{dt^2} = 0$; thus $(a_P)_t = \dot{\omega} \times r = 0$

$\omega \times (\omega \times r) = 5^2 k \times (k \times 200\sqrt{3}i) = -8660i$ mm/s²

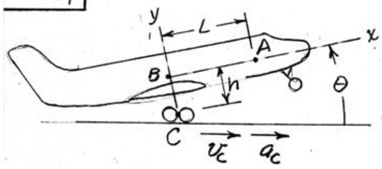
$2\omega \times v_{rel} = 2(5k) \times 1000(-i) = -10\,000j$ mm/s²

$a_{rel} = \ddot{x}i$; $a_A = 200(10)^2(-0.866i - 0.5j)$ mm/s²

Thus $20\,000(-0.866i - 0.5j) = 0 + 0 - 8660i - 10000j + \ddot{x}i$

$\ddot{x} = -8660$ mm/s², $\underline{a_{rel} = -8660i \text{ mm/s}^2}$

►5/184



$\underline{\omega} = \omega\underline{k} = \dot{\theta}\underline{k}$, $\underline{\alpha} = \dot{\omega}\underline{k}$

$\underline{r} = L\underline{i}$, $\underline{v}_{rel} = \dot{L}\underline{i}$

$\underline{a}_{rel} = \ddot{L}\underline{i}$

$\underline{v}_C = v_C(\underline{i}\cos\theta - \underline{j}\sin\theta)$

$\underline{a}_C = a_C(\underline{i}\cos\theta - \underline{j}\sin\theta)$

$\underline{v}_B = \underline{v}_C + \underline{\omega}\times\underline{r}_{CB} = \underline{v}_C + \omega\underline{k}\times h\underline{j} = (v_C\cos\theta - \omega h)\underline{i} - (v_C\sin\theta)\underline{j}$

$\underline{a}_B = \underline{a}_C + \dot{\underline{\omega}}\times\underline{r}_{CB} + \underline{\omega}\times(\underline{\omega}\times\underline{r}_{CB}) = \underline{a}_C + \alpha\underline{k}\times h\underline{j} + \omega\underline{k}\times(\omega\underline{k}\times h\underline{j})$
$= (a_C\cos\theta - \alpha h)\underline{i} - (a_C\sin\theta + h\omega^2)\underline{j}$

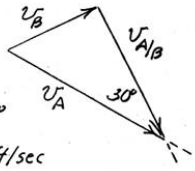
$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel} = \underline{v}_B + \omega\underline{k}\times L\underline{i} + \dot{L}\underline{i}$

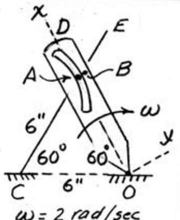
$\underline{v}_A = (v_C\cos\theta - \omega h + \dot{L})\underline{i} + (\omega L - v_C\sin\theta)\underline{j}$

$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$
$= \underline{a}_B + \dot{\omega}\underline{k}\times L\underline{i} + \omega\underline{k}\times(\omega\underline{k}\times L\underline{i}) + 2\omega\underline{k}\times\dot{L}\underline{i} + \ddot{L}\underline{i}$

$\underline{a}_A = (a_C\cos\theta - \alpha h - L\omega^2 + \ddot{L})\underline{i} + (-a_C\sin\theta - h\omega^2 + L\alpha + 2\omega\dot{L})\underline{j}$

►5/185 Let B = point on DO coincident with A





$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

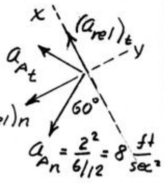
$v_B = \frac{6}{12}2 = 1$ ft/sec

$v_{A/B} = v_{rel} = v_B/\tan 30°$

$v_A = 2$ ft/sec, $v_{rel} = \sqrt{3}$ ft/sec

$\omega = 2$ rad/sec
$\dot{\omega} = 6$ rad/sec^2

$\underline{a}_A = \underline{a}_O + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$



$\underline{a}_O = 0$, $\dot{\underline{\omega}}\times\underline{r} = 6\underline{k}\times\frac{6}{12}\underline{i} = 3\underline{j}$ ft/sec^2

$\underline{\omega}\times(\underline{\omega}\times\underline{r}) = 2\underline{k}\times(2\underline{k}\times\frac{6}{12}\underline{i}) = -2\underline{i}$ ft/sec^2

$2\underline{\omega}\times\underline{v}_{rel} = 4\underline{k}\times(-\sqrt{3}\underline{i}) = -4\sqrt{3}\underline{j}$ ft/sec^2

$(\underline{a}_{rel})_n = \frac{v_{rel}^2}{\rho}(-\underline{j}) = -\frac{3}{6/12}\underline{j} = -6\underline{j}$ ft/sec^2

$a_{A_n} = \frac{2^2}{6/12} = 8\ \frac{ft}{sec^2}$

$\underline{a}_{A_n} = -8\cos 60°\underline{i} - 8\sin 60°\underline{j}$ ft/sec^2

$\underline{a}_{A_t} = a_{A_t}\cos 30°\underline{i} - a_{A_t}\sin 30°\underline{j}$

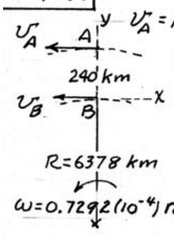
substitute & get

$a_{A_t}\frac{\sqrt{3}}{2}\underline{i} - a_{A_t}\frac{1}{2}\underline{j} - 4\underline{i} - 8\frac{\sqrt{3}}{2}\underline{j} = 3\underline{j} - 2\underline{i} - 4\sqrt{3}\underline{j} - 6\underline{j} + (a_{rel})_t\underline{i}$

Equate $\underline{i}$ & $\underline{j}$ terms & get $(a_{rel})_t = 3.20$ ft/sec^2

$(a_A)_t = 6$ ft/sec^2 so $\alpha_{EC} = 6/6/12 = \underline{12 \text{ rad/sec}^2 \text{ CCW}}$

►5/186 For circular orbit ; At equator $g = 9.814$ m/s^2



$v_A = R\sqrt{g/(R+h)} = 6378\sqrt{\frac{9.814/1000}{6378+240}}(3600) = 27\,960\ \frac{km}{h}$

$a_A = \frac{v_A^2}{R+h} = g\left(\frac{R}{R+h}\right)^2 = 9.814\left(\frac{6378}{6378+240}\right)^2 = 9.115$ m/s^2

$v_B = R\omega = 6378(0.7292)(10^{-4})(3600) = 1674$ km/h

$\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

$\underline{\omega}\times\underline{r} = 0.7292(10^{-4})(3600)(240)(-\underline{i})$
$= -63.00\underline{i}$ km/h

$\underline{v}_{rel} = -27\,960\underline{i} - (-1674\underline{i}) - (-63.00\underline{i})$
$= -26\,220\underline{i}$ km/h

$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_B = -R\omega^2\underline{j} = -6378(10^3)(0.7292)^2(10^{-4})^2\underline{j} = -0.03391\underline{j}$ m/s^2

$\dot{\underline{\omega}} = \underline{0}$; $\underline{\omega}\times(\underline{\omega}\times\underline{r}) = (0.7292)(10^{-4})\underline{k}\times(-63.00\underline{i})\frac{1000}{3600} = -0.001276\underline{j}$

$2\underline{\omega}\times\underline{v}_{rel} = 2(0.7292)(10^{-4})\underline{k}\times(-26\,220\underline{i})\frac{1000}{3600} = -1.0623\underline{j}$ m/s^2

so $-9.115\underline{j} = -0.03391\underline{j} - 0.001276\underline{j} - 1.0623\underline{j} + \underline{a}_{rel}$

$\underline{a}_{rel} = \underline{-8.018\underline{j}\text{ m/s}^2}$

5/187 $\underline{\omega}\times\underline{r} = \underline{v}$; $2\underline{k}\times(x\underline{i} + y\underline{j}) = -0.8\underline{i} - 0.6\underline{j}$

$2x\underline{j} - 2y\underline{i} = -0.8\underline{i} - 0.6\underline{j}$

So $2x = -0.6$, $\underline{x = -0.3\text{ m}}$
$-2y = -0.8$, $\underline{y = 0.4\text{ m}}$

$r = \sqrt{0.3^2 + 0.4^2} = \underline{0.5\text{ m}}$

5/188 $\underline{\omega} = 3\underline{k}$ rad/s $\underline{\alpha} = -6\underline{k}$ rad/s

$\underline{r}_P = \underline{r} = -0.1\underline{i} + 0.15\underline{j}$ m

$\underline{v} = \underline{\omega}\times\underline{r} = 3\underline{k}\times(-0.1\underline{i} + 0.15\underline{j}) = -0.45\underline{i} - 0.3\underline{j}$ m/s

$\underline{a}_t = \underline{\alpha}\times\underline{r} = -6\underline{k}\times(-0.1\underline{i} + 0.15\underline{j}) = 0.9\underline{i} + 0.6\underline{j}$ m/s^2

$\underline{a}_n = \underline{\omega}\times\underline{v} = 3\underline{k}\times(-0.45\underline{i} - 0.3\underline{j}) = \underline{0.9\underline{i} - 1.35\underline{j}\text{ m/s}^2}$

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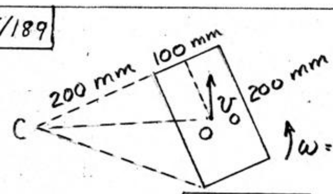


$v_O = \overline{OC}\,\omega$, $\overline{OC} = \sqrt{100^2 + 250^2} = 269$ mm

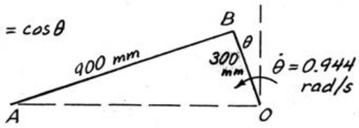
$v_O = 269(4) = 1077$ mm/s or $\underline{1.077\text{ m/s}}$

5/190

$900\sin\beta = 300\cos\theta$, $3\sin\beta = \cos\theta$

$3\dot{\beta}\cos\beta = -\dot{\theta}\sin\theta$,

$\dot{\beta} = -\frac{0.944}{3}\frac{\sin\theta}{\cos\beta}$



$9\sin^2\beta = \cos^2\theta$, $9(1-\cos^2\beta) = \cos^2\theta$, $\cos\beta = \sqrt{1 - \frac{1}{9}\cos^2\theta}$

So $\dot{\beta} = -\frac{0.944}{3}\frac{\sin\theta}{\sqrt{1-\frac{1}{9}\cos^2\theta}} = -\frac{0.944}{3}\frac{\sin 20°}{\sqrt{1-\frac{1}{9}\cos^2 20°}}$

$= -0.1133$ rad/s

$\underline{\omega_{AB} = 0.1133\text{ rad/s CW}}$

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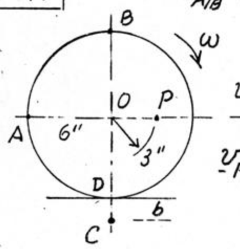
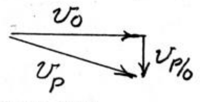


$v_{A/B} = \overline{AB}\,\omega$, $3\sqrt{2} = \frac{1}{2}\sqrt{2}\,\omega$, $\omega = 6$ rad/s

$v_O/\overline{OC} = \omega$, $\frac{4(12)}{6+b} = 6$

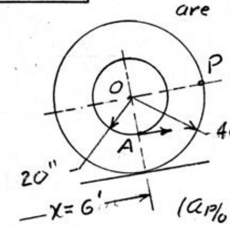
$v_O = 4$ ft/sec $\underline{b = 2\text{ in}}$

$\underline{v}_P = \underline{v}_O + \underline{v}_{P/O}$, $v_{P/O} = \overline{PO}\,\omega = \frac{3}{12}6 = \frac{3}{2}$ ft/sec



$v_P = \sqrt{4^2 + (3/2)^2} = \underline{4.27\text{ ft/sec}}$

5/192 Displacement, velocity, & acceleration of truck are 20/40 = 0.5 of x, v_0, a_0



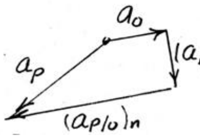
Thus $a_0 = 2(a_{truck}) = 2(2) = 4\ ft/sec^2$

$v_0^2 = 2a_0 x = 2(4)(6) = 48 (\frac{ft}{sec})^2$

$\underline{a}_P = \underline{a}_0 + (\underline{a}_{P/0})_n + (\underline{a}_{P/0})_t$

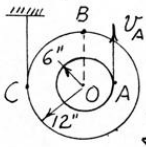
$(a_{P/0})_n = \overline{PO}\omega^2 = \overline{PO}(\frac{v_0}{\overline{PO}})^2 = \frac{48}{40/12} = 14.4\ ft/sec^2$

$(a_{P/0})_t = \overline{PO}\alpha = a_0 = 4\ ft/sec^2$



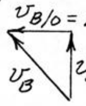
$a_P = \sqrt{(14.4-4)^2 + 4^2} = \underline{11.14\ \frac{ft}{sec^2}}$

5/193 $v_0 = \frac{12}{18} v_A = \frac{2}{3}(3) = \underline{2\ ft/sec}$

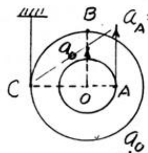


$\underline{v}_B = \underline{v}_0 + \underline{v}_{B/0}$

$v_{B/0} = \overline{BO}\,\omega_{BO} = \overline{BO}\frac{v_0}{\overline{CO}} = 2\ ft/sec$



$v_B = 2\sqrt{2} = \underline{2.83\ ft/sec}$

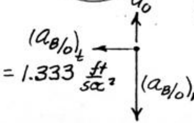


$a_0 = \frac{12}{18}(2) = \underline{1.333\ ft/sec^2}$

$\underline{a}_B = \underline{a}_0 + (\underline{a}_{B/0})_n + (\underline{a}_{B/0})_t$

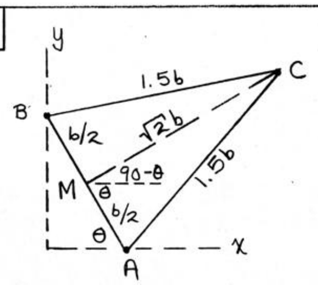
$(a_{B/0})_n = \overline{BO}\,\omega_{BO}^2 = \frac{12}{12}(\frac{2}{1})^2 = 4\ ft/sec^2$

$(a_{B/0})_t = \overline{BO}\,\alpha_{BO} = \frac{12}{12}\,\frac{1.333}{1} = 1.333\ ft/sec^2$



$a_B = \sqrt{(4-1.333)^2 + (1.333)^2} = \underline{2.98\ ft/sec^2}$

5/194



Note: $\overline{CM}^2 = (1.5b)^2 - (\frac{b}{2})^2$

$\overline{CM} = \sqrt{2}b$

$x_C = \frac{b}{2}\cos\theta + \sqrt{2}b\cos(90° - \theta)$

$= \frac{b}{2}\cos\theta + \sqrt{2}b\sin\theta$

$\dot{x}_C = -\frac{b}{2}\dot{\theta}\sin\theta + \sqrt{2}b\dot{\theta}\cos\theta$

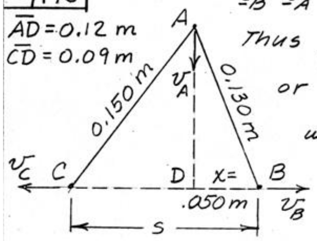
$\dot{x}_C = 0$ when $\frac{1}{2}\sin\theta = \sqrt{2}\cos\theta$

$\tan\theta = 2\sqrt{2}$, $\underline{\theta = 70.5°}$

5/195

$\overline{AD} = 0.12\ m$

$\overline{CD} = 0.09\ m$



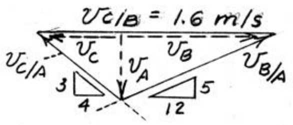
$v_{C/B} = \dot{s} = 1.6\ m/s$

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$, $\underline{v}_C = \underline{v}_A + \underline{v}_{C/A}$

Thus $\underline{v}_B = \underline{v}_C - \underline{v}_{C/A} + \underline{v}_{B/A}$

or $\underline{v}_{C/A} = \underline{v}_{C/B} + \underline{v}_{B/A}$

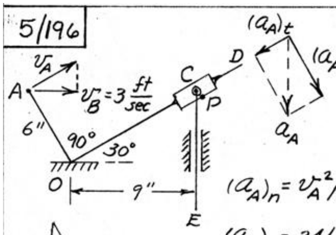
where $\underline{v}_{C/B} = \underline{v}_C - \underline{v}_B$



From diagram $\frac{5}{12}v_B = \frac{3}{4}v_C$; $v_B + v_C = 1.6$

$\frac{5}{12}v_B = \frac{3}{4}(1.6 - v_B)$, $\underline{v_B = 1.029\ m/s}$

5/196



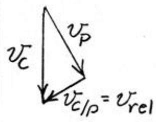
$v_A = v_B/\cos 30° = 3/0.866 = 3.46\ ft/sec$

$(a_A)_{horiz.} = 0$ since $(v_A)_{horiz}$ is constant

Thus a_A is vertical

$(a_A)_n = v_A^2/\overline{AO} = (2\sqrt{3})^2/\frac{6}{12} = 24\ ft/sec^2$

$(a_A)_t = 24/\sqrt{3} = 13.86\ ft/sec^2$



$\alpha_{OD} = \alpha_{OA} = \frac{(a_A)_t}{\overline{OA}} = \frac{13.86}{6/12} = 27.7\ rad/sec^2\ ccw$

$v_P = \frac{\overline{OP}}{\overline{OA}}v_A = \frac{9(2/\sqrt{3})}{6}(3.46) = 6\ ft/sec$

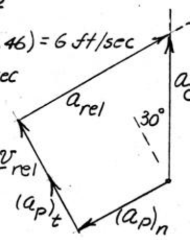
$\underline{a}_C = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$v_{rel} = 6/\sqrt{3}\ ft/sec$

$(a_P)_n = v_P^2/\overline{OP} = 6^2/\frac{9}{12}\frac{2}{\sqrt{3}} = 24\sqrt{3}\ ft/sec^2$

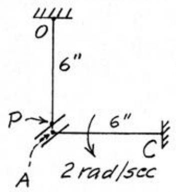
$(a_P)_t = \overline{OP}\alpha_{OD} = \frac{9}{12}\frac{2}{\sqrt{3}}(27.7) = 24\ ft/sec^2$

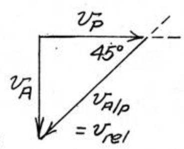
$|2\underline{\omega}\times\underline{v}_{rel}| = 2\frac{2\sqrt{3}}{6/12}\frac{6}{\sqrt{3}} = 48\ ft/sec^2$



From diag. $a_C = (24+48)\frac{2}{\sqrt{3}} = \underline{83.1\ \frac{ft}{sec^2}\ up}$

5/197 Let P be a point on EBO coincident with A





$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$

$v_A = 6(2) = 12\ in./sec$

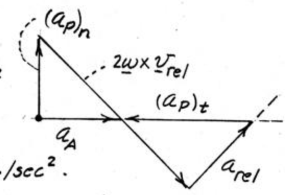
$\omega_{PO} = \omega = \frac{v_P}{\overline{PO}} = \frac{12}{6} = 2\ \frac{rad}{sec}$ $\underline{ccw}$

$\underline{a}_A = \underline{a}_P + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$a_A = (a_A)_n = 6(2^2) = 24\ in./sec^2$

$|2\underline{\omega}\times\underline{v}_{rel}| = 2(2)(12\sqrt{2}) = 48\sqrt{2}\ \frac{in.}{sec^2}$

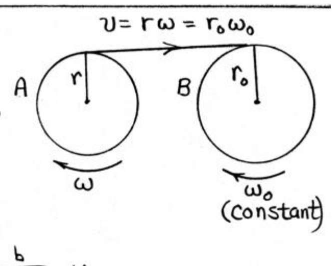
$(a_P)_n = 6(2^2) = 24\ in./sec^2$



From diagram $(a_P)_t = 48\ in./sec^2$.

$\alpha_{PO} = \alpha = \frac{48}{6} = \underline{8\ rad/sec^2\ CW}$

5/198



From $r\omega = r_0\omega_0$,

$\dot{r}\omega + r\dot{\omega} = \dot{r}_0\omega_0 + r_0\dot{\omega}_0$; $\dot{\omega}_0 = 0$

But $\dot{r} = -\frac{b}{2\pi/\omega}$

and $\dot{r}_0 = +\frac{b}{2\pi/\omega_0}$

So $-\frac{b}{2\pi/\omega}\omega + r\dot{\omega} = \frac{b}{2\pi/\omega_0}\omega_0$

$\dot{\omega} = \alpha = \frac{b}{2\pi r}\left[\omega_0^2 + \omega^2\right] = \underline{\frac{b\omega_0^2}{2\pi r}\left(1 + \frac{r_0^2}{r^2}\right)}$

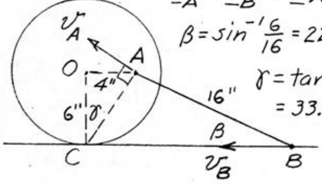
5/199 $v_B = 10$ in./sec, constant

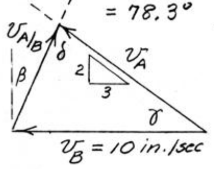
$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$

$\beta = \sin^{-1}\frac{6}{16} = 22.0°$

$\gamma = \tan^{-1}\frac{2}{3} = 33.7°$

$\delta = 90 + \beta - \gamma = 78.3°$





$$\frac{v_{A/B}}{\sin 33.7°} = \frac{10}{\sin 78.3°}$$

$$v_{A/B} = 10\frac{0.555}{0.979} = 5.66 \text{ in./sec.}$$

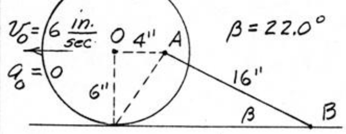
$$\omega_{AB} = v_{A/B}/\overline{AB} = \frac{5.66}{16} = \underline{0.354 \text{ rad/sec CW}}$$

$$\frac{v_A}{\sin(90°-\beta)} = \frac{10}{\sin 78.3°}, \quad v_A = 10\frac{0.927}{0.979} = 9.47 \text{ in./sec}$$

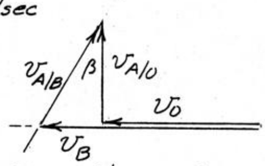
$$v_O = \frac{\overline{OC}}{\overline{AC}}v_A = \frac{6}{\sqrt{4^2+6^2}}(9.47) = \underline{7.88 \text{ in./sec}}$$

5/200 $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$, $\underline{v}_O + \underline{v}_{A/O} = \underline{v}_B + \underline{v}_{A/B}$

$v_{A/O} = \frac{4}{6}6 = 4$ in./sec



$\beta = 22.0°$



$v_{A/B} = 4/\cos 22.0° = 4.31$ in./sec

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$\underline{a}_A = \underline{a}_O + \underline{a}_{A/O} = 0 + (\underline{a}_{A/O})_n$

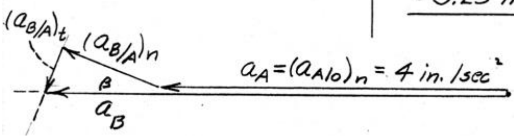
$(a_{A/O})_n = 4^2/4 = 4$ in./sec²

$(a_{B/A})_n = 4.31^2/16 = 1.16$ in./sec²

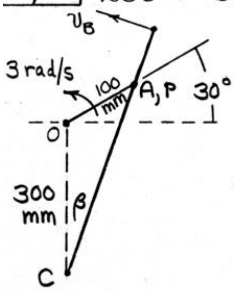
From diagram

$a_B = 4 + 1.16/\cos 22.0°$

$= \underline{5.25 \text{ in./sec}^2}$



5/201 Let P be a point on BC coincident with A.

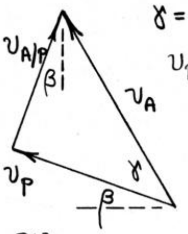


$CA^2 = 300^2 + 100^2 - 2(300)(100)\cos 120°$

$= 361$ mm

$\frac{100}{\sin\beta} = \frac{361}{\sin 120°}$, $\beta = 13.90°$

$\gamma = 60 - \beta = 46.1°$



$v_P = v_A\cos\gamma$

$= 300\cos 46.1°$

$= 208$ mm/s

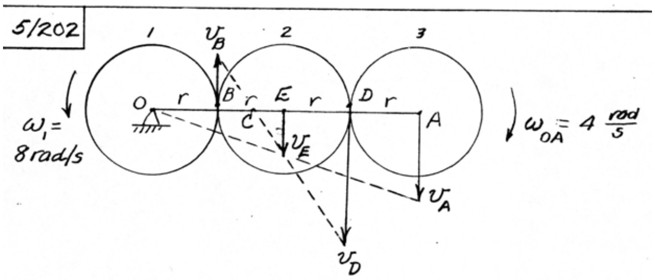
$\underline{v}_A = \underline{v}_P + \underline{v}_{A/P}$

$v_A = 100(3)$

$= 300$ mm/s

$$v_B = \overline{CB}\frac{v_P}{\overline{CA}} = 500\frac{208}{361} = \underline{288 \text{ mm/s}}$$

5/202



Let B be contact point common to gears 1 & 2

" D " " " " gears 2 & 3

Point C is instantaneous center of zero velocity for gear 2. By similar triangles, $v_D = 3(8r) = 24r$

$v_B = r\omega_1 = 8r$ $\quad v_A = \overline{OA}\,\omega_{OA} = 4r(4) = 16r$

$v_E = 2r\omega_{OA} = 8r$ $\quad \omega_3 = \frac{v_{D/A}}{\overline{DA}} = \frac{24r - 16r}{r} = \underline{8 \text{ rad/s CCW}}$

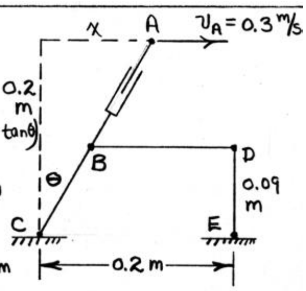
5/203 $x = 0.2\tan\theta$

$\dot{x} = 0.2\dot{\theta}\sec^2\theta$

$\ddot{x} = 0.2\ddot{\theta}\sec^2\theta + 0.2(2)\dot{\theta}^2\sec\theta(\sec\theta\tan\theta)$

For $\dot{x} = 0.3\frac{m}{s}$, $\ddot{x} = 0$, $\theta = 30°$,

$\dot{\theta} = 1.125\frac{rad}{s}$, $\ddot{\theta} = -1.461\frac{rad}{s^2}$

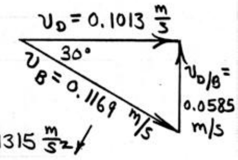


$\overline{BD} = 0.2 - 0.09\tan 30° = 0.1480$ m

$\overline{CB} = \frac{0.09}{\cos 30°} = 0.1039$ m

$v_B = \overline{CB}\dot{\theta} = 0.1039(1.125) = 0.1169\frac{m}{s}$

$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$



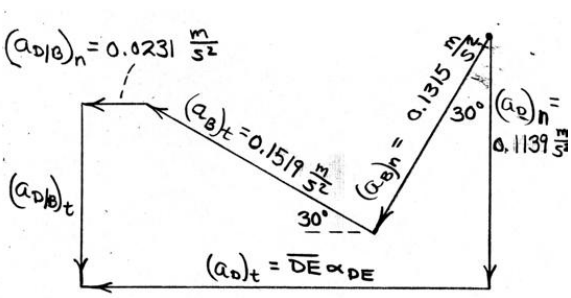
$\underline{a}_D = \underline{a}_B + (\underline{a}_{D/B})_n + (\underline{a}_{D/B})_t$

$(a_B)_n = \overline{CB}\dot{\theta}^2 = 0.1039(1.125)^2 = 0.1315\frac{m}{s^2}$

$(a_B)_t = \overline{CB}\ddot{\theta} = 0.1039(-1.461) = -0.1519\frac{m}{s^2}$

$(a_{D/B})_n = \frac{v_{D/B}^2}{\overline{DB}} = \frac{0.0585^2}{0.1480} = 0.0231\frac{m}{s^2}$

$(a_D)_n = \frac{0.1013^2}{0.09} = 0.1139\frac{m}{s^2}$

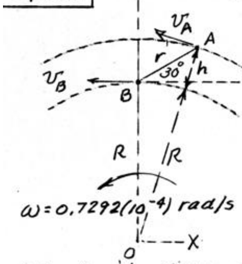


$(a_D)_t = \overline{DE}\alpha_{DE} = 0.1315\sin 30° + 0.1519\cos 30° + 0.0231$

$= 0.220$ m/s²

$$\alpha_{DE} = \frac{0.220}{0.09} = \underline{2.45 \text{ rad/s}^2 \text{ CCW}}$$

5/204



$R = 6378$ km, $h = 200$ km

Law of cosines gives

$(R+h)^2 = R^2 + r^2 - 2Rr\cos 120°$

$(6378+200)^2 = (6378)^2 + r^2 - 2(6378)(-0.5)r$

$r^2 + 6378r - 2\,591\,200 = 0$

$r = -\frac{6378}{2} \pm \frac{1}{2}\sqrt{(6378)^2 + 4(2591200)}$

$= 383.2$ km (or -6761 km)

Rel. velocity from nonrotating system X-Y at O is

$\underline{v}_A - \underline{v}_B$. But $\underline{v}_A = \underline{v}_B + \underline{\omega}\times\underline{r} + \underline{v}_{rel}$

so $\underline{\omega}\times\underline{r} = (\underline{v}_A - \underline{v}_B) - (\underline{v}_{rel})$

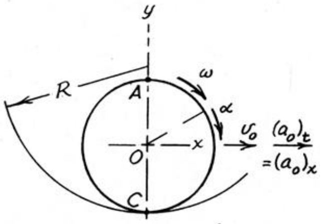
$\underline{\omega}\times\underline{r} = 0.7292(10^{-4})(3600)\underline{k} \times 383.2(0.866\underline{i} + 0.5\underline{j})$

so $\Delta\underline{v}_{rel} = \underline{-50.3\underline{i} + 87.1\underline{j}}$ km/h

5/205

$v_O = r\omega$, $(a_O)_t = r\alpha$

$\underline{a}_C = \underline{a}_O + (\underline{a}_{C/O})_n + (\underline{a}_{C/O})_t$



$\underline{a}_O = r\alpha\underline{i} + \frac{(r\omega)^2}{R-r}\underline{j}$

$(\underline{a}_{C/O})_n = r\omega^2\underline{j}$, $(\underline{a}_{C/O})_t = -r\alpha\underline{i}$

$\underline{a}_C = r\alpha\underline{i} + \frac{(r\omega)^2}{R-r}\underline{j} + r\omega^2\underline{j} - r\alpha\underline{i}$, $\underline{\underline{a}_C = \frac{r\omega^2}{1-r/R}\underline{i}}$

$\underline{a}_A = \underline{a}_O + (\underline{a}_{A/O})_n + (\underline{a}_{A/O})_t$

$(\underline{a}_{A/O})_n = -r\omega^2\underline{j}$, $(\underline{a}_{A/O})_t = r\alpha\underline{i}$

$\underline{a}_A = r\alpha\underline{i} + \frac{(r\omega)^2}{R-r}\underline{j} - r\omega^2\underline{j} + r\alpha\underline{i}$, $\underline{\underline{a}_A = 2r\alpha\underline{i} + r\omega^2\frac{2r/R - 1}{1-r/R}\underline{j}}$

***5/206**

$\dot\theta = 120\frac{2\pi}{60} = 4\pi$ rad/sec

$5\sin\theta = (25 - 5\cos\theta)\tan\beta$ ---(1)

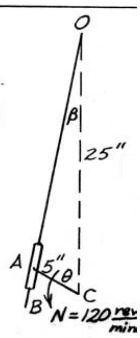
$5\dot\theta\cos\theta = 5\dot\theta\sin\theta\tan\beta + (25-5\cos\theta)\dot\beta\sec^2\beta$

$= 5\dot\theta\sin\theta\tan\beta + (25-5\cos\theta)\dot\beta(1+\tan^2\beta)$

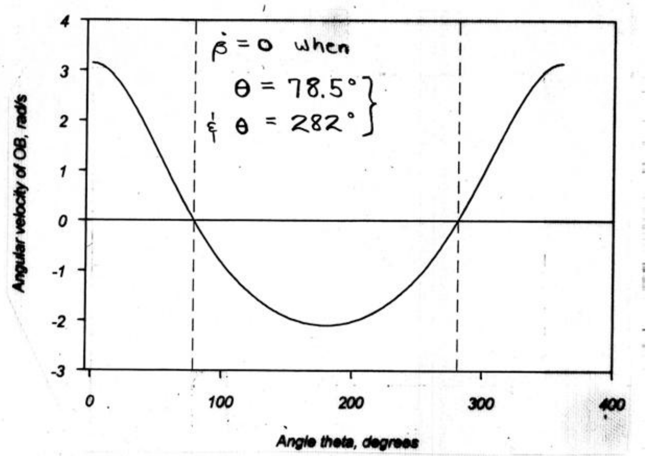
$\dot\beta = \frac{(4\pi)(\cos\theta - \sin\theta\tan\beta)}{(5-\cos\theta)(1+\tan^2\beta)}$

Substitute Eq. (1) & get

$\dot\beta = 4\pi\frac{5\cos\theta - 1}{26 - 10\cos\theta} = 2\pi\frac{5\cos\theta - 1}{13 - 5\cos\theta}$



Calculate & plot $\dot\beta$ vs θ:



***5/207**

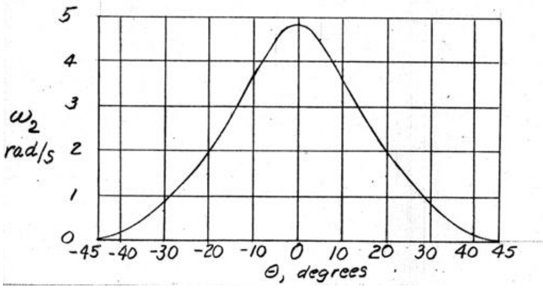
From Prob. 5/56 we have

$\omega_2 = \dot\theta\frac{\cos(\theta+\beta)}{\cos(\theta+\beta) - \sqrt{2}\cos\beta}$ where $\beta = \angle O_1O_2P$

Also $\tan\beta = \frac{\sin\theta}{\sqrt{2} - \cos\theta}$

For $\dot\theta = -2$ rad/s, $\omega_2 = 2\frac{\cos(\theta+\beta)}{\sqrt{2}\cos\beta - \cos(\theta+\beta)}$

Set up program to compute β & ω_2 & plot results.



***5/208**

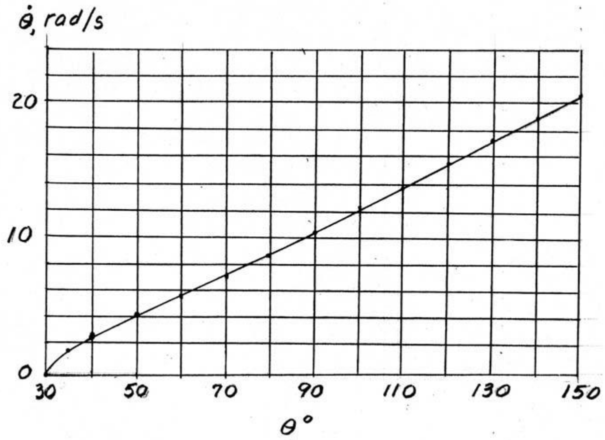
$\ddot\theta = 100(1-\cos\theta)$ rad/s²

$\dot\theta\,d\dot\theta = \ddot\theta\,d\theta$ so $\int_0^{\dot\theta}\dot\theta\,d\dot\theta = 100\int_{\pi/6}^{\theta}(1-\cos\theta)\,d\theta$

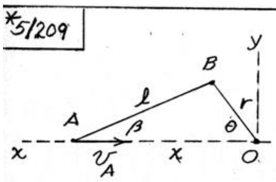
$\dot\theta^2 = 200(\theta - \sin\theta)\Big|_{\pi/6}^{\theta} = 200(\theta - \sin\theta - 0.0236)$

$\dot\theta = \frac{d\theta}{dt} = 10\sqrt{2}\sqrt{\theta - \sin\theta - 0.0236}$ rad/s

$\int_0^t dt = \int_{\pi/2}^{5\pi/6}\frac{d\theta}{10\sqrt{2}\sqrt{\theta - \sin\theta - 0.0236}}$ Numerical integration gives $\underline{t = 0.0701}$ s



*5/209



$x = \ell\cos\beta + r\cos\theta$

$r\sin\theta = \ell\sin\beta$

$v_A = -\dot{x} = \ell\dot{\beta}\sin\beta + r\dot{\theta}\sin\theta$

$r\dot{\theta}\cos\theta = \ell\dot{\beta}\cos\beta$

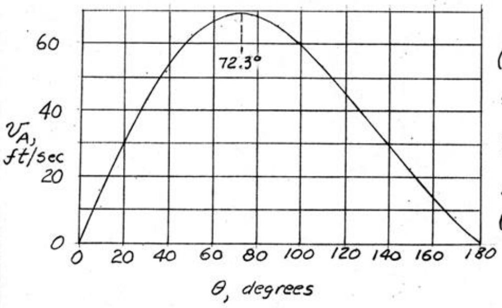
$\dot{\beta} = \frac{r}{\ell}\frac{\dot{\theta}\cos\theta}{\sqrt{1-\sin^2\beta}} = \frac{\omega\cos\theta}{\sqrt{(\ell/r)^2-\sin^2\theta}}$

$v_A = \ell\left[\frac{\omega\cos\theta}{\sqrt{(\ell/r)^2-\sin^2\theta}}\right]\frac{r}{\ell}\sin\theta + r\omega\sin\theta = r\omega\sin\theta\left(1+\frac{\cos\theta}{\sqrt{(\ell/r)^2-\sin^2\theta}}\right)$

From Sample Problem 5/15 substitute

$\ell = 14/12$ ft, $r = 5/12$ ft, $\omega = 1500(2\pi)/60 = 157.1$ rad/sec & get

$v_A = 65.45\sin\theta\left(1+\frac{\cos\theta}{\sqrt{7.84-\sin^2\theta}}\right)$, Set up computer program & solve for $0<\theta<180°$



$(v_A)_{max}$ = 69.6 ft/sec at $\theta = 72.3°$

By symmetry $(v_A)_\theta = -(v_A)_{-\theta}$

*5/210 From the results of Prob. 5/209, we may write

$a_A = \dot{v}_A = \frac{d}{dt}\left\{r\omega\sin\theta + r\omega\frac{\sin\theta\cos\theta}{\sqrt{(\ell/r)^2-\sin^2\theta}}\right\}$

$= r\omega\left\{\dot{\theta}\cos\theta + \frac{\sqrt{(\ell/r)^2-\sin^2\theta}\,(\dot{\theta}\cos 2\theta) - \frac{1}{2}\sin 2\theta\,\frac{(-\frac{1}{2}\sin 2\theta)\dot{\theta}}{\sqrt{(\ell/r)^2-\sin^2\theta}}}{(\ell/r)^2-\sin^2\theta}\right\}$

which reduces to

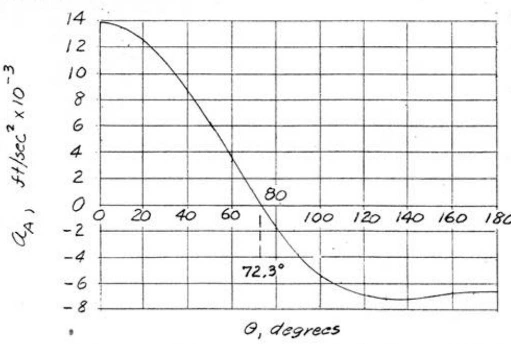
$a_A = r\omega^2\left[\cos\theta + \frac{r}{\ell}\,\frac{1-2\sin^2\theta+\frac{r^2}{\ell^2}\sin^4\theta}{(1-\frac{r^2}{\ell^2}\sin^2\theta)^{3/2}}\right]$

From Sample Problem 5/15 Substitute

$\ell = 14/12$ ft, $r = 5/12$ ft, $\omega = 1500(2\pi)/60 = 157.1$ rad/sec & get

$a_A = 1.028(10^4)\left[\cos\theta + 0.357\,\frac{1-2\sin^2\theta+0.1276\sin^4\theta}{(1-0.1276\sin^2\theta)^{3/2}}\right]$

Set up computer program & solve for $0<\theta<180°$



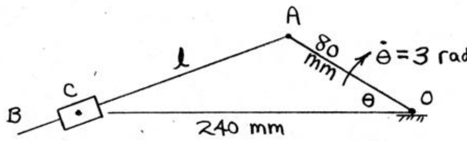
$(a_A)_{max} = 13.95(10^3)$ ft/sec² at $\theta = 0$

$a_A = -6.61(10^3)$ ft/sec² at $\theta = 180°$

$a_A = 0$ at $\theta = 72.3°$

$(a_A)_{+\theta} = +(a_A)_{-\theta}$

*5/211

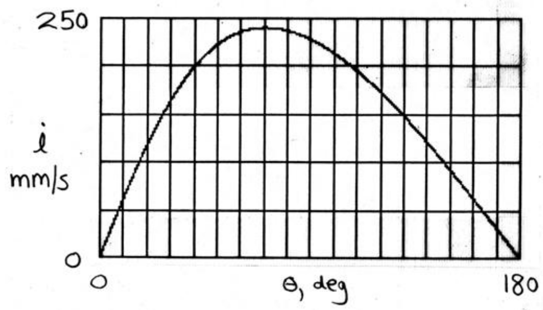


Velocity of AB through collar C is $\dot{\ell}$.

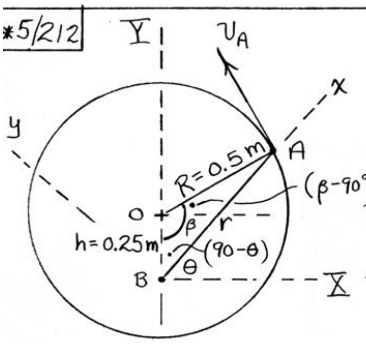
$\ell^2 = 240^2 + 80^2 - 2(240)(80)\cos\theta$, $\ell = 80\sqrt{2}\sqrt{5-3\cos\theta}$

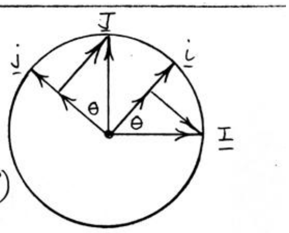
$2\ell\dot{\ell} = 2(240)(80)\dot{\theta}\sin\theta$

$\dot{\ell} = \frac{720\sin\theta}{\sqrt{2}\sqrt{5-3\cos\theta}}$ mm/s, $\dot{\ell}_{max} = 240\ \frac{mm}{s}$ @ $\theta = 70.5°$



*5/212





$$\begin{cases} \underline{I} = \underline{i}\cos\theta - \underline{j}\sin\theta \\ \underline{J} = \underline{i}\sin\theta + \underline{j}\cos\theta \end{cases}$$

$$r^2 = h^2 + R^2 - 2hR\cos\beta$$

$$\frac{\sin(90-\theta)}{R} = \frac{\cos\theta}{R} = \frac{\sin\beta}{r}, \quad \theta = \cos^{-1}\left[\frac{R}{r}\sin\beta\right]$$

$$\underline{v}_A = R\dot{\beta}\left[-\sin(\beta-90°)\underline{I} + \cos(\beta-90°)\underline{J}\right]$$

$$\underline{a}_A = R\dot{\beta}^2\left[-\cos(\beta-90°)\underline{I} - \sin(\beta-90°)\underline{J}\right]$$

Substitute the above transformation equations into the expressions for $\underline{v}_A$ & $\underline{a}_A$ and simplify to obtain (with c = cos, s = sin)

$$\underline{v}_A = R\dot{\beta}\Big\{\left[-c\theta\, s(\beta-90°) + s\theta\, c(\beta-90°)\right]\underline{i} + \left[s\theta\, s(\beta-90°) + c\theta\, c(\beta-90°)\right]\underline{j}\Big\}$$

$$\underline{a}_A = R\dot{\beta}^2\Big\{\left[-c\theta\, c(\beta-90°) - s\theta\, s(\beta-90°)\right]\underline{i} + \left[s\theta\, c(\beta-90°) - c\theta\, s(\beta-90°)\right]\underline{j}\Big\}$$

Eqs. 5/12 & 5/14, Bxy attached to BD:

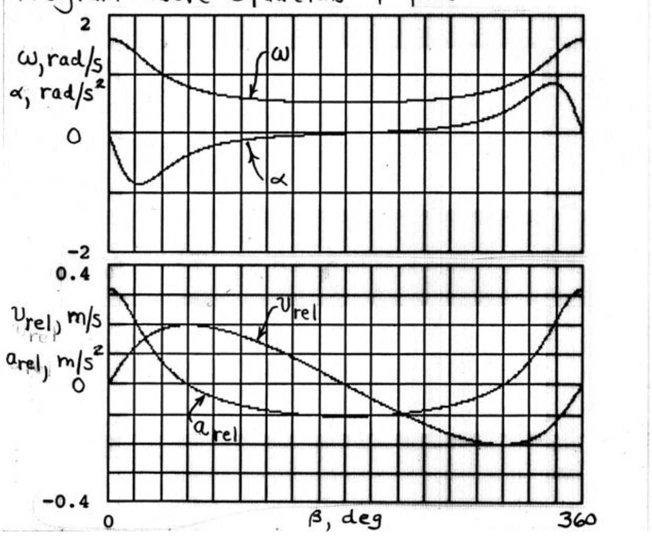
$$\begin{cases} \underline{v}_A = \underline{v}_B^{\nearrow 0} + \underline{\omega}\times\underline{r} + \underline{v}_{rel} \\ \underline{a}_A = \underline{a}_B^{\nearrow 0} + \underline{\alpha}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\underline{v}_{rel} + \underline{a}_{rel} \end{cases}$$

$$\text{Note: } \begin{cases} \underline{r} = r\underline{i}, & \underline{v}_{rel} = v_{rel}\underline{i} \\ \underline{a}_{rel} = a_{rel}\underline{i}, & \underline{\omega} = \omega\underline{k}, \quad \underline{\alpha} = \alpha\underline{k} \end{cases}$$

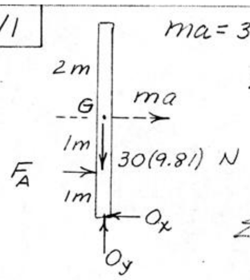
$$\Rightarrow v_{rel} = v_{A_x}, \quad \omega = \frac{v_{A_y}}{r}, \quad a_{rel} = a_{A_x} + r\omega^2,$$

$$\text{and} \quad \alpha = \frac{1}{r}\left(a_{A_y} - 2\omega v_{rel}\right)$$

Program above equations & plot:



6/1



$ma = 30(20) = 600$ N

$\Sigma M_A = mad_A$

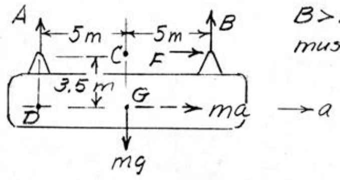
$F_A(1) = 600(2)$

$\underline{F_A = 1200 \text{ N}}$

$\Sigma F_x = ma_x;\quad 1200 - O_x = 600$

$\underline{O_x = 600 \text{ N}}$

6/2



$B > A$ since ΣM_C must be CCW.

$F_{max} = \mu B = 0.25B$

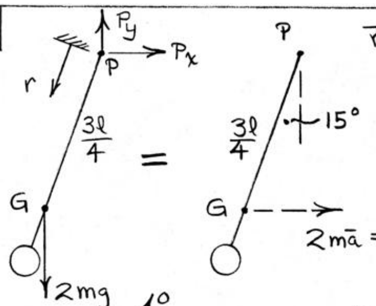
$\Sigma M_D = 0;\quad 5mg + 0.25B(3.5) - 10B = 0$

$B = 5mg/9.125 = 0.548\,mg$

$\Sigma F = ma;\quad 0.25B = ma,\quad a = \dfrac{0.25(0.548)mg}{m}$

$a = 0.25(0.548)(9.81) = \underline{1.344 \text{ m/s}^2}$

6/3

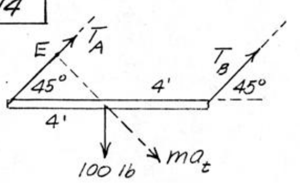


$\bar{r} = \dfrac{\Sigma m_i r_i}{\Sigma m_i} = \dfrac{m(\ell/2) + m(\ell)}{m + m} = \dfrac{3}{4}\ell$

$\Sigma M_P = \bar{I}\alpha + m\bar{a}d:\quad 2mg\left(\frac{3\ell}{4}\sin 15^\circ\right) = 2ma\left(\frac{3\ell}{4}\cos 15^\circ\right)$

$\Rightarrow a = g\tan 15^\circ = \underline{0.268g}$

6/4

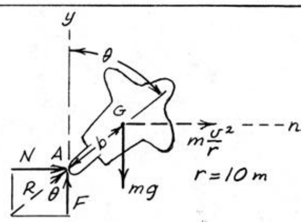


$\Sigma M_E = 0;$

$100(2) - T_B\dfrac{8}{\sqrt{2}} = 0$

$T_B = 25\sqrt{2} = \underline{35.4 \text{ lb}}$

6/5



$\Sigma M_A = m\bar{a}d:\ mgb\sin\theta = m\dfrac{v^2}{r}b\cos\theta,\quad v^2 = gr\tan\theta$

But $\tan\theta = N/F = 1/\mu$ so $v^2 = \dfrac{gr}{\mu}$, $v = \sqrt{\dfrac{9.81 \times 10}{0.70}}$

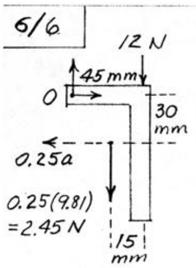
$= 11.84$ m/s

or $\underline{v = 42.6 \text{ km/h}}$

$\theta = \tan^{-1}\dfrac{v^2}{gr} = \tan^{-1}\dfrac{11.84^2}{9.81 \times 10} = \underline{55.0^\circ}$

Note: The fact that in reality this is a rigid body rotating about the central axis does not invalidate the plane-motion analysis as a translating body so long as $\dot{\theta} = 0$.

6/6

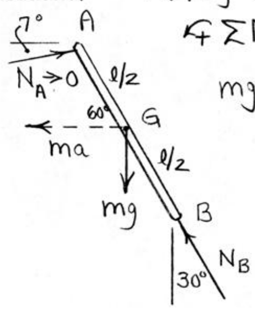


$\Sigma M_O = mad_O$

$12(45) + 0.25(9.81)(30) = 0.25a(30)$

$a = 81.8 \text{ m/s}^2 = \underline{8.34g}$

6/7



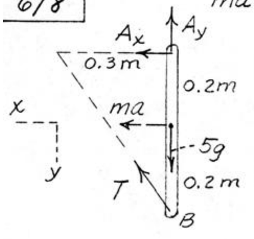
Tipping impends when $N_A \to 0$.

$\Sigma M_B = mad:$

$mg\dfrac{\ell}{2}\sin 30^\circ = ma\dfrac{\ell}{2}\cos 30^\circ$

$\underline{a = g\tan 30^\circ = 5.66 \text{ m/s}^2}$

6/8



$ma = 5(0.6g) = 3g$

$\Sigma M_B = mad_B$

$0.4A_x = 3g(0.2)$

$A_x = \dfrac{0.6}{0.4}(9.81) = 14.72$ N

$\Sigma M_A = mad_A$

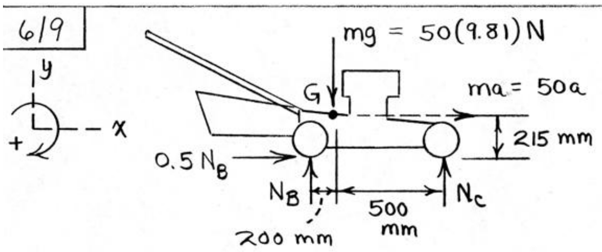
$\left(\frac{3}{5}T\right)0.4 = 3g(0.2)$

$\underline{T = 24.5 \text{ N}}$

$\Sigma F_y = 0;\quad \frac{4}{5}(24.5) + A_y - 5(9.81) = 0,\quad A_y = 29.4$ N

$A = \sqrt{29.4^2 + 14.72^2} = \underline{32.9 \text{ N}}$

6/9



$\Sigma F_x = ma: \quad 0.5N_B = 50a$

$\Sigma F_y = 0: \quad N_B + N_C - 50(9.81) = 0$

$\Sigma M_B = mad: \quad 50(9.81)(0.2) - N_C(0.7) = 50a(0.215)$

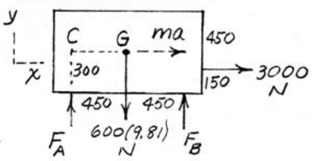
Simultaneous solution : $\begin{cases} N_B = 414 \text{ N} \\ N_C = 76.6 \text{ N} \\ a = 4.14 \text{ m/s}^2 \end{cases}$

6/10

$\Sigma M_C = 0; \quad 600(9.81)450 - 900F_B - 3000(150) = 0$

$F_B = 2440$ N

Dimensions in mm



$\Sigma F_y = 0;$

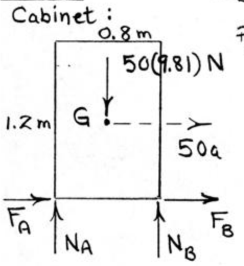
$F_A + 2440 - 600(9.81) = 0$

$F_A = 3440$ N

6/11

N_B & $F_B \to 0$ when tipping impends

Cabinet :



$\Sigma M_A = mad: \quad mg(0.4) = ma(0.6)$

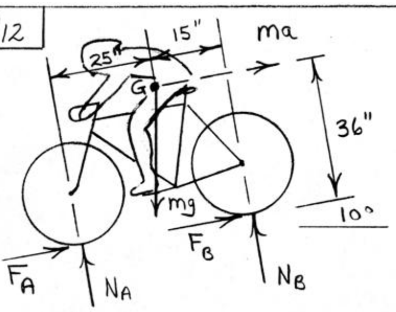
$a = \frac{2}{3}g$ or 6.54 m/s^2

As a whole : $\Sigma F = ma$

$P = 60(6.54) = 392$ N

$\mu_s > \frac{a}{g} = \frac{2}{3}$

6/12

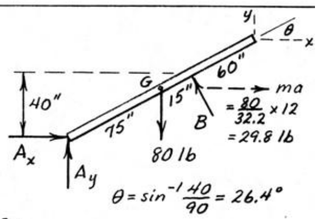


Tipping at front wheel : $N_B, F_B \to 0$

$\Sigma M_A = mad: \quad mg(25\cos 10° - 36\sin 10°) = ma(36)$

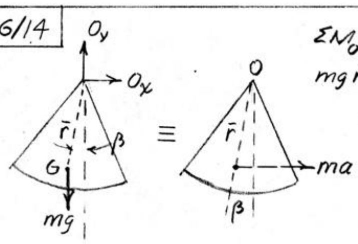
Solve to obtain $a = 0.510g$ (16.43 ft/sec^2)

6/13



$\Sigma M_A = mad: \quad 80 \times \frac{75}{12} \times \cos 26.4° - \frac{90}{12}B = 29.8 \times \frac{75}{12}\sin 26.4°, \quad B = 48.7 \text{ lb}$

6/14



$\Sigma M_O = mad$

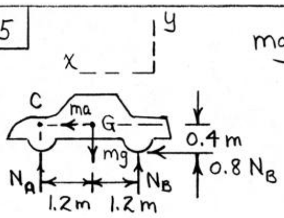
$mg\bar{r}\sin\beta = ma\bar{r}\cos\beta$

$a = g\tan\beta$

But $\theta = \frac{120}{20}\beta = 6\beta$

so $a = g\tan\theta/6$

6/15



$mg = 1650(9.81) = 16.19(10^3)$ N

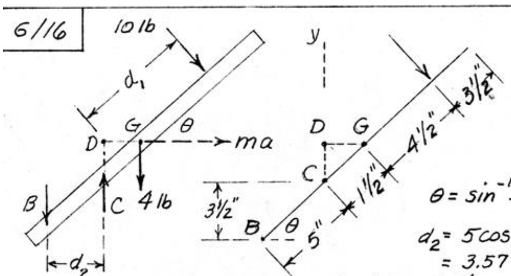
$\Sigma M_C = mad = 0: \quad N_B(2.4) - 0.8N_B(0.4) - 16.19(10^3)1.2 = 0$

$N_B = 9.34(10^3)$ N or $N_B = 9.34$ kN

$\Sigma F_y = 0: \quad N_A + 9.34(10^3) - 16.19(10^3) = 0$

$N_A = 6.85(10^3)$ N or $N_A = 6.85$ kN

6/16



$\theta = \sin^{-1}\frac{3.5}{5} = 44.4°$

$d_2 = 5\cos 44.4° = 3.57$ in.

$d_1 = 4.5 + 1.5(\cos 44.4°)^2 = 5.27$ in.

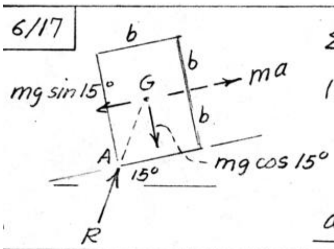
$\Sigma M_D = 0; \quad 10(5.27) + 4(1.5\cos 44.4°) - 3.57B = 0$

$B = 15.94$ lb

$\Sigma F_y = 0; \quad C - 4 - 15.94 - 10\cos 44.4° = 0$

$C = 27.1$ lb

6/17



$\Sigma M_A = m a d$

$(mg\cos 15°)\frac{b}{2} - (mg\sin 15°)b = m a b$

$g\left(\frac{0.966}{2} - 0.259\right) = a$

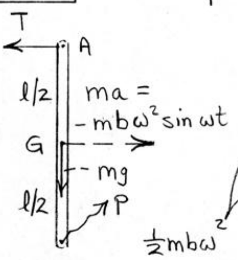
$\underline{a = 0.224g}$

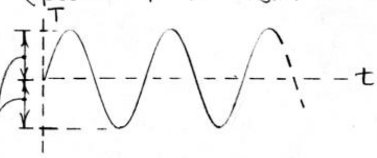
6/18

$\circlearrowleft \Sigma M_P = m a d: \; -Tl = -mb\omega^2 \sin\omega t\left(\frac{l}{2}\right)$

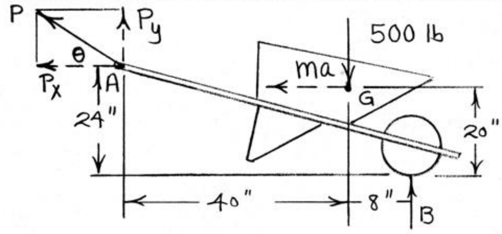
$\Rightarrow T = \frac{1}{2} m b\omega^2 \sin\omega t$

(positive T is tension in link)





6/19



Static equilibrium : $P_x = ma = 0$

$\circlearrowleft \Sigma M_A = 0: \; 500(40) - B(48) = 0, \quad \underline{B_{st} = 417 \text{ lb}}$

Dynamic : $\circlearrowleft \Sigma M_A = m a d$:

$500(40) - B(48) = \frac{500}{32.2}(5)(4), \quad \underline{B = 410 \text{ lb}}$

$\overset{+}{\leftarrow} \Sigma F_x = ma: \; P_x = \frac{500}{32.2}(5) = 77.6 \text{ lb}$

$+\uparrow \Sigma F_y = 0: \; B - 500 + P_y = 0, \; P_y = 89.8 \text{ lb}$

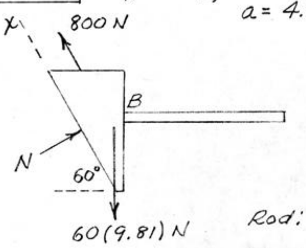
$\therefore \; \underline{P = 118.7 \text{ lb}}, \quad \underline{\theta = 49.2°}$

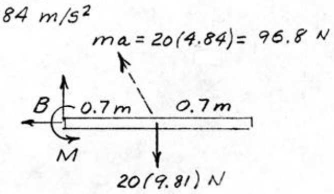
6/20

$\Sigma F_x = m a_x; \; 800 - 60(9.81)\sin 60° = 60a$

$a = 4.84 \text{ m/s}^2$

$ma = 20(4.84) = 96.8 \text{ N}$





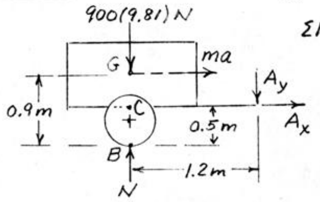
Rod: $\Sigma M_B = m a d$

$M - 20(9.81)0.7 = 96.8(0.7\sin 60°)$

$\underline{M = 196.0 \text{ N·m}}$

6/21

$v^2 = 2as, \quad a = \frac{v^2}{2s} = \frac{(60/3.6)^2}{2(30)} = 4.63 \text{ m/s}^2$



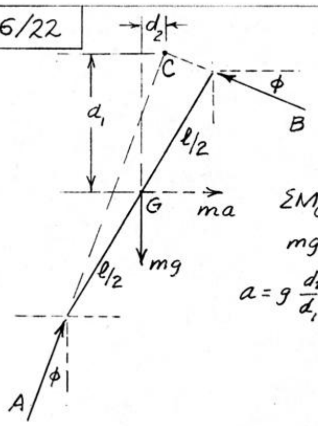
$\Sigma M_C = m a d;$

$1.2A_y = 900(4.63)(0.9 - 0.5)$

$\underline{A_y = 1389 \text{ N}}$

6/22

$\phi = \tan^{-1}\mu = \tan^{-1} 0.4 = 21.8°$



Moment arms measure

$d_1 = 0.486l$

$d_2 = 0.118l$

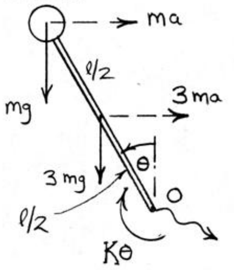
$\Sigma M_C = m a d$

$mg\,d_2 = m a d_1$

$a = g\frac{d_2}{d_1} = 9.81\frac{0.118l}{0.486l} = \underline{2.38 \text{ m/s}^2}$

6/23

$\circlearrowright \Sigma M_O = \Sigma m a d: \; K\theta - 3mg\left(\frac{l}{2}\sin\theta\right) - mg(l\sin\theta) = 3ma\left(\frac{l}{2}\cos\theta\right) + ma(l\cos\theta)$

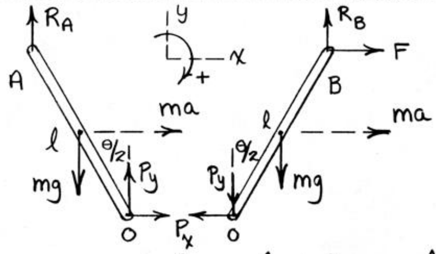


Simplify to

$K\theta - \frac{5}{2}mgl\sin\theta = \frac{5}{2}mal\cos\theta$

With $m = 0.5$ kg, $l = 0.6$ m, $a = 2g$, and $\theta = 20°$, K is found to be $\underline{K = 46.8 \frac{\text{N·m}}{\text{rad}}}$

6/24



Ⓐ $\Sigma M_O = m a d: \; R_A l\sin\frac{\theta}{2} - mg\frac{l}{2}\sin\frac{\theta}{2} = ma\frac{l}{2}\cos\frac{\theta}{2}$

Ⓑ $\Sigma M_O = m a d: \; Fl\cos\frac{\theta}{2} + mg\frac{l}{2}\sin\frac{\theta}{2} - R_B l\sin\frac{\theta}{2} = ma\frac{l}{2}\cos\frac{\theta}{2}$

Two bars together :

$\Sigma F_y = 0: \; R_A + R_B - 2mg = 0$

Subtract Eq. Ⓐ from Ⓑ, combine with y-eq. to obtain $\underline{\theta = 2\tan^{-1}\frac{F}{mg}}$

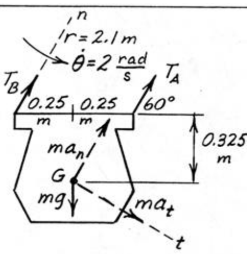
Both bars together: $\Sigma F_x = m a_x: \; F = 2ma, \; a = \frac{g}{2}\tan\frac{\theta}{2}$

From Ⓑ: $mg\tan\frac{\theta}{2}\,l\cos\frac{\theta}{2} + mg\frac{l}{2}\sin\frac{\theta}{2} - R_B l\sin\frac{\theta}{2} = m\left(\frac{g}{2}\tan\frac{\theta}{2}\right)\frac{l}{2}\cos\frac{\theta}{2}$

$\Rightarrow \underline{R_B = \frac{5}{4}mg}$

Finally, from y-eq., $\underline{R_A = \frac{3}{4}mg}$

6/25

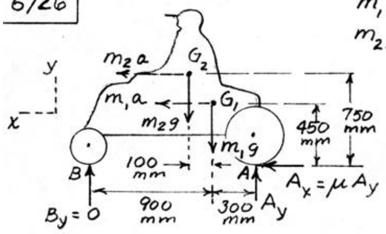


$\Sigma M_G = 0: (T_A \sin 60°)(0.25) - (T_B \sin 60°)(0.25)$
$-(T_A \cos 60°)(0.325) - (T_B \cos 60°)(0.325) = 0$
$0.0540\, T_A = 0.379\, T_B$ --- (1)

$\Sigma F_n = ma_n: \; T_A + T_B - 10(9.81)\sin 60°$
$= 10(2.1)(2^2)$ --- (2)

Solve (1) & (2) & get $\underline{T_A = 147.9 \text{ N}}$, $\underline{T_B = 21.1 \text{ N}}$

6/26



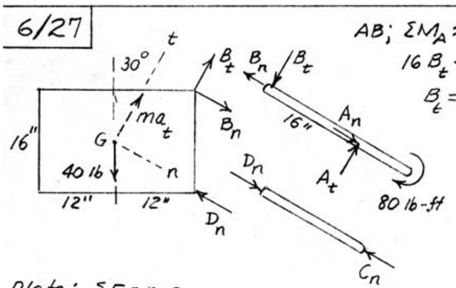
$m_1 g = 140(9.81) = 1373$ N
$m_2 g = 90(9.81) = 883$ N

$\Sigma F_y = 0$
$A_y - 1373 - 883 = 0$
$A_y = 2256$ N

$\Sigma M_A = \Sigma mad; \; 883(0.4) + 1373(0.3) = 90a(0.75) + 140a(0.45)$
$765.2 = 130.5a$
$a = 5.86 \text{ m/s}^2$

$\Sigma F_x = \Sigma ma_x; \; \mu(2256) = (140 + 90)5.86$
$\underline{\mu = 0.598}$

6/27



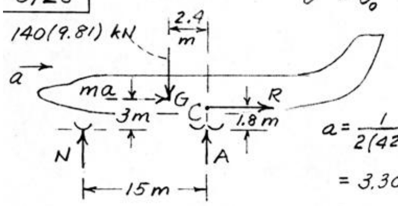
AB; $\Sigma M_A \cong 0$;
$16 B_t - 80(12) = 0$
$B_t = 60$ lb

Plate; $\Sigma F_t = ma_t$; $60 - 40\cos 30° = \frac{40}{32.2} a_t$, $a_t = 20.4$ ft/sec²

$\Sigma M_B = mad$; $D_n(16\cos 30°) - 40(12)$
$= \frac{40}{32.2}(20.4)[12\cos 30° - 8\sin 30°]$

$D = D_n = \underline{46.3 \text{ lb}}$

6/28

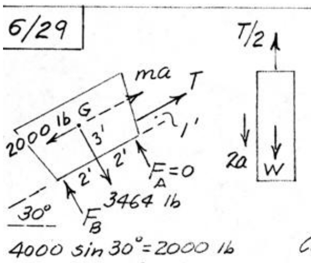


$v^2 = v_0^2 + 2as$

$a = \frac{1}{2(425)}\left[(200)^2 - (60)^2\right]\frac{1}{(3.6)^2}$
$= 3.30 \text{ m/s}^2$

$\Sigma M_C = mad; \; 15N - 140(9.81)(2.4) = 140(3.30)(3 - 1.8)$
$\underline{N = 257 \text{ kN}}$

6/29



4000 sin 30° = 2000 lb
4000 cos 30° = 3464 lb

Car: $\Sigma M_B = mad_B$
$T(1) + 3464(2) - 2000(3)$
$= \frac{4000}{32.2} a(3)$

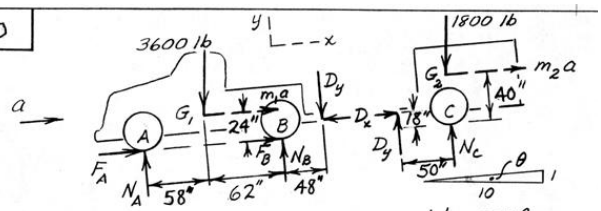
$\Sigma F = ma; \; T - 2000 = \frac{4000}{32.2} a$

Solve & get $T = 3464$ lb
$a = 11.79$ ft/sec²

Counterweight:
$\Sigma F = ma; \; W - 3464/2$
$= \frac{W}{32.2}(2 \times 11.79)$

$\underline{W = 6460 \text{ lb}}$

6/30

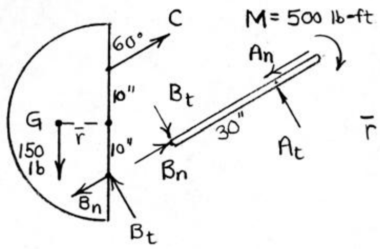


For const. accel., $\theta = \tan^{-1}\frac{1}{10} = 5.71°$
$v^2 = v_0^2 + 2as$: $44^2 = 88^2 - 2a(360)$, $a = 8.07$ ft/sec² decel.
$m_1 a = \frac{3600}{32.2} \times 8.07 = 902$ lb, $m_2 a = \frac{1800}{32.2} \times 8.07 = 451$ lb
Trailer: $\Sigma F_x = ma_x$: $D_x - 1800\sin 5.71° = 451$, $\underline{D_x = 630 \text{ lb}}$
$\Sigma M_C = mad$: $50D_y + 630(18) - 1800\sin 5.71°(40) = 451(40)$, $\underline{D_y = 277 \text{ lb}}$
$\Sigma F_y = 0$: $N_C - 1800\cos 5.71° + 277 = 0$, $\underline{N_C = 1514 \text{ lb}}$
Truck:
$\Sigma M_A = mad$: $3600\cos 5.71° \times 58 - 3600\sin 5.71° \times 24 - 120 N_B$
$+ 277(168) - 630(18) = 902(24)$
$\underline{N_B = 1773 \text{ lb}}$

6/31

AB: $\begin{cases} \Sigma M_A = 0: \; 30 B_t = 500(12), \; B_t = 200 \text{ lb} \\ \Sigma F_t = 0 \Rightarrow \underline{A_t = 200 \text{ lb}} \end{cases}$

Plate: $\Sigma F_t = ma_t$: $200 - 150\frac{\sqrt{3}}{2} = \frac{150}{32.2} a_t$
$a_t = 15.05$ ft/sec²

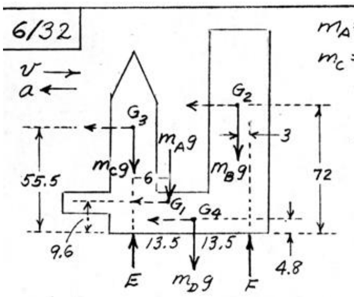


$\bar{r} = \frac{4r}{3\pi} = \frac{4(20)}{3\pi}$
$= 8.49$ in.

$\Sigma M_C = mad: \; 200(20)(\frac{1}{2}) + B_n(20\frac{\sqrt{3}}{2})$
$-150(8.49) = \frac{150}{32.2} 15.05(\frac{\sqrt{3}}{2} 8.49 + \frac{1}{2} 10)$

$A_n = B_n = \underline{8.03 \text{ lb}}$

6/32



$m_A = 3$ Gg, $m_B = 3.3$ Gg

$m_C = 0.23$ Gg, $m_D = 3$ Gg

$v^2 = 2as$

$a = \frac{v^2}{2s} = \frac{(1.5/3.6)^2}{2(0.1)} = 0.868 \frac{m}{s^2}$

$\Sigma M_E = \Sigma mad$

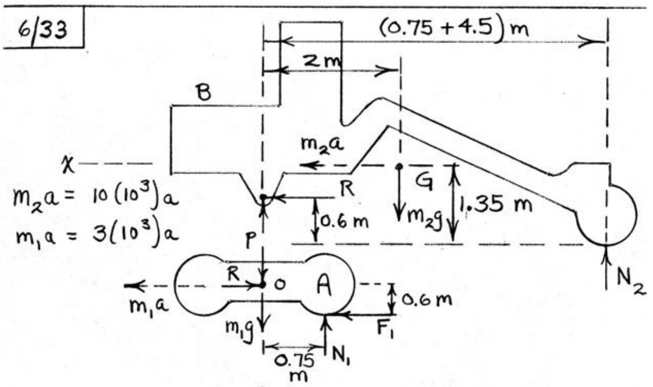
Dimensions in meters

$27F - [3(6) + 3.3(27-3) + 3(13.5)]9.81$

$= [3(9.6) + 3.3(72) + 0.23(55.5) + 3(4.8)]0.868$

$27F = 1350.8 + 254.8$, $\underline{F = 59.5 \text{ MN}}$

6/33



$m_2 a = 10(10^3)a$

$m_1 a = 3(10^3)a$

For rear wheels of unit A to lift off ground:

(A) $\Sigma M_{N_1} = m_1 a d_1 : [P + 3(10^3)(9.81)](0.750) - 0.6R = 3(10^3)a(0.6)$

(B) $\Sigma M_{N_2} = m_2 a d_2 : 10(10^3)(9.81)(4.5 + 0.75 - 2)$

$-P(4.5 + 0.75) + 0.6R = 10(10^3)a(1.35)$

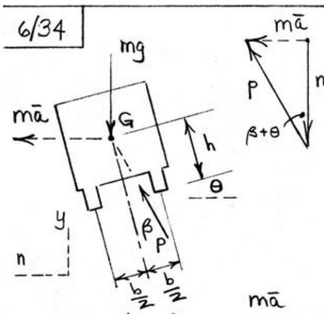
$\Sigma F_x = ma_x : R = 10(10^3)a$

Solve the above three equations to obtain

$R = 76.2$ kN, $P = 49.8$ kN, $a = 7.62$ m/s^2

For constant acceleration, $s = \frac{v^2}{2a} = \frac{(40/3.6)^2}{2(7.62)} = \underline{8.10 \text{ m}}$

6/34



(a) For no tendency to slip, $\beta = 0$.

From diagram,

$\tan\theta = \frac{m\bar{a}}{mg} = \frac{v^2/r}{g}$

$\underline{\theta = \tan^{-1}\frac{v^2}{gr}}$

(b) $\tan(\beta + \theta) = \frac{m\bar{a}}{mg} = \frac{v^2/r}{g}$

$v^2 = gr\tan(\beta+\theta) = gr\frac{\tan\beta + \tan\theta}{1 - \tan\beta\tan\theta}$

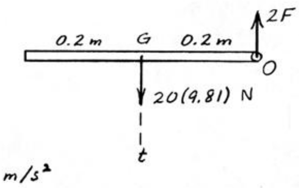
Slips first if $\mu < \frac{b/2}{h}$ & $\mu = \tan\beta$

So $\underline{v^2 = gr\frac{\mu + \tan\theta}{1 - \mu\tan\theta}}$

Tips first if $\mu > \frac{b/2}{h}$ & $\tan\beta = \frac{b}{2h}$:

$\underline{v^2 = gr\frac{\frac{b}{2h} + \tan\theta}{1 - \frac{b}{2h}\tan\theta}}$

6/35



$\Sigma M_O = I_O\alpha$, $20(9.81)(0.2)$

$= \frac{1}{3}20(0.4)^2\alpha$

$\alpha = 36.8$ rad/s^2

$\bar{a} = \bar{r}\alpha$, $\bar{a} = 0.2 \times 36.8 = 7.36$ m/s^2

$\Sigma F_t = m\bar{a}_t$, $20(9.81) - 2F = 20 \times 7.36$

$2F = 49.0$, $\underline{F_A = F_B = F = 24.5 \text{ N}}$

6/36 Accelerating force on rear wheels is

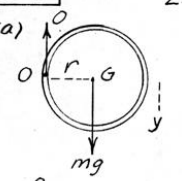
$F = ma = \frac{5200}{g}0.5g = 2600$ lb

$\alpha_{drum} = \frac{a_t}{r} = \frac{0.5(32.2)}{3} = 5.37$ rad/sec^2

$\Sigma M_O = I_O\alpha$; $I_O = \frac{2600(3)}{5.37} = \underline{1453 \text{ lb-ft-sec}^2}$

6/37

(a)



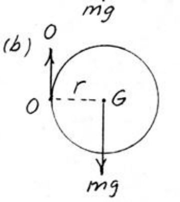
$\Sigma M_O = I_O\alpha$; $mgr = 2mr^2\alpha$

$\underline{\alpha = g/2r}$

$\Sigma F_y = m\bar{a}_y$; $mg - O = mr\left(\frac{g}{2r}\right)$

$\underline{O = mg/2}$

(b)



$\Sigma M_O = I_O\alpha$; $mgr = \left(\frac{1}{2}mr^2 + mr^2\right)\alpha$

$\underline{\alpha = 2g/3r}$

$\Sigma F_y = m\bar{a}_y$; $mg - O = mr\left(\frac{2g}{3r}\right)$

$\underline{O = mg/3}$

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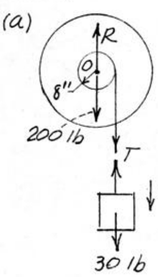
(a)



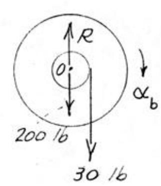
$\Sigma M_O = I_O\alpha$; $T\frac{8}{12} = \frac{200}{32.2}\left(\frac{15}{12}\right)^2\alpha_a$

$\Sigma F = ma$; $30 - T = \frac{30}{32.2}\left(\frac{8}{12}\alpha_a\right)$

Solve simultaneously & get

$T = 28.77$ lb $\underline{\alpha_a = 1.976 \text{ rad/sec}^2}$

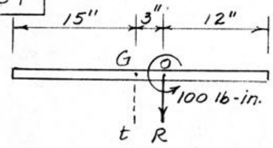
(b)



$\Sigma M_O = I_O\alpha$; $30\frac{8}{12} = \frac{200}{32.2}\left(\frac{15}{12}\right)^2\alpha_b$

$\underline{\alpha_b = 2.06 \text{ rad/sec}^2}$

6/39



$I_O = I_G + md^2$

$I_G = \frac{1}{12}ml^2 = \frac{1}{12}\frac{20}{32.2}\left(\frac{30}{12}\right)^2$

$= 0.323$ lb-ft-sec^2

$I_O = 0.323 + \frac{20}{32.2}\left(\frac{3}{12}\right)^2$

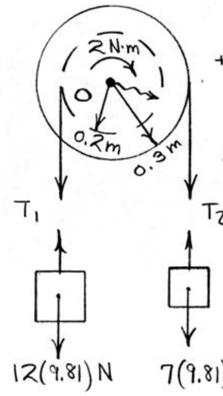
$= 0.3623$ lb-ft-sec^2

$\Sigma M_O = I_O\alpha$; $\frac{100}{12} = 0.3623\alpha$, $\alpha = 23.0$ rad/sec^2

$\bar{a}_t = \bar{r}\alpha$; $\bar{a}_t = \frac{3}{12}(23.0) = 5.75$ ft/sec^2

$\Sigma F_t = m\bar{a}_t$; $R = \frac{20}{32.2}(5.75) = \underline{3.57 \text{ lb}}$

6/40



$\curvearrowleft \Sigma M_O = I_O \alpha$ for drum:

$T_1(0.2) - T_2(0.3) - 2 = 8(0.225)^2 \alpha$ (1)

$+\downarrow \Sigma F = ma$ for 12-kg cylinder:

$12(9.81) - T_1 = 12(0.2\alpha)$ (2)

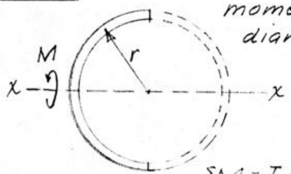
$+\uparrow \Sigma F = ma$ for 7-kg cylinder:

$T_2 - 7(9.81) = 7(0.3\alpha)$ (3)

Solution of Eqs. (1)-(3):

$$\begin{cases} T_1 = 116.2 \text{ N} \\ T_2 = 70.0 \text{ N} \\ \alpha = 0.622 \text{ rad/s}^2 \end{cases}$$

6/41



For complete ring of mass $2m$ moment of inertia about diameter x-x $= \frac{1}{2}(2mr^2)$

So moment of inertia of half ring about x-x is $\frac{1}{2}mr^2$

$\Sigma M_x = I_x \alpha$; $M = \frac{1}{2}mr^2\alpha$, $\alpha = \frac{2M}{mr^2}$

6/42

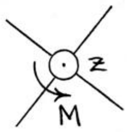
$\alpha = \frac{\omega}{\tau}$ (H = hub ; B = blades)

$$I_{zz} = \frac{1}{2}m_H r^2 + 4\left[\frac{1}{12}m_B l^2 + m_B\left(r + \frac{l}{2}\right)^2\right]$$

$$= \frac{1}{2}(\rho\pi r^2 d)r^2 + 4(\rho l d t)\left[\frac{1}{12}l^2 + r^2 + rl + \frac{l^2}{4}\right]$$

$$= \frac{1}{2}\rho\pi d r^4 + 4\rho l d t\left[\frac{1}{3}l^2 + rl + r^2\right]$$

$$= \rho d\left[\frac{1}{2}\pi r^4 + 4lt\left(\frac{1}{3}l^2 + rl + r^2\right)\right]$$



$\Sigma M_z = I_{zz}\alpha$:

$$M = \frac{\omega\rho d}{\tau}\left[\frac{1}{2}\pi r^4 + 4lt\left(\frac{1}{3}l^2 + rl + r^2\right)\right]$$

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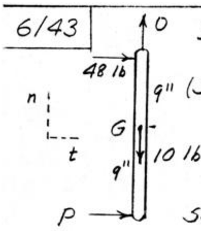


$\bar{I} = \frac{1}{12}ml^2 = \frac{1}{12}\frac{10}{32.2}\left(\frac{18}{12}\right)^2 = 0.0582$ ft-lb-sec²

(Sol. I)

$\Sigma M_G = \bar{I}\alpha$; $\frac{9}{12}P - 48\frac{9}{12} = 0.0582\alpha$

$\Sigma F_t = m\bar{a}_t$; $P + 48 = \frac{10}{32.2}\frac{9}{12}\alpha$

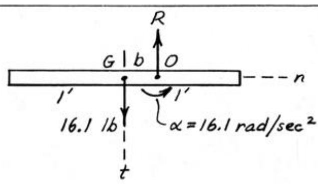
Solve simultaneously & get

$P = 96.0$ lb, $\alpha = 618.2$ rad/sec²

(Sol. II) $q = k_o^2/\bar{r} = \frac{l^2}{3}/\frac{l}{2} = \frac{2}{3}l = \frac{2}{3}\frac{18}{12} = 1$ ft

$\Sigma M_Q = 0$; $P\left(\frac{18}{12} - 1\right) - 48(1) = 0$, $P = 48/0.5 = 96$ lb

6/44



$I_O = \frac{1}{12}mL^2 + mb^2$

$= \frac{16.1}{32.2}\left(\frac{2^2}{12} + b^2\right)$

$= \frac{1}{6} + \frac{b^2}{2}$ lb-ft-sec²

$\Sigma M_O = I_O\alpha$: $16.1b = \left(\frac{1}{6} + \frac{b^2}{2}\right)16.1$, $3b^2 - 6b + 1 = 0$

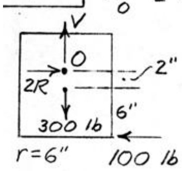
$b = 1 \pm \sqrt{24}/6$, $b = 0.1835$ ft (1.817 ft),

$b = 2.20$ in.

$\Sigma F_t = m\bar{r}\alpha$: $16.1 - R = \frac{16.1}{32.2}0.1835(16.1)$, $R = 14.62$ lb

6/45

$I_O = \bar{I} + m\bar{r}^2 = \left(\frac{1}{4}mr^2 + \frac{1}{12}ml^2\right) + m\bar{r}^2$



$= \frac{300}{32.2}\left[\frac{1}{4}\left(\frac{6}{12}\right)^2 + \frac{1}{12}\left(\frac{12}{12}\right)^2 + \left(\frac{2}{12}\right)^2\right]$

$= 1.617$ lb-ft-sec²

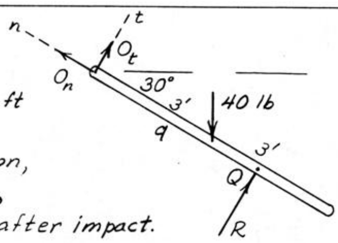
$\Sigma M_O = I_O\alpha$; $100\left(\frac{8}{12}\right) = 1.617\alpha$

$\alpha = 41.2$ rad/sec²

$\Sigma F_t = m\bar{r}\alpha$; $100 - 2R = \frac{300}{32.2}\frac{2}{12}(41.2)$

$R = 18$ lb

6/46



For slender rod,

$q = \frac{k_o^2}{\bar{r}} = \frac{\frac{1}{3}L^2}{\frac{L}{2}} = \frac{2}{3}(6) = 4$ ft

For fixed-axis rotation,

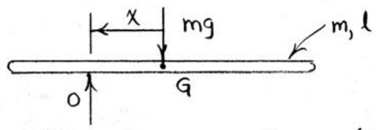
$\Sigma M_Q = 0$ at all times,

before, during, and after impact.

Thus

$40(1)\cos 30° - 4O_t = 0$, $O_t = 8.66$ lb at all times

6/47



$I_O = I_G + mx^2 = \frac{1}{12}ml^2 + mx^2 = m\left(\frac{l^2}{12} + x^2\right)$

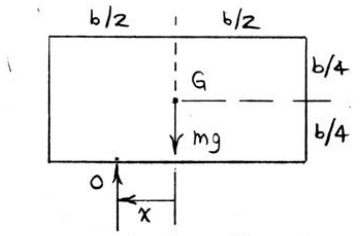
$\circlearrowright \Sigma M_O = I_O\alpha$: $mgx = m\left(\frac{l^2}{12} + x^2\right)\alpha$

$$\alpha = \frac{gx}{\frac{1}{12}l^2 + x^2}$$

$$\frac{d\alpha}{dx} = \frac{\left(\frac{1}{12}l^2 + x^2\right)g - gx(2x)}{\left(\frac{1}{12}l^2 + x^2\right)^2} = 0 \Rightarrow x = \frac{l}{2\sqrt{3}}$$

$$\alpha = \frac{g\frac{l}{\sqrt{12}}}{\frac{1}{12}l^2 + \frac{1}{12}l^2} = \sqrt{3}\,\frac{g}{l}$$

6/48



$$I_G = \frac{1}{12}m\left[b^2 + \left(\frac{b}{2}\right)^2\right] = \frac{5}{48}mb^2$$

$$I_O = I_G + m\left[\left(\frac{b}{4}\right)^2 + x^2\right] = \frac{1}{6}mb^2 + mx^2$$

$$\Sigma M_O = I_O\alpha: \quad mgx = \left(\frac{1}{6}mb^2 + mx^2\right)\alpha$$

$$\alpha = \frac{gx}{\frac{1}{6}b^2 + x^2}$$

$$\frac{d\alpha}{dx} = \frac{\left(\frac{1}{6}b^2 + x^2\right)g - gx(2x)}{\left(\frac{1}{6}b^2 + x^2\right)} = 0 \Rightarrow \underline{x = \frac{b}{\sqrt{6}}}$$

$$\alpha = \frac{g\frac{b}{\sqrt{6}}}{\frac{1}{6}b^2 + \frac{1}{6}b^2} = \underline{\sqrt{\frac{3}{2}}\,\frac{g}{b}}$$

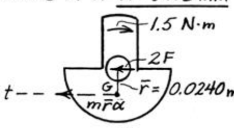
6/49 $\omega^2 = \omega_0^2 + 2\bar{\alpha}\theta$, $\left(\frac{1200\times 2\pi}{60}\right)^2 = 0 + 2\bar{\alpha}\,(18\times 2\pi)$, $\bar{\alpha} = 69.8\ \text{rad/s}^2$

Static test $\Sigma M = 0$: $0.660 - 2.8(9.81)\bar{r}$, $\bar{r} = 0.0240$ m

(a) $\Sigma M = I\alpha$: $1.5 = 2.8\,k^2 \times 69.8$, $k = 0.0876$ m or $\underline{k = 87.6\ \text{mm}}$

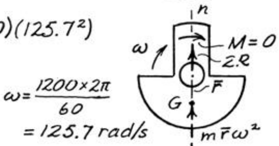
(b) $\Sigma F_t = m\bar{r}\alpha$: $2F = 2.8\,(0.0240)69.8$

$\underline{F = 2.35\ \text{N}}$



(c) $\Sigma F_n = m\bar{r}\omega^2$: $2R = 2.8(0.0240)(125.7^2)$

$\underline{R = 531\ \text{N}}$

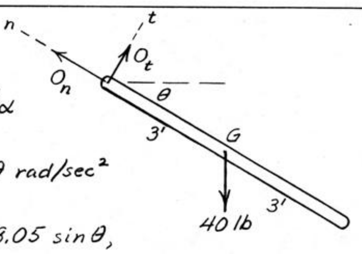


$\omega = \frac{1200\times 2\pi}{60} = 125.7$ rad/s

6/50

$\Sigma M_O = I_O\alpha$:

$$40(3)\cos\theta = \frac{1}{3}\frac{40}{32.2}6^2\alpha$$



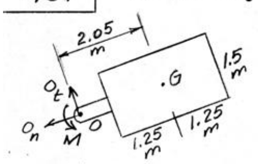
$\alpha = 8.05\cos\theta\ \text{rad/sec}^2$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta: \quad \frac{\omega^2}{2} = 8.05\sin\theta,$$

$$\omega^2_{\theta=30°} = 2(8.05)0.5 = 8.05\ (\text{rad/sec})^2$$

$\Sigma F_n = m\bar{a}_n$: $O_n - 40\sin 30° = \frac{40}{32.2}(3)(8.05)$, $\underline{O_n = 50\ \text{lb}}$

6/51 $m = 6000$ kg; From Table D/4



$$I_G = \frac{1}{12}(6000)\left[(1.5)^2 + (2.5)^2\right] = 4250\ \text{kg}\cdot\text{m}^2$$

$$I_O = I_G + md^2 = 4250 + 6000(2.05)^2 = 29\,465\ \text{kg}\cdot\text{m}^2$$

$M = 30$ N·m

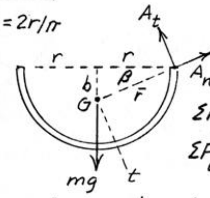
$\Sigma M_O = I_O\alpha$; $30 = 29\,465\,\alpha$, $\alpha = 1.018(10^{-3})\ \text{rad/s}^2$

$\theta = \frac{1}{2}\alpha t^2$, $\frac{\pi}{4} = \frac{1}{2}1.018(10^{-3})\,t_1^2$, $t_1 = 39.28$ s

Total time $t = 2t_1 = \underline{78.6\ \text{s}}$

6/52 $I_A = \frac{1}{2}(2mr^2 + 2mr^2) = 2mr^2$

$b = 2r/\pi$



$\Sigma M_A = I_A\alpha$; $mgr = 2mr^2\alpha$

$\underline{\alpha = g/2r}$

$\Sigma F_n = m\bar{a}_n = 0$; $A_n = mg\sin\beta$

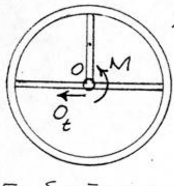
$\Sigma F_t = m\bar{a}_t$; $mg\cos\beta - A_t = m\bar{r}\alpha$

or $A_n = mg\frac{b}{\bar{r}}$, $A_t = mg\left(\frac{r}{\bar{r}} - \frac{\bar{r}}{2r}\right)$

$$\text{so } A = mg\sqrt{\frac{b^2}{\bar{r}^2} + \frac{r^2}{\bar{r}^2} - 1 + \frac{\bar{r}^2}{4r^2}} = mg\frac{\bar{r}}{2r} = \frac{mg}{2}\sqrt{1 + 4/\pi^2}$$

or $\underline{A = 0.593\ mg}$

6/53 Rim: $I_O = mr^2 = \frac{100}{32.2}\left(\frac{18}{12}\right)^2 = 6.99\ \text{lb-ft-sec}^2$



Each spoke: $I_O = \frac{1}{3}ml^2 = \frac{1}{3}\frac{15}{32.2}\left(\frac{18}{12}\right)^2$

$= 0.349\ \text{lb-ft-sec}^2$

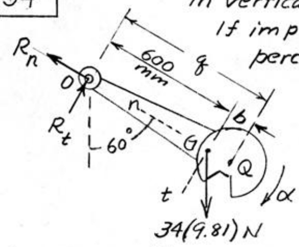
$\Sigma M_O = I_O\alpha$; $\frac{400}{12} = [6.99 + 3(0.349)]\alpha$

$\alpha = 4.15\ \text{rad/sec}^2$

$\Sigma F_t = \Sigma m\bar{r}\alpha$; $O_t = \frac{15}{32.2}\left(\frac{9}{12}\right)(4.15) + 0$ (middle spoke only)

$\underline{O_t = 1.449\ \text{lb}}$

6/54



In vertical position $R_t = 0$

If impact occurs at center of percussion so

$q = k_O^2/\bar{r}$,

$0.600 + b = \frac{(0.620)^2}{0.600}$

$b = 0.0407$ m

or $\underline{b = 40.7\ \text{mm}}$

$\Sigma M_Q = 0$; $34(9.81)(0.0407\sin 60°) - (0.6407)R_t = 0$

$R_t = 18.35$ N

$\Sigma F_n = m\bar{r}\omega^2 = 0$; $R_n - 34(9.81)\cos 60° = 0$

$R_n = 166.8$ N

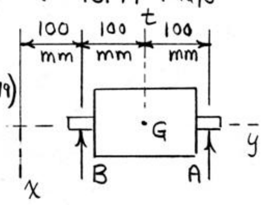
$R = \sqrt{(166.8)^2 + (18.35)^2} = \underline{167.8\ \text{N}}$

6/55 For entire assembly,

$I_{zz} = 0.60 + \left(0.080 + 12(0.2)^2\right) = 1.160\ \text{kg}\cdot\text{m}^2$

$\Sigma M_z = I_{zz}\alpha$: $16 = 1.160\,\alpha$, $\alpha = 13.79\ \text{rad/s}^2$

For cylinder :



$\Sigma F_t = ma_t$: $A + B = 12(0.2)(13.79)$

$= 33.1$ N

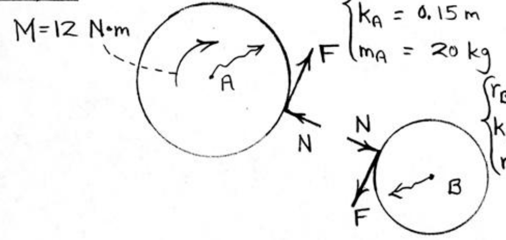
$\Sigma M_O = I_{zz}\alpha$:

$0.3A + 0.1B = [0.080 + 12(0.2)^2]\ 13.79$

Simultaneous solution : $\underline{A = 22.1\ \text{N}}$, $\underline{B = 11.03\ \text{N}}$

6/56

$$\begin{cases} r_A = 0.24\text{ m} \\ k_A = 0.15\text{ m} \\ m_A = 20\text{ kg} \end{cases} \qquad \begin{cases} r_B = 0.18\text{ m} \\ k_B = 0.1\text{ m} \\ m_B = 10\text{ kg} \end{cases}$$



$\circlearrowright \Sigma M_A = I_A \alpha_A: \quad 12 - F(0.24) = 20(0.15)^2 \alpha_A \quad (1)$

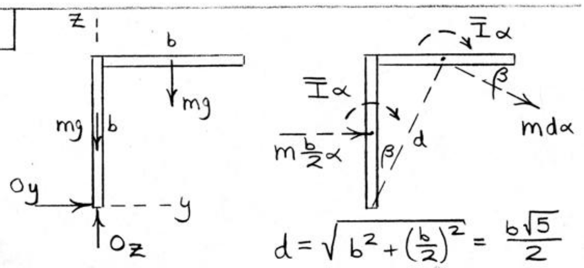
$\circlearrowleft \Sigma M_B = I_B \alpha_B: \quad F(0.18) = 10(0.1)^2 \alpha_B \quad (2)$

Tangential accelerations match: $r_A \alpha_A = r_B \alpha_B$

$0.24\,\alpha_A = 0.18\,\alpha_B \quad (3)$

Solution of Eqs. (1)-(3): $\begin{cases} F = 14.16\text{ N} \\ \alpha_A = 19.12\text{ rad/s}^2\text{(CW)} \\ \alpha_B = 25.5\text{ rad/s}^2\text{(CCW)} \end{cases}$

6/57



$d = \sqrt{b^2 + \left(\frac{b}{2}\right)^2} = \frac{b\sqrt{5}}{2}$

$m = \rho bc\,; \quad I_O = \frac{1}{3}mb^2 + \frac{1}{12}mb^2 + m\left[\left(\frac{b}{2}\right)^2 + b^2\right]$

$= \frac{5}{3}mb^2 = \frac{5}{3}\rho b^3 c$

$\Sigma M_O = I_O\alpha: \quad g\rho bc\left(\frac{b}{2}\right) = \frac{5}{3}\rho b^3 c\,\alpha, \quad \alpha = \frac{3g}{10b}$

For each plate, $\bar{I} = \frac{1}{12}mb^2 = \frac{1}{12}\rho b^3 c$

$\cos\beta = \frac{b}{\frac{b}{2}\sqrt{5}} = \frac{2}{\sqrt{5}}, \quad \sin\beta = \frac{b/2}{\frac{b}{2}\sqrt{5}} = \frac{1}{\sqrt{5}}$

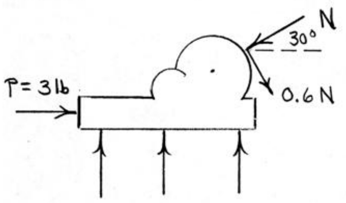
$\Sigma F_y = m\bar{a}_y: \quad O_y = m\frac{b}{2}\alpha + m\frac{b}{2}\sqrt{5}\,\alpha\frac{2}{\sqrt{5}}$

$= \frac{3}{2}mb\alpha = \frac{9}{20}\rho bcg$

$\Sigma F_z = \Sigma m\bar{a}_z: \quad O_z - 2mg = -md\alpha\sin\beta$

$O_z = 2\rho gbc - \rho bc\left(\frac{b}{2}\sqrt{5}\right)\frac{3g}{10b}\frac{1}{\sqrt{5}} = \frac{37}{20}\rho bcg$

6/58 Power unit C:

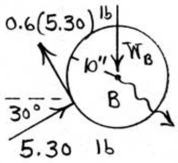


$\overset{+}{\rightarrow}\Sigma F = 0:$

$3 - N\cos 30° + 0.6N\sin 30° = 0$

$N = 5.30\text{ lb}$

Wheel B : $\circlearrowright \Sigma M_B = I_B\alpha: \quad 0.6(5.30)\left(\frac{10}{12}\right) = \frac{50}{32.2}\left(\frac{8}{12}\right)^2\alpha$



$\alpha = 3.84\text{ rad/sec}^2$

Steady-state speed:

$r_A\omega_A = r_B\omega_B$

$\omega_B = \frac{r_A\omega_A}{r_B} = \frac{8\left[1600\ \frac{2\pi}{60}\right]}{10}$

$= 134.0\text{ rad/sec}$

$\omega_B = \cancelto{0}{\omega_{B_0}} + \alpha t\ : \quad t = \frac{\omega_B}{\alpha} = \frac{134.0}{3.84} = 34.9\text{ sec}$

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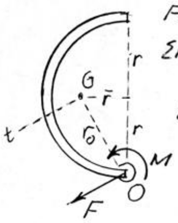
For complete ring $I_O = 2(2m)r^2$

For the half ring $I_O = 2mr^2$



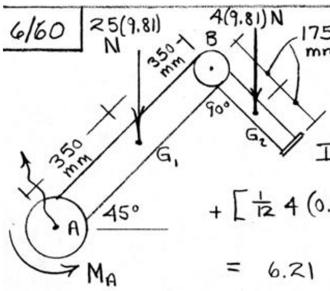
$\Sigma M_O = I_O\alpha\,; \quad M = 2mr^2\alpha, \quad \alpha = \frac{M}{2mr^2}$

$\Sigma F_t = m\bar{a}_t\,; \quad F = m\,\bar{r}_O\,\alpha$

$= m\sqrt{r^2 + \bar{r}^2}\,\frac{M}{2mr^2}$

$F = \frac{M}{r}\sqrt{\frac{1}{4} + \frac{1}{\pi^2}} = 0.593\,M/r$

6/60



$I_B = \frac{1}{3}(4)(0.350)^2$

$= 0.1633\text{ kg·m}^2$

$I_A = \frac{1}{3}(25)(0.7)^2$

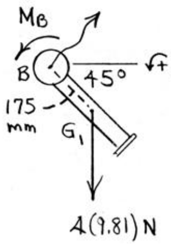
$+ \left[\frac{1}{12}\,4\,(0.350)^2 + 4\left(0.7^2 + 0.175^2\right)\right]$

$= 6.21\text{ kg·m}^2$

$\circlearrowleft \Sigma M_A = I_A\alpha: \quad M_A - 25(9.81)(0.350\cos 45°)$

$-4(9.81)(0.700 + 0.175)\cos 45° = 6.21(4)$

$M_A = 109.8\text{ N·m}$

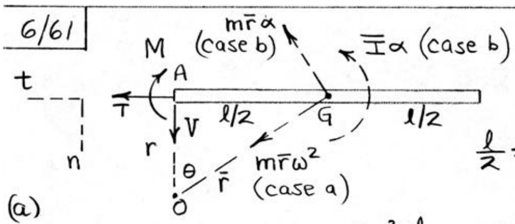


$\circlearrowleft \Sigma M_B = I_B\alpha:$

$M_B - 4(9.81)(0.175\cos 45°) = 0.1633(4)$

$M_B = 5.51\text{ N·m}$

6/61



$\frac{l}{2} = \bar{r}\sin\theta$

(a)

$\circlearrowleft \Sigma M_A = m\bar{a}d: \quad M = m\bar{r}\omega^2 \frac{l}{2}\cos\theta$

$= m\frac{r}{\cos\theta}\omega^2\frac{l}{2}\cos\theta = \underline{mrl\omega^2/2}$

$\Sigma F_n = m\bar{a}_n: \quad V = m\bar{r}\omega^2\cos\theta = m\frac{r}{\cos\theta}\omega^2\cos\theta$

$= \underline{mr\omega^2}$

$\Sigma F_t = m\bar{a}_t: \quad T = m\bar{r}\omega^2\sin\theta = \underline{ml\omega^2/2}$

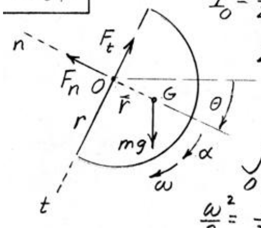
(b) $\Sigma M_A = m\bar{a}d: \quad M = -m\bar{r}\alpha\frac{l}{2}\sin\theta - \bar{I}\alpha$

$M = -m\alpha\frac{l^2}{4} - \frac{1}{12}ml^2\alpha = \underline{-ml^2\alpha/3}$

$\Sigma F_n = m\bar{a}_n: \quad V = -m\bar{r}\alpha\sin\theta = \underline{-ml\alpha/2}$

$\Sigma F_t = m\bar{a}_t: \quad T = m\bar{r}\alpha\cos\theta = \underline{mr\alpha}$

6/62



$I_O = \frac{1}{2}mr^2; \quad \bar{r} = \frac{4r}{3\pi}$

$\Sigma M_O = I_O\alpha; \quad mg\bar{r}\cos\theta = I_O\alpha$

$\alpha = mg\cos\theta\,\frac{\bar{r}}{I_O} = \frac{8}{3\pi}\frac{g}{r}\cos\theta$

$\int_0^\omega \omega\,d\omega = \int_0^\theta \alpha\,d\theta$

$\frac{\omega^2}{2} = \frac{8}{3\pi}\frac{g}{r}\sin\theta\Big]_0^\theta, \quad \omega^2 = \frac{16}{3\pi}\frac{g}{r}\sin\theta$

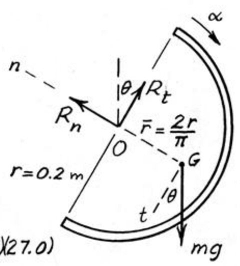
$\Sigma F_n = m\bar{r}\omega^2; \quad F_n - mg\sin\theta = m\frac{4r}{3\pi}\frac{16}{3\pi}\frac{g}{r}\sin\theta$

$F_n = \left(\frac{64}{9\pi^2}+1\right)mg\sin\theta = \underline{1.721\,mg\sin\theta}$

$\Sigma F_t = m\bar{r}\alpha; \quad mg\cos\theta - F_t = m\frac{4r}{3\pi}\frac{8}{3\pi}\frac{g}{r}\cos\theta$

$F_t = \left(1-\frac{32}{9\pi^2}\right)mg\cos\theta = \underline{0.640\,mg\cos\theta}$

6/63



$\circlearrowleft \Sigma M_O = I_O\alpha: \; mg\bar{r}\cos\theta = mr^2\alpha,$

$24.5(0.1273)\cos 30° = 0.1\alpha,$

$\alpha = 2.70/0.1 = 27.0 \text{ rad/s}$

$\Sigma F_t = m\bar{r}\alpha: \; mg\cos\theta - R_t = m\bar{r}\alpha,$

$R_t = 24.5\cos 30° - 2.5(0.1273)(27.0)$

$= 12.63 \text{ N}$

$mg = 2.5(9.81) = 24.5 \text{ N}$

$\Sigma F_n = m\bar{r}\omega^2: \; R_n - mg\sin\theta = 0,$

$R_n = 24.5\sin 30° = 12.26 \text{ N}$

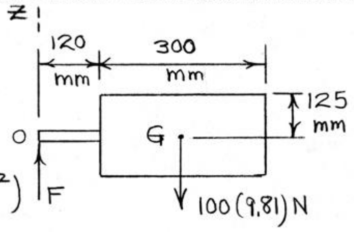
$\bar{r} = \frac{2(0.2)}{\pi} = 0.1273 \text{ m}$

$I_O = mr^2 = 2.5(0.2)^2 = 0.1 \text{ kg}\cdot\text{m}^2$

$R = \sqrt{R_n^2+R_t^2}: \; R = \sqrt{12.26^2+12.63^2} = \underline{17.60 \text{ N}}$

6/64

From Table D/4,



$I_G = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$

$= \frac{1}{4}100\left(0.125^2+\frac{1}{3}0.3^2\right)$

$= 1.141 \text{ kg}\cdot\text{m}^2$

$I_O = I_G + md^2 = 1.141 + 100(0.120+0.150)^2 = 8.43 \text{ kg}\cdot\text{m}^2$

$\Sigma M_O = I_O\alpha: \; 981(0.120+0.150) = 8.43\alpha$

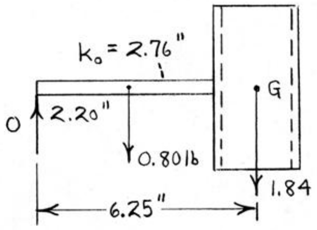
$\underline{\alpha = 31.4 \text{ rad/s}^2}$

$\Sigma F_z = m\bar{a}_z: \quad F - 981 = 100(-0.27)(31.4)$

$\underline{F = 132.7 \text{ N}}$

6/65

From Appendix D, for tube:



$I_G = \frac{1}{2}m\left(r^2+\frac{l^2}{6}\right)$

$= \frac{1}{2}\frac{1.84}{32.2}\left[\left(\frac{1.25}{12}\right)^2+\frac{1}{6}\left(\frac{4}{12}\right)^2\right]$

$= 0.000839 \text{ lb-ft-sec}^2$

$I_o = I_G + m\bar{r}^2$

$= 0.000839 + \frac{1.84}{32.2}\left(\frac{6.25}{12}\right)^2$

$= 0.01634 \text{ lb-ft-sec}^2$

Link : $I_o = \frac{0.80}{32.2}\left(\frac{2.76}{12}\right)^2 = 0.001314 \text{ lb-ft-sec}^2$

$\Sigma M_o = I_o\alpha: \; 1.84\left(\frac{6.25}{12}\right) + 0.08\left(\frac{2.20}{12}\right)$

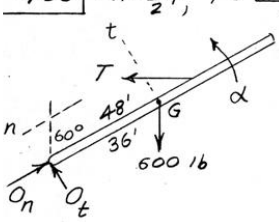
$= (0.001314 + 0.01634)\alpha, \quad \underline{\alpha = 62.6 \text{ rad/sec}^2}$

$\Sigma F_t = m\bar{a}_t: \; 1.84 + 0.80 - O = \left[\frac{1.84}{32.2}\frac{6.25}{12} + \frac{0.80}{32.2}\frac{2.20}{12}\right]62.6$

$\underline{O = 0.492 \text{ lb}}$

6/66

$M = \frac{T}{2}r, \quad T = \frac{2(9.00)}{2} = 900 \text{ lb}$



$\Sigma M_O = I_O\alpha;$

$900(48\cos 60°) - 600(36\sin 60°)$

$= \frac{1}{3}\frac{600}{32.2}\overline{72}^2\alpha$

$\underline{\alpha = 0.0899 \text{ rad/sec}^2}$

$\Sigma F_t = m\bar{a}_t; \; O_t + 900\cos 60° - 600\sin 60° = \frac{600}{32.2}(36)(0.0899)$

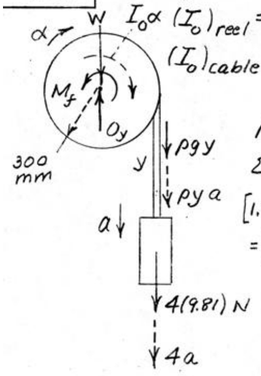
$O_t = 129.9 \text{ lb}$

$\Sigma F_n = m\bar{a}_n = 0; \; 900\sin 60° + 600\cos 60° - O_n = 0$

$O_n = 1079.4 \text{ lb}$

$O = \sqrt{\overline{129.9}^2 + \overline{1079.4}^2} = \underline{1087 \text{ lb}}$

6/67



$\rho = 0.16$ kg/m ; $M_f = 4(9.81)(0.3) = 11.772$ N·m

$I_O\alpha$ $(I_O)_{reel} = mk_O^2 = 16(0.200)^2 = 0.64$ kg·m²

$(I_O)_{cable} = mr^2 = (60-y)(0.16)(0.300)^2$

$= 0.0144(60-y)$ kg·m²

$\rho g y = 0.16(9.81)y = 1.5696y$

$\Sigma M_O = I_O\alpha + \Sigma m a r$ for system

$[1.5696y + 4(9.81)]0.300 - 11.772$

$= [0.64 + 0.0144(60-y)]\frac{a}{0.300}$

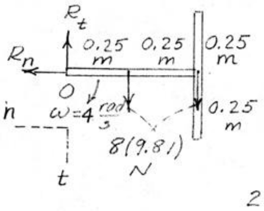
$+ [0.16y + 4]a(0.300)$

solve for a & get

$\underline{a = 0.0758y}$, a in m/s², y in m

6/68

$I_O = \frac{1}{3}ml^2 + (\frac{1}{12}ml^2 + ml^2) = \frac{17}{12}(8)(0.5)^2$ kg·m²



$\Sigma M_O = I_O\alpha$

$8(9.81)(0.50 + 0.25) = \frac{17}{12}(8)(0.5)^2\alpha$

$\alpha = 20.8$ rad/s²

$\Sigma F_t = \Sigma m\bar{a}_t$

$2(8)(9.81) - R_t = 8(0.25)(20.8) + 8(0.50)(20.8)$

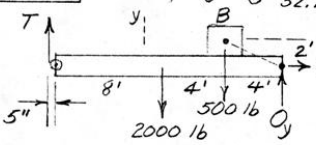
$R_t = 32.3$ N

$\Sigma F_n = \Sigma m\bar{a}_n$; $R_n = 8(0.25)4^2 + 8(0.50)4^2 = 96.0$ N

$R = \sqrt{32.3^2 + 96.0^2} = \underline{101.3 \text{ N}}$

6/69

Beam, $I_O = \frac{1}{3}\frac{2000}{32.2}16^2 + \frac{500}{32.2}(2^2 + 4^2)$



$= 5300 + 310.6$

$= 5611$ lb-ft-sec²

$M = Tr$; $T = \frac{600}{6/12} = 1200$ lb

$\Sigma M_O = I_O\alpha$; $1200(16 + \frac{5}{12}) - 2000(8) - 500(4) = 5611\alpha$

$\alpha = 0.303$ rad/sec²

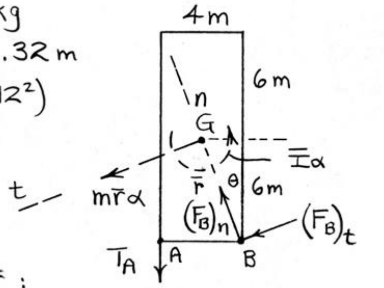
$\Sigma F_y = m\bar{r}\alpha$; $O_y + 1200 - 2000 - 500 = \frac{2000}{32.2}(8)(0.303) + \frac{500}{32.2}(4)(0.303)$

$\underline{O_y = 1469 \text{ lb}}$

The $m\bar{r}\alpha$ resultant for the winch has an x-component, so that $O_x \neq 0$.

6/70

$m = 3000$ kg



$\bar{r} = \sqrt{6^2 + 2^2} = 6.32$ m

$\bar{I} = \frac{1}{12}(3000)(4^2 + 12^2)$

$= 40\,000$ kg·m²

$\theta = \tan^{-1}\frac{2}{6} = 18.43°$

$\Sigma M_B = \bar{I}\alpha + m\bar{a}\bar{r}$:

$4T_A = 40\,000\alpha + 3000(6.32\alpha)(6.32)$

$\underline{\alpha = 0.05 \text{ rad/s}^2}$

$\Sigma F_n = m\bar{a}_n = 0 : (F_B)_n - 2000\cos 18.43° = 0$

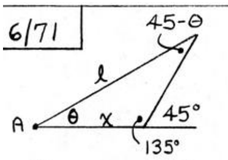
$(F_B)_n = 1897$ N

$\Sigma F_t = m\bar{a}_t$: $2000\sin 18.43° + (F_B)_t = 3000(6.32)(0.05)$

$(F_B)_t = 316$ N

$F_B = \sqrt{1897^2 + 316^2} = \underline{1924 \text{ N}}$

6/71



$\frac{\sin(45°-\theta)}{x} = \frac{\sin 135°}{l}$

Differentiate WRT time t :

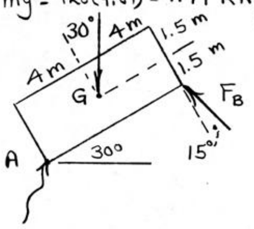
$-\dot{\theta}\cos(45°-\theta) = \dot{x}\frac{\sin 135°}{l}$

$-\ddot{\theta}\cos(45°-\theta) + \dot{\theta}^2\sin(45°-\theta) = \ddot{x}\frac{\sin 135°}{l}$

So $\ddot{\theta} = -\frac{\ddot{x}}{l}\frac{\sin 135°}{\cos(45°-\theta)}$; For $l = 8$ m & $\theta = 35°$,

and $\ddot{x} = 3$ m/s², $\ddot{\theta} = -0.275$ rad/s² (CW)

$mg = 120(9.81) = 1177$ kN



From Table D/4,

$I_A = \frac{1}{3}m(b^2 + l^2)$

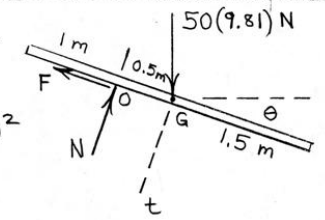
$= \frac{1}{3}120(10^3)[3^2 + 8^2]$

$= 2920(10^3)$ kg·m²

$\Sigma M_A = I_A\alpha$: $1177(10^3)\cos 30°(4) - 1177(10^3)\sin 30°(1.5) - 8F_B\cos 15° = 2920(10^3)(0.275)$

$F_B = 310{,}000$ N or $\underline{310 \text{ kN}}$

6/72



$I_O = \bar{I} + mr^2$

$= \frac{1}{12}50(3)^2 + 50(0.5)^2$

$= 50\ kg\cdot m^2$

$\Sigma M_O = I_O\alpha: \quad 50(9.81)(0.5\cos\theta) = 50\alpha$

$\alpha = 4.905\cos\theta = \omega\frac{d\omega}{d\theta}, \quad \int_0^\omega \omega\, d\omega = \int_0^\theta 4.905\cos\theta\, d\theta$

$\omega^2 = 9.81\sin\theta$

$\begin{cases}\Sigma F_t = m\bar{a}_t: & 50(9.81)\cos\theta - N = 50(0.5)(4.905\cos\theta)\\ \Sigma F_n = ma_n: & F - 50(9.81)\sin\theta = 50(0.5)(9.81\sin\theta)\end{cases}$

Slipping occurs when $F = 0.30N$

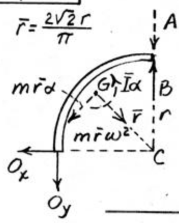
2^{nd} eq: $0.3N = 75(9.81)\sin\theta$

1^{st} eq: $N = \frac{75}{2}(9.81)\cos\theta$

Divide to obtain $\underline{\theta = 8.53°}$

6/73

$\bar{r} = \frac{2\sqrt{2}r}{\pi}$



(a) $\alpha = 0$; $\Sigma M_C = 0$; $O_y = 0$

$\Sigma M_O = m\bar{a}d$; $Ar = m\frac{2\sqrt{2}r}{\pi}\omega^2\frac{r}{\sqrt{2}}$

$\underline{A = \frac{2}{\pi}mr\omega^2}$

By symmetry, $\Sigma M_A = m\bar{a}d$ or $\Sigma F_n = m\bar{a}_n$

$O = -O_x = A = \underline{\frac{2}{\pi}mr\omega^2}$

(b) $\omega = 0$; $\Sigma M_C = I_C\alpha$; $O_y r = mr^2\alpha$, $O_y = mr\alpha$

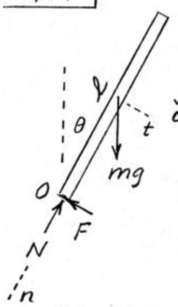
$\Sigma F_x = m\bar{a}_x$; $O_x = m\frac{2\sqrt{2}r}{\pi}\alpha\frac{1}{\sqrt{2}} = \frac{2}{\pi}mr\alpha$

$\Sigma F_y = m\bar{a}_y$; $O_y - B = m\frac{2\sqrt{2}r}{\pi}\alpha\frac{1}{\sqrt{2}}$, $B = mr\alpha - \frac{2mr\alpha}{\pi}$

$= \underline{mr\alpha(1 - \frac{2}{\pi})}$

Thus $O = \sqrt{O_x^2 + O_y^2} = \underline{mr\alpha\sqrt{1 + \frac{4}{\pi^2}}}$

6/74



$\Sigma M_O = I_O\alpha$; $mg\frac{l}{2}\sin\theta = \frac{1}{3}ml^2\alpha$

$\alpha = \frac{3g}{2l}\sin\theta$

$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta$, $\omega^2 = \frac{3g}{l}(-\cos\theta)\Big|_0^\theta$

$\omega^2 = \frac{3g}{l}(1 - \cos\theta)$

$\Sigma F_n = m\bar{a}_n$; $mg\cos\theta - N = m\frac{l}{2}\omega^2$

$N = mg\cos\theta - m\frac{l}{2}\frac{3g}{l}(1-\cos\theta)$

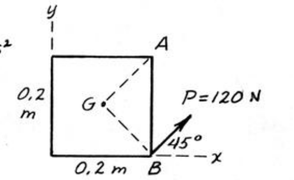
$= mg[\cos\theta - \frac{3}{2}(1-\cos\theta)] = \frac{mg}{2}(5\cos\theta - 3)$

$\Sigma F_t = m\bar{a}_t$; $mg\sin\theta - F = m\frac{l}{2}\frac{3g}{2l}\sin\theta$, $F = \frac{mg}{4}\sin\theta$

(a) Slips at $\theta = 30°$, $\mu_s = F/N = \dfrac{mg\sin 30°/4}{\frac{mg}{2}(5\cos 30° - 3)} = \underline{0.188}$

(b) No slip: $N = 0$ when $\cos\theta = 3/5$, $\underline{\theta = 53.1°}$

6/75



$\Sigma F = m\bar{a}: \ 120 = 6\bar{a}$, $\bar{a} = a_G = 20\ m/s^2$

$\Sigma M_G = \bar{I}\alpha: \ 120\frac{0.2}{\sqrt{2}} = \frac{1}{6}(6)(0.2^2)\alpha$,

$\alpha = 424\ rad/s^2$

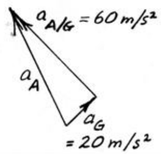
$\bar{I} = \frac{1}{6}ml^2$

$\mathbf{a}_A = \mathbf{a}_G + \mathbf{a}_{A/G}$ where $a_{A/G} = (a_{A/G})_t$

$= \overline{AG}\alpha$

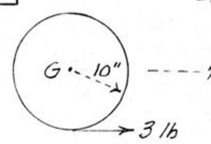
$= \frac{0.2}{\sqrt{2}}\times 424$

$= 60\ m/s^2$



$a_A = \sqrt{60^2 + 20^2} = \underline{63.2\ m/s^2}$

6/76



$\Sigma F_x = m\bar{a}_x$; $3 = \frac{64.4}{32.2}a$, $a = 1.5\ ft/sec^2$

$v^2 = 2ax$, $v^2 = 2(1.5)(3) = 9$

$\underline{v = 3\ ft/sec}$

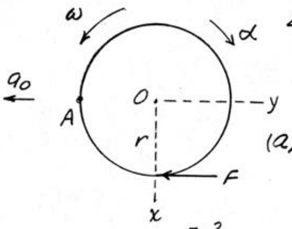
$\Sigma M_G = \bar{I}\alpha$; $3\frac{10}{12} = \frac{1}{2}\frac{64.4}{32.2}\left(\frac{10}{12}\right)^2\alpha$

$\alpha = 3.6\ rad/sec^2$

$\omega = \alpha t$, $\underline{\omega = 3.6(2) = 7.2\ rad/sec}$

6/77

$\Sigma M_O = I_O\alpha$; $Fr = mk^2\alpha$, $\alpha = \frac{Fr}{mk^2}$



$\Sigma F_y = ma_y$; $-F = m(-a_0)$

$a_0 = F/m$

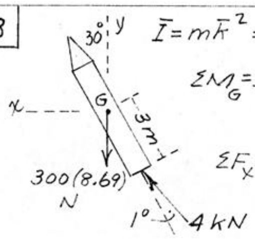
$\mathbf{a}_A = \mathbf{a}_0 + \mathbf{a}_{A/0}$

$(a_{A/0})_n = r\omega^2\mathbf{j}$, $(a_{A/0})_t = -r\alpha\mathbf{i} = -\frac{Fr^2}{mk^2}\mathbf{i}$

$\mathbf{a}_0 = -\frac{F}{m}\mathbf{j}$

so $\underline{\mathbf{a}_A = -\frac{Fr^2}{mk^2}\mathbf{i} - (\frac{F}{m} - r\omega^2)\mathbf{j}}$

6/78



$\bar{I} = m\bar{k}^2 = 300(1.5)^2 = 675\ kg\cdot m^2$

$\Sigma M_G = \bar{I}\alpha$; $4000\sin 1°(3) = 675\alpha$

$\underline{\alpha = 0.310\ rad/s^2}$

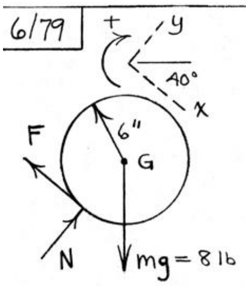
$\Sigma F_x = m\bar{a}_x$; $4000\sin 31° = 300\bar{a}_x$

$\underline{\bar{a}_x = 6.87\ m/s^2}$

$\Sigma F_y = m\bar{a}_y$; $4000\cos 31° - 300(8.69) = 300\bar{a}_y$

$\underline{\bar{a}_y = 2.74\ m/s^2}$

6/79



$$\begin{cases} mg = 8\text{ lb}, & \bar{I} = \frac{1}{2}mr^2 \\ \mu_s = 0.3, & \mu_k = 0.20 \\ \theta = 40° \end{cases}$$

$\Sigma F_x = m\bar{a}_x:\quad -F + 8\sin 40° = \frac{8}{32.2}a \qquad (1)$

$\Sigma F_y = 0:\quad N - 8\cos 40° = 0 \qquad (2)$

$\Sigma M_G = \bar{I}\alpha:\quad F\left(\frac{6}{12}\right) = \frac{1}{2}\frac{8}{32.2}\left(\frac{6}{12}\right)^2\alpha \qquad (3)$

Assume rolling with no slip: $a = \frac{6}{12}\alpha \qquad (4)$

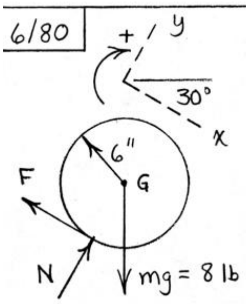
Solution of (1) - (4): $\underline{F = 1.714\text{ lb}}$ $\quad \underline{a = 13.80\ \frac{ft}{sec^2}}$

$N = 6.13\text{ lb} \quad \alpha = 27.6\ \frac{rad}{sec^2}$

$F_{max} = \mu_s N = 0.3\,(6.13) = 1.839\text{ lb} > F$

Assumption valid.

6/80



$$\begin{cases} mg = 8\text{ lb}, & \bar{I} = \frac{1}{2}mr^2 \\ \mu_s = 0.15, & \mu_k = 0.10 \\ \theta = 30° \end{cases}$$

$\Sigma F_x = m\bar{a}_x:\quad -F + 8\sin 30° = \frac{8}{32.2}a \qquad (1)$

$\Sigma F_y = 0:\quad N - 8\cos 30° = 0 \qquad (2)$

$\Sigma M_G = \bar{I}\alpha:\quad F\left(\frac{6}{12}\right) = \frac{1}{2}\frac{8}{32.2}\left(\frac{6}{12}\right)^2\alpha \qquad (3)$

Assume rolling with no slip: $a = \frac{6}{12}\alpha \qquad (4)$

Solution of (1) - (4): $F = 1.333\text{ lb} \quad a = 10.73\ \frac{ft}{sec^2}$

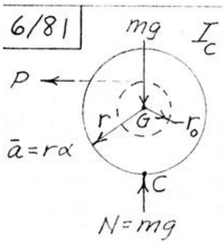
$N = 6.93\text{ lb} \quad \alpha = 21.5\ \frac{rod}{sec^2}$

$F_{max} = \mu_s N = 0.15\,(6.93) = 1.039\text{ lb} < F \Rightarrow$ slips

$F = \mu_k N = 0.10\,(6.93) = \underline{0.693\text{ lb}}$

From Eqs. (1) & (3): $\underline{a = 13.31\text{ ft/sec}^2}, \ \underline{\alpha = 11.15\ \frac{rad}{sec^2}}$

6/81



$I_C = \bar{I} + mr^2 = m(\bar{k}^2 + r^2)$

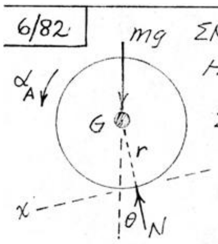
$\Sigma M_C = I_C\alpha;\quad P(r + r_o) = m(\bar{k}^2 + r^2)\alpha$

$\Sigma F = m\bar{a};\quad P = m\bar{a} = mr\alpha$

Thus $mr\alpha(r + r_o) = m(\bar{k}^2 + r^2)\alpha$

$r^2 + rr_o = \bar{k}^2 + r^2;\quad \underline{r_o = \frac{\bar{k}^2}{r}}$

6/82

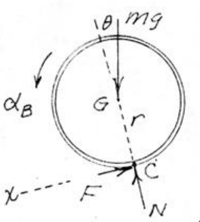


$\Sigma M_G = \bar{I}\alpha$, but $\bar{I} = 0$ so $\Sigma \bar{M} = 0$

Hence no friction force & $\underline{\mu_s = 0}$

$\Sigma F_x = ma_x;\quad mg\sin\theta = mr\alpha_A$

$\underline{\alpha_A = \frac{g}{r}\sin\theta}$



$\Sigma M_C = I_C\alpha;\quad mgr\sin\theta = 2mr^2\alpha_B$

$\underline{\alpha_B = \frac{g}{2r}\sin\theta}$

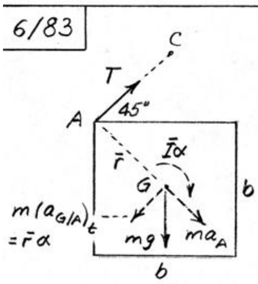
$\Sigma \bar{M} = \bar{I}\alpha;\quad Fr = mr^2\frac{g}{2r}\sin\theta$

$F = \frac{1}{2}mg\sin\theta$

$\mu_s = \frac{F}{N} = \frac{1}{2}mg\sin\theta / mg\cos\theta$

$\underline{\mu_s = \frac{1}{2}\tan\theta}$

6/83



$\Sigma M_A = \bar{I}\alpha + m\bar{a}d$

$\frac{mg\,b}{2} = \frac{1}{6}mb^2\alpha + m\frac{b}{\sqrt{2}}\alpha\frac{b}{\sqrt{2}}$

$\alpha = \frac{3g}{4b}$

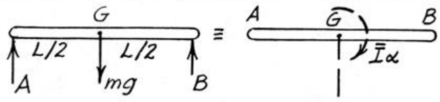
$\Sigma M_G = \bar{I}\alpha$

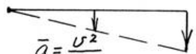
$T\frac{b}{\sqrt{2}} = \frac{1}{6}mb^2\left(\frac{3g}{4b}\right)$

$T = \frac{\sqrt{2}}{8}mg = \frac{\sqrt{2}}{8}(12)(9.81)$

$= \underline{20.8\ N}$

6/84





$a_A = 0$

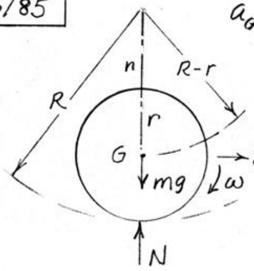
$\omega_{AB} = 0 \quad \bar{a} = \frac{v^2}{2r} \quad a_B = \frac{v^2}{r} \quad \alpha = \frac{a_B}{L} = \frac{v^2}{Lr}$

$\Sigma M_A = m\bar{a}\frac{L}{2} + \bar{I}\alpha:\ mg\frac{L}{2} - BL = m\frac{v^2}{2r}\frac{L}{2} + \frac{1}{12}mL^2\frac{v^2}{Lr}$

$\underline{B = m\left(\frac{g}{2} - \frac{v^2}{3r}\right)}$

$B = 0$ if $\frac{g}{2} - \frac{v^2}{3r} = 0,\quad \underline{v = \sqrt{3gr/2}}$

6/85



$a_G = \frac{\bar{v}^2}{R-r} = \frac{r^2\omega^2}{R-r} = \bar{a}_n$

$\Sigma F_n = m\bar{a}_n$

$N - mg = m\frac{r^2\omega^2}{R-r}$

$\underline{N = m\left(g + \frac{r^2\omega^2}{R-r}\right)}$

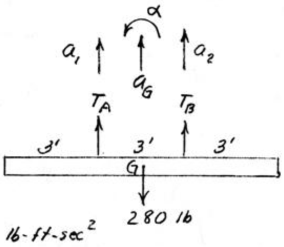
6/86

$a_1 = r\alpha_1 = \frac{15/2}{12}4 = 2.5 \text{ ft/sec}^2$

$a_2 = r\alpha_2 = \frac{15/2}{12}6 = 3.75 \text{ ft/sec}^2$

$a_G = (2.5 + 3.75)/2 = 3.13 \text{ ft/sec}^2$

$\alpha = \frac{a_{B/A}}{\overline{AB}} = \frac{3.75-2.5}{3} = 0.417 \frac{\text{rad}}{\text{sec}^2}$



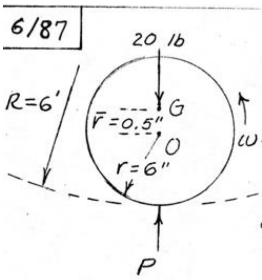
$I_G = \frac{1}{12}mL^2 = \frac{1}{12}\frac{280}{32.2}9^2 = 58.7 \text{ lb-ft-sec}^2$

$\Sigma F = m a_G$: $T_A + T_B - 280 = \frac{280}{32.2}(3.13)$, $T_A + T_B = 307$

$\Sigma M_G = I_G\alpha$: $(T_B - T_A)\frac{3}{2} = 58.7(0.417)$, $T_B - T_A = 16.30$

Solve & get $\underline{T_A = 145.4 \text{ lb}}$, $\underline{T_B = 161.7 \text{ lb}}$

6/87



$\bar{a} = a_G = a_O + a_{G/O}$

$v_O = r\omega = (6/12)(10) = 5 \text{ ft/sec}$

$a_O = v_O^2/(R-r)$
$= \frac{5^2}{6 - 6/12} = 4.55 \frac{\text{ft}}{\text{sec}^2}\uparrow$

$a_{G/O} = (a_{G/O})_n = \bar{r}\omega^2 = (0.5/12)\overline{10}^2$
$= 4.17 \text{ ft/sec}^2 \downarrow$

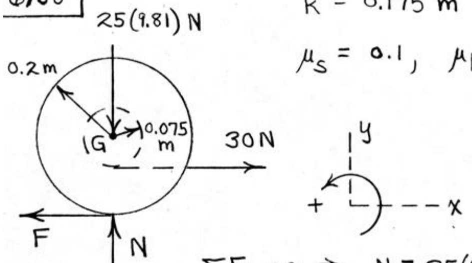
$\bar{a} = 4.55 - 4.17 = 0.38 \text{ ft/sec}^2 \uparrow$

$\Sigma F = m\bar{a}$; $P - 20 = \frac{20}{32.2}0.38$, $\underline{P = 20.2 \text{ lb}}$

6/88

$\bar{k} = 0.175 \text{ m}$

$\mu_s = 0.1$, $\mu_k = 0.08$



$\Sigma F_y = 0 \Rightarrow N = 25(9.81) = 245 \text{ N}$

$\Sigma F_x = m\bar{a}_x$: $30 - F = 25a$ (1)

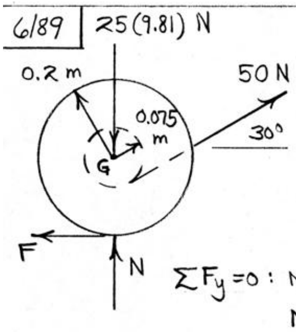
$\Sigma M_G = \bar{I}\alpha$: $30(0.075) - F(0.2) = 25(0.175)^2\alpha$ (2)

Assume rolling with no slip : $a = -r\alpha$ (3)

Solution of Eqs. (1)-(3) : $\begin{cases} a = 0.425 \text{ m/s}^2 \\ \alpha = -2.12 \text{ rad/s}^2 \\ \underline{F = 19.38 \text{ N}} \end{cases}$

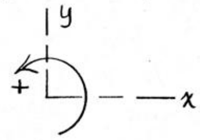
$F_{max} = \mu_s N = 0.1(245) = 24.5 \text{ N} > F$ (assumption OK)

6/89



$\bar{k} = 0.175 \text{ m}$

$\mu_s = 0.1$, $\mu_k = 0.08$



$\Sigma F_y = 0$: $N - 25(9.81) + 50 \sin 30° = 0$
$N = 220 \text{ N}$

$\Sigma F_x = m\bar{a}_x$: $50\cos 30° - F = 25a$ (1)

$\Sigma M_G = \bar{I}\alpha$: $50(0.075) - F(0.2) = 25(0.175)^2\alpha$ (2)

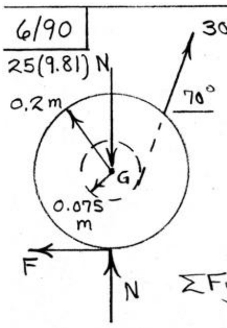
Assume rolling with no slip : $a = -r\alpha$ (3)

Solution of (1)-(3) : $\begin{cases} a = 0.556 \text{ m/s}^2, \ \alpha = -2.78 \text{ rad/s}^2, \\ F = 29.4 \text{ N} \end{cases}$

$F_{max} = \mu_s N = 0.1(220) = 22.0 \text{ N} < F$: slips, $\underline{F = \mu_k N = 17.62 \text{ N}}$

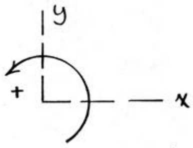
From Eqs. (1) & (2): $\underline{a = 1.027 \text{ m/s}^2, \ \alpha = 0.295 \text{ rad/s}^2}$

6/90



$\bar{k} = 0.0175 \text{ m}$

$\mu_s = 0.1$, $\mu_k = 0.08$



$\Sigma F_y = 0$: $N - 25(9.81) + 30\sin 70° = 0$
$N = 217 \text{ N}$

$\Sigma F_x = m\bar{a}_x$: $30\cos 70° - F = 25a$ (1)

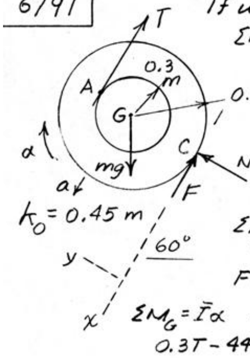
$\Sigma M_G = \bar{I}\alpha$: $30(0.075) - F(0.2) = 25(0.175)^2\alpha$ (2)

Assume rolling with no slip ; $a = -r\alpha$ (3)

Solution of Eqs. (1)-(3) : $\begin{cases} a = -0.0224 \text{ m/s}^2 \\ \alpha = 0.1121 \text{ rad/s}^2 \\ \underline{F = 10.82 \text{ N}} \end{cases}$

$F_{max} = \mu_s N = 0.1(217) = 21.7 > F$ (assumption OK)

6/91



If wheel were in equilibrium,

$\Sigma M_A = 0$; $0.9F - 0.3mg \sin 60°$

$\Sigma F_y = 0$; $N - mg\cos 60° = 0$

min. $\mu_s = \frac{F}{N} = \frac{(mg/3)\sin 60°}{mg\cos 60°} = \frac{\tan 60°}{3}$

$= 0.577 > (\mu_s = 0.40)$

so wheel slips & $\mu = \mu_k = 0.30$

$\Sigma F_y = 0$; $N = mg\cos 60° = 30(9.81)(0.5)$
$= 147.2 \text{ N}$

$F = \mu_k N = 0.3(147.2) = 44.1 \text{ N}$

$\Sigma M_G = \bar{I}\alpha$ where $\alpha = a/0.3$

$0.3T - 44.1(0.6) = 30(0.450)^2\frac{a}{0.3}$ ----(1)

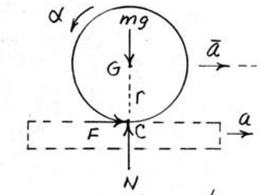
$\Sigma F_x = ma_x$

$30(9.81)\sin 60° - T - 44.1 = 30a$ ----(2)

Solve (1) & (2) & get $\underline{a = 1.256 \text{ m/s}^2}$

6/92

$\alpha = \frac{a_{C/G}}{r} = \frac{a-\bar{a}}{r}$



$\Sigma F_x = m a_x;\ F = m\bar{a}$

$\Sigma M_G = \bar{I}\alpha;\ Fr = \frac{1}{2}mr^2\frac{a-\bar{a}}{r}$

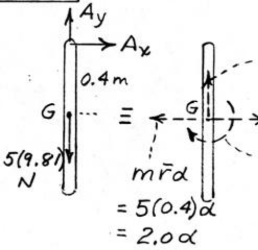
solve & get $\bar{a} = \frac{1}{3}a$

$a_{G/C} = a - \bar{a} = \frac{2}{3}a$ to the left

Rel. to truck, $s = \frac{1}{2}a_{rel}t^2$, $d = \frac{1}{2}\left(\frac{2}{3}a\right)t^2$, $t^2 = \frac{3d}{a}$

Truck $s = \frac{1}{2}at^2$, $s = \frac{1}{2}a\frac{3d}{a} = \underline{\frac{3d}{2}}$

6/93



$m\bar{r}\omega^2 = 5(0.4)2^2 = 8.0\ N$

$ma_A = 5(4) = 20\ N$

$\bar{I}\alpha = \frac{1}{12}5(0.8)^2\alpha = 0.267\alpha$

$m\bar{r}\alpha = 5(0.4)\alpha = 2.0\alpha$

$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d;\ 0 = 0.267\alpha + 2.0\alpha(0.4) - 20(0.4)$

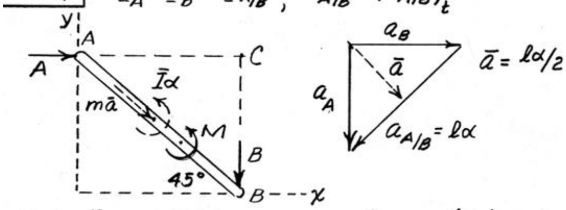
$\alpha = 7.50\ rad/s^2$

$\Sigma F_x = m\bar{a}_x;\ A_x = 20 - 2.0(7.50) = \underline{5\ N}$

$\Sigma F_y = m\bar{a}_y;\ A_y - 5(9.81) = 8,\ \ \underline{A_y = 57.1\ N}$

6/94

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$, $a_{A/B} = (a_{A/B})_t = l\alpha$



$\bar{a} = l\alpha/2$

$\Sigma M_C = \bar{I}\alpha + m\bar{a}d;\ M = \frac{1}{12}ml^2\alpha + m\frac{l\alpha}{2}\frac{l}{2} = \frac{1}{3}ml^2\alpha$

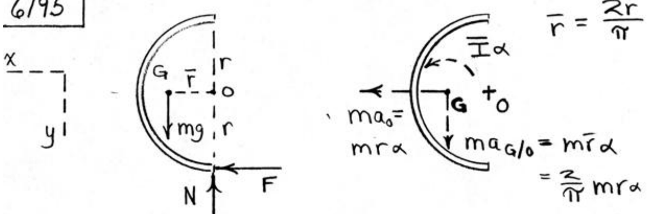
$\alpha = \frac{3M}{ml^2}$

$\Sigma F_x = m\bar{a}_x;\ A = m\frac{l\alpha}{2}\frac{1}{\sqrt{2}} = \frac{ml}{2\sqrt{2}}\frac{3M}{ml^2},\ \ \underline{A = \frac{3M}{2\sqrt{2}l}\underline{i}}$

$\Sigma F_y = m\bar{a}_y;\ -B = m\left(-\frac{l\alpha}{2}\frac{1}{\sqrt{2}}\right)\ \ \ \ \underline{B = -\frac{3M}{2\sqrt{2}l}\underline{j}}$

6/95

$\bar{r} = \frac{2r}{\pi}$



$ma_O = mr\alpha$

$ma_{G/O} = m\bar{r}\alpha = \frac{2}{\pi}mr\alpha$

$\bar{I} = I_O - m\bar{r}^2 = mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$

$\Sigma F_x = m\bar{a}_x:\ \ F = mr\alpha$

$\Sigma F_y = m\bar{a}_y:\ \ mg - N = \frac{2}{\pi}mr\alpha,\ N = m\left(g - \frac{2r\alpha}{\pi}\right)$

$\Sigma M_G = \bar{I}\alpha:\ \ N\left(\frac{2r}{\pi}\right) - Fr = mr^2\left(1 - \frac{4}{\pi^2}\right)\alpha$

Solve simultaneously : $\alpha = \frac{g}{\pi r}$

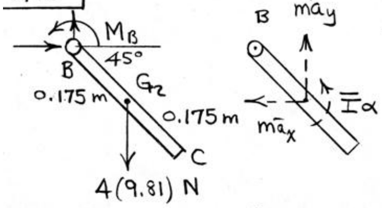
$\therefore\ F = mr\alpha = mr\frac{g}{\pi r} = m\frac{g}{\pi}$ & $N = mg\left(1 - \frac{2}{\pi^2}\right)$

Thus $\mu_s = \frac{F}{N} = \frac{mg/\pi}{mg\left(1-\frac{2}{\pi^2}\right)} = \frac{\pi}{\pi^2 - 2} = \underline{0.399}$

6/96

$\omega_{BC} = \omega_{AB} = 2\underline{k}$ rad/s

$\alpha_{BC} = \alpha_{AB} = 4\underline{k}$ rad/s²

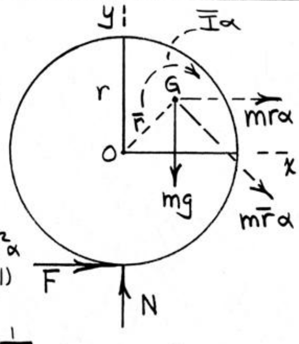


$\underline{a}_{G_2} = \underline{\alpha} \times \underline{r}_{AG_2} - \omega^2\underline{r}_{AG_2} = 4\underline{k} \times [(0.7 + 0.175)\cos 45°\underline{i}$
$+ (0.7 - 0.175)\sin 45°\underline{j}] - 2^2[(0.7 + 0.175)\cos 45°\underline{i}$
$+ (0.7 - 0.175)\sin 45°\underline{j}] = -3.96\underline{i} + 0.990\underline{j}$ m/s²

$\Sigma M_B = \bar{I}\alpha + \Sigma m\bar{a}d:\ \ M_B - 4(9.81)(0.175\sin 45°) =$
$\frac{1}{12}(4)(0.35)^2(4) + 4(0.990)(0.175\cos 45°)$
$-4(3.96)(0.175\sin 45°)$, $\underline{M_B = 3.55\ N\cdot m}$ (CCW)

6/97

$\bar{r} = \frac{4r}{3\pi}\sqrt{2}$



$\bar{I} = I_O - m\bar{r}^2$
$= \frac{1}{2}mr^2 - m\left[\frac{4r}{3\pi}\sqrt{2}\right]^2$
$= 0.1397mr^2$

$+\circlearrowright\ \Sigma M_G = \bar{I}\alpha:$

$N\left(\frac{4r}{3\pi}\right) - F\left(r + \frac{4r}{3\pi}\right) = 0.1397mr^2\alpha$ (1)

$\Sigma F_x = m\bar{a}_x:$

$F = mr\alpha + m\left(\frac{4r}{3\pi}\sqrt{2}\right)\frac{1}{\sqrt{2}}\alpha$ (2)

$\Sigma F_y = m\bar{a}_y:$

$N - mg = -m\left(\frac{4r}{3\pi}\sqrt{2}\right)\frac{1}{\sqrt{2}}\alpha$ (3)

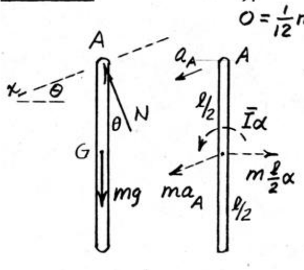
Solve Eqs. (1)-(3) to obtain

$$\begin{cases} F = 0.257mg \\ N = 0.923mg \\ \alpha = 0.1807\frac{g}{r} \end{cases}$$

6/98

$\Sigma M_A = \bar{I}\alpha + \Sigma m\bar{a}d$

$0 = \frac{1}{12}ml^2\alpha + m\frac{l}{2}\alpha\frac{l}{2} - ma_A\frac{l}{2}\cos\theta$

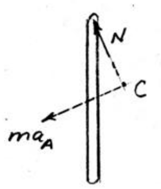


$\Sigma F_x = m\bar{a}_x$

$mg\sin\theta = m\left(a_A - \frac{l}{2}\alpha\cos\theta\right)$

Eliminate α & get

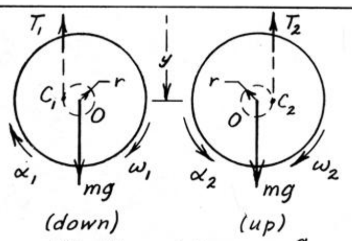
$\underline{a_A = \frac{g\sin\theta}{1 - \frac{3}{4}\cos^2\theta}}$



Alternative solution:

Pt. C may be used as a moment center thus eliminating reference to a_A & N giving one equation in α.

6/99



Down: $\Sigma M_{C_1} = I_{C_1}\alpha_1$: $mgr = m(k^2+r^2)\alpha_1$, $(a_O)_1 = r\alpha_1 = \dfrac{g}{k^2/r^2+1}$ const.

$\Sigma F_y = m\bar{a}_y$: $mg - T_1 = \dfrac{mg}{k^2/r^2+1}$, $T_1 = \dfrac{mg}{1+r^2/k^2}$

$v^2 = v_0^2 + 2as$: $\underline{v = \sqrt{v_0^2 + \dfrac{2gL}{k^2/r^2+1}}}$

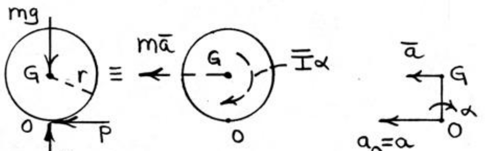
Up: $\Sigma M_{C_2} = I_{C_2}\alpha_2$: $mgr = m(k^2+r^2)\alpha_2$, $(a_O)_2 = r\alpha_2 = \dfrac{g}{k^2/r^2+1}$ const.

$\Sigma F_y = m\bar{a}_y$: $mg - T_2 = \dfrac{mg}{k^2/r^2+1}$, $T_2 = \dfrac{mg}{1+r^2/k^2}$

Thus $\underline{T = \dfrac{mg}{1+r^2/k^2}}$ & $\underline{a = \dfrac{g}{k^2/r^2+1}}$ for both motions

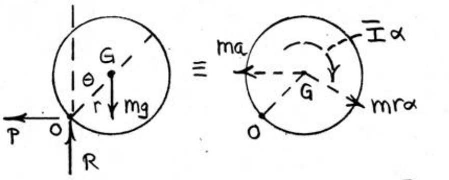
6/100

$\bar{I} = \frac{2}{5}mr^2$



$\Sigma M_O = \bar{I}\alpha - m\bar{a}r$: $0 = \frac{2}{5}mr^2\alpha - m\bar{a}r$, $\bar{a} = \frac{2}{5}r\alpha$

$a_G = a_O + a_{G/O}$: $\bar{a} = a - r\alpha = \frac{2}{5}r\alpha \Rightarrow \underline{\alpha = \frac{5}{7}\frac{a}{r}}$



$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$: $mgr\sin\theta = \frac{2}{5}mr^2\alpha + mr^2\alpha - mar\cos\theta$

$\alpha = \frac{5}{7r}(g\sin\theta + a\cos\theta)$

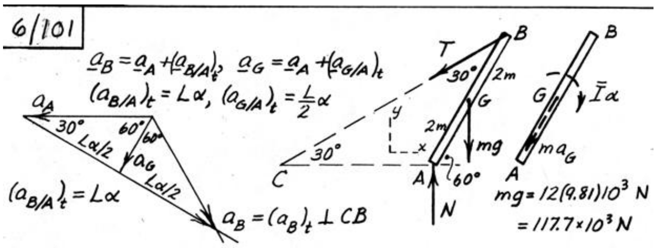
$\omega\,d\omega = \alpha\,d\theta$: $\int_0^\omega \omega\,d\omega = \frac{5}{7r}\int_0^\theta (g\sin\theta + a\cos\theta)d\theta$

$\underline{\omega = \sqrt{\frac{10}{7r}}\sqrt{g(1-\cos\theta) + a\sin\theta}}$

6/101

$a_B = a_A + (a_{B/A})_t$, $a_G = a_A + (a_{G/A})_t$

$(a_{B/A})_t = L\alpha$, $(a_{G/A})_t = \frac{L}{2}\alpha$



From accel. diag. $a_B = \frac{L}{2}\alpha\sec 30° = \frac{4}{2}\sec 30°\alpha = 2.30\,\alpha$ m/s²

Since a_G passes through A,

$\Sigma M_A = \bar{I}\alpha$: $117.7(10^3)2\cos 60° - T\times 4\sin 30° = \frac{1}{12}12(10^3)4^2\alpha$

$117.7(10^3) - 2T = 16(10^3)\alpha$ --(a)

$\Sigma F_x = m\bar{a}_x$: $T\cos 30° = 12(10^3)(a_G)_x$ where $a_G = \frac{L}{2}\alpha\tan 30°$,

$(a_G)_x = \frac{L}{2}\tan 30°\alpha\cos 60° = 0.577\alpha$ m/s²

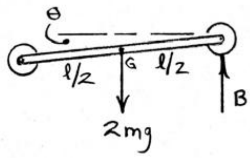
so $T = 8(10^3)\alpha$ ---(b)

Solve (a) & (b) & get $\alpha = 3.68$ rad/s², $\underline{T = 29.4\text{ kN}}$

$a_A = \frac{L}{2}\alpha/\cos 30° = \frac{4}{2}(3.68)/\cos 30°$, $\underline{a_A = 8.50\text{ m/s}^2}$

6/102 Assume that the angle θ present as B clears the surface is very small and that the speed of B is constant (while on surface).

Time t between A & B leaving surface : $t = \frac{l}{v}$.



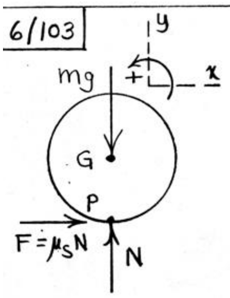
$\bar{I} = 2m\left(\frac{l}{2}\right)^2 = ml^2/2$

$I_B = \frac{ml^2}{2} + 2m\left(\frac{l}{2}\right)^2 = ml^2$

$\Sigma M_B = I_B\ddot{\theta}$: $2mg\frac{l}{2} = ml^2\ddot{\theta}$, $\ddot{\theta} = \frac{g}{l}$ (CCW)

$\omega = \cancel{\omega_0}^{\,0} + \ddot{\theta}t = \frac{g}{l}\frac{l}{v} = \underline{\frac{g}{v}}$

6/103



The no-slip constraint is found by equating the horizontal acceleration of point P to the acceleration a_C of the cart.

$(a_P)_{hor} = a_G + r\alpha = a_C$ (1)

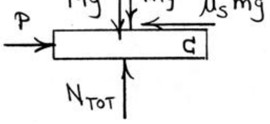
$\Sigma F_y = 0 \Rightarrow N = mg$ (2)

$\Sigma F_x = ma_G$: $\mu_s \cancel{m}g = \cancel{m}a_G$ (3)

$\Sigma M_G = \bar{I}\alpha$: $+\mu_s\cancel{m}gr = \cancel{m}\bar{k}^2\alpha$ (4)

Solution of (1), (3), & (4) : $\begin{cases} a_G = \mu_s g \\ a_C = \mu_s g\left[1 + \frac{r^2}{\bar{k}^2}\right] \\ \alpha = +\mu_s gr/\bar{k}^2 \end{cases}$

Cart :



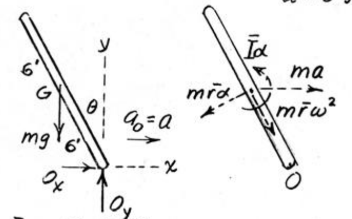
$\Sigma F_x = Ma_C$:

$P - \mu_s mg = M\mu_s g\left[1 + \frac{r^2}{\bar{k}^2}\right]$

$\underline{P = \mu_s g\left[m + M\left(1 + \frac{r^2}{\bar{k}^2}\right)\right]}$

6/104 $m\bar{a} = ma_O + ma_{G/O} = ma + m\bar{r}\omega^2 + m\bar{r}\alpha$

$a = 3$ ft/sec², $\bar{r} = 6$ ft



$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$;

$mg(6\sin\theta) = \frac{1}{12}m(12^2)\alpha + m(6\alpha)(6) - m(3)6\cos\theta$

$\alpha = \frac{1}{8}(g\sin\theta + 3\cos\theta)$

$\int\omega\,d\omega = \int\alpha\,d\theta$; $\int_0^\omega \omega\,d\omega = \frac{1}{8}\int_0^{\pi/2}(g\sin\theta + 3\cos\theta)d\theta$

$\omega^2 = \frac{1}{4}\left[-32.2\cos\theta + 3\sin\theta\right]_0^{\pi/2} = \frac{1}{4}[32.2 + 3] = \frac{35.2}{4}$

$\omega = \frac{1}{2}\sqrt{35.2} = \underline{2.97\text{ rad/sec}}$

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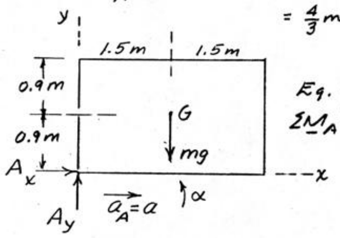
$I_A = \frac{1}{12}m([1.8]^2 + [3.0]^2) + m([0.9]^2 + [1.5]^2)$

$= \frac{4}{3}m([0.9]^2 + [1.5]^2) = 4.08\ m\ \text{kg}\cdot\text{m}^2$



Eq. 6/3

$\Sigma \underline{M}_A = \underline{I}_A \underline{\alpha} + \bar{\rho} \times m\underline{a}_A$

$-m(9.81)(1.5)\underline{k} = 4.08m\alpha\underline{k} + (1.5\underline{i} + 0.9\underline{j}) \times ma\underline{i}$

$-14.72 = 4.08\alpha - 0.9a,$

when $\alpha = 0$, $a = a_{min} = 14.72/0.9 = \underline{16.35\ m/s^2}$

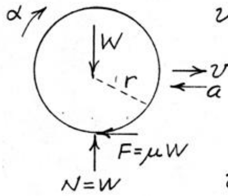
(Max. feasible accel. for $\mu_s \cong 1$ is approx. $g = 9.81\ m/s^2$) so a_{min} is not feasible)

If accel is $1.2a = 1.2(16.35)\ m/s^2$,

$\alpha = \frac{1}{4.08}(0.9[1.2]16.35 - 14.72) = \underline{0.721\ rad/s^2}$

6/106

$\Sigma F = ma$; $\mu W = \frac{W}{g}a$, $a = \mu g$



$v = v_0 - \mu g t$, $t = \frac{v_0 - v}{\mu g}$ = time to slow down to vel. v

$\Sigma \bar{M} = \bar{I}\alpha$; $\mu W r = \frac{W}{g}k^2\alpha$

$\alpha = \frac{\mu g r}{k^2}$

$v = r\omega = r(0 + \alpha t)$

so $t = \frac{v}{r\alpha} = \frac{vk^2}{\mu g r^2}$ = time to acquire $v = r\omega$.

Equate t's & set $\frac{v_0 - v}{\mu g} = \frac{vk^2}{\mu g r^2}$, $v = \frac{v_0 r^2}{k^2 + r^2}$

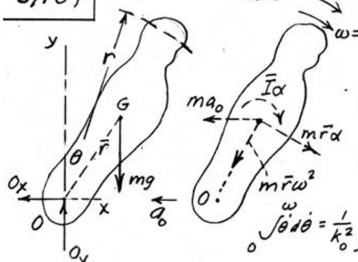
& $v^2 = v_0^2 - 2as$,

so $s = \frac{-v^2 + v_0^2}{2a} = \frac{v_0^2}{2\mu g}\frac{k^2(k^2 + 2r^2)}{(k^2 + r^2)^2}$;

Substitute $v_0 = 20$ ft/sec, $k = 3.28/12$ ft, $r = \frac{1}{12}\left(\frac{27}{2\pi}\right)$ ft, $\mu = 0.20$

& get $\underline{s = 18.66\ ft}$

6/107



$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$

$mg\bar{r}\sin\theta = m(k_O^2 - \bar{r}^2)\alpha + m\bar{r}\alpha(\bar{r}) - ma_O\bar{r}\cos\theta$

$\alpha = \frac{1}{k_O^2}(g\bar{r}\sin\theta + a_O\bar{r}\cos\theta)$

$\dot{\theta}d\dot{\theta} = \ddot{\theta}d\theta$

$\int_0^\omega \dot{\theta}d\dot{\theta} = \frac{1}{k_O^2}\int(g\bar{r}\sin\theta + a_O\bar{r}\cos\theta)d\theta$

$\frac{\omega^2}{2} = \frac{1}{k_O^2}\left[g\bar{r}(1-\cos\theta) + a_O\bar{r}\sin\theta\right]$ where $a_O = a$

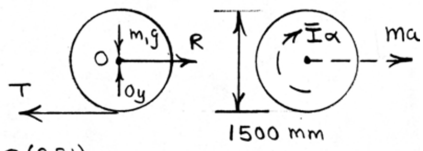
$v = r\omega = \frac{r\sqrt{2}}{k_O}\sqrt{g\bar{r}(1-\cos\theta) + a\bar{r}\sin\theta}$

For $\bar{r} = 0.45\ m$, $r = 0.8\ m$, $k_O = 0.55\ m$, $\theta = 45°$, $a = 10g$,

$v = \frac{0.80\sqrt{2}}{0.55}\sqrt{9.81(0.45)(1 - 1/\sqrt{2}) + 10(9.81)(0.45)1/\sqrt{2}} = \underline{11.73\ m/s}$

(Alternatively apply Eq. 6/3 with moment center at O)

6/108



$a = r\alpha$: $\alpha = \frac{0.2(9.81)}{1.500/2} = 2.62\ rad/s^2$

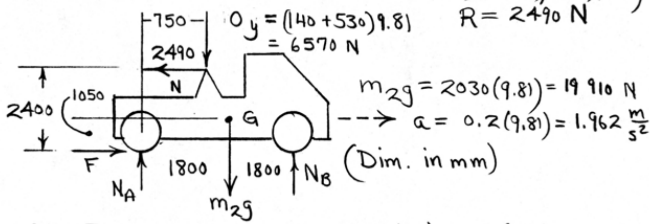
Spool & cable: $\bar{I} = 140(0.530)^2 + 150\pi(1.5)(0.75)\left(\frac{1.5}{2}\right)^2$

$= 338\ kg\cdot m^2$

$\Sigma M_O = I_O\alpha$: $T\left(\frac{1.500}{2}\right) = 338(2.62)$, $\underline{T = 1177\ N}$

$\Sigma F = m\bar{a}$: $R - 1177 = (140 + 150\pi(1.5)(0.75))(0.2)(9.81)$

$O_y = (140 + 530)9.81 = 6570\ N$ $\quad R = 2490\ N$



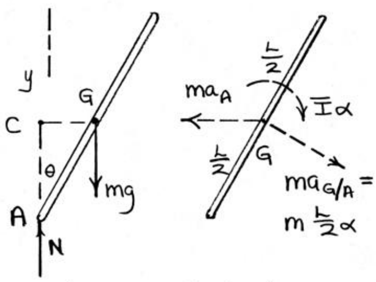
$m_2g = 2030(9.81) = 19\,910\ N$

$a = 0.2(9.81) = 1.962\ \frac{m}{s^2}$

$\Sigma M_A = \Sigma m\bar{a}d$: $6570(0.750) + 19\,910(1.8) - 2490(2.4) - 3.6N_B = 2030(1.962)(1.05)$

$\Sigma F_y = 0$: $N_A + N_B - 6570 - 19\,910 = 0 \Rightarrow \underline{N_A = 17\,980\ N,\ N_B = 8500\ N}$

6/109



$\Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d$:

$mg\frac{L}{2}\sin\theta = \frac{1}{12}mL^2\alpha + \frac{L}{2}\sin^2\theta\left(m\frac{L}{2}\alpha\right)$

$\alpha = \frac{2g}{L}\frac{\sin\theta}{\frac{1}{3} + \sin^2\theta}$

$\Sigma F_y = m\bar{a}_y$: $mg - N = m\frac{L}{2}\left(\frac{2g}{L}\frac{\sin\theta}{\frac{1}{3} + \sin^2\theta}\right)\sin\theta$

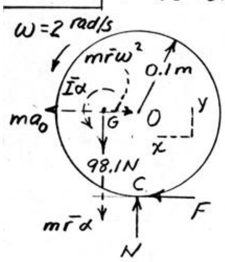
$\underline{N = \frac{mg}{1 + 3\sin^2\theta}}$

6/110

$\bar{r} = \overline{OG} = 0.040\ m$; $m\bar{r}\omega^2 = 10(0.040)(2^2) = 1.6\ N$

$\bar{I} = m\bar{k}^2 = 10(0.064)^2 = 0.0410\ kg\text{-}m^2$

$m\bar{r}\alpha = 10(0.040)\frac{a_O}{0.1} = 4a_O\ N$



$\Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d$

$98.1(0.040) = 0.0410\frac{a_O}{0.1} + 4a_O(0.040) + (10a_O - 1.6)(0.1)$

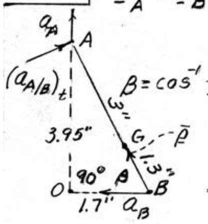
$a_O = 2.60\ m/s^2$

$\Sigma F_x = m\bar{a}_x$; $F = 10(2.60) - 1.6 = \underline{24.4\ N}$

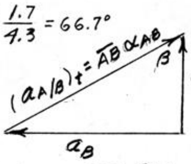
$\Sigma F_y = m\bar{a}_y$; $N - 98.1 = -4(2.60)$, $\underline{N = 87.7\ N}$

6/111 $a_A = a_B + (a_{A/B})_t$; $(a_{A/B})_n = \overline{AB}\,\omega_{AB}^2 = 0$ since $v_A = v_B$

$a_B = r\omega^2 = \frac{1.7}{12}(100\pi)^2 = 13.98(10^3)$ ft/sec²



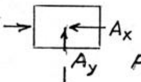
$\beta = \cos^{-1}\frac{1.7}{4.3} = 66.7°$



$(a_{A/B})_t = \frac{13.98(10^3)}{\sin 66.7°} = 15.22(10^3)$ $\frac{ft}{sec^2}$

$\alpha_{AB} = \frac{15.22(10^3)}{4.3/12} = 42.5(10^3)$ rad/sec² CW

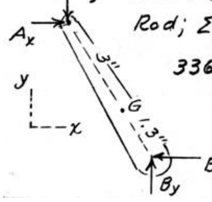
$a_A = 13.98(10^3)/\tan\beta = 6.02(10^3)$ ft/sec²



Piston; $\Sigma F_y = ma_y$; $A_y = \frac{1.80}{32.2}6.02(10^3) = 336$ lb

Rod; $\Sigma M_B = \bar{I}_B\alpha + \bar{\rho}\times m a_B$ (Eq. 6/3)

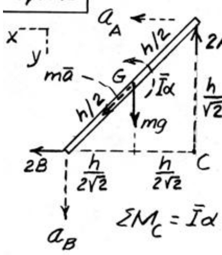
$$336(1.7) - A_x(3.95) = \frac{1.20}{32.2(12)}[(1.12)^2+(1.3)^2](-42.5)10^3$$
$$+ 1.3\frac{1.20}{32.2(12)}(13.98)(10^3)12\sin\beta$$

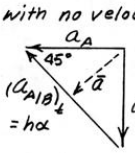


$A_x = 85.6$ lb, $A = \sqrt{(85.6)^2 + (336)^2} = \underline{347\text{ lb}}$

6/112 $a_A = a_B + a_{A/B}$

with no velocity, $a_{A/B} = (a_{A/B})_t = h\alpha$





$a_A = h\alpha/\sqrt{2}$

$a_B = h\alpha/\sqrt{2}$

$\bar{a} = h\alpha/2$

$\Sigma M_C = \bar{I}\alpha + m\bar{a}d$; $mg\frac{h}{2\sqrt{2}} = \frac{1}{12}mh^2\alpha + m\frac{h\alpha}{2}\frac{h}{2}$

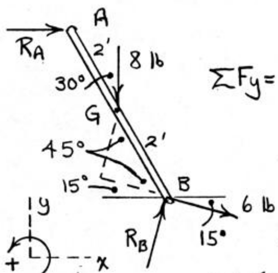
$\alpha = \frac{3g}{2\sqrt{2}h}$

$\Sigma F_x = m\bar{a}_x$; $2B = m\frac{h\alpha}{2}\frac{1}{\sqrt{2}}$, $B = \frac{mh}{4\sqrt{2}}\frac{3g}{2\sqrt{2}h} = \underline{\frac{3}{16}mg}$

$\Sigma F_y = m\bar{a}_y$; $mg - 2A = m\frac{h\alpha}{2}\frac{1}{\sqrt{2}}$

$2A = mg - \frac{mh}{2\sqrt{2}}\frac{3g}{2\sqrt{2}h}$, $A = \underline{\frac{5}{16}mg}$

▶6/113 $\Sigma F_x = m\bar{a}_x$: $R_A + 6\cos 15° + R_B \sin 15° = \frac{8}{32.2}\bar{a}_x$ (1)



$\Sigma F_y = m\bar{a}_y$: $R_B\cos 15° - 6\sin 15° - 8 = \frac{8}{32.2}\bar{a}_y$ (2)

$\Sigma M_G = \bar{I}\alpha$:

$-R_A(2\cos 30°) + R_B(2\cos 45°) + 6(2\sin 45°) = \frac{1}{12}\frac{8}{32.2}(4)^2\alpha$ (3)

Kinematics :

$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \underline{\alpha}\times\underline{r}_{A/G} - \omega^2\underline{r}_{A/G}$

With $\underline{r}_{A/G} = 2[-\sin 30°\underline{i} + \cos 30°\underline{j}]$, we have

$a_A\underline{j} = [\bar{a}_x - 2\cos 30°\alpha - 2^2\cdot(-2\sin 30°)]\underline{i}$
$+ [\bar{a}_y - 2\sin 30°\alpha - 2^2\cdot(2\cos 30°)]\underline{j}$

$$\Rightarrow \begin{cases} 0 = \bar{a}_x - \sqrt{3}\alpha + 4 & (4) \\ a_A = \bar{a}_y - \alpha - 4\sqrt{3} & (5) \end{cases}$$

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = \bar{a}_x\underline{i} + \bar{a}_y\underline{j} + \alpha\underline{k}\times\underline{r}_{B/G} - \omega^2\underline{r}_{B/G}$

With $\underline{r}_{B/G} = 2[\sin 30°\underline{i} - \cos 30°\underline{j}]$, we have

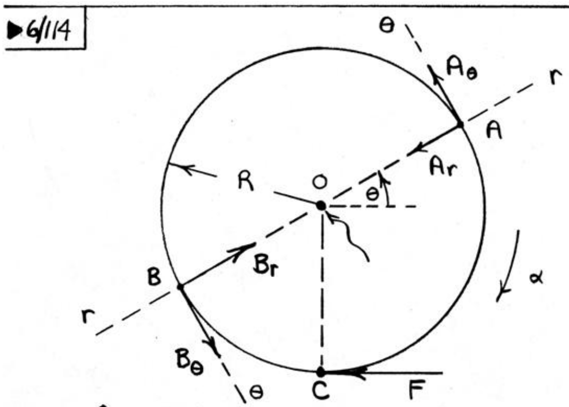
$a_B[\cos 15°\underline{i} - \sin 15°\underline{j}] =$
$[\bar{a}_x + 2\cos 30°\alpha - 2^2\cdot(2\sin 30°)]\underline{i}$
$+ [\bar{a}_y + 2\sin 30°\alpha - 2^2\cdot(-2\cos 30°)]\underline{j}$

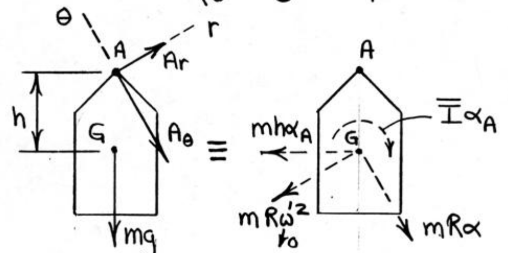
$$\Rightarrow \begin{cases} a_B\cos 15° = \bar{a}_x + \sqrt{3}\alpha - 4 & (6) \\ -a_B\sin 15° = \bar{a}_y + \alpha + 4\sqrt{3} & (7) \end{cases}$$

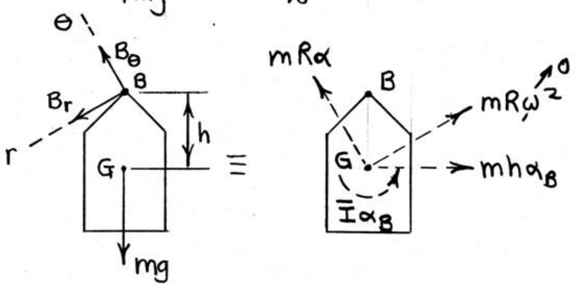
Solution of Eqs. (1)–(7) :

$$\begin{cases} \underline{R_A = 1.128\text{ lb}} & \underline{\alpha = 18.18\text{ rad/sec}^2} \\ \underline{R_B = -0.359\text{ lb}} & \underline{a_A = -65.0\text{ ft/sec}^2} \\ \bar{a}_x = 27.5\text{ ft/sec}^2 & \underline{a_B = 56.9\text{ ft/sec}^2} \\ \bar{a}_y = -31.8\text{ ft/sec}^2 & \end{cases}$$

▶6/114







• Gondola A – $\sum M_A = \bar{I}\alpha_A + \sum m a_A d$:

$0 = \bar{I}\alpha_A + mh^2\alpha_A - mR\alpha h \sin\theta$

But $\bar{I} + mh^2 = I_A = mk^2$

So $I_A\alpha_A = mRh\alpha \sin\theta$ or $m\alpha_A = mRh\alpha\sin\theta / k^2$

$\sum F_\theta = m\bar{a}_\theta$: $A_\theta + mg\cos\theta = mR\alpha - mh\alpha_A \sin\theta$

$A_\theta = m(R\alpha - g\cos\theta) - mR\alpha\left(\frac{h\sin\theta}{k}\right)^2$ (1)

• Gondola B – $\sum M_B = \bar{I}\alpha_B + \sum m a_B d$:

$0 = \bar{I}\alpha_B + mh^2\alpha_B - mR\alpha h \sin\theta$

So $I_B\alpha_B = mRh\alpha\sin\theta$ or $m\alpha_B = mRh\alpha\sin\theta/k^2$

(where $I_B = I_A = mk^2$, as above)

$\sum F_\theta = m\bar{a}_\theta$: $B_\theta - mg\cos\theta = mR\alpha - mh\alpha_B\sin\theta$

$B_\theta = m(R\alpha + g\cos\theta) - mR\alpha\left(\frac{h\sin\theta}{k}\right)^2$ (2)

• Wheel – $\sum M_O = I_O\alpha$:

$[F - \sum(A_\theta + B_\theta)]R = I_O\alpha$ (3)

Substitute (1) & (2) into (3)

$FR - \sum_{1}^{n/2}\left[2mR\alpha - 2mR\alpha\left(\frac{h\sin\theta_n}{k}\right)^2\right]R = I_O\alpha$

Simplify & solve for F :

$F = \left\{mR\left[n - 2\frac{h^2}{k^2}(\sin^2\theta_1 + \sin^2\theta_2 + \cdots \sin^2\theta_{n/2})\right] + \frac{I_O}{R}\right\}\alpha$

($n = 0$ corresponds to $\theta = 0$; $n/2$ corresponds to $\theta < \pi$)

Note: The above expression for F simplifies to

$F = \left\{mRn\left(1 - \frac{h^2}{2k^2}\right) + \frac{I_O}{R}\right\}\alpha$

6/115 $I_O = \frac{1}{12}ml^2 + m\left(\frac{l}{4}\right)^2 + 2m\left(\frac{3l}{4}\right)^2$

$= \frac{61}{48}ml^2$

$T_1 + U_{1-2} = T_2$: $0 + mg\left(\frac{l}{4}\right) + 2mg\left(\frac{3l}{4}\right) = \frac{1}{2}\frac{61}{48}ml^2\omega^2$

$\omega = 1.660\sqrt{\frac{g}{l}}$ CW

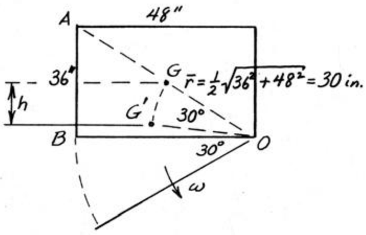
6/116

$\angle AOB = \tan^{-1}\frac{36}{48} = 36.9°$

$\angle G'OB = 36.9° - 30° = 6.87°$

$h = 30\sin 36.9° - 30\sin 6.87°$

$= 14.41$ in.



$U = \Delta T$: $mgh = \frac{1}{2}I_O\omega^2$

$250\frac{14.41}{12} = \frac{1}{2}\frac{1}{3}\frac{250}{32.2}(3^2 + 4^2)\omega^2$

$\omega^2 = 9.28\ (\text{rad/sec})^2$, $\omega = 3.05$ rad/sec

Weight cancels so does not influence the results.

6/117 $T_1 + U_{1-2} = T_2$

$T_1 = \frac{1}{2}8(0.3)^2 + \frac{1}{2}12(0.210)^2\left(\frac{0.3}{0.2}\right)^2 = 0.955$ J

$U_{1-2} = 8(9.81)(1.5) - 3\left(\frac{1.5}{0.2}\right) = 95.2$ J

$T_2 = \frac{1}{2}8v^2 + \frac{1}{2}12(0.210)^2\left(\frac{v}{0.2}\right)^2 = 10.62v^2$

So $0.955 + 95.2 = 10.62v^2$, $v = 3.01$ m/s

6/118 $U'_{1-2} = \Delta T + \Delta V_g$

$0 = \frac{1}{2}m(4^2 - 0^2) - mg(5)(1 - \cos\theta)$

$\theta = 33.2°$

6/119

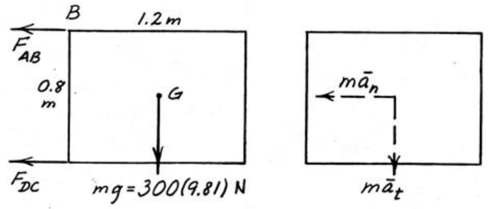


Plate has curvilinear translation so $T = \frac{1}{2}mv^2$

$U = \Delta T$: $300(9.81)(0.8\cos 60°) = \frac{1}{2}(300)v^2$, $v = 2.80$ m/s

$\omega = v/r$: Angular velocity of links is $\omega = 2.80/0.8 = 3.50$ rad/s

$\sum F_t = m\bar{a}_t$: $\bar{a}_t = 9.81$ m/s²

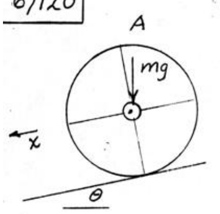
$\bar{a}_n = v^2/r$: $\bar{a}_n = 2.80^2/0.8 = 9.81$ m/s²

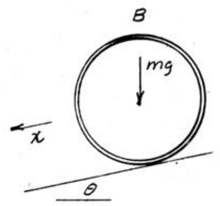
$\sum M_B = m\bar{a}d$: $300(9.81)(0.6) + F_{DC}(0.8)$

$= 300(9.81)(0.6) + 300(9.81)(0.4)$

$F_{DC} = 1472$ N

6/120





$U = \Delta T$

$U = mgx\sin\theta$

$\Delta T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

Case A: $\Delta T = \frac{1}{2}mv^2 + 0$

Case B: $\Delta T = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\left(\frac{v}{r}\right)^2$

$= mv^2$

Case A: $mgx\sin\theta = \frac{1}{2}mv^2$

$v_A = \sqrt{2gx\sin\theta}$

Case B: $mgx\sin\theta = mv^2$

$v_B = \sqrt{gx\sin\theta}$

6/121 $U = \Delta T$; $U = 40(2\times 3) = 240$ J

$\Delta T_{hoop} = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}m\omega^2(r^2 + r^2) = mr^2\omega^2$

$= 10(0.3)^2\omega^2 = 0.9\omega^2$

$\Delta T_{\text{each pair spokes}} = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}\frac{1}{12}m(2r)^2\omega^2$

$= \frac{2}{3}mr^2\omega^2$

$\Delta T_{\text{both pair}} = \frac{4}{3}mr^2\omega^2 = \frac{4}{3}4(0.3)^2\omega^2 = 0.48\omega^2$

Thus $240 = 0.9\omega^2 + 0.48\omega^2$, $\omega^2 = 173.9$

$\omega = 13.19$ rad/s

6/122

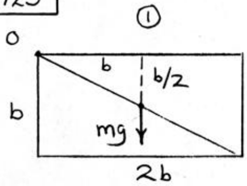
For rotation: $T_{rot} = \frac{1}{2} I_C \omega^2 = \frac{1}{2}(4)\left(\frac{1}{12}mb^2 + m\left(\frac{b}{2}\right)^2\right)\omega^2$
$= \frac{2}{3}mb^2\omega^2$

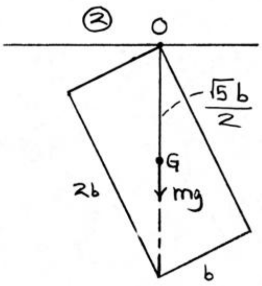
For translation: $T_{tran} = \frac{1}{2}(4m)v^2 = 2mv^2$

For $T_{tran} = T_{rot}$: $2mv^2 = \frac{2}{3}mb^2\omega^2$

$$v = \frac{b\omega}{\sqrt{3}}$$

6/123





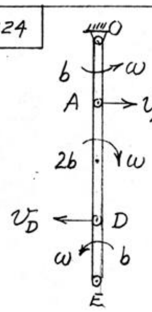
$I_O = \bar{I} + md^2 = \frac{1}{12}m\left[b^2 + (2b)^2\right] + m\left[b^2 + \left(\frac{b}{2}\right)^2\right]$
$= \frac{5}{3}mb^2$

$T_1 + U_{1\text{-}2} = T_2$:

$0 + mgb\left[\frac{\sqrt{5}}{2} - \frac{1}{2}\right] = \frac{1}{2}\left[\frac{5}{3}mb^2\right]\omega^2$

$\omega^2 = \frac{3g}{5b}(\sqrt{5} - 1)$, $\quad \omega = 0.861\sqrt{\frac{g}{b}}$

6/124



Final position of OADE (all bars)

$\Delta V_g = 8b\rho g(-2b) = -16\rho g b^2$

$\Delta T = 8\left(\frac{1}{3}mb^2\right)\omega^2$
$= \frac{8}{3}\rho b^3\omega^2$

$U = 0 = \Delta T + \Delta V_g$

$0 = \frac{8}{3}\rho b^3\omega^2 - 16\rho g b^2$

$\omega^2 = 6g/b$, $\quad \omega = \sqrt{6g/b}$

6/125 *Note: the wheel has no motion in initial or final positions so* $\Delta T_{wheel} = 0$

$U' = \Delta V_g + \Delta T$; $\quad U' = Fb\sin\theta$

$\Delta V_g = -2m_o g\frac{b}{2}\sin\theta$

$\Delta T = 2\left(\frac{1}{2}I_C\omega^2\right) = \frac{1}{3}m_o b^2\omega^2$

Thus $Fb\sin\theta = -m_o g b\sin\theta + \frac{1}{3}m_o b^2\omega^2$

$$\omega = \sqrt{\frac{3(F + m_o g)\sin\theta}{m_o b}}$$

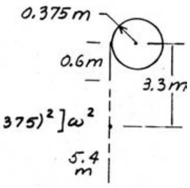
6/126 Power $P = \frac{d(\text{Energy})}{dt} = \frac{\Delta E}{t}$

$\Delta E = \frac{1}{2}\bar{I}(\omega_2^2 - \omega_1^2) = \frac{1}{2}(1200)(0.4)^2([5000]^2 - [3000]^2)\left(\frac{2\pi}{60}\right)^2$
$= 16.84(10^6)$ J

$P = \frac{16.84(10^6)}{2(60)} = 140.4(10^3)$ J/s or W

so $P = 140.4$ kW $\quad$ or $\quad P = \frac{140.4(10^3)}{7.457(10^2)} = 188$ hp

6/127 $\Delta V_g + \Delta T = 0$



$\Delta V_g = -5.4(3.08)(9.81)(3.3) = -538$ J

$\Delta T = \frac{1}{2}6.0(3.08)(0.375\omega)^2$
$+ \frac{1}{2}\left[41(0.30)^2 + (3.08)(18-6)(0.375)^2\right]\omega^2$
$= 1.299\omega^2 + 4.44\omega^2 = 5.74\omega^2$

Thus $-538 + 5.74\omega^2 = 0$, $\omega^2 = 93.8$,
$\omega = 9.68$ rad/s

6/128 $U'_{1\text{-}2} = \Delta T + \Delta V_g + \Delta V_e$

$U'_{1\text{-}2} = M\theta = \frac{\pi}{2}M = 1.571M$ in.-lb

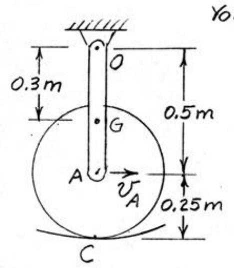
$\Delta T = \frac{1}{2}I_O\omega^2 - 0 = \frac{1}{2}\left(\frac{12}{32.2\times 12}\times 10^2\right)4^2 = 24.8$ in.-lb

$\Delta V_g = Wh = 12(-8) = -96$ in.-lb

$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}3([30 - 15\sqrt{2}]^2 - 0) = 115.8$ in.-lb

Thus $1.571M = 24.8 - 96 + 115.8$, $\quad M = 28.4$ lb-in.

6/129 For system $\Delta T + \Delta V_g = 0$ since $U = 0$



Yoke: $\Delta V_g = 3\times 9.81(-0.3) = -8.83$ J

$\Delta T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}(3\times\overline{0.35}^2)\left(\frac{v_A}{0.5}\right)^2$
$= 0.735v_A^2$

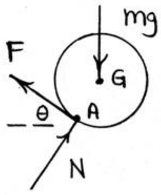
Hoop: $\Delta V_g = 4\times 9.81(-0.5) = -19.62$ J

$\Delta T = \frac{1}{2}I_C\omega^2 = \frac{1}{2}(2\times 4\times\overline{0.25}^2)\left(\frac{v_A}{0.25}\right)^2$
$= 4v_A^2$

Thus $0.735v_A^2 + 4v_A^2 - 8.83 - 19.62 = 0$

$4.735v_A^2 = 28.45$, $\quad v_A^2 = 6.01$, $\quad v_A = 2.45$ m/s

6/130 (a) $v = 0$



$v_A = 0$, so F and N are applied at a stationary point and thus do no work.

(b) $v \neq 0$, $v_A \neq 0$: F and N do work.

6/131

For the top position $\omega_B = \frac{v}{0.080}$, $\omega_{OA} = \frac{v}{0.280}$

For entire system $U'_{1\text{-}2} = \Delta T + \Delta V_g$

$U'_{1\text{-}2} = M\theta = 4(\pi/2) = 6.28$ J

$\Delta T_{OA} = \frac{1}{2}I_O\omega_{OA}^2 = \frac{1}{2}0.8(0.140^2)(v/0.280)^2 = 0.1v^2$ J

$\Delta T_B = \frac{1}{2}mv^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}0.9v^2 + \frac{1}{2}\left[\frac{1}{2}0.9\times 0.080^2\right]\left(\frac{v}{0.080}\right)^2$
$= 0.675v^2$ J

$(\Delta V_g)_{OA} = mgh = 0.8(9.81)(0.100) = 0.785$ J

$(\Delta V_g)_B = mgh = 0.9(9.81)(0.280) = 2.47$ J

Thus $6.28 = 0.1v^2 + 0.675v^2 + 0.785 + 2.47$,

$v^2 = 3.90$ (m/s)^2

$v = 1.976$ m/s

6/132 $T_1 + U_{1-2} = T_2$

$T_1 = 0$

$$U_{1-2} = \int_1^2 M\,d\theta = \int_0^{5(2\pi)} 2(1-e^{-0.1\theta})\,d\theta$$

$$= (2\theta + 20e^{-0.1\theta})\Big|_0^{5(2\pi)}$$

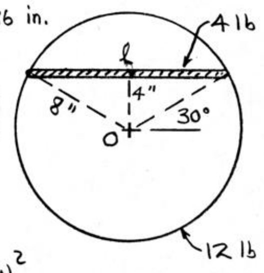
$$= 2(5)(2\pi) + 20e^{-0.1(5)(2\pi)} - 20$$

$$= 43.7 \text{ J}$$

$$T_2 = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(50)(0.4)^2\omega^2 = 4\omega^2$$

So $0 + 43.7 = 4\omega^2$, $\underline{\omega = 3.31 \text{ rad/s}}$

6/133 $l = 2(8)\cos 30° = 13.86$ in.



$$I_O = \tfrac{1}{2}\tfrac{12}{32.2}\left(\tfrac{8}{12}\right)^2 + \left[\tfrac{1}{12}\tfrac{4}{32.2}\left(\tfrac{13.86}{12}\right)^2 + \tfrac{4}{32.2}\left(\tfrac{4}{12}\right)^2\right]$$

$$= 0.1104 \text{ lb-sec}^2\text{-ft}$$

$T_1 + U_{1-2} = T_2$

$$0 + 4\left(\tfrac{8}{12}\right) = \tfrac{1}{2}(0.1104)\omega^2$$

$\underline{\omega = 6.95 \text{ rad/sec}}$

6/134 $I = mk^2 = 10(0.090)^2 = 0.081 \text{ kg}\cdot\text{m}^2$

$M = I\dot{\omega}$, $\dot{\omega} = M/I = -2.10/0.081 = -25.9 \text{ rad/s}^2$

$\omega_0 = 80\,000\left(\frac{2\pi}{60}\right) = 8380$ rad/s

$P = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = I\omega\dot{\omega}$

(a) $t = 0$: $P = I\omega\dot{\omega} = (0.081)(8380)(25.9)$
$= 17\,590$ W or $\underline{17.59 \text{ kW}}$

(b) $t = 120$ s : $\omega = \omega_0 + \dot{\omega}t = 8380 - 25.9(120)$
$= 5270$ rad/s

$P = I\omega\dot{\omega} = (0.081)(5270)(25.9) = 11\,060$ W

or $\underline{P = 11.06 \text{ kW}}$

6/135 $T_1 + U_{1-2} = T_2$

$$mg\left(\tfrac{l}{2} - x\right) = \tfrac{1}{2}\left[\tfrac{1}{12}ml^2 + m\left(\tfrac{l}{2}-x\right)^2\right]\omega^2$$

$$\omega^2 = \frac{g\left(\frac{l}{2}-x\right)}{\frac{l^2}{6} - \frac{lx}{2} + \frac{x^2}{2}}$$

Set $\frac{d\omega^2}{dx} = 0$ & obtain $\underline{x = 0.789l}$

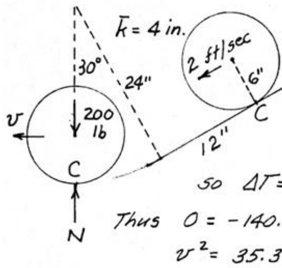
or $\underline{x = 0.211l}$

$$\omega_{max} = \omega_{x=0.211} = \sqrt{\frac{g\left(\frac{l}{2} - 0.211l\right)}{\frac{l^2}{6} - \frac{0.211l^2}{2} + \frac{(0.211l)^2}{2}}}$$

$$= \underline{1.861\sqrt{\tfrac{g}{l}}}$$

(The solution $x = 0.789l$ would yield the same ω_{max}, only then the motion is CCW.)

6/136 $0 = \Delta V_g + \Delta T$; $\Delta V_g = -200\left[\frac{12}{12}\sin 30° + \frac{18}{12}(1-\cos 30°)\right]$



$= -140.2$ ft-lb

$\Delta T = \frac{1}{2}I_C(\omega_2^2 - \omega_1^2)$

$I_C = \frac{200}{32.2}\left(\left[\frac{4}{12}\right]^2 + \left[\frac{6}{12}\right]^2\right) = 2.24$ lb-ft-sec^2

$\omega_1 = \frac{2}{6/12} = 4$ rad/sec, $\omega_2 = \frac{v}{6/12} = 2v$

so $\Delta T = \frac{1}{2}2.24(4v^2 - 4^2) = 4.49v^2 - 17.94$ ft-lb

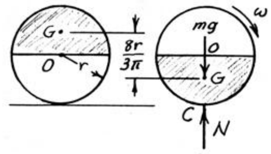
Thus $0 = -140.2 + 4.49v^2 - 17.94$

$v^2 = 35.30$, $v = 5.94$ ft/sec

$\Sigma F_n = m\bar{a}_n$; $N - 200 = \frac{200}{32.2}\,\frac{35.30}{18/12}$, $\underline{N = 346 \text{ lb}}$

6/137

$U = \Delta T$: $mg\left(\frac{8r}{3\pi}\right) = \frac{1}{2}I_C\omega^2$



$I_G = I_O - m\bar{r}^2$, $I_C = I_G + m(r-\bar{r})^2$

so $I_C = I_O - m\bar{r}^2 + m(r-\bar{r})^2$

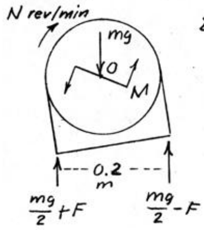
$$I_C = m\left(\tfrac{1}{2}r^2 - \bar{r}^2 + r^2 - 2r\bar{r} + \bar{r}^2\right) = m\left(\tfrac{3}{2}r^2 - 2r\left[\tfrac{4r}{3\pi}\right]\right) = mr^2\left(\tfrac{3}{2} - \tfrac{8}{3\pi}\right)$$

So $mg\left(\frac{8r}{3\pi}\right) = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\omega^2$, $\omega^2 = \frac{32}{9\pi - 16}\frac{g}{r}$, $\underline{\omega = \sqrt{\frac{g}{r}\frac{32}{9\pi - 16}} \ \frac{\text{rad}}{\text{s}}}$

$\Sigma F_n = m\bar{a}_n$: $N - mg = m\bar{r}\omega^2$, $N = mg + m\frac{4r}{3\pi}\omega^2$

$$\underline{N = mg\left(1 + \frac{128}{3\pi(9\pi - 16)}\right)}$$

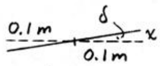
6/138 Power $P = M\omega = M\frac{2\pi N}{60}$, $M = \frac{4000(60)}{2\pi(1725)} = 22.14$ N·m



$\Sigma M_O = 0$; $2F(0.2) - 22.14 = 0$ $F = 55.4$ N

$F = kx$, $x = \frac{55.4}{15(10^3)} = 0.00369$ m

or $x = 3.69$ mm



$\delta = \tan^{-1}\frac{x}{0.1} = \tan^{-1}\frac{3.69}{100} = \underline{2.11°}$

$\underline{\text{Motor shaft turns CW}}$

6/139 $U = \Delta T$

For treads $T = 2(T_{hoop} + T_{top\ section})$, $T_{bottom\ section} = 0$

$T_{hoop} = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left[2\pi r\rho(r^2 + r^2)\frac{v^2}{r^2}\right] = 2\pi\rho r v^2$

$T_{top\ section} = \frac{1}{2}(\rho b)(2v)^2 = 2\rho b v^2$

$U = M\theta = M\frac{s}{r}$

Thus $M\frac{s}{r} = 2(2\pi\rho r v^2 + 2\rho b v^2)$, $\underline{M = 4\rho\frac{r}{s}v^2(\pi r + b)}$

6/140 $\Delta V_e = \frac{1}{2}(1500)\left[(0.1 + 2\times 0.05)^2 - \overline{0.1}^2\right] = 22.5$ J

$\Delta V_g = -(150)(9.81)(0.05) = -73.58$ J

$\Delta T = \Sigma\frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(150)v^2 + \frac{1}{2}(50)(0.3)^2\left(\frac{v}{0.4}\right)^2$

$= 75v^2 + 14.06v^2 = 89.06v^2$

$\Delta T + \Delta V_g + \Delta V_e = 0$; $89.06v^2 - 73.58 + 22.5 = 0$

$v^2 = 0.573$, $\underline{v = 0.757 \text{ m/s}}$

6/141

For dropping, $U' = \Delta T + \Delta V_g + \Delta V_e$

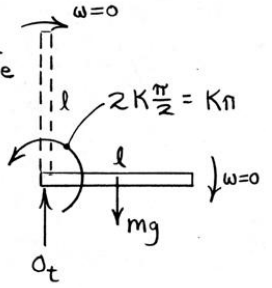
$U' = \Delta T = 0$, $\Delta V_g = -mg\frac{l}{2}$

$\Delta V_e = 2\int_0^{\pi/2} K\theta\, d\theta$

$= K\theta^2\Big|_0^{\pi/2} = \frac{\pi^2}{4}K$

So $0 = -mg\frac{l}{2} + \frac{\pi^2}{4}K$

$K = \frac{2l}{\pi^2}mg$

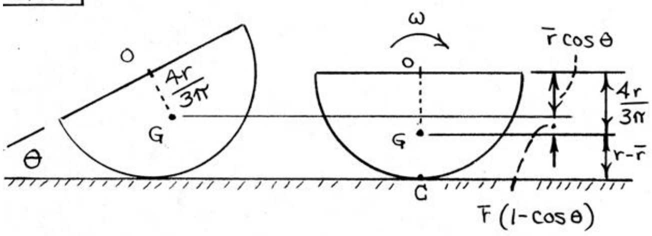


Release from rest at $\theta = \pi/2$:

$\Sigma M_O = I_O\alpha$: $\frac{2l}{\pi^2}mg\pi - mg\frac{l}{2} = \frac{1}{3}ml^2\alpha$, $\alpha = 0.410\frac{g}{l}$

Lid would not stay down; $K = \frac{2l}{\pi^2}mg$ is not practical.

6/142



$U' = \Delta T + \Delta V_g = 0$

$I_C = \bar{I} + m(r-\bar{r})^2 = (I_O - m\bar{r}^2) + m(r-\bar{r})^2$

$= I_O + m(r^2 - 2r\bar{r}) = \frac{1}{2}mr^2 + mr^2\left(1 - \frac{8}{3\pi}\right)$

$= mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)$

$\Delta T = \frac{1}{2}I_C\omega^2 - 0 = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\omega^2$

$\Delta V_g = -mg\frac{4r}{3\pi}(1-\cos\theta)$

So $0 = \frac{1}{2}mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\omega^2 - mg\frac{4r}{3\pi}(1-\cos\theta)$

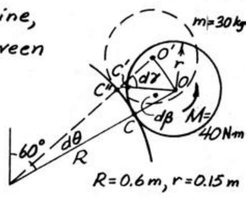
$\omega = 4\sqrt{\frac{g(1-\cos\theta)}{(9\pi - 16)r}}$

6/143 During rotation $d\theta$ of radial line, disk rotates through angle $d\gamma$ between lines OC' and $O'C''$. $CC' = CC''$ so

$R\,d\theta = r\,d\beta$ & $d\gamma = d\theta + d\beta$

$= \left(1 + \frac{R}{r}\right)d\theta$

or $\gamma = \left(1 + \frac{R}{r}\right)\theta$



For $\theta = \frac{\pi}{3}$, $\gamma = (1 + 0.6/0.15)\frac{\pi}{3} = 5\pi/3$ rad

$U' = \Delta T + \Delta V_g$: $U' = M\gamma = 40\frac{5\pi}{3} = 209$ J

$\Delta T = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left(\frac{3}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{3}{4}mv^2 = \frac{3}{4}(30)v^2$

$= 22.5v^2$

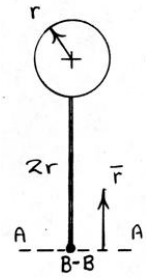
$\Delta V_g = mgh = mg(R+r)(1-\cos 60°)$

$= 30(9.81)(0.75)(\frac{1}{2}) = 110.4$ J

$209 = 22.5v^2 + 110.4$, $v^2 = 4.40\ (m/s)^2$, $v = 2.10$ m/s

6/144 Total mass $m = 2r\rho + 2\pi r\rho = 2r\rho(1+\pi)$

where ρ = mass per unit length.



$\bar{r} = \frac{\Sigma\bar{r}m}{\Sigma m} = \frac{2r\rho(r) + 2\pi r\rho(3r)}{2r\rho + 2\pi r\rho}$

$= r\frac{1+3\pi}{1+\pi}$

$I_{B-B} = \frac{1}{3}(2r\rho)(2r)^2 + \left[2\pi r\rho r^2 + 2\pi r\rho(3r)^2\right]$

$= \frac{4+30\pi}{3(1+\pi)}mr^2$

$I_{A-A} = \frac{1}{3}(2r\rho)(2r)^2 + \left[\frac{1}{2}2\pi r\rho r^2 + 2\pi r\rho(3r)^2\right]$

$= \frac{8+57\pi}{6(1+\pi)}mr^2$

$T_1 + U_{1-2} = T_2$

(a) $0 + mgr\frac{1+3\pi}{1+\pi} = \frac{1}{2}\frac{8+57\pi}{6(1+\pi)}mr^2\omega^2$

$\omega = 2\sqrt{\frac{3+9\pi}{8+57\pi}\frac{g}{r}}$

(b) $0 + mgr\frac{1+3\pi}{1+\pi} = \frac{1}{2}\frac{4+30\pi}{3(1+\pi)}mr^2\omega^2$

$\omega = \sqrt{\left(\frac{3+9\pi}{2+15\pi}\right)\frac{g}{r}}$

6/145 $P = \frac{dU}{dt} = \frac{d}{dt}(T + V_g) + Rv$

$P = \frac{d}{dt}\left[\Sigma\frac{1}{2}mv^2 + \Sigma\frac{1}{2}\bar{I}\omega^2\right] + \frac{d}{dt}(mgh) + Rv$

$= \Sigma mv\frac{dv}{dt} + \Sigma\bar{I}\omega\frac{d\omega}{dt} + mgv\sin\theta + Rv$

$= mva + 4\bar{I}\omega\alpha + (mg\sin\theta + R)v$

$= (mv + 4\bar{I}\frac{v}{r^2})a + (mg\sin\theta + R)v$

(a) with $a = 0$, $P = 0 + (500\times 9.81\times\frac{1}{\sqrt{101}} + 400)\frac{72}{3.6}$

$= 17\,761$ W or $P = 17.76$ kW

(b) with $a = 3$ m/s²

$P = \left(500\frac{72}{3.6} + 4(40)(0.4)^2\frac{72/3.6}{0.6^2}\right)3 + 17\,761$

$= 30\,000 + 4\,267 + 17\,761$

$= 52\,028$ W or $P = 52.0$ kW

6/146 $\Delta T_{translational} = \frac{1}{2}mv^2 - 0 = \frac{1}{2}(10000)(\frac{72}{3.6})^2 - 0$

$= 2(10^6)$ J

$\Delta T_{rotation} = \frac{1}{2}I(\omega_2^2 - \omega_1^2)$

$= \frac{1}{2}(1500)(0.5)^2(\omega_2^2 - [\frac{4000\times 2\pi}{60}]^2)$

$= 187.5\omega_2^2 - 32.90\times 10^6$ J

$\Delta E = 0.1(187.5\omega_2^2 - 32.90\times 10^6) = 18.75\omega_2^2 - 3.29\times 10^6$ J

$\Delta V_g = mgh = 10000(9.81)(20) = 1.96\times 10^6$ J

$\Delta E = \Delta T + \Delta V_g$;

$18.75\omega_2^2 - 3.29\times 10^6 = 2\times 10^6 + 187.5\omega_2^2 - 32.90\times 10^6 + 1.96\times 10^6$

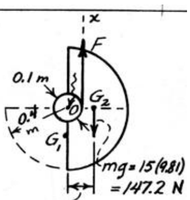
$168.75\omega_2^2 = 25.65\times 10^6$, $\omega_2^2 = 152\,000\ (rad/s)^2$

$\omega_2 = 390$ rad/s or $N = \frac{390\times 60}{2\pi} = 3720$ rev/min

6/147 For equil. $\Sigma M_O = 0$,

$0.05F_o - 147.2(0.1698) = 0$, $F_o = 500$ N

$F_o = 2k\delta$, where δ = initial spring stretch in equil. position. $\delta = \dfrac{500}{2 \times 2.6 \times 10^3} = 0.0961$ m



$U' = \Delta T + \Delta V_e + \Delta V_g$ where $U' = 0$

$\Delta T = \frac{1}{2}I_o\omega^2 - 0 = \frac{1}{2}(\frac{1}{2} \times 15 \times 0.4^2)\omega^2 = 0.6\omega^2$ $\quad \left(\bar{r} = \dfrac{4 \times 0.4}{3\pi} = 0.1698 \text{ m}\right)$

$\Delta V_e = 2(\frac{1}{2}k\Delta[x^2]) = 2.6 \times 10^3(0.0961^2 - [0.0961 + 0.05\pi/2]^2)$

$= 2.6 \times 10^3(0.0961^2 - 0.1746^2) = -55.3$ J

$\Delta V_g = mg\Delta h = mg\bar{r} = 15 \times 9.81 \times 0.1698 = 25.0$ J

Thus $0 = 0.6\omega^2 - 55.3 + 25.0$, $\omega^2 = 50.5\,(\text{rad/s})^2$,

$\underline{\omega = 7.11 \text{ rad/s}}$

6/148 Each spring stretches 4 ft.

so $\Delta V_e = 2(\frac{1}{2}kx^2) = 2(\frac{1}{2}50[4]^2) = 800$ ft-lb

$\Delta V_g = -200(9-4) = -1000$ ft-lb

$U' = \Delta T + \Delta V_g + \Delta V_e$: $\quad 0 = \frac{1}{2}\frac{200}{32.2}v^2 - 1000 + 800$

$v^2 = 64.4$, $\underline{v = 8.02 \text{ ft/sec}}$

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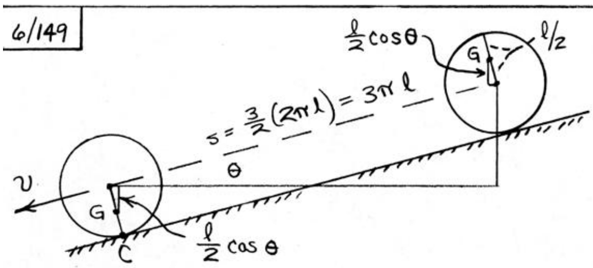

Mass center drops $h = 2(\frac{l}{2}\cos\theta) + (3\pi l)\sin\theta$

$= l(\cos\theta + 3\pi\sin\theta)$

$U' = \Delta T + \Delta V_g$: $U' = 0$

$T = \frac{1}{2}I_C\omega^2 = \frac{1}{2}(\frac{1}{3}ml^2)(\frac{v}{l})^2 = \frac{1}{6}mv^2$

$\Delta V_g = -mgh = -mgl(\cos\theta + 3\pi\sin\theta)$

So $0 = \frac{1}{6}mv^2 - mgl(\cos\theta + 3\pi\sin\theta)$

$\underline{v = \sqrt{6gl(\cos\theta + 3\pi\sin\theta)}}$

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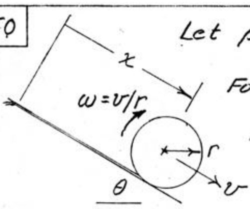


Let ρ = mass of paper per unit length

For general position $m = \rho(L-x)$

$\Delta T = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2$

$= \frac{1}{2}\frac{1}{2}mr^2\omega^2 + \frac{1}{2}mv^2$

$= \frac{3}{4}mv^2 = \frac{3}{4}\rho(L-x)v^2$

$\Delta V_g = -\rho g(L-x)x\sin\theta - \rho gx\frac{x}{2}\sin\theta$

$= -\rho gx(L - \frac{x}{2})\sin\theta$

$U' = 0 = \Delta T + \Delta V_g$; $\quad 0 = \frac{3}{4}\rho(L-x)v^2 - \rho gx(L - \frac{x}{2})\sin\theta$

$v^2 = \dfrac{4}{3}\dfrac{gx(L - x/2)\sin\theta}{L-x}$, $\quad \underline{v = 2\sqrt{\dfrac{gx}{3}\dfrac{L - x/2}{L-x}\sin\theta}}$

As $x \to L$, $v \to \infty$ so that the loss of potential energy $-\rho gL\sin\theta/2$ is concentrated in the kinetic energy of the last bit of moving paper. Abrupt termination of motion causes abrupt energy loss at the end.

6/151 Let x = distance moved by center O in m.

$\theta = \tan^{-1}\frac{1}{5} = 11.31°$, $\sin\theta = 0.1961$

$\Delta V_g = mg\Delta h = mgx\sin\theta = 10(9.81)x(0.1961) = 19.24x$

$\Delta V_e = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(600)\left[\left(0.225 - \frac{275}{200}x\right)^2 - (0.225)^2\right]$

$567.2x^2 - 185.6x$

$\Delta T = \frac{1}{2}mv^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(10)v^2 + \frac{1}{2}(10)(0.125)^2\left(\frac{v}{0.2}\right)^2$

$= 6.95v^2$. For system, $U' = \Delta T + \Delta V_g + \Delta V_e$:

$0 = 6.95v^2 + 19.24x + 567.2x^2 - 185.6x$

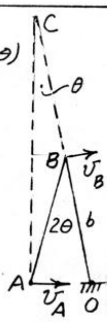
$v^2 = 23.93x - 81.57x^2$

Set $\dfrac{dv^2}{dx} = 0$ to get $x = 0.1467$ m for v_{max}

$v_{max}^2 = 23.93(0.1467) - 81.57(0.1467)^2$, $\underline{v_{max} = 1.325 \frac{\text{m}}{\text{s}}}$

6/152 $U' = M\theta$

$\Delta V_g = 2mg(\frac{b}{2} - \frac{b}{2}\cos\theta) = mgb(1-\cos\theta)$



Bar BO is rotating about O so

$\Delta T_{BO} = \frac{1}{2}I_O\omega^2 - 0 = \frac{1}{2}\frac{1}{3}mb^2\left(\frac{v_B}{b}\right)^2$

But in the limit as $\theta \to 0$, $v_B = \frac{1}{2}v_A$

so $\Delta T_{BO} = \frac{1}{6}m\frac{v_A^2}{4} = \frac{1}{24}mv_A^2$

Also AB is rotating about C so

$\Delta T_{AB} = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\left[\frac{1}{12}mb^2 + m\left(\frac{3b}{2}\right)^2\right]\left(\frac{v_A}{2b}\right)^2$

$= \frac{7}{24}mv_A^2$

$U' = \Delta T + \Delta V_g$; $\quad M\theta = \frac{7}{24}mv_A^2 + \frac{1}{24}mv_A^2 + mgb(1-\cos\theta)$

$\underline{v_A = \sqrt{3}\sqrt{\dfrac{M\theta}{m} - gb(1-\cos\theta)}}$

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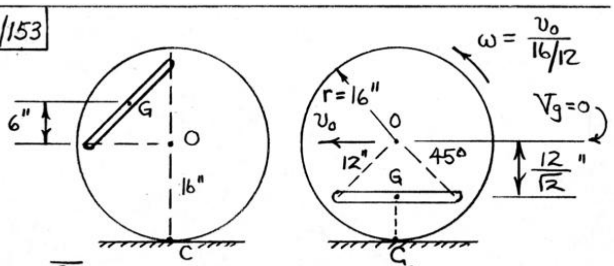

$v_G = \dfrac{\overline{CG}}{r}v_0$, $\quad I_G = \frac{1}{12}\frac{20}{32.2}\left(\frac{12\sqrt{2}}{12}\right)^2 = 0.1035$ lb-ft-sec²

$U_{1-2}' = 0 = \Delta T + \Delta V_g \quad (1)$

$\Delta T_{disk} = \frac{1}{2}\frac{100}{32.2}v_0^2 + \frac{1}{2}\left[\frac{1}{2}\frac{100}{32.2}\left(\frac{16}{12}\right)^2\right]\left(\frac{v_0}{16/12}\right)^2 - 0$

$= 2.33v_0^2$

$\Delta T_{bar} = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}\frac{20}{32.2}\left[\dfrac{16 - \frac{12}{\sqrt{2}}}{16}v_0\right]^2$

$+ \frac{1}{2}(0.1035)\left(\frac{v_0}{16/12}\right)^2 - 0 = 0.0976v_0^2$

$\Delta V_g = -mgh = -20\left(\dfrac{6 + 12/\sqrt{2}}{12}\right) = -24.1$ ft-lb

Eq. (1): $0 = (2.33 + 0.0976)v_0^2 - 24.1$

$\underline{v_0 = 3.15 \text{ ft/sec}}$

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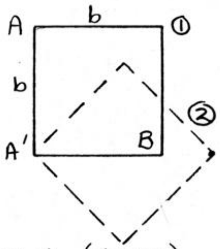
$\bar{I} = 4\left[\frac{1}{12}\frac{m}{4}b^2 + \frac{m}{4}\left(\frac{b}{2}\right)^2\right]$

$= \frac{1}{3}mb^2$

$I_B = \frac{1}{3}mb^2 + m\left(\frac{b\sqrt{2}}{2}\right)^2$

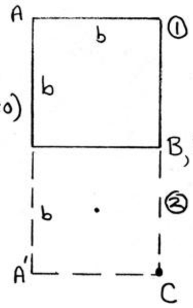
$= \frac{5}{6}mb^2$



(a) A has dropped distance b $(v_{B'} = 0)$

$T_1 + U_{1-2} = T_2 : \quad 0 + \frac{mgb}{2} = \frac{1}{2}\left[\frac{5}{6}mb^2\right]\omega^2$

$\omega = \sqrt{\frac{6g}{5b}}$, $v_A = b\omega\sqrt{2} = \underline{\sqrt{\frac{12}{5}gb}}$



(b) A has dropped distance $2b$ $(v_c = 0)$

$T_1 + U_{1-2} = T_2$:

$0 + mgb = \frac{1}{2}\left[\frac{5}{6}mb^2\right]\omega^2$

$\omega = \sqrt{\frac{12g}{5b}}$, $v_A = b\omega = \underline{\sqrt{\frac{12}{5}gb}}$

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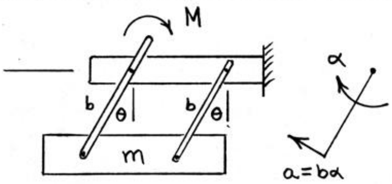

$dU' = dT + dV_g$

$dU' = M\,d\theta$

$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv = m\underline{a}\cdot d\underline{s}$

$= mb\alpha(b\,d\theta) = mb^2\alpha\,d\theta$

$dV_g = d(-mgb\cos\theta) = mgb\sin\theta\,d\theta$

Thus $M\,d\theta = mb^2\alpha\,d\theta + mgb\sin\theta\,d\theta$

$\underline{\alpha = \frac{M}{mb^2} - \frac{g}{b}\sin\theta}$

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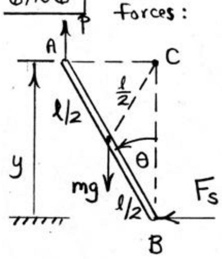
Active forces :



$I_C = \frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}ml^2$

$dU' = dT + dV$

Due to the equilibrium condition, The work due to the weight and the spring add to zero,

So that $dU' = P\,dy$

$= P\,d(l\cos\theta) = -Pl\sin\theta\,d\theta$

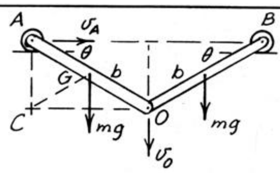
$dT = d\left(\frac{1}{2}I_C\omega^2\right) = I_C\omega\,d\omega = I_C\alpha\,d\theta$

So $-Pl\sin\theta\,d\theta = I_C\alpha\,d\theta = \frac{1}{3}ml^2\alpha\,d\theta$

$\underline{\alpha = -\frac{3P\sin\theta}{ml}}$ (minus sign indicates α is CW)

6/157 $dU = dT$

C = instantaneous center of zero velocity for AO



$dU = 2mg\,d\left(\frac{b}{2}\sin\theta\right)$

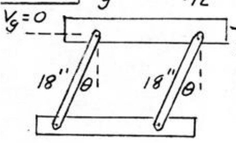
$= mgb\cos\theta\,d\theta$

$dT = 2d\left(\frac{1}{2}I_C\omega^2\right) = 2I_C\omega\,d\omega = \frac{2}{3}mb^2\alpha\,d\theta$

So $mgb\cos\theta\,d\theta = \frac{2}{3}mb^2\ddot{\theta}\,d\theta$ where $\alpha \equiv \frac{d^2\theta}{dt^2}$

$\underline{\alpha = \ddot{\theta} = \frac{3g\cos\theta}{2b}}$

6/158 $V_g = -2(8)\frac{9}{12}\cos\theta - 12\frac{18}{12}\cos\theta = -30\cos\theta$ ft-lb



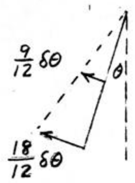
$\delta V_g = 30\sin\theta\,\delta\theta$

$\delta T = \Sigma m\bar{a}\,\delta s$

$= 2\frac{8}{32.2}4\left(-\frac{9}{12}\delta\theta\cos\theta\right)$

$+ \frac{12}{32.2}4\left(-\frac{18}{12}\delta\theta\cos\theta\right)$

$= -3.73\cos\theta\,\delta\theta$

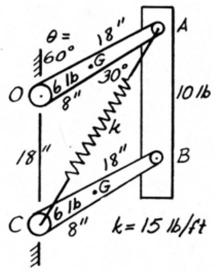


$\delta T + \delta V_g = 0$; $-3.73\cos\theta\,\delta\theta + 30\sin\theta\,\delta\theta = 0$

$\tan\theta = \frac{3.73}{30} = 0.1242$

$\underline{\theta = 7.08°}$

6/159 $dU' = 0 = dT + dV_e + dV_g$



$dV_g = 2(6)d(8\cos\theta) + 10d(18\cos\theta)$

$= 276\,d(\cos\theta) = -276\sin\theta\,d\theta$ in.-lb

$dT_{bar} = d\left(\frac{1}{2}mv^2\right) = mv\,dv = ma_t\,ds$

$= \frac{10}{32.2\times 12}(18\alpha)18\,d\theta = 8.39\,\alpha\,d\theta$

where $a_t = r\alpha$, $ds = r\,d\theta$

$dT_{links} = 2d\left(\frac{1}{2}I_O\omega^2\right) = 2I_O\omega\,d\omega = 2I_O\alpha\,d\theta$

$= 2\left(\frac{6}{32.2\times 12}10^2\right)\alpha\,d\theta = 3.11\,\alpha\,d\theta$

$\overline{CA} = 2(18)\cos 30° = 31.2$ in.; stretch $x = 2(18)\cos\frac{\theta}{2} - 18$,

$dx = 36\left(-\sin\frac{\theta}{2}\frac{d\theta}{2}\right)$

$dV_e = d\left(\frac{1}{2}kx^2\right) = kx\,dx = -\frac{15}{12}18\left(2\cos\frac{\theta}{2} - 1\right)36\sin\frac{\theta}{2}\frac{d\theta}{2}$

$= -148.2\,d\theta$ in.-lb

Thus $0 = (8.39 + 3.11)\alpha\,d\theta - 148.2\,d\theta - 276\sin 60°\,d\theta$

$\underline{\alpha = 33.7 \text{ rad/sec}^2}$

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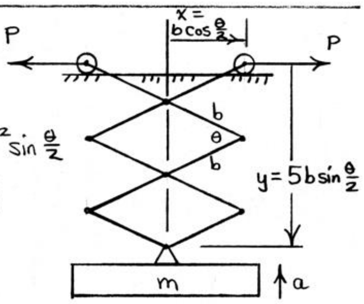


$\dot{y} = \frac{5}{2} b\dot{\theta}\cos\frac{\theta}{2}$

$\ddot{y} = \frac{5}{2} b\ddot{\theta}\cos\frac{\theta}{2} - \frac{5}{4} b\dot{\theta}^2 \sin\frac{\theta}{2}$

For $\dot{\theta} = 0$ & $a = -\ddot{y}$

$a = -\frac{5}{2} b\ddot{\theta}\cos\frac{\theta}{2}$

$dU' = dT + dV_g$

$dU' = +2P\,d\left(b\cos\frac{\theta}{2}\right) = -Pb\sin\frac{\theta}{2}\,d\theta$

$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv = ma\,(-dy)$

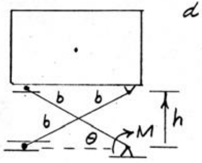
$= -ma\left(\frac{5}{2}b\cos\frac{\theta}{2}\right)d\theta$

$dV_g = d(-mgy) = -mg\frac{5}{2}b\cos\frac{\theta}{2}\,d\theta$

Thus $-Pb\sin\frac{\theta}{2}\,d\theta = -\frac{5}{2}mab\cos\frac{\theta}{2}d\theta - \frac{5}{2}mgb\cos\frac{\theta}{2}d\theta$

$$\underline{a = \frac{2P}{5m}\tan\frac{\theta}{2} - g}$$

6/161 $dU' = dT + dV_g$; $dU' = M\,d\theta$



$dT = ma\,dh = ma\,d(2b\sin\theta)$

$= 2mba\cos\theta\,d\theta$

$dV_g = mg\,dh = 2mbg\cos\theta\,d\theta$

Thus $M\,d\theta = 2mb\cos\theta\,(a+g)\,d\theta$

$a + g = \frac{M}{2mb\cos\theta}$

But $2b\sin\theta = h$

so $\cos\theta = \sqrt{4b^2 - h^2}/2b$ so $\underline{a = \frac{M}{2mb\sqrt{1-(h/2b)^2}} - g}$

$= \sqrt{1-(h/2b)^2}$

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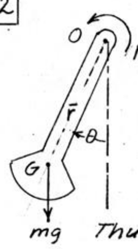

For a virtual displacement $\delta\theta$ from the steady-state configuration,

$\delta U = \delta T$

$\delta U = -M\delta\theta + mg\,\delta(\bar{r}\cos\theta)$

$= -M\delta\theta - mg\bar{r}\sin\theta\,\delta\theta$

$\delta T = m\underline{a}\cdot\delta\underline{s} = ma(-\bar{r}\delta\theta\cos\theta)$

Thus $-K\theta\delta\theta - mg\bar{r}\sin\theta\,\delta\theta = -ma\bar{r}\cos\theta\,\delta\theta$

$$\underline{K = \frac{m\bar{r}}{\theta}(a\cos\theta - g\sin\theta)}$$

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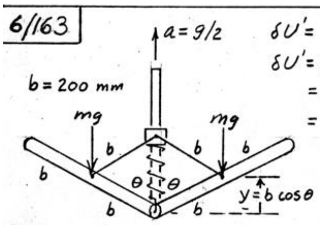


$\delta U' = \delta T + \delta V_e$

$\delta U' = 2mg(-\delta y)$

$= 2(2 \times 9.81)(-\delta[b\cos\theta])$

$= -39.24(0.200)(-\sin\theta)\delta\theta$

$= 7.85\sin\theta\,\delta\theta$ J

$\delta T = \Sigma m\underline{\bar{a}}\cdot\delta\underline{s} = 2(2)\frac{9.81}{2}\delta y = 19.62\,\delta(b\cos\theta)$

$= -19.62(0.200)\sin\theta\,\delta\theta$

$= -3.92\sin\theta\,\delta\theta$

$\delta V_e = kx\,\delta x = 130\,(2b - 2b\cos\theta)\,\delta(2b - 2b\cos\theta)$

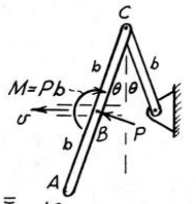
$= 520\,b^2(1-\cos\theta)\sin\theta\,\delta\theta$

Thus $7.85\sin\theta\,\delta\theta = -3.92\sin\theta\,\delta\theta + 520b^2(1-\cos\theta)\sin\theta\,\delta\theta$

$[(7.85 + 3.92) - 520(0.200)^2(1-\cos\theta)]\sin\theta\,\delta\theta = 0$

$1 - \cos\theta = \frac{11.77}{520\,(0.200)^2}$, $\cos\theta = 1 - 0.5660 = 0.4340$, $\underline{\theta = 64.3°}$

6/164 Replace P by force P at B and couple $M = Pb$



$dU = dT$

$dU = P\cos\theta\,d(2b\sin\theta) + Pb\,d\theta$

$= Pb(2\cos^2\theta + 1)d\theta$

$dT_{AC} = d\left(\frac{1}{2}2mv^2 + \frac{1}{2}\bar{I}\omega^2\right)$

$= 2mv\,dv + \bar{I}\omega\,d\omega = 2ma\,dx + \bar{I}\alpha\,d\theta$

where $x = 2b\sin\theta$, $v = 2b\dot{\theta}\cos\theta$, $a = 2b(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)$

$= 2b\ddot{\theta}\cos\theta$ since $\dot{\theta} = 0$

So $dT_{AC} = 2m(2b\ddot{\theta}\cos\theta)d(2b\sin\theta) + \frac{1}{12}(2m)(2b)^2\ddot{\theta}\,d\theta$

$= 2mb^2(4\cos^2\theta + \frac{1}{3})\ddot{\theta}\,d\theta$

$dT_{OC} = d\left(\frac{1}{2}I_o\omega^2\right) = I_o\omega\,d\omega = I_o\alpha\,d\theta = \frac{1}{3}mb^2\ddot{\theta}\,d\theta$

So $dT = 2mb^2(4\cos^2\theta + \frac{1}{3})\ddot{\theta}\,d\theta + \frac{1}{3}mb^2\ddot{\theta}\,d\theta$

$= mb^2(8\cos^2\theta + 1)\ddot{\theta}\,d\theta$

$Pb(2\cos^2\theta + 1)d\theta = mb^2(8\cos^2\theta + 1)\ddot{\theta}\,d\theta$,

$$\ddot{\theta} = \alpha = \underline{\frac{P(2\cos^2\theta + 1)}{mb(8\cos^2\theta + 1)}}$$

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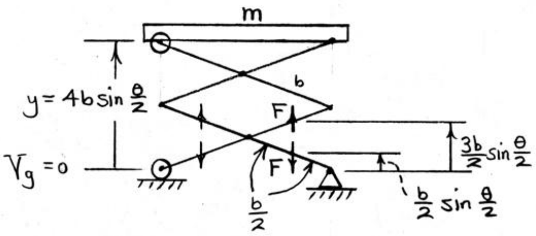

$dU' = dT + dV_g$

$dU' = 2F\,d\left(\frac{3b}{2}\sin\frac{\theta}{2}\right) - 2F\,d\left(\frac{b}{2}\sin\frac{\theta}{2}\right)$

$= 2F\,d\left(b\sin\frac{\theta}{2}\right) = Fb\cos\frac{\theta}{2}\,d\theta$

$dV_g = d\left(mg\,4b\sin\frac{\theta}{2}\right) = 2mgb\cos\frac{\theta}{2}\,d\theta$

$dT = d\left(\frac{1}{2}mv^2\right) = mv\,dv = ma\,dy$

$= ma\left(2b\cos\frac{\theta}{2}\,d\theta\right)$

Thus $Fb\cos\frac{\theta}{2}\,d\theta = 2mgb\cos\frac{\theta}{2}\,d\theta + 2mab\cos\frac{\theta}{2}\,d\theta$

$\underline{a = \frac{F}{2m} - g}$ } Both b and θ cancel so a is independent of both b and θ.

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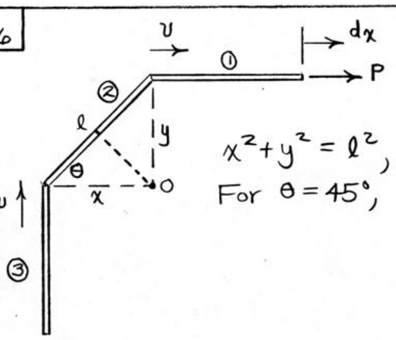

$x^2+y^2=l^2, \; x\,dx+y\,dy=0$

For $\theta=45°$, $x=y$, $dx=-dy$

$dU' = dT + dV_g \;;\quad dU' = P\,dx$

$dT = d(T_1+T_2+T_3) = d\left(\frac{1}{2}mv^2+\frac{1}{2}I_O\omega^2+\frac{1}{2}mv^2\right)$

$= 2mv\,dv + I_O\omega\,d\omega = 2ma\,dx + I_O\alpha\,|d\theta|$

$= 2ma\,dx + \frac{1}{3}ml^2\frac{a}{l/\sqrt{2}}\frac{dx}{l/\sqrt{2}} = \frac{8}{3}ma\,dx$

$dV_g = d(V_{g_1}+V_{g_2}+V_{g_3}) = 0 + mg\frac{dx}{2} + mg\,dx$

$= \frac{3}{2}mg\,dx$

Thus $P\,dx = \frac{8}{3}ma\,dx + \frac{3}{2}mg\,dx$

$\underline{a = \frac{3}{8}\left(\frac{P}{m}-\frac{3g}{2}\right)}$

6/167 Radius to each weight is $r = 0.25+1.5\sin\theta$ in.

$\delta T = 2(mr\omega^2)(-\delta r) = 2\frac{12}{16(32.2)}\frac{0.25+1.5\sin\theta}{12}\omega^2(-\delta r)$ ft-lb

But $\delta r = 1.5\cos\theta\,\delta\theta$ in.

$\not{c}$ $2(1.5)-2(1.5)\cos\theta = 0.625\sin\beta$, $\beta=15°$

so $\cos\theta = \frac{3-0.625\sin 15°}{3} = 0.9461$, $\theta = 18.90°$

so $\delta T = \frac{0.25+1.5\sin 18.90°}{8(32.2)}\omega^2\left(-\frac{1.5\cos 18.90°}{12}\right)\delta\theta$

$= -0.3378(10^{-3})\omega^2\delta\theta$ ft-lb

$\delta V_e = kx\delta x = 5(12)\frac{2(1.5)}{12}(1-\cos\theta)\,\delta\left\{\frac{2(1.5)}{12}(1-\cos\theta)\right\}$

$= 3.75(1-\cos\theta)\sin\theta\,\delta\theta =$

$= 3.75(1-\cos 18.90°)\sin 18.9°\,\delta\theta = 65.50(10^{-3})\,\delta\theta$

$\delta U = \delta T + \delta V_e = 0; \; -0.3378(10^{-3})\omega^2\delta\theta + 65.50(10^{-3})\delta\theta = 0$

$\omega^2 = \frac{65.50}{0.3378} = 193.9 \;(\text{rad/sec})^2$

$\omega = 13.92$ rad/sec, $N = \frac{13.92(60)}{2\pi} = \underline{133.0\,\frac{\text{rev}}{\text{min}}}$

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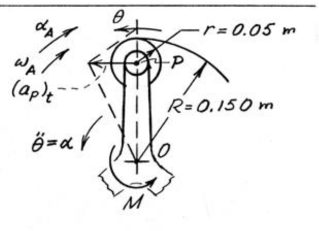


$ds_P = (R-r)d\theta, \; d\theta_A = ds_P/r$

$= \left(\frac{R}{r}-1\right)d\theta$

$v_P = (R-r)\dot{\theta}, \; \omega_A = \frac{v_P}{r} = \left(\frac{R}{r}-1\right)\dot{\theta}$

$(a_P)_t = (R-r)\alpha, \; \alpha_A = \frac{(a_P)_t}{r} = \left(\frac{R}{r}-1\right)\alpha$

$dU = dT_{spider} + dT_{gears}$

$dU = M\,d\theta$

$dT_{spider} = d\left(\frac{1}{2}I_O\omega^2\right) = I_O\omega\,d\omega = I_O\alpha\,d\theta$

$dT_{gears} = 3\left\{d\left(\frac{1}{2}I_A\omega_A^2\right) + d\left(\frac{1}{2}m_A v_P^2\right)\right\} = 3\left\{I_A\alpha_A\,d\theta_A + m_A(a_P)_t\,ds_P\right\}$

$= 3\left\{I_A\left(\frac{R}{r}-1\right)^2\alpha\,d\theta + m_A(R-r)^2\alpha\,d\theta\right\}$

$= 3(R-r)^2\left(\frac{I_A}{r^2}+m_A\right)\alpha\,d\theta$

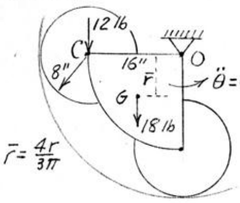
So $M\,d\theta = \left[I_O + 3(R-r)^2\left(\frac{I_A}{r^2}+m_A\right)\right]\alpha\,d\theta$

$5 = \left[1.2\times0.06^2 + 3(0.150-0.050)^2\left(\frac{0.8\times0.030^2}{0.050^2}+0.8\right)\right]\alpha$

$= [0.00432 + 0.03\times1.088]\,\alpha,$

$\underline{\alpha = 135.3 \text{ rad/s}^2}$

6/169 Each wheel: $dT = m_w\bar{a}_w\,ds_w + \bar{I}_w\alpha_w\,d\theta_w$



$\bar{r} = \frac{4r}{3\pi}$

$= \frac{12}{32.2}\frac{16}{12}\alpha\frac{16}{12}d\theta$

$+ \frac{1}{2}\frac{12}{32.2}\left(\frac{8}{12}\right)^2(2\alpha)(2\,d\theta)$

$= \frac{32}{32.2}\alpha\,d\theta$

where $d\theta_w = 2\,d\theta$

$\alpha_w = 2\alpha$

Sector: $dT = I_O\alpha\,d\theta = \frac{1}{2}\frac{18}{32.2}\left(\frac{16}{12}\right)^2\alpha\,d\theta = \frac{16}{32.2}\alpha\,d\theta$

Combined $dT = 2\left(\frac{32}{32.2}\alpha\,d\theta\right) + \frac{16}{32.2}\alpha\,d\theta = \frac{80}{32.2}\alpha\,d\theta$

$dU = 12\frac{16}{12}d\theta + 18\frac{4\times16}{3\pi\,12}d\theta = 16\left(1+\frac{2}{\pi}\right)d\theta = 26.19\,d\theta$

$dU = dT; \; 26.19\,d\theta = \frac{80}{32.2}\alpha\,d\theta, \; \alpha = \frac{26.19(32.2)}{80} = \underline{10.54\,\frac{\text{rad}}{\text{sec}^2}}$

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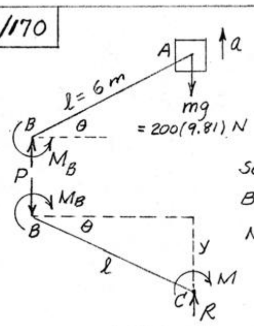


Upper arm; $dU = dT$

$P\,dy - mg\,2\,dy + M_B\,d\theta = d\left(\frac{1}{2}mv^2\right)$

$y = l\sin\theta, \; dy = l\cos\theta\,d\theta$

$d\left(\frac{1}{2}ml^2\right) = ma\,d(2y)$

$= 2ma\,l\cos\theta\,d\theta$

So $(P-2mg)l\cos\theta + M_B = 2mal\cos\theta$

But $P - mg = ma$ so

$M_B = mgl\cos\theta\left(\frac{a}{g}+1\right)$

$= 200(9.81)(6)(0.866)\left(\frac{1.2}{9.81}+1\right)$

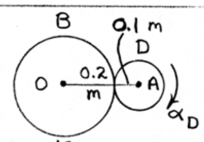
$= 11\,440$ N·m or $\underline{11.44 \text{ kN·m}}$

Lower arm; $\Sigma M = 0; \; M + M_B - Pl\cos\theta = 0$

$M = -mgl\cos\theta\left(\frac{a}{g}+1\right) + mg\left(\frac{a}{g}+1\right)l\cos\theta, \; \underline{M=0}$

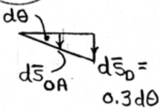
$M=0$ can be obtained by inspection since m is directly above C. Also, problem can be solved directly by F-m-a equations.

6/171 $dU' = dT + dV_g$



$dU' = \Sigma m_i a_i \cdot ds_i + \Sigma I_i\alpha_i\cdot d\theta_i$

$+ \Sigma m_i g\,dh_i$



Let $\begin{cases} \alpha = \text{angular acceleration of } OA \\ d\theta = \text{angular displacement of } OA \end{cases}$

Arm OA : $\bar{a} = \frac{0.3}{2}\alpha, \; d\bar{s} = \frac{0.3}{2}d\theta, \; dh = -\frac{0.3}{2}d\theta$

$\bar{I} = \frac{1}{12}(4)(0.3)^2 = 0.03$ kg·m^2

$dU'_{arm} = 4\left(\frac{0.3}{2}\alpha\right)\left(\frac{0.3}{2}d\theta\right) + 0.03\alpha\,d\theta - 4(9.81)\left(\frac{0.3}{2}d\theta\right)$

$= 0.12\alpha\,d\theta - 5.89\,d\theta$

Gear D : $\bar{a} = a_A = 0.3\alpha, \; d\bar{s}_D = 0.3\,d\theta, \; dh = -0.3\,d\theta$

$\alpha_D = 3\alpha, \; d\theta_D = 3\,d\theta$

$\bar{I} = m\bar{k}^2 = 5(0.064)^2 = 0.0205$ kg·m^2

$dU'_D = 5(0.3\alpha)(0.3\,d\theta) + 0.0205(3\alpha)(3\,d\theta)$

$-5(9.81)(0.3\,d\theta) = 0.634\alpha\,d\theta - 14.72\,d\theta$

For system : $dU' = dU'_{arm} + dU'_D = 0$

$0.12\alpha\,d\theta - 5.89\,d\theta + 0.634\alpha\,d\theta - 14.72\,d\theta = 0$

$\underline{\alpha = 27.3 \text{ rad/s}^2}$

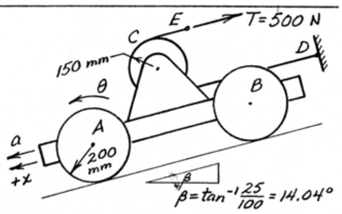
6/172 $dU'_{1-2} = dT + dV_g$

Let x = displacement of vehicle down slope

$\bar{I}_A = \bar{I}_B = mk^2 = 140(0.150)^2$

$= 3.15\ kg\cdot m^2$

$\bar{I}_C = 40(0.100)^2 = 0.4\ kg\cdot m^2$



$a = r\alpha$

$dU'_{1-2} = -500(2dx) = -1000dx$

$(dT_{wheels})_{rotation\ only} = 2d(\frac{1}{2}\bar{I}_A\omega^2) = 2\bar{I}_A\omega d\omega = 2\bar{I}_A\alpha d\theta = 2\bar{I}_A\frac{a}{r_A}\frac{dx}{r_A}$

$= 2\times 3.15\frac{a\,dx}{0.2^2} = 157.5\,a\,dx$

$(dT_{drum})_{rotation\ only} = d(\frac{1}{2}\bar{I}_C\omega_C^2)$

$= \bar{I}_C\omega_C d\omega_C = \bar{I}_C\alpha_C d\theta_C = \bar{I}_C\frac{a}{r_C}\frac{dx}{r_C} = 0.4\frac{a\,dx}{0.150^2} = 17.78\,a\,dx$

$dT_{vehicle\ translation} = d(\frac{1}{2}mv^2) = mv\,dv = ma\,dx = 520\,a\,dx$

$dV_g = -mg\,dh = -520(9.81)dx\sin 14.04° = -1237\,dx$

Thus $-1000dx = (157.5 + 17.78 + 520)a\,dx - 1237\,dx$,

$\underline{a = 0.341\ m/s^2}$

6/173 $\int_{t_1}^{t_2} M_O\,dt = H_{O_2} - H_{O_1}$

$\int_0^3 90\cos 15°(0.8)\,dt = 4(\frac{1}{3})(60)(1.2)^2\omega$

$\underline{\omega = 1.811\ rad/s}$

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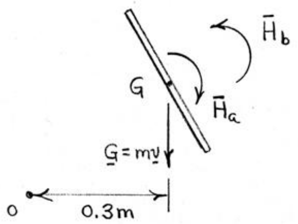


$\bar{H} = \bar{I}\omega = \frac{1}{12}ml^2\omega = \frac{1}{12}0.8(0.4)^2 10$

$= 0.1067\ kg\cdot m^2/s$

$G = mv = 0.8(2) = 1.6\ kg\cdot m/s$

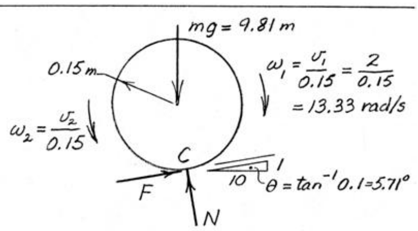
(a) $H_O = \bar{H}_a + Gr = 0.1067 + 1.6(0.3)$

$= \underline{0.587\ kg\cdot m^2/s}$

(b) $H_O = -\bar{H}_b + Gr = -0.1067 + 1.6(0.3)$

$= \underline{0.373\ kg\cdot m^2/s}$

6/175



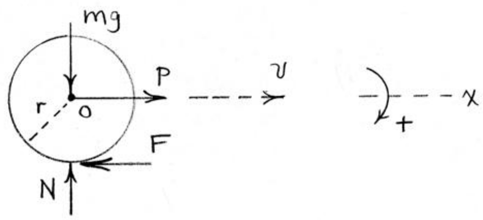
$\Sigma M_C = I_C\alpha$:

$\Sigma M_C t = I_C(\omega_2 - \omega_1)$

$9.81\cancel{m}\sin 5.71°(0.15)6 = \cancel{m}(0.090^2 + 0.150^2)(\frac{v_2}{0.15} - [-13.33])$

$\underline{v_2 = 2.31\ m/s}$

6/176



$\int\Sigma F_x\,dt = \Delta mv_x$: $(P - F)t = mv - 0$

$\int\Sigma M_O\,dt = \Delta I_O\omega$: $Frt = \frac{1}{2}mr^2(\frac{v}{r} - 0)$

Eliminate F & obtain $v = \underline{\frac{2Pt}{3m}}$

6/177 $\int_{t_1}^{t_2}\Sigma M_G\,dt = \bar{I}(\omega_2 - \cancel{\omega_1}^{\,0}) = m\bar{k}^2\omega$

$\int_0^3 10(1 - e^{-t})dt = 75(0.5)^2\omega$

$10[t + e^{-t}]\Big|_0^3 = 75(0.5)^2\omega$, $\underline{\omega = 1.093\ rad/s}$

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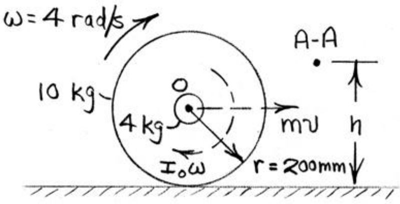


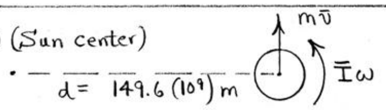
$v = r\omega = 0.2(4) = 0.8\ m/s$

$I_O = 10(0.180)^2 + \cancel{I_{shaft}}^{\,\sim 0} = 0.324\ kg\cdot m^2$

$\circlearrowleft H_{A\text{-}A} = I_O\omega - mvd$: $0.324(4) - (10+4)(0.8)(h - 0.2) = 0$

$h = 0.316\ m$ or $\underline{316\ mm}$

6/179



$\bar{H} = \bar{I}\omega = \frac{2}{5}mr^2(\frac{2\pi}{\tau})$

$= \frac{2}{5}(5.976\cdot 10^{24})(6.371\cdot 10^6)^2\ \frac{2\pi}{23.9344(3600)}$

$= 7.08(10^{33})\ kg\cdot m^2/s$

$\bar{v} = \sqrt{\frac{Gm_s}{d}} = \sqrt{\frac{6.673(10^{-11})(333{,}000)(5.976\cdot 10^{24})}{149.6(10^9)}}$

$= 29\,800\ m/s$

$m\bar{v}d = 5.976(10^{24})(29\,800)(149.6\cdot 10^9)$

$= 2.66(10^{40})\ kg\cdot m^2/s$

$\bar{H} = \bar{I}\omega + m\bar{v}d = \underline{2.66(10^{40})\ kg\cdot m^2/s}$

(The $\bar{I}\omega$ term is insignificant compared with the $m\bar{v}d$ term.)

6/180

System $\xrightarrow{+}$ $\int_0^{10} \Sigma F\,dt = \Delta G : 400(10) = (1200+800)[v-(-1.5)]$

$v = 0.5$ m/s (right)

Drum ↻+ $\int \Sigma M_O\,dt = \Delta H_O : 400(0.500)(10) = 800(0.480)^2[\omega-(-3)]$

$\omega = 7.85$ rad/s CW

The rotation of the drum does not affect the linear momentum of the system, so $v = 0.5$ m/s is independent of ω.

6/181

$M\,dt = d(I\omega) = I\,d\omega$

$M = -k\omega^2$ so $-k\omega^2 dt = I\,d\omega$

$-k\int_0^t dt = I\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega^2}$, $-kt = I\left(-\frac{1}{\omega}\right)_{\omega_0}^{\omega_0/2} = I\left(\frac{-1}{\omega_0/2}+\frac{1}{\omega_0}\right) = -I/\omega_0$

so $t = \frac{I}{\omega_0 k}$

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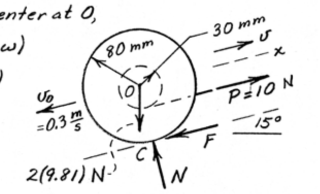
For no slipping & mass center at O,

$\Sigma M_C = I_C\alpha$ so $\int \Sigma M_C\,dt = \Delta H_C = \Delta(I_C\omega)$

$I_C = m(k_O^2+r^2) = 2(0.060^2+0.080^2)$

$= 0.02$ kg·m²



↻+ $\int_0^5 (10[0.080-0.030]-2(9.81)(0.080)\sin 15°)\,dt$

$= 0.02\left(\frac{v}{0.080}-\left[\frac{-0.3}{0.080}\right]\right)$

$0.469 = 0.25(v+0.3)$, $v = 1.575$ m/s up the incline

Alternative sol. if preferred:

↻+ $\int \Sigma M_G\,dt = \Delta H_G : \int_0^5 (F\times 0.080 - 10\times 0.030)dt = 2\times 0.060^2\left(\frac{v}{0.080}-\left[\frac{-0.3}{0.080}\right]\right)$

$0.080F(5) - 0.3(5) = 0.09(v+0.3)$ (1)

$\int \Sigma F_x\,dt = \Delta G_x : \int_0^5 (10-F-2(9.81)\sin 15°)dt = 2(v-[-0.3])$

$50 - 25.4 - 5F = 2v + 0.6$ (2)

Solve & get $v = 1.575$ m/s ($F = 4.17$ N,

$N = 2(9.81)\cos 15° = 18.95$ N

$(\mu_s)_{min} = 4.17/18.95 = 0.220$)

6/183

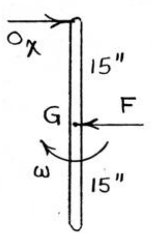
$H_{O_1} = H_{O_2}$: $mvb = (I_O + mb^2)\omega$

$\frac{2}{16}\frac{1}{32.2}(1500)\frac{15}{12} = \left[\frac{1}{3}\frac{20}{32.2}\left(\frac{30}{12}\right)^2 + \frac{2}{16}\frac{1}{32.2}\left(\frac{15}{12}\right)^2\right]\omega$

$\omega = 5.60$ rad/sec

6/184

$\int \Sigma M_G\,dt = \bar{H}_2 - \bar{H}_1$:



$O_x \frac{15}{12}(0.001) = \frac{1}{12}\frac{20}{32.2}\left(\frac{30}{12}\right)^2(\omega - 0)$,

where $\omega = 5.60$ rad/sec from Prob. 6/183.

$\Rightarrow O_x = 1449$ lb

6/185

↻+ $H_{O_1} = H_{O_2}$ for system

$mvh = (I_O + mh^2)\omega$

$\left(\frac{1/16}{32.2}\right)(1600)\left(\frac{43}{12}\right) = \left[\frac{55}{32.2}\left(\frac{37}{12}\right)^2 + \frac{1/16}{32.2}\left(\frac{43}{12}\right)^2\right]\omega$

$\omega = 0.684$ rad/sec

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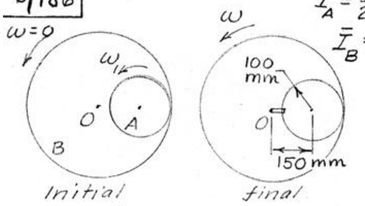

$\bar{I}_A = \frac{1}{2}mr^2 = \frac{1}{2}8(0.1)^2 = 0.04$ kg·m²

$\bar{I}_B = mk^2 = 60(0.2)^2 = 2.4$ kg·m²

Initial

$\bar{H}_A = (H_A)_O = \bar{I}_A\omega_1$

$= 0.04(80)$

$= 3.2$ kg·m²/s

Final $H_O = (I_{A_O} + \bar{I}_B)\omega = (0.04 + 8\times\overline{0.15}^2 + 2.4)\omega$

$= 2.62\omega$

$\Delta H_O = 0$; $3.2 = 2.62\omega$, $\omega = 1.221$ rad/s

(Note: Overbars refer to center of mass.)

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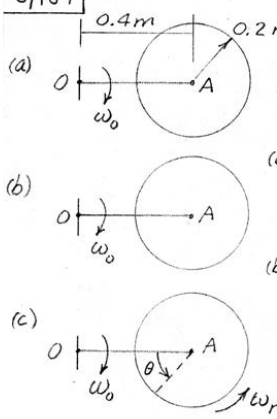


$\omega_0 = 4$ rad/s

ω = angular velocity of disk

$\bar{I} = I_A = \frac{1}{2}mr^2 = \frac{1}{2}25(0.2)^2$

$= \frac{1}{2}$ kg·m²

(a) $\omega = \omega_0$

$H_O = I_O\omega = \left(\frac{1}{2}+25[0.4]^2\right)4$

$= 18$ kg·m²/s

(b) $\omega = 0$

$H_O = m\bar{v}d = 25(0.4)(4)(0.4)$

$= 16$ kg·m²/s

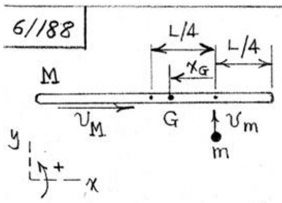
(c) $\omega = \omega_0 - \omega_r = 4 - 8 = -4$ rad/s

$H_O = \bar{I}\omega + m\bar{v}d$

$= \frac{1}{2}(-4) + 16$

$= 14$ kg·m²/s

6/188



G is system mass center.

$x_G = \frac{ML/4}{M+m}$

x mom. : $Mv_M = (M+m)v_x$, $v_x = \frac{Mv_M}{M+m}$

y mom : $mv_m = (M+m)v_y$, $v_y = \frac{mv_m}{M+m}$

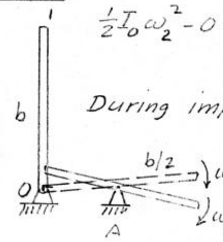
ang. mom$_G$: $mv_m\left(\frac{ML/4}{M+m}\right) = \left[\frac{1}{12}ML^2 + M\left(\frac{L}{4}-\frac{ML/4}{M+m}\right)^2 + m\left(\frac{ML/4}{M+m}\right)^2\right]\omega$

Solving, $\omega = \frac{12v_m}{L}\left(\frac{m}{4M+7m}\right)$ ↺

6/189 From 1 to 2, $\Delta T + \Delta V_g = 0$

$\frac{1}{2} I_O \omega_2^2 - 0 - mg\frac{b}{2} = 0$, $\frac{1}{3} m b^2 \omega_2^2 = mgb$

$\omega_2 = \sqrt{3g/b}$



During impact with A, $\Delta H_A = 0$, $H_{A_2} = H_{A_3}$

$H_{A_2} = \bar{I}\omega_2$

$H_{A_3} = \bar{I}\omega_3$

so $\omega_3 = \omega_2$

$= \sqrt{3g/b}$

6/190 Approximate the diver's body as a uniform slender bar in the first case and as a sphere in the second case. Conservation of angular momentum $H_1 = H_2$:

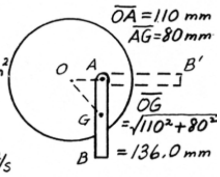
$\frac{1}{12} \cancel{m} l^2 N_1 = \frac{2}{5} \cancel{m} r^2 N_2$

$\frac{1}{12}(2)^2(0.3) = \frac{2}{5}\left(\frac{0.7}{2}\right)^2 N_2$

$N_2 = 2.04$ rev/s

6/191 $\Sigma M_O = 0 = \Delta H_O$ so $H_{O_1} = H_{O_2}$

Initial conditions:



$(I_O)_{\text{each rod}} = 0.84\left(\frac{1}{12} \times 0.160^2 + 0.136^2\right) = 0.01733$ kg·m²

$(I_O)_{\text{disk}} = 30(0.090)^2 = 0.243$ kg·m²

$\omega_1 = 600 \times 2\pi/60 = 62.8$ rad/s

$H_{O_1} = [4(0.01733) + 0.243]62.8 = 19.62$ kg·m²/s

Final conditions:

$(I_O)_{\text{each rod}} = 0.84\left(\frac{1}{12} \times 0.160^2 + [0.110 + 0.080]^2\right) = 0.0321$ kg·m²

$(I_O)_{\text{disk}} = 0.243$ kg·m²

$H_{O_2} = [4(0.0321) + 0.243]\,\omega_2 = 0.371\,\omega_2$

Thus $19.62 = 0.371\,\omega_2$, $\omega_2 = 52.8$ rad/s, $N = 504$ rev/min

Energy loss:

$T_1 = \Sigma \frac{1}{2} I_O \omega^2 = \frac{1}{2}(4 \times 0.01733 + 0.243)(62.8)^2 = 617$ J

$T_2 = \Sigma \frac{1}{2} I_O' \omega'^2 = \frac{1}{2}(4 \times 0.0321 + 0.243)(52.8)^2 = 518$ J

$|\Delta E| = T_1 - T_2 = 617 - 518 = 98.1$ J loss

Direction of rotation & sequence of rod release do not affect the results.

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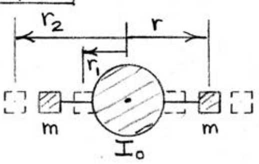

$H = I_o \omega_o + 2mr^2\omega_o$

$\dot{H} = 4mr\dot{r}\omega_o$

$r = r_1 + \frac{\Delta r}{\Delta t} t$

$= 1.2 + \frac{4.5 - 1.2}{120} t$

$= 1.2 + 0.02750\,t$

$\dot{r} = 0.02750$ m/s

$M = \dot{H}$, $2T(1.1) = 4(10)(1.2 + 0.0275t)(0.0275) \times (1.25)$

$T = 0.750 + 0.01719\,t$ N

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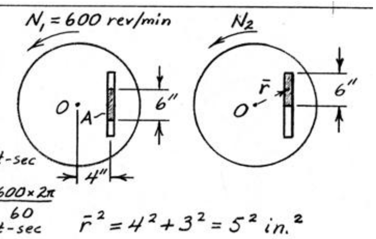

For system, $\Sigma M_O = 0$ so $\Delta H_O = 0$, $H_1 = H_2$

$(H_1)_{\text{disk}} = \frac{8}{32.2}\left(\frac{6}{12}\right)^2 \frac{600 \times 2\pi}{60}$ lb-ft-sec

$(H_1)_{\text{bar}} = \left[\frac{1}{12}\frac{2}{32.2}\left(\frac{6}{12}\right)^2 + \frac{2}{32.2}\left(\frac{4}{12}\right)^2\right]\frac{600 \times 2\pi}{60}$ lb-ft-sec

$\bar{r}^2 = 4^2 + 3^2 = 5^2$ in.²

$(H_2)_{\text{disk}} = \frac{8}{32.2}\left(\frac{6}{12}\right)^2 \frac{2\pi N_2}{60}$ lb-ft-sec, N_2 in rev/min

$(H_2)_{\text{bar}} = \left[\frac{1}{12}\frac{2}{32.2}\left(\frac{6}{12}\right)^2 + \frac{2}{32.2}\left(\frac{5}{12}\right)^2\right]\frac{2\pi N_2}{60}$ lb-ft-sec

Factor out $\frac{1}{32.2} \times \frac{1}{12^2} \times \frac{2\pi}{60}$ & get

$(8 \times 6^2 + \frac{2}{12} \times 6^2 + 2 \times 4^2)600 = (8 \times 6^2 + \frac{2}{12} \times 6^2 + 2 \times 5^2)N_2$,

$N_2 = 569$ rev/min

Friction forces in the slot are internal so have no effect on ΣM_O. Hence the final value of N_2, as well as the loss of energy, is unaffected.

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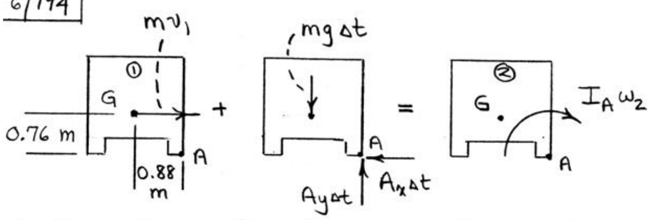
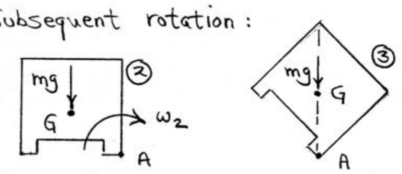


$+\circlearrowleft$ $H_{A_1} = H_{A_2}$ (ignoring nonimpulsive $mg\Delta t$)

$2300v_1(0.76) = [900 + 2300(0.76^2 + 0.88^2)]\,\omega_2$

$\omega_2 = 0.436\,v_1$

Subsequent rotation:



$T_2 + V_{2-3} = T_3$:

$\frac{1}{2} I_A \omega_2^2 - mgh = 0$

$\frac{1}{2}[900 + 2300(0.76^2 + 0.88^2)][0.436v_1]^2 - 2300(9.81)[\sqrt{0.76^2 + 0.88^2} - 0.76] = 0$

$v_1 = 4.88$ m/s (not very fast!)

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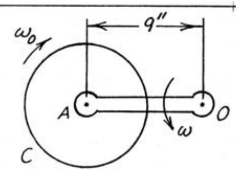
Let ω_o = true angular velocity of disk & armature $= \omega_{rel} - \omega$



$\Sigma M_O = 0$ so $\Delta H_O = 0$;

$H_{O_{\text{initial}}} = 0$ so $H_{O_{\text{final}}} = 0$

$\omega_{rel} = \frac{300 \times 2\pi}{60} = 31.4$ rad/sec

OA: $H_O = I_O\omega = \frac{10}{32.2}\left(\frac{7}{12}\right)^2\omega = 0.1057\,\omega$ lb-ft-sec CCW

C: $H_O = I_{z'}\omega_o - mr\omega = \frac{15}{32.2}\left(\frac{4}{12}\right)^2[31.4 - \omega] - \frac{15}{32.2}\left(\frac{9}{12}\right)^2\omega$

$= 1.626 - 0.314\,\omega$ lb-ft-sec CW

$0.1057\,\omega = 1.626 - 0.314\,\omega$, $\omega = 3.88$ rad/sec

$N = \frac{3.88 \times 60}{2\pi} = 37.0$ rev/min

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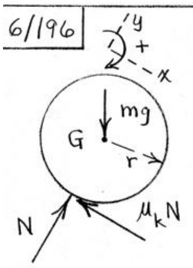


$$\int_0^t \Sigma F_y\,dt = m(v_y - v_{y_0}) = 0 \Rightarrow N = mg\cos\theta$$

$$\int_0^t \Sigma F_x\,dt = m(v_x - v_{x_0}):$$

$$(-\mu_k mg\cos\theta + mg\sin\theta)t = m(v - v_0) \quad (1)$$

$$\int_0^t \Sigma M_G\,dt = \bar{I}(\omega - \omega_0):$$

$$(\mu_k mg\cos\theta\, r)t = \frac{2}{5}mr^2\omega \quad (2)$$

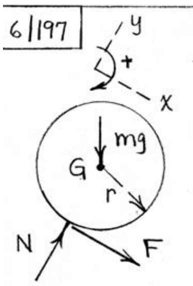
We desire the time t when $v = r\omega$ (3)

Solution of Eqs. (1)-(3):
$$t = \frac{2v_0}{g(7\mu_k\cos\theta - 2\sin\theta)}, \quad v = \frac{5v_0\mu_k}{7\mu_k - 2\tan\theta}, \quad \omega = \frac{5v_0\mu_k/r}{7\mu_k - 2\tan\theta}$$

For slipping to cease, $7\mu_k\cos\theta > 2\sin\theta$

or $\mu_k > \frac{2}{7}\tan\theta$

6/197



$$\int_0^t \Sigma F_y\,dt = m(v_y - v_{y_0}) = 0 \Rightarrow N = mg\cos\theta$$

$$\int_0^t \Sigma F_x\,dt = m(v_x - v_{x_0}):$$

$$(+\mu_k mg\cos\theta + mg\sin\theta)t = mv \quad (1)$$

$$\int_0^t \Sigma M_G\,dt = \bar{I}(\omega - \omega_0):$$

$$(-\mu_k mgr\cos\theta)t = \frac{2}{5}mr^2(\omega - \omega_0) \quad (2)$$

Slipping ceases when $v = r\omega$ (3)

Solution of Eqs. (1)-(3): $t = \dfrac{2r\omega_0}{g(2\sin\theta + 7\mu_k\cos\theta)}$

Note that the effect of the ramp is to decrease t.

$$v = \frac{2r\omega_0(\sin\theta + \mu_k\cos\theta)}{(2\sin\theta + 7\mu_k\cos\theta)}$$

$$\omega = \frac{2\omega_0(\sin\theta + \mu_k\cos\theta)}{(2\sin\theta + 7\mu_k\cos\theta)}$$

6/198 Conservation of angular momentum about the vertical spin axis of the platform:

$$H_1 = H_2$$

$$[10(0.3)^2]\left(250\,\frac{2\pi}{60}\right) = \left[I + \frac{1}{2}(10)(0.3)^2 + 10(0.6)^2\right] \times \left(30\,\frac{2\pi}{60}\right)$$

$$I = 3.45 \text{ kg}\cdot\text{m}^2$$

6/199 Conservation of angular momentum about the vertical spin axis of the platform:

$$H_1 = H_2$$

$$[10(0.3)^2][250] = [3.45 + 10(0.6)^2]N - 10(0.3)^2[250]$$

$$N = 63.8 \text{ rev/min}$$

6/200 Bar B: $U'_{1-2} = 0 = \Delta T + \Delta V_g$

$\Delta V_g = -mgh = -8(9.81)(0.180) = -14.13$ J

$\Delta T = \frac{1}{2}I\omega_B^2 = \frac{1}{2}(8)(0.220)^2\omega_B^2 = 0.1936\,\omega_B^2$

So $0 = 0.1936\,\omega_B^2 - 14.13$, $\omega_B = 8.54$ rad/s

Prior to impact: $H_O = I\omega_B = 8(0.220)^2(8.54) = 3.31\,\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$

For system after impact:

$H_O = I_{tot}\,\omega = [2\cdot 20(0.3)^2 + 8(0.220)^2]\,\omega = 3.99\omega$

$\Delta H_O = 0$: $3.99\omega - 3.31 = 0$, $\omega = 0.830$ rad/s

After impact: $U'_{1-2} = 0 = \Delta T + \Delta V_g$

$\Delta V_g = mgh = 2\cdot 20(9.81)(0.25)(1-\cos\theta) + 8(9.81)(0.18)(1-\cos\theta) = 112.2(1-\cos\theta)$

$\Delta T = 0 - \frac{1}{2}I\omega^2 = -\frac{1}{2}[2\cdot 20(0.3)^2 + 8(0.220)^2](0.830)^2 = -1.372$ J

So $0 = 112.2(1-\cos\theta) - 1.372$, $\theta = 8.97°$

Loss of energy $|\Delta E| = (V_g)_{before} - (V_g)_{after} = 14.13 - 112.2(1-\cos 8.97°) = 12.75$ J

6/201 $\Delta H = 0$;

Initial: $H_{rods} = 2I\omega = 2(1.5)(0.060)^2\,\frac{300\times 2\pi}{60}$ N·m·s

$H_{base} = mk^2\omega = 4(0.040)^2\,\frac{300\times 2\pi}{60}$ N·m·s

Final: $H_{rods} = 2[\bar{I} + md^2]\omega = 2m\left[\frac{\ell^2}{12} + d^2\right]\frac{2\pi N}{60}$

$= 2(1.5)\left[\frac{0.3^2}{12} + (0.150 + 0.060)^2\right]\frac{2\pi N}{60}$

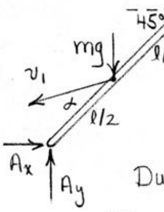
$= 0.1548\left(\frac{2\pi N}{60}\right)$ N·m·s

$H_{base} = 4(0.040)^2\,\frac{2\pi N}{60} = 0.0064\left(\frac{2\pi N}{60}\right)$

Thus $[3(0.06)^2 + 4(0.04)^2]300 = [0.1548 + 0.0064]N$

$0.0172(300) = 0.1612\,N$, $N = 32.0$ rev/min

6/202 Neglecting impulse of weight, $\Delta H_A = 0$ during impact:



$$mv_1\frac{\ell}{2}\sin\alpha = \frac{1}{3}m\ell^2\omega_2$$

$$\omega_2 = \frac{3v_1}{2\ell}\sin\alpha$$

During subsequent rotation about A,

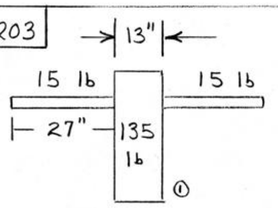
$U = \Delta T$ or $-mg\frac{\ell}{2}(1-\cos 45°) = 0 - \frac{1}{2}I_A\omega_2^2$

$$\omega_2 = \sqrt{\frac{3g}{\ell}\left(1 - \frac{\sqrt{2}}{2}\right)}$$

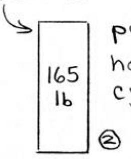
So $\sqrt{\dfrac{3g}{\ell}\left(1-\dfrac{\sqrt{2}}{2}\right)} = \dfrac{3v_1}{2\ell}\sin\alpha$

$$\sin\alpha = \frac{0.625}{v_1}\sqrt{g\ell} \quad (0 \le \alpha \le 45°)$$

6/203



Assume arms become part of homogeneous cylinder



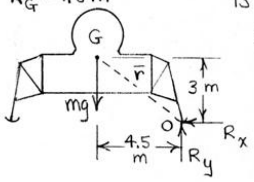
Conservation of angular momentum about a vertical axis : $H_1 = H_2$

$$\left\{\frac{1}{2}\frac{135}{32.2}\left(\frac{13}{2\cdot 12}\right)^2 + 2\left[\frac{1}{12}\frac{15}{32.2}\left(\frac{27}{12}\right)^2 + \frac{15}{32.2}\left(\frac{13+27}{2\cdot 12}\right)^2\right]\right\}\times 1 = \left\{\frac{1}{2}\frac{165}{32.2}\left(\frac{13}{2\cdot 12}\right)^2\right\} N$$

$N = 4.78$ rev/sec

6/204

$k_G = 1.8$ m



Impulse of mg during impact interval is small and is neglected.

Before impact, $\bar{v} = \frac{8}{3.6} = 2.22\ \frac{m}{s}$

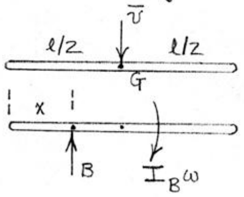
$\Delta H_O = 0$:

$$m\bar{v}d = m(\bar{k}^2 + \bar{r}^2)\,\omega$$

$$2.22(4.5) = \left((1.8)^2 + (4.5)^2 + (3)^2\right)\omega$$

$\omega = 0.308$ rad/s

6/205 Velocity of bar at impact $= \sqrt{2gh} = \bar{v}$



Neglect small impulse of weight.

$\Delta H_B = 0$

$$I_B\,\omega = m\bar{v}\left(\frac{l}{2} - x\right)$$

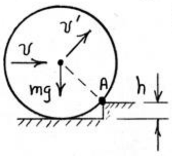
$$I_B = \frac{1}{12}ml^2 + m\left(\frac{l}{2} - x\right)^2 = \frac{1}{3}ml^2 - mlx + mx^2$$

Thus $$\omega = \frac{\left(\frac{l}{2} - x\right)\sqrt{2gh}}{\left(\frac{1}{3}l^2 - lx + x^2\right)}$$

$$\omega_{x=0} = \frac{3}{2l}\sqrt{2gh}\ , \quad \omega_{x=l/2} = 0$$

$$\omega_{x=l} = -\frac{3}{2l}\sqrt{2gh}$$

6/206 Angular impulse of mg is negligible.



Before impact: $H_A = \bar{I}\omega + mv(r-h)$
$= mk^2\frac{v}{r} + mv(r-h)$

Just after impact :

$$H_A' = I_A\frac{v'}{r} = m(k^2 + r^2)\frac{v'}{r}$$

$$\Delta H_A = 0 : \quad mv\left(\frac{k^2}{r} + r - h\right) = m(k^2 + r^2)\frac{v'}{r}$$

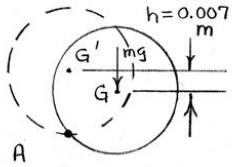
$$v' = v\left(1 - \frac{rh}{k^2 + r^2}\right)$$

During roll on curb point, $\Delta T + \Delta V_g = 0$

$$\left[0 - \frac{1}{2}m(k^2 + r^2)\frac{v'^2}{r^2}\right] + \left[mgh - 0\right] = 0$$

Solve for v : $$v = \frac{r}{k^2 + r^2 - rh}\sqrt{2gh(k^2 + r^2)}$$

6/207 Process II – roll about fixed point A



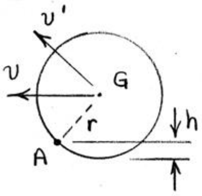
$U_{1\text{-}2} = \Delta T$

$$-mgh = \frac{1}{2}\left(\frac{3}{2}mr^2\right)(\omega'^2 - 0)$$

$$\omega' = \sqrt{\frac{4gh}{3r^2}} = \sqrt{\frac{4(9.81)(0.007)}{3(0.035)^2}}$$

$= 8.65$ rad/s

Process I – impact at A



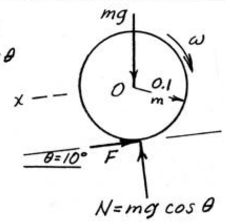
$$\Delta H_A = 0 : \quad mv(r-h) = I_A\omega'$$
$$= \left(\frac{3}{2}mr^2\right)\omega'$$

With $v = 0.5\Omega$:

$$\frac{1}{2}\Omega(r-h) = \frac{3}{2}r^2\omega'$$

$$\Omega = \frac{3r^2\omega'}{r-h} = \frac{3(0.035)^2(8.65)}{0.035 - 0.007} = 1.135\ \frac{rad}{s}$$

▶6/208 During slipping $(a_o)_x = 0$, so



$\Sigma F_x = 0,\ F - mg\sin\theta = 0;\ F = \mu_k mg\cos\theta$

so $mg\sin\theta = \mu_k mg\cos\theta$,

$\mu_k = \tan\theta = \tan 10°$

$\mu_k = 0.1763$

$\Sigma M_o \times t = \Delta H_o$:

$0.1763(30)(9.81)\cos 10°(0.1)t$

$= 0 - (-30\times 0.075^2)\frac{2\pi\times 300}{60}$, $t = 1.037$ s

During rolling (assume no slip)

$\int_0^4 \Sigma F_x\,dt = m\Delta v_x$: $(30\times 9.81\sin 10° - F)4 = 30(v - 0)$, $204 - 4F = 30v$

$\int_0^4 \Sigma M_o\,dt = I_o\Delta\omega$: $0.1F\times 4 = 30\times 0.075^2(v/0.1)$, $4F = 16.88v$

Combine & get $F = 18.40$ N, $v = 4.36$ m/s

Check: $F_{max} = \mu_s N$, & $\mu_k N = 0.1763\times 30\times 9.81\cos 10° = 51.1\ N < \mu_s N$
so $18.40 < \mu_k N < \mu_s N$ & assumption of no slip is valid.

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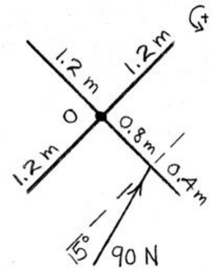
$$I_o = 4\left(\frac{1}{3}ml^2\right) = 4\left(\frac{1}{3}60\,(1.2)^2\right) = 115.2\ kg{\cdot}m^2$$



$\Sigma M_o = I_o\alpha$:

$$(90\cos 15°)(0.8) = 115.2\,\alpha$$

$\alpha = 0.604$ rad/s²

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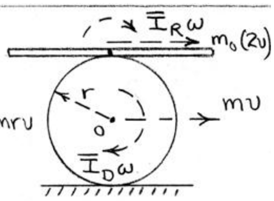


$\omega = v/r$

Disk: $\bar{I}_D\omega = \frac{1}{2}mr^2\left(\frac{v}{r}\right) = \frac{1}{2}mrv$

Rod : $\bar{I}_R\omega = \frac{1}{12}m_o l^2\frac{v}{r}$

Combined : $H_o = \frac{1}{2}mrv + \frac{1}{12}m_o l^2\frac{v}{r} + m_o(2v)r$
$$= vr\left[\frac{m}{2} + m_o\left(2 + \frac{l^2}{12r^2}\right)\right]$$

6/211

$\circlearrowleft \Sigma M_O = I_O \ddot{\theta}$:

$-mg\frac{l}{2}\sin\theta = \frac{1}{3}ml^2\ddot{\theta}$

$\ddot{\theta} = -\frac{3g}{2l}\sin\theta$

$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta$

$\int_{\omega_0}^{\omega}\dot{\theta}\,d\dot{\theta} = \int_0^{\theta} -\frac{3g}{2l}\sin\theta\,d\theta$

$\Rightarrow \omega^2 = \omega_0^2 - \frac{3g}{l}(1-\cos\theta)$

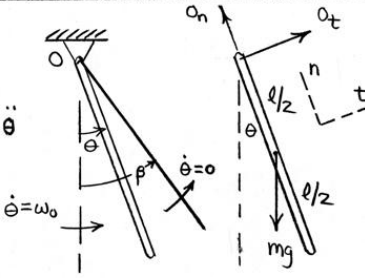
When $\theta = \beta$, $\omega = 0$: $0 = \omega_0^2 - \frac{3g}{l}(1-\cos\beta)$

$\omega_0^2 = \frac{3g}{l}(1-\cos\beta)$

So $\omega^2 = \frac{3g}{l}(\cos\theta - \cos\beta)$

$\Rightarrow \frac{d\theta}{dt} = \sqrt{\frac{3g}{l}}\sqrt{\cos\theta - \cos\beta}$

So $t = \sqrt{\frac{l}{3g}}\int_0^{\beta}\frac{d\theta}{\sqrt{\cos\theta - \cos\beta}}$



6/212 Max. power occurs when dV_g/dt is greatest, which occurs when $\bar{v}_y$ is max. at the start.

$\bar{v}_y = 1.500\,\omega = 1.500\,\frac{4\pi}{180} = 0.1047$ m/s

$P = mg\bar{v}_y = 1600(5)\,9.81(0.1047) = 8218$ W

or $P = 8.22$ kW

6/213 $\Delta V_g + \Delta V_e + \Delta T = 0$

$\Delta V_g = -15(2) = -30$ ft-lb

$\Delta V_e = \frac{1}{2}3\left(\sqrt{6^2+4^2}-2\right)^2 = 40.74$ ft-lb

$\Delta T = 0 - \frac{1}{2}\frac{1}{3}\frac{15}{32.2}4^2\omega^2 = -1.242\,\omega^2$

$-30 + 40.74 - 1.242\,\omega^2 = 0$, $\omega^2 = 8.64$, $\omega = 2.94$ rad/sec

6/214

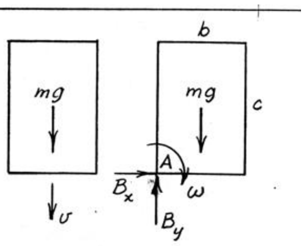
$v = \sqrt{2gh}$

$\Delta H_B = 0$

$I_A\omega - mv\frac{b}{2} = 0$

$I_A = \frac{1}{12}m(b^2+c^2) + m\left[\left(\frac{b}{2}\right)^2 + \left(\frac{c}{2}\right)^2\right]$

$= \frac{1}{3}m(b^2+c^2)$



$\frac{1}{3}m(b^2+c^2)\omega - m\sqrt{2gh}\,\frac{b}{2} = 0$, $\omega = \frac{3b\sqrt{2gh}}{2(b^2+c^2)}$

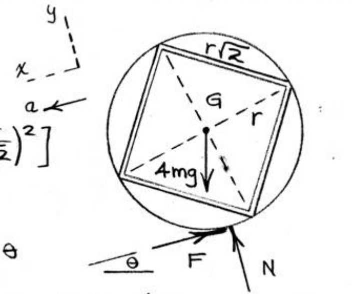
Percentage loss of energy $n = \frac{|\Delta E|}{E} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}I_A\omega^2}{\frac{1}{2}mv^2}$

$= 1 - \frac{I_A\omega^2}{mv^2}$

and for $b = c$, $n = 1 - \frac{2c^2/3}{2gh}\,\frac{9c^2(2gh)}{4(2c^2)}$

$= 1 - \frac{3}{8} = \frac{5}{8}$ or $n = 62.5\%$ loss

6/215



$\bar{I} = 4\left[\frac{1}{12}m(2r^2) + m\left(\frac{r}{\sqrt{2}}\right)^2\right]$

$= \frac{8}{3}mr^2$

$\Sigma F_y = 0$: $N = 4mg\cos\theta$

$\Sigma F_x = ma_{G_x}$: $4mg\sin\theta - F = 4ma$ (1)

$\Sigma M_G = \bar{I}\alpha$: $Fr = \frac{8}{3}mr^2\alpha$ (2)

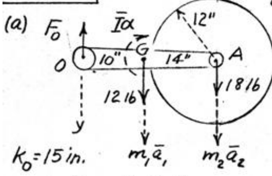
No slipping : $a = r\alpha$ (3)

Solution of (1)-(3) : $\begin{cases} a = \frac{3}{5}g\sin\theta, \quad \alpha = \frac{3g}{5r}\sin\theta \\ F = \frac{8}{5}mg\sin\theta \end{cases}$

$\mu_s = \frac{F}{N} = \frac{\frac{8}{5}mg\sin\theta}{4mg\cos\theta} = \frac{2}{5}\tan\theta$

6/216 Disk has no moment about its center so undergoes curvilinear translation with no $\bar{I}\alpha$.

(a)



For OA; $m_1\bar{a}_1 = \frac{12}{32.2}\frac{10}{12}\alpha = 0.311\alpha$

$\bar{I}\alpha = \frac{12}{32.2}\left[\left(\frac{15}{12}\right)^2 - \left(\frac{10}{12}\right)^2\right]\alpha = 0.3235\alpha$

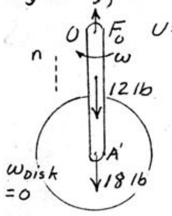
For disk; $m_2\bar{a}_2 = \frac{18}{32.2}\frac{24}{12}\alpha = 1.1180\,\alpha$

$\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d$;

$12\frac{10}{12} + 18\frac{24}{12} = 0.3235\alpha + 0.311\alpha\left(\frac{10}{12}\right) + 1.1180\alpha\left(\frac{24}{12}\right)$

$\alpha = 46/2.818 = 16.32$ rad/sec²

$\Sigma F_y = \Sigma m\bar{a}_y$; $18 + 12 - F_O = (0.311 + 1.1180)16.32$, $F_O = 6.68$ lb



$U = \Delta T$; $12\frac{10}{12} + 18\frac{24}{12} = \frac{1}{2}\frac{12}{32.2}\left(\frac{15}{12}\right)^2\omega^2 + \frac{1}{2}\frac{18}{32.2}\left(\frac{24}{12}\omega\right)^2$

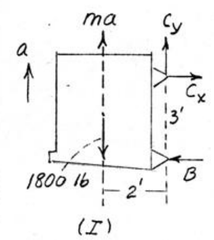
$\omega^2 = 32.6$ (rad/s)²

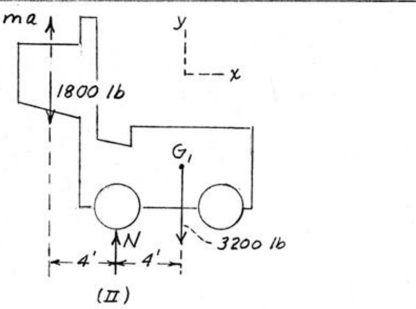
$\Sigma F_n = \Sigma m\bar{a}_n$; $F_O - 12 - 18 = \frac{12}{32.2}\frac{10}{12}(32.6)$

$+ \frac{18}{32.2}\frac{24}{12}(32.6)$

$F_O = 76.6$ lb

6/217



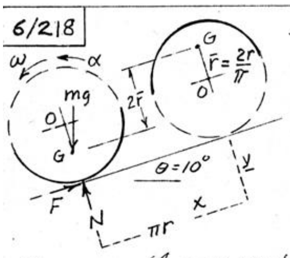


(II) $\Sigma M_N = m\bar{a}d$; $3200(4) - 1800(4) = \frac{1800}{32.2}a(4)$, $a = 25.04$ ft/sec²

(I) $\Sigma M_C = m\bar{a}d$; $3B - 2(1800) = \frac{1800}{32.2}(25.04)(2)$

$B = 2130$ lb

6/218



$\bar{I} = I_O - m\bar{r}^2 = mr^2 - m(\frac{2r}{\pi})^2$
$= mr^2(1 - 4/\pi^2)$

$U = \Delta T;\ U = mg(2\bar{r}\cos\theta + \pi r\sin\theta)$
$= mgr(\frac{4}{\pi}\cos\theta + \pi\sin\theta)$

$\Delta T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$
$= \frac{1}{2}m[(r-\bar{r})\omega]^2 + \frac{1}{2}mr^2(1-\frac{4}{\pi^2})\omega^2$
$= mr^2\omega^2(1 - 2/\pi)$

Thus $mgr(\frac{4}{\pi}\cos\theta + \pi\sin\theta) = mr^2\omega^2(1 - 2/\pi)$

$\omega^2 = \frac{(\frac{4}{\pi}\cos\theta + \pi\sin\theta)\frac{g}{r}}{(1 - 2/\pi)}$, $\omega = \sqrt{\frac{g}{r}\frac{4\cos\theta + \pi^2\sin\theta}{\pi - 2}}$

$\Sigma F_y = m\bar{a}_y;\ N - mg\cos\theta = m\bar{r}\omega^2$

$N = mg\cos\theta + m\frac{2r}{\pi}\ \frac{\frac{4}{\pi}\cos\theta + \pi\sin\theta}{1 - 2/\pi}\ \frac{g}{r}$

$N = mg\left[\frac{\pi^2 - 2\pi + 8}{\pi(\pi - 2)}\cos\theta + \frac{2\pi}{\pi - 2}\sin\theta\right]$

For $\theta = 10°$, $N = mg[3.231\cos 10° + 5.504\sin 10°] = \underline{4.14\,mg}$

6/219 For the entire spacecraft,

$\Sigma M_x = I_x\alpha:\quad 10^{-6} = 150{,}000\,\alpha$

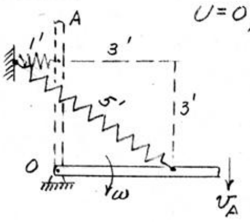
$\alpha = 6.67 \times 10^{-12}$ rad/s^2

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\frac{1}{3600}\left(\frac{\pi}{180}\right) = 0 + 0 + \frac{1}{2}(6.67 \times 10^{-12})t^2$

$\underline{t = 1206\ s}$

6/220 For system $U = \Delta T + \Delta V_g + \Delta V_e$



$U = 0,\ \Delta T = \frac{1}{2}I_O\omega^2 = \frac{1}{2}(\frac{1}{3}ml^2)(\frac{v_A}{l})^2$
$= \frac{1}{6}mv_A^2 = \frac{1}{6}\frac{60}{32.2}v_A^2 = 0.3106\,v_A^2$

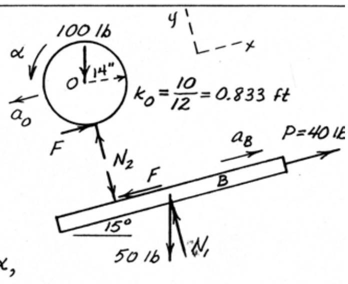
$\Delta V_e = \frac{1}{2}kx^2 - 0$
$= \frac{1}{2}10(5-1)^2 = 80$ ft-lb

$\Delta V_g = -60(2) = -120$ ft lb

Thus $0 = 0.3106\,v_A^2 - 120 + 80$

$v_A^2 = 128.8$, $\underline{v_A = 11.35\ ft/sec}$

6/221



Slab

$\Sigma F_x = ma_x:\ 40 - 50\sin 15° - F$
$= \frac{50}{32.2}a_B\quad --(1)$

$k_0 = \frac{10}{12} = 0.833$ ft

Wheel

$\Sigma F_x = m\bar{a}_x:\ F - 100\sin 15°$
$= \frac{100}{32.2}(-a_o)\quad --(2)$

$\Sigma M_o = I_o\alpha:\ F\frac{14}{12} = \frac{100}{32.2}(0.833)^2\alpha,$

$F = \frac{1200}{451}(0.833)^2\alpha = 1.849\,\alpha\quad --(3)$

Relative accel.: $(a_B + a_o)/\frac{14}{12} = \alpha,$

$a_o + a_B = 1.167\,\alpha\quad --(4)$

Solve (1), (2), (3), (4) & get $\underline{a_B = 7.04\ ft/sec^2\ (+x\text{-dir})}$

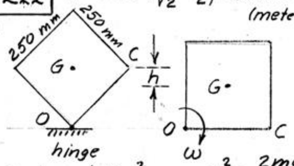
$\underline{a_o = 3.14\ ft/sec^2\ (-x\text{-dir})}$

$F = 16.13$ lb

$(\mu_s)_{min} = F/N_2 = \frac{16.13}{100\cos 15°} = \underline{0.1670}$

6/222 $h = 0.25(\frac{1}{\sqrt{2}} - \frac{1}{2}) = 0.05178$ m (meters); $\bar{I} = \frac{1}{6}m(0.25)^2$
$= 0.01042\,m$, (m = mass)

(a)



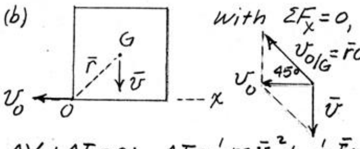
$I_O = \bar{I} + m(0.25/\sqrt{2})^2$
$= 0.04167\,m$

$\Delta V_g + \Delta T = 0$

$-mgh + \frac{1}{2}I_O\omega^2 = 0$, $\omega^2 = \frac{2mgh}{I_O} = \frac{2m(9.81)(0.05178)}{0.04167\,m} = 24.38$ (rad/s)^2

$\underline{\omega = 4.94\ rad/s}$

(b)



With $\Sigma F_x = 0$, $\bar{a}$ & hence $\bar{v}$ remain vertical

$v_{O/G} = \bar{r}\omega$

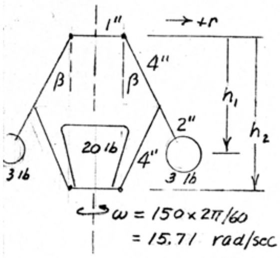
$\bar{v} = \frac{\bar{r}\omega}{\sqrt{2}} = \frac{0.25}{2}\omega = 0.125\omega$

$\Delta V_g + \Delta T = 0;\quad \Delta T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}m(0.125\omega)^2$
$+ \frac{1}{2}(0.01042\,m)\omega^2$
$= 13.02\,m\omega^2$

So $-mg(0.05178) + 0.01302\,m\omega^2 = 0$, $\omega^2 = \frac{9.81(0.05178)}{0.01302}$
$= 39.01$ (rad/s)^2

$\underline{\omega = 6.25\ rad/s}$

6/223 $\delta T_{both\ balls} = 2ma_r\delta r$, $r = \frac{1}{12}(1 + 6\sin\beta)$ ft



$a_r = -r\omega^2 = -r(15.71)^2$
$= -20.56(1 + 6\sin\beta)$ ft/sec^2

$\delta T = -2\frac{3}{32.2}(20.56)(1 + 6\sin\beta)\frac{\cos\beta}{2}\delta\beta$
$= -1.916(1 + 6\sin\beta)\cos\beta\,\delta\beta$

$\delta V_g = -20\,\delta h_2 - 2(3)\delta h_1$

$\delta h_1 = \delta(6\cos\beta) = -6\sin\beta\,\delta\beta$
$\delta h_2 = \delta(2 \times 4\cos\beta) = -8\sin\beta\,\delta\beta$

$\delta U = \delta T + \delta V_g = 0$

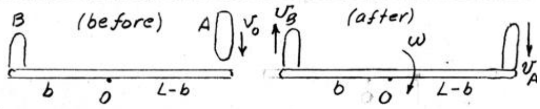
$\delta V_g = [20(8) + 6(6)]\sin\beta\,\delta\beta/12$
$= 16.33\sin\beta\,\delta\beta$

$-1.916(1 + 6\sin\beta)\cos\beta\,\delta\beta + 16.33\sin\beta\,\delta\beta = 0$

$1 + 6\sin\beta = 8.526\tan\beta$

Solve by Newton's method of approximations & get $\underline{\beta = 19.26°}$

6/224



Before: $H_O = m_A v_o(L - b)$

After: $H_O = m_A v_A(L - b) + m_B v_B b$

$\Delta H_O = 0$ along with $\omega = v_B/b = v_A/(L-b)$ give

$m_A v_o(L - b) = m_A\frac{L-b}{b}v_B(L - b) + m_B v_B b$

$v_B = v_o\frac{1}{\frac{L-b}{b} + n\frac{b}{L-b}}$ where $n = m_B/m_A$

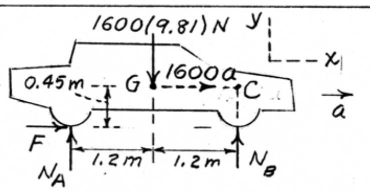
$\frac{dv_B}{db} = v_o\frac{-(\frac{L}{b^2} + n\frac{L-b-b(-1)}{(L-b)^2})}{(\frac{L-b}{b} + n\frac{b}{L-b})^2} = v_o\frac{L(\frac{1}{b^2} - \frac{n}{(L-b)^2})}{(\frac{L-b}{b} + n\frac{b}{L-b})^2} = 0$ for $v_{B_{max}}$

So $\frac{1}{b^2} = \frac{n}{(L-b)^2}$, $b = \frac{L}{1 \pm \sqrt{n}}$ (+ sign gives positive v_B)

Thus $\underline{b = \frac{L}{1 + \sqrt{n}}}$ which gives $v_B = \frac{v_o}{2\sqrt{n}}$

6/225

(a) Max. acceleration occurs when $F = \mu N_A = 0.8 N_A$



$\circlearrowleft + \Sigma M_C = m a d = 0$: $1600(9.81)(1.2) - 2.4 N_A + 0.8 N_A(0.45) = 0$

$N_A = 9233$ N, $F = 0.8(9233) = 7386$ N

$\Sigma F_x = m a_x$: $7386 = 1600 a$, $\underline{a = 4.62 \text{ m/s}^2}$

(b) Each rear wheel:

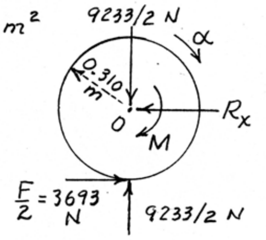
$\bar{I}_O = m k^2 = 32(0.210)^2 = 1.411 \text{ kg·m}^2$

$\alpha = \dfrac{a}{r} = \dfrac{4.62}{0.310} = 14.89 \text{ rad/s}^2$

$\Sigma M_O = \bar{I}_O \alpha$:

$M - 3693(0.310) = 1.411(14.90)$

$\underline{M = 1166 \text{ N·m}}$



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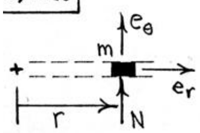


Conservation of angular momentum : $I_o \omega_o = (I_o + m r^2)\omega$

$\dot{\theta} = \omega = \dfrac{I_o \omega_o}{I_o + m r^2}$

$\Sigma F_r = m a_r = m(\ddot{r} - r\dot{\theta}^2)$: $0 = m(\ddot{r} - r\dot{\theta}^2)$

$\ddot{r} = \dot{r}\dfrac{d\dot{r}}{dr} = r\left(\dfrac{I_o \omega_o}{I_o + m r^2}\right)^2$

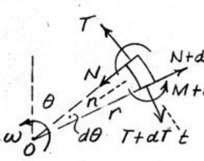
$\displaystyle\int_0^{\dot{r}} \dot{r}\, d\dot{r} = I_o^2 \omega_o^2 \int_0^r \frac{r\,dr}{(I_o + m r^2)^2}$

Integrating and solving for $\dot{r}$:

$\dot{r} = \left(\dfrac{I_o \omega_o^2 r^2}{I_o + m r^2}\right)^{1/2} = \underline{\omega_o r \sqrt{\dfrac{I_o}{I_o + m r^2}}}$

▸6/227 $\Sigma F_n = m\bar{a}_n$; $2T \sin\frac{d\theta}{2} + dT \sin\frac{d\theta}{2} + N\cos\frac{d\theta}{2} - (N + dN)\cos\frac{d\theta}{2} = \rho r d\theta (r\omega^2)$

Simplify & get $T - \rho r^2 \omega^2 = \dfrac{dN}{d\theta}$ ----(1)



$\Sigma F_t = m\bar{a}_t = 0$; $-T\cos\frac{d\theta}{2} + (T + dT)\cos\frac{d\theta}{2} + N\sin\frac{d\theta}{2} + (N + dN)\sin\frac{d\theta}{2} = 0$

Simplify & get $N = -dT/d\theta$ ------- (2)

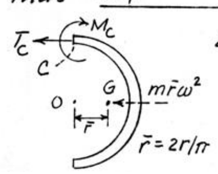
Combine (1) & (2) & get $\dfrac{d^2N}{d\theta^2} + N = 0$

Sol. $N = A\sin\theta + B\cos\theta$

By symmetry $N = 0$ for $\theta = 0$ so $B = 0$ & $N = A\sin\theta$

From (1) $T = \rho r^2\omega^2 + A\cos\theta$; $T = 0$ when $\theta = \pi$ so $A = \rho r^2\omega^2$

Thus $\underline{N = \rho r^2\omega^2 \sin\theta}$ & $\underline{T = \rho r^2\omega^2(1 + \cos\theta)}$

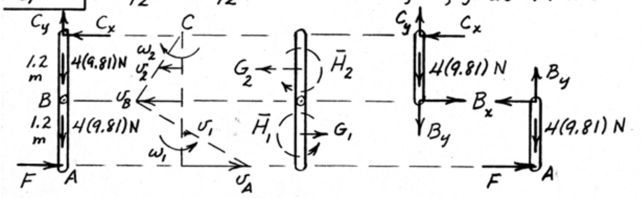


$\Sigma M_c = m\bar{a}d$; $M_c = m\frac{2r}{\pi}\omega^2 r$

$= \rho\pi r\left(\frac{2r^2}{\pi}\omega^2\right)$

$\underline{M_c = 2\rho r^3\omega^2}$

▸6/228 $\bar{I} = \frac{1}{12} m l^2 = \frac{1}{12}(4)(1.2)^2 = 0.48 \text{ kg·m}^2$, $\int F dt = 14$ N·s



$\omega_2 = v_2/0.6$, $\omega_1 = (v_1 + v_B)/0.6 = (v_1 + 2v_2)/0.6$, $m = 4$ kg

System: $\int \Sigma M_c\, dt = \Sigma \Delta H_c$: $14(2.4) = 4v_1(1.8) + 0.48\omega_1 - 4v_2(0.6) - 0.48\omega_2$ (a)

AB: $\int \Sigma M_c\, dt = \Delta H_c$: $14(2.4) - \int 1.2 B_x dt = 4v_1(1.8) + 0.48\omega_1$ (b)

$\int \Sigma F_x\, dt = \Delta G_x$: $14 - \int B_x dt = 4v_1$ (c)

(b) & (c) & ω_1 give $2v_1 + v_2 = 10.5$

(a) & ω_1 & ω_2 give $5v_1 - v_2 = 21$

Combine & get $v_1 = 4.5$ m/s, $v_2 = 1.5$ m/s

& $\underline{\omega_2 = 2.50 \text{ rad/s}}$

▸6/229 Fixed-axis rotation

$\Sigma F_n = m\bar{a}_n$: $T - 150 = \dfrac{150}{32.2}\dfrac{13^2}{92/12}$,

$T = 253$ lb

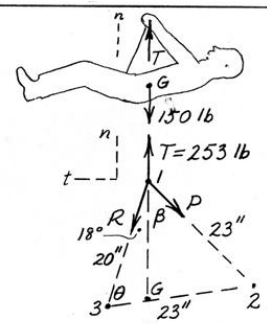
$\theta = \cos^{-1}(10/23) = 64.2°$

$\beta = \theta - 18° = 46.2°$

$\Sigma F_n \doteq 0$: $253 - R\cos 18° - P\cos 46.2° = 0$

$\Sigma F_t \doteq 0$: $R\sin 18° - P\cos 46.2° = 0$

Solve & get $P = 86.7$ lb, $R = 203$ lb



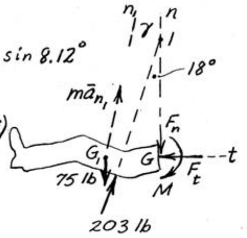
$\gamma = \sin^{-1}\dfrac{13}{92} = 8.12°$

$\Sigma F_t = m\bar{a}_t$: $203\sin 18° - F_t = \dfrac{75}{32.2}\dfrac{13^2}{92/12}\sin 8.12°$

$F_t = 55.4$ lb

$\circlearrowleft + \Sigma M_O = \bar{I}_O\alpha = 0$: $203\sin 18°(92 - 18.18) - 75(13) + 55.4(92) + M = 0$

$\underline{M = 504 \text{ lb-in.}}$



*6/230 $\Sigma M_O = I_O\alpha$: $78.5(0.220\cos\theta) = 8(0.235^2)\alpha$

$\alpha = 39.1\cos\theta$

$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta$: $\omega^2/2 = 39.1\sin\theta$

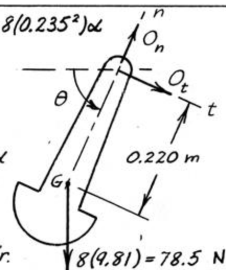
$\omega^2 = 78.2\sin\theta$

$\Sigma F_t = m\bar{a}_t$: $O_t + 78.5\cos\theta = 8 \times 0.220\alpha$

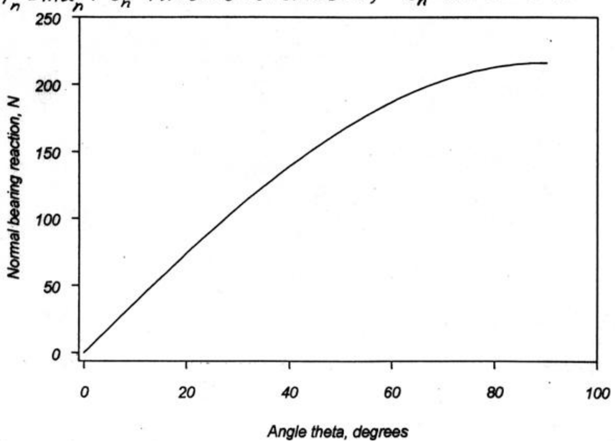
$O_t = 8(0.220)39.1\cos\theta - 78.5\cos\theta$

$O_t = -9.70\cos\theta$ N

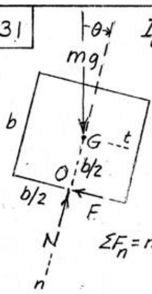
$\underline{(O_t)_{max} = 9.70 \text{ N at } \theta = 0, \text{ -t-dir.}}$



$\Sigma F_n = m\bar{a}_n$: $O_n - 78.5\sin\theta = 8 \times 0.220\omega^2$, $O_n = 216\sin\theta$ N



***6/231**



$I_O = \frac{1}{12}m(b^2+b^2) + m(b/2)^2 = \frac{5}{12}mb^2$

$\Sigma M_O = I_O\alpha;\quad mg\frac{b}{2}\sin\theta = \frac{5}{12}mb^2\alpha$

$\alpha = \frac{6}{5}\frac{g}{b}\sin\theta$

$\int_0^\omega \omega\,d\omega = \int_0^\theta \alpha\,d\theta;\quad \omega^2 = \frac{12}{5}\frac{g}{b}\int_0^\theta \sin\theta\,d\theta$

$\omega^2 = \frac{12}{5}\frac{g}{b}(1-\cos\theta)$

$\Sigma F_n = m\bar{a}_n;\quad mg\cos\theta - N = m\frac{b}{2}\left(\frac{12}{5}\frac{g}{b}\right)(1-\cos\theta)$

$N = \frac{mg}{5}(11\cos\theta - 6)$

$\Sigma F_t = m\bar{a}_t;\quad mg\sin\theta - F = m\frac{b}{2}\left(\frac{6}{5}\frac{g}{b}\sin\theta\right)$

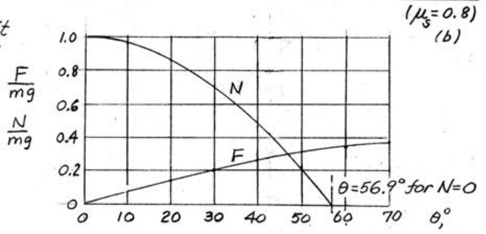
$F = \frac{2}{5}mg\sin\theta$

Compute & plot N/mg & F/mg

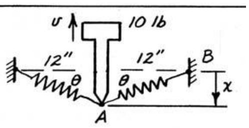
For $F = 0.8N$, $\frac{2}{5}\sin\theta = 0.8(\frac{1}{5})(11\cos\theta - 6)$, $5\sin\theta = 22\cos\theta - 12$

Solve by Newton's method & get slip at $\underline{\theta = 45.1°}$ $(\mu_s = 0.8)$ (b)

(a) for no limit on F, contact ceases when $N = 0$ so $\theta = \cos^{-1}6/11 = \underline{56.9°}$



***6/232** $U' = \Delta T + \Delta V_e + \Delta V_g;\ U' = 0$



$\Delta T = \frac{1}{2}mv^2 - 0 = \frac{1}{2}\frac{10}{32.2}v^2$ ft-lb

$\Delta V_e = \frac{2}{2}k(x_2^2 - x_1^2) = 6\left[(\sqrt{x^2+12^2} - 12)^2 - (15-12)^2\right]\frac{1}{12}$

$= \frac{1}{2}\left[x^2 - 24\sqrt{x^2+144} + 279\right]$ ft-lb

$\overline{AB} = 15''$ when $x = 9''$

where x is in inches

$\Delta V_g = 10(9-x)/12 = \frac{5}{6}(9-x)$ ft-lb

$\frac{5}{32.2}v^2 + \frac{x^2}{2} - 12\sqrt{x^2+144} + \frac{279}{2} + \frac{15}{2} - \frac{5x}{6} = 0$

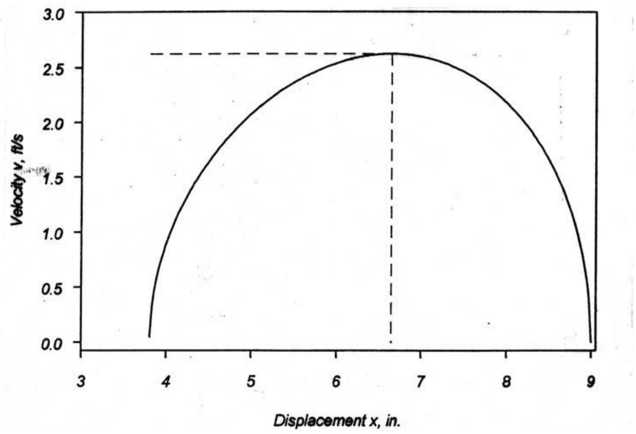
$v^2 = \frac{32.2}{5}\left\{12\sqrt{x^2+144} - \frac{x^2}{2} + \frac{5x}{6} - 147\right\}$ $(ft/sec)^2$

(where x is in inches)

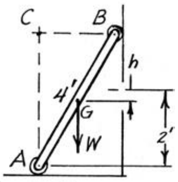
Plot v vs. x (see continuation)

$v = 0$ at $x = 3.81$ in.

$v_{max} = 2.62$ ft/sec at $x = 6.65$ in.



***6/233** $U = \Delta T:\quad T = \frac{1}{2}I_C\omega^2 = \frac{1}{2}\frac{1}{3}\frac{W}{g}4^2\omega^2$



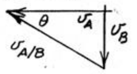
$U = Wh = W(2 - 2\cos\theta)$

$= 2W(1-\cos\theta)$

Thus $2W(1-\cos\theta) = \frac{8W}{3g}\omega^2,\quad \omega^2 = \frac{3g}{4}(1-\cos\theta)$

$\omega = \sqrt{(3\times32.2/4)(1-\cos\theta)} = 4.91\sqrt{1-\cos\theta}$ rad/sec

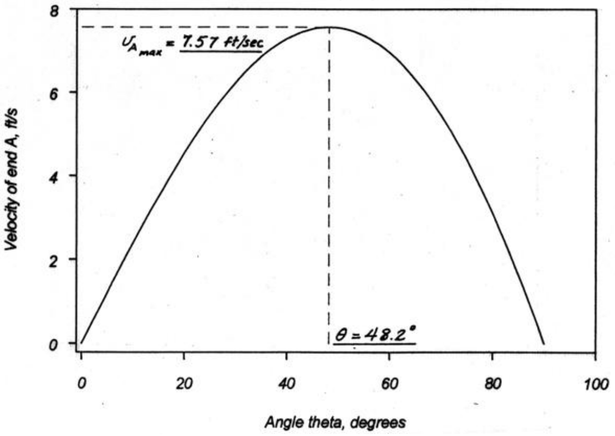
$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}:$



$v_A = v_{A/B}\cos\theta = L\omega\cos\theta$

$= 4(4.91)\sqrt{1-\cos\theta}\cos\theta$ ft/sec

$= 19.66\cos\theta\sqrt{1-\cos\theta}$ ft/sec



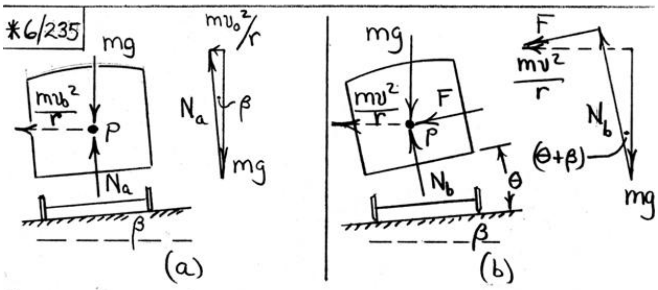
***6/234** From the solution of Prob. 6/23,

$K\theta - \frac{5}{2}mgl\sin\theta - \frac{5}{2}mal\cos\theta = 0$

With numbers : $75\theta - 7.36\sin\theta - 14.72\cos\theta = 0$

Numerical solution : $\underline{\theta = 12.17°}$

***6/235**



(Passenger is shown as particle P above)

Note that $F = 0.3mv^2/r$

(a) $\tan\beta = \frac{mv_0^2/r}{mg} = \frac{v_0^2}{gr} = \frac{(160/3.6)^2}{9.81(1900)}$

$\underline{\beta = 6.05°}$

(b) From the force polygon,

$mg\sin(\theta+\beta) + \frac{0.3mv^2}{r} = \frac{mv^2}{r}\cos(\theta+\beta)$

$9.81\sin(\theta+\beta) + \frac{(260/3.6)^2}{1900}\left(0.3 - \cos(\theta+\beta)\right) = 0$

$9.81\sin(\theta+\beta) + 2.75\left[0.3 - \cos(\theta+\beta)\right] = 0$

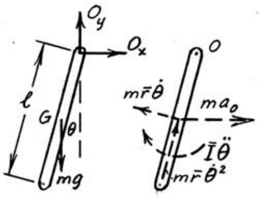
Numerical solution : $\underline{\theta = 4.95°}$

***6/236**

$\Sigma M_O = \bar{I}\alpha + m\bar{a}d$:

$-mg\frac{l}{2}\sin\theta = \frac{1}{12}ml^2\ddot{\theta} + m\frac{l}{2}\ddot{\theta}\left(\frac{l}{2}\right) - ma_o\frac{l}{2}\cos\theta$

$\ddot{\theta} = \frac{3}{l}\left(\frac{a_o}{2}\cos\theta - \frac{g}{2}\sin\theta\right)$



$\int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = \int_0^{\theta} \ddot{\theta}\,d\theta$:

$\frac{\dot{\theta}^2}{2} = \frac{3}{l}\int_0^{\theta}\left(\frac{a_o}{2}\cos\theta - \frac{g}{2}\sin\theta\right)d\theta$

$\dot{\theta}^2 = \frac{6}{l}\left(\frac{a_o}{2}\sin\theta - \frac{g}{2}[1-\cos\theta]\right) = 1.5\left(\sin\theta - \frac{9.81}{2}[1-\cos\theta]\right)\frac{rad^2}{s^2}$

$\dot{\theta} = 0$ when $R = \sin\theta - \frac{9.81}{2}(1-\cos\theta) = 0$

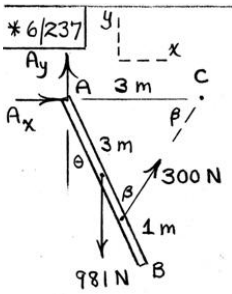
Solve numerically & get

$\underline{\theta_{max} = 23.0°}$

$\dot{\theta}$ is max. when $\ddot{\theta} = 0$: $\frac{a_o}{2}\cos\theta - \frac{g}{2}\sin\theta = 0$ or $\theta = \tan^{-1}\frac{a_o}{g}$

$\underline{\theta = 11.52°}$, $(\dot{\theta}^2)_{max} = 0.1513\ (rad/s)^2$, $\underline{\dot{\theta}_{max} = 0.389\ rad/s}$

***6/237**



$2\beta + \left(\frac{\pi}{2} - \theta\right) = \pi$, $\beta = \frac{\pi}{4} + \frac{\theta}{2}$

$\sin\beta = \sin\frac{\pi}{4}\cos\frac{\theta}{2} + \cos\frac{\pi}{4}\sin\frac{\theta}{2}$

$= \frac{1}{\sqrt{2}}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)$

$I_A = \frac{1}{3}ml^2 = \frac{1}{3}(100)(4)^2$

$= 533\ kg\cdot m^2$

$\Sigma M_A = I_A\alpha$: $300(3\sin\beta) - 981(2\sin\theta) = 533\alpha$

$\frac{900}{\sqrt{2}}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right) - 1962\sin\theta = 533\alpha$ (1)

$\int_0^{\omega}\omega\,d\omega = \int_0^{\theta}\alpha\,d\theta$: $\omega^2 = \frac{2}{533}\int_0^{\theta}\left[\frac{900}{\sqrt{2}}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right) - 1962\sin\theta\right]d\theta$

or $\omega^2 = \frac{2}{533}\left[\frac{1800}{\sqrt{2}}\left(1 - \cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) - 1962(1-\cos\theta)\right]$ (2)

(a) For max ω, set $\alpha = 0$ in (1) & solve for θ:

$\underline{\theta = 22.4°}$ From (2): $\underline{\omega_{max} = 0.680\ \frac{rad}{s}}$

(b) Solve (2) for $\omega = 0$: $\underline{\theta_{max} = 45.9°}$

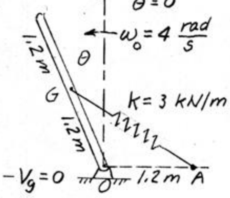
***6/238**

$m = 30$ kg

$V_e = 0$ when $\theta = 0$

$\omega_o = 4\ \frac{rad}{s}$

$k = 3$ kN/m



$U = 0 = \Delta T + \Delta V_g + \Delta V_e$

$\overline{AG} = 1.2\sqrt{2(1+\sin\theta)}$

$\Delta V_e = \frac{1}{2}k\left[\overline{GA} - 1.2\sqrt{2}\right]^2$

$= \frac{3000}{2}\left[1.2\sqrt{2(1+\sin\theta)} - 1.2\sqrt{2}\right]^2$

$= 4320\left(2 + \sin\theta - 2\sqrt{1+\sin\theta}\right)$

$\Delta T = \frac{1}{2}I_o(\omega^2 - \omega_o^2)$

$= \frac{1}{2}\left[\frac{1}{3}30(2.4)^2(\omega^2 - 16)\right]$

$= 28.8(\omega^2 - 16)$

$\Delta V_g = -30(9.81)(1.2)(1-\cos\theta)$

$= -353(1-\cos\theta)$

Thus

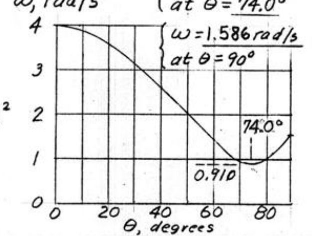
$0 = 28.8(\omega^2 - 16) - 353(1-\cos\theta) + 4320\left(2 + \sin\theta - 2\sqrt{1+\sin\theta}\right)$

$\omega^2 = 300\sqrt{1+\sin\theta} - 150\sin\theta - 12.26\cos\theta - 271.7\ (rad/s)^2$

Set up computer program & solve for ω vs θ & get

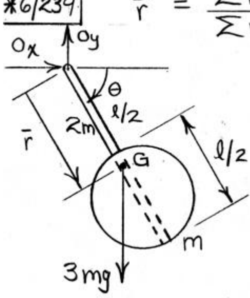
$\omega_{min} = 0.910\ \frac{rad}{s}$ at $\theta = \underline{74.0°}$

$\omega = 1.586\ rad/s$ at $\theta = 90°$



***6/239**

$\bar{r} = \frac{\Sigma m\bar{r}}{\Sigma m} = \frac{2m(l/2) + m(3l/4)}{3m} = \frac{7}{12}l$



$I_o = \frac{1}{3}(2m)l^2 + \left[\frac{1}{2}m\left(\frac{l}{4}\right)^2 + m\left(\frac{3l}{4}\right)^2\right]$

$= \frac{121}{96}ml^2$

$\Sigma M_o = I_o\alpha$:

$3mg\left(\frac{7}{12}l\cos\theta\right) = \frac{121}{96}ml^2\alpha$

$\alpha = \frac{168}{121}\frac{g}{l}\cos\theta$

$\alpha = \omega\frac{d\omega}{d\theta} = \frac{168}{121}\frac{g}{l}\cos\theta$

$\int_{\omega_o}^{\omega}\omega\,d\omega = \frac{168}{121}\frac{g}{l}\int_0^{\theta}\cos\theta\,d\theta$

$\Rightarrow \omega = \frac{d\theta}{dt} = \left[\omega_o^2 + \frac{336}{121}\frac{g}{l}\sin\theta\right]^{1/2}$

$\Rightarrow t = \int_0^{\theta}\frac{d\theta}{\left[\omega_o^2 + \frac{336}{121}\frac{g}{l}\sin\theta\right]^{1/2}}$

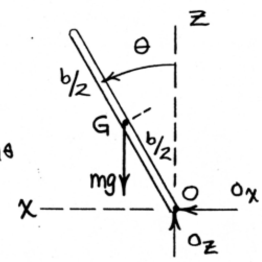
Numerical solution with $\begin{cases}\omega_o = 3\ rad/s\\ l = 0.8\ m\\ \theta = \pi/2\end{cases}$: $\underline{t = 0.302\,s}$

***6/240**

$\Sigma M_o = I_o\ddot{\theta}$:

$mg\frac{b}{2}\sin\theta = \frac{1}{3}mb^2\ddot{\theta}$

$\ddot{\theta} = \frac{3}{2}\frac{g}{b}\sin\theta$



$\dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta$: $\int_{\dot{\theta}_o}^{\dot{\theta}}\dot{\theta}\,d\dot{\theta} = \frac{3g}{2b}\int_{\theta_o}^{\theta}\sin\theta\,d\theta$

$\frac{\dot{\theta}^2}{2} - \frac{\dot{\theta}_o^2}{2} = \frac{3g}{2b}(\cos\theta_o - \cos\theta)$

$\frac{d\theta}{dt} = \left[\dot{\theta}_o^2 + \frac{3g}{b}(\cos\theta_o - \cos\theta)\right]^{1/2}$

$\int_0^t dt = \int_{\theta_o}^{\theta}\frac{d\theta}{\left[\dot{\theta}_o^2 + \frac{3g}{b}(\cos\theta_o - \cos\theta)\right]^{1/2}}$

With $\theta_o = 10°$ (0.1745 rad), $b = 60'$, $g = 32.2\frac{ft}{sec^2}$ and $\dot{\theta}_o = \frac{(v_A)_o}{b} = \frac{4.5}{60} = 0.0750$ rad/sec, a numerical solution yields $\underline{t = 2.85\ sec}$.

Energy considerations from $\theta_o = 10°$ to $\theta = 90°$:

$\Delta T + \Delta V_g = 0$

$\Delta T = \frac{1}{2}I_o\left[\frac{v_A}{b}\right]^2 - \frac{1}{2}I_o\left[\frac{(v_A)_o}{b}\right]^2 = \frac{1}{6}m\left[v_A^2 - (v_A)_o^2\right]$

$\Delta V_g = -mgh = -mg\frac{b}{2}\cos 10°$

So $\frac{1}{6}\cancel{m}\left[v_A^2 - 4.5^2\right] - \cancel{m}(32.2)\frac{60}{2}\cos 10° = 0$

$\underline{v_A = 75.7\ ft/sec}$

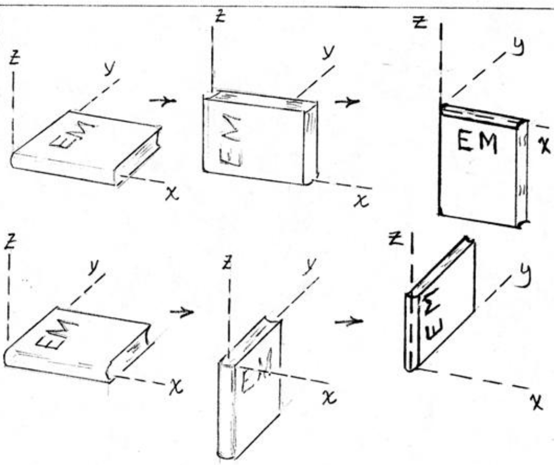
7/1 Line O-1 can rotate through π radians (180°) about any axis through O which lies in the x-z plane.
Line O-2 can rotate through π radians about any axis through O which lies in a plane perpendicular to the line from 2 to 2'.
The intersection of these planes is the unique axis along the 45° line in the x-z plane. Thus $\underline{\theta} = \pi\left(\frac{\underline{i}}{\sqrt{2}} + \frac{\underline{k}}{\sqrt{2}}\right)$

$$\underline{\theta} = \frac{\pi}{\sqrt{2}}(\underline{i} + \underline{k})$$

7/2

First Sequence (x, y)

Second Sequence (y, x)

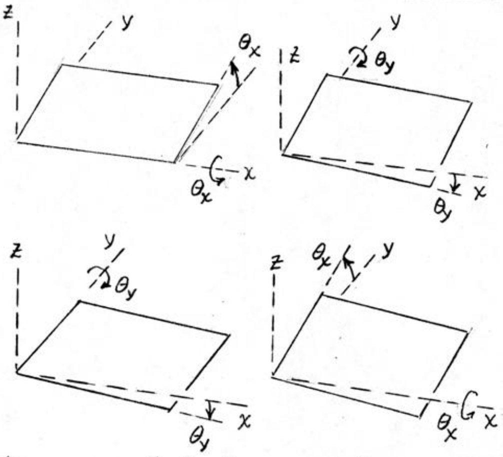


Final positions are different so finite rotations cannot be added as proper vectors

7/3

First Sequence

Second Sequence



Final positions essentially the same - the more so the smaller the angle. Infinitesimal angles add as proper vectors.

7/4 $\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$, $\underline{r} = \vec{OC}$, $\dot{\underline{\omega}} = \underline{0}$

$\underline{r} = 10(2\underline{i} + 0\underline{j} + 8\underline{k})$ mm, $\underline{\omega} = 30(3\underline{i} + 2\underline{j} + 6\underline{k})$ rad/s

$$\underline{v} = \underline{\omega} \times \underline{r} = 300 \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 2 & 6 \\ 2 & 0 & 8 \end{vmatrix} = 300(16\underline{i} - 12\underline{j} - 4\underline{k}) \ \frac{\text{mm}}{\text{s}}$$

$$\underline{a} = \underline{\omega} \times \underline{v} = 30(300) \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & 2 & 6 \\ 16 & -12 & -4 \end{vmatrix} = 9000(64\underline{i} + 108\underline{j} - 68\underline{k}) \ \text{mm/s}^2$$

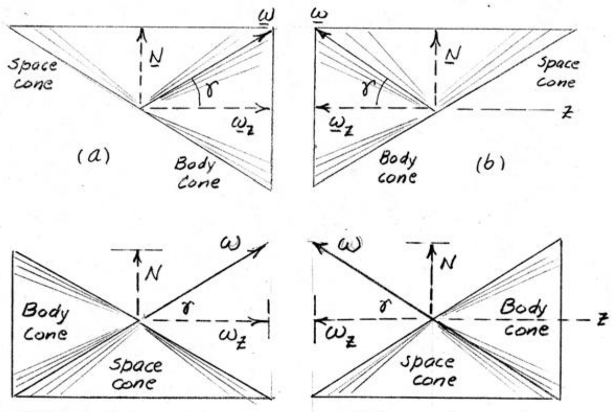
$$a = 9\sqrt{64^2 + 108^2 + (-68)^2} = 9\sqrt{20\,384} = \underline{1285 \text{ m/s}^2}$$

7/5 $v_A = \underline{\omega} \times \underline{r} = (-4\underline{j} - 3\underline{k}) \times (0.5\underline{i} + 1.2\underline{j} + 1.1\underline{k})$
$= \underline{-0.8\underline{i} - 1.5\underline{j} + 2\underline{k} \text{ m/s}}$

The rim speed of any point B is

$$v_B = \sqrt{0.8^2 + 1.5^2 + 2^2} = \underline{2.62 \text{ m/s}}$$

7/6 $\tan\gamma = \frac{N}{\omega_z} = \frac{10}{15} = 0.667$, $\gamma = 33.7°$



7/7 $\underline{r}_{OA} = \underline{r} = 0.260\underline{i} + 0.240\underline{j} + 0.473\underline{k}$ m

Unit vector along OB is

$\underline{n} = (0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k})/\sqrt{0.2^2 + 0.4^2 + 0.3^2}$

$\underline{\omega} = \omega\underline{n} = \frac{1200(2\pi)}{60} \ \frac{0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}}{0.539}$

$= 233(0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k})$ rad/s

$\underline{v} = \underline{\omega} \times \underline{r} = 233(0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}) \times (0.260\underline{i} + 0.240\underline{j} + 0.473\underline{k})$

$= 233(0.1172\underline{i} - 0.0166\underline{j} - 0.0566\underline{k})$ m/s

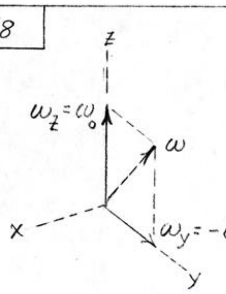
$= \underline{27.3\underline{i} - 3.87\underline{j} - 13.07\underline{k} \text{ m/s}}$

$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = 0 + \underline{\omega} \times \underline{v}$

$= 233(0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}) \times (27.3\underline{i} - 3.87\underline{j} - 13.07\underline{k})$

$= \underline{-949\underline{i} + 2520\underline{j} - 2730\underline{k} \text{ m/s}^2}$

7/8



$\underline{\underline{\omega} = p\underline{j} + \omega_o\underline{k}}$

$\underline{\alpha} = \underline{\omega}_z \times \underline{\omega} = \underline{\omega}_z \times \underline{\omega}_y$

$= \omega_o\underline{k} \times p\underline{j}$

$\underline{\underline{\alpha} = -p\omega_o\underline{i}}$

7/9

$\underline{\omega}\cdot\underline{v} = 0, \quad 10(\underline{i}+2\underline{j}+2\underline{k})\cdot(120\underline{i}-80\underline{j}+v_z\underline{k}) = 0$

$120 - 160 + 2v_z = 0, \quad v_z = 20$ in./sec

$v = \sqrt{120^2+80^2+20^2} = \underline{145.6 \text{ in./sec}}$

$v = R\omega, \quad R = \frac{145.6}{30} = \underline{4.85 \text{ in.}}$

where $\omega = 10\sqrt{1^2+2^2+2^2} = 10(3) = 30$ rad/sec

$\underline{a} = \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) = \underline{0} + \underline{\omega}\times\underline{v}$

$= 10(\underline{i}+2\underline{j}+2\underline{k})\times(120\underline{i}-80\underline{j}+20\underline{k})$

$= 10(200\underline{i}+220\underline{j}-320\underline{k})$

$a = 10\sqrt{200^2+220^2+320^2} = 10\sqrt{190800} = \underline{4370 \text{ in./sec}^2}$

(or simply $a = a_n = r\omega^2 = 4.85(30^2) = 4370$ in./sec^2)

7/10

$\underline{\alpha} = \underline{\Omega}\times\underline{\omega} = 0.6\underline{k}\times 2\underline{j} = \underline{-1.2\underline{i} \text{ rad/sec}^2}$

$\underline{a}_P = \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}), \quad \underline{\omega} = \underline{\Omega}+\underline{\omega}_0$

$\dot{\underline{\omega}} = \underline{\alpha} = -1.2\underline{i}$ rad/sec^2

$\underline{r} = 34\underline{j}+20\underline{k}$ in. (for $\beta = 90°$)

Carry out algebra to obtain

$\underline{a}_P = 35.8\underline{j} - 80\underline{k}$ in./sec^2

7/11

$\underline{v} = \underline{\omega}\times\underline{r} \qquad \underline{a} = \underline{\omega}\times(\underline{\omega}\times\underline{r})$ where $\dot{\underline{\omega}} = \underline{0}$

$\underline{\omega} = 20(\frac{4}{5}\underline{j}+\frac{3}{5}\underline{k}) = 4(4\underline{j}+3\underline{k})$ rad/sec

$\underline{r}_A = 1.5\underline{i}+4.75\underline{j}+2\underline{k}$ in.

Thus $\underline{v} = 4(4\underline{j}+3\underline{k})\times(1.5\underline{i}+4.75\underline{j}+2\underline{k})$

$= 4(-6.25\underline{i}+4.5\underline{j}-6\underline{k})$ in./sec

$v = 4\sqrt{6.25^2+4.5^2+6^2} = 4(9.76) = \underline{39.1 \text{ in./sec}}$

$\underline{a} = \underline{\omega}\times\underline{v} = 4(4\underline{j}+3\underline{k})\times 4(-6.25\underline{i}+4.5\underline{j}-6\underline{k})$

$= 16(-37.5\underline{i}-18.75\underline{j}+25\underline{k})$

$a = 16\sqrt{37.5^2+18.75^2+25^2}$

$= 16(48.8) = \underline{781 \text{ in./sec}^2}$

$v = R\omega, \quad R = 39.1/20 = \underline{1.953 \text{ in.}}$

7/12

$\underline{\alpha} = \underline{\omega}_x\times\underline{\omega}_z = -\dot{\gamma}\underline{i}\times\omega_0\underline{k} = -3\pi\underline{i}\times 4\pi\underline{k}$

$= \underline{12\pi^2\underline{j} \text{ rad/sec}^2}$

$\underline{r} = 5\underline{j}+10\underline{k}$ in.

$\underline{v} = \underline{\omega}\times\underline{r} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3\pi & 0 & 4\pi \\ 0 & 5 & 10 \end{vmatrix} = \underline{5\pi(-4\underline{i}+6\underline{j}-3\underline{k}) \text{ in./sec}}$

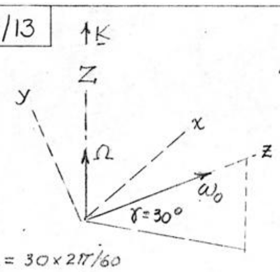
$\underline{a} = \dot{\underline{\omega}}\times\underline{r}+\underline{\omega}\times(\underline{\omega}\times\underline{r}) = \underline{\alpha}\times\underline{r}+\underline{\omega}\times\underline{v}$

$= 12\pi^2\underline{j}\times(5\underline{j}+10\underline{k}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3\pi & 0 & 4\pi \\ -4 & 6 & -3 \end{vmatrix} 5\pi$

$= 120\pi^2\underline{i} - 120\pi^2\underline{i} - 125\pi^2\underline{j} - 90\pi^2\underline{k}$

$= \underline{-5\pi^2(25\underline{j}+18\underline{k}) \text{ in./sec}^2}$

7/13



$\Omega = 30\times 2\pi/60$
$= \pi$ rad/sec

$\omega_0 = \alpha_0 t$

when $t = 2$ sec, $\omega_0 = \frac{3000(2\pi)}{60}$
$= 100\pi$ rad/sec

So $\alpha_0 = 100\pi/2 = 50\pi$ rad/sec^2

$\omega_0 = 50\pi t$

$\underline{\omega}_0 = 50\pi t\underline{k}$ rad/sec

$\underline{\alpha} = \dot{\underline{\omega}}_0 = 50\pi\underline{k} + 50\pi t\dot{\underline{k}}$

But $\dot{\underline{k}} = \underline{\Omega}\times\underline{k}$
$= \pi\underline{K}\times\underline{k}$
$= (\sqrt{3}/2)\pi\underline{i}$

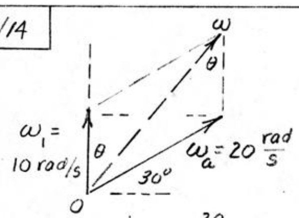
Thus for $t = 1/3$ s,

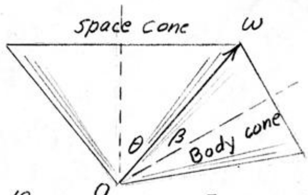
$\underline{\alpha} = 50\pi\underline{k} + 50\pi(\frac{1}{3})(\sqrt{3}/2)\pi\underline{i}$

$= \underline{50\pi(\frac{\pi}{2\sqrt{3}}\underline{i}+\underline{k}) \text{ rad/sec}^2}$

(Note: Total angular velocity is $\underline{\omega} = \underline{\Omega}+\underline{\omega}_0$
& $\underline{\alpha} = \dot{\underline{\omega}} = \dot{\underline{\Omega}}+\dot{\underline{\omega}}_0 = \underline{0}+\dot{\underline{\omega}}_0$)

7/14

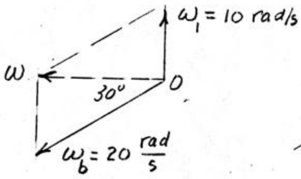


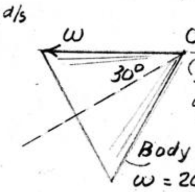


Law of sines $\frac{20}{\sin\theta} = \frac{10}{\sin(60-\theta)}$, $\theta = \tan^{-1}\frac{\sqrt{3}}{2} = 40.9°$

$\omega = \sqrt{(20\cos 30°)^2+(20\sin 30°+10)^2}$, $\beta = 60°-\theta = 19.1°$

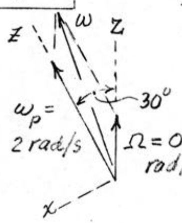
$\underline{\omega = 26.5 \text{ rad/s}}$





$\omega = 20\sin 60° = \underline{17.32 \text{ rad/s}}$

7/15



$\underline{\omega} = \underline{\omega}_p + \underline{\Omega}$

$= 2\underline{k} + 0.8\cos 30°\underline{k} - 0.8\sin 30°\underline{i}$

$= \underline{-0.4\underline{i}+2.69\underline{k} \text{ rad/s}}$

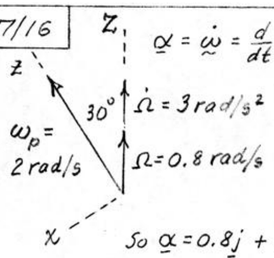
$\underline{\alpha} = \underline{\Omega}\times\underline{\omega}_p$

$= 0.8(-0.5\underline{i}+0.866\underline{k})\times 2\underline{k}$

$= 1.6(0.5\underline{j}+0)$

$\underline{\alpha} = \underline{0.8\underline{j} \text{ rad/s}^2}$

7/16



$\underline{\alpha} = \dot{\underline{\omega}} = \frac{d}{dt}(\underline{\omega}_p+\underline{\Omega}) = \underline{\Omega}\times\underline{\omega}_p + \dot{\underline{\Omega}}$

$\underline{\Omega}\times\underline{\omega}_p =$

$0.8(\cos 30°\underline{k} - \sin 30°\underline{i})\times 2\underline{k}$

$= 0.8\underline{j}$ rad/s^2

So $\underline{\alpha} = 0.8\underline{j} + 3(\cos 30°\underline{k} - \sin 30°\underline{i})$

$= \underline{-1.5\underline{i}+0.8\underline{j}+2.60\underline{k} \text{ rad/s}^2}$

7/17

$\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_2 = 2\underline{k}+1.5\underline{i}$

$\omega = \sqrt{2^2+1.5^2} = \underline{2.5 \text{ rad/s}}$

$\underline{\alpha} = \underline{\omega}_1\times\underline{\omega}_2 = 2\underline{k}\times 1.5\underline{i} = \underline{3\underline{j} \text{ rad/s}^2}$

7/18 $\underline{\omega} = \underline{\omega}_1 + \underline{\omega}_5$

$= 2\underline{k} + 0.8(\underline{j}\cos 30^\circ + \underline{k}\sin 30^\circ)$

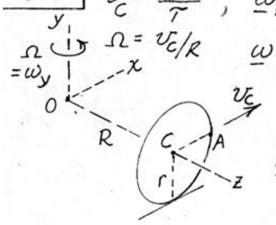
$\underline{\omega} = 0.693\underline{j} + 2.40\underline{k}$ rad/s

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega}_5 = 2\underline{k} \times 0.8(\underline{j}\cos 30^\circ + \underline{k}\sin 30^\circ)$

$\underline{\alpha} = -1.386\,\underline{i}$ rad/s²

7/19 $v_C = \frac{2\pi R}{\tau}$; $\underline{\omega}_z = -\frac{v_c}{r}\underline{k} = -\frac{2\pi R}{\tau r}\underline{k}$

$\Omega = v_C/R$ $\quad \underline{\omega} = \underline{\omega}_y + \underline{\omega}_z = \frac{v_c}{R}\underline{j} - \frac{v_c}{r}\underline{k}$

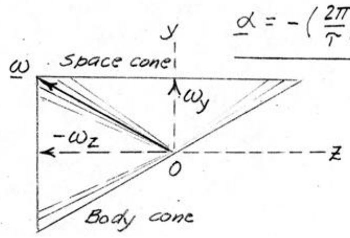


$|\underline{\omega}| = const.$ so $\underline{\alpha} = \underline{\Omega} \times \underline{\omega}$

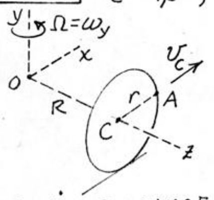
$\underline{\alpha} = \frac{v_c}{R}\underline{j} \times \left(\frac{v_c}{R}\underline{j} - \frac{v_c}{r}\underline{k}\right)$

$= -\frac{v_c^2}{rR}\underline{i} = -\left(\frac{2\pi R}{\tau}\right)^2 \frac{1}{rR}\underline{i}$

$\underline{\alpha} = -\left(\frac{2\pi}{\tau}\right)^2 \frac{R}{r}\underline{i}$



7/20 $v_C = \frac{2\pi R}{\tau}$; $\omega_y = \Omega = \frac{v_c}{R}$, $\omega_z = -\frac{v_c}{R}$



$\underline{\omega} = \underline{\omega}_y + \underline{\omega}_z = v_c\left(\frac{1}{R}\underline{j} - \frac{1}{r}\underline{k}\right)$

$\underline{v}_A = \underline{\omega} \times \underline{r}_A = v_c\left(\frac{1}{R}\underline{j} - \frac{1}{r}\underline{k}\right) \times (r\underline{i} + R\underline{k})$

$= \frac{2\pi R}{\tau}\left(\underline{i} - \underline{j} - \frac{r}{R}\underline{k}\right)$

$\underline{a} = \dot{\underline{\omega}} \times \underline{r}_A + \underline{\omega} \times \underline{v}_A$

$\dot{\underline{\omega}} = v_c\left(\frac{1}{R}\dot{\underline{j}} - \frac{1}{r}\dot{\underline{k}}\right) = v_c\left(0 - \frac{1}{r}[\Omega\underline{j} \times \underline{k}]\right) = -\left(\frac{2\pi}{\tau}\right)^2 \frac{R}{r}\underline{i}$

$\dot{\underline{\omega}} \times \underline{r}_A = -\left(\frac{2\pi}{\tau}\right)^2 \frac{R}{r}\underline{i} \times (r\underline{i} + R\underline{k}) = \left(\frac{2\pi}{\tau}\right)^2 \frac{R^2}{r}\underline{j}$

$\underline{\omega} \times \underline{v}_A = \frac{2\pi R}{\tau}\left(\frac{1}{R}\underline{j} - \frac{1}{r}\underline{k}\right) \times \frac{2\pi R}{\tau}\left(\underline{i} - \underline{j} - \frac{r}{R}\underline{k}\right)$

$= \left(\frac{2\pi}{\tau}\right)^2 R^2\left(-\left[\frac{1}{r} + \frac{r}{R^2}\right]\underline{i} - \frac{1}{r}\underline{j} - \frac{1}{R}\underline{k}\right)$

Combine & get $\quad \underline{a} = -\left(\frac{2\pi}{\tau}\right)^2 R\left[\left(\frac{R}{r} + \frac{r}{R}\right)\underline{i} + \underline{k}\right]$

7/21 $\underline{r} = \vec{OB} = -120\sin 30^\circ\underline{i} + 120\cos 30^\circ\underline{j} + 200\underline{k}$ mm

$= -60\underline{i} + 103.9\underline{j} + 200\underline{k}$ mm

$\underline{\omega} = \underline{\omega}_x + \underline{\omega}_z = 10\underline{i} + 20\underline{k}$ rad/s

$\underline{v} = \underline{\omega} \times \underline{r} = 10(\underline{i} + 2\underline{k}) \times (-60\underline{i} + 103.9\underline{j} + 200\underline{k})$

$= 10(-208\underline{i} - 320\underline{j} + 103.9\underline{k})$

$v = 10\sqrt{208^2 + 320^2 + 103.9^2} = 3950$ mm/s

or $v = 3.95$ m/s

$a = \dot{\omega} \times r + \omega \times (\omega \times r)$

where $\dot{\underline{\omega}} = \underline{\alpha} = \underline{\omega}_x \times \underline{\omega} = \underline{\omega}_x \times \underline{\omega}_z = 10\underline{i} \times 20\underline{k} = -200\underline{j}$ rad/s²

$\dot{\underline{\omega}} \times \underline{r} = -200\underline{j} \times (-60\underline{i} + 103.9\underline{j} + 200\underline{k})$

$= -4000(10\underline{i} + 3\underline{k})$ mm/s²

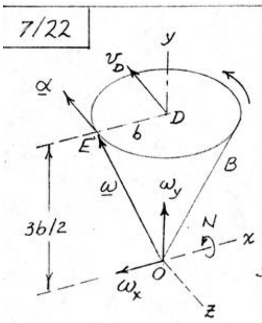
$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\omega} \times \underline{v} = 10(\underline{i} + 2\underline{k}) \times 10(-208\underline{i} - 320\underline{j} + 103.9\underline{k})$

$= 100(640\underline{i} - 520\underline{j} - 320\underline{k})$

$\underline{a} = 24.0\underline{i} - 52.0\underline{j} - 44.0\underline{k}$ m/s²

$a = \sqrt{24.0^2 + 52.0^2 + 44.0^2} = 72.2$ m/s²

7/22 $v_D = \frac{3b}{\ell}\frac{60(2\pi)}{60} = 3\pi b$; $v_E = 0$ (Gear C fixed)



$\omega_y = \frac{v_D}{b} = 3\pi$ rad/s

$\omega_x = -\frac{60(2\pi)}{60} = -2\pi$ rad/s

$\underline{\omega} = -2\pi\underline{i} + 3\pi\underline{j} = \pi(-2\underline{i} + 3\underline{j})$ rad/s

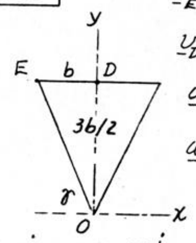
$\underline{\alpha} = \dot{\underline{\omega}}$; $\dot{\underline{i}} = 0$, $\dot{\underline{j}} = \underline{\omega}_x \underline{j} = -2\pi\underline{k}$

so $\underline{\alpha} = 0 + 3\pi(-2\pi\underline{k}) = -6\pi^2\underline{k}$ rad/s²

Body cone is the pitch cone of gear B

Space " " " " " " C

7/23 $\underline{v}_E = \frac{3b}{2}\frac{20(2\pi)}{60}(-\underline{k}) = -b\pi\underline{k}$



$\underline{v}_D = \frac{3b}{2}\frac{60(2\pi)}{60}(-\underline{k}) = -3b\pi\underline{k}$

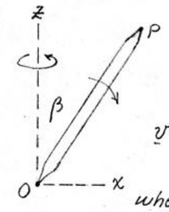
$\underline{\omega}_y = \frac{v_D - v_E}{b}\underline{j} = \frac{3\pi - \pi}{b}b\underline{j} = 2\pi\underline{j}$ rad/s

$\underline{\omega}_x = -\frac{60(2\pi)}{60}\underline{i} = -2\pi\underline{i}$ rad/s

$\underline{\omega} = -2\pi\underline{i} + 2\pi\underline{j} = 2\pi(-\underline{i} + \underline{j})$ rad/s

$\underline{\alpha} = \dot{\underline{\omega}} = 0 + 2\pi\dot{\underline{j}} = 2\pi(-2\pi\underline{k}) = -4\pi^2\underline{k}$ rad/s²

7/24 $\overline{OP} = 24$ m, $\dot{\beta} = 0.10$ rad/s const., $\beta = 30^\circ$



$\underline{r} = \vec{OP} = (24\sin 30^\circ)\underline{i} + (24\cos 30^\circ)\underline{k}$

$= 12\underline{i} + 20.78\underline{k}$ m

$\underline{\omega} = \frac{2(2\pi)}{60}\underline{k} + 0.10\underline{j} = 0.209\underline{k} + 0.10\underline{j}$ rad/s

$\underline{v} = \underline{\omega} \times \underline{r} = (0.209\underline{k} + 0.10\underline{j}) \times (12\underline{i} + 20.78\underline{k})$

$= 2.078\underline{i} + 2.513\underline{j} - 1.2\underline{k}$ m/s

where $v = |\underline{v}| = \sqrt{(2.078)^2 + (2.513)^2 + (-1.2)^2} = 3.48$ m/s

$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\alpha} \times \underline{r} + \underline{\omega} \times \underline{v}$

$\underline{\alpha} = \dot{\underline{\omega}} = \underline{\omega}_z \times \underline{\omega}_y = 0.209\underline{k} \times 0.10\underline{j} = -0.0209\underline{i}$ rad/s²

$\dot{\underline{\omega}} \times \underline{r} = \underline{\alpha} \times \underline{r} = -0.0209\underline{i} \times (12\underline{i} + 20.78\underline{k}) = 0.435\underline{j}$ m/s²

$\underline{\omega} \times \underline{v} = (0.209\underline{k} \times 0.10\underline{j}) \times (2.078\underline{i} + 2.513\underline{j} - 1.2\underline{k})$

$= -0.646\underline{i} + 0.435\underline{j} - 0.208\underline{k}$ m/s²

$\underline{a} = -0.646\underline{i} + 0.870\underline{j} - 0.208\underline{k}$ m/s²

$a = |\underline{a}| = \sqrt{(-0.646)^2 + (0.870)^2 + (-0.208)^2} = 1.104$ m/s²

7/25 $\underline{\omega} = \Omega\underline{k} + \dot{\gamma}\underline{i} - \omega_0\cos\gamma\,\underline{j} - \omega_0\sin\gamma\,\underline{k}$

$\underline{\alpha} = \dot{\underline{\omega}} = \Omega\dot{\underline{k}} + \dot{\gamma}\dot{\underline{i}} + \omega_0\dot{\gamma}\sin\gamma\,\underline{j} - \omega_0\cos\gamma\,\dot{\underline{j}}$

$-\omega_0\dot{\gamma}\cos\gamma\,\underline{k} - \omega_0\sin\gamma\,\dot{\underline{k}}$

where $\Omega = 4$ rad/s const.

$\omega_0 = 3$ rad/s " $\quad \gamma = 30^\circ$

$\dot{\gamma} = -\pi/4$ rad/s "

& $\dot{\underline{i}} = \underline{\Omega} \times \underline{i} = \Omega\underline{k} \times \underline{i} = \Omega\underline{j}$; $\dot{\underline{j}} = \underline{\Omega} \times \underline{j} = \Omega\underline{k} \times \underline{j} = -\Omega\underline{i}$; $\dot{\underline{k}} = \Omega\underline{k} \times \underline{k} = 0$

so $\underline{\alpha} = 0 + \dot{\gamma}\Omega\underline{j} + \omega_0\dot{\gamma}\sin\gamma\,\underline{j} + \omega_0\Omega\cos\gamma\,\underline{i} - \omega_0\dot{\gamma}\cos\gamma\,\underline{k} + 0$

$= \omega_0\Omega\cos\gamma\,\underline{i} + \dot{\gamma}(\Omega + \omega_0\sin\gamma)\underline{j} - \omega_0\dot{\gamma}\cos\gamma\,\underline{k}$

$= 3(4)(0.866)\underline{i} - \frac{\pi}{4}(4 + 3\times 0.5)\underline{j} + 3(\pi/4)(0.866)\underline{k}$

$= 10.392\underline{i} - 4.320\underline{j} + 2.040\underline{k}$ rad/s²

$\alpha = |\underline{\alpha}| = \sqrt{(10.392)^2 + (4.320)^2 + (2.040)^2} = 11.44$ rad/s²

$\underline{\omega} = -\frac{\pi}{4}\underline{i} - 3(0.866)\underline{j} + (4 - 3\times 0.5)\underline{k}$

$= -0.785\underline{i} - 2.60\underline{j} + 2.5\underline{k}$ rad/s

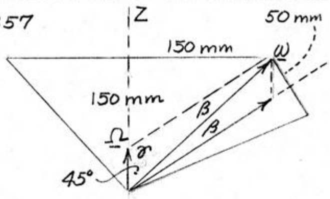
▶7/26 $\sin\beta = \frac{50}{150\sqrt{2}} = 0.2357$

$\beta = 13.63°$

$\Omega = \frac{2\pi}{4} = \pi/2$ rad/s

Law of sines

$\frac{\omega}{\sin\gamma} = \frac{\Omega}{\sin\beta}$, $\omega = \Omega\frac{\sin\gamma}{\sin\beta}$



$|\underline{\alpha}| = |\underline{\Omega}\times\underline{\omega}| = \Omega\omega\sin 45° = \Omega^2\frac{\sin\gamma}{\sin\beta}\sin 45°$

$\sin\gamma = \sin(180-45-13.63) = 0.8539$

so $\alpha = (\frac{\pi}{2})^2\frac{0.8539}{0.2357}0.7071 = \underline{6.32 \text{ rad/s}^2}$

▶7/27 For $t=0$ $\theta=0$ and position vector of B is $\underline{r} = 4\underline{i} - 8\underline{k}$ in.

$\omega_x = -\dot\theta = -\frac{\pi}{6}3\pi\cos 3\pi t = -\frac{\pi^2}{2}$ rad/sec for $t=0$

$\omega_z = 2\pi$ rad/sec

$\underline{\omega} = \omega_x\underline{i} + \omega_z\underline{k} = -\frac{\pi^2}{2}\underline{i} + 2\pi\underline{k}$ rad/sec for $t=0$

$\underline{v} = \underline{\omega}\times\underline{r} = (-\frac{\pi^2}{2}\underline{i} + 2\pi\underline{k})\times(4\underline{i} - 8\underline{k}) = -4\pi^2\underline{j} + 8\pi\underline{j} = 4\pi(2-\pi)\underline{j}$ in./sec

or $\underline{v} = -14.35\underline{j}$ in./sec

$\underline{a} = \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r})$

$\dot{\underline{\omega}} = \dot\omega_x\underline{i} + \omega_x\dot{\underline{i}} + \dot\omega_z\underline{k} + \omega_z\dot{\underline{k}} = +\frac{\pi^2}{2}(3\pi)\sin 3\pi t\,\underline{i} - \frac{\pi^2}{2}\cos 3\pi t(\omega_z\underline{j}) + \underline{0} + \underline{0}$

$\dot{\underline{\omega}}_{t=0} = \underline{0} - \frac{\pi^2}{2}2\pi\underline{j} = -\pi^3\underline{j}$, $\underline{\alpha} = \dot{\underline{\omega}} = -\pi^3\underline{j} = \underline{-31.0\underline{j} \text{ rad/sec}^2}$

so $\underline{a} = -\pi^3\underline{j}\times(4\underline{i} - 8\underline{k}) + (-\frac{\pi^2}{2}\underline{i} + 2\pi\underline{k})\times 4\pi(2-\pi)\underline{j}$

$= 16\pi^2(\pi-1)\underline{i} + 2\pi^4\underline{k}$ in./sec²

$\underline{a} = 338\underline{i} + 194.8\underline{k}$ in./sec²

▶7/28 Angular velocity $\underline{\omega}$ of link A cannot have a component along the y-axis so $\underline{\omega}\cdot\underline{j} = 0$. A vector in the j-direction is $\underline{h}\times\underline{n}$ as is $\underline{h}\times(\underline{r}\times\underline{h})$. The magnitude is immaterial. Thus

$\underline{\omega}\cdot(\underline{h}\times\underline{n}) = 0$ or $\underline{\omega}\cdot(\underline{h}\times[\underline{r}\times\underline{h}]) = 0$

or $\underline{\omega}\cdot\underline{h}\times(\underline{r}\times\underline{h}) = 0$

7/29 $\underline{p} = \omega\cos 20°\underline{k} = 30(0.9397)\underline{k}$ rad/s

$p = 28.2$ rad/s

$\underline{v}_{B/A} = \underline{\omega}\times\underline{r}_{B/A} = \underline{\omega}_y\times\underline{r}_{B/A} = 30\sin 20°\underline{j}\times 0.4\underline{k}$

$= 4.10\underline{i}$ m/s

7/30 Angular velocity of rotor is

$\underline{\omega} = p\underline{k} - q\underline{i}$, $\underline{\alpha} = \dot{\underline{\omega}} = p\dot{\underline{k}} - q\dot{\underline{i}} = \underline{\Omega}\times(p\underline{k} - q\underline{i})$

where $\underline{\Omega}$ = angular velocity of axes $= -q\underline{i}$

Thus $\underline{\alpha} = -q\underline{i}\times(p\underline{k} - q\underline{i}) = \underline{pq\underline{j}}$

or from Eq. 7/7, $\underline{\alpha} = (\frac{d\underline{\omega}}{dt})_{XYZ} = \underline{0} + \underline{\Omega}\times\underline{\omega}$

$= -q\underline{i}\times(p\underline{k} - q\underline{i}) = \underline{pq\underline{j}}$

7/31 $\underline{\omega} = \underline{\Omega} + \underline{p} = 4\underline{i} + 10\underline{k}$, $\omega = \sqrt{4^2+10^2} = 10.77\ \frac{\text{rad}}{\text{s}}$

$\underline{\alpha} = \underline{\Omega}\times\underline{p} = 4\underline{i}\times 10\underline{k} = \underline{-40\underline{j} \text{ rad/s}^2}$

7/32 $\underline{\alpha} = \frac{d}{dt}\underline{\omega} = \frac{d}{dt}(\underline{\Omega} + \underline{p}) = 0 + \frac{d}{dt}(p\underline{k})$

$= \dot p\underline{k} + p\dot{\underline{k}} = \dot p\underline{k} + p(\underline{\Omega}\times\underline{k}) = \dot p\underline{k} + p\Omega(-\underline{j})$

$\underline{\alpha} = 6\underline{k} - 10(4)\underline{j} = \underline{-40\underline{j} + 6\underline{k} \text{ rad/s}^2}$

7/33 Angular velocity of x-y-z axes is $\underline{\Omega} = 4\underline{i}$ rad/s

$\underline{v}_A = \underline{v}_C + \underline{\Omega}\times\underline{r}_{A/C} + \underline{v}_{rel}$

$\underline{v}_C = 0.4(4)(-\underline{j}) = -1.6\underline{j}$ m/s

$\underline{\Omega}\times\underline{r}_{A/C} = 4\underline{i}\times 0.3\underline{j} = 1.2\underline{k}$ m/s

$\underline{v}_{rel} = 0.3(10)(-\underline{i}) = -3\underline{i}$ m/s

So $\underline{v}_A = -1.6\underline{j} + 1.2\underline{k} - 3\underline{i}$, $\underline{v_A = -3\underline{i} - 1.6\underline{j} + 1.2\underline{k} \text{ m/s}}$

$\underline{a}_A = \underline{a}_C + \dot{\underline{\Omega}}\times\underline{r}_{A/C} + \underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/C}) + 2\underline{\Omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_C = 0.4(4^2)(-\underline{k}) = -6.4\underline{k}$ m/s², $\dot{\underline{\Omega}} = \underline{0}$

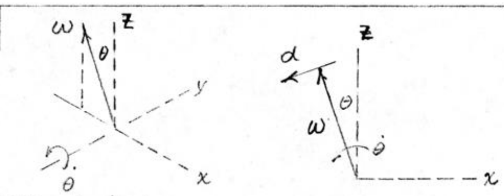
$\underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/C}) = 4\underline{i}\times 1.2\underline{k} = -4.8\underline{j}$ m/s²

$2\underline{\Omega}\times\underline{v}_{rel} = 2(4\underline{i})\times(-3\underline{i}) = \underline{0}$

$\underline{a}_{rel} = 0.3(10^2)(-\underline{j}) = -30\underline{j}$ m/s²

So $\underline{a}_A = -6.4\underline{k} - 4.8\underline{j} - 30\underline{j}$, $\underline{a_A = -34.8\underline{j} - 6.4\underline{k} \text{ m/s}^2}$

7/34



$\omega = \frac{2\pi N}{60} = \frac{2\pi(360)}{60} = 12\pi$ rad/s

$\underline{\alpha} = -\dot\theta\underline{j}\times\underline{\omega} = -0.2\underline{j}\times 12\pi(-\sin\theta\underline{i} + \cos\theta\underline{k})$

$= 2.4\pi(-0.5\underline{k} - 0.866\underline{i})$

$= \underline{-1.2\pi(\sqrt{3}\underline{i} + \underline{k}) \text{ rad/s}^2}$

7/35 $\overline{OB} = \sqrt{7^2 - 2^2 - 3^2} = 6$ ft; $\underline{v}_A = -3\underline{i}$ ft/sec

$\underline{v}_B = \underline{v}_A + \underline{\omega}_n\times\underline{r}_{B/A}$, $\underline{r}_{B/A} = -2\underline{i} - 3\underline{j} + 6\underline{k}$ ft

so $v_B\underline{k} = -3\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -2 & -3 & 6 \end{vmatrix}$; Equate coefficients of $\underline{i}, \underline{j}, \underline{k}$ terms & get

$1 = 2\omega_y + \omega_z$, $\omega_z = -3\omega_x$, $v_B = -3\omega_x + 2\omega_y$

Eliminate ω's & get $\underline{v}_B = 1.0\underline{k}$ ft/sec

Now ω_n is ⊥ to AB, so $\underline{\omega}_n\cdot\underline{r}_{B/A} = 0$ which gives $-2\omega_x - 3\omega_y + 6\omega_z = 0$. Combine with above & get

$\omega_x = -3/49$ rad/sec, $\omega_y = 20/49$ rad/sec, $\omega_z = 9/49$ rad/sec

so $\underline{\omega}_n = \frac{1}{49}(-3\underline{i} + 20\underline{j} + 9\underline{k})$ rad/sec

(Alternative solution for v_B

$x^2 + y^2 + z^2 = \ell^2$, $x\dot x + y\dot y + z\dot z = 0$; $\dot y = 0$, $\dot x = -3$ ft/sec

so $\dot z = -\frac{x\dot x}{z} = -\frac{2(-3)}{6} = 1.0$ ft/sec)

7/36 Angular velocity of OA is $\underline{\omega} = -\dot\beta\underline{i} + p\sin\beta\underline{j} + (p\cos\beta + \Omega)\underline{k}$

Eq. 7/7a, $[\,] = \underline{\omega}$, $(\frac{d[\,]}{dt})_{XYZ} = (\frac{d[\,]}{dt})_{xyz} + \underline{\Omega}\times[\,]$

$(\frac{d\underline{\omega}}{dt})_{xyz} = 0 + p\dot\beta\cos\beta\underline{j} + (-p\dot\beta\sin\beta + 0)\underline{k}$

$\underline{\Omega}\times\underline{\omega} = \Omega\underline{k}\times(-\dot\beta\underline{i} + p\sin\beta\underline{j} + [p\cos\beta + \Omega]\underline{k})$

$= -\Omega\dot\beta\underline{j} - \Omega p\sin\beta\underline{i}$

so $\underline{\alpha} = (p\dot\beta\cos\beta - \Omega\dot\beta)\underline{j} - \Omega p\sin\beta\underline{i} - p\dot\beta\sin\beta\underline{k}$

$\underline{\alpha} = -\Omega p\sin\beta\underline{i} + \dot\beta(p\cos\beta - \Omega)\underline{j} - p\dot\beta\sin\beta\underline{k}$

7/37 $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$\underline{a}_A = \underline{a}_B + \dot{\underline{\omega}} \times \underline{r}_{A/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{A/B})$

$\underline{\omega} = 1.4\underline{i} + 1.2\underline{j}$ rad/sec ; $\dot{\underline{\omega}} = 2\underline{i} + 3\underline{j}$ rad/sec^2

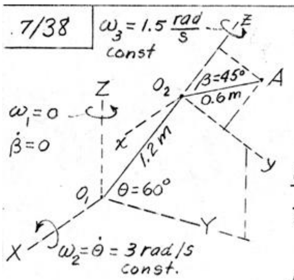
$\underline{r}_{A/B} = 5\underline{i}$ ft , $\underline{v}_B = 3.2\underline{j}$ ft/sec , $\underline{a}_B = 4\underline{j}$ ft/sec^2

Substitution and simplification yield

$\underline{v}_A = 3.2\underline{j} - 6\underline{k}$ ft/sec $\Rightarrow$ $v_A = 6.8$ ft/sec

$\underline{a}_A = -7.2\underline{i} + 12.4\underline{j} - 15\underline{k}$ ft/sec^2 $\Rightarrow$ $a_A = 20.8$ ft/sec^2

7/38



Attach axes x-y-z with origin at O_2 and x parallel to X. So x-y-z axes have angular velocity $\underline{\Omega} = \dot{\theta}\underline{i} = 3\underline{i}$ rad/s

$\underline{v}_A = \underline{v}_{O_2} + \underline{\Omega} \times \underline{r}_{A/O_2} + \underline{v}_{rel}$

$\underline{v}_{O_2} = \underline{\omega}_2 \times \underline{r}_{O_2} = 3\underline{i} \times 1.2\underline{k} = -3.6\underline{j}$ m/s

$\underline{v}_{rel} = \underline{\omega}_3 \times \underline{r}_{A/O_2} = 1.5\underline{k} \times \frac{0.6}{\sqrt{2}}(\underline{j} + \underline{k})$
$= -0.636\underline{i}$ m/s

$\underline{\Omega} \times \underline{r}_{A/O_2} = 3\underline{i} \times \frac{0.6}{\sqrt{2}}(\underline{j} + \underline{k}) = 1.273(\underline{k} - \underline{j})$ $\frac{m}{s}$

So $\underline{v}_A = -3.6\underline{j} + 1.273(\underline{k} - \underline{j}) - 0.636\underline{i} = -0.636\underline{i} - 4.873\underline{j} + 1.273\underline{k}$ m/s

7/39 Sol. I $X^2 + Y^2 + Z^2 = L^2$

$X\dot{X} + Y\dot{Y} + 0 = 0$, $Z =$ const, $L =$ const.

$\dot{Y} = v_A = -\frac{X}{Y}\dot{X} = -\frac{0.3}{0.2}4 = -6$ m/s $(-Y\text{-dir.})$

Sol. II $\underline{v}_A = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$, $\underline{\omega} \cdot \underline{r}_{A/B} = 0$ taking $\omega \perp AB$

$$v_A\underline{j} = 4\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -0.3 & 0.2 & 0.6 \end{vmatrix}$$

$(\underline{i}\omega_x + \underline{j}\omega_y + \underline{k}\omega_z) \cdot (-0.3\underline{i} + 0.2\underline{j} + 0.6\underline{k}) = 0$

Expand, equate coefficients & get

$0.6\omega_y - 0.2\omega_z = -4$ (1)

$-0.6\omega_x - 0.3\omega_z = v_A$ (2)

$0.2\omega_x + 0.3\omega_y = 0$ (3)

$-0.3\omega_x + 0.2\omega_y + 0.6\omega_z = 0$ (4)

Solve simultaneously & get

$\omega_x = 7.35$ rad/s, $\omega_y = -4.90$ rad/s, $\omega_z = 5.31$ rad/s

$\underline{v}_A = -6\underline{j}$ m/s

7/40 Angular velocity of axes is $\underline{\Omega} = p\underline{k}$

" " " A " $\underline{\omega} = \underline{\Omega} - \dot{\beta}\underline{i}$

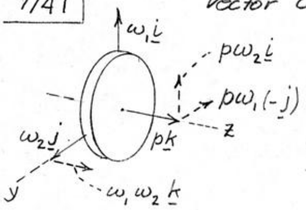
$\underline{\alpha} = \dot{\underline{\omega}} = \dot{\underline{\Omega}} - \ddot{\beta}\underline{i} - \dot{\beta}\dot{\underline{i}} = \dot{\underline{\Omega}} - \ddot{\beta}\underline{i} - \dot{\beta}\,\underline{\Omega} \times \underline{i}$
$= 0 - \ddot{\beta}\underline{i} - \dot{\beta}p\underline{j}$

(a) before; $\dot{\beta}\,d\dot{\beta} = \ddot{\beta}\,d\beta$, $\ddot{\beta} = \dot{\beta}\frac{d\dot{\beta}}{d\beta} = \left(2\frac{2\pi}{360}\right)\frac{2}{18}$
$= 0.00388$ rad/s^2

$\underline{\alpha} = -0.00388\underline{i} - \frac{2\pi}{180}\frac{1}{10}\underline{j} = -(3.88\underline{i} + 3.49\underline{j})10^{-3}$ $\frac{rad}{s^2}$

(b) after; $\ddot{\beta} = 0$, $\underline{\alpha} = -3.49(10^{-3})\underline{j}$ rad/s^2

7/41



Vector $\omega_1\underline{i}$ does not change orientation so $\frac{d}{dt}(\underline{i}\omega_1) = 0$

Accel. components give

$\underline{\alpha} = p\omega_2\underline{i} - p\omega_1\underline{j} + \omega_1\omega_2\underline{k}$

7/42 Let γ = angle between AB & y-axis

Angular velocity of AB is $\underline{\omega} = -\dot{\gamma}\underline{i} + \Omega\underline{k}$

So $\underline{\alpha} = \dot{\underline{\omega}} = -\ddot{\gamma}\underline{i} - \dot{\gamma}\dot{\underline{i}} + \underline{0}$

But $z = \ell\sin\gamma$, $v_A = \dot{z} = \ell\dot{\gamma}\cos\gamma$

& $\dot{v}_A = 0 = -\ell\dot{\gamma}^2\sin\gamma + \ell\ddot{\gamma}\cos\gamma$

So $\dot{\gamma} = \frac{v_A}{\ell\cos\gamma} = \frac{8}{5(4/5)} = 2$ rad/sec

$\ddot{\gamma} = \dot{\gamma}^2\tan\gamma = 2^2(3/4) = 3$ rad/sec^2

Also $\dot{\underline{i}} = \Omega\underline{k} \times \underline{i} = \Omega\underline{j} = 2\underline{j}$ rad/sec

Thus $\underline{\alpha} = -3\underline{i} - 2(2\underline{j}) = -3\underline{i} - 4\underline{j}$ rad/sec^2

7/43 $\underline{\Omega}$ = angular velocity of axes x-y-z $= \frac{2\pi N}{60}\underline{j} = \pi\underline{j}$ $\frac{rad}{s}$

$\underline{v} = \underline{v}_A = \underline{v}_B + \underline{\Omega} \times \underline{r}_{A/B} + \underline{v}_{rel}$

where $\underline{v}_B = \pi\underline{j} \times \underline{r}_{OB} = \pi\underline{j} \times (-0.18\underline{i} + 0.1\underline{k}) = \pi(0.1\underline{i} + 0.18\underline{k})$ m/s

$\underline{\Omega} \times \underline{r}_{A/B} = \pi\underline{j} \times 0.1\underline{i} = -0.1\pi\underline{k}$ m/s

$\underline{v}_{rel} = p\underline{k} \times \underline{r}_{A/B} = \frac{240(2\pi)}{60}\underline{k} \times 0.1\underline{i} = 0.8\pi\underline{j}$ m/s

Collect terms & get $\underline{v} = \pi(0.1\underline{i} + 0.8\underline{j} + 0.08\underline{k})$ m/s

$\underline{a} = \underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}} \times \underline{r}_{A/B} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$; $\dot{\underline{\Omega}} = \underline{0}$

where $\underline{a}_B = \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{B/O}) = \pi\underline{j} \times (\pi\underline{j} \times [-0.18\underline{i} + 0.1\underline{k}])$
$= \pi^2(0.18\underline{i} - 0.1\underline{k})$ m/s^2

$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) = \pi\underline{j} \times (-0.1\pi\underline{k}) = -0.1\pi^2\underline{i}$ m/s^2

$2\underline{\Omega} \times \underline{v}_{rel} = 2\pi\underline{j} \times 0.8\pi\underline{j} = \underline{0}$

$\underline{a}_{rel} = \dot{p}\underline{k} \times \underline{r}_{A/B} + \underline{r}_{A/B}\,p^2(-\underline{i}) = \underline{0} - (8\pi)^2 0.1\underline{i} = -6.4\pi^2\underline{i}$ $\frac{m}{s^2}$

Collect terms & get

$\underline{a} = -0.1\pi^2\underline{i} - 6.4\pi^2\underline{i} + 0.18\pi^2\underline{i} - 0.1\pi^2\underline{k}$

$\underline{a} = -\pi^2(6.32\underline{i} + 0.1\underline{k})$ m/s^2

7/44 $\underline{\Omega}$ = angular velocity of disk & axes x-y-z
$= \frac{2\pi}{60}(N\underline{j} + p\underline{k}) = \frac{\pi}{30}(30\underline{j} + 240\underline{k}) = \pi(\underline{j} + 8\underline{k})$ rad/s

$\underline{v} = \underline{v}_A = \underline{v}_B + \underline{\Omega} \times \underline{r}_{A/B} + \underline{v}_{rel}$

Where $\underline{v}_B = \pi(0.1\underline{i} + 0.18\underline{k})$ m/s from Prob. 7/43

$\underline{\Omega} \times \underline{r}_{A/B} = \pi(\underline{j} + 8\underline{k}) \times 0.1\underline{i} = \pi(0.8\underline{j} - 0.1\underline{k})$ m/s

$\underline{v}_{rel} = \underline{0}$

Thus $\underline{v} = \pi(0.1\underline{i} + 0.18\underline{k}) + \pi(0.8\underline{j} - 0.1\underline{k})$
$= \pi(0.1\underline{i} + 0.8\underline{j} + 0.08\underline{k})$ m/s (agrees with 7/43)

$\underline{a} = \underline{a}_A = \underline{a}_B + \dot{\underline{\Omega}} \times \underline{r}_{A/B} + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) + 2\underline{\Omega} \times \underline{v}_{rel} + \underline{a}_{rel}$

where $\dot{\underline{\Omega}} = \pi(\dot{\underline{j}} + 8\dot{\underline{k}})$ with $\dot{\underline{j}} = \underline{\Omega} \times \underline{j} = \pi(\underline{j} + 8\underline{k}) \times \underline{j} = -8\pi\underline{i}$

$\dot{\underline{k}} = \underline{\Omega} \times \underline{k} = \pi(\underline{j} + 8\underline{k}) \times \underline{k} = \pi\underline{i}$

so $\dot{\underline{\Omega}} = \pi(-8\pi\underline{i} + 8\pi\underline{i}) = \underline{0}$

$\underline{\Omega} \times (\underline{\Omega} \times \underline{r}_{A/B}) = \pi(\underline{j} + 8\underline{k}) \times \pi(0.8\underline{j} - 0.1\underline{k}) = -6.5\pi^2\underline{i}$ m/s^2

$2\underline{\Omega} \times \underline{v}_{rel} = 2\pi(\underline{j} + 8\underline{k}) \times \underline{0} = \underline{0}$

$\underline{a}_{rel} = \underline{0}$

Collect terms & get

$\underline{a} = -\pi^2(6.32\underline{i} + 0.1\underline{k})$ m/s^2 (agrees with 7/43)

7/45 From Eqs. 7/6

$\mathbf{v}_A = \mathbf{v}_O + \boldsymbol{\Omega}\times\mathbf{r}_{A/O} + \mathbf{v}_{rel}$

$\mathbf{v}_O = -R\Omega\mathbf{i},\ \boldsymbol{\Omega} = \Omega\mathbf{k},\ \mathbf{r}_{A/O} = b\sin\beta\,\mathbf{j} + b\cos\beta\,\mathbf{k},\ \mathbf{v}_{rel} = b\dot\beta(\cos\beta\,\mathbf{j} - \sin\beta\,\mathbf{k})$

$\mathbf{v}_A = -R\Omega\mathbf{i} + \Omega\mathbf{k}\times b(\sin\beta\,\mathbf{j} + \cos\beta\,\mathbf{k}) + b\dot\beta(\cos\beta\,\mathbf{j} - \sin\beta\,\mathbf{k})$

$\underline{\mathbf{v}_A = -\Omega(R + b\sin\beta)\mathbf{i} + b\dot\beta\cos\beta\,\mathbf{j} - b\dot\beta\sin\beta\,\mathbf{k}}$

$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\Omega}}\times\mathbf{r}_{A/O} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{A/O}) + 2\boldsymbol{\Omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}$

$\mathbf{a}_O = -R\Omega^2\mathbf{j},\ \dot{\boldsymbol{\Omega}} = \mathbf{0},\ \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{A/O}) = \Omega\mathbf{k}\times(\Omega\mathbf{k}\times b[\sin\beta\,\mathbf{j} + \cos\beta\,\mathbf{k}])$

$2\boldsymbol{\Omega}\times\mathbf{v}_{rel} = 2\Omega\mathbf{k}\times b\dot\beta(\cos\beta\,\mathbf{j} - \sin\beta\,\mathbf{k}),\ \mathbf{a}_{rel} = b\dot\beta^2(\sin\beta\,\mathbf{j} + \cos\beta\,\mathbf{k})$

Combine, collect terms, & get

$\underline{\mathbf{a}_A = -2b\Omega\dot\beta\cos\beta\,\mathbf{i} - (\Omega^2[R + b\sin\beta] + b\dot\beta^2\sin\beta)\mathbf{j} - b\dot\beta^2\cos\beta\,\mathbf{k}}$

7/46 Precession is steady so $\boldsymbol{\alpha} = \boldsymbol{\Omega}\times\mathbf{p}$

$\boldsymbol{\alpha} = 4\pi\mathbf{k}\times 10\pi\mathbf{j} = \underline{-40\pi^2\mathbf{i}\ \text{rad/s}^2}$

$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\Omega}}\times\mathbf{r}_{A/O} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{A/O}) + 2\boldsymbol{\Omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}$

$\mathbf{a}_O = \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_O) = -r_O\Omega^2\mathbf{i} = -0.3(4\pi)^2\mathbf{i} = -4.8\pi^2\mathbf{i}\ \text{m/s}^2$

$\dot{\boldsymbol{\Omega}} = 0;\ \boldsymbol{\Omega}\times\mathbf{r}_{A/O} = 4\pi\mathbf{k}\times 0.1\mathbf{k} = \mathbf{0}$

$\mathbf{v}_{rel} = \mathbf{p}\times\mathbf{r}_{A/O} = 10\pi\mathbf{j}\times 0.1\mathbf{k} = \pi\mathbf{i}\ \text{m/s}$

$2\boldsymbol{\Omega}\times\mathbf{v}_{rel} = 2(4\pi\mathbf{k})\times\pi\mathbf{i} = 8\pi^2\mathbf{j}\ \text{m/s}^2$

$\mathbf{a}_{rel} = \mathbf{p}\times(\mathbf{p}\times\mathbf{r}_{A/O}) = -0.1(10\pi)^2\mathbf{k} = -10\pi^2\mathbf{k}\ \text{m/s}^2$

$\mathbf{a}_A = -4.8\pi^2\mathbf{i} + 8\pi^2\mathbf{j} - 10\pi^2\mathbf{k}$

$= \underline{2\pi^2(-2.4\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})\ \text{m/s}^2}$

7/47 Angular velocity of axes $\boldsymbol{\Omega} = \Omega\mathbf{k}$

" " " panels $\boldsymbol{\omega} = -\dot\theta\mathbf{j} + \Omega\mathbf{k}$

$\dot{\boldsymbol{\omega}} = -\dot\theta\dot{\mathbf{j}} + \Omega\dot{\mathbf{k}} = -\dot\theta(\boldsymbol{\Omega}\times\mathbf{j}) + \Omega(\boldsymbol{\Omega}\times\mathbf{k}) = \boldsymbol{\Omega}\times\boldsymbol{\omega} = \Omega\dot\theta\mathbf{i}$

$= \frac{1}{2}\frac{1}{4}\mathbf{i} = \underline{\frac{1}{8}\mathbf{i}\ \text{rad/sec}^2}$

$\mathbf{a}_A = \mathbf{a}_O + \dot{\boldsymbol{\Omega}}\times\mathbf{r}_{A/O} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{A/O}) + 2\boldsymbol{\Omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}$

$\mathbf{a}_O = \mathbf{0};\quad \boldsymbol{\Omega}\times\mathbf{r}_{A/O} = \frac{1}{2}\mathbf{k}\times(-\mathbf{i} + 8\mathbf{j} + \sqrt{3}\mathbf{k}) = -\frac{1}{2}\mathbf{j} - 4\mathbf{i}\ \frac{\text{ft}}{\text{sec}}$

$\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{A/O}) = \frac{1}{2}\mathbf{k}\times(-\frac{1}{2}\mathbf{j} - 4\mathbf{i}) = \frac{1}{4}\mathbf{i} - 2\mathbf{j}\ \text{ft/sec}^2$

$2\boldsymbol{\Omega}\times\mathbf{v}_{rel} = 2(\frac{1}{2}\mathbf{k})\times(-\frac{\sqrt{3}}{4}\mathbf{i} - \frac{1}{4}\mathbf{k}) = -\frac{\sqrt{3}}{4}\mathbf{j}\ \text{ft/sec}^2$

$\mathbf{a}_{rel} = 2(\frac{1}{4})^2(\frac{1}{2}\mathbf{i} - \frac{\sqrt{3}}{2}\mathbf{k}) = \frac{1}{16}\mathbf{i} - \frac{\sqrt{3}}{16}\mathbf{k}\ \text{ft/sec}^2$

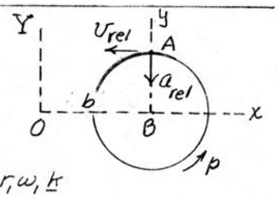
$\mathbf{a}_A = (\frac{1}{4} + \frac{1}{16})\mathbf{i} + (-2 - \frac{\sqrt{3}}{4})\mathbf{j} - \frac{\sqrt{3}}{16}\mathbf{k}$

$= \underline{0.313\mathbf{i} - 2.43\mathbf{j} - 0.1083\mathbf{k}\ \text{ft/sec}^2}$

with $a_A = 2.45\ \text{ft/sec}^2$

7/48 Angular velocity of x-y-z axes is

$\boldsymbol{\Omega} = -\omega_1\mathbf{i} + \omega_2\mathbf{J}$



$\mathbf{v} = \mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\Omega}\times\mathbf{r}_{A/B} + \mathbf{v}_{rel}$

where $\mathbf{v}_B = b\omega_2(-\mathbf{k}) = -b\omega_2\mathbf{k}$

$\boldsymbol{\Omega}\times\mathbf{r}_{A/B} = (-\omega_1\mathbf{i} + \omega_2\mathbf{J})\times r\mathbf{j} = -r\omega_1\mathbf{k}$

$\mathbf{v}_{rel} = -rp\mathbf{i}$

Thus $\mathbf{v} = -b\omega_2\mathbf{k} - r\omega_1\mathbf{k} - rp\mathbf{i} = \underline{-rp\mathbf{i} - (r\omega_1 + b\omega_2)\mathbf{k}}$

$\mathbf{a} = \mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\Omega}}\times\mathbf{r}_{A/B} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{A/B}) + 2\boldsymbol{\Omega}\times\mathbf{v}_{rel} + \mathbf{a}_{rel}$

where $\mathbf{a}_B = -b\omega_2^2\mathbf{i}$

$\dot{\boldsymbol{\Omega}} = -\omega_1\dot{\mathbf{i}} + \omega_2\dot{\mathbf{J}} = -\omega_1\boldsymbol{\Omega}\times\mathbf{i} = \omega_1\omega_2\mathbf{k}$

$\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{A/B}) = (-\omega_1\mathbf{i} + \omega_2\mathbf{J})\times(-r\omega_1\mathbf{k}) = -r\omega_1(\omega_1\mathbf{j} + \omega_2\mathbf{i})$

$2\boldsymbol{\Omega}\times\mathbf{v}_{rel} = 2(-\omega_1\mathbf{i} + \omega_2\mathbf{J})\times(-rp\mathbf{i}) = 2rp\omega_2\mathbf{k}$

$\mathbf{a}_{rel} = -rp^2\mathbf{j},\quad \dot{\boldsymbol{\Omega}}\times\mathbf{r}_{A/B} = \omega_1\omega_2\mathbf{k}\times r\mathbf{j} = -r\omega_1\omega_2\mathbf{i}$

Substitute, combine & get

$\underline{\mathbf{a} = -\omega_2(b\omega_2 + 2r\omega_1)\mathbf{i} - r(\omega_1^2 + p^2)\mathbf{j} + 2rp\omega_2\mathbf{k}}$

▸ 7/49 From Sample Problem 7/2

$\Omega = 2\pi$ rad/sec, $\omega_y = \sqrt{3}\pi$ rad/sec, $\omega_z = 5\pi$ rad/sec, $\omega_0 = 4\pi\frac{\text{rad}}{\text{sec}}$

Also $\omega_x = -\dot\gamma = -3\pi$ rad/sec

In general $\boldsymbol{\omega} = (-\dot\gamma\mathbf{i} + \Omega\cos\gamma\,\mathbf{j} + [\omega_0 + \Omega\sin\gamma]\mathbf{k})$

For $\gamma = 30°$, $\underline{\boldsymbol{\omega} = \pi(-3\mathbf{i} + \sqrt{3}\mathbf{j} + 5\mathbf{k})\ \text{rad/sec}}$

From Eq. 7/7 $\boldsymbol{\alpha} = [d\boldsymbol{\omega}/dt]_{XYZ} = [d\boldsymbol{\omega}/dt]_{xyz} + \boldsymbol{\omega}_{axes}\times\boldsymbol{\omega}$

But $[d\boldsymbol{\omega}/dt]_{xyz} = (\mathbf{0} - \Omega\dot\gamma\sin\gamma\,\mathbf{j} + \Omega\dot\gamma\cos\gamma\,\mathbf{k})$

$= 6\pi^2(-\frac{1}{2}\mathbf{j} + \frac{\sqrt{3}}{2}\mathbf{k}) = 3\pi^2(-\mathbf{j} + \sqrt{3}\mathbf{k})\ \text{rad/sec}^2$

$\boldsymbol{\omega}_{axes} = \boldsymbol{\omega} - \omega_0\mathbf{k}$ & $\boldsymbol{\omega}_{axes}\times\boldsymbol{\omega} = (\boldsymbol{\omega} - \omega_0\mathbf{k})\times\boldsymbol{\omega} = -\omega_0\mathbf{k}\times\boldsymbol{\omega}$

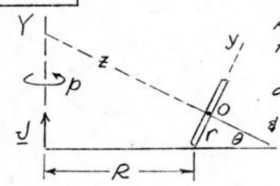
so $\boldsymbol{\omega}_{axes}\times\boldsymbol{\omega} = -4\pi\mathbf{k}\times\pi(-3\mathbf{i} + \sqrt{3}\mathbf{j} + 5\mathbf{k}) = 4\pi^2(\sqrt{3}\mathbf{i} + 3\mathbf{j})\ \frac{\text{rad}}{\text{sec}^2}$

Thus $\boldsymbol{\alpha} = 3\pi^2(-\mathbf{j} + \sqrt{3}\mathbf{k}) + 4\pi^2(\sqrt{3}\mathbf{i} + 3\mathbf{j})$

$= \underline{\pi^2(4\sqrt{3}\mathbf{i} + 9\mathbf{j} + 3\sqrt{3}\mathbf{k})\ \text{rad/sec}^2}$

▸ 7/50

$\boldsymbol{\omega} = \mathbf{J}p + \frac{Rp}{r}\mathbf{k} = (p\cos\theta)\mathbf{j} + (p\sin\theta + \frac{Rp}{r})\mathbf{k}$



Angle $d\phi$ measured in x-y-z turned by wheel in time dt is

$d\phi = \frac{R(p\,dt)}{r}$ so $\dot\phi = \frac{Rp}{r}$

& $\underline{\boldsymbol{\omega} = p[\mathbf{j}\cos\theta + \mathbf{k}(\sin\theta + \frac{R}{r})]}$

Angular velocity of axes is $\boldsymbol{\Omega} = \mathbf{J}p$ so

$\boldsymbol{\omega} = \boldsymbol{\Omega} + (\frac{Rp}{r})\mathbf{k}$; Now use $\left(\frac{d[\ ]}{dt}\right)_{XYZ} = \left(\frac{d[\ ]}{dt}\right)_{xyz} + \boldsymbol{\Omega}\times[\ ]$

Noting $\boldsymbol{\Omega}$ is constant in XYZ & xyz.

Thus $\boldsymbol{\alpha} = \left(\frac{d\boldsymbol{\omega}}{dt}\right)_{XYZ} = 0 + \boldsymbol{\Omega}\times[\boldsymbol{\Omega}\times\frac{Rp}{r}\mathbf{k}] = \boldsymbol{\Omega}\times\frac{Rp}{r}\mathbf{k}$

$\alpha = [(p\cos\theta)\mathbf{j} + (p\sin\theta)\mathbf{k}]\times\frac{Rp}{r}\mathbf{k}$, $\underline{\boldsymbol{\alpha} = (\frac{Rp^2}{r}\cos\theta)\mathbf{i}}$

or merely $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = 0 + \frac{Rp}{r}\dot{\mathbf{k}} = \frac{Rp}{r}(\boldsymbol{\Omega}\times\mathbf{k})$, etc.

►7/51 Angular velocity of axes $= \underline{\Omega}$

" " " rotor $= \underline{\omega} = \underline{\Omega} + p\underline{k}$

where $p = 100(2\pi)/60 = 10\pi/3$ rad/s

$\underline{\Omega} = -\dot{\gamma}\underline{i} + \underline{j}\,\omega_1 \cos\gamma + \underline{k}\,\omega_1 \sin\gamma$, $\omega_1 = \frac{2\pi}{60}20 = \frac{2\pi}{3}\frac{\text{rad}}{\text{s}}$

$\underline{\alpha} = \left(\frac{d\underline{\omega}}{dt}\right)_{XYZ} = \left(\frac{d\underline{\omega}}{dt}\right)_{xyz} + \underline{\Omega}\times\underline{\omega}$ (Eq. 8/7)

$\left(\frac{d\underline{\omega}}{dt}\right)_{xyz} = \left(\frac{d\underline{\Omega}}{dt}\right)_{xyz} + 0 = 0 - \underline{j}\dot{\gamma}\omega_1 \sin\gamma + \underline{k}\dot{\gamma}\omega_1\cos\gamma$

$\underline{\Omega}\times\underline{\omega} = \underline{\Omega}\times(\underline{\Omega}+p\underline{k}) = \underline{\Omega}\times p\underline{k} = \dot{\gamma}p\underline{j} + p\omega_1\cos\gamma\,\underline{i}$

$\underline{\alpha} = (\dot{\gamma}p - \dot{\gamma}\omega_1\sin\gamma)\underline{j} + p\omega_1\cos\gamma\,\underline{i} + \dot{\gamma}\omega_1\cos\gamma\,\underline{k}$

substitute $\dot{\gamma} = 4$ rad/s, $p = 10\pi/3$ rad/s, $\omega_1 = 2\pi/3$ rad/s & get

$\underline{\alpha} = \left(4\frac{10\pi}{3} - 4\frac{2\pi}{3}\frac{1}{2}\right)\underline{j} + \frac{10\pi}{3}\frac{2\pi}{3}\frac{\sqrt{3}}{2}\underline{i} + 4\frac{2\pi}{3}\frac{\sqrt{3}}{2}\underline{k}$

$= 12\pi\underline{j} + \frac{10\pi^2}{3\sqrt{3}}\underline{i} + \frac{4\pi}{\sqrt{3}}\underline{k} = 18.99\underline{i} + 37.70\underline{j} + 7.25\underline{k}\ \frac{\text{rad}}{\text{s}^2}$

$\alpha = \sqrt{18.99^2 + 37.70^2 + 7.25^2} = \underline{42.8 \text{ rad/s}^2}$

►7/52 $\underline{v}_A = \underline{v}_B + \underline{\omega}_n \times \underline{r}_{A/B}$ where $\underline{\omega}_n \cdot \underline{r}_{A/B} = 0$

$200^2 + 300^2 + z^2 = 700^2$, $z = 600$ mm

$\underline{r}_{A/B} = 100(3\underline{i} + 2\underline{j} - 6\underline{k})$ mm

$$2\underline{j} = v_B\underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_{n_x} & \omega_{n_y} & \omega_{n_z} \\ 3 & 2 & -6 \end{vmatrix}(0.1) \quad \text{m/s}$$

Equate coefficients of like terms and get

$\omega_{n_z} + 3\omega_{n_y} = 0$, $2\omega_{n_x} + \omega_{n_z} = 20/3$, $v_B = -0.2\omega_{n_x} + 0.3\omega_{n_y}$

Eliminate ω_n's & get $v_B = -\frac{2}{3}$ m/s, $\underline{\underline{v}_B = -\frac{2}{3}\underline{k}\text{ m/s}}$

$(\omega_{n_x}\underline{i} + \omega_{n_y}\underline{j} + \omega_{n_z}\underline{k})\cdot(3\underline{i} + 2\underline{j} - 6\underline{k}) = 0$

$3\omega_{n_x} + 2\omega_{n_y} - 6\omega_{n_z} = 0$. Combine with above & get

$\omega_{n_x} = \frac{1}{3}\frac{400}{49}$, $\omega_{n_y} = -\frac{20}{49}$, $\omega_{n_z} = \frac{60}{49}$ rad/s

$\underline{\underline{\omega}_n = \frac{10}{49}\left(\frac{40}{3}\underline{i} - 2\underline{j} + 6\underline{k}\right)\text{ rad/s}}$

7/53 $\omega_x = \omega_y = 0$, $\omega_z = \omega$

For these conditions, Eq. 7/11 is

$\underline{H} = \omega\left[-I_{xz}\underline{i} - I_{yz}\underline{j} + I_{zz}\underline{k}\right]$

$$\begin{cases} I_{xz} = 0 \\ I_{yz} = mR\left(\frac{L}{3}\right) - mR\left(\frac{2L}{3}\right) = -mRL/3 \\ I_{zz} = 2mR^2 \end{cases}$$

So $\underline{\underline{H} = mR\omega\left[\frac{L}{3}\underline{j} + 2R\underline{k}\right]}$

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_o = \frac{1}{2}\omega\underline{k}\cdot mR\omega\left[\frac{L}{3}\underline{i} + 2R\underline{k}\right] = \underline{mR^2\omega^2}$

(By inspection, $T = \frac{1}{2}I_{zz}\omega_z^2 = \frac{1}{2}(2mR^2)\omega^2 = \underline{mR^2\omega^2}$)

7/54 x-y-z are principal axes so

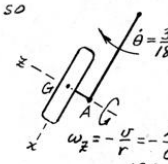
$\underline{H} = I_{xx}\omega_x\underline{i} + I_{yy}\omega_y\underline{j} + I_{zz}\omega_z\underline{k}$

$I_{zz} = mk^2$

$= 45(0.370)^2 = 6.16$ kg·m²

$I_{xx} + I_{yy} = I_{zz}$ & $I_{xx} = I_{yy}$

so $I_{yy} = \frac{1}{2}I_{zz} = 3.08$ kg·m²



$\dot{\theta} = \frac{30}{180}\pi = 0.524$ rad/s $= -\omega_y$

$\omega_z = -\frac{v}{r} = -\frac{200(10^3)/3600}{0.920/2} = -120.8$ rad/s

$\omega_x = 0$

About G, $\underline{H}_G = \underline{0} + 3.08(-0.524)\underline{j} + 6.16(-120.8)\underline{k}$

$\underline{\underline{H}_G = -1.613\underline{j} - 744\underline{k}\ \text{kg·m}^2/\text{s}}$

About A, $I_{yy} = \bar{I}_{yy} + md^2 = 3.08 + 45(0.215)^2 = 5.16$ kg·m²

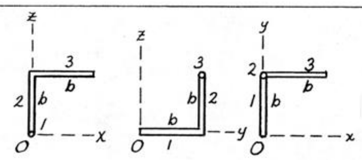
$\underline{H}_A = \underline{0} + 5.16(-0.524)\underline{j} + 6.16(-120.8)\underline{k}$

$\underline{\underline{H}_A = -2.70\underline{j} - 744\underline{k}\ \text{kg·m}^2/\text{s}}$

7/55 With $\omega_x = \omega_y = 0$,

Eq. 7/11 gives

$\underline{H}_o = (-I_{xz}\underline{i} - I_{yz}\underline{j} + I_{zz}\underline{k})\omega$



Part	I_{xz}	I_{yz}	I_{zz}
1	0	0	$\frac{1}{3}\rho b^3$
2	0	$\frac{1}{2}\rho b^3$	ρb^3
3	$\frac{1}{2}\rho b^3$	ρb^3	$\frac{4}{3}\rho b^3$
Totals	$\frac{1}{2}\rho b^3$	$\frac{3}{2}\rho b^3$	$\frac{8}{3}\rho b^3$

$\left\{(I_{zz})_3 = \frac{1}{12}\rho b\, b^2 + \rho b\left(b^2 + \left[\frac{b}{2}\right]^2\right) = \frac{4}{3}\rho b^3\right\}$

so $\underline{\underline{H}_o = \rho b^3\left(-\frac{1}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{8}{3}\underline{k}\right)\omega}$

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_o = \frac{1}{2}\omega\cdot\frac{8}{3}\rho b^3\omega$, $\underline{T = \frac{4}{3}\rho b^3\omega^2}$

7/56 From Eq. 7/14 using O for A,

$\underline{H}_o = \underline{H}_G + \bar{\underline{r}}\times m\bar{\underline{v}}$ where $\bar{\underline{r}} = \Sigma m\underline{r}/\Sigma m$

$\bar{r}_x = \rho b(0 + 0 + \frac{b}{2})/3\rho b = b/6$, $\bar{r}_y = \rho b(\frac{b}{2} + b + b)/3\rho b = \frac{5}{6}b$,

$\bar{r}_z = \rho b(0 + \frac{b}{2} + b)/3\rho b = \frac{1}{2}b$

$\bar{\underline{v}} = \underline{\omega}\times\bar{\underline{r}} = \omega\underline{k}\times b(\frac{1}{6}\underline{i} + \frac{5}{6}\underline{j} + \frac{1}{2}\underline{k}) = \frac{\omega b}{6}(-5\underline{i} + \underline{j})$

$\bar{\underline{r}}\times m\bar{\underline{v}} = b(\frac{1}{6}\underline{i} + \frac{5}{6}\underline{j} + \frac{1}{2}\underline{k})\times 3\rho b\left(\frac{\omega b}{6}\right)(-5\underline{i} + \underline{j}) = \frac{\rho b^3\omega}{4}(-\underline{i} - 5\underline{j} + \frac{26}{3}\underline{k})$

From Prob. 7/55 $\underline{H}_o = \rho b^3(-\frac{1}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{8}{3}\underline{k})\omega$

Thus $\underline{H}_G = \underline{H}_o - \bar{\underline{r}}\times m\bar{\underline{v}} = \rho b^3\omega(-\frac{1}{2}\underline{i} - \frac{3}{2}\underline{j} + \frac{8}{3}\underline{k} + \frac{1}{4}\underline{i} + \frac{5}{4}\underline{j} - \frac{13}{6}\underline{k})$

$\underline{\underline{H}_G = \rho b^3\omega(-\frac{1}{4}\underline{i} - \frac{1}{4}\underline{j} + \frac{1}{2}\underline{k})}$, $\underline{\underline{H}_G = \frac{1}{4}\rho b^3\omega(-\underline{i} - \underline{j} + 2\underline{k})}$

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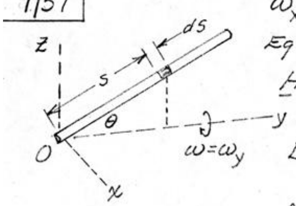


$\omega_x = \omega_z = 0$, $\omega_y = \omega$, so

Eq. 7/11 gives

$\underline{H} = (-\underline{i}I_{xy} + \underline{j}I_{yy} - \underline{k}I_{yz})\omega$

$\omega = \omega_y$ But $I_{xy} = 0$

$I_{yy} = \frac{1}{3}m(l\sin\theta)^2$

& $I_{yz} = \int yz\,dm = \int_0^l (s\cos\theta)(s\sin\theta)\rho\,ds$

where ρ = mass per unit length

so $I_{yz} = \rho\sin\theta\cos\theta\frac{l^3}{3} = \frac{1}{3}ml^2\sin\theta\cos\theta$

& $\underline{H} = \left[\underline{i}(0) + \underline{j}\frac{1}{3}ml^2\sin^2\theta - \underline{k}\frac{1}{3}ml^2\sin\theta\cos\theta\right]\omega$

$= \underline{\frac{1}{3}ml^2\omega\sin\theta(\underline{j}\sin\theta - \underline{k}\cos\theta)}$

7/58 $\omega_x = \omega_y = 0, \quad \omega_z = \omega$

$I_{xz} = 0, \; I_{yz} = 0 + m\left(\frac{4r}{3\pi}\right)\left(c + \frac{b}{2}\right), \; I_{zz} = \frac{1}{2}mr^2$

So $\underline{H} = -I_{yz}\omega_z\underline{j} + I_{zz}\omega_z\underline{k}$

$$\underline{H} = mr\omega\left[-\frac{2(2c+b)}{3\pi}\underline{j} + \frac{r}{2}\underline{k}\right]$$

7/59 Eq. 7/14: $\underline{H}_o = \underline{H}_G + \bar{\underline{r}} \times m\bar{\underline{v}}$

$\underline{H}_G = \bar{I}_{xx}\omega_x\underline{i} + \bar{I}_{yy}\omega_y\underline{j} + \bar{I}_{zz}\omega_z\underline{k}$

$\omega_x = \omega, \; \omega_y = p, \; \omega_z = 0$

$\bar{I}_{xx} = \frac{3}{20}mr^2 + \frac{3}{80}mb^2 = \bar{I}_{zz}$ (from Table D/4)

$\bar{I}_{yy} = \frac{3}{10}mr^2$

$\bar{\underline{r}} = h\underline{k} - \frac{b}{4}\underline{j}, \quad \bar{\underline{v}} = -h\omega\underline{j} - \frac{b}{4}\omega\underline{k}$

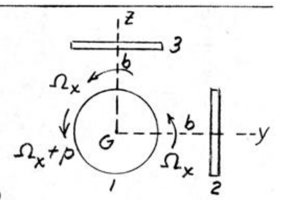
Substitution and simplification yield

$$\underline{H}_o = \left[m\omega\left(\frac{3}{20}r^2 + \frac{1}{10}b^2 + h^2\right)\underline{i} + \frac{3}{10}mr^2p\,\underline{j}\right]$$

From $T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_o$,

$$T = \frac{1}{2}m\omega^2\left(\frac{3}{20}r^2 + \frac{1}{10}b^2 + h^2\right) + \frac{3}{20}mr^2p^2$$

7/60 About G,



$H_{x_1} = I(\Omega_x + p)$

$H_{x_2} = \left(\frac{I}{2} + mb^2\right)\Omega_x$

$H_{x_3} = \left(\frac{I}{2} + mb^2\right)\Omega_x$

So $H_x = I(\Omega_x + p) + (I + 2mb^2)\Omega_x$

$\quad = Ip + 2(I + mb^2)\Omega_x$

similarly

$H_y = Ip + 2(I + mb^2)\Omega_y$

$H_z = Ip + 2(I + mb^2)\Omega_z$

Thus $\underline{H}_G = Ip(\underline{i} + \underline{j} + \underline{k}) + 2(I + mb^2)\underline{\Omega}$

where $\underline{\Omega} = \Omega_x\underline{i} + \Omega_y\underline{j} + \Omega_z\underline{k}$

7/61 $\underline{\omega} = p\underline{k} - \dot{\gamma}\underline{i} + N(\cos\gamma\,\underline{j} + \sin\gamma\,\underline{k})$

$p = 100\left(\frac{2\pi}{60}\right) = \frac{10\pi}{3}$ rad/sec, $\gamma = 30°$

$\dot{\gamma} = 4$ rad/sec, $N = 20\left(\frac{2\pi}{60}\right) = \frac{2\pi}{3}$ rad/sec

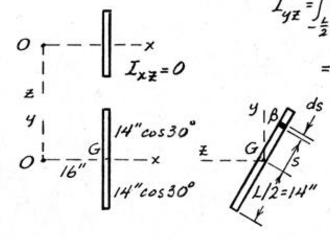
So $\underline{\omega} = -4\underline{i} + 1.814\underline{j} + 11.52\underline{k}$ rad/sec

Eq. 7/11 yields $\underline{H} = I_{xx}\omega_x\underline{i} + I_{yy}\omega_y\underline{j} + I_{zz}\omega_z\underline{k}$

With numbers: $\underline{H}_o = -0.01\underline{i} + 0.0045\underline{j} + 0.0576\underline{k}$ lb-ft-sec

7/62 From Eq. 7/11 with $\omega_x = \omega_y = 0$,

$\underline{H}_o = -I_{xz}\omega_z\underline{i} - I_{yz}\omega_z\underline{j} + I_{zz}\omega_z\underline{k}$



$I_{yz} = \int_{-L/2}^{L/2}(s\cos\beta)(-s\sin\beta)\rho\,ds$

where ρ = mass/unit length

$= -\rho\sin\beta\cos\beta\,\frac{s^3}{3}\Big|_{-L/2}^{L/2} = -\rho\frac{L^3}{24}\sin 2\beta$

$= -\frac{6.20/32.2}{28/12}\,\frac{(28/12)^3}{24}\sin 60°$

$= -0.0378$ lb-ft-sec^2

$I_{zz} = I_o = \frac{1}{12}mL^2 + md^2$

$= \frac{6.20}{32.2}\left[\left(\frac{28\cos 30°}{12}\right)^2\frac{1}{12} + \left(\frac{16}{12}\right)^2\right]$

$= 0.408$ lb-ft-sec^2

$\underline{H}_o = (-I_{xz}\underline{i} - I_{yz}\underline{j} + I_{zz}\underline{k})\omega_z = (\underline{0} - [-0.0378]\underline{j} + 0.408\underline{k})\frac{600 \times 2\pi}{60}$

$\underline{H}_o = 2.38\underline{j} + 25.6\underline{k}$ lb-ft-sec

From Eq. 7/18 $T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_o = \frac{1}{2}\omega_z\underline{k}\cdot\underline{H}_o$

$= \frac{1}{2}\,\frac{600 \times 2\pi}{60} \times 25.6 = 805$ ft-lb

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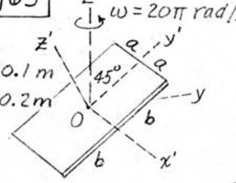


a = 0.1 m
b = 0.2 m

Introduce axes x'-y'-z'

$\omega_{x'} = 0, \; \omega_{y'} = \frac{\omega}{\sqrt{2}}, \; \omega_{z'} = \frac{\omega}{\sqrt{2}}$

$I_{x'y'} = 0, \; I_{y'y'} = \frac{1}{12}m(2a)^2 = \frac{1}{3}ma^2$

$I_{y'z'} = 0, \; I_{x'z'} = 0$

$I_{z'z'} = \frac{1}{12}m([2a]^2 + [2b]^2) = \frac{1}{3}m(a^2 + b^2)$

Eq. 7/11 applied to x'-y'-z' gives $\underline{H} = \underline{j}'I_{y'y'}\omega_{y'} + \underline{k}'I_{z'z'}\omega_{z'}$

$= \underline{j}'\left(\frac{1}{3}ma^2\right)\frac{\omega}{\sqrt{2}} + \underline{k}'\left(\frac{1}{3}m[a^2 + b^2]\right)\frac{\omega}{\sqrt{2}}$

But $\underline{j}' = \underline{j}\cos 45° + \underline{k}\sin 45° = \frac{1}{\sqrt{2}}(\underline{j} + \underline{k})$

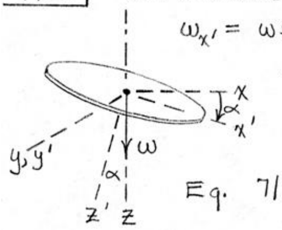
$\underline{k}' = -\underline{j}\sin 45° + \underline{k}\cos 45° = \frac{1}{\sqrt{2}}(-\underline{j} + \underline{k})$

So $\underline{H} = \frac{1}{6}m\omega\left(-b^2\underline{j} + [2a^2 + b^2]\underline{k}\right) = \frac{3}{6}20\pi\left(-0.04\underline{j} + 0.06\underline{k}\right)$

$= \pi(-0.4\underline{j} + 0.6\underline{k})$ N·m·s

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H} = \frac{1}{2}(20\pi\underline{k})\cdot\pi(-0.4\underline{j} + 0.6\underline{k}) = 6.0\pi^2 = 59.2$ ✓

7/64 Introduce axes x'-y'-z' as shown.



$\omega_{x'} = \omega\sin\alpha, \; \omega_{y'} = 0, \; \omega_{z'} = \omega\cos\alpha$

$I_{x'x'} = I_{y'y'} = \frac{1}{4}mr^2$

$I_{z'z'} = \frac{1}{2}mr^2$

Eq. 7/11 yields

$\underline{H} = \left(\frac{1}{4}mr^2\right)\omega\sin\alpha\,\underline{i}' + \left(\frac{1}{2}mr^2\right)\omega\cos\alpha\,\underline{k}'$

But $\begin{cases} \underline{i}' = \underline{i}\cos\alpha + \underline{k}\sin\alpha \\ \underline{k}' = -\underline{i}\sin\alpha + \underline{k}\cos\alpha \end{cases}$

Thus $\underline{H} = \frac{1}{4}mr^2\omega\left[(-\sin\alpha\cos\alpha)\underline{i} + (\sin^2\alpha + 2\cos^2\alpha)\underline{k}\right]$

$\beta = \cos^{-1}\left(\frac{\underline{H}\cdot\underline{k}}{H}\right) = 4.96°$ for $\alpha = 10°$

7/65 $\omega_x = \Omega,\ \omega_y = 0,\ \omega_z = p$

$I_{xx} = I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$

$I_{zz} = \frac{3}{10}mr^2,\quad I_{xy} = I_{xz} = I_{yz} = 0$

Eq. 7/11 yields $\underline{H} = I_{xx}\omega_x\underline{i} + I_{yy}\omega_y\underline{j} + I_{zz}\omega_z\underline{k}$

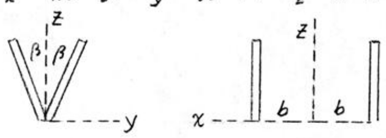
$\underline{H}_O = \left(\frac{3}{20}r^2 + \frac{3}{5}h^2\right)m\Omega\underline{i} + \frac{3}{10}mr^2p\underline{k}$

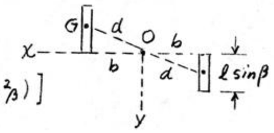
or $\underline{H}_O = \frac{3}{10}mr^2\left[\left(\frac{1}{2} + 6\frac{h^2}{r^2}\right)\Omega\underline{i} + p\underline{k}\right]$

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{3}{10}mr^2\left[\left(\frac{1}{4} + \frac{h^2}{r^2}\right)\Omega^2 + \frac{1}{2}p^2\right]$

7/66 With $\omega_x = \omega_y = 0,\ \omega_z = \omega$, the components of $\underline{H}_O$ are $H_{O_x} = -I_{xz}\omega_z,\ H_{O_y} = -I_{yz}\omega_z,\ H_{O_z} = I_{zz}\omega_z$

By inspection $I_{yz} = 0,\ I_{xz} = 0$





$I_{zz} = 2(I_G + md^2)$

$= 2\left[\frac{1}{12}m(l\sin\beta)^2 + m\left(b^2 + \frac{l^2}{4}\sin^2\beta\right)\right]$

$= 2m\left[\frac{1}{3}l^2\sin^2\beta + b^2\right]$

Thus $\underline{H}_O = 2m\left[\frac{1}{3}l^2\sin^2\beta + b^2\right]\omega\underline{k}$

7/67 Let Ω = angular velocity of x-y-z about z_0

For axes: $\Omega_x = -\Omega\sin\theta,\ \Omega_y = \dot\theta = 0,\ \Omega_z = \Omega\cos\theta;\ \Omega = 2\pi f$

Capsule: $\omega_x = -\Omega\sin\theta,\ \omega_y = 0,\ \omega_z = \Omega\cos\theta + p$

$H_{G_x} = I_{xx}\omega_x = mk'^2(-2\pi f\sin\theta),\quad H_{G_y} = I_{yy}\omega_y = 0$

$H_{G_z} = I_{zz}\omega_z = mk^2(2\pi f\cos\theta + p)$

$\underline{H}_G = 2\pi mf(-k'^2\sin\theta\,\underline{i} + k^2\cos\theta\,\underline{k}) + mk^2p\underline{k}$

7/68 $\omega_x = -\omega_1,\ \omega_y = \omega_2,\ \omega_z = p$

Eq. 7/14, $\underline{H}_O = \underline{H}_B + \vec{OB}\times\underline{G},\ \vec{OB} = b\underline{i},\ \underline{G} = m\underline{v}_B = -mb\omega_2\underline{k}$

$\vec{OB}\times\underline{G} = b\underline{i}\times(-mb\omega_2\underline{k}) = mb^2\omega_2\underline{j}$

$I_{xx} = \frac{1}{4}mr^2,\ I_{yy} = \frac{1}{4}mr^2,\ I_{zz} = \frac{1}{2}mr^2,\ I_{xy} = I_{xz} = I_{yz} = 0$

Eq. 7/11, $\underline{H}_B = \frac{1}{4}mr^2(-\omega_1)\underline{i} + \frac{1}{4}mr^2\omega_2\underline{j} + \frac{1}{2}mr^2p\underline{k}$

so $\underline{H}_O = -\frac{1}{4}mr^2\omega_1\underline{i} + m\omega_2\left(b^2 + \frac{r^2}{4}\right)\underline{j} + \frac{1}{2}mr^2p\underline{k}$

$= \frac{1}{4}mr^2\left\{-\omega_1\underline{i} + \left(1 + \frac{4b^2}{r^2}\right)\omega_2\underline{j} + 2p\underline{k}\right\}$

From Eq. 7/15 $T = \frac{1}{2}\bar{\underline{v}}\cdot m\bar{\underline{v}} + \frac{1}{2}\underline{\omega}\cdot\underline{H}_B$

So $T = \frac{1}{2}mb^2\omega_2^2 + \frac{1}{2}(-\omega_1\underline{i} + \omega_2\underline{j} + p\underline{k})\cdot\left(-\frac{1}{4}mr^2\omega_1\underline{i} + \frac{1}{4}mr^2\omega_2\underline{j} + \frac{1}{2}mr^2p\underline{k}\right)$

$= \frac{1}{2}mb^2\omega_2^2 + \frac{1}{8}mr^2(\omega_1^2 + \omega_2^2 + 2p^2)$

$= \frac{mr^2}{8}\left\{\omega_1^2 + \left(1 + \frac{4b^2}{r^2}\right)\omega_2^2 + 2p^2\right\}$

7/69 x'-y'-z' are principal axes of inertia

so $\underline{H}_{O'} = \underline{i}I_{x'x'}\omega_x + \underline{j}I_{y'y'}\omega_y + \underline{k}I_{z'z'}\omega_z$

where $I_{x'x'} = I_{z'z'} = \frac{1}{4}mr^2,\ I_{y'y'} = \frac{1}{2}mr^2$

$\omega_x = \omega,\ \omega_y = p,\ \omega_z = 0$

so $\underline{H}_{O'} = \frac{1}{4}mr^2\omega\underline{i} + \frac{1}{2}mr^2p\underline{j} = \frac{1}{2}mr^2\left(\frac{\omega}{2}\underline{i} + p\underline{j}\right)$

$= \frac{1}{2}\frac{6}{32.2}\left(\frac{4}{12}\right)^2\left(\frac{10\pi}{2}\underline{i} + 40\pi\underline{j}\right) = 0.1626(\underline{i} + 8\underline{j})$ lb-ft-sec

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_{O'} + \frac{1}{2}\bar{\underline{v}}\cdot\underline{G} = \frac{1}{2}(\omega\underline{i} + p\underline{j})\cdot\frac{1}{2}mr^2\left(\frac{\omega}{2}\underline{i} + p\underline{j}\right)$

$+ \frac{1}{2}(-\bar r\omega\underline{j})\cdot(-m\bar r\omega\underline{j})$ where $\bar r = 10\underline{k}$ in.

$= \frac{1}{4}mr^2\left(\frac{1}{2}\omega^2 + p^2\right) + \frac{1}{2}m\bar r^2\omega^2$

$= \frac{1}{4}\frac{6}{32.2}\left(\frac{4}{12}\right)^2\left(\frac{1}{2}\overline{10\pi}^2 + \overline{40\pi}^2\right) + \frac{1}{2}\frac{6}{32.2}\left(\frac{10}{12}10\pi\right)^2$

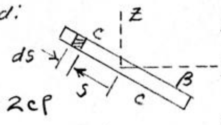
$= 84.29 + 63.85$

$= 148.1$ ft-lb

7/70 With $\omega_x = \omega_y = 0$ & $\omega_z = \omega$, the components of angular momentum become

$H_{O_x} = -I_{xz}\omega_z,\ H_{O_y} = -I_{yz}\omega_z,\ H_{O_z} = I_{zz}\omega_z$

Rod:



Let ρ = mass per unit length

$dI_{yz} = yz\,dm = yz\rho\,ds$

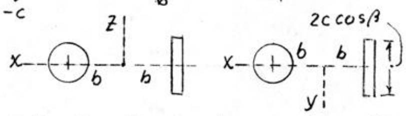
$m = 2c\rho$ $= (-s\cos\beta)(s\sin\beta)\rho\,ds$

$I_{yz} = -\rho\sin\beta\cos\beta\int_{-c}^{c}s^2ds = -\frac{1}{6}mc^2\sin2\beta$

Sphere: $I_{yz} = 0$

By inspection

$I_{xz} = I_{xy} = 0$



Sphere $I_{zz} = \frac{2}{5}mr^2 + mb^2$, Rod $I_{zz} = \frac{1}{12}m(2c\cos\beta)^2 + mb^2$

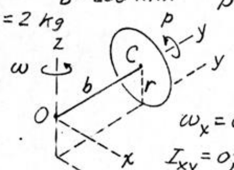
Thus $\underline{H}_O = 0\underline{i} - \left(-\frac{1}{6}mc^2\sin2\beta\right)\omega\underline{j} + \left(\frac{2}{5}mr^2 + \frac{1}{3}mc^2\cos^2\beta + 2mb^2\right)\omega\underline{k}$

$\underline{H}_O = m\omega\left[\frac{1}{6}c^2\sin2\beta\,\underline{j} + \left(\frac{2}{5}r^2 + \frac{1}{3}c^2\cos^2\beta + 2b^2\right)\underline{k}\right]$

7/71 $r = 100$ mm $\quad \omega = 4\pi$ rad/s

$b = 200$ mm $\quad p = \frac{v_C}{r} = \frac{b}{r}\omega = 8\pi$ rad/s

$m = 2$ kg



Eq. 7/11 holds for point O as a fixed point on axis of disk

$\omega_x = 0,\ \omega_y = -p = -8\pi$ rad/s, $\omega_z = \omega = 4\pi$ rad/s

$I_{xy} = 0;\ I_{yy} = \frac{1}{2}mr^2 = \frac{1}{2}(2)(0.1)^2 = 0.01$ kg·m²

$I_{yz} = 0,\ I_{xz} = 0,\ I_{zz} = \frac{1}{4}mr^2 + mb^2 = 2\left(\frac{1}{4}\overline{0.1}^2 + \overline{0.2}^2\right)$

$= 0.085$ kg·m²

So $\underline{H}_O = \underline{j}I_{yy}\omega_y + \underline{k}I_{zz}\omega_z = \underline{j}\left(-\frac{1}{2}mr^2p\right) + \underline{k}\left(\frac{1}{4}mr^2 + mb^2\right)\omega$

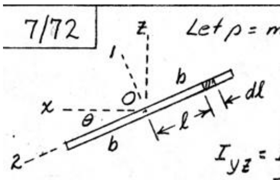
$= mr^2\omega\left(-\frac{1}{2}\frac{b}{r}\underline{j} + \left[\frac{1}{4} + \frac{b^2}{r^2}\right]\underline{k}\right)$

$= 2(0.1)^2 4\pi\left(-\frac{1}{2}2\underline{j} + \left[\frac{1}{4} + 4\right]\underline{k}\right) = 0.251(-\underline{j} + 4.25\underline{k})$ N·m·s

$T = \frac{1}{2}\underline{\omega}\cdot\underline{H}_O = \frac{1}{2}(-8\pi\underline{j} + 4\pi\underline{k})\cdot0.251(-\underline{j} + 4.25\underline{k})$

$= 3.15 + 6.71 = 9.87$ J

7/72



Let ρ = mass per unit of panel area

$I_{xz} = \int_{-b}^{b} (-l\cos\theta)(l\sin\theta)2c\rho\, dl$

$= -\frac{2}{3}\rho c b^3 \sin 2\theta$ for 2 panels

$I_{yz} = I_{xy} = 0$ by symmetry

$I_{zz} = \bar{I}_{zz} + md^2$ for each panel

For total, $I_{zz} = 2\left\{\frac{2bc\rho}{12}\left[c^2 + (2b\cos\theta)^2\right] + 2bc\rho\left[a + \frac{c}{2}\right]^2\right\}$

$= 4bc\rho\left\{\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac\right\}$

$\underline{H}_O = -I_{xz}\omega_z\underline{i} + I_{zz}\omega_z\underline{k}$, $m = 4bc\rho$ (total)

$\underline{H}_O = \frac{m}{6}b^2\omega\sin 2\theta\,\underline{i} + m\omega\left\{\frac{c^2}{3} + \frac{b^2}{3}\cos^2\theta + a^2 + ac\right\}\underline{k}$

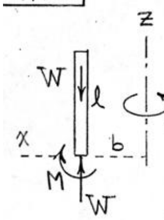
By symmetry, principal axes are O-1, O-2, O-y

$I_1 = m\left\{\frac{c^2 + b^2}{3} + a^2 + ac\right\}$ (max)

$I_2 = m\left\{\frac{1}{3}c^2 + a^2 + ac\right\}$ (intermediate)

$I_3 = \frac{1}{3}mb^2$ (minimum)

7/73



$\sum M_y = -I_{xz}\omega_z^2$:

$-M = -m\frac{bl}{2}\omega^2$

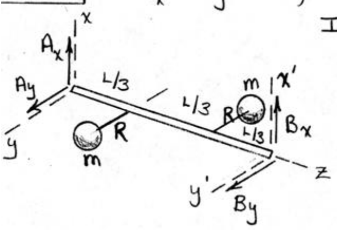
$M = \frac{mbl}{2}\omega^2$

7/74

$\omega_x = \omega_y = 0$, $\omega_z = \omega$, $\dot{\omega}_z = 0$

$I_{yz} = m\frac{L}{3}R - m\frac{2L}{3}R$

$= -mLR/3$



$I_{xz} = 0$

$I_{y'z} = -mLR/3$

$I_{x'z} = 0$

x-y axes: $\sum M_x = I_{yz}\omega_z^2$ (from Eq. 7/23)

$-B_y L = -\frac{mLR}{3}\omega^2$, $B_y = \frac{mR\omega^2}{3}$

$\sum M_y = 0$, $B_x = 0$

x'-y' axes: $\sum M_{x'} = I_{y'z}\omega_z^2$

$A_y L = -\frac{mLR}{3}\omega^2$, $A_y = -\frac{mR\omega^2}{3}$

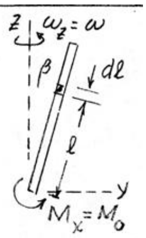
$\sum M_{y'} = 0$, $A_x = 0$

7/75

$dI_{yz} = yz\,dm = (l\sin\beta)(l\cos\beta)\,dm$

$I_{yz} = \frac{1}{2}\sin 2\beta \int l^2 dm = \frac{1}{2}\sin 2\beta (I_{xx})$

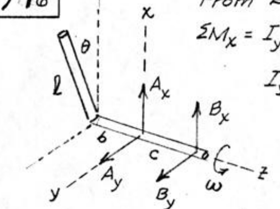
$= \frac{1}{6}mL^2\sin 2\beta$



Eq. 7/23 $\sum M_x = I_{yz}\omega_z^2$

$M_O = \frac{1}{6}mL^2\omega^2\sin 2\beta$

7/76



From Eqs. 7/23, with $\dot{\omega}_z = \dot{\omega} = 0$,

$\sum M_x = I_{yz}\omega_z^2$, $\sum M_y = -I_{xz}\omega_z^2$, $\sum M_z = 0$

$I_{yz} = -m\frac{bl}{2}\sin\theta$, $I_{xz} = -m\frac{bl}{2}\cos\theta$

So $-B_y c = -m\frac{bl}{2}\sin\theta\,(\omega^2)$

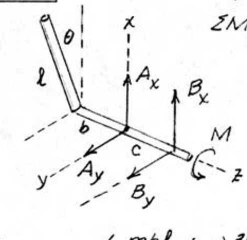
$B_y = \frac{mbl\omega^2}{2c}\sin\theta$

& $+B_x c = m\frac{bl}{2}\cos\theta\,(\omega^2)$

$B_x = \frac{mbl\omega^2}{2c}\sin\theta$

& $\underline{B} = \frac{mbl\omega^2}{2c}(\underline{i}\sin\theta + \underline{j}\cos\theta)$, $B = |\underline{B}| = \frac{mbl\omega^2}{2c}$

7/77



From Eq. 7/23 with $\omega_z = 0$

$\sum M_z = I_{zz}\dot{\omega}_z$; $M = \frac{1}{3}ml^2\dot{\omega}$

$\dot{\omega} = \frac{3M}{ml^2}$

$\sum M_x = -I_{xz}\dot{\omega}_z$, $\sum M_y = -I_{yz}\dot{\omega}_z$

So $-B_y c = -\left(-m\frac{bl}{2}\cos\theta\right)\frac{3M}{ml^2}$

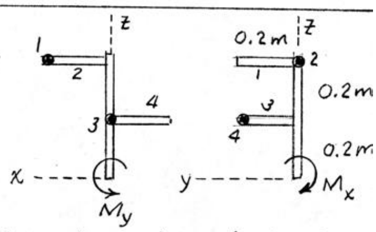
$B_y = -\frac{3Mb}{2lc}\cos\theta$

& $B_x c = -\left(-\frac{mbl}{2}\sin\theta\right)\frac{3M}{ml^2}$, $B_x = \frac{3Mb}{2lc}\sin\theta$

7/78

$\omega_z = \frac{1200(2\pi)}{60}$

$= 125.7\ \frac{rad}{s}$



Eq. 7/23, $\sum M_x = I_{yz}\omega_z^2$

$\sum M_y = -I_{xz}\omega_z^2$

$I_{yz} = m_1(0.1)(0.4) + m_2(0) + m_3(0.1)(0.2) + m_4(0.2)(0.2)$

$= 0.12(0.04 + 0.02 + 0.04) = 0.012\ kg\cdot m^2$

$I_{xz} = m_1(0.2)(0.4) + m_2(0.1)(0.4) + m_3(0) + m_4(-0.1)(0.2)$

$= 0.12(0.08 + 0.04 - 0.02) = 0.012\ kg\cdot m^2$

Thus $M_x = 0.012(125.7)^2 = 189.5\ N\cdot m$

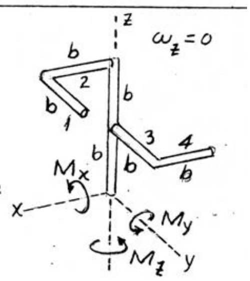
$M_y = -0.012(125.7)^2 = -189.5\ N\cdot m$

$M = \sqrt{M_x^2 + M_y^2} = 189.5\sqrt{2} = 268\ N\cdot m$

7/79

$m_1 = m_2 = m_3 = m_4 = 0.12$ kg

$b = 0.2$ m, $M_z = .64$ N·m



Eq. 7/23 $\sum M_x = -I_{xz}\dot{\omega}_z$

$\sum M_y = -I_{yz}\dot{\omega}_z$

$\sum M_z = I_{zz}\dot{\omega}_z$

For ① $I_{zz} = \frac{1}{12}mb^2 + m(b^2 + \frac{b^2}{4}) = \frac{4}{3}mb^2$

② & ③ $I_{zz} = \frac{1}{3}mb^2$

④ $I_{zz} = \frac{4}{3}mb^2$

Total $I_{zz} = \frac{10}{3}mb^2 = \frac{10}{3}(0.12)(0.2)^2 = 0.016\ kg\cdot m^2$

From Sol. to Prob. 7/78, $I_{yz} = I_{xz} = 0.012\ kg\cdot m^2$

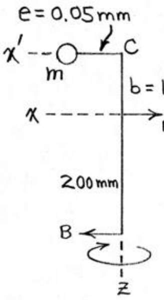
So $64 = 0.016\dot{\omega}_z$, $\dot{\omega}_z = 4000\ rad/s^2$

$M_x = -0.012(4000) = -48\ N\cdot m$

$M_y = -0.012(4000) = -48\ N\cdot m$

$M = \sqrt{M_x^2 + M_y^2} = 48\sqrt{2}\ N\cdot m$

7/80



$\Sigma M_y = -I_{yz}\dot{\omega}_z - I_{xz}\omega_z^2$, $\dot{\omega}_z = 0$

$\omega_z = \omega = 10{,}000\left(\frac{2\pi}{60}\right) = 1047 \ \frac{rad}{sec}$

$I_{xz} = -mbe = -6(0.15)(50)(10^{-6})$

$= -45(10^{-6})$ kg·m^2

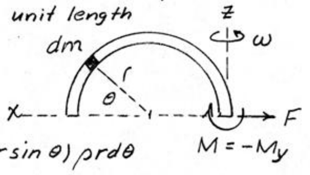
Thus $B(0.20) = 45(10^{-6})(1047)$

$= \underline{247 \text{ N}}$

For origin of coordinates x'-y'-z at C, $\Sigma M_{y'} = 0$, since $I_{x'z} = 0$.

Thus $0.35B - 0.15A = 0$, $A = \frac{0.35}{0.15}(247) = \underline{576 \text{ N}}$

7/81 Let ρ = mass per unit length



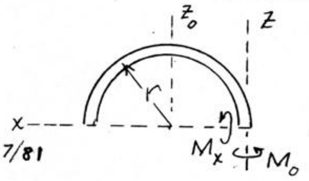
$\Sigma M_y = -I_{xz}\omega_z^2$

$I_{xz} = \int xz\,dm = \int_0^\pi (r + r\cos\theta)(r\sin\theta)\rho r\,d\theta$

$= \rho r^3\left[-\cos\theta - \frac{1}{4}\cos 2\theta\right]_0^\pi = 2\rho r^3 = \frac{2}{\pi}mr^2$

so $-M = -\frac{2}{\pi}mr^2\omega^2$, $\underline{M = \frac{2}{\pi}mr^2\omega^2}$

7/82 $m = \rho\pi r$



$I_{zz} = I_{z_0z_0} + mr^2$

$= \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$

$I_{xz} = \frac{2}{\pi}mr^2$ from Prob. 7/81

Eq. 7/23 with $\omega = \omega_z = 0$, $\dot{\omega} = \dot{\omega}_z$

$\Sigma M_z = I_{zz}\dot{\omega}_z$: $M_0 = \frac{3}{2}mr^2\dot{\omega}_z$, $\dot{\omega}_z = \frac{2M_0}{3mr^2}$

$\Sigma M_x = -I_{xz}\dot{\omega}_z$: $M = M_x = -\frac{2}{\pi}mr^2\left(\frac{2M_0}{3mr^2}\right) = \underline{-\frac{4M_0}{3\pi}}$

7/83 $\Sigma M_{z_1} = I_{z_1}\alpha$ where I_{z_1} is given by Eq. B/10 with $l = \cos\theta$, $m = 0$, $n = \sin\theta$

$I_{xy} = I_{xz} = I_{yz} = 0$

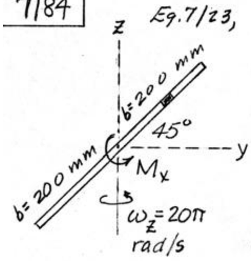
Thus $I_{z_1} = I_{xx}l^2 + I_{yy}m^2 + I_{zz}n^2 + 0$

$= I_0\cos^2\theta + 0 + I\sin^2\theta$

so $M = (I_0\cos^2\theta + I\sin^2\theta)\alpha$

$\underline{\alpha = \dfrac{M}{I_0\cos^2\theta + I\sin^2\theta}}$

7/84



Eq. 7/23, $\Sigma M_x = I_{yz}\omega_z^2$

$I_{yz} = \int yz\,dm = \int y^2\rho\,dl = \int_{-b/\sqrt{2}}^{b/\sqrt{2}} y^2\rho\sqrt{2}\,dy$

$= \frac{\rho\sqrt{2}}{3}\left(\frac{b^3}{2\sqrt{2}} - \frac{-b^3}{2\sqrt{2}}\right) = \frac{mb^2}{6} = \frac{3(0.2)^2}{6} = 0.02$ kg·m^2

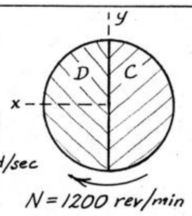
$M_x = 0.02(20\pi)^2$

$= 79.0$ N·m

On plate, $M_x = 79.0\,\underline{i}$ N·m

but acting on shaft, $\underline{\underline{M} = -79.0\,\underline{i} \text{ N·m}}$

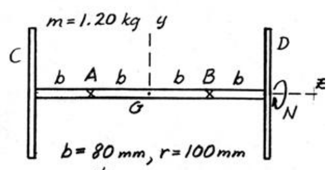
7/85



$\omega_x = \omega_y = 0$,

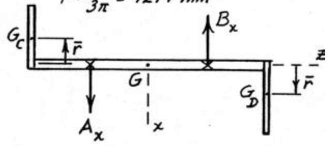
$\omega_z = \frac{1200 \times 2\pi}{60}$

$= 125.7$ rad/sec



$b = 80$ mm, $r = 100$ mm

$\bar{r} = \frac{4r}{3\pi} = 42.4$ mm



From Eqs. 7/23, about G,

$\Sigma M_x = I_{yz}\omega_z^2$, $\Sigma M_y = -I_{xz}\omega_z^2$,

$\Sigma M_z = 0$

$I_{yz} = 0$, $I_{xz} = \{0 + m(2b)(\bar{r})\} + \{0 + m(-2b)(-\bar{r})\} = 4mb\bar{r}$

$= 4(1.20)(0.080)(0.0424) = 0.01630$ kg·m^2

$\Sigma F_x = 0$ so $A_x = B_x$

$\Sigma M_y = -A_xb - B_xb = -4mb\bar{r}\omega_z^2$, $A_x = B_x = 2m\bar{r}\omega_z^2/b$

$A_x = B_x = \frac{1}{2}(0.01630)(125.7)^2/0.080$

$= 1608$ N

$\Sigma M_x = 0$, $A_y = B_y = 0$

$\underline{\underline{F}_A = 1608\,\underline{i} \text{ N}}$, $\underline{\underline{F}_B = -1608\,\underline{i} \text{ N}}$

7/86 With $\omega_x = \omega_y = \omega_z = \dot{\omega}_x = \dot{\omega}_y = 0$, $\dot{\omega}_z = 900$ rad/s^2,

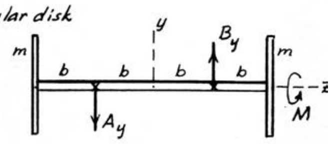
Eqs. 7/23 become

$\Sigma M_x = -I_{xz}\alpha$, $\Sigma M_y = -I_{yz}\alpha$, $\Sigma M_z = I_{zz}\alpha$

From the solution to Prob. 7/85, $I_{yz} = 0$, $I_{xz} = 0.01630$ kg·m^2

Also $I_{zz} = \frac{1}{2}(2m)r^2 = 1.20(0.100)^2 = 0.012$ kg·m^2

where m = mass of semicircular disk



$\Sigma F_y = 0$ so $A_y = B_y$

$\Sigma M_x = -0.080A_y - 0.080B_y$

$= -0.01630(900)$

$A_y = B_y = 91.7$ N

so $\underline{\underline{F}_A = -91.7\,\underline{j} \text{ N}, \underline{F}_B = 91.7\,\underline{j} \text{ N}}$

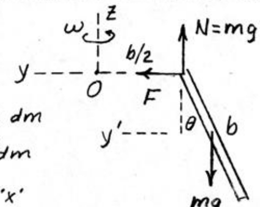
$b = 80$ mm

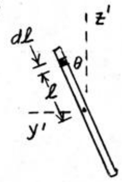
$m = 1.20$ kg

$\alpha = \dot{\omega}_z = 900$ rad/s^2

$M = \Sigma M_z = 0.012(900) = \underline{10.8 \text{ N·m}}$

7/87





$I_{yz} = I_{y'z'} + md_yd_z$

$I_{y'z'} = \int l\sin\theta\, l\cos\theta\,dm$

$= \sin\theta\cos\theta\int l^2dm$

$= \sin\theta\cos\theta\, I_{x'x'}$

$= \frac{1}{2}\sin 2\theta\,\frac{1}{12}mb^2 = \frac{1}{24}mb^2\sin 2\theta$

$I_{yz} = \frac{1}{24}mb^2\sin 2\theta + m\left(-\frac{b}{2} - \frac{b}{2}\sin\theta\right)\left(-\frac{b}{2}\cos\theta\right)$

$= \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)$

Eq. 7/23 $\Sigma M_x = 0 + I_{yz}\omega_z^2$

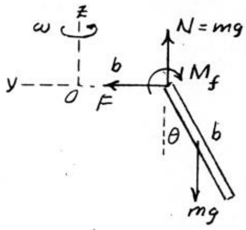
$mg\left(\frac{b}{2} + \frac{b}{2}\sin\theta\right) - mg\frac{b}{2} = \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)$

$g\tan\theta = b\left(\frac{2}{3}\sin\theta + \frac{1}{2}\right)\omega^2$

$\underline{\omega = \sqrt{\frac{1}{b}\,\frac{6g\tan\theta}{4\sin\theta + 3}}}$

7/88

Eq. 7/23 $\Sigma M_x = 0 + I_{yz}\,\omega_z^2$



From Sol. to Prob. 7/87

$I_{yz} = \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)$

$M_f + mg\left(\frac{b}{2} + \frac{b}{2}\sin\theta\right) - mg\frac{b}{2}$
$= \frac{mb^2}{4}\left(\frac{2}{3}\sin 2\theta + \cos\theta\right)\omega^2$

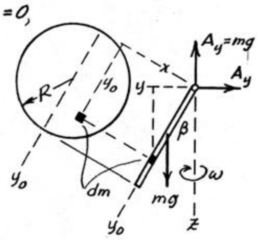
$M_f = \frac{mb}{2}\left\{\cos\theta\left[\frac{2}{3}\sin\theta + \frac{1}{2}\right]b\omega^2 - g\sin\theta\right\}$

where $\omega^2 > \frac{6g\tan\theta}{b(4\sin\theta + 3)}$

7/89 From Eq. 7/23 with $\omega_z = \omega$, $\dot{\omega}_z = 0$,

$\Sigma M_x = I_{yz}\,\omega^2$

$I_{yz} = \int yz\,dm = \int (y_0\sin\beta)(y_0\cos\beta)\,dm$
$= \sin\beta\cos\beta\int y_0^2\,dm$
$= \frac{1}{2}\sin 2\beta\, I_{xx}$
$= \frac{1}{2}\sin 2\beta\left(\frac{1}{4}mR^2 + mR^2\right)$
$= \frac{5}{8}mR^2\sin 2\beta$



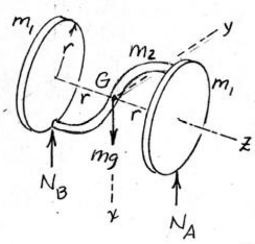
So $mgR\sin\beta = \left(\frac{5}{8}mR^2\sin 2\beta\right)\omega^2$;

$\sin\beta\left(g - \frac{5}{8}R\omega^2 \times 2\cos\beta\right) = 0$, $\beta = \cos^{-1}\frac{4g}{5R\omega^2}$ if $\omega^2 \geq \frac{4g}{5R}$;

otherwise $\beta = 0$

7/90 From Sample Problem 7/7:

$I_{xz} = -\rho r^3 = -\frac{m_2 r^2}{\pi}$



$\Sigma M_y = -I_{xz}\,\omega_z^2$, $\omega_z = v/r$

$N_B r - N_A r = \frac{m_2 r^2}{\pi}\frac{v^2}{r^2} = \frac{m_2 v^2}{\pi}$

Also $\Sigma F_x = 0$; $N_A + N_B - mg = 0$

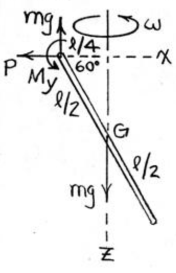
Combine & get

$N_A = \frac{mg}{2} - \frac{m_2 v^2}{2\pi r}$

$m = 2m_1 + m_2$

$N_B = \frac{mg}{2} + \frac{m_2 v^2}{2\pi r}$

7/91 $\Sigma F_x = m\bar{a}_x$: $P = 0$



$\Sigma M_y = -I_{xz}\,\omega_z^2$

$I_{xz} = \int xz\,dm = \int_{-l/4}^{l/4} x\sqrt{3}\left(\frac{l}{4} + x\right)\rho\,dx$

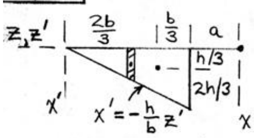
where ρ = mass/(x-comp. of length)

$I_{xz} = \frac{\sqrt{3}}{48}\,ml^2$, where $m = \frac{\rho l}{2}$

So $M_y - mg\frac{l}{4} = -\frac{\sqrt{3}}{48}ml^2\omega^2$

& for $M_y = 0$, $\omega = 2\sqrt{\frac{\sqrt{3}\,g}{l}}$

►7/92



$I_{x'z'} = \int x_c' z_c'\,dm$

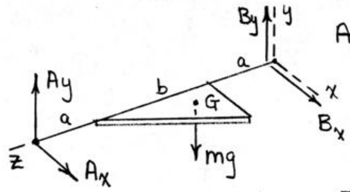
$= \int\left(-\frac{h}{2b}z'\right)(z')\,\rho\,(x'dz')$

$= \int\left(-\frac{h}{2b}z'\right)(z')\,\rho\left(+\frac{h}{b}z'dz'\right)$

$I_{x'z'} = -\frac{h^2\rho}{2b^2}\int_0^{-b} z'^3\,dz' = -\frac{1}{4}mhb$, since $m = \frac{\rho hb}{2}$

$\bar{I}_{x'z'} = I_{x'z'} - m\,d_{x'}d_{y'} = -\frac{1}{4}mhb - m\left(+\frac{h}{3}\right)\left(-\frac{2b}{3}\right)$

$= -\frac{1}{36}mhb$. Similarly, $I_{xz} = \frac{1}{12}mhb + \frac{1}{3}mha$



Also, $I_{zz} = \frac{1}{6}mh^2$, $I_{yz} = 0$

Eqs. 7/23: $\Sigma M_z = I_{zz}\dot{\omega}_z$

$-mg\frac{h}{3} = \frac{1}{6}mh^2\dot{\omega}_z$

$\dot{\omega}_z = -2g/h$

$\Sigma M_y = 0 \Rightarrow A_x = 0$

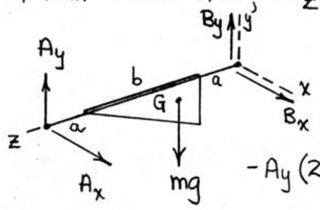
$\Sigma M_x = -I_{xz}\dot{\omega}_z$: $-A_y(2a+b) + mg\left(a + \frac{b}{3}\right) = -I_{xz}\dot{\omega}_z$

Simplifying, $A_y = A = mg/6$

►7/93 $U = \Delta T + \Delta V_e + \Delta V_g$

$0 = \frac{1}{2}I_{zz}\,\omega_z^2 - mg(h/3)$

From Prob. 7/92, $I_{zz} = \frac{1}{6}mh^2$, so $\omega_z = 2\sqrt{\frac{g}{h}}$



$\Sigma M_z = 0$, $\dot{\omega}_z = 0$

$\Sigma M_x = I_{yz}\,\omega_z^2$,

$-A_y(2a+b) + mg\left(a + \frac{b}{3}\right) =$
$-mh\left(\frac{b}{12} + \frac{a}{3}\right)4\frac{g}{h}$

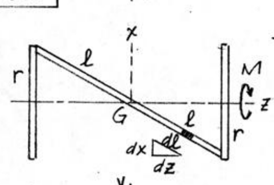
$A_y = \frac{mg}{3}\left[\frac{7a + 2b}{2a + b}\right]$

$\Sigma M_y = 0$: $A_x(2a+b) = 0$, $A_x = 0$

$A = \sqrt{A_x^2 + A_y^2} = \frac{mg}{3}\left[\frac{7a + 2b}{2a + b}\right]$

►7/94 ↑a

$\Sigma M_x = -I_{xz}\dot{\omega}_z$ from Eq. 7/23

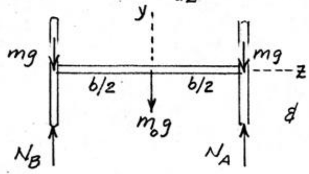


$I_{xz} = \int xz\,dm = \int(-x)\left(\frac{b}{2r}x\right)\rho\,dl$

$= -\frac{b}{2r}\rho\int_{-r}^{r} x^2\frac{l}{r}\,dx = -\frac{1}{6}brm_0$

where $m_0 = 2\rho l$

Thus $N_B\frac{b}{2} - N_A\frac{b}{2} = +\frac{1}{6}brm_0\left(-\frac{a}{r}\right)$



$N_B - N_A = -\frac{1}{3}m_0 a$

& $\Sigma F_y = 0$; $N_A + N_B = (m_0 + 2m)g$

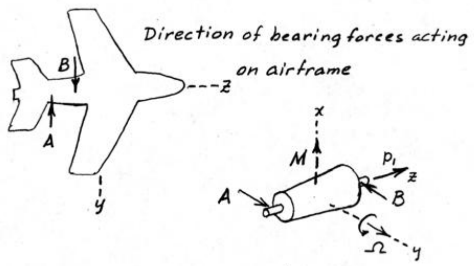
Combine & get

$\dot{\omega}_z = -\frac{a}{r}k$, $\omega_z = \omega_x = \omega_y = 0$

$N_A = mg + \frac{m_0 g}{2}\left(1 + \frac{a}{3g}\right)$

$N_B = mg + \frac{m_0 g}{2}\left(1 - \frac{a}{3g}\right)$

7/95



To satisfy $\underline{M} = I\underline{\Omega}\times\underline{p}$

$\underline{p}$ must be p_1

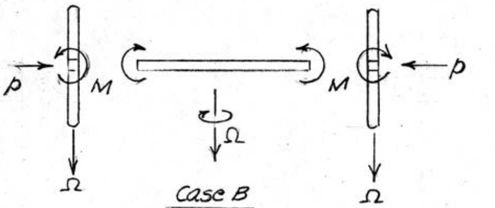
7/96

$\underline{M} = I\underline{\Omega}\times\underline{p}$: $-M\underline{i} = I\underline{\Omega}\times p\underline{j}$

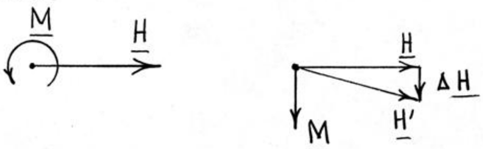
$\underline{\Omega}$ is in $+\underline{k}$ direction

So precession is CCW when viewed from above.

7/97



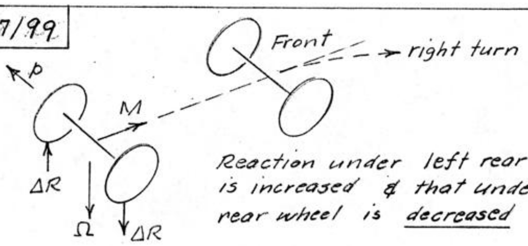
7/98



(Side view) (Overhead view)

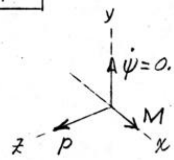
$\underline{M}$ is the moment exerted on the handle by the student ; $\underline{H}$ is the wheel angular momentum. From $\underline{M} = \dot{\underline{H}} \cong \frac{\Delta \underline{H}}{\Delta t}$, we see that $\Delta\underline{H}$ is in the same direction as $\underline{M}$. $\underline{H}'$ is the new angular momentum. The student will sense a tendency of the wheel to rotate to her right.

7/99



Reaction under left rear wheel is increased & that under right rear wheel is decreased

7/100



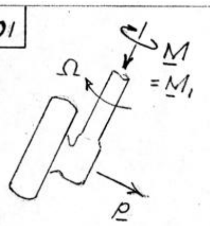
$M = I\dot{\psi}p$

$0.8(9.81)(b-0.180) = 2.2(0.06)^2(0.2)\frac{1725(2\pi)}{60}$

$b - 0.180 = 0.0364$

$b = 0.216$ m or $b = 216$ mm

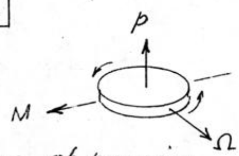
7/101

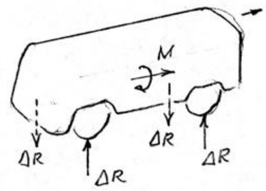


$M = I\Omega p$

$M = M_1 = mk^2\Omega\frac{v}{r}$

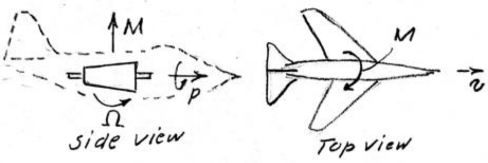
7/102





Because of precession Ω, gyroscopic moment on rotor points to the rear and reacting moment on bus is forward. Result is that the force under the right-hand tires is increased.

7/103



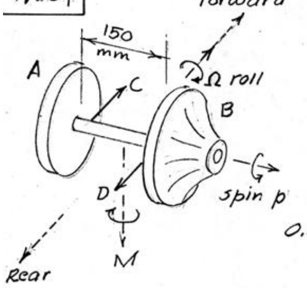
Pilot would apply left rudder to counter the clockwise (viewed from above) reaction to the gyroscopic moment

$M = I\Omega p = 210(0.220)^2\left[\frac{1200(1000)}{3600}\Big/3800\right]\frac{18000\times 2\pi}{60}$

$= (10.16)(0.0877)(1885)$

$= 1681$ N·m

7/104



$p = 20\,000\frac{2\pi}{60} = 2094$ rad/s

$\Omega = 2$ rad/s

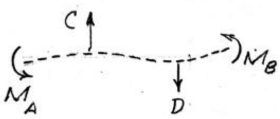
$I = 3.5(0.079)^2 + 2.4(0.071)^2$

$= 0.0339$ kg·m²

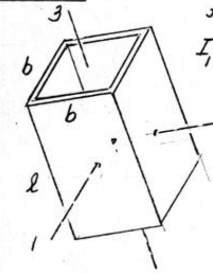
$M = I\Omega p \quad (= M_A + M_B)$

$0.15C = 0.0339(2)(2094)$

$C = D = 948$ N



7/105 Let m = mass of each of the four sides



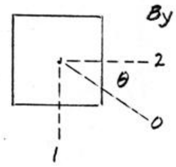
$I_1 = I_2 = 2\left\{\frac{m}{12}(b^2+\ell^2) + \frac{1}{12}m\ell^2 + m(\frac{b}{2})^2\right\}$

$= \frac{m}{3}(2b^2+\ell^2)$

$I_3 = 4\left\{\frac{m}{12}b^2 + m(\frac{b}{2})^2\right\} = \frac{4}{3}mb^2$

$I_1 = I_3$ if $\frac{m}{3}b^2(2+[\ell/b]^2) = \frac{4}{3}mb^2$

or $\ell/b = \sqrt{2}$

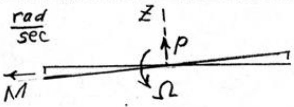


By Eq. B/10, $I_0 = I_1 = I_2 = \frac{m}{3}(2b^2+\ell^2)$

If $\underline{\ell > b\sqrt{2}}$, $I_0 > I_3$ direct precession

If $\underline{\ell < b\sqrt{2}}$, $I_0 < I_3$ retrograde precession

7/106 $\Omega = \frac{10}{180}\pi = 0.1745\ \frac{\text{rad}}{\text{sec}}$



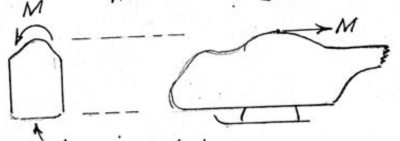
$p = \frac{500}{60}2\pi = 52.4$ rad/sec

$I = \frac{140}{32.2}10^2 = 435$ lb-ft-sec^2

$M = I\Omega p$

$= 435(0.1745)52.4$

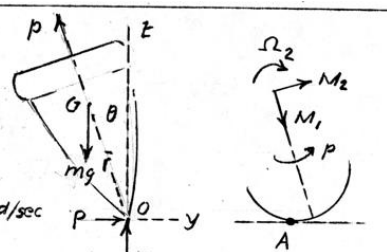
$= \underline{3970\ \text{lb-ft}}$



As viewed by passenger looking forward

Conclusion: $\underline{\text{CCW deflection}}$

7/107 Neglect momentum about z-axis compared with that about spin axis.



$\bar{r} = 2.5$ in., $k = 0.62$ in.

$p = 3600(2\pi)/60 = 377$ rad/sec

Eq. 7/24a

$\underline{M}_O = I\underline{\Omega}\times\underline{p}$: $mg\bar{r}\sin\theta\,\underline{i} = I\Omega\underline{k}\times p(\cos\theta\,\underline{k} - \sin\theta\,\underline{j})$

$mg\bar{r}\sin\theta = I\Omega p\sin\theta$

or $g\bar{r} = k^2\Omega p$, $\Omega = \frac{g\bar{r}}{k^2 p}$ (Eq. 7/25)

So $\Omega = \frac{32.2(2.5/12)}{(0.62/12)^2 377} = 6.67$ rad/sec or $\underline{\Omega} = 6.67\underline{k}$ rad/sec

Friction force at A is into the paper (−x-dir) which produces a moment M_1 to slow the spin and a moment M_2 which causes a precession Ω_2 that decreases θ.

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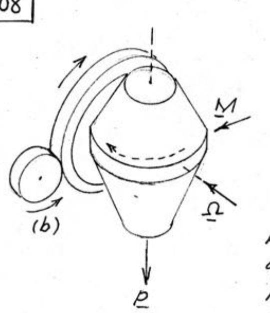

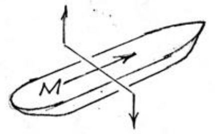

$\underline{M}$ needed on structure of ship to counteract roll to port (left).

Reaction on gyro is opposite to $\underline{M}$ on ship.

Proper directions of $\underline{p}$, $\underline{\Omega}$, $\underline{M}$ shown – requiring rotation (b) of motor.

$M = I\Omega p = 80(1.45)^2\,960\,\frac{2\pi}{60}\,0.320 = \underline{5410\ \text{kN}\cdot\text{m}}$

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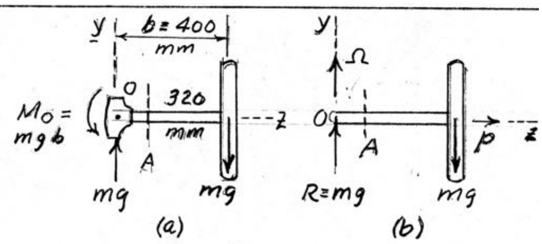


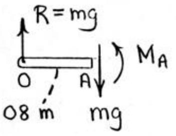
Case (a) $\Sigma M_x = 0$: so no precession

$M_A = 4(9.81)0.320 = \underline{12.56\ \text{N}\cdot\text{m}}$

Case (b) $\Sigma M_x = mgb = 4(9.81)0.4 = 15.70$ N·m

$\Sigma M_x = I_{zz}\Omega p$: $15.70 = 4(0.12)^2\Omega\,\frac{3600(2\pi)}{60}$

$\underline{\Omega = 0.723\ \text{rad/s}}$



$\Sigma M_{A_x} = 0$: $M_A = mg(0.08)$

$M_A = 4(9.81)(0.08) = \underline{3.14\ \text{N}\cdot\text{m}}$

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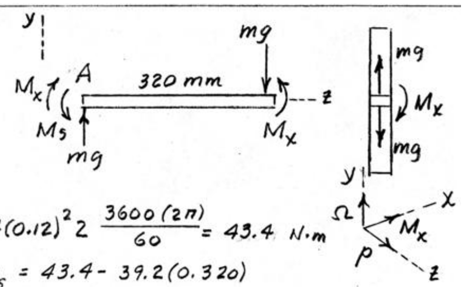
$mg = 4(9.81) = 39.2$ N

$\Omega = 2$ rad/s const



For rotor

$M_x = I_{zz}\Omega p = 4(0.12)^2\,2\,\frac{3600(2\pi)}{60} = 43.4$ N·m

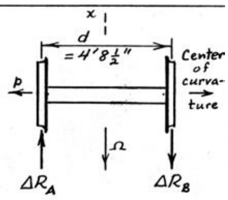
So $M_A = M_x - M_s = 43.4 - 39.2(0.320)$

$= \underline{30.9\ \text{N}\cdot\text{m}}$

$\Sigma M_y = I_{yy}\dot{\Omega}$ but Ω = const. so $\dot{\Omega} = 0$ & $\underline{M_y = M_o = 0}$

7/111 $\underline{M} = I\underline{\Omega}\times\underline{p} = I(-\Omega\underline{i}\times p\underline{k})$

$= I\Omega p\underline{j}$



so $\underline{M}$ is into the paper &

$\underline{\Delta R_A}$ is up (increase of normal force)

& $\underline{\Delta R_B}$ is down (decrease of normal force)

For wheel & axle unit,

(view from rear)

$I_{zz} = \Sigma\frac{1}{2}mr^2 = 2\left(\frac{1}{2}\frac{560}{32.2}\left[\frac{33/2}{12}\right]^2\right) + \frac{1}{2}\frac{300}{32.2}\left(\frac{5/2}{12}\right)^2$

$= 32.9 + 0.202 = 33.1$ lb-ft-sec^2

$p = v/r = \left(\frac{80}{30}44\right)/\frac{33/2}{12} = 85.3$ rad/sec

$\Omega = v/\rho = \left(\frac{80}{30}44\right)/717 = 0.1636$ rad/sec

$d = 4'8\frac{1}{2}'' = 4.71$ ft

So $M = \Delta R(4.71) = 33.1\times 85.3\times 0.1636$, $\underline{\Delta R = 98.1\ \text{lb}}$

7/112 From Eq. 7/30 with θ small so that $\cos\theta \approx 1$, the precessional rate is

$\dot{\psi} = \frac{Ip}{I_0 - I} = \frac{p}{(I_0/I)-1} = \frac{3}{\frac{1}{2}-1} = \underline{-6\ \text{rev/min}}$

Where the minus sign indicates $\underline{\text{retrograde precession}}$

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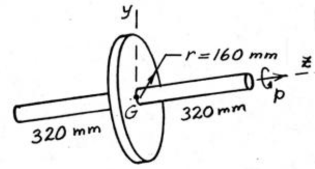

$I_{zz_{disk}} = \frac{1}{2}mr^2 = \frac{1}{2}8(0.160)^2 = 0.1024\ kg\cdot m^2$

$I_{yy_{disk}} = \frac{1}{4}mr^2 = 0.0512\ kg\cdot m^2$

$I_{zz_{rod}} \approx 0,\ I_{yy_{rod}} = \frac{1}{12}3(0.640)^2 = 0.1024\ kg\cdot m^2$

From Eq. 7/30 $\dot\psi = \dfrac{Ip}{(I_o - I)\cos\theta}$

where $I = I_{zz} = 0.1024\ kg\cdot m^2$

$I_o = I_{yy} = 0.0512 + 0.1024 = 0.1536\ kg\cdot m^2$

$\theta = 15°,\ p = 60\ rad/s$

so $\dot\psi = \dfrac{0.1024(60)}{(0.1536 - 0.1024)\cos 15°} = \underline{124.2\ rad/s}$

$I_o - I$ is plus, so precession is direct & $\underline{\dot\psi \text{ is } \dot\psi_1}$

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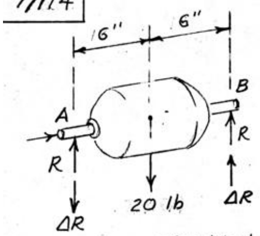
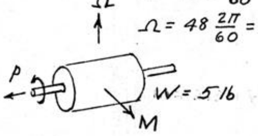




$p = 1725\frac{2\pi}{60} = 180.6\ \frac{rad}{sec}$

$\Omega = 48\frac{2\pi}{60} = 5.03\ \frac{rad}{sec}$

Static reactions

$R = \frac{1}{2}20 = 10\ lb$

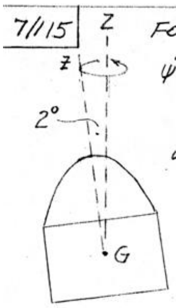
$M = I\Omega p;\quad 2(\Delta R)(6/12) = \frac{5}{32.2}(\frac{1.5}{12})^2(5.03)(180.6)$

$\Delta R = 2.20\ lb$

$R_A = 10 - 2.20 = \underline{7.80\ lb}$

$R_B = 10 + 2.20 = \underline{12.20\ lb}$

7/115



For zero moment Eq. 7/30 is

$\dot\psi = \dfrac{Ip}{(I_o - I)\cos\theta} = \dfrac{p}{(\frac{k_o^2}{\bar k^2} - 1)\cos\theta}$

where $\bar k = 0.72\ m$

$k_o = 0.54\ m$

$p = 1.5\ rad/s$

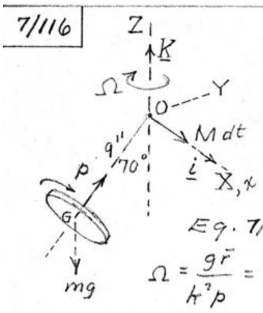
$\theta = 2°$

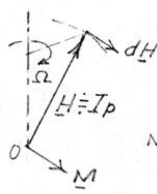
$(I = \bar k^2 m) > (I_o = k_o^2 m)$ so retrograde precession with p in negative z-dir.

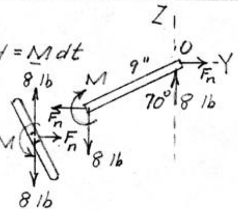
Period $\tau = |\frac{2\pi}{\dot\psi}| = 2\pi\left|\dfrac{(k_o^2/\bar k^2 - 1)}{p}\cos\theta\right|$

$= 2\pi\left|\dfrac{(0.54/0.72)^2 - 1}{1.5}\cos 2°\right| = \underline{1.831\ s}$

7/116







Eq. 7/25 applies

$\Omega = \dfrac{g\bar r}{k^2 p} = \dfrac{(32.2\times12)9}{3^2(3000\times 2\pi/60)},\ \underline{\Omega = -1.23\underline{K}\ \frac{rad}{sec}}$

Results are independent of 70°-angle. $\left(\text{or } \frac{1.23\times60}{2\pi} = 11.75\ rev/min\right)$

$M = I\Omega p \sin 70° = mk^2\left(\dfrac{g\bar r}{k^2 p}\right)p \sin 70° = mg\bar r \sin 70°$

which agrees with static analysis of shaft where $\Sigma M_O = 0$ gives $M = 8\times9\sin 70°$

$\underline{\underline{M} = 67.7\underline{i}\ lb\text{-}in.}$

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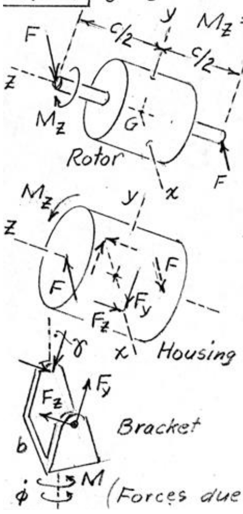

$\dot\theta = 0$ where $\theta = \pi/2 - \gamma$; $\dot\psi = 0$, $p = p$ const.

M_z = torque of electric field to maintain $\dot\phi$ constant (ω_z is not constant)

Apply Eq. 7/26 with

$I = mk_z^2,\ I_o = mk_x^2,\ \gamma = \pi/2 - \theta$

$\Sigma M_x = 0 = I_o(\ddot\theta - 0) + 0,\ \ddot\theta = -\ddot\gamma = 0$

$\Sigma M_y = Fc = F_z b = mk_x^2(\ddot\psi\cos\gamma + 0) + 0$

$\Sigma M_z = -M_z = -F_y b = mk_z^2\frac{d}{dt}(\dot\psi\sin\gamma + p)$

$-F_y b = mk_z^2(\ddot\psi\sin\gamma + 0 + 0)$

From bracket $\Sigma M_z = 0$ so

$M = F_z b\cos\gamma - F_y b\sin\gamma$

Substitute $F_z b$ & $F_y b$ & get

$\ddot\psi = \dfrac{M/m}{k_x^2\cos^2\gamma + k_z^2\sin^2\gamma}$

(Forces due to gyroscopic action only are shown)

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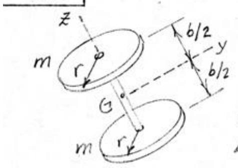


$I = I_{zz} = 2(\frac{1}{2}mr^2) = mr^2$

$I_o = I_{yy} = 2\left(\frac{1}{4}mr^2 + m[\frac{b}{2}]^2\right)$

$= \frac{1}{2}mr^2 + \frac{1}{2}mb^2$

Precession is not possible when $I = I_o$ $(\theta = \beta = 0)$

So $\frac{1}{2}mr^2 + \frac{1}{2}mb^2 = mr^2,\quad \underline{b = r}$

7/119

From Eq. 7/30,

$\dot\psi = \dfrac{Ip}{(I_o - I)\cos\theta} = \dfrac{p}{[(I_o/I) - 1]\cos\theta}$

where $I_o/I = \dfrac{\frac{1}{4}mr^2}{\frac{1}{2}mr^2} = \frac{1}{2}$, $p = \frac{300(2\pi)}{60} = 10\pi\ rad/s$

$\tau = 2\pi/|\dot\psi|$ $\quad\cos\theta = \cos 5° = 0.9962$

$\tau = 2\pi\dfrac{|(1/2 - 1|0.9962}{10\pi} = \underline{0.0996\ s}$

Precession is $\underline{\text{retrograde}}$ since $I > I_o$

7/120

Case (a) $p = \frac{120\times2\pi}{60} = \underline{4\pi\ rad/s}$

$\underline{\theta = \beta = 0\ ,\ \dot\psi = 0}$

Case (b) $p = 4\pi$, $\theta = 10°$, $I_o/I = 1/0.3$

From Eq. 7/30, the precessional rate is

$\dot\psi = \dfrac{p}{(\frac{I_o}{I} - 1)\cos\theta} = \dfrac{4\pi}{(\frac{1}{0.3} - 1)\cos 10°}$

$= \underline{5.47\ rad/s}$

From Eq. 7/29,

$\tan\beta = \frac{I}{I_o}\tan\theta = 0.3\tan 10°,\ \underline{\beta = 3.03°}$

Case (c) $\underline{\theta = \beta = 90°,\ p = 0}$

$\underline{\dot\psi = 4\pi\ rad/s}$

7/121 I = moment of inertia about its longitudinal axis $= \frac{1}{12}m(a^2+a^2)$, $a = 4''$

I_o = moment of inertia about transverse axis through O $= \frac{1}{12}m(a^2+\ell^2)$, $\ell = 8'' = 2a$

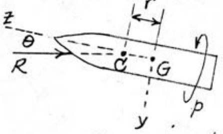
$I_o/I = \frac{1}{12}m(a^2+4a^2)/\frac{1}{6}ma^2 = 5/2$

Eq. 7/30 $\dot\psi = \dfrac{p}{\left(\frac{I_o}{I}-1\right)\cos\theta} = \dfrac{200}{\left(\frac{5}{2}-1\right)\cos 10°} = 135.4$ rev/min

period of wobble $\tau = \dfrac{60}{135.4} = 0.443$ sec

▸7/122 $\Sigma M_x = R\bar r \sin\theta$ & from Eq. 7/27 we have

$R\bar r = \dot\psi[I(\dot\psi\cos\theta + p) - I_o\dot\psi\cos\theta]$



or $\dot\psi^2 + \dfrac{Ip}{(I-I_o)\cos\theta}\dot\psi - \dfrac{R\bar r}{(I-I_o)\cos\theta} = 0$

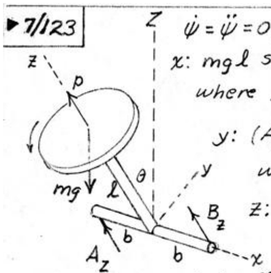
Solve for $\dot\psi$ & rearrange to give

$$\dot\psi = \frac{Ip}{2(I_o-I)\cos\theta}\left[1 \pm \sqrt{1 - \frac{4R\bar r(I_o-I)\cos\theta}{I^2p^2}}\right]$$

Expression under radical is (+) if

$p > \frac{2}{I}\sqrt{R\bar r(I_o-I)\cos\theta}$ = min. value of p for which precession at constant θ can occur.

▸7/123 $\dot\psi = \ddot\psi = 0$; From moment Eqs. 7/26



x: $mg\ell\sin\theta = m\left(\frac{r^2}{4}+\ell^2\right)\ddot\theta$ ------- (a)

where $I_o = I_{xx} = \frac{1}{4}mr^2 + m\ell^2$

y: $(A_z - B_z)b = -\frac{1}{2}mr^2\dot\theta p$ ------- (b)

where $I = \frac{1}{2}mr^2$

z: $0 = I\dot p$ where $\omega_z = 0 + p$ --- (c)

From (c), p = const

From (a) with $\dot\theta d\dot\theta = \ddot\theta d\theta$, $\int_0^{\pi/2} g\ell\sin\theta\, d\theta = \left(\frac{r^2}{4}+\ell^2\right)\int_0^{\dot\theta}\dot\theta\, d\dot\theta$

which gives $\dot\theta^2 = 8g\ell/(r^2+4\ell^2)$

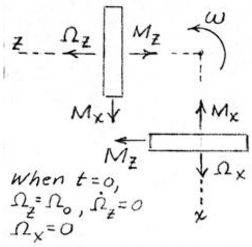
From (b) $-A_z + B_z = \frac{1}{2}m\frac{r^2}{b}\dot\theta p$ for $\theta = \pi/2$ ------ (d)

Also for $\theta = \pi/2$, $\Sigma F_z = m\bar a_z$; $-A_z - B_z = m\ell\dot\theta^2$ ------ (e)

Solve (d) & (e) & get

$$\left.\begin{aligned} A_z &= -\frac{m\dot\theta}{2}\left(\frac{r^2p}{2b}+\ell\dot\theta\right) \\ B_z &= \frac{m\dot\theta}{2}\left(\frac{r^2p}{2b}-\ell\dot\theta\right)\end{aligned}\right\} \text{where } \dot\theta = 2\sqrt{\frac{2g\ell}{r^2+4\ell^2}}$$

▸7/124 $\omega = \frac{2\pi}{\tau}$ = constant precessional rate about y-axis



When $t = 0$, $\Omega_z = \Omega_o$, $\dot\Omega_z = 0$, $\Omega_x = 0$

M_x = gyroscopic moment on z-wheel $= I\Omega_z\omega$

$-M_x$ = moment to accelerate x-wheel $= I\dot\Omega_x$

so $I\Omega_z\omega = -I\dot\Omega_x$, $\dot\Omega_x + \omega\Omega_z = 0$ (a)

M_z = gyroscopic moment on x-wheel $= -I\Omega_x\omega$

$-M_z$ = moment to accelerate z-wheel $= +I\dot\Omega_z$

so $I\Omega_x\omega = +I\dot\Omega_z$, $\dot\Omega_z - \omega\Omega_x = 0$ (b)

Combine (a) & (b) & get $\ddot\Omega_z + \omega^2\Omega_z = 0$ & $\ddot\Omega_x + \omega^2\Omega_x = 0$

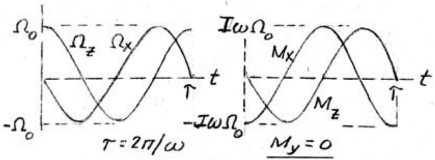
For given conditions at $t = 0$, $\begin{cases}\Omega_x = -\Omega_o\sin\omega t \\ \Omega_z = \Omega_o\cos\omega t\end{cases}$

Thus motor torques on shafts are

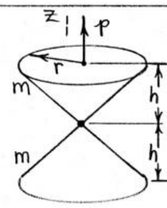
$M_x = -I\omega\Omega_o\cos\omega t$

$M_z = -I\omega\Omega_o\sin\omega t$

$M = \sqrt{M_x^2 + M_z^2} = I\omega\Omega_o$ constant



▸7/125 $\dot\psi = \dfrac{Ip}{(I_o-I)\cos\theta}$



(a) No precession if $I_o = I$

From Table D4,

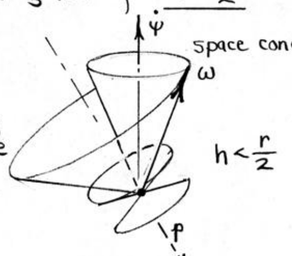
$I = I_{zz} = 2\left(\frac{3}{10}mr^2\right) = \frac{3}{5}mr^2$

$I_o = I_{xx} = 2\left(\frac{3}{20}mr^2 + \frac{3}{5}mh^2\right) = \frac{3}{10}mr^2 + \frac{6}{5}mh^2$

$I = I_o$: $\frac{3}{5}mr^2 = \frac{3}{10}mr^2 + \frac{6}{5}mh^2$, $h = \frac{r}{2}$

(b) For $h < \frac{r}{2}$, $I_o < I$;

retrograde precession



(c) $h = r$, $I_o = \frac{3}{10}mr^2 + \frac{6}{5}mr^2 = \frac{3}{2}mr^2$

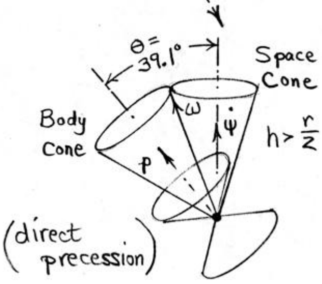
$\dfrac{I}{I_o - I} = \dfrac{3/5}{3/2 - 3/5} = \dfrac{2}{3}$

$p = 200\left(\frac{2\pi}{60}\right) = 20.9$ rad/s

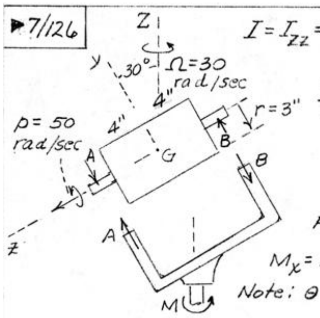
$\theta = \cos^{-1}\left[\frac{I}{I_o-I}\frac{p}{\dot\psi}\right]$

$= \cos^{-1}\left[\frac{2}{3}\,\frac{20.9}{18}\right]$

$= 39.1°$



▶7/126



$I = I_{zz} = \frac{1}{2}mr^2 = \frac{1}{2}\,\frac{64.4}{32.2}\left(\frac{3}{12}\right)^2 = 0.0625$ lb-ft-sec²

$I_o = I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$

$= \frac{64.4}{32.2}\left(\frac{1}{4}\left[\frac{3}{12}\right]^2 + \frac{1}{12}\left[\frac{8}{12}\right]^2\right)$

$= 0.1053$ lb-ft-sec²

From Eq. 7/27 with $\dot\psi = \Omega$

$M_x = \Omega \sin\theta\left[I(\Omega\cos\theta + p) - I_o\Omega\cos\theta\right]$

Note: $\theta = \pi/2 + 30°$; $\sin\theta = \sqrt{3}/2$, $\cos\theta = -\frac{1}{2}$

$M_x = 30\frac{\sqrt{3}}{2}\left[0.0625(30[-\frac{1}{2}] + 50) - 0.1053(30)(-\frac{1}{2})\right]$

$= 25.98\,[2.188 + 1.580] = 97.9$ lb-ft, so $\underline{\mathbf{M} = 97.9\mathbf{i}}$ lb-ft

$\omega_x = 0$, $\omega_y = 30\frac{\sqrt{3}}{2} = 25.98\ \frac{rad}{sec}$, $\omega_z = 50 - 30(\frac{1}{2}) = 35$ rad/sec

$H_x = 0$, $H_y = I_o\omega_y = 0.1053(25.98) = 2.736$ lb-ft-sec

$H_z = I\omega_z = 0.0625(35) = 2.188$ lb-ft-sec

$T = \frac{1}{2}\boldsymbol{\omega}\cdot\mathbf{H} = \frac{1}{2}(25.98\mathbf{j} + 35\mathbf{k})\cdot(2.736\mathbf{j} + 2.188\mathbf{k}) = \underline{73.8 \text{ ft-lb}}$

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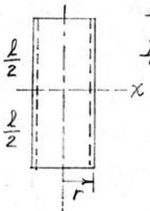

$I = I_{zz} = mr^2$

$I_o = I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$

$\frac{I_o}{I} = \frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2$

Direct precession if $I_o/I > 1$; $\frac{1}{2} + \frac{1}{12}\left(\frac{l}{r}\right)^2 > 1$, $\underline{\frac{l}{r} > \sqrt{6}}$

Retrograde " if $I_o/I < 1$; $\underline{\frac{l}{r} < \sqrt{6}}$

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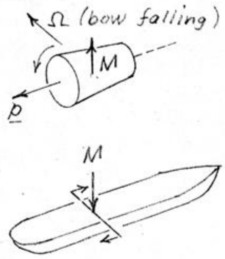
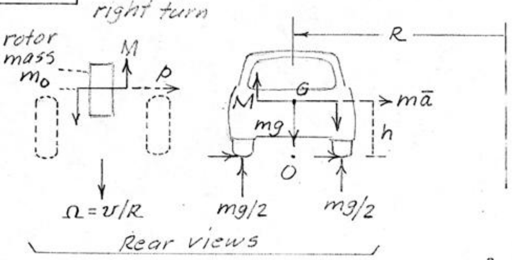


Reaction of M on hull tends to swing bow to starboard (right)

7/129 Assume right turn

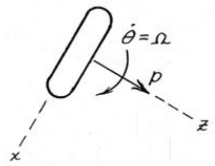


$m\bar a = mv^2/R$; $\Sigma M_O = m\bar a h$ so $M = mv^2h/R$

$M = I\Omega p$; $\frac{mv^2h}{R} = m_o k^2 \frac{v}{R} p$

$\underline{p = \frac{m}{m_o}\,\frac{vh}{k^2}}$ opposite direction to rotation of wheels

7/130



$p = \frac{v}{r} = \frac{150(10^3)}{60^2 \times 0.560/2} = 148.8$ rad/s

$\mathbf{p} = 148.8\,\mathbf{k}$ rad/s

$\Omega = \frac{30\pi}{180} = 0.524$ rad/s

$\boldsymbol{\Omega} = 0.524\,\mathbf{j}$ rad/s

$\boldsymbol{\alpha} = \boldsymbol{\Omega}\times\mathbf{p} = 0.524\mathbf{j} \times 148.8\mathbf{k} = 77.9\,\mathbf{i}$ rad/s², $\underline{\boldsymbol{\alpha} = 77.9\,\mathbf{i} \text{ rad/s}^2}$

7/131 Angular velocity $\boldsymbol{\omega}$ and velocity $\mathbf{v}$ of point A are perpendicular.

Thus $\boldsymbol{\omega}\cdot\mathbf{v} = 0$

$\boldsymbol{\omega} = \omega(300\mathbf{i} + 150\mathbf{j} + 300\mathbf{k})/\sqrt{300^2+150^2+300^2} = \frac{\omega}{3}(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$

$\mathbf{v} = 15\mathbf{i} - 20\mathbf{j} + v_z\mathbf{k}$ m/s

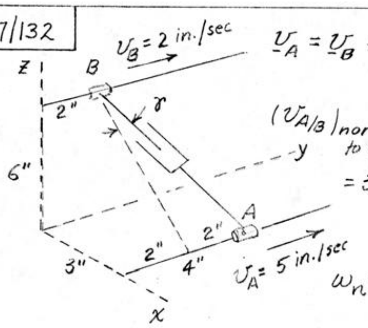
Thus $\frac{\omega}{3}(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})\cdot(15\mathbf{i} - 20\mathbf{j} + v_z\mathbf{k}) = 0$

$30 - 20 + 2v_z = 0$, $v_z = -5$ m/s

$v = \sqrt{15^2 + 20^2 + 5^2} = \underline{25.5 \text{ m/s}}$

$v = \frac{d}{2}\omega$, $d = \frac{2v}{\omega} = \frac{2(25.5)}{1720\times 2\pi/60} = 0.283$ m or $\underline{d = 283 \text{ mm}}$

7/132



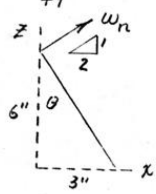
$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$, $\mathbf{v}_{A/B} = (5-2)\mathbf{j}$

$= 3\mathbf{j}$ in./sec

$(v_{A/B})_{\substack{normal\\ to\ AB}} = 3\cos\gamma$

$= 3\,\frac{\sqrt{6^2+3^2}}{\sqrt{6^2+3^2+2^2}} = \frac{9\sqrt5}{7}$ in./sec

$\omega_n = \frac{9\sqrt5/7}{7} = \frac{9\sqrt5}{49}$ rad/sec



$\omega_n = \frac{9\sqrt5}{49}(\mathbf{i}\cos\theta + \mathbf{k}\sin\theta)$

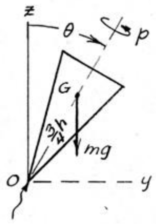
$= \frac{9\sqrt5}{49}\left(\frac{2}{\sqrt5}\mathbf{i} + \frac{1}{\sqrt5}\mathbf{k}\right)$

$\underline{\omega_n = \frac{9}{49}(2\mathbf{i} + \mathbf{k})\ \text{rad/sec}}$

7/133

$M_O = mg\frac{3}{4}h\sin\theta\,(-\mathbf{i})$

so change in angular-momentum vector is in $-x$ direction and precession is designated by $\Omega\mathbf{k}$. Eq. 7/25 gives the precession, so the period is



$\tau = 2\pi/\Omega$

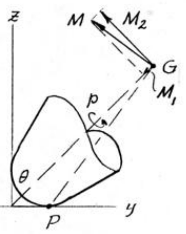
$\tau = 2\pi\Big/\left(\frac{g\bar r}{k^2p}\right)$. For the solid cone, $\bar r = \frac{3}{4}h$

& from Table D/4, $I = \frac{3}{10}mr^2$ so $k^2 = \frac{3}{10}r^2$

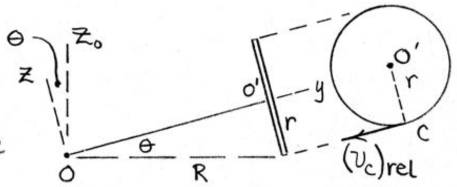
Thus

$\tau = \frac{2\pi}{\frac{3gh/4}{\frac{3}{10}r^2p}} = \underline{\frac{4\pi r^2 p}{5gh}}$ independent of θ for large p.

7/134 For the given direction of spin p, the friction force acting on the cone at P will be in the $+x$-direction. This force produces a moment M about G, a small component of which, M_1, is along the spin axis and tends to reduce the spin. The other component M_2 causes a change in the principal angular momentum $I p$ in the direction of M_2, thus causing θ to decrease.



7/135



Let $\underline{\Omega}$ be the angular velocity of the axes xyz.

$$\underline{\Omega} = \frac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta)$$

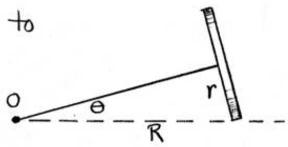
Relative to the xyz axes, O' is fixed and C moves with speed $(v_c)_{rel} = R\dfrac{2\pi}{\tau}$

So $\underline{\omega}_{rel} = \dfrac{(v_c)_{rel}}{r}(-\underline{j}) = -\dfrac{2\pi}{\tau}\dfrac{R}{r}\underline{j}$

Thus $\underline{\omega} = \dfrac{2\pi}{\tau}\left[-\dfrac{R}{r}\underline{j} + \underline{j}\sin\theta + \underline{k}\cos\theta\right]$

$$= \frac{2\pi}{\tau}\left[\left(-\frac{R}{r} + \frac{r}{R}\right)\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right]$$

7/136 From the solution to Prob. 7/135, the absolute angular velocity of the disk is



$$\underline{\omega} = \frac{2\pi}{\tau}\left[\left(-\frac{R}{r} + \frac{r}{R}\right)\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right]$$

$\underline{\alpha} = \underline{\dot{\omega}}$; Need $\dot{\underline{j}} = \underline{\Omega}\times\underline{j} = \dfrac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta)\times\underline{j}$

$$= -\frac{2\pi}{\tau}\cos\theta\,\underline{i} = \frac{2\pi}{\tau}\left(-\frac{\sqrt{R^2-r^2}}{R}\underline{i}\right)$$

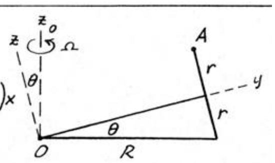
and $\dot{\underline{k}} = \underline{\Omega}\times\underline{k} = \dfrac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta)\times\underline{k}$

$$= \frac{2\pi}{\tau}\sin\theta\,\underline{i} = \frac{2\pi}{\tau}\frac{r}{R}\underline{i}$$

So $\underline{\alpha} = \left(\dfrac{2\pi}{\tau}\right)^2\left\{\left[\dfrac{r}{R} - \dfrac{R}{r}\right]\left(-\dfrac{\sqrt{R^2-r^2}}{R}\right)\underline{i} + \dfrac{\sqrt{R^2-r^2}}{R}\dfrac{r}{R}\underline{i}\right\}$

$$= \left(\frac{2\pi}{\tau}\right)^2\frac{\sqrt{R^2-r^2}}{r}\underline{i}$$

7/137 From Eq. 7/6



$\underline{v}_A = \underline{v}_O + \underline{\Omega}\times\underline{r}_{A/O} + \underline{v}_{rel}$

$\underline{v}_O = \underline{0}$, $\underline{\Omega}\times\underline{r}_{A/O} = \dfrac{2\pi}{\tau}\left(\dfrac{r}{R}\underline{j} + \dfrac{\sqrt{R^2-r^2}}{R}\underline{k}\right)\times(\sqrt{R^2-r^2}\,\underline{j} + r\underline{k})$

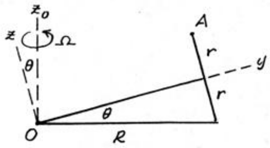
$$= \frac{2\pi}{\tau}\left(\frac{2r^2}{R} - R\right)\underline{i}$$

$\underline{\Omega} = \dfrac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta) = \dfrac{2\pi}{\tau}\left(\dfrac{r}{R}\underline{j} + \dfrac{\sqrt{R^2-r^2}}{R}\underline{k}\right)$

$\underline{v}_{rel} = -r\omega_{rel}\underline{i} = -r\left(\dfrac{R}{r}\dfrac{2\pi}{\tau}\right)\underline{i} = \dfrac{2\pi}{\tau}R\underline{i}$

$\underline{v}_A = \dfrac{2\pi}{\tau}\left[\dfrac{2r^2}{R} - R - R\right]\underline{i}$, $\underline{v}_A = -\dfrac{4\pi}{\tau}\left(R - \dfrac{r^2}{R}\right)\underline{i}$

7/138 Using Eqs. 7/6



$\underline{a}_A = \underline{a}_O + \dot{\underline{\Omega}}\times\underline{r}_{A/O} + \underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/O}) + 2\underline{\Omega}\times\underline{v}_{rel} + \underline{a}_{rel}$

$\underline{a}_O = \underline{0}$, $\dot{\underline{\Omega}} = \underline{0}$

$\underline{\Omega}\times\underline{r}_{A/O} = \dfrac{2\pi}{\tau}\left(\dfrac{r}{R}\underline{j} + \dfrac{\sqrt{R^2-r^2}}{R}\underline{k}\right)\times(\sqrt{R^2-r^2}\,\underline{j} + r\underline{k})$

$$= \frac{2\pi}{\tau}\left(\frac{2r^2}{R} - R\right)\underline{i}$$

$\underline{\Omega} = \dfrac{2\pi}{\tau}(\underline{j}\sin\theta + \underline{k}\cos\theta) = \dfrac{2\pi}{\tau}\left(\dfrac{r}{R}\underline{j} + \dfrac{\sqrt{R^2-r^2}}{R}\underline{k}\right)$

$$\underline{\Omega}\times(\underline{\Omega}\times\underline{r}_{A/O}) = \left(\frac{2\pi}{\tau}\right)^2\left(\frac{r}{R}\underline{j} + \frac{\sqrt{R^2-r^2}}{R}\underline{k}\right)\times\left(\frac{2r^2}{R} - R\right)\underline{i}$$

$$= \left(\frac{2\pi}{\tau}\right)^2\left(\frac{2r^2}{R^2} - 1\right)\left(\sqrt{R^2-r^2}\,\underline{j} - r\underline{k}\right)$$

$\underline{v}_{rel} = -r\omega_{rel}\underline{i} = -r\left(\dfrac{R}{r}\dfrac{2\pi}{\tau}\right)\underline{i} = -\dfrac{2\pi}{\tau}R\underline{i}$

$2\underline{\Omega}\times\underline{v}_{rel} = \dfrac{4\pi}{\tau}\left(\dfrac{r}{R}\underline{j} + \dfrac{\sqrt{R^2-r^2}}{R}\underline{k}\right)\times\left(-\dfrac{2\pi}{\tau}R\underline{i}\right) = -2\left(\dfrac{2\pi}{\tau}\right)^2\left(\sqrt{R^2-r^2}\,\underline{j} - r\underline{k}\right)$

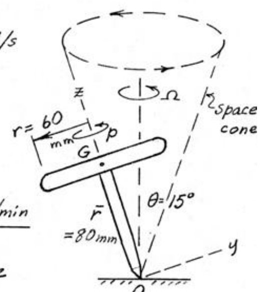
$\underline{a}_{rel} = -r\omega_{rel}^2\underline{k} = -r\left(\dfrac{R}{r}\dfrac{2\pi}{\tau}\right)^2\underline{k} = -\left(\dfrac{2\pi}{\tau}\right)^2\dfrac{R^2}{r}\underline{k}$

Substitute, simplify, & get

$$\underline{a}_A = \left(\frac{2\pi}{\tau}\right)^2\left[\sqrt{R^2-r^2}\left(\frac{2r^2}{R^2} - 3\right)\underline{j} + \left(3r - \frac{R^2}{r} - \frac{2r^3}{R^2}\right)\underline{k}\right]$$

7/139 $I_{zz} = mr^2$, $k = r = 0.060$ m

$p = 10\,000\,(2\pi/60) = 1047$ rad/s



From Eq. 7/25,

$$\Omega \approx \frac{g\bar{r}}{k^2 p} = \frac{9.81(0.080)}{(0.060)^2(1047)}$$

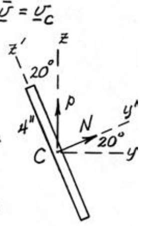
$$= 0.208 \text{ rad/s}$$

$N = \dfrac{\Omega}{2\pi}60 = \dfrac{0.208}{2\pi}\times 60 = 1.988$ cycles/min

With $\Omega = \dot{\psi}$ very small, the body cone is too small to observe, so space cone is the only relatively apparent cone.

(Note direction of precession on diagram.)

7/140 Eq. 7/14 becomes $\underline{H}_O = \underline{H}_C + \bar{\underline{r}}\times m\bar{\underline{v}}$, $\bar{\underline{r}} = \overline{OC}$, $\bar{\underline{v}} = \underline{v}_C$



For disk, $\omega_{y'} = \dfrac{300\times 2\pi}{60} + \dfrac{60\times 2\pi}{60}\sin 20° = 33.6$ rad/sec

$\omega_{z'} = \dfrac{60\times 2\pi}{60}\cos 20° = 5.90$ rad/sec

$\omega_{x'} = 0$

$I_{y'y'} = \frac{1}{2}mr^2 = \frac{1}{2}\frac{8}{32.2}\left(\frac{4}{12}\right)^2 = 0.01380$ lb-ft-sec²

$I_{z'z'} = \frac{1}{4}mr^2 = 0.00690$ lb-ft-sec²

With $\omega_x = 0$ & principal axes x-y'-z', Eq. 7/13 gives

$\underline{H}_C = I_{y'y'}\omega_{y'}\underline{j}' + I_{z'z'}\omega_{z'}\underline{k}' = 0.01380(33.6)\underline{j}' + 0.00690(5.90)\underline{k}'$

$= 0.463\underline{j}' + 0.0407\underline{k}' = 0.421\underline{j} + 0.1967\underline{k}$

$\bar{\underline{r}} = \frac{10}{12}\underline{i} = 0.833\underline{i}$ ft

$\bar{\underline{v}} = p\underline{k}\times\bar{\underline{r}} = \dfrac{60\times 2\pi}{60}\underline{k}\times 0.833\underline{i} = 5.24\underline{j}$ ft/sec

$\bar{\underline{r}}\times m\bar{\underline{v}} = 0.833\underline{i}\times\frac{8}{32.2}(5.24\underline{j}) = 1.084\underline{k}$ lb-ft-sec

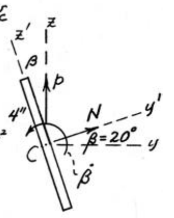
$\underline{H}_O = 0.421\underline{j} + 0.1967\underline{k} + 1.084\underline{k} = 0.421\underline{j} + 1.281\underline{k}$ lb-ft-sec

$T = \frac{1}{2}\bar{\underline{v}}\cdot\underline{G} + \frac{1}{2}\underline{\omega}\cdot\underline{H}_G$ ($G = C$ here)

$= \frac{1}{2}\,5.24\underline{j}\cdot\frac{8}{32.2}(5.24\underline{j}) + \frac{1}{2}(29.5\underline{j} + 17.03\underline{k})\cdot(0.421\underline{j} + 0.1967\underline{k})$

$= 11.30$ ft-lb

7/141 Eq. 7/14 becomes $H_O = H_C + \bar{r}\times m\bar{v}$, $\bar{r} = \overline{OC}$, $\bar{v} = v_C$



For disk, $\omega_x = \dot\beta = \frac{120\times 2\pi}{60} = 12.57$ rad/sec

$\omega_{y'} = \frac{300\times 2\pi}{60} + \frac{60\times 2\pi}{60}\sin 20° = 33.6$ rad/sec

$\omega_{z'} = \frac{60\times 2\pi}{60}\cos 20° = 5.90$ rad/sec

$I_{xx} = I_{z'z'} = \frac{1}{4}mr^2 = \frac{1}{4}\frac{8}{32.2}\left(\frac{4}{12}\right)^2 = 0.00690$ lb-ft-sec²

$I_{y'y'} = \frac{1}{2}mr^2 = 0.01380$ lb-ft-sec²

For principal axes x-y'-z' Eq. 7/13 gives

$H_C = I_{xx}\omega_x i + I_{y'y'}\omega_{y'} j' + I_{z'z'}\omega_{z'} k'$

$= 0.00690(12.57)i + 0.01380(33.8)j' + 0.00690(5.90)k'$

$H_C = 0.0867i + 0.463j' + 0.0407k'$

$= 0.0867i + 0.421j + 0.1967k$ lb-ft-sec

$\bar{r} = \frac{10}{12}i = 0.833i$ ft

$\bar{v} = pk\times\bar{r} = \frac{60\times 2\pi}{60}k\times 0.833i = 5.24j$ ft/sec

$\bar{r}\times m\bar{v} = 0.833i\times\frac{8}{32.2}(5.24j) = 1.084k$ lb-ft-sec

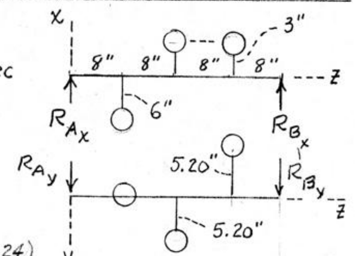
$H_O = 0.0867i + 0.421j + 0.1967k + 1.084k = \underline{0.0867i + 0.421j + 1.281k}$ lb-ft-sec

$T = \frac{1}{2}\bar{v}\cdot G + \frac{1}{2}\omega\cdot H_G$ (G = C here)

$= \frac{1}{2}5.24j\cdot\frac{8}{32.2}(5.24j) + \frac{1}{2}(12.57i + 29.5j + 17.03k)\cdot(0.0867i + 0.421j + 0.1967k)$

$= \underline{11.85 \text{ ft-lb}}$

7/142 $\omega_z = \frac{1200(2\pi)}{60} = 40\pi$ rad/sec



Eq. 7/23

$\Sigma M_x = I_{yz}\omega_z^2$

$\Sigma M_y = -I_{xz}\omega_z^2$

Where

$I_{yz} = m(5.20\times 16 - 5.20\times 24) = -161.4(10^{-3})$ in.-lb-sec²

$m = \frac{1.5}{32.2}\frac{1}{12} = 3.88(10^{-3})$ lb-sec²/in.

$I_{xz} = m(-6\times 8 + 3\times 16 + 3\times 24) = 280(10^{-3})$ in.-lb-sec²

$\Sigma M_x = -32R_{B_y} = -0.1614(40\pi)^2$, $R_{B_y} = 79.6$ lb

$\Sigma M_y = +32R_{B_x} = -0.280(40\pi)^2$, $R_{B_x} = -137.9$ lb

Because mass center has no acceleration

$R_{A_y} = -R_{B_y}$, $R_{A_x} = R_{B_x}$

$|R_A| = |R_B| = \sqrt{79.6^2 + 137.9^2} = \underline{159.3 \text{ lb}}$

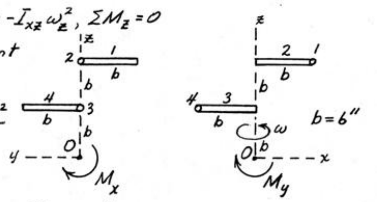
7/143 With $\omega_x = \omega_y = 0$, $\omega_z = \frac{1200\times 2\pi}{60} = 125.7$ rad/sec,

$\dot\omega_x = \dot\omega_y = \dot\omega_z = 0$, Eqs. 7/23 about O become

$\Sigma M_x = I_{yz}\omega_z^2$, $\Sigma M_y = -I_{xz}\omega_z^2$, $\Sigma M_z = 0$

Let m = mass of each segment of length b

$= \frac{1.4}{32.2} = 0.0435\ \frac{\text{lb-sec}^2}{\text{ft}}$



Static forces produce no moment so are not shown.

① ② ③ ④

$I_{xz} = m(b)(2b) + m\left(\frac{b}{2}\right)(2b) + m\left(-\frac{b}{2}\right)(b) + m(-b)(b) = \frac{3}{2}mb^2$

$M_y = -\frac{3}{2}mb^2\omega_z^2 = -\frac{3}{2}(0.0435)\left(\frac{6}{12}\right)^2(125.7)^2 = -257$ lb-ft

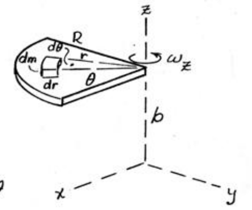
① ② ③ ④

$I_{yz} = m\left(-\frac{b}{2}\right)(2b) + m(0) + m(0) + m\left(\frac{b}{2}\right)(b) = -\frac{1}{2}mb^2$

$M_x = -\frac{1}{2}mb^2\omega_z^2 = -\frac{1}{2}(0.0435)\left(\frac{6}{12}\right)^2(125.7)^2 = -85.8$ lb-ft

$M = \sqrt{M_x^2 + M_y^2} = \sqrt{85.8^2 + 257^2} = \underline{271 \text{ lb-ft}}$

7/144 Let m = mass of each plate



mass per unit area $= m/(\pi R^2/4)$

$= 4m/\pi R^2$

$dm = \frac{4m}{\pi r^2} r\,dr\,d\theta$

$I_{xz} = \int xz\,dm = \frac{4m}{\pi R^2}\int_0^{\frac{\pi}{2}}\int_0^R (r\cos\theta)b\,r\,dr\,d\theta$

$= \frac{4mbR}{3\pi}$

$I_{yz} = \int yz\,dm = \frac{4m}{\pi R^2}\int_0^{\frac{\pi}{2}}\int_0^R (-r\sin\theta)b\,r\,dr\,d\theta = -\frac{4mbr}{3\pi}$

Top plate $I_{xz} = -I_{yz} = \frac{4(2)(0.150)(0.150)}{3\pi} = 0.01910$ kg·m²

Lower plate $I_{xz} = -\frac{4mbR}{3\pi}$, $I_{yz} = \frac{4mbR}{3\pi}$ where $b = 0.075$ m ($\frac{1}{2}$ of 0.150)

$I_{xz} = -I_{yz} = -0.01910/2 = -0.00955$ kg·m²

From Eq. 7/23 with $\omega_x = \omega_y = 0$, $\omega_z = \frac{2\pi(300)}{60} = 10\pi$ rad/s, $\dot\omega_z = 0$

$\Sigma M_x = I_{yz}\omega_z^2 = (-0.01910 + 0.00955)(10\pi)^2 = -9.42$ N·m

$\Sigma M_y = -I_{xz}\omega_z^2 = -(0.01910 - 0.00955)(10\pi)^2 = -9.42$ N·m

$M = \sqrt{9.42^2 + 9.42^2} = \underline{13.33 \text{ N·m}}$

7/145 With $\omega_x = \omega_y = \omega_z = 0$ & $\dot\omega_z = 200$ rad/s², Eq. 7/23 gives

$\Sigma M_x = -I_{xz}\dot\omega_z$, $\Sigma M_y = -I_{yz}\dot\omega_z$

From solution to Prob. 7/144,

$I_{xz} = 0.01910 - 0.00955 = 0.00955$ kg·m²

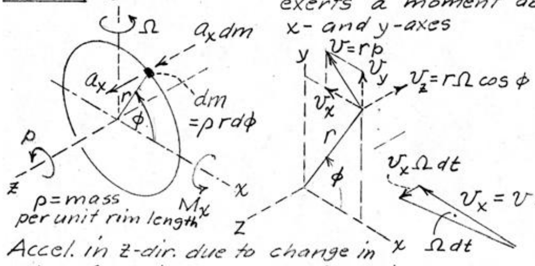
$I_{yz} = -0.01910 + 0.00955 = -0.00955$ kg·m²

So $\Sigma M_x = -0.00955(200) = -1.910$ N·m

$\Sigma M_y = 0.00955(200) = 1.910$ N·m

$M = \sqrt{1.910^2 + 1.910^2} = \underline{2.70 \text{ N·m}}$

▶7/146 $a_x dm$ is the only force on dm which exerts a moment about x- and y-axes



Accel. in z-dir. due to change in dir. of v_x is $v_x\Omega = rp\Omega\sin\phi$

Accel. in z-dir. due to change in mag. of v_z is

$-\frac{d}{dt}(r\Omega\cos\phi) = r\Omega\dot\phi\sin\phi = r\Omega p\sin\phi$

Thus $a_z = 2rp\Omega\sin\phi$

$M_x = \int r\sin\phi\,(a_x dm) = 2\rho r^3 p\,\Omega\int_0^{2\pi}\sin^2\phi\,d\phi = 2\rho r^3 p\,\Omega\pi$

$= mr^2\Omega p = \underline{I\Omega p}$

$M_y = -\int r\cos\phi\,(a_x dm) = 2\rho r^3 p\,\Omega\int_0^{2\pi}\sin\phi\cos\phi\,d\phi = 0$

8/1

$$k = \frac{W}{\delta_{st}} = \frac{3(9.81)}{0.042} = \underline{701 \text{ N/m}}$$

$$k = 701 \frac{\text{N}}{\text{m}}\left(\frac{1 \text{ lb/in.}}{175.13 \text{ N/m}}\right) = \underline{4.00 \text{ lb/in.}}$$

$$k = 4.00 \frac{\text{lb}}{\text{in.}}\left(\frac{12 \text{ in.}}{\text{ft}}\right) = \underline{48.0 \text{ lb/ft}}$$

8/2

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{g}{W/k}} = \underline{\sqrt{\frac{g}{\delta_{st}}}}$$

8/3

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{54(12)}{2}} = \underline{18 \text{ rad/sec}}$$

$$f_n = \left(18 \frac{\text{rad}}{\text{sec}}\right)\left(\frac{1 \text{ cycle}}{2\pi \text{ rad}}\right) = \underline{2.86 \text{ Hz}}$$

8/4

$$x = x_0 \cos\omega_n t + \frac{\dot{x}_0}{\omega_n}\sin\omega_n t$$

$x_0 = -2$ in., $\dot{x}_0 = 0$, $\omega_n = 18$ rad/sec, so

$$\underline{x = -2\cos 18t \text{ in.}}$$

$$\dot{x} = +36\sin 18t \text{ in./sec} \Rightarrow \underline{v_{max} = 36 \text{ in./sec}}$$

$$\ddot{x} = 36(18)\cos 18t \text{ in./sec}^2 \Rightarrow \underline{a_{max} = 648 \text{ in./sec}^2}$$

(or $\underline{v_{max} = 3 \text{ ft/sec}}$, $\underline{a_{max} = 54 \text{ ft/sec}^2}$)

8/5

$$x = C\sin(\omega_n t + \psi)$$

$$C = \left[x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2\right]^{1/2} = \left[2^2 + \left(\frac{-9}{18}\right)^2\right]^{1/2} = \underline{2.06 \text{ in.}}$$

$$\psi = \tan^{-1}\left(\frac{x_0\omega_n}{\dot{x}_0}\right) = \tan^{-1}\left(\frac{2(18)}{-9}\right) = 1.816 \text{ rad}$$

So $\underline{x = 2.06\sin(18t + 1.816) \text{ in.}}$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{18} = \underline{0.349 \text{ sec}}$$

(ω_n from solution to Prob. 8/3)

8/6

$$\delta_{st} = \frac{W}{k} = \frac{2(9.81)}{98} = \underline{0.200 \text{ m}}$$

$$\omega_n = \sqrt{k/m} = \sqrt{98/2} = 7 \text{ rad/s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{7} = \underline{0.898 \text{ s}}$$

$$v_{max} = C\omega_n = 0.1(7) = \underline{0.7 \text{ m/s}}$$

8/7 $\omega_n = 7$ rad/s (from Prob. 8/6)

$$y = y_0\cos\omega_n t + \frac{\dot{y}_0}{\omega_n}\sin\omega_n t$$

$$= 0.1\cos 7t \text{ m}$$

$$v = \dot{y} = -0.7\sin 7t \text{ m/s}$$

$$a = \ddot{y} = -4.9\cos 7t \text{ m/s}^2$$

When $t = 3$ s:

$$y = 0.1\cos(7\cdot 3) = \underline{-0.0548 \text{ m}}$$

$$v = -0.7\sin(7\cdot 3) = \underline{-0.586 \text{ m/s}}$$

$$a = -4.9\cos(7\cdot 3) = \underline{+2.68 \text{ m/s}^2}$$

$$\underline{a_{max} = 4.9 \text{ m/s}^2}$$

8/8 $F = kx$: $30\times 9.81 = k(0.050)$, $k = 5890$ N/m

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{5890}{30}} = \underline{2.23 \text{ Hz}}$$

8/9 $\ddot{x} + \omega_n^2 x = 0$ where $\omega_n = \sqrt{k/m} = 2\pi(2.23) = 14.01$ rad/s (Prob. 8/8)

$x = A\cos\omega_n t + B\sin\omega_n t$, $\dot{x} = -A\omega_n\sin\omega_n t + B\omega_n\cos\omega_n t$

When $t = 0$, $\dot{x} = 0$, $0 = 0 + B\omega_n$, $B = 0$

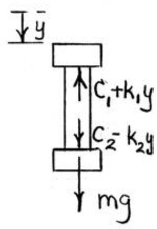
" $t = 0$, $x = 0.025$ m, $0.025 = A\times 1$, $A = 0.025$ m

$x = 0.025\cos 14.01t$ meters

or $\underline{x = 25\cos 14.01t \text{ mm}}$ (t in seconds)

8/10

Equil. pos.



$\Sigma F_y = m\ddot{y}$: $C_2 - k_2 y + mg - C_1 - k_1 y = m\ddot{y}$

At equilibrium, $C_2 + mg - C_1 = 0$

So $-(k_1 + k_2)y = m\ddot{y}$

$$\ddot{y} + \frac{k_1 + k_2}{m}y = 0$$

$$\omega_n = \sqrt{\frac{k_1 + k_2}{m}} = \sqrt{\frac{3600 + 1800}{2.5}} = 46.5 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{46.5}{2\pi} = \underline{7.40 \text{ Hz}}$$

8/11 $\omega_n = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2(180{,}000)}{100}} = 60$ rad/s

$$x = x_0\cos\omega_n t + \frac{\dot{x}_0}{\omega_n}\sin\omega_n t$$

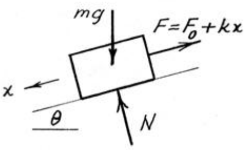
$$= 0 + \frac{0.5}{60}\sin 60t = 8.33(10^{-3})\sin 60t$$

$$\dot{x} = 60(8.33)(10^{-3})\cos 60t = 0.5\cos 60t$$

$$\ddot{x} = -60(0.5)\sin 60t = -30\sin 60t$$

$$\underline{a_{max} = 30 \text{ m/s}^2}$$

8/12



For equilibrium position
$F_0 = mg\sin\theta$

$\Sigma F_x = m\ddot{x}: \quad mg\sin\theta - (mg\sin\theta + kx) = m\ddot{x}$

$\ddot{x} + \frac{k}{m}x = 0$ so $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ independent of θ

8/13

$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(17{,}500)}{1000}} = 8.37\ \frac{\text{rad}}{\text{sec}}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{8.37}{2\pi} = 1.332\ \text{Hz}$$

We have assumed the unsprung mass (wheels, axles, etc.) to be a small fraction of the total car mass.

8/14

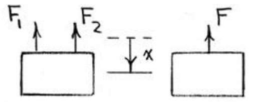
(a) From Eq. 8/3 frequency $f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

$3 = \frac{1}{2\pi}\sqrt{\frac{3k}{4000}}$, $k = 474\times10^3$ N/m or $k = 474$ kN/m

(b) For $m = 4000 + 40\,000 = 44\,000$ kg,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3(474\times10^3)}{44\times10^3}} = 0.905\ \text{Hz}$$

8/15



(a) $F = F_1 + F_2$

$kx = k_1x + k_2x$

$k = k_1 + k_2$

(b) $F = F_1 = F_2$

$x_1 = \frac{F_1}{k_1}$, $x_2 = \frac{F_2}{k_2}$, $x = \frac{F}{k}$

From $x = x_1 + x_2$, we have $\frac{F}{k} = \frac{F_1}{k_1} + \frac{F_2}{k_2}$

or $\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2}$. Thus $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

8/16

Deflect the system as shown and experimentally observe its natural frequency ω_1. Then add m_2 and observe ω_2.

$\omega_1 = \sqrt{\frac{k}{m_1}}$, $\omega_2 = \sqrt{\frac{k}{m_1+m_2}}$

Solve simultaneously to obtain

$$m_1 = \frac{m_2\omega_2^2}{\omega_1^2 - \omega_2^2}$$

$$k = \frac{m_2\omega_1^2\omega_2^2}{\omega_1^2 - \omega_2^2}$$

We could have used the natural frequencies in Hz – f_1 and f_2 – rather than ω_1 and ω_2 in rad/sec. Alternative solutions:

(a) Place m_1, then m_2 (but not both) on spring.

(b) Place m_2, then $m_1 + m_2$ on spring.

8/17

$$f_n = \frac{1}{2\pi}\sqrt{k/m}$$

$\frac{1}{0.6} = \frac{1}{2\pi}\sqrt{\frac{k}{90}}$, $k = 9870$ N/m

$$\delta_{st} = \frac{W}{k} = \frac{90(9.81)}{9870} = 0.0895\ \text{m or } 89.5\ \text{mm}$$

8/18

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{k/m_{Tot}}$$

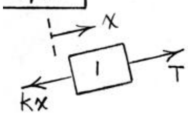
$\frac{1}{0.75} = \frac{1}{2\pi}\sqrt{600/(m+6)}$, $m = 2.55$ kg

$$\omega_n = \sqrt{k/m_{Tot}} = \sqrt{600/(6+2.55)} = 8.38\ \text{rad/s}$$

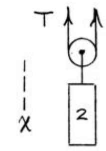
$$a_{max} = \omega_n^2 C = 8.38^2(0.050) = 3.51\ \text{m/s}^2$$

$a_{max} = \mu_s g$: $3.51 = \mu_s(9.81)$, $\mu_s = 0.358$

8/19



Free body diagrams show dynamic forces only. x is the displacement from equilibrium. From constraint, $a_2 = \frac{1}{2}a_1$



$\Sigma F_x = m\ddot{x}$:

① $-kx + T = m\ddot{x}$

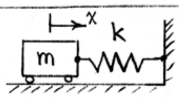
② $-2T = m(\frac{1}{2}\ddot{x})$

Eliminating T: $\ddot{x} + \frac{4k}{5m}x = 0$

$$\omega_n = \sqrt{\frac{4k}{5m}}$$

8/20

Equivalent system:



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{(3000)(12)}{2500/32.2}} = 21.5\ \text{rad/sec}$$

$$x = \cancel{x_0}^{\,0}\cos\omega_n t + \frac{\dot{x}_0}{\omega_n}\sin\omega_n t$$

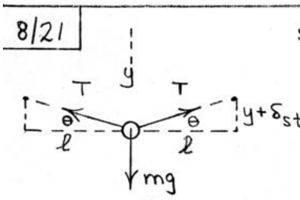
$$= \frac{5(5280/3600)}{21.5}\sin 21.5t = 0.341\sin 21.5t$$

$x_{max} = 0.341$ ft or 4.09 in.

$v = (0.341)(21.5)\cos 21.5t = 7.33\cos 21.5t$ (in ft/sec)

or $v = 88.0\cos 21.5t$ in./sec

8/21



$\sin\theta \cong \theta \cong \dfrac{y+\delta_{st}}{\ell}$

δ_{st} = static deflection
y = dynamic deflection

$\Sigma F_y = m\ddot{y}: \quad -2T\sin\theta + mg = m\ddot{y}$

$-2T\left(\dfrac{y+\delta_{st}}{\ell}\right) + mg = m\ddot{y}$

$-2T\dfrac{y}{\ell} - 2T\dfrac{\delta_{st}}{\ell} + mg = m\ddot{y}$

$\ddot{y} + \left(\dfrac{2T}{m\ell}\right)y = 0, \quad \omega_n = \sqrt{\dfrac{2T}{m\ell}}$

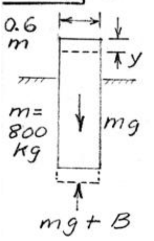
Although done above, the inclusion of the forces $+mg$ and $-2T\dfrac{\delta_{st}}{\ell}$, which sum to zero, is not necessary.

8/22 $f_n = \dfrac{1}{2\pi}\sqrt{k/m}$

$\dfrac{1}{0.5} = \dfrac{1}{2\pi}\sqrt{k/4000}, \quad k = 632 \text{ kN/m}$

$\delta_{st} = \dfrac{W}{k} = \dfrac{4000(9.81)}{632(10^3)} = 0.0621 \text{ m or } 62.1 \text{ mm}$

8/23 B = added buoyancy due to deflection y below equil. position



B = Vol. × density × g

$= \dfrac{\pi d^2}{4} y\rho g = \dfrac{\pi(0.6^2)}{4} y\,(1.03\times10^3)(9.81)$

$= 2857y \quad N$

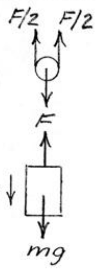
$\Sigma F_y = ma_y; \quad mg - (mg+B) = m\ddot{y}$

$\ddot{y} + \dfrac{2857}{800}y = 0$

$\omega_n = \sqrt{\dfrac{2857}{800}} = 1.890 \dfrac{rad}{sec}, \quad f_n = \dfrac{\omega_n}{2\pi} = \dfrac{1.890}{2\pi}$

$= 0.301$ Hz

8/24



When $y = 0$, $F_0 = mg$

For $y = y$, spring stretched $2y$

So $F/2 = \dfrac{mg}{2} + k(2y)$

Hence $F = mg + 4ky$

$\Sigma F_y = m\ddot{y}; \quad mg - (mg + 4ky) = m\ddot{y}$

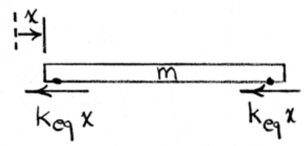
$\ddot{y} + 4\dfrac{k}{m}y = 0$

$\tau = \dfrac{2\pi}{\omega_n} = \dfrac{2\pi}{\sqrt{\dfrac{4k}{m}}} = \pi\sqrt{\dfrac{m}{k}}$

8/25 For one upright $P = \left(\dfrac{12EI}{L^3}\right)\delta = k_{eq}\delta$

So $k_{eq} = \dfrac{12EI}{L^3}$.

FBD of top mass:
(dynamic forces only)

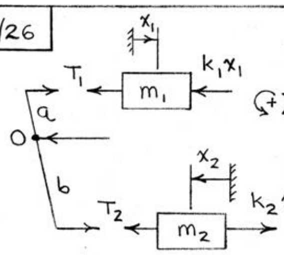


$\Sigma F_x = m\ddot{x}: \quad -2k_{eq}x = m\ddot{x}$

$\ddot{x} + \dfrac{2k_{eq}}{m}x = 0 \quad \text{or} \quad \ddot{x} + \dfrac{24EI}{mL^3}x = 0$

$\omega_n = \sqrt{\dfrac{24EI}{mL^3}} = 2\sqrt{\dfrac{6EI}{mL^3}}$

8/26



$\dfrac{x_1}{a} = \dfrac{x_2}{b}, \quad x_2 = \dfrac{b}{a}x_1$

$\circlearrowleft \Sigma M_O = 0: \quad -T_1 a + T_2 b = 0$

$T_2 = \dfrac{a}{b}T_1$

$\Sigma F_x = m\ddot{x}: \quad -T_1 - k_1x_1 = m_1\ddot{x}_1 \qquad (a)$

$T_2 - k_2x_2 = m_2\ddot{x}_2$

Second eq.: $\dfrac{a}{b}T_1 - k_2\left(\dfrac{b}{a}x_1\right) = m_2\dfrac{b}{a}\ddot{x}_1 \qquad (b)$

Solve (b) for T_1 and substitute into (a):

$\left[m_1 + \dfrac{b^2}{a^2}m_2\right]\ddot{x}_1 + \left[k_1 + \dfrac{b^2}{a^2}k_2\right]x_1 = 0$

For $k_1 = k_2 = k$, $m_1 = m_2 = m$: $m\ddot{x}_1 + kx_1 = 0$, $\omega_n' = \sqrt{k/m}$

8/27 The velocity of the putty after dropping 2 m is

$v = \sqrt{2gh} = \sqrt{2(9.81)(2)} = 6.26$ m/s

The additional static deflection due to the 3-kg mass is

$\delta_{st} = \dfrac{W}{4k} = \dfrac{3(9.81)}{4(800)} = 9.20(10^{-3})$ m

Velocity after impact: $\dot{x}_0 = \dfrac{3(6.26)}{31} = 0.606$ m/s

Natural frequency of motion: $\omega_n = \sqrt{\dfrac{4k}{m}} = 10.16 \dfrac{rad}{s}$

$x = \delta_{st} + x_0\cos\omega_n t + \dfrac{\dot{x}_0}{\omega_n}\sin\omega_n t$

$= 9.20(10^{-3}) - 9.20(10^{-3})\cos 10.16t + 59.7(10^{-3})\sin 10.16t$

$= 9.20(10^{-3})(1-\cos 10.16t) + 59.7(10^{-3})\sin 10.16t$ m

8/28 $\omega_n = \sqrt{k/m} = \sqrt{\dfrac{3(12)}{8/32.2}} = 12.04 \dfrac{rad}{sec}$

$\zeta = \dfrac{c}{2m\omega_n} = \dfrac{2.5}{2\left(\frac{8}{32.2}\right)(12.04)} = 0.418$

8/29 $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{30\,000}{35}} = 29.3$ rad/s

$\zeta = \dfrac{c}{2m\omega_n}, \quad c = 2m\omega_n = 2(35)(29.3)$

$= 2050 \; \dfrac{N\cdot s}{m}$

8/30 $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3(12)}{8/32.2}} = 12.04$ rad/sec

$\zeta = \frac{c}{2m\omega_n} = \frac{2.5}{2\left(\frac{8}{32.2}\right)(12.04)} = 0.418$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 12.04\sqrt{1-0.418^2} = 10.94\ \frac{\text{rad}}{\text{sec}}$

$x = (A_3 \cos\omega_d t + A_4 \sin\omega_d t)e^{-\zeta\omega_n t}$

$= (A_3 \cos 10.94t + A_4 \sin 10.94t)e^{-5.03t}$

Initial conditions: $x_0 = A_3$

$\dot{x} = -5.03(A_3 \cos 10.94t + A_4 \sin 10.94t)e^{-5.03t}$
$+ 10.94(-A_3 \sin 10.94t + A_4 \cos 10.94t)e^{-5.03t}$

$0 = -5.03A_3 + 10.94A_4$

$A_4 = 0.460A_3 = 0.460x_0$

So $x = x_0(\cos 10.94t + 0.460 \sin 10.94t)e^{-5.03t}$

8/31 $\tau_d = 1.25\tau$

$\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.25\frac{2\pi}{\omega_n}$, $\zeta = 0.6$

8/32 Log decrement $\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln\left(\frac{4.65}{4.3}\right)$
$= 0.0783$

Then $\zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}} = 0.01245$

From $\zeta = \frac{c}{2m\omega_n}$, $c = 2m\omega_n\zeta$

$c = 2(1.1)(10\cdot 2\pi)(0.01245) = 1.721\ \frac{\text{N}\cdot\text{s}}{\text{m}}$

8/33 $\omega_n = \sqrt{k/m} = \sqrt{\frac{200(12)}{80/32.2}} = 31.1$ rad/sec

$\overset{1}{\zeta} = \frac{c}{2m\omega_n}$, $c = 2m\omega_n = 2\left(\frac{80}{32.2}\right)(31.1)$
$= 154.4$ lb-sec/ft

8/34 $\frac{x_1}{x_2} = 4$, so log decrement δ is

$\delta = \ln\frac{x_1}{x_2} = \ln 4 = 1.386$

Viscous damping factor $\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$

$= \frac{1.386}{\sqrt{4\pi^2 + 1.386^2}} = 0.215$

Natural frequency $\omega_n = \frac{2\pi}{\tau_n} = \frac{2\pi}{0.75} = 8.38\ \frac{\text{rad}}{\text{s}}$

From $\omega_n = \sqrt{\frac{k}{m}}$, $k = m\omega_n^2 = 2.5(8.38)^2 = 175.5\ \frac{\text{N}}{\text{m}}$

Damping ratio $\zeta = \frac{c}{2m\omega_n}$

So $c = 2m\omega_n\zeta = 2(2.5)(8.38)(0.215) = 9.02\ \frac{\text{N}\cdot\text{s}}{\text{m}}$

8/34 $\frac{x_1}{x_2} = 4$, so log decrement δ is

$\delta = \ln\frac{x_1}{x_2} = \ln 4 = 1.386$

Viscous damping factor $\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$

$= \frac{1.386}{\sqrt{4\pi^2 + 1.386^2}} = 0.215$

Natural frequency $\omega_n = \frac{2\pi}{\tau_n} = \frac{2\pi}{0.75} = 8.38\ \frac{\text{rad}}{\text{s}}$

From $\omega_n = \sqrt{\frac{k}{m}}$, $k = m\omega_n^2 = 2.5(8.38)^2 = 175.5\ \frac{\text{N}}{\text{m}}$

Damping ratio $\zeta = \frac{c}{2m\omega_n}$

So $c = 2m\omega_n\zeta = 2(2.5)(8.38)(0.215) = 9.02\ \frac{\text{N}\cdot\text{s}}{\text{m}}$

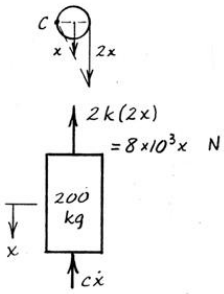
8/35 $\frac{x_0}{x_N} = \frac{Ce^{-\zeta\omega_n t_0}}{Ce^{-\zeta\omega_n(t_0+N\tau_d)}} = e^{\zeta\omega_n N\tau_d}$

Define $\delta_N = \ln\frac{x_0}{x_N} = \zeta\omega_n N\tau_d$, $\tau_d = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$

so $\delta_N = \zeta\cancel{\omega_n} N\frac{2\pi}{\cancel{\omega_n}\sqrt{1-\zeta^2}} = \frac{2\pi N\zeta}{\sqrt{1-\zeta^2}}$

Solve for ζ and get $\zeta = \frac{\delta_N}{\sqrt{(2\pi N)^2+\delta_N^2}}$

8/36 *Dynamic forces shown only.*
x measured from equilibrium position



$\Sigma F_x = m\ddot{x}:\ -c\dot{x} - 8\times10^3x = 200\ddot{x}$

$200\ddot{x} + c\dot{x} + 8\times10^3x = 0$

$\omega_n = \sqrt{\frac{8\times10^3}{200}} = 2\sqrt{10}$ rad/s

Let $\zeta = \frac{c}{2m\omega_n} = \frac{c}{400\times2\sqrt{10}}$,

$\zeta = 1$ for critical damping

so $c = 800\sqrt{10} = 2530$ N·s/m

8/37 Eq. 8/11: $x = (A_3 \cos\omega_d t + A_4 \sin\omega_d t)e^{-\zeta\omega_n t}$

At time $t = 0$: $x_0 = A_3$

$\dot{x} = -\zeta\omega_n(A_3\cos\omega_d t + A_4 \sin\omega_d t)e^{-\zeta\omega_n t}$
$+ \omega_d(-A_3\sin\omega_d t + A_4\cos\omega_d t)e^{-\zeta\omega_n t}$

At time $t = 0$: $\dot{x}_0 = -\zeta\omega_n A_3 + \omega_d A_4 = 0$

$A_4 = x_0\omega_n\zeta/\omega_d$

Thus $x = x_0\left[\cos\omega_d t + \frac{\zeta\omega_n}{\omega_d}\sin\omega_d t\right]e^{-\zeta\omega_n t}$

At time $t = \tau_d = 2\pi/(\omega_n\sqrt{1-\zeta^2})$

$x_{\tau_d} = x_0[1+0]e^{-\zeta\omega_n\left(\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}\right)}$

$\frac{x_0}{2} = x_0 e^{-2\pi\zeta/\sqrt{1-\zeta^2}}$

$\ln\left(\frac{1}{2}\right) = -\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$, $\zeta = 0.1097$

8/38

$$\frac{x_8}{x_{20}} = \frac{e^{-\zeta\omega_n t_8}}{e^{-\zeta\omega_n(t_8+12\tau_d)}} = e^{\zeta\omega_n(12\tau_d)}$$

Then $\ln\frac{x_8}{x_{20}} = \zeta\omega_n(12\tau_d)$

But $\omega_n\tau_d = \frac{2\pi}{\sqrt{1-\zeta^2}}$

So $\ln(16) = 12\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$, $\underline{\zeta = 0.0367}$

8/39

Combined $c = 2m\omega_n\zeta$ where $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{3\times474\times10^3}{4000}} = 18.85$ rad/s

and $\zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$

But the logarithmic decrement $\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln 4 = 1.386$

so the viscous damping factor $\zeta = 1.386/\sqrt{(2\pi)^2+1.386^2} = 0.215$

Thus combined $c = 2(4000)(18.85)(0.215) = 32.5\times10^3$

& for each damper $c = 32.5\times10^3/2 = \underline{16.24\times10^3 \text{ N·s/m}}$

8/40

$\omega_n = \sqrt{k/m} = \sqrt{98/2} = 7$ rad/s

$\zeta = \frac{c}{2m\omega_n} = \frac{42}{2(2)(7)} = 1.5$

$s_{1,2} = \omega_n\left[-\zeta \pm \sqrt{\zeta^2-1}\right]$

$= 7\left[-1.5 \pm \sqrt{1.5^2-1}\right] = \begin{cases}-2.674\\-18.326\end{cases}$

So $x = A_1 e^{-2.674t} + A_2 e^{-18.326t}$

At $t=0$: $x_0 = A_1 + A_2$ (a)

$\dot{x} = -2.674A_1 e^{-2.674t} - 18.326A_2 e^{-18.326t}$

At $t=0$: $0 = -2.674A_1 - 18.326A_2$ (b)

Solve (a) and (b) for A_1 and A_2.

Then $\underline{x = x_0\left[1.171e^{-2.67t} - 0.1708e^{-18.33t}\right]}$

8/41

$\omega_n = \sqrt{k/m} = \sqrt{108/3} = 6$ rad/s

$\zeta = \frac{c}{2m\omega_n} = \frac{18}{2(3)(6)} = 0.5$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6\sqrt{1-0.5^2} = 5.196$ rad/s

$x = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\zeta\omega_n t}$

$x(t=0) = A_1 = x_0$

$\dot{x} = -\zeta\omega_n(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\zeta\omega_n t}$
$\quad + \omega_d(-A_1\sin\omega_d t + A_2\cos\omega_d t)e^{-\zeta\omega_n t}$

$\dot{x}(t=0) = -\zeta\omega_n A_1 + A_2\omega_d = 0$

$A_2 = \zeta\omega_n A_1/\omega_d = 0.5(6)x_0/5.196 = 0.577x_0$

So $x = x_0[\cos 5.196t + 0.577\sin 5.196t]e^{-3t}$

and $x(t=\frac{\tau_d}{2}) = x(t=0.605) = \underline{-0.1630x_0}$

8/42

$x = (A_1 + A_2 t)e^{-\omega_n t}$

$x(t=0) = A_1 = x_0$

$\dot{x} = A_2 e^{-\omega_n t} - \omega_n(A_1 + A_2 t)e^{-\omega_n t}$

$\dot{x}(t=0) = A_2 - \omega_n A_1 = \dot{x}_0$

$A_2 = \dot{x}_0 + \omega_n x_0$

So $x = [x_0 + (\dot{x}_0 + \omega_n x_0)t]e^{-\omega_n t}$

For x to become negative with $x_0 > 0$,

$\dot{x}_0 + \omega_n x_0 < 0$, $\dot{x}_0 < -\omega_n x_0$ or $\underline{(\dot{x}_0)_c = -\omega_n x_0}$

8/43

$\omega_n = \sqrt{k/m} = \sqrt{\frac{12}{96.6/32.2}} = 2$ rad/sec

(a) $\zeta = \frac{c}{2m\omega_n} = \frac{12}{2(3)(2)} = 1$ (Critical damping)

$x = (A_1 + A_2 t)e^{-\omega_n t}$

Consideration of initial conditions yields

$x = (0.5+t)e^{-2t}$, $\underline{x(t=0.5) = 0.368 \text{ ft}}$ $\underline{(4.42 \text{ in.})}$

(b) $\zeta = \frac{c}{2m\omega_n} = \frac{18}{2(3)(2)} = 1.5$ (Overdamped)

$x = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

Determine A_1 and A_2 in usual fashion:

$x = \left(\frac{\lambda_2 x_0}{\lambda_2-\lambda_1}\right)e^{\lambda_1 t} + \left(\frac{\lambda_1 x_0}{\lambda_1-\lambda_2}\right)e^{\lambda_2 t}$, where

$\lambda_{1,2} = \omega_n\left[-\zeta \pm \sqrt{\zeta^2-1}\right] = -0.7639, -5.236$

$x = 0.585e^{-0.764t} - 0.085e^{-5.24t}$

$x(t=0.5) = \underline{0.393 \text{ ft } (4.72 \text{ in.})}$

8/44

$\zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$ where $\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln\frac{3}{1/2} = 1.792$

$\zeta = \frac{1.792}{\sqrt{(2\pi)^2+1.792^2}} = \underline{0.274}$

$c = 2m\omega_n\zeta$ where equivalent m for each absorber is

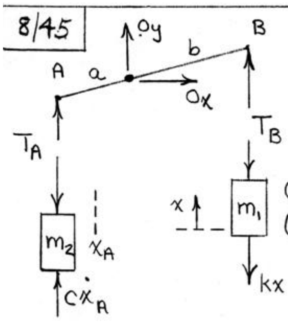
$\frac{1}{2}\left(\frac{1}{2}\frac{3400}{32.2}\right) = 26.4$ lb-sec²/ft

$k = F/\delta_{st} = 100/\frac{3}{12} = 400$ lb/ft (for both)

$\omega_n = \sqrt{k/m} = \sqrt{400/52.8} = 2.75$ rad/sec

so for each shock, $c = 2(26.4)(2.75)(0.274) = \underline{39.9 \text{ lb-sec/ft}}$

8/45



$\dfrac{x}{b} = \dfrac{x_A}{a}, \quad \dot{x}_A = \left(\dfrac{a}{b}\right)\dot{x}, \quad \ddot{x}_A = \left(\dfrac{a}{b}\right)\ddot{x}$

$\Sigma F_x = m\ddot{x}$:

① $-kx - T_B = m_1\ddot{x}$

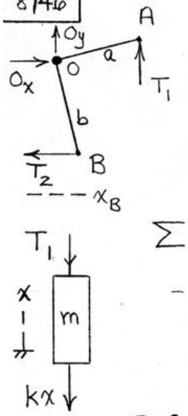
② $T_A - c\dot{x}_A = m_2\ddot{x}_A$

$\Sigma M_O = 0: \; T_B b - T_A a = 0$

Elimination of T_B from Eq. ① yields

$$\left[m_1 + \frac{a^2}{b^2}m_2\right]\ddot{x} + \frac{a^2}{b^2}c\dot{x} + kx = 0$$

8/46



$\dfrac{x}{a} = \dfrac{x_B}{b}, \quad T_2 = c\dot{x}_B = c\dfrac{b}{a}\dot{x}$

$\Sigma M_O = 0: \; T_1 a - T_2 b = 0$

$T_1 = \dfrac{b}{a}T_2 = \dfrac{b}{a}\left(c\dfrac{b}{a}\dot{x}\right) = c\dfrac{b^2}{a^2}\dot{x}$

$\Sigma F_x = m\ddot{x}: \; -T_1 - kx = m\ddot{x}$

$-c\dfrac{b^2}{a^2}\dot{x} - kx = m\ddot{x}$

$$\ddot{x} + \frac{b^2}{a^2}\frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

$2\zeta\omega_n = \dfrac{b^2}{a^2}\dfrac{c}{m}$

$$\zeta = \frac{b^2}{a^2}\frac{c}{m}\frac{1}{2\sqrt{\frac{k}{m}}} = \frac{1}{2}\frac{b^2}{a^2}\frac{c}{\sqrt{km}}$$

8/47 With negligible damping,

$$\frac{X}{F_0/k} = \frac{1}{|1-(\omega/\omega_n)^2|}$$

$\omega_n^2 = \dfrac{k}{m} = \dfrac{k}{24}$

$X = 0.30$ mm

$F_0/k = \delta_{st} = 0.60$ mm

$\omega = 2\pi(4) = 8\pi$ rad/s

For $\dfrac{X}{F_0/k} = M = \frac{1}{2} < 1$, $1-\left(\dfrac{\omega}{\omega_n}\right)^2$ is negative.

Thus $-\dfrac{0.30}{0.60} = \dfrac{1}{1-(8\pi)^2/(k/24)}$, $\underline{k = 5050 \text{ N/m}}$

8/48 $\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{100{,}000}{10}} = 100$ rad/s

$\zeta = \dfrac{c}{2m\omega_n} = \dfrac{500}{2(10)(100)} = 0.25$ for (a)

$$X = \frac{F_0/k}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}} = \frac{1000/100{,}000}{\left\{[1-1.2^2]^2 + [2(0.25)(1.2)]^2\right\}}$$

$$= \frac{1000/100\,000}{\left\{[1-1.2^2]^2 + [2(0.25)(1.2)]^2\right\}^{1/2}} = \underline{1.344(10^{-2}) \text{ m}}$$

(b) With $\zeta = 0$, $\underline{X = 2.27(10^{-2}) \text{ m}}$

8/49 $(M)_{\frac{\omega}{\omega_n}=1} = 8(M)_{\frac{\omega}{\omega_n}=2}$

$$\frac{1}{\left\{[1-1^2]^2 + [2\zeta(1)]^2\right\}^{1/2}} = \frac{8}{\left\{[1-2^2]^2 + [2\zeta(2)]^2\right\}^{1/2}}$$

Square both sides and solve for ζ to obtain $\underline{\zeta = 0.1936}$

8/50 $\omega_n = \sqrt{k/m} = \sqrt{6(12)\Big/\frac{64.4}{32.2}} = 6$ rad/sec

$$X = \frac{F_0/k}{1-(\omega/\omega_n)^2} = \frac{5/6(12)}{1-\frac{\omega^2}{6^2}}$$

Because $|X| < \frac{3}{12}$, we set $X < \frac{3}{12}$ & $X > -\frac{3}{12}$

and obtain $\underline{\omega > 6.78 \text{ rad/sec}}$ & $\underline{\omega < 5.10 \text{ rad/sec}}$

8/51 $\omega_n = 6$ rad/sec (from Prob. 8/50)

$$X = \frac{F_0/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$\boxed{\zeta = \dfrac{c}{2m\omega_n} = \dfrac{2.4}{2(2)6} = 0.1}$

$$= \frac{5/6(12)}{\sqrt{\left[1-\left(\frac{\omega}{6}\right)^2\right]^2 + \left[2(0.1)\frac{\omega}{6}\right]^2}} < \frac{3}{12}$$

Square both sides to obtain a quadratic in ω^2. Solution: $\omega < 5.32$ rad/sec

$\underline{\omega > 6.50 \text{ rad/sec}}$

As expected, damping allows a wider range of ω than when $\zeta = 0$ (Prob. 8/50).

8/52 $\omega_n = 6$ rad/sec (from Prob. 8/50)

$$X = \frac{F_0/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$= \frac{5/6(12)}{\sqrt{\left[1-\left(\frac{6}{6}\right)^2\right]^2 + \left[2\zeta\frac{6}{6}\right]^2}} = \frac{3}{12}$$

$\zeta = 0.1389$

$\zeta = \dfrac{c}{2m\omega_n}$, $c = 2\zeta m\omega_n = 2(0.1389)(2)(6)$

$= \underline{3.33 \text{ lb-sec/ft}}$

8/53 $\omega_n = \sqrt{2k/m} = \sqrt{\dfrac{2(6)(12)}{4/32.2}} = 34.0$ rad/sec

Steady-state amplitude is

$X = \dfrac{b}{1-\left(\frac{\omega}{\omega_n}\right)^2}$, where $X = \frac{1}{2}(10-8) = 1$ in.

So $1 = \dfrac{0.5}{1-(\omega/34.0)^2}$, $\omega = 24.1$ rad/sec

Shaker frequency $f = \dfrac{\omega}{2\pi} = \dfrac{24.1}{2\pi} = \underline{3.83 \text{ Hz}}$

8/54 $\omega_n = \sqrt{2k/m} = \sqrt{\dfrac{2(200)}{100/32.2}} = 11.35$ rad/sec

(a) $\zeta = 0$: $X = \left| \dfrac{F_0/k_{eff}}{1-\left(\frac{\omega}{\omega_n}\right)^2} \right|$

$= \left| \dfrac{75/(2\cdot 200)}{1-\left(\frac{15}{11.35}\right)^2} \right| = \underline{0.251 \text{ ft}}$

(b) $\zeta \neq 0$: $\zeta = \dfrac{c}{2m\omega_n} = \dfrac{60}{2\left(\frac{100}{32.2}\right)(11.35)}$

$\zeta = 0.851$

$X = \dfrac{F_0/k_{eff}}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$

$= \dfrac{75/(2\cdot 200)}{\left\{\left[1-\left(\frac{15}{11.35}\right)^2\right]^2+\left[2(0.851)\left(\frac{15}{11.35}\right)\right]^2\right\}^{1/2}} = \underline{0.0791 \text{ ft}}$

$\delta_{st} = \dfrac{W}{k_{eff}} = \dfrac{100}{2(200)} = \underline{0.25 \text{ ft}}$

8/55 $M = \dfrac{1}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$

$M_1 = \dfrac{1}{\{[1-1^2]^2+[2(0.1)(1)]^2\}^{1/2}} = 5$

$M_1' = \dfrac{1}{\{[1-1^2]^2+[2(0.2)(1)]^2\}^{1/2}} = 2.5$

$R_1 = \dfrac{M_1-M_1'}{M_1}(100) = \underline{50\%}$

$M_2 = \dfrac{1}{\{[1-2^2]^2+[2(0.1)(2)]^2\}^{1/2}} = 0.3304$

$M_2' = \dfrac{1}{\{[1-2^2]^2+[2(0.2)(2)]^2\}^{1/2}} = 0.3221$

$R_2 = \dfrac{M_2-M_2'}{M_2}(100) = \underline{2.52\%}$

8/56 $\omega_n = \sqrt{\dfrac{k}{m}}$, $\zeta = 1 = \dfrac{c}{2m\omega_n}$

$c = 2m\omega_n = 2m\sqrt{\dfrac{k}{m}} = 2\times 90\sqrt{\dfrac{4\times 30\times 10^6}{90}} = 180\times 1155 = \underline{208\times 10^3 \text{ N}\cdot\text{s/m}}$

$\omega = \frac{1}{2}\left(3600\times\frac{2\pi}{60}\right) = 188.5$ rad/s, $\omega/\omega_n = \dfrac{188.5}{1155} = 0.1632$

Eq. 8/23 with $\zeta = 1$ becomes $M = \dfrac{1}{1+(\omega/\omega_n)^2} = \dfrac{1}{1+0.1632^2} = \underline{0.974}$

8/57 The condition for the maxima is

$\dfrac{dM}{d\left(\frac{\omega}{\omega_n}\right)} = \dfrac{d}{d\left(\frac{\omega}{\omega_n}\right)}\left[\dfrac{1}{\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}\right] = 0$

Differentiate to obtain $\dfrac{\omega}{\omega_n} = \underline{\sqrt{1-2\zeta^2}}$

8/58 $F = m\bar{a}_n$: $k\delta = m(\delta+e)\omega^2$

$\delta = \dfrac{\frac{m}{k}e\omega^2}{1-\frac{m}{k}\omega^2}$

$= \underline{\dfrac{e(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}}$

$\underline{\omega_c = \sqrt{k/m}}$

8/59 *Equivalent spring constant*

$k = P/\delta_{st} = \dfrac{2}{4\times 10^{-3}} = 500$ N/m

$y_0 = \dfrac{b}{1-(\omega/\omega_n)^2}$ where $\omega_n^2 = \dfrac{k}{m} = \dfrac{500}{0.5} = 1000 \left(\frac{rad}{s}\right)^2$

& $\omega^2 = (4\times 2\pi)^2 = 632 \left(\frac{rad}{s}\right)^2$

Thus $y_0 = \dfrac{3}{1-\frac{632}{1000}} = \underline{8.15 \text{ mm}}$

8/60 $x_i = x_B + x$, $x = x_i - x_B$

From text: $-c\dot{x} - kx = m\dfrac{d^2}{dt^2}(x+x_B)$

$-c(\dot{x}_i - \dot{x}_B) - k(x_i - x_B) = m\ddot{x}_i$

$\ddot{x} + \frac{c}{m}\dot{x}_i + \frac{k}{m}x_i = \frac{k}{m}x_B + \frac{c}{m}\dot{x}_B$

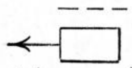
$x_B = b\sin\omega t$, $\dot{x}_B = b\omega\cos\omega t$

So $\underline{\ddot{x}_i + 2\zeta\omega_n\dot{x}_i + \omega_n^2 x_i = \frac{k}{m}b\sin\omega t + \frac{c}{m}b\omega\cos\omega t}$

With 2 forcing terms, we must find the particular solution corresponding to each term and then add (legal for a linear system). Alternatively, we could first combine the two forcing terms into one.

8/61 Let x_m be the absolute cart displacement.

Then $x_m = x_B + x$, x = mass relative displacement $= (x_m - x_B)$



$k(x_m - x_B) = kx$ $\quad \Sigma F_x = ma_x$: $-k(x_m - x_B) = m(\ddot{x}_B + \ddot{x})$

$\ddot{x} + \frac{k}{m}x = b\omega^2\sin\omega t$

Assume $x = X\sin\omega t$ & obtain $X = \dfrac{b(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2}$

Requirement : $|X| < 2b$

So $\dfrac{b(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2} < 2b$ and $\dfrac{b(\omega/\omega_n)^2}{1-(\omega/\omega_n)^2} > -2b$

$\Downarrow$ $\quad\quad\quad \Downarrow$

$\underline{\dfrac{\omega}{\omega_n} < \sqrt{2/3}}$ and $\underline{\dfrac{\omega}{\omega_n} > \sqrt{2}}$

See Fig. 8/14.

8/62 $\omega_n = \sqrt{k/m} = \sqrt{2(2.1)(10^3)/20} = 14.49 \text{ rad/s}$

$\zeta = \dfrac{c}{2m\omega_n} = \dfrac{2(58)}{2(20)(14.49)} = 0.200$

$M^2 = \dfrac{1}{[1-(\frac{\omega}{\omega_n})^2]^2 + [2\zeta\frac{\omega}{\omega_n}]^2}$; let $r = \dfrac{\omega}{\omega_n} = \dfrac{\omega}{14.49}$

$2^2 = \dfrac{1}{[1-r^2]^2 + 4(0.200)^2 r^2}$

$r^4 - 1.84r^2 + 0.75 = 0$, $r^2 = 0.610$ or $r^2 = 1.230$

Thus $\dfrac{\omega}{14.49} = r = \sqrt{0.610}$, $\omega = 11.32$ rad/s or 108.1 rev/min

or $\dfrac{\omega}{14.49} = r = \sqrt{1.230}$, $\omega = 16.07$ rad/s or 153.5 rev/min

Summary : $\underline{N \le 108.1 \dfrac{\text{rev}}{\text{min}} \text{ or } N \ge 153.5 \dfrac{\text{rev}}{\text{min}}}$

8/63 $W = k_{eq}\delta_{st}$; $k_{eq} = \dfrac{W}{\delta_{st}} = \dfrac{mg}{\delta_{st}}$

$\omega_n = \sqrt{k_{eq}/m} = \sqrt{\dfrac{mg/\delta_{st}}{m}} = \sqrt{g/\delta_{st}}$

For maximum response , $\omega = \omega_n = \sqrt{g/\delta_{st}}$

and $f = \dfrac{\omega}{2\pi} = \underline{\dfrac{1}{2\pi}\sqrt{\dfrac{g}{\delta_{st}}}}$

8/64 $\dfrac{X}{b} = \left|\dfrac{1}{1-(\omega/\omega_n)^2}\right| = \dfrac{0.15}{0.10} = 1.5$

For $\omega < \omega_n$, $\dfrac{\omega}{\omega_n} = 0.577$

For $\omega > \omega_n$, $\dfrac{\omega}{\omega_n} = 1.291$

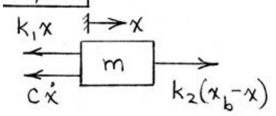
$\omega_n = \sqrt{\dfrac{4k}{m}} = \sqrt{\dfrac{4(7200)}{43}} = 25.9$ rad/s

For $\omega < \omega_n$, $\omega = 0.577(25.9) = 14.94$ rad/s

For $\omega > \omega_n$, $\omega = 1.291(25.9) = 33.4$ rad/s

Thus prohibited range is $\underline{2.38 < f_n < 5.32 \text{ Hz}}$

8/65



$\Sigma F_x = m\ddot{x}$:

$-k_1x - c\dot{x} + k_2(x_b - x) = m\ddot{x}$

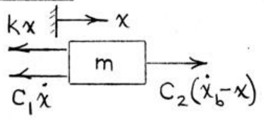
$\underline{m\ddot{x} + c\dot{x} + (k_1+k_2)x = k_2x_b}$

$= \underline{k_2 b \cos \omega t}$

$\underline{\omega_c = \sqrt{\dfrac{k_1+k_2}{m}}}$

(It is assumed that the damping is light so that the forced response is a maximum at $(\omega/\omega_n) \cong 1$.)

8/66



$\Sigma F_x = m\ddot{x}$:

$-kx - c_1\dot{x} + c_2(\dot{x}_b - \dot{x}) = m\ddot{x}$

$\underline{m\ddot{x} + (c_1+c_2)\dot{x} + kx = c_2\dot{x}_b}$

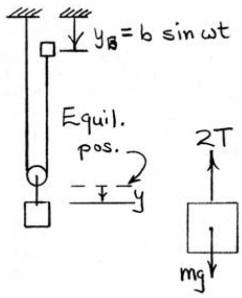
$= \underline{-c_2 b\omega \sin \omega t}$

$\underline{\omega_c = \sqrt{\dfrac{k}{m}}}$

$2\zeta\omega_n = \dfrac{c_1+c_2}{m}$, $\underline{\zeta = \dfrac{c_1+c_2}{2\sqrt{km}}}$

(See assumption in solution of Prob. 8/65)

8/67



In equilibrium position, the spring tension is $T_0 = \frac{1}{2}mg$

In displaced position, spring is stretched $2y - y_B$, so spring force is

$T = \frac{1}{2}mg + k(2y - y_B)$

For $\Sigma F_y = m\ddot{y}$ on m:

$mg - 2[\frac{1}{2}mg + k(2y - y_B)] = m\ddot{y}$

or $\ddot{y} + \dfrac{4k}{m}y = \dfrac{2k}{m} b \sin \omega t$

For particular solution, assume $y = Y \sin \omega t$ and obtain $Y = \dfrac{b/2}{1-(\omega/\omega_n)^2}$ where $\omega_n = 2\sqrt{k/m}$.

Thus $\underline{\omega_c = 2\sqrt{k/m}}$

8/68 The maximum value of the force transmitted to the base, from Sample Problem 8/6, is $(F_{tr})_{max} = X\sqrt{k^2 + c^2\omega^2}$

$= (F_0/k)M\sqrt{k^2 + (4\zeta^2m^2\omega_n^2)\omega^2}$

$= (F_0/k)M\sqrt{k^2 + \dfrac{4\zeta^2m^2\omega_n^2\omega^2}{1} \cdot \dfrac{k^2/m^2}{\omega_n^4}}$

$= (F_0/k)Mk\sqrt{1 + (2\zeta\frac{\omega}{\omega_n})^2}$

$= MF_0\sqrt{1 + (2\zeta\frac{\omega}{\omega_n})^2}$

Then transmission ratio T is

$T = \dfrac{(F_{tr})_{max}}{F_0} = \underline{M\sqrt{1+(2\zeta\frac{\omega}{\omega_n})^2}}$

(M = magnification factor)

8/69 $F_o = 2m_o e\omega^2 = 2(1)(0.012)\left(1800\,\frac{2\pi}{60}\right)^2$
$= 853$ N

Force transmitted $kX = 1500$ N

But $kX = \left|\dfrac{F_o}{1-(\omega/\omega_n)^2}\right|$ or $1500 = \left|\dfrac{853}{1-(\omega/\omega_n)^2}\right|$

Solving, $\left(\dfrac{\omega}{\omega_n}\right)^2 = 1.568$ or 0.432

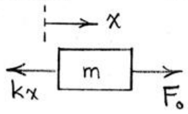
With $\omega_n^2 = k/m$, we obtain

$k = \dfrac{m\omega^2}{1.568}$ or $k = \dfrac{m\omega^2}{0.432}$

With $m = 10$ kg and $\omega = 1800\left(\frac{2\pi}{60}\right)$, we obtain

$\underline{k = 227 \text{ kN/m}}$ or $\underline{823 \text{ kN/m}}$

8/70 $\Sigma F = m\ddot{x}: \quad -kx + F_o = m\ddot{x}$



$$\ddot{x} + \frac{k}{m}x = \frac{F_o}{m}$$

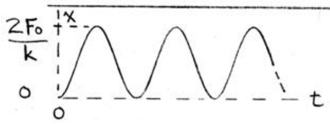
$$x = x_c + x_p$$

or $x = \left(A_1\cos\omega_n t + A_2\sin\omega_n t\right) + \dfrac{F_o}{k}$

$x(t=0) = A_1 + \dfrac{F_o}{k} = 0, \quad A_1 = -\dfrac{F_o}{k}$

$\dot{x}(t=0) = A_2\omega_n = 0, \quad A_2 = 0$

So $\underline{x = \dfrac{F_o}{k}\left(1-\cos\omega_n t\right)}$



8/71 For steady-state motion,

$$\frac{X}{b} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$\dfrac{X}{b} = \dfrac{24}{18} = \dfrac{4}{3}$; $\omega_n = \sqrt{k/m} = \sqrt{\dfrac{1500}{2}} = 27.4\ \dfrac{\text{rad}}{\text{s}}$

$\omega = 5(2\pi) = 31.4$ rad/s $\Rightarrow \dfrac{\omega}{\omega_n} = 1.147$, $\left(\dfrac{\omega}{\omega_n}\right)^2 = 1.316$

So $\left(\dfrac{4}{3}\right)^2 = \dfrac{1.316^2}{\left[1-1.316\right]^2 + \left[4(1.316)\zeta^2\right]}$, $\zeta = 0.408$

From $\zeta = \dfrac{c}{2m\omega_n}$, $c = 2\zeta m\omega_n = 2(0.408)(2)(27.4)$
$= \underline{44.6 \text{ N·s/m}}$

▸8/72 Take road contour to be $x = b\sin\omega t$

Wavelength $L = v\tau = v\dfrac{2\pi}{\omega} \Rightarrow \omega = \dfrac{2\pi v}{L}$

Thus $x = b\sin\dfrac{2\pi v}{L}t$.

$X = \left|\dfrac{b}{1-(\omega/\omega_n)^2}\right|$. Set $b = 25$ mm

$\omega = \dfrac{2\pi\left(\frac{25}{3.6}\right)}{1.2} = 36.4$ rad/s, and $\omega_n = \sqrt{k/m}$

$= \left(\dfrac{75(9.81)/0.003}{500}\right)^{1/2} = 22.1$ rad/s to

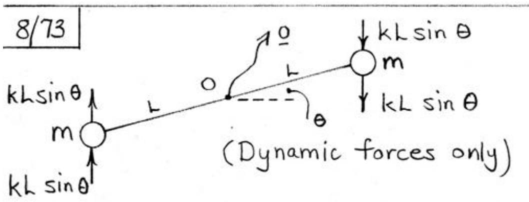
obtain $\underline{X = 14.75 \text{ mm}}$

Critical speed: $\omega_c = \omega_n$

$\dfrac{2\pi v_c}{L} = \sqrt{k/m} = 22.1$

$v_c = 4.23$ m/s or $\underline{15.23\ \frac{\text{km}}{\text{h}}}$

8/73



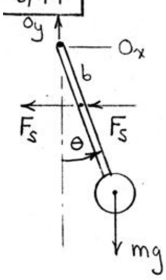
$\Sigma M_o = I_o\ddot{\theta}: \quad -4kL\sin\theta(L\cos\theta) = 2mL^2\ddot{\theta}$

For small θ: $\quad \ddot{\theta} + \dfrac{2k}{m}\theta = 0$

$\underline{\omega_n = \sqrt{\dfrac{2k}{m}}}, \quad \tau = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{\dfrac{m}{2k}}$

$= \underline{\pi\sqrt{\dfrac{2m}{k}}}$

8/74 Each spring force is $F_s = kb\sin\theta$ (Springs are assumed to remain horizontal)



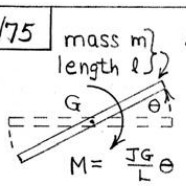
$\Sigma M_o = I_o\ddot{\theta}: -mgl\sin\theta - 2F_s b\cos\theta$
$= ml^2\ddot{\theta}$

Simplify to obtain (for small θ)

$\underline{\ddot{\theta} + \left(\dfrac{g}{l} + \dfrac{2kb^2}{ml^2}\right)\theta = 0}$

$\tau = \dfrac{2\pi}{\omega_n} = \underline{2\pi\Big/\sqrt{\dfrac{g}{l} + \dfrac{2kb^2}{ml^2}}}$

8/75



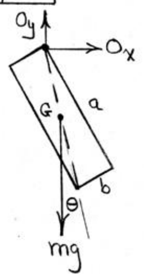
$\Sigma M_G = \bar{I}\ddot{\theta}: -\dfrac{JG}{L}\theta = \dfrac{1}{12}ml^2\ddot{\theta}$

$\ddot{\theta} + \left(\dfrac{12JG}{ml^2L}\right)\theta = 0$

$\omega_n = \left(\dfrac{12JG}{ml^2L}\right)^{1/2}$

$\tau = \dfrac{2\pi}{\omega_n} = \underline{2\pi\left(\dfrac{ml^2L}{12JG}\right)^{1/2}}$

8/76



$$\sum M_O = I_O\ddot{\theta} : -mg\frac{\sqrt{a^2+b^2}}{2}\sin\theta$$

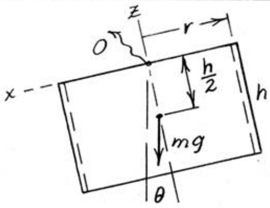
$$= \left[\frac{1}{12}m(a^2+b^2) + m\left(\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2\right)\right]\ddot{\theta}$$

For small θ:

$$\ddot{\theta} + \frac{\frac{3}{2}g}{\sqrt{a^2+b^2}}\theta = 0$$

$$\omega_n = \frac{\sqrt{\frac{3g}{2}}}{\sqrt[4]{a^2+b^2}}$$

8/77



$\Sigma M_O = I_O\alpha : I_O = I_{yy}$

$= \frac{1}{2}mr^2 + \frac{1}{3}mh^2$

from Table D/4

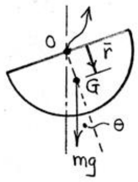
$$-mg\frac{h}{2}\sin\theta = \left(\frac{1}{2}mr^2 + \frac{1}{3}mh^2\right)\ddot{\theta}$$

For small angles $\sin\theta \approx \theta$

$$\ddot{\theta} + \frac{gh}{2}\frac{1}{\frac{r^2}{2}+\frac{h^2}{3}}\theta = 0, \quad \omega_n = \sqrt{\frac{gh}{2}}\Big/\sqrt{\frac{r^2}{2}+\frac{h^2}{3}}$$

8/78

$\bar{r} = \frac{4r}{3\pi}$, $I_O = \frac{1}{2}mr^2$



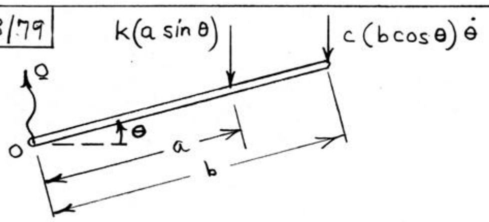
$$\sum M = I_O\ddot{\theta} : -mg\frac{4r}{3\pi}\sin\theta = \frac{1}{2}mr^2\ddot{\theta}$$

For small θ, $\sin\theta \cong \theta$, so

$$\ddot{\theta} + \frac{8g}{3\pi r}\theta = 0$$

$$\omega_n = \sqrt{\frac{8g}{3\pi r}}, \quad f_n = \frac{\omega_n}{2\pi} = \frac{1}{\pi}\sqrt{\frac{2g}{3\pi r}}$$

8/79



$$\sum M_O = I_O\ddot{\theta} : -(ka\sin\theta)a\cos\theta - cb\cos\theta\,\dot{\theta}(b\cos\theta) = \frac{1}{3}mb^2\ddot{\theta}$$

Small θ: $\ddot{\theta} + \frac{3c}{m}\dot{\theta} + \frac{3a^2}{b^2}\frac{k}{m}\theta = 0$

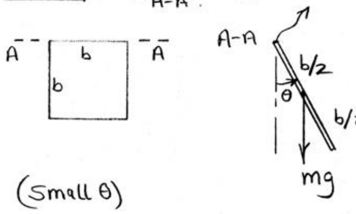
$\omega_n = \sqrt{\frac{3a^2k}{b^2m}}$; $2\zeta\omega_n = \frac{3c}{m}$, $\zeta = \frac{3c}{2m\omega_n}$

or $\zeta = \frac{3c}{2m}\sqrt{\frac{b^2m}{3a^2k}} = \frac{cb}{2a}\sqrt{\frac{3}{km}}$

For $\zeta = 1$, $c_{cr} = \frac{2a}{b}\sqrt{\frac{km}{3}}$

8/80

$$I_{A\text{-}A} = \frac{1}{12}mb^2 + m\left(\frac{b}{2}\right)^2 = \frac{1}{3}mb^2$$



(Small θ)

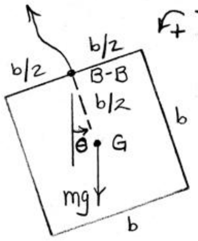
$$\sum M_{A\text{-}A} = I_{A\text{-}A}\ddot{\theta} : -mg\frac{b}{2}\theta = \frac{1}{3}mb^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{3g}{2b}\theta = 0, \quad \omega_n = \sqrt{\frac{3g}{2b}}$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2b}{3g}}$$

8/81

$$I_{B\text{-}B} = \frac{1}{6}mb^2 + m\left(\frac{b}{2}\right)^2 = \frac{5}{12}mb^2$$



$\sum M_{B\text{-}B} = I_{B\text{-}B}\ddot{\theta}$: (small θ)

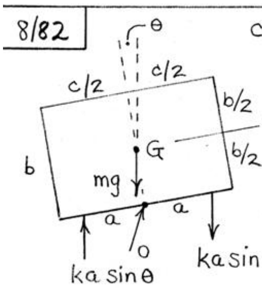
$$-mg\frac{b}{2}\theta = \frac{5}{12}mb^2\ddot{\theta}$$

$$\ddot{\theta} + \frac{6}{5}\frac{g}{b}\theta = 0$$

$$\omega_n = \sqrt{\frac{6g}{5b}}, \quad \tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{5b}{6g}}$$

(11.8 % higher than result of Prob. 8/80)

8/82



$c = 0.9$ m, $b = 0.6$ m, $a = 0.375$ m

$m = 250$ kg

$$\sum M_O = I_O\ddot{\theta} : mg\frac{b}{2}\sin\theta - 2ka\sin\theta(a\cos\theta) = I_O\ddot{\theta}$$

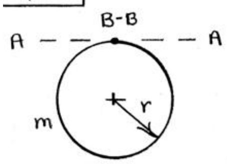
For small θ:

$$\ddot{\theta} + \frac{2ka^2 - mg\frac{b}{2}}{I_O}\theta = 0$$

For harmonic oscillation, coefficient of θ must be positive.

Thus $k_{min} = \frac{mgb}{4a^2} = \frac{250(9.81)(0.6)}{4(0.375)^2} = 2620$ N/m

8/83



$I_{A\text{-}A} = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$

$I_{B\text{-}B} = mr^2 + mr^2 = 2mr^2$

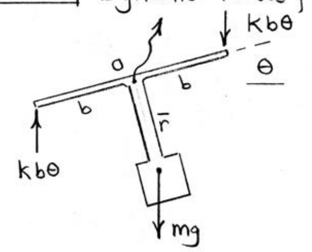
Desired ratio $R = \frac{\tau_{B\text{-}B}}{\tau_{A\text{-}A}}$

or $R = \frac{\omega_{A\text{-}A}}{\omega_{B\text{-}B}}$

Each natural frequency is proportional to $\frac{1}{\sqrt{I}}$, so

$$R = \frac{1/\sqrt{I_{A\text{-}A}}}{1/\sqrt{I_{B\text{-}B}}} = \frac{\sqrt{2mr^2}}{\sqrt{\frac{3}{2}mr^2}} = \frac{2}{\sqrt{3}}$$

8/84 Dynamic forces, small θ:



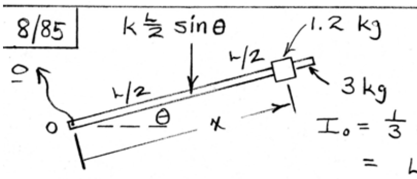
$$\curvearrowleft + \Sigma M_O = I_O\ddot\theta : -2kb\theta(b) - mg\bar r\theta = mk_o^2\ddot\theta$$

$$\ddot\theta + \left[\frac{2kb^2 + mg\bar r}{mk_o^2}\right]\theta = 0$$

$$\omega_n = \frac{1}{k_o}\sqrt{\frac{2kb^2}{m} + g\bar r}$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi k_o \Big/ \sqrt{\frac{2kb^2}{m} + g\bar r}$$

8/85



Let L be the 0.8-m rod length

$$I_O = \frac{1}{3}(3)L^2 + 1.2x^2 = L^2 + 1.2x^2$$

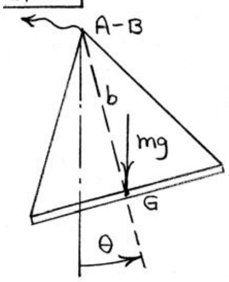
$$\curvearrowleft \Sigma M_O = I_O\ddot\theta : -k\frac{L}{2}\sin\theta\left(\frac{L}{2}\cos\theta\right) = (L^2 + 1.2x^2)\ddot\theta$$

$$\Rightarrow \ddot\theta + \frac{kL^2}{4(L^2+1.2x^2)}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{4(L^2+1.2x^2)}{kL^2}} = 4\pi\sqrt{\frac{L^2+1.2x^2}{kL^2}}$$

So $4\pi\sqrt{\dfrac{0.8^2+1.2x^2}{250\,(0.8)^2}} = 1$, $\underline{x = 0.558\text{ m}}$

8/86



$$I_{A\text{-}B} = I_G + mb^2 = \frac{1}{12}mb^2 + mb^2 = \frac{13}{12}mb^2$$

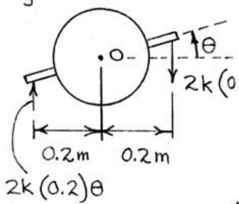
$$\curvearrowleft \Sigma M_{A\text{-}B} = I_{A\text{-}B}\ddot\theta : -mgb\sin\theta = \frac{13}{12}mb^2\ddot\theta$$

Small θ: $\ddot\theta + \dfrac{12g}{13b}\theta = 0$

$$\omega_n = \sqrt{\frac{12g}{13b}}, \quad \tau = \frac{2\pi}{\omega_n} = \frac{1}{\pi}\sqrt{\frac{3g}{13b}}$$

8/87

Dynamic forces:



$$I_O = mk_o^2 = 43(0.1)^2 = 0.43\text{ kg}\cdot\text{m}^2$$

$$\curvearrowleft + \Sigma M_O = I_O\ddot\theta : -4k(0.2)^2\theta = 0.43\ddot\theta$$

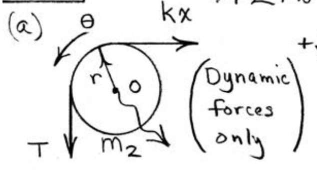
$$\ddot\theta + \frac{0.16k}{0.43}\theta = 0$$

$$\omega_n = \left(\frac{0.16k}{0.43}\right)^{1/2} = 360\left(\frac{2\pi}{60}\right)$$

$$\underline{k = 3820\text{ N/m}}$$

8/88

(a)

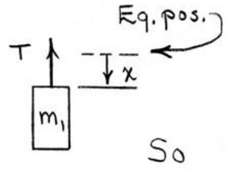


$$\curvearrowleft + \Sigma M_O = I_O\ddot\theta : r(T - kx) = \frac{1}{2}m_2r^2\ddot\theta$$

$$+\downarrow \Sigma F = ma : -T = m_1\ddot x$$

With $x = r\theta$:

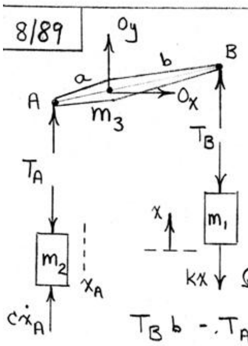
$$\ddot x\left[m_1 + \frac{1}{2}m_2\right] + kx = 0$$



Equation of motion for (b) is $m_{eff}\ddot x + kx = 0$

So $\underline{m_{eff} = m_1 + \frac{1}{2}m_2}$

8/89



$$\frac{x}{b} = \frac{x_A}{a}, \quad x_A = \frac{a}{b}x, \quad \ddot x_A = \frac{a}{b}\ddot x$$

$$x = b\theta, \quad \ddot x = b\ddot\theta$$

$\Sigma F_x = m\ddot x$:

① $-kx - T_B = m_1\ddot x$

② $T_A - c\dot x_A = m_2\ddot x_A$

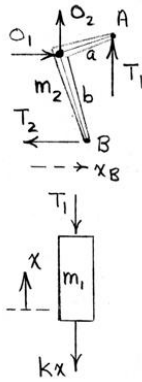
$\curvearrowleft \Sigma M_O = I_O\ddot\theta$:

$$T_Bb - T_Aa = m_3k_o^2\ddot\theta$$

Elimination of T_B from Eq. (1) yields

$$\left[m_1 + \frac{a^2}{b^2}m_2 + \frac{k_o^2}{b^2}m_3\right]\ddot x + \left[\frac{a^2}{b^2}c\right]\dot x + kx = 0$$

8/90



$$\frac{x}{a} = \frac{x_B}{b}, \quad x_B = \frac{b}{a}x, \quad \dot x_B = \frac{b}{a}\dot x$$

$$\frac{x}{a} = \theta, \quad \frac{\ddot x}{a} = \ddot\theta, \quad T_2 = c\dot x_B$$

$$\begin{cases}\Sigma F_x = m\ddot x : -T_1 - kx = m\ddot x \\ \Sigma M_O = I_O\ddot\theta : aT_1 - b_2T_2 = m_2k_o^2\ddot\theta\end{cases}$$

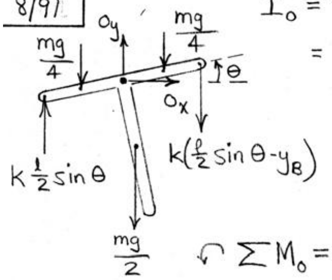
Elimination of T_1 from the x-equation yields

$$\left[m_1 + m_2\frac{k_o^2}{a^2}\right]\ddot x + \frac{b^2}{a^2}c\dot x + kx = 0$$

$$2\zeta\omega_n = \frac{cb^2/a^2}{m_1 + k_o^2m_2/a^2}, \quad \omega_n = \sqrt{\frac{k}{m_1 + m_2(k_o/a)^2}}$$

$$\zeta = \frac{cb^2/a^2}{2\sqrt{k\left(m_1 + \left(\frac{k_o}{a}\right)^2 m_2\right)}}$$

8/91



$$I_o = \frac{1}{3}\frac{m}{2}l^2 + 2\left(\frac{1}{3}\frac{m}{4}\left(\frac{l}{2}\right)^2\right) = \frac{5}{24}ml^2$$

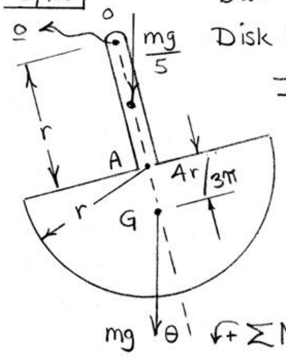
$\circlearrowleft \Sigma M_o = I_o\ddot{\theta}$:

$$-\left(k\frac{l}{2}\sin\theta\right)\frac{l}{2}\cos\theta - k\left(\frac{l}{2}\sin\theta - y_B\right)\frac{l}{2}\cos\theta - \frac{mg}{2}\left(\frac{l}{2}\sin\theta\right) = \frac{5}{24}ml^2\ddot{\theta}$$

Small θ : $\ddot{\theta} + \left[\frac{12}{5}\frac{k}{m} + \frac{6}{5}\frac{g}{l}\right]\theta = \frac{12}{5}\frac{kb}{ml}\sin\omega t$

$$\omega_c = \sqrt{\frac{6}{5}\left(\frac{2k}{m} + \frac{g}{l}\right)}$$

8/92



Bar : $I_o = \frac{1}{3}\left(\frac{m}{5}\right)r^2 = \frac{1}{15}mr^2$

Disk : $I_A = \frac{1}{2}mr^2$

$$I_G = \frac{1}{2}mr^2 - \left(\frac{4r}{3\pi}\right)^2 m$$

$$I_O = I_G + md^2 = \frac{1}{2}mr^2 - \left(\frac{4r}{3\pi}\right)^2 m + m\left(r + \frac{4r}{3\pi}\right)^2 = mr^2\left[\frac{3}{2} + \frac{8}{3\pi}\right]$$

$\circlearrowleft + \Sigma M_o = I_o\ddot{\theta}$:

$$-\frac{mg}{5}\left(\frac{r}{2}\sin\theta\right) - mg\left(r + \frac{4r}{3\pi}\right)\sin\theta = mr^2\left(\frac{1}{15} + \frac{3}{2} + \frac{8}{3\pi}\right)\ddot{\theta}$$

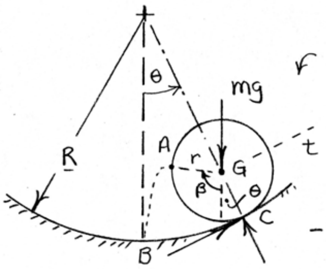
Small θ : $\ddot{\theta} + \left(\frac{33\pi + 40}{47\pi + 80}\right)\frac{g}{r}\theta = 0$

$$f_n = \frac{\omega_n}{2\pi} = \sqrt{\left(\frac{33\pi + 40}{47\pi + 80}\right)\frac{g}{r}} \Big/ 2\pi$$

8/93

Arc AC = Arc BC : $r(\beta + \theta) = R\theta$

$$\beta = \left(\frac{R}{r} - 1\right)\theta$$



$\circlearrowleft \Sigma M_G = \bar{I}\ddot{\beta}$:

$$Fr = \frac{1}{2}mr^2(-\ddot{\beta})$$

$\Sigma F_t = ma_t$:

$$-mg\sin\theta + F = mr\ddot{\beta}$$

Eliminate F , substitute $\ddot{\beta} = \left(\frac{R}{r} - 1\right)\ddot{\theta}$, & assume small θ : $\ddot{\theta} + \frac{2g}{3(R-r)}\theta = 0$

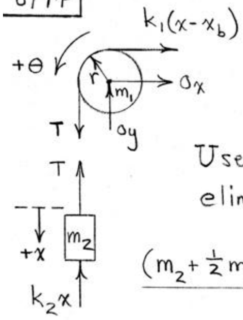
$\omega_n = \sqrt{\frac{2g}{3(R-r)}}$, $\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{3(R-r)}{2g}}$

Solution of differential eq. : $\theta = \theta_0 \sin\omega_n t$

$\dot{\theta} = \theta_0\omega_n\cos\omega_n t$, $\dot{\theta}_{max} = \theta_0\omega_n$.

$$\omega = \dot{\beta}_{max} = \left(\frac{R}{r} - 1\right)\theta_0\omega_n = \frac{\theta_0}{r}\sqrt{2g(R-r)/3}$$

8/94



$\Sigma M_o = I_o\ddot{\theta}$:

$$r(T - k_1(x - x_b)) = \frac{1}{2}m_1 r^2\ddot{\theta}$$

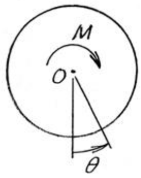
$$\Sigma F_x = m\ddot{x} : -T - k_2 x = m_2\ddot{x}$$

Use the constraint $x = r\ddot{\theta}$ and eliminate T to obtain

$$\left(m_2 + \frac{1}{2}m_1\right)\ddot{x} + (k_1 + k_2)x = k_1 b\cos\omega t$$

8/95

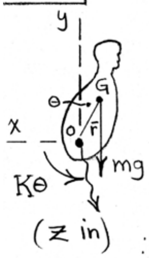
$\circlearrowleft \Sigma M_O = I_O\alpha$: $-\frac{JG}{L}\theta = I\ddot{\theta}$



$\ddot{\theta} + \frac{JG}{IL}\theta = 0$, $\omega_n = \sqrt{\frac{JG}{IL}}$

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{JG}{IL}}$$

8/96



$$\Sigma \underline{M}_o = \dot{\underline{H}}_o + \bar{\underline{\rho}} \times m\ddot{\underline{r}}_o$$

$$\Sigma \underline{M}_o = (mg\bar{r}\sin\theta - K\theta)\,\underline{k}$$

$$\dot{\underline{H}}_o = I_o\ddot{\theta}\,\underline{k} = mk_o^2\ddot{\theta}\,\underline{k}$$

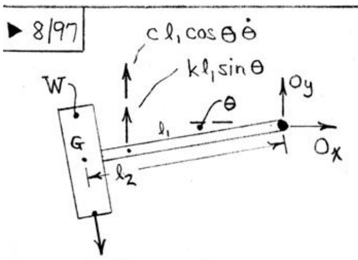
$$\bar{\underline{\rho}} = \bar{r}(-\sin\theta\,\underline{i} + \cos\theta\,\underline{j})$$

$$\ddot{\underline{r}}_o = a\,\underline{i}$$

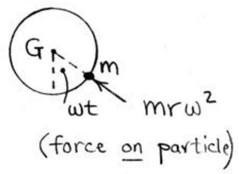
$$\therefore \; mg\bar{r}\sin\theta - K\theta = mk_o^2\ddot{\theta} - ma\bar{r}\cos\theta$$

Rearrange : $mk_o^2\ddot{\theta} + K\theta - m\bar{r}(g\sin\theta + a\cos\theta) = 0$

▶ 8/97



View of wheel from left side :



$$I_o = \bar I + \frac{W}{g} l_2^2 = 1 + \frac{100}{32.2}(3)^2$$
$$= 28.95 \text{ lb-sec}^2\text{-ft}$$

$\Sigma M_o = I_o \ddot\theta$:

$$-(k l_1 \sin\theta)(l_1 \cos\theta) - (c l_1 \dot\theta \cos\theta)(l_1 \cos\theta)$$
$$+(m r\omega^2 \cos\omega t)\cos\theta\, l_2 = I_o \ddot\theta$$

For small θ :

$$\ddot\theta + \frac{c l_1^2}{I_o}\dot\theta + \frac{k l_1^2}{I_o}\theta = \frac{m r\omega^2 l_2 \cos\omega t}{I_o}$$

$$\omega_n = \sqrt{\frac{k l_1^2}{I_o}} = \sqrt{\frac{(50)(12)(\frac{27}{12})^2}{28.95}} = \underline{10.24 \ \frac{\text{rad}}{\text{sec}}}$$

$$v = r\omega_n = \frac{14}{12}(10.24) = \underline{11.95 \text{ ft/sec}}$$

$$2\zeta\omega_n = \frac{c l_1^2}{I_o}, \quad \zeta = \frac{c l_1^2}{2 I_o \omega_n}$$

$$\zeta = \frac{(200)(\frac{27}{12})^2}{2(10.24)(28.95)} = \underline{1.707}$$

▶ 8/98 The particular solution to the differential equation of the previous solution is

$$\theta = \Theta \cos(\omega t - \phi), \quad \text{where}$$

$$\Theta = \left(\frac{M_o}{k_t}\right) M = \left(\frac{m r\omega^2 l_2}{k l_1^2}\right) M$$

$$= \frac{m r\omega^2 l_2 / k l_1^2}{\left\{[1-(\frac{\omega}{\omega_n})^2]^2 + [2\zeta\frac{\omega}{\omega_n}]^2\right\}^{1/2}}$$

(a) $\frac{\omega}{\omega_n} = \frac{10.24}{10.24} = 1$, $\quad \Theta = 5.50(10^{-4})$ rad

Vertical motion $X = l_2 \Theta = 3(5.50)(10^{-4})$
$= 1.650 (10^{-3})$ ft
$= \underline{1.980 (10^{-2}) \text{ in.}}$

(b) $v = 55 \ \frac{\text{mi}}{\text{hr}} = 80.67$ ft/sec, $\quad \omega = \frac{v}{r} = 69.14 \ \frac{\text{rad}}{\text{sec}}$

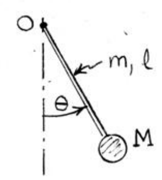
$\frac{\omega}{\omega_n} = \frac{69.14}{10.24} = 6.75$

$\Theta = 1.705 (10^{-3})$ rad, $\quad X = 5.12(10^{-3})$ ft
$= \underline{6.14 (10^{-2}) \text{ in.}}$

8/99

$$I_o = \frac{1}{3} m l^2 + M l^2 = \left(M + \frac{m}{3}\right) l^2$$

$$E = T + V = \frac{1}{2} I_o \dot\theta^2$$
$$+ mg\frac{l}{2}(1-\cos\theta) + Mgl(1-\cos\theta)$$



Differentiate with respect to time and let θ be small to obtain

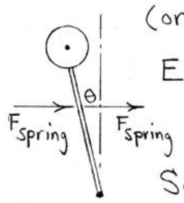
$$\underline{\ddot\theta + \frac{3}{2}\frac{g}{l}\left(\frac{m+2M}{m+3M}\right)\theta = 0}$$

8/100 *Energy* $E = T + V = 8\dot x^2 + 64 x^2 = constant$

so $dE/dt = 16\dot x \ddot x + 128 x \dot x = 0$, $\quad \underline{\ddot x + 8x = 0}$

$\omega_n = \sqrt{8} = 2\sqrt{2}$ *rad/sec*, $\quad \tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{2\sqrt{2}} = \underline{2.22 \text{ sec}}$

8/101 Let the initial spring stretch (or compression) be δ.



$$E = T + V = \frac{1}{2} m (l\dot\theta)^2 - mgl(1-\cos\theta)$$
$$+ \frac{1}{2}k(\delta + b\sin\theta)^2 + \frac{1}{2}k(\delta - b\sin\theta)^2$$

Set $\frac{dE}{dt} = 0$ and assume small θ to obtain $\ddot\theta + \left[\frac{2kb^2 - mgl}{m l^2}\right]\theta = 0$

$$f_n = \frac{\omega_n}{2\pi} = \underline{\frac{1}{2\pi}\sqrt{\frac{2kb^2}{m l^2} - \frac{g}{l}}}$$

$$\frac{2kb^2}{m l^2} > \frac{g}{l}, \quad \underline{k > \frac{mgl}{2b^2}}$$

8/102 $V_{max} = T_{max}$

Take $V = V_e + V_g = 0$ *at equil. position* $\theta = 0$

For small θ_o *spring deflection* δ *is*

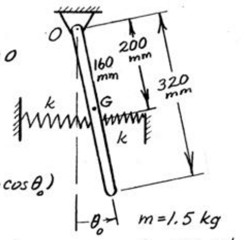
$\delta \approx 0.200\,\theta_o$

so $V_{max} = \frac{1}{2}k\delta^2 - \frac{1}{2}k(-\delta)^2 + mgh$

$= 120(0.200\,\theta_o)^2 + 1.5(9.81)(0.160)(1-\cos\theta_o)$

& with $\cos\theta_o = 1 - \frac{\theta_o^2}{2!} + \dots$,

$V_{max} = 4.80\theta_o^2 + 1.177\theta_o^2 = 5.98\,\theta_o^2$ J



$T_{max} = \frac{1}{2} I_o \dot\theta_{max}^2$ *where* $\dot\theta_{max} = \theta_o \omega_n$

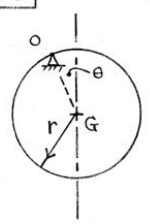
$= \frac{1}{2}\left(\frac{1}{3} \times 1.5 \times 0.320^2\right)\theta_o^2 \omega_n^2 = 0.0256\,\theta_o^2 \omega_n^2$ J

Thus $5.98\,\theta_o^2 = 0.0256\,\theta_o^2 \omega_n^2$, $\quad \omega_n = 15.28$ *rad/s*

$$f_n = \frac{\omega_n}{2\pi} = \frac{15.28}{2\pi} = \underline{2.43 \text{ Hz}}$$

8/103

$$I_o = \bar I + md^2 = mr^2 + mr^2$$
$$= 2mr^2$$



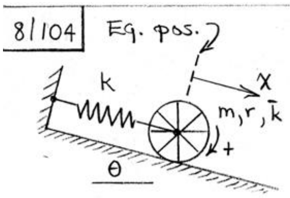
$$E = T + V = \frac{1}{2} I_o \dot\theta^2 + mgr(1-\cos\theta)$$

Set $\frac{dE}{dt} = 0$ and assume small θ to obtain

$$\ddot\theta + \frac{g}{2r}\theta = 0$$

$$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{2r}{g}}}$$

8/104 Eq. pos.



Choose $V = 0$ @ $x = 0$

Then $V = \frac{1}{2}kx^2$

$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\bar{I}\omega^2$

$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}(m\bar{k}^2)\left(\frac{\dot{x}}{r}\right)^2$

$= \frac{1}{2}m\left(1+\frac{\bar{k}^2}{r^2}\right)\dot{x}^2$

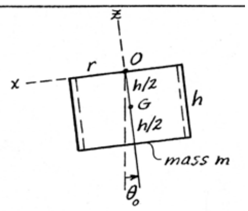
$E = T + V = \frac{1}{2}m\left(1+\frac{\bar{k}^2}{r^2}\right)\dot{x}^2 + \frac{1}{2}kx^2$

$\frac{dE}{dt} = m\left(1+\frac{\bar{k}^2}{r^2}\right)\dot{x}\ddot{x} + kx\dot{x} = 0$

$\ddot{x} + \frac{k}{m\left(1+\frac{\bar{k}^2}{r^2}\right)}x = 0$

$\omega_n = \sqrt{\frac{k}{m(1+\bar{k}^2/r^2)}}$ $\begin{cases} \bar{k} = 0: & \omega_n = \sqrt{k/m} \\ \bar{k} = r: & \omega_n = \sqrt{k/2m} \end{cases}$

8/105 $V_{max} = T_{max}$



$V_{max} = mg\frac{h}{2}(1-\cos\theta_o)$

where for small θ_o, $\cos\theta_o \approx 1 - \frac{\theta_o^2}{2!}$

so $V_{max} = mg\frac{h}{2}\frac{\theta_o^2}{2} = \frac{1}{4}mgh\theta_o^2$

$T_{max} = \frac{1}{2}I_o\dot{\theta}^2_{max}$

with $\theta = \theta_o \sin\omega_n t$, $\dot{\theta}_{max} = \theta_o\omega_n$

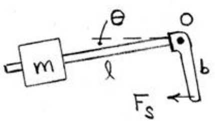
From Table D/4, $I_o = \frac{1}{2}mr^2 + \frac{1}{3}mh^2$

so $T_{max} = \frac{1}{2}m\left(\frac{r^2}{2}+\frac{h^2}{3}\right)\theta_o^2\omega_n^2$

Thus $\frac{1}{4}mgh\theta_o^2 = \frac{1}{2}m\left(\frac{r^2}{2}+\frac{h^2}{3}\right)\theta_o^2\omega_n^2$, $\omega_n^2 = \frac{gh/2}{\frac{r^2}{2}+\frac{h^2}{3}}$

$\tau = 2\pi/\omega_n = 2\pi\frac{\sqrt{2}}{\sqrt{gh}}\sqrt{\frac{r^2}{2}+\frac{h^2}{3}}$

8/106 At equilibrium, $\Sigma M_O = 0$ to obtain



$\delta_{st} = \frac{mgl}{kb}$

$E = T + V$

$= \frac{1}{2}m(l\dot{\theta})^2 + \frac{1}{2}k\left(\frac{mgl}{kb} + b\sin\theta\right)^2$

$- mgl\sin\theta$

Set $\frac{dE}{dt} = 0$ and assume θ small

to obtain $\ddot{\theta} + \frac{kb^2}{ml^2}\theta = 0$

$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\frac{b}{l}\sqrt{\frac{k}{m}}$

8/107 Let y be the downward displacement from the equilibrium position where $V = V_e + V_g$ is taken to be zero.

$(T_{max})_{y=0} = (V_{max})_{y=y_{max}}$

$T_{max} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(70)\dot{y}^2_{max} + \frac{1}{2}(40)(0.2)^2\left(\frac{\dot{y}_{max}}{0.3}\right)^2$

$= 43.9\,\dot{y}^2_{max}$

But $y = y_{max}\sin\omega_n t$, $\dot{y}_{max} = y_{max}\omega_n$

So $T_{max} = 43.9\,y^2_{max}\,\omega_n^2$

$V_{max} = \frac{1}{2}(2000)(2y_{max})^2 = 4000\,y^2_{max}$

Thus $43.9\,y^2_{max}\,\omega_n^2 = 4000\,y^2_{max}$

$\omega_n = 9.55$ rad/s, $f_n = \frac{\omega_n}{2\pi} = 1.519$ Hz

8/108 Let $\theta = 0$ be the angular position of static equilibrium & choose $V = 0$ there.

$T = \frac{1}{2}I_o\dot{\theta}^2 + \frac{1}{2}(2mx^2)\dot{\theta}^2 = \frac{1}{2}[I_o + 2mx^2]\dot{\theta}^2$

$V = \frac{1}{2}K\theta^2$

Set $\frac{dE}{dt} = 0$: $(I_o + 2mx^2)\ddot{\theta} + K\theta = 0$

$\omega_n = \sqrt{\frac{K}{(I_o + 2mx^2)}}$; $\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I_o + 2mx^2}{K}}$

Solve for x as $x = \sqrt{\frac{\tau^2 K/4\pi^2 - I_o}{2m}}$

8/109 Let x be the displacement (downward) from the equilibrium position, where V is taken to be zero.

$T = \frac{1}{2}m\dot{x}^2$, $V = 2\left[\frac{1}{2}k(2x)^2\right]$

$E = T + V = \frac{1}{2}m\dot{x}^2 + 4kx^2$

$\frac{dE}{dt} = m\dot{x}\ddot{x} + 8kx\dot{x} = 0$, $\ddot{x} + \frac{8k}{m}x = 0$

$\omega_n = \sqrt{\frac{8k}{m}}$, $\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{m}{2k}}$

Numbers: $\tau = \pi\sqrt{\frac{50/32.2}{2(6)(12)}} = 0.326$ sec

8/110 Take $\theta = 0$ to be the position where $V = 0$.

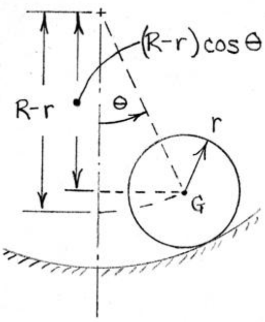
$E = T + V = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}m(r\dot{\theta})^2 + mgr(1-\cos\theta)\sin\alpha$

Set $\frac{dE}{dt} = 0$ to obtain, for small angles,

$\ddot{\theta} + \left[\frac{mgr\sin\alpha}{I + mr^2}\right]\theta = 0$

So $\omega_n = \sqrt{\frac{mgr\sin\alpha}{I + mr^2}}$

8/111 $E = T + V = \frac{1}{2}mv_G^2 + \frac{1}{2}\bar{I}\omega^2 + mg(R-r)(1-\cos\theta)$



Now, $v_G = (R-r)\dot\theta$

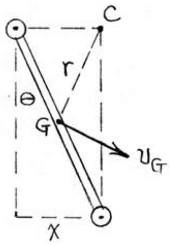
$\omega = \frac{v_G}{r} = \frac{(R-r)\dot\theta}{r}$

Thus $E = \frac{3}{4}m(R-r)^2\dot\theta^2 + mg(R-r)(1-\cos\theta)$

Set $\frac{dE}{dt} = 0$: $\ddot\theta + \frac{2}{3}\frac{g}{R-r}\sin\theta = 0$

For small θ, $\tau = \frac{2\pi}{\omega_n} = \pi\sqrt{\frac{6(R-r)}{g}}$

8/112 $v_G = r\dot\theta = \left[\left(\frac{l}{2}\cos\theta\right)^2 + \left(\frac{l}{2}\sin\theta\right)^2\right]^{1/2}\dot\theta = \frac{l}{2}\dot\theta$



$E = T + V = \frac{1}{2}mv_G^2 + \frac{1}{2}\bar{I}\dot\theta^2 + \frac{1}{2}k(\delta_{st} - l\sin\theta)^2 + \frac{1}{2}k(\delta_{st} + l\sin\theta)^2 - mg\frac{l}{2}(1-\cos\theta)$, where δ_{st} is the spring deflection at $\theta = 0$.

Substitute $v_G = \frac{l}{2}\dot\theta$, $\bar{I} = \frac{1}{12}ml^2$ into expression for E, set $\frac{dE}{dt} = 0$ and assume θ small to obtain

$\ddot\theta + \left[\frac{6k}{m} - \frac{3g}{2l}\right]\theta = 0$

$\omega_n = \sqrt{\frac{6k}{m} - \frac{3}{2}\frac{g}{l}}$, $k > \frac{mg}{4l}$

8/113 For the bar, $I_O = \frac{1}{12}m_2l^2 + m_2\left(\frac{3}{10}l\right)^2 = \frac{13}{75}m_2l^2$

Combined : $I_O = \frac{1}{2}m_1\left(\frac{l}{5}\right)^2 + \frac{13}{75}m_2l^2 = \frac{1}{50}m_1l^2 + \frac{13}{75}m_2l^2$

Let $\theta = 0$ be the equilibrium position shown & choose $V = 0$ @ $\theta = 0$; $V = \frac{1}{2}k\left(\frac{3l}{5}\theta\right)^2 = \frac{9}{50}kl^2\theta^2$

$E = T + V = \frac{1}{2}I_O\dot\theta^2 + \frac{9}{50}kl^2\theta^2 = l^2\left[\left(\frac{1}{100}m_1 + \frac{13}{150}m_2\right)\dot\theta^2 + \frac{9}{50}k\theta^2\right]$

Set $\frac{dE}{dt} = 0$ to obtain

$\ddot\theta + \frac{54k}{3m_1 + 26m_2}\theta = 0$, $\omega_n = 3\sqrt{\frac{6k}{3m_1 + 26m_2}}$

$\theta = \theta_0\sin\omega_nt$, $\dot\theta = \theta_0\omega_n\cos\omega_nt$

$\dot\theta_{max} = \omega = 3\theta_0\sqrt{\frac{6k}{3m_1 + 26m_2}}$

8/114 $E = T + V = 2\left(\frac{1}{2}Mv_0^2\right) + 2\left(\frac{1}{2}\bar{I}\dot\theta^2\right) + \frac{1}{2}mv_A^2 + mgr_0(1-\cos\theta)$

But $v_0 = r\dot\theta$ and $\bar{I} = \frac{1}{2}Mr^2$. Also, from kinematics, $v_A^2 = \dot\theta^2(r^2 + r_0^2) - 2rr_0\dot\theta^2\cos\theta$

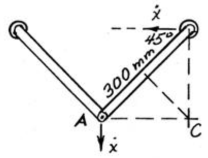
Thus $E = \left[\frac{3}{2}Mr^2 + \frac{1}{2}m(r^2 + r_0^2 - 2rr_0\cos\theta)\right]\dot\theta^2 + mgr_0[1-\cos\theta] = \text{constant}$

Set $\frac{dE}{dt} = 0$: $2\left[\frac{3}{2}Mr^2 + \frac{1}{2}m(r^2 + r_0^2 - 2rr_0\cos\theta)\right]\ddot\theta + \dot\theta^2[mrr_0\sin\theta] + mgr_0\sin\theta = 0$

Small $\theta, \dot\theta$: $\ddot\theta + \left[\frac{mgr_0}{3Mr^2 + m(r^2 + r_0^2 - 2rr_0)}\right]\theta = 0$

$f_n = \frac{1}{2\pi}\left[\frac{mgr_0}{3Mr^2 + m(r - r_0)^2}\right]^{1/2}$

8/115 *Take $V = V_g + V_e = 0$ at equilibrium position, where spring tension is $2(W/2) = W$*



so for downward displacement x_0 from equilibrium position,

$V_{max} = \Delta V_e + \Delta V_g = (Wx_0 + \frac{1}{2}kx_0^2) + (-2W\frac{x_0}{2})$
$= \frac{1}{2}kx_0^2 = \frac{1}{2}1050\,x_0^2$

$T_{max} = 2\left(\frac{1}{2}I_c\omega^2\right) = I_c\left(\frac{\dot x_{max}}{0.300/\sqrt{2}}\right)^2$

where $I_c = I_A = \frac{1}{3}ml^2 = \frac{1}{3}1.5\times0.300^2 = 0.045\ kg\cdot m^2$

$T_{max} = 0.045\frac{2\dot x_{max}^2}{0.09} = \dot x_{max}^2$, *at* $x = 0$

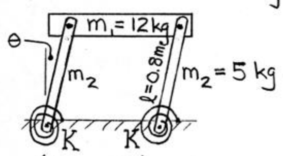
But $\dot x_{max} = x_0\omega_n$ *for harmonic motion*

so $T_{max} = V_{max}$ *gives* $x_0^2\omega_n^2 = 525\,x_0^2$

$\omega_n = \sqrt{525} = 22.9\ rad/s$,

$f_n = \frac{\omega_n}{2\pi} = \frac{22.9}{2\pi} = 3.65\ Hz$

8/116



(θ small)

$V_g = V_{g_1} + V_{g_2} = -m_1gl(1-\cos\theta) - 2m_2g\frac{l}{2}(1-\cos\theta) = -(m_1+m_2)gl(1-\cos\theta) \cong -(m_1+m_2)gl\frac{\theta^2}{2}$

$V_e = 2\left(\frac{1}{2}K\theta^2\right) = K\theta^2$

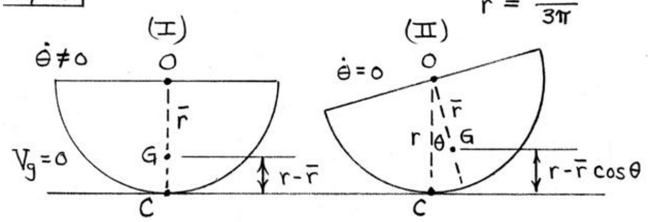
$V_{max} = -(m_1+m_2)gl\frac{\theta_{max}^2}{2} + K\theta_{max}^2$
$= \left[K - \frac{m_1+m_2}{2}gl\right]\theta_{max}^2$
$= \left[500 - \frac{12+5}{2}(9.81)(0.8)\right]\theta_{max}^2 = 433.3\,\theta_{max}^2$

$T = \frac{1}{2}m_1v_1^2 + 2\left(\frac{1}{2}I\dot\theta^2\right) = \frac{1}{2}m_1(l\dot\theta)^2 + 2\left(\frac{1}{2}\cdot\frac{1}{3}m_2l^2\dot\theta^2\right)$
$= \left(\frac{1}{2}m_1 + \frac{1}{3}m_2\right)l^2\dot\theta^2$

$T_{max} = \left(\frac{12}{2} + \frac{5}{3}\right)(0.8)^2\dot\theta_{max}^2 = 4.907\dot\theta_{max}^2$
$= 4.907(\theta_{max}\omega_n)^2 = 4.907\omega_n^2\theta_{max}^2$

Set $T_{max} = V_{max}$ & obtain $\omega_n = 9.40$ rad/s , $f_n = 1.496$ Hz

8/117 $\bar{r} = \frac{4r}{3\pi}$



In position II, $(V_g)_{max} = mg[(r-\bar{r}\cos\theta)-(r-\bar{r})]$
$= mg\left(\frac{4r}{3\pi}\right)(1-\cos\theta)$

In position I, $T_{max} = \frac{1}{2}I_C\dot\theta^2$

$I_C = \bar{I} + m(r-\bar{r})^2 = I_O - m\bar{r}^2 + m(r-\bar{r})^2$
$= \frac{1}{2}mr^2 + mr^2 - 2mr\bar{r} = \left(\frac{3}{2}-\frac{8}{3\pi}\right)mr^2$

$T_{max} = (V_g)_{max}$: $mg\frac{4r}{3\pi}(1-\cos\theta) = \frac{1}{2}\left(\frac{3}{2}-\frac{8}{3\pi}\right)mr^2\dot\theta^2$

For small θ, replace $\cos\theta$ by $1-\frac{\theta^2}{2}$

For harmonic oscillation, $\dot\theta_{max} = \theta\omega_n$

So $\frac{4g}{3\pi}\left[1-\left(1-\frac{\theta^2}{2}\right)\right] = \left(\frac{3}{4}-\frac{4}{3\pi}\right)r\theta^2\omega_n^2$

$\omega_n = 0.807\sqrt{g/r}$, $\tau = \frac{2\pi}{\omega_n} = 7.78\sqrt{r/g}$

8/118 $E = T+V \neq$ constant. Because ω = constant, the system will have more kinetic energy of rotation when the blocks are in outer positions than when the blocks are in inner positions of the same potential energy.

8/119 Let y_o = amplitude of vertical deflection of frame & body

$\frac{12}{18}y_o = \frac{2}{3}y_o$ = corresponding spring deflection

ΔV = change in $V_e + V_g$ due only to y_o so

$\Delta V = 2\left(\frac{1}{2}\right)(270)\left(\frac{2}{3}y_o\right)^2 = 120y_o^2$ in.-lb

$\Delta T = T_{max} = \frac{1}{2}\frac{1800}{32.2\times 12}\dot{y}_{max}^2$ but $\dot{y}_{max} = y_o\omega_n$

so $T_{max} = 2.33y_o^2\omega_n^2$

Thus with $T_{max} = \Delta V$, $2.33y_o^2\omega_n^2 = 120y_o^2$

& $\omega_n^2 = \frac{120}{2.33}$, $\omega_n = 7.18\ \frac{rad}{sec}$, $f_n = \frac{\omega_n}{2\pi} = \frac{7.18}{2\pi}$

$= 1.142$ Hz

8/120 $V_{max} = T_{max}$

$V_{max} = mgh = mg\ell(1-\cos\beta_o) = mg\ell\left(1-\left[1-\frac{\beta_o^2}{2!}+\dots\right]\right) = \frac{1}{2}mg\ell\beta_o^2$

But $\ell\beta_o \approx b\theta_o$ so $V_{max} = \frac{1}{2}mg\frac{b^2\theta_o^2}{\ell}$ where θ_o = max. angular twist

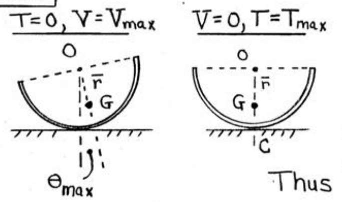
$T_{max} = \frac{1}{2}I_o\dot\theta_{max}^2$ where $\dot\theta = \theta_o\omega_n\cos\omega_n t$ & $\dot\theta_{max}^2 = \theta_o^2\omega_n^2$

& $T_{max} = \frac{1}{2}\left(\frac{1}{12}m[2b]^2\right)\theta_o^2\omega_n^2 = \frac{1}{6}mb^2\theta_o^2\omega_n^2$

Thus $\frac{1}{2}mg\frac{b^2\theta_o^2}{\ell} = \frac{1}{6}mb^2\theta_o^2\omega_n^2$, $\omega_n = \sqrt{3g/\ell}$

so $\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{\ell}{3g}}$

8/121



$V_{max} = mg\bar{r}(1-\cos\theta_{max})$
$= mg\frac{2r}{\pi}(1-\cos\theta_{max})$

For θ small, $\cos\theta \cong 1-\theta^2/2$

Thus $V_{max} = mgr\,\theta_{max}^2/\pi$

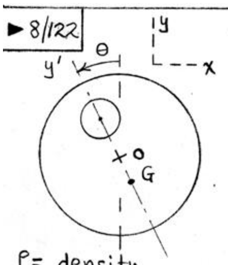
$T = \frac{1}{2}I_C\omega^2 = \frac{1}{2}[\bar{I}+m(r-\bar{r})^2]\omega^2$
$= \frac{1}{2}[I_o - m\bar{r}^2 + m(r-\bar{r})^2]\omega^2 = mr^2\left(1-\frac{2}{\pi}\right)\omega^2$

$T_{max} = mr^2\left(1-\frac{2}{\pi}\right)(\omega_n\theta_{max})^2$

$T_{max} = V_{max}$: $mr^2\left(1-\frac{2}{\pi}\right)\omega_n^2\theta_{max}^2 = mgr\,\theta_{max}^2/\pi$

$\omega_n^2 = \frac{g}{\pi r(1-2/\pi)}$, $\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{(\pi-2)r}{g}}$

►8/122



ρ = density
t = length

$\bar{Y}' = \frac{\Sigma\bar{y}'A}{\Sigma A} = \frac{-\frac{R}{2}(\pi)\left(\frac{R}{4}\right)^2}{\pi R^2 - \pi\left(\frac{R}{4}\right)^2}$
$= -\frac{R}{30}$, So $\overline{OG} = R/30$

$I_o = I_{whole} - I_{hole}$
$= \frac{1}{2}m_1R^2 - \left[\frac{1}{2}m_2\left(\frac{R}{4}\right)^2 + m_2\left(\frac{R}{2}\right)^2\right]$
$= \frac{247}{512}\pi t\rho R^4$

But $m = \pi\rho t\left[\pi R^2 - \pi\left(\frac{R}{4}\right)^2\right] = \frac{15}{16}\pi\rho tR^2$

$\therefore I_o = \frac{247}{512}\pi t\rho R^4\left(\frac{m}{\frac{15}{16}\pi\rho tR^2}\right) = \frac{247}{480}mR^2$

$\bar{I} = I_o - m(\overline{OG}^2) = \frac{247}{480}mR^2 - m\left(\frac{R}{30}\right)^2$
$= 0.5135\,mR^2$

$\underline{v}_G = \underline{v}_o + \underline{\omega}\times\underline{r}_{G/O} = -R\omega\underline{i} + \omega\underline{k}\times\frac{R}{30}[\sin\theta\underline{i} - \cos\theta\underline{j}]$
$= \omega\left[\left(\frac{R}{30}\cos\theta - R\right)\underline{i} + \left(\frac{R}{30}\sin\theta\right)\underline{j}\right]$

$v_G^2 = \omega^2\left[1.001R^2 - \frac{1}{15}R^2\cos\theta\right]$

Now, $E = T+V$

$E = \frac{1}{2}\bar{I}\dot\theta^2 + \frac{1}{2}mv_G^2 + mg\frac{R}{30}(1-\cos\theta)$

$E = \frac{1}{2}(0.5135\,mR^2)\dot\theta^2 + \frac{1}{2}m\left[\dot\theta^2\left(1.001R^2 - \frac{1}{15}R^2\cos\theta\right)\right]$
$+ mg\frac{R}{30}(1-\cos\theta)$

$\frac{dE}{dt} = (0.5135\,mR^2)\dot\theta\ddot\theta + m\dot\theta\ddot\theta\left(1.001R^2 - \frac{1}{15}R^2\cos\theta\right)$
$+ \frac{1}{2}m\dot\theta^2\left(\frac{1}{15}R^2\sin\theta\,\dot\theta\right) + mg\frac{R}{30}\sin\theta\,\dot\theta = 0$

We now assume small θ:

$1.448R^2\ddot\theta + 0.0333gR\theta = 0$

$\ddot\theta + 0.0230\frac{g}{R}\theta = 0$

$\tau = \frac{2\pi}{\omega_n} = 41.4\sqrt{\frac{R}{g}}$

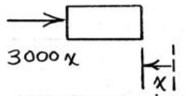
8/123 Linear momentum is conserved during impact

$G_1 = G_2$: $0.1(500) = 10.1\,v$, $v = 4.95$ m/s

After impact, energy is conserved

$T_1 = V_e$: $\frac{1}{2}(10.1)(4.95)^2 = \frac{1}{2}(3000)X^2$

$\underline{X = 0.287 \text{ m}}$



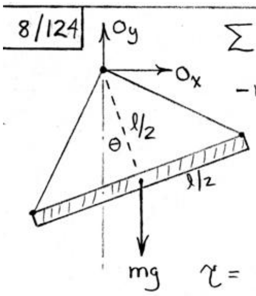
$\Sigma F_x = m\ddot{x}$: $-3000x = 10.1\ddot{x}$

$\ddot{x} + 297x = 0$

$\omega_n = \sqrt{297} = 17.23$ rad/s

$\tau = \frac{2\pi}{\omega_n} = \underline{0.365 \text{ s}}$

8/124



$\Sigma M_O = I_O\ddot{\theta}$:

$-mg\frac{l}{2}\sin\theta = \left[\frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2\right]\ddot{\theta}$

For small θ,

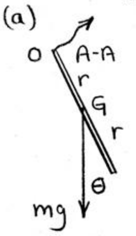
$\ddot{\theta} + \frac{3g}{2l}\theta = 0$

$\tau = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2l}{3g}} = \underline{7.33 \text{ s}}$

8/125 $I_{A-A} = \frac{1}{4}mr^2 + mr^2 = \frac{5}{4}mr^2$

$I_{B-B} = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$

(a)

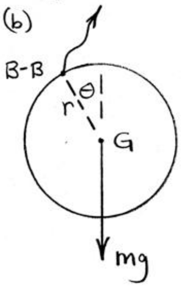


$\Sigma M_O = I_O\ddot{\theta}$: $-mgr\sin\theta = \frac{5}{4}mr^2\ddot{\theta}$

Small θ : $\ddot{\theta} + \frac{4g}{5r}\theta = 0$

$\underline{\omega_n = 2\sqrt{\frac{g}{5r}}}$

(b)

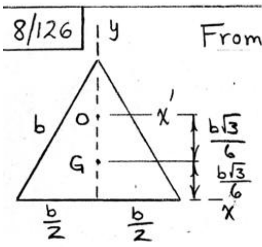


$\Sigma M_O = I_O\ddot{\theta}$: $-mgr\sin\theta = \frac{3}{2}mr^2\ddot{\theta}$

Small θ : $\ddot{\theta} + \frac{2}{3}\frac{g}{r}\theta = 0$

$\underline{\omega_n = \sqrt{\frac{2g}{3r}}}$

8/126



From Table D/3, $I_x = \frac{1}{12}b\left(\frac{b\sqrt{3}}{2}\right)^3 = \frac{\sqrt{3}}{32}b^4$

$I_{xx} = I_x\rho t = \frac{\sqrt{3}}{32}b^4\rho t$

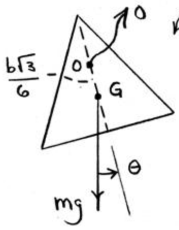
$= \frac{\sqrt{3}}{32}b^4\rho t\left(\frac{m}{\rho(\frac{1}{2}b\,\frac{\sqrt{3}}{2}b)t}\right)$

$= \frac{1}{8}mb^2 = I_{x'x'}$

Also, $I_y = 2\frac{1}{12}\left(\frac{\sqrt{3}}{2}b\right)\left(\frac{b}{2}\right)^3 = \frac{\sqrt{3}}{96}b^4$

$I_{yy} = I_y\rho t = \frac{\sqrt{3}}{96}b^4\rho t\left(\frac{m}{\rho(\frac{1}{2}b\,\frac{\sqrt{3}}{2}b)t}\right) = \frac{1}{24}mb^2$

$I_{zz} = I_O = I_{x'x'} + I_{yy} = \frac{1}{6}mb^2$

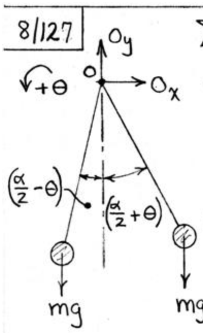


$\Sigma M_O = I_O\ddot{\theta}$: $mg\frac{b\sqrt{3}}{6}\theta = \frac{1}{6}mb^2\ddot{\theta}$

$\ddot{\theta} + \frac{g\sqrt{3}}{b}\theta = 0$

$\underline{\omega_n = \sqrt{\frac{g\sqrt{3}}{b}}}$

8/127



$\Sigma M_O = I_O\ddot{\theta}$:

$-mgl\sin\left(\frac{\alpha}{2}+\theta\right) + mgl\sin\left(\frac{\alpha}{2}-\theta\right)$

$= 2ml^2\ddot{\theta}$ (a)

$\sin\left(\frac{\alpha}{2}+\theta\right) = \sin\frac{\alpha}{2}\cos\theta + \cos\frac{\alpha}{2}\sin\theta$

(for θ small) $= \sin\frac{\alpha}{2} + \theta\cos\frac{\alpha}{2}$

Similarly,

$\sin\left(\frac{\alpha}{2}-\theta\right) = \sin\frac{\alpha}{2} - \theta\cos\frac{\alpha}{2}$

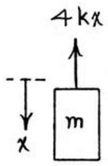
The equation of motion (a) becomes

$\ddot{\theta} + \left(\frac{g}{l}\cos\frac{\alpha}{2}\right)\theta = 0$

$\tau = \frac{2\pi}{\omega_n} = \underline{2\pi\sqrt{\frac{l}{g\cos(\alpha/2)}}}$

For $\alpha \to 0$, $\underline{\tau \to 2\pi\sqrt{l/g}}$; For $\alpha \to 180°$, $\underline{\tau \to \infty}$

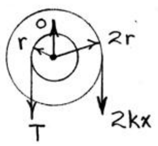
8/128



$\Sigma F_x = m\ddot{x}$: $-4kx = m\ddot{x}$

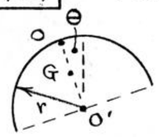
$\ddot{x} + \frac{4k}{m}x = 0$

$f_n = \frac{\omega_n}{2\pi} = \underline{\frac{1}{\pi}\sqrt{\frac{k}{m}}}$



Note that $\Sigma M_O = 0$ yields $T = 4kx$.

8/129 From Appendix D, $\overline{O'G} = 2r/\pi$



$$\bar{I} = I_{O'} - m(\overline{O'G})^2$$
$$= mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$$
$$I_O = \bar{I} + m(\overline{OG}^2)$$
$$= mr^2\left(2 - \frac{4}{\pi}\right)$$

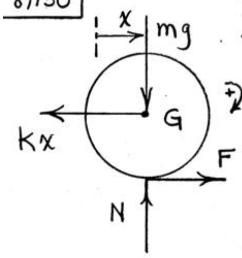
$$E = T + V = \frac{1}{2}I_O\dot\theta^2 + mg\,\overline{OG}(1-\cos\theta)$$
$$= \frac{1}{2}mr^2\left(2 - \frac{4}{\pi}\right)\dot\theta^2 + mgr\frac{\pi-2}{\pi}(1-\cos\theta)$$

Set $\frac{dE}{dt} = 0$ and assume small θ:

$$\ddot\theta + \frac{g}{2r}\theta = 0, \quad f_n = \frac{1}{2\pi}\sqrt{\frac{g}{2r}}$$

(Same result as for full circular shape!)

8/130



$\Sigma F_y = 0 \Rightarrow N = mg$

$\Sigma F_x = m\ddot x: \; F - kx = m\ddot x \quad (1)$

$\Sigma M_G = \bar I\ddot\theta: \; -Fr = \frac{1}{2}mr^2\ddot\theta \quad (2)$

Constraint: $\ddot x = r\ddot\theta \quad (3)$

Combine (1), (2), & (3):

$$\ddot x + \frac{2k}{3m}x = 0$$

$x = x_0 \sin\omega_n t$, $\ddot x = -x_0\omega_n^2 \sin\omega_n t$, $\ddot x_{max} = x_0\omega_n^2$

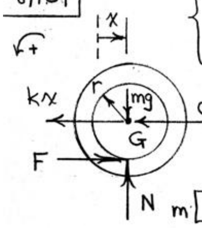
$$\ddot\theta_{max} = \frac{\ddot x_{max}}{r} = \frac{x_0\omega_n^2}{r}$$

Eq. (2): $|F_{max}|r = \frac{1}{2}mr^2\ddot\theta_{max}$

$$\mu_s mg\,r = \frac{1}{2}mr^2\frac{x_0\omega_n^2}{r}$$

$$x_0 = \frac{2\mu_s g}{\omega_n^2} = \frac{2\mu_s g}{2k/3m} = \frac{3\mu_s mg}{k}$$

8/131



$\Sigma F_x = m\ddot x: \; -kx - c\dot x + F = m\ddot x$

$\Sigma M_G = \bar I\alpha: \; Fr = mk_G^2\alpha$

Roll with no slip: $\ddot x = -r\alpha$

The x-equation reduces to

$$m\left[1 + \frac{k_G^2}{r^2}\right]\ddot x + c\dot x + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{m(1 + k_G^2/r^2)}} = \sqrt{\frac{15(12)}{\frac{20}{32.2}\left(1 + \frac{5.5^2}{6^2}\right)}} = 12.55 \frac{\text{rad}}{\text{sec}}$$

$$\zeta = \frac{c}{2\omega_n m\left(1 + \frac{k_G^2}{r^2}\right)} = \frac{2}{2(12.55)\left(\frac{20}{32.2}\right)\left(1 + \frac{5.5^2}{6^2}\right)}$$

$\zeta = 0.0697$

8/132 $Q = (T+V)_1 - (T+V)_2$

But at x_1 & x_2, $\dot x = 0$ so $T_1 = T_2 = 0$ & $Q = V_1 - V_2 = \frac{1}{2}k(x_1^2 - x_2^2)$

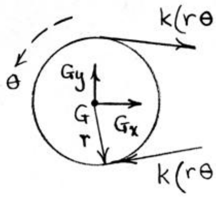
For the damped linear oscillator (case III, underdamped, of Art. 8/2b)

$$\frac{x_1}{x_2} = e^{\delta}$$

So $Q = \frac{1}{2}kx_1^2\left(1 - \left[\frac{x_2}{x_1}\right]^2\right) = \frac{1}{2}kx_1^2(1 - e^{-2\delta})$

where $\delta = \frac{c}{2m}\tau_d = \pi\Big/\sqrt{km/c^2 - \frac{1}{4}}$

8/133 $\Sigma M_G = \bar I\ddot\theta$:



$$-2k(r\theta - r_0\phi)r = m\bar k^2\ddot\theta$$

$$\ddot\theta + \frac{2kr^2}{m\bar k^2}\theta = \frac{2krr_0\phi_0}{m\bar k^2}\cos\omega t$$

Assume $\theta = \theta_{max}\cos\omega t$, substitute, and solve for $\theta_{max} = \phi_0\dfrac{r_0/r}{1 - (\frac{\omega}{\omega_n})^2}$,

where $\omega_n = \frac{r}{\bar k}\sqrt{\frac{2k}{m}}$

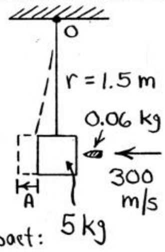
8/134 For seismic instruments,

$$\frac{X}{b} = \frac{(\omega/\omega_n)^2}{\left\{\left[1 - (\omega/\omega_n)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

For $X = 0.75$ mm, $\zeta = 0.5$, $\frac{\omega}{\omega_n} = \frac{\frac{180}{60}(3)}{3} = 3$,

solve for $b = \delta_0 = 0.712$ mm

8/135 Angular momentum about O is conserved during impact:



$H_{O_2} = H_{O_1}: \; (5 + 0.06)vr = 0.06(300)r$

$v = 3.56$ m/s

With neglect of energy loss after impact:

$T_{max} = V_{max}: \; \frac{1}{2}(5.06)(3.56)^2 = \frac{1}{2}kA^2$

But $f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$, so $4 = \frac{1}{2\pi}\sqrt{\frac{k}{5.06}}$

$k = 3200$ N/m

Then $A = 3.56\sqrt{\frac{5.06}{3200}} = 0.1415$ m

For damped vibration, $\ln\left(\frac{x_0}{x_n}\right) = n\zeta\omega_n\tau_d$

$\ln\left(\frac{x_0}{x_n}\right) = n\zeta\omega_n\frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$, where $n = 10$

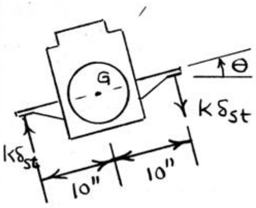
$\ln\left(\frac{1}{0.6}\right) = 10\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$, $\zeta = 0.00813$

$c = 2m\omega_n\zeta = 2(5.06)\left(\sqrt{\frac{3200}{5.06}}\right)0.00813 = 2.07 \frac{\text{N}\cdot\text{s}}{\text{m}}$

8/136 Vertical vibration:

$$(f_n)_y = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{k/m} = \frac{1}{2\pi}\sqrt{\frac{2(600)(12)}{480/32.2}} = \underline{4.95 \text{ Hz}}$$

Rotation about G:



Force changes for position of static equilibrium.

$k\delta_{st} = 600(10\theta)$
$= 6000\,\theta$ lb
(θ small)

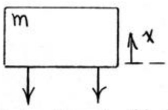
$$\circlearrowleft\, \Sigma M_G = I_G\ddot\theta : -2(6000\theta)\left(\frac{10}{12}\right) = \frac{48.0}{32.2}\left(\frac{4.60}{12}\right)^2\ddot\theta$$

$\ddot\theta + 4570\theta = 0$, $\omega_n = 67.6$ rad/sec

$(f_n)_\theta = \omega_n/2\pi = \underline{10.75 \text{ Hz}}$

Critical speed: $N = \omega_n = 10.75(60) = \underline{645 \frac{\text{rev}}{\text{min}}}$

▶8/137 Let $y = y_0 \sin\omega t$ be the floor motion.



$\Sigma F_x = m\ddot x$ yields

$m\ddot x + 4c\dot x + 4kx = 4ky + 4c\dot y$
$= 4ky_0 \sin\omega t + 4c\omega y_0 \cos\omega t$

$4k(x-y)$ $4c(\dot x - \dot y)$

$$\omega_n = \sqrt{\frac{4k}{m}} = \sqrt{\frac{4(250{,}000)}{200}} = 70.71 \text{ rad/s}$$

$$\zeta = \frac{4c}{2m\omega_n} = \frac{4(1000)}{2(200)(70.71)} = 0.1414$$

Assume $x_p = X\sin(\omega t - \alpha)$ to obtain

$$X = \frac{\left[1 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right]^{1/2} y_0}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}}$$

Originally, $X = 0.325\, y_0$

With damping doubled, $X' = 0.418\, y_0$

So amplitude increases by $\underline{28.9\,\%}$!

***8/138** $\omega_n = \sqrt{k/m} = \sqrt{\frac{100(12)}{50/32.2}} = 27.8$ rad/sec

$$\zeta = \frac{c}{2m\omega_n} = \frac{18}{2\left(\frac{50}{32.2}\right)(27.8)} = 0.208$$

$\frac{\omega}{\omega_n} = \frac{60}{27.8} = 2.158$, $\omega_d = 27.8\sqrt{1 - 0.208^2} = 27.2$ rad/sec

$$X = \frac{160/1200}{\left\{[1 - 2.158^2]^2 + [2(0.208)(2.158)]^2\right\}^{1/2}} = 0.03539 \text{ ft}$$

$$\phi = \tan^{-1}\left[\frac{2(.208)(2.158)}{1 - 2.158^2}\right] = 2.90 \text{ rad}$$

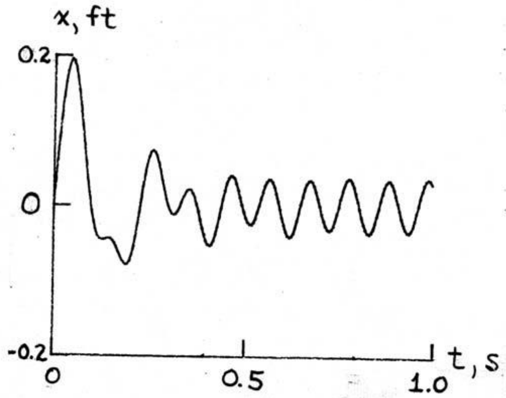
So $x = Ce^{-\zeta\omega_n t}\cos(\omega_d t - \psi) + X\cos(\omega t - \phi)$

$= Ce^{-5.796t}\cos(27.2t - \psi) + 0.0354\cos(60t - 2.9)$

Determine C and ψ: $\begin{cases} C = 0.212 \text{ ft} \\ \psi = 1.408 \text{ rad} \end{cases}$

$x_{max} = 0.1955$ ft @ $t = 0.046$ sec

$x_{min} = -0.079$ ft @ $t = 0.192$ sec



***8/139** For $\zeta = 1$, $x = (A_1 + A_2 t)e^{-\omega_n t}$

$x_0 = A_1$

$\dot x = -\omega_n(A_1 + A_2 t)e^{-\omega_n t} + A_2 e^{-\omega_n t}$

$\dot x_0 = -\omega_n A_1 + A_2 = -\omega_n x_0 + A_2 = 0$, $A_2 = \omega_n x_0$

So $x = x_0(1 + \omega_n t)e^{-\omega_n t} = x_0(1 + 4t)e^{-4t}$

When $x = 0.1x_0$, $0.1x_0 = x_0(1 + 4t)e^{-4t}$

or $f(t) = 4te^{-4t} + e^{-4t} - 0.1 = 0$

$f'(t) = -16te^{-4t}$

By Newton's method, $\underline{t = 0.972 \text{ s}}$

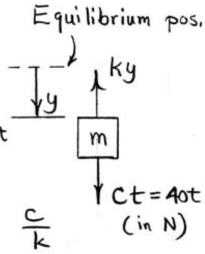
***8/140** $\Sigma F_y = m\ddot y$: $Ct - ky = m\ddot y$

$\ddot y + \omega_n^2 y = \frac{Ct}{m}$, $\omega_n^2 = \frac{k}{m}$

$y = y_H + y_P$, $y_H = A\cos\omega_n t + B\sin\omega_n t$

Try $y_P = B_2 t$: $\omega_n^2 B_2 t = \frac{Ct}{m}$

$B_2 = \frac{C}{m\omega_n^2} = \frac{C}{k}$

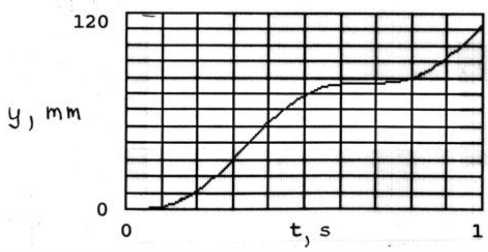


So $y = A\cos\omega_n t + B\sin\omega_n t + \frac{C}{k}t$

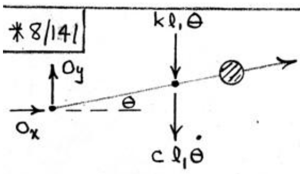
Quiet initial conditions yield $y = \frac{C}{k}\left[t - \frac{1}{\omega_n}\sin\omega_n t\right]$

With $C = 40\ \frac{N}{s}$, $k = 350$ N/m, $\omega_n = \sqrt{k/m} = 9.35$ rad/s:

$\underline{y = 114.3\,[t - 0.1069\sin 9.35t]}$ (in mm)



***8/141**



Eq. 4/13 : (with $P=0$)

$$\Sigma \underline{M}_O = \dot{\underline{H}}_{O_r} + \underline{\bar{\rho}} \times m\underline{a}_O$$

For small θ,

$$-kl_1^2\theta - cl_1^2\dot{\theta} = ml_2^2\ddot{\theta} + ml_2\ddot{y}_B$$

or $$\ddot{\theta} + \frac{cl_1^2}{ml_2^2}\dot{\theta} + \frac{kl_1^2}{ml_2^2}\theta = \frac{b}{l_2}\omega^2 \sin \omega t$$

Steady-state amplitude :

$$\Theta = Mb\left(\frac{\omega}{\omega_n}\right)^2 \frac{1}{l_2}, \quad \text{where } M = \text{magnification factor}$$

$$\text{Pen amplitude} = l_3\Theta = Mb\left(\frac{\omega}{\omega_n}\right)^2 \frac{l_3}{l_2} = A$$

Set up computer program to determine range of k for which $A \leq 1.5b$. Note that $\omega_n = \frac{l_1}{l_2}\sqrt{\frac{k}{m}}$, $2\zeta\omega_n = \frac{cl_1^2}{ml_2^2}$ or

$$\zeta = \frac{cl_1}{2l_2}\sqrt{\frac{1}{km}}.$$

Answer : $\underline{0 < k < 1.895 \ \frac{\text{lb}}{\text{ft}}}$

8/142 Eq. 8/9 : $\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0$

Solution : $y = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$

where $\lambda_{1,2} = \omega_n\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$

$\omega_n = \sqrt{k/m} = \sqrt{800/4} = 14.14$ rad/s ; $\zeta = \frac{c}{2m\omega_n}$

(a) $\zeta = \frac{124}{2(4)(14.14)} = 1.096$; (b) $\zeta = \frac{80}{2(4)(14.14)} = 0.707$

(a) $\zeta > 1$ (overdamped)

$\lambda_1 = 14.14\left(-1.096 + \sqrt{1.096^2 - 1}\right) = -9.16 \ \text{s}^{-1}$

$\lambda_2 = 14.14\left(-1.096 - \sqrt{1.096^2 - 1}\right) = -21.8 \ \text{s}^{-1}$

Initial condition considerations :

$$\left.\begin{aligned} y_0 &= 0.1 = A_1 + A_2 \\ \dot{y}_0 &= 0 = A_1\lambda_1 + A_2\lambda_2 \end{aligned}\right\} \Rightarrow \begin{aligned} A_1 &= 0.1722 \text{ m} \\ A_2 &= -0.0722 \text{ m} \end{aligned}$$

Solution : $\underline{y = 0.1722e^{-9.16t} - 0.0722e^{-21.8t} \text{ m}}$

(b) $\zeta < 1$ (underdamped)

Eq. (8/12) : $y = Ce^{-\zeta\omega_n t}\sin\left[\omega_d t + \psi\right]$

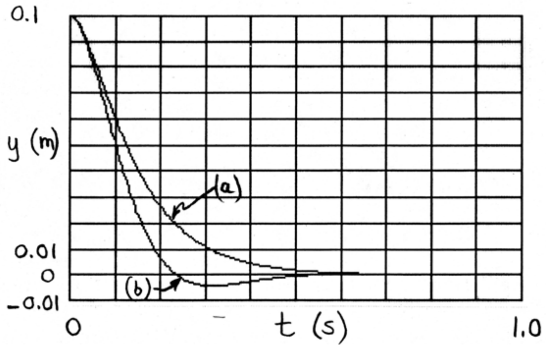
$\omega_d = \omega_n\sqrt{1-\zeta^2} = 14.14\sqrt{1 - 0.707^2} = 10$ rad/s

Initial condition considerations :

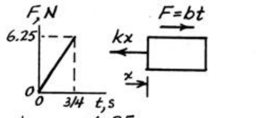
$$\left.\begin{aligned} y_0 &= 0.1 = C\sin\psi \\ \dot{y}_0 &= 0 = -\zeta\omega_n C\sin\psi + C\omega_d\cos\psi \end{aligned}\right\} \begin{aligned} C &= 0.1414 \text{ m} \\ \psi &= 0.785 \text{ rad} \end{aligned}$$

Solution : $y = 0.1414e^{-0.707(14.14)t}\sin\left[10t + 0.785\right]$

$\underline{= 0.1414e^{-10t}\sin\left[10t + 0.785\right] \text{ m}}$



***8/143** $\Sigma F_x = m\ddot{x}$: $bt - kx = m\ddot{x}$,



$\ddot{x} + \frac{k}{m}x = \frac{bt}{m}$

Sol. is $x = x_c + x_p$ where

$x_c = C_1\sin\omega_n t + C_2\cos\omega_n t$, $x_p = C_3 t$ with $C_3 = \frac{b}{k}$ $\quad b = \frac{6.25}{3/4} = 8.33$ N/s

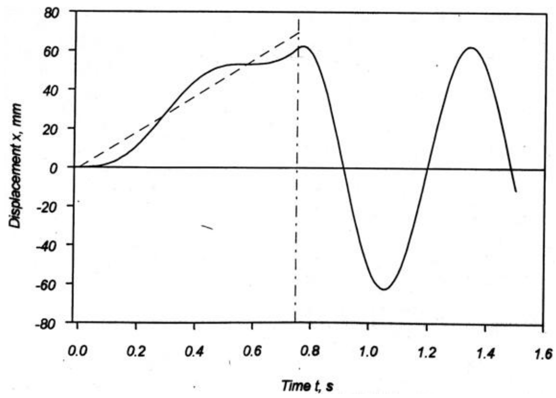
so $x = C_1\sin\omega_n t + C_2\cos\omega_n t + \frac{b}{k}t$ $\quad k = 90$ N/m

When $t=0$, $\dot{x} = 0$ & $x = 0$ giving $C_1 = -\frac{b}{\omega_n k}$, $C_2 = 0$ $\quad b/k = \frac{8.33}{90} = 0.0926$ m/s

& $x = -\frac{b}{\omega_n k}\sin\omega_n t + \frac{b}{k}t = \frac{b}{k}\left(t - \frac{1}{\omega_n}\sin\omega_n t\right)$

where $\omega_n = \sqrt{k/m} = \sqrt{90/0.75} = 10.95$ rad/s, $\frac{1}{\omega_n} = 0.0913$ s

Thus $\underline{x = 0.0926\left(t - 0.0913\sin 10.95t\right) \text{ m}}$ for first 3/4 s



***8/144** $I = \int F dt = m\dot{x}$, $8 = 4\dot{x}$, $\dot{x} = 2$ m/s at $t \approx 0$

After impulse, oscillator obeys Eq. 8/9 with $\zeta = 0.1 < 1$

so underdamped with solution given by Eq. 8/12

$x = Ce^{-\zeta\omega_n t}\sin(\omega_d t + \psi)$, C, ψ constants

$\dot{x} = -C\zeta\omega_n e^{-\zeta\omega_n t}\sin(\omega_d t + \psi) + Ce^{-\zeta\omega_n t}\omega_d\cos(\omega_d t + \psi)$

where $\omega_n = \sqrt{k/m} = \sqrt{200/4} = 7.07$ rad/s,

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 7.07\sqrt{1-0.1^2} = 7.04$ rad/s

When $t=0$, $x=0$, so $0 = C\sin\psi$, $\psi = 0$

" " , $\dot{x} = 2$ m/s, so $2 = -C(0.1)(7.07)(0) + C \times 7.04$, $C = 0.284$ m

Thus $\underline{x = 0.284e^{-0.707t}\sin 7.04t}$