SOLUTIONS MANUAL TO ACCOMPANY

INTRODUCTION TO FLIGHT
7th Edition

By

John D. Anderson, Jr.

McGraw-Hill Higher Education

Boston Burr Ridge, IL Dubuque, IA New York San Francisco St. Louis
Bangkok Bogotá Caracas Kuala Lumpur Lisbon London Madrid Mexico City
Milan Montreal New Delhi Santiago Seoul Singapore Sydney Taipei Toronto

Chapter 2

2.1 $\rho = p/RT = (1.2)(1.01\times10^5)/(287)(300)$

$\rho = 1.41\ \text{kg/m}^2$

$v = 1/\rho = 1/1.41 = 0.71\ \text{m}^3/\text{kg}$

2.2 Mean kinetic energy of each atom $= \dfrac{3}{2}k\,T = \dfrac{3}{2}(1.38\times10^{-23})(500) = 1.035\times10^{-20}\,\text{J}$

One kg-mole, which has a mass of 4 kg, has 6.02×10^{26} atoms. Hence 1 kg has $\dfrac{1}{4}(6.02\times10^{26}) = 1.505\times10^{26}$ atoms.

$$\begin{aligned}\text{Total internal energy} &= \text{(energy per atom)(number of atoms)}\\ &= (1.035\times10^{-20})(1.505\times10^{26}) = 1.558\times10^{6}\,\text{J}\end{aligned}$$

2.3 $\rho = \dfrac{p}{RT} = \dfrac{2116}{(1716)(460+59)} = 0.00237\dfrac{\text{slug}}{\text{ft}^3}$

$$\begin{aligned}\text{Volume of the room} &= (20)(15)(8) = 2400\,\text{ft}^3\\ \text{Total mass in the room} &= (2400)(0.00237) = 5.688\,\text{slug}\\ \text{Weight} &= (5.688)(32.2) = 183\,\text{lb}\end{aligned}$$

2.4 $\rho = \dfrac{p}{RT} = \dfrac{2116}{(1716)(460-10)} = 0.00274\dfrac{\text{slug}}{\text{ft}^3}$

Since the volume of the room is the same, we can simply compare densities between the two problems.

$\Delta\rho = 0.00274 - 0.00237 = 0.00037\dfrac{\text{slug}}{\text{ft}^3}$

$\%\,\text{change} = \dfrac{\Delta\rho}{\rho} = \dfrac{0.00037}{0.00237}\times(100) = 15.6\%\ \text{increase}$

2.5 First, calculate the density from the known mass and volume, $\rho = 1500/900 = 1.67\,\text{lb}_\text{m}/\text{ft}^3$

In consistent units, $\rho = 1.67/32.2 = 0.052\,\text{slug/ft}^3$. Also, $T = 70\,F = 70 + 460 = 530\,R$.

Hence,

$$p = \rho RT = (0.52)(1716)(530)$$

$$p = 47{,}290\,\text{lb/ft}^2$$

or $$p = 47{,}290/2116 = 22.3\ \text{atm}$$

2.6 $p = \rho RT$

$\ell np = \ell np + \ell nR + \ell nT$

Differentiating with respect to time,

$$\frac{1}{p}\frac{dp}{dt} = \frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{T}\frac{dT}{dt}$$

or, $$\frac{dp}{dt} = \frac{p}{\rho}\frac{d\rho}{dt} + \frac{p}{\mathrm{T}}\frac{dT}{dt}$$

or, $$\frac{dp}{dt} = RT\frac{d\rho}{dt} + \rho R\frac{dT}{dt} \tag{1}$$

At the instant there is 1000 $\mathrm{lb_m}$ of air in the tank, the density is

$$\rho = 1000/900 = 1.11\,\mathrm{lb_m/ft^3}$$
$$\rho = 1.11/32.2 = 0.0345\,\mathrm{slug/ft^3}$$

Also, in consistent units, is given that

$$T = 50 + 460 = 510\ R$$

and that

$$\frac{dT}{dt} = 1F/\mathrm{min} = 1R/\mathrm{min} = 0.016R/\mathrm{sec}$$

From the given pumping rate, and the fact that the volume of the tank is 900 $\mathrm{ft^3}$, we also have

$$\frac{d\rho}{dt} = \frac{0.5\ \mathrm{lb_m/sec}}{900\ \mathrm{ft^3}} = 0.000556\ \mathrm{lb_m/(ft^3)(sec)}$$
$$\frac{d\rho}{dt} = \frac{0.000556}{32.2} = 1.73\times10^{-5}\,\mathrm{slug/(ft^3)(sec)}$$

Thus, from equation (1) above,

$$\frac{d\rho}{dt} = (1716)(510)(1.73\times10^{-5}) + (0.0345)(1716)(0.0167)$$
$$= 15.1 + 0.99 = 16.1\ \mathrm{lb/(ft^2)(sec)} = \frac{16.1}{2116}$$
$$= 0.0076\ \mathrm{atm/sec}$$

2.7 In consistent units,

$$T = -10 + 273 = 263\ \mathrm{K}$$

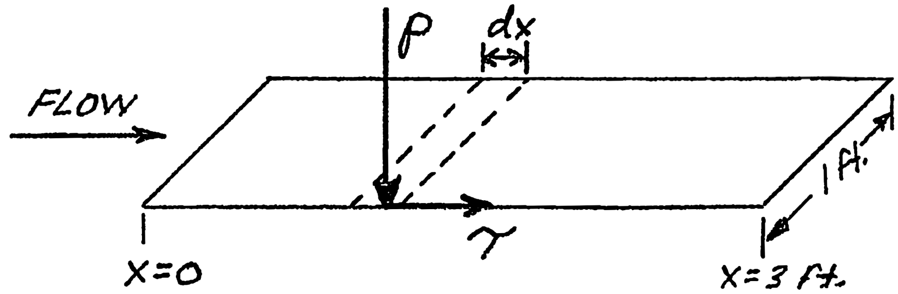
Thus,

$$\rho = p/RT = (1.7\times10^4)/(287)(263)$$
$$\rho = 0.225\ \mathrm{kg/m^3}$$

2.8 $\rho = p/RT = 0.5\times10^5/(287)(240) = 0.726\ \mathrm{kg/m^3}$

$v = 1/\rho = 1/0.726 = 1.38\ \mathrm{m^3/kg}$

2.9

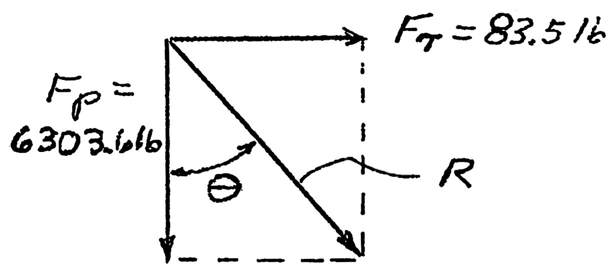


$$F_p = \text{Force due to pressure} = \int_0^3 p\,dx = \int_0^3 (2116 - 10x)\,dx$$

$$= [2116x - 5x^2]_0^3 = 6303 \text{ lb perpendicular to wall.}$$

$$F_\tau = \text{Force due to shear stress} = \int_0^3 \tau\,dx = \int_0^3 \frac{90}{(x+9)^{\frac{1}{2}}}\,dx$$

$$= [180\,(x+9)^{\frac{1}{2}}]_0^3 = 623.5 - 540 = 83.5 \text{ lb tangential to wall.}$$



Magnitude of the resultant aerodynamic force =

$$R = \sqrt{(6303)^2 + (835)^2} = 6303.6 \text{ lb}$$

$$\theta = \text{Arc Tan}\left(\frac{83.5}{6303}\right) = 0.76^\circ$$

2.10 $V = \dfrac{3}{2} V_\infty \sin\theta$

Minimum velocity occurs when $\sin\theta = 0$, i.e., when $\theta = 0°$ and 180°.

$V_{\min} = 0$ at $\theta = 0°$ and 180°, i.e., at its most forward and rearward points.

Maximum velocity occurs when $\sin\theta = 1$, i.e., when $\theta = 90°$. Hence,

$$V_{\max} = \frac{3}{2}(85)(1) = 127.5 \text{ mph at } \theta = 90°,$$

i.e., the entire rim of the sphere in a plane perpendicular to the freestream direction.

2.11 The mass of air displaced is

$$M = (2.2)(0.002377) = 5.23 \times 10^{-3} \text{ slug}$$

The weight of this air is

$$W_{\text{air}} = (5.23 \times 10^{-3})(32.2) = 0.168 \text{ lb}$$

This is the lifting force on the balloon due to the outside air. However, the helium inside the balloon has weight, acting in the downward direction. The weight of the helium is less than that of air by the ratio of the molecular weights

$$W_{H_c} = (0.168)\frac{4}{28.8} = 0.0233 \text{ lb.}$$

Hence, the maximum weight that can be lifted by the balloon is

$$0.168 - 0.0233 = 0.145 \text{ lb.}$$

2.12 Let p_3, ρ_3, and T_3 denote the conditions at the beginning of combustion, and p_4, ρ_4, and T_4 denote conditions at the end of combustion. Since the volume is constant, and the mass of the gas is constant, then $p_4 = \rho_3 = 11.3$ kg/m^3. Thus, from the equation of state,

$$p_4 = \rho_4 RT_4 = (11.3)(287)(4000) = 1.3 \times 10^7 \text{ N/m}^2$$

or,

$$p_4 = \frac{1.3 \times 10^7}{1.01 \times 10^5} = \boxed{129 \text{ atm}}$$

2.13 The area of the piston face, where the diameter is 9 cm = 0.09 m, is

$$A = \frac{\pi(0.09)^2}{4} = 6.36 \times 10^{-3} \text{m}^2$$

(a) The pressure of the gas mixture at the beginning of combustion is

$$p_3 = \rho_3 RT_3 = 11.3(287)(625) = 2.02 \times 10^6 \text{ N/m}^2$$

The force on the piston is

$$F_3 = p_3 A = (2.02 \times 10^6)(6.36 \times 10^{-3}) = 1.28 \times 10^4 \text{N}$$

Since 4.45 N = l lb*f*,

$$F_3 = \frac{1.28 \times 10^4}{4.45} = \boxed{2876 \text{ lb}}$$

(b) $p_4 = \rho_4 RT_4 = (11.3)(287)(4000) = 1.3 \times 10^7 \text{N/m}^2$

The force on the piston is

$$F_4 = p_4 A = (1/3 \times 10^7)(6.36 \times 10^{-3}) = \boxed{8.27 \times 10^4 \text{N}}$$

$$F_4 = \frac{8.27 \times 10^4}{4.45} = \boxed{18{,}579 \text{ lb}}$$

2.14 Let p_3 and T_3 denote conditions at the inlet to the combustor, and T_4 denote the temperature at the exit. Note: $p_3 = p_4 = 4 \times 10^6 \text{N/m}^2$

(a) $\rho_3 = \dfrac{p_3}{RT_3} = \dfrac{4 \times 10^6}{(287)(900)} = \boxed{15.49 \text{ kg/m}^3}$

(b) $\rho_4 = \dfrac{p_4}{RT_4} = \dfrac{4 \times 10^6}{(287)(1500)} = \boxed{9.29 \text{ kg/m}^3}$

2.15 1 mile = 5280 ft, and 1 hour = 3600 sec.

So:

$$\left(60 \frac{\text{miles}}{\text{hour}}\right)\left(\frac{5280 \text{ ft}}{\text{mile}}\right)\left(\frac{1 \text{ hour}}{3600 \text{ sec}}\right) = 88 \text{ ft/sec.}$$

A very useful conversion to remember is that

$$\boxed{60 \text{ mph} = 88 \text{ ft/sec}}$$

also, 1 ft = 0.3048 m

$$\left(88 \frac{\text{ft}}{\text{sec}}\right)\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 26.82 \frac{\text{m}}{\text{sec}}$$

Thus $\boxed{88 \dfrac{\text{ft}}{\text{sec}} = 26.82 \dfrac{\text{m}}{\text{sec}}}$

2.16 $692 \dfrac{\text{miles}}{\text{hour}}\left(\dfrac{88 \text{ ft/sec}}{60 \text{ mph}}\right) = \boxed{1015 \text{ ft/sec}}$

$$692 \frac{\text{miles}}{\text{hour}}\left(\frac{26.82 \text{ m/sec}}{60 \text{ mph}}\right) = \boxed{309.3 \text{ m/sec}}$$

2.17 On the front face

$$F_f = p_f A = (1.0715 \times 10^5)(2) = 2.143 \times 10^5 \text{ N}$$

On the back face

$$F_b = p_b A = (1.01 \times 10^5)(2) = 2.02 \times 10^5 \text{ N}$$

The net force on the plate is

$$F = F_f - F_b = (2.143 - 2.02) \times 10^5 = 0.123 \times 10^5 \text{ N}$$

From Appendix C,

$$1 \; lb_f = 4.448 \text{ N.}$$

So,

$$F = \frac{0.123 \times 10^5}{4.448} = \boxed{2765 \text{ lb}}$$

This force acts in the same direction as the flow (i.e., it is aerodynamic drag.)

2.18 Wing loading $= \dfrac{W}{s} = \dfrac{10{,}100}{233} = \boxed{43.35 \text{ lb/ft}^2}$

In SI units:

$$\frac{W}{s} = \left(43.35 \frac{\text{lb}}{\text{ft}^2}\right)\left(\frac{4.448 \text{ N}}{1 \text{ lb}}\right)\left(\frac{1 \text{ ft}}{0.3048 \text{ m}}\right)^2$$

$$\frac{W}{s} = \boxed{2075.5 \frac{\text{N}}{\text{m}^2}}$$

In terms of kilogram force,

$$\frac{W}{s} = \left(2075.5 \frac{\text{N}}{\text{m}^2}\right)\left(\frac{1 \text{ k}_f}{9.8 \text{ N}}\right) = \boxed{211.8 \frac{\text{kg}_f}{\text{m}^2}}$$

2.19 $V = \left(437 \dfrac{\text{miles}}{\text{hr}}\right)\left(\dfrac{5280 \text{ ft}}{\text{mile}}\right)\left(\dfrac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 7.033 \times 10^5 \dfrac{\text{m}}{\text{hr}} = \boxed{703.3 \dfrac{\text{km}}{\text{hr}}}$

$$\text{Altitude} = (25{,}000 \text{ ft})\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 7620 \text{ m} = \boxed{7.62 \text{ km}}$$

2.20 $V = \left(26{,}000 \dfrac{\text{ft}}{\text{sec}}\right)\left(\dfrac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 7.925 \times 10^3 \dfrac{\text{m}}{\text{sec}} = \boxed{7.925 \dfrac{\text{km}}{\text{sec}}}$

2.21 From Fig. 2.16,
length of fuselage = 33 ft, 4.125 inches = 33.34 ft

$$= 33.34 \text{ ft}\left(\frac{0.3048 \text{ m}}{\text{ft}}\right) = \boxed{10.16 \text{ m}}$$

wing span = 40 ft, 11.726 inches = 40.98 ft

$$= 40.98 \text{ ft}\left(\frac{0.3048 \text{ m}}{\text{ft}}\right) = \boxed{12.49 \text{ m}}$$

Chapter 3

3.1 An examination of the standard temperature distribution through the atmosphere given in Figure 3.3 of the text shows that both 12 km and 18 km are in the same constant temperature region. Hence, the equations that apply are Eqs. (3.9) and (3.10) in the text. Since we are in the same isothermal region with therefore the same base values of p and ρ, these equations can be written as

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} = e^{-(g_0/RT)(h_2-h_1)}$$

where points 1 and 2 are any two arbitrary points in the region. Hence, with $g_0 = 9.8$ m/sec^2 and $R = 287$ joule/kgK, and letting points 1 and 2 correspond to 12 km and 18 km altitudes, respectively,

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1} = e^{\frac{9.8}{(287)(216.66)}(6000)} = 0.3884$$

Hence:

$$p_2 = (0.3884)(1.9399 \times 10^4) = 7.53 \times 10^3 \text{ N/m}^2$$

$$\rho_2 = (0.3884)(3.1194 \times 10^{-1}) = 0.121 \text{ kg/m}^3$$

and, of course,

$$T_2 = 216.66 \text{ K}$$

These answers check the results listed in Appendix A of the text within round-off error.

3.2 From Appendix A of the text, we see immediately that $p = 2.65 \times 10^4$ N/m^2 corresponds to 10,000 m, or 10 km, in the standard atmosphere. Hence,

pressure altitude = 10 km

The outside air density is

$$\rho = \frac{p}{RT} = \frac{2.65 \times 10^4}{(287)(220)} = 0.419 \text{ kg/m}^3$$

From Appendix A, this value of ρ corresponds to 9.88 km in the standard atmosphere. Hence,

density altitude = 9.88 km

3.3 At 35,000 ft, from Appendix B, we find that $p = 4.99 \times 10^2 = 499$ lb/ft^2.

3.4 From Appendix B in the text,

33,500 ft corresponds to $p = 535.89$ lb/ft^2

32,000 ft corresponds to $\rho = 8.2704 \times 10^{-4}$ slug/ft^3

Hence,

$$T = \frac{p}{\rho R} = \frac{535.89}{(8.2704 \times 10^{-4})(1716)} = 378 \, R$$

3.5 $$\frac{|h - h_G|}{h} = 0.02 = \left|1 - \frac{h_G}{h}\right|$$

From Eq. (3.6), the above equation becomes

$$\left|1 - \left(\frac{r + h_G}{r}\right)\right| = \left|1 - 1 - \frac{h_G}{r}\right| = 0.02$$

$$h_G = 0.02\, r = 0.02(6.357 \times 10^6)$$

$$h_G = 1.27 \times 10^5 \text{m} = 127 \text{ km}$$

3.6 $T = 15 - 0.0065h = 15 - 0.0065(5000) = -17.5°\text{C} = 255.5°\text{K}$

$$a = \frac{dT}{dh} = -0.0065$$

From Eq. (3.12)

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-g_0/aR} = \left(\frac{255.5}{288}\right)^{-(2.8)/(-0.0065)(287)} = 0.533$$

$$p = 0.533\, p_1 = 0.533\,(1.01 \times 10^5) = 5.38 \times 10^4 \text{N/m}^2$$

3.7 $$\ell n \frac{p}{p_1} = -\frac{g}{RT}(h - h_1)$$

$$h - h_1 = -\frac{1}{g} RT\, \ell n \frac{p}{p_1} = -\frac{1}{24.9}(4157)(150)\, \ell n\, 0.5$$

Letting $h_1 = 0$ (the surface)

h = 17,358 m = 17.358 km

3.8 A standard altitude of 25,000 ft falls within the first gradient region in the standard atmosphere. Hence, the variation of pressure and temperature are given by:

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-\frac{g}{aR}} \tag{1}$$

and

$$T = T_1 + a\,(h - h_1) \tag{2}$$

Differentiating Eq. (1) with respect to time:

$$\frac{1}{p_1}\frac{dp}{dt} = \left(\frac{1}{T_1}\right)^{-\frac{g}{aR}}\left(-\frac{g}{AR}\right)T^{\left(-\frac{g}{aR}-1\right)}\frac{dT}{dt} \tag{3}$$

Differentiating Eq. (2) with respect to time:

$$\frac{dT}{dt} = a\frac{dh}{dt} \tag{4}$$

Substitute Eq. (4) into (3)

$$\frac{dp}{dt} = -p_1(T_1)^{\frac{g}{aR}}\left(\frac{g}{R}\right)T^{-\left(\frac{g}{aR}+1\right)}\frac{dh}{dt} \tag{5}$$

In Eq. (5), dh/dt is the rate-of-climb, given by $dh/dt = 500$ ft/sec. Also, in the first gradient region, the lapse rate can be calculated from the tabulations in Appendix B. For example, take 0 ft and 10,000 ft, we find

$$a = \frac{T_2 - T_1}{h_2 - h_1} = \frac{483.04 - 518.69}{10{,}000 - 0} = -0.00357\frac{°R}{\text{ft}}$$

Also from Appendix B, $p_1 = 2116.2$ lb/ft^2 at sea level, and $T = 429.64$ °R at 25,000 ft.

Thus,

$$\frac{g}{aR} = \frac{32.2}{(-0.00357)(1716)} = -5.256$$

Hence, from Eq. (5)

$$\frac{dp}{dt} = -(2116.2)(518.69)^{-5.256}\left(\frac{32.2}{1716}\right)(429.64)^{4.256}(500)$$

$$\frac{dp}{dt} = -17.17\frac{\text{lb}}{\text{ft}^2\text{sec}}$$

3.9 From the hydrostatic equation, Eq. (3.2) or (3.3),

$$dp = -\rho g_0 dh$$

or
$$\frac{dp}{dt} = -\rho\, g_0 \frac{dh}{dt}$$

The upward speed of the elevator is dh/dt, which is

$$\frac{dp}{dt} = \frac{dp/dt}{-\rho\, g_0}$$

At sea level, $\rho = 1.225$ kg/m^3. Also, a one-percent change in pressure per minute starting from sea level is

$$\frac{dp}{dt} = -(1.01 \times 10^5)(0.01) = -1.01 \times 10^3 \text{ N/m}^2 \text{ per minute}$$

Hence,

$$\frac{dh}{dt} = \frac{-1.01 \times 10^3}{(1.225)(9.8)} = 84.1 \text{ meter per minute}$$

3.10 From Appendix B:

At 35,500 ft: $p = 535.89$ lb/ft^2

At 34,000 ft: $p = 523.47$ lb/ft^2

For a pressure of 530 lb/ft^2, the pressure altitude is

$$33{,}500 + 500\left(\frac{535.89 - 530}{535.89 - 523.47}\right) = 33737 \text{ ft}$$

The density at the altitude at which the airplane is flying is

$$\rho = \frac{\rho}{RT} = \frac{530}{(1716)(390)} = 7.919 \times 10^{-4} \text{ slug/ft}^3$$

From Appendix B:

$$\text{At 33,000 ft: } \rho = 7.9656 \times 10^{-4} \text{ slug/ft}^3$$

$$\text{At 33,500 ft: } \rho = 7.8165 \times 10^{-4} \text{ slug/ft}^3$$

Hence, the density altitude is

$$33{,}000 + 500\left(\frac{7.9656 - 7.919}{7.9656 - 7.8165}\right) = 33{,}156 \text{ ft}$$

3.11 Let ℓ be the length of one wall of the tank, $\ell = 30$ ft. Let d be the depth of the pool, $d = 10$ ft. At the water surface, the pressure is atmospheric pressure, p_a. The water pressure increases with increasing depth; the pressure as a function of distance below the surface, h, is given by the hydrostatic equation

$$dp = \rho g\, dh \tag{1}$$

Note: The hydrostatic equation given by Eq. (3.2) in the text has a minus sign because h_G is measured positive in the upward direction. In Eq. (1), h is measured positive in the downward direction, with $h = 0$ at the surface of the water. Hence, no minus sign appears in Eq. (1); as h increases (as we go deeper into the water), p increases. Eq. (1) is consistent with this fact. Integrating Eq. (1) from $h = 0$ where $p = p_a$ to some local depth h where the pressure is p, and noting that ρ is constant for water, we have

$$\int_{p_a}^{\rho} dp = \rho g \int_0^h dh$$

or, $\quad p - p_a = \rho g h$

or, $$p = \rho g h + p_a \tag{2}$$

Eq. (2) gives the water pressure exerted on the wall at an arbitrary depth h.

Consider an elementary small sliver of wall surface of length ℓ and height dh. The water force on this sliver of area is

$$dF = p\, \ell\, dh$$

Total force, F, on the wall is

$$F = \int_0^F dF = \int_0^d p\,\ell\, dh \tag{3}$$

where p is given by Eq. (2). Inserting Eq. (2) into (3),

$$F = \int_0^d (\rho g h + p_a)\,\ell\, dh$$

or, $$F = \rho g \ell \left(\frac{d^2}{2}\right) + p_a \ell d \qquad (4)$$

In Eq. (4), the product ρg is the specific weight (weight per unit volume) of water; $\rho g = 62.4$ lb/ft^3. From Eq. (4),

$$F = (62.4)(30)\frac{(10)^2}{2} + (2116)(30)(10)$$

$$F = 93{,}600 + 634{,}800 = \boxed{728{,}400\,\text{lb}}$$

Note: This force is the combined effect of the force due to the weight of the water, 93,600 lb, and the force due to atmospheric pressure transmitted through the water, 634,800 Ib. In this example, the latter is the larger contribution to the force on the wall. If the wall were freestanding with atmospheric pressure exerted on the opposite side, then the net force exerted on the wall would be that due to the weight of the water only, i.e., 93,600 Ib. In tons, the force on the side of the wall in contact with the water is

$$F = \frac{728{,}400}{2000} = \boxed{364.2\,\text{tons}}$$

In the case of a freestanding wall, the net force, that due only to the weight of the water, is

$$F = \frac{93{,}600}{2000} = \boxed{46.8\,\text{tons}}$$

3.12 For the exponential atmosphere model,

$$\frac{\rho}{\rho_0} = e^{-g_0 h/(RT)}$$

$$\frac{\rho}{\rho_0} = e^{-(9.8)(45,000)/(287)(240)} = e^{-6.402}$$

Hence,

$$\rho = \rho_0 e^{-6.402} = (1225)(1.6575 \times 10^{-3}) = \boxed{2.03 \times 10^{-3}\ \mathrm{kg/m^3}}$$

From the standard atmosphere, at 45 km, $\rho = 2.02 \times 10^{-3}$ kg/m^3. The exponential atmosphere model gives a remarkably accurate value for the density at 45 km when a value of 240 K is used for the temperature.

3.13 At 3 km, from App. A, $T = 268.67$ K, $p = 7.0121 \times 10^4$ N/m^2, and $\rho = 0.90926$ kg/m^3. At 3.1 km, from App. A, $T = 268.02$ K, $p = 6.9235 \times 10^4$ N/m^2, and $\rho = 0.89994$ kg/m^3. At $h = 3.035$ km, using linear interpolation:

$$T = 268.67 - (268.67 - 268.02)\left(\frac{0.035}{0.1}\right) = \boxed{268.44\ \mathrm{K}}$$

$$p = 7.0121\times10^4 - (7.0121\times10^4 - 6.9235\times10^4)\left(\frac{0.035}{0.1}\right)$$

$$\boxed{= 6.98099\times10^4\ \frac{\mathrm{N}}{\mathrm{m}^2}}$$

$$\rho = 0.09026 - (0.90926 - 0.89994)\left(\frac{0.035}{0.1}\right) = \boxed{0.906\ \frac{\mathrm{kg}}{\mathrm{m}^3}}$$

3.14 The altitude of 3.035 km is in the gradient region. From Example 3.1, a = –0.0065 K/m. From Eq. (3.14),

$$T = T_1 + a(h - h_1) = T_s + a(h - 0) = 288.16 - (0.0065)(3035) = \boxed{268.43 \text{ K}}$$

From Eq. (3.12),

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-g_0/aR}$$

$$\frac{-g_0}{aR} = \frac{-9.8}{(-0.0065)(28.7)} = 5.25328$$

So

$$\frac{p}{p_s} = \left(\frac{T}{T_s}\right)^{5.25328} = \left(\frac{268.43}{288.16}\right)^{5.25328} = 0.688946$$

$$p = 0.688946(1.01325 \times 10^5) = 6.9807 \times 10^4 \ \frac{\text{N}}{\text{m}^2}$$

By comparison the approximate result from the solution of Prob. 3.13 differs from the exact result above by

$$\frac{6.980990 - 06.9807}{6.9807} = 4.15 \times 10^{-5} = \boxed{0.00415\%}$$

A very small amount.

From Eq. (3.13),

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-[(g_0/aR)+1]}$$

$$\frac{g_0}{aR} + 1 = -5.25328 + 1 = -4.25328$$

$$\frac{\rho}{\rho_s} = \left(\frac{T}{T_s}\right)^{4.2538} = \left(\frac{268.43}{288.16}\right)^{4.24328} = 0.73958$$

$$\rho = 0.73598 \ \ \rho_s = 0.73958\ (1.2250) = \boxed{0.90599 \ \frac{\text{kg}}{\text{m}^3}}$$

3.15 We want to calculate h_G when

$$\frac{h_G - h}{h_G} = 0.01 = 1 - \frac{h}{h_G}$$

From Eq. (3.6)

$$h = \frac{r h_G}{r + h_G}, \text{ where } r = 6.356766 \times 10^6 \text{ m}$$

Thus,

$$0.01 = 1 - \frac{r}{r + h_G}$$

$$\frac{r}{r + h_G} = 1 - 0.01 = 0.99$$

$$0.99(r + h_G) = r$$

$$h_G = \left(\frac{1 - 0.99}{0.99}\right) r = 0.0101\ (6.356766 \times 10^5)$$

$$h_G = 64.21 \times 10^3 \text{ m} = \boxed{64.21 \text{ km}}$$

3.16 From Eq. (3.1),

$$g = g_0 \left(\frac{r}{r + h_G}\right)^2$$

where $r = 6.356766 \times 10^6$ m, $g_0 = 9.8$ m/sec^2, and

$$h_G = (70{,}000 \text{ ft})\left(\frac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 21{,}336 \text{ m}$$

Thus,

$$g = 9.8\left(\frac{6.356766 \times 10^6}{6.356766 \times 10^6 + 21{,}336}\right) = 9.767 \ \frac{\text{m}}{\text{sec}^2}$$

$$\left(\frac{9.8 - 9.767}{98}\right) 100 = 0.34\%$$

Thus, at 70,000 ft, the acceleration of gravity has decreased only by 0.34% compared to its sea level value.

Chapter 4

4.1 $A_1V_1 = A_2V_2$

Let points 1 and 2 denote the inlet and exit conditions, respectively. Then,

$$V_2 = V_1\left(\frac{A_1}{A_2}\right) = (5)\left(\frac{1}{4}\right) = 1.25\,\text{ft/sec}$$

4.2 From Bernoulli's equation,

$$p_1 + \rho\frac{V_1^2}{2} = p_2 + \rho\frac{V_2^2}{2}$$

$$p_2 - p_1 = \frac{\rho}{2}(V_1^2 - V_2^2)$$

In consistent units,

$$\rho = \frac{62.4}{32.2} = 1.94\,\text{slug/ft}^3$$

Hence,

$$p_2 - p_1 = \frac{1.94}{2}[(5)^2 - (1.25)^2]$$

$$p_2 - p_1 = 0.97(23.4) = 22.7\,\text{lb/ft}^2$$

4.3 From Appendix A; at 3000m altitude,

$$p_1 = 7.01\times10^4\ \text{N/m}^2$$

$$\rho = 0.909\,\text{kg/m}^3$$

From Bernoulli's equation,

$$p_2 = p_1 + \frac{\rho}{2}(V_1^2 - V_2^2)$$

$$p_2 = 7.01\times10^4 + \frac{0.909}{2}[60^2 - 70^2]$$

$$p_2 = 7.01\times10^4 - 0.059\times10^4 = 6.95\times10^4\ \text{N/m}^2$$

4.4 From Bernoulli's equation, $p_1 + \frac{\rho}{2}V_1^2 = p_2 + \frac{\rho}{2}V_2^2$

Also from the incompressible continuity equation

$$V_2 = V_1(A_1/A_2)$$

Combining, $$p_1 + \frac{\rho}{2}V_1^2 = p_2 + \frac{\rho}{2}(A_1/A_2)^2$$

$$V_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[(A_1/A_2)^2 - 1\right]}}$$

At standard sea level, $\rho = 0.002377$ slug/ft^3. Hence,

$$V_1 = \sqrt{\frac{2(80)}{(.002377)[(4)^2 - 1]}} = 67\,\text{ft/sec}$$

Note that also $V_1 = 67\left(\frac{60}{80}\right) = 46\,\text{mi/h}$. (This is approximately the landing speed of World War I vintage aircraft).

4.5 $p_1 + \frac{1}{2}\rho V^2 = p_3 + \frac{1}{2}\rho V^2$

$$V_1^2 = \frac{2(p_3 - p_1)}{\rho} + V_3^2 \tag{1}$$

$$A_1 V_1 = A_3 V_3, \text{ or } V_3 = \frac{A_1}{A_3}V_1 \tag{2}$$

Substitute (2) into (1)

$$V_1^2 = \frac{2(p_3 - p_1)}{\rho} + \left(\frac{A_1}{A_3}\right)^2 V_1^2$$

or, $$V_1 = \sqrt{\frac{2(p_3 - p_1)}{\rho\left[1 - \left(\frac{A_1}{A_3}\right)^2\right]}} \tag{3}$$

Also, $$A_1 V_1 = A_2 V_2$$

or, $$V_2 = \left(\frac{A_1}{A_2}\right)V_1 \tag{4}$$

Substitute (3) into (4)

$$V_1 = \frac{A_1}{A_2}\sqrt{\frac{2(p_3 - p_1)}{\rho\left[1 - \left(\frac{A_1}{A_3}\right)^2\right]}}$$

$$V_2 = \frac{3}{1.5}\sqrt{\frac{2(1.00 - 1.02)\times 10^5}{(1.225)\left[1 - \left(\frac{3}{2}\right)^2\right]}} \qquad V_2 = 102.22\,\text{m/sec}$$

Note: It takes a pressure difference of <u>only</u> 0.02 atm to produce such a high velocity.

4.6 $V_1 = 130\,\text{mph} = 130\left(\frac{88}{60}\right) = 190.7\,\text{ft/sec}$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$
$$V_2^2 = \frac{2}{\rho}(p_1 - p_2) + V_1^2$$
$$V_2^2 = \frac{2(1760.9 - 1750.0)}{0.0020482} + (190.7)^2$$
$$V_2 = 216.8\,\text{ft/sec}$$

4.7 From Bernoulli's equation,

$$p_1 - p_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$

And from the incompressible continuity equation,

$$V_2 = V_1(A_1/A_2)$$

Combining: $$p_1 - p_2 = \frac{\rho}{2} V_1^2[(A_1/A_2)^2 - 1]$$

Hence, the maximum pressure difference will occur when simultaneously:

1. V_1 is maximum
2. ρ is maximum i.e., sea level

The design maximum velocity is 90 m/sec, and $\rho = 1.225$ kg/m^3 at sea level. Hence,

$$p_1 - p_2 = \frac{1.225}{2}(90)^2[(1.3)^2 - 1] = 3423\,\text{N/m}^2$$

Please note: In reality the airplane will most likely exceed 90 m/sec in a dive, so the airspeed indicator should be designed for a maximum velocity somewhat above 90 m/sec.

4.8 The isentropic relations are

$$\frac{p_c}{p_0} = \left(\frac{\rho_e}{\rho_0}\right)^{\gamma} = \left(\frac{T_c}{T_0}\right)^{\frac{\gamma}{\gamma-1}}$$

Hence, $$T_e = T_0\left(\frac{\rho_e}{\rho_0}\right)^{\frac{\gamma}{\gamma-1}} = (300)\left(\frac{1}{10}\right)^{\frac{14-1}{1.4}} = 155\,\text{K}$$

From the equation of state:

$$\rho_0 = \frac{p_0}{RT_0} = \frac{(10)(1.01\times 10^5)}{(287)(300)} = 11.73\,\text{kg/m}^3$$

Thus, $$\rho_e = \rho_0\left(\frac{p_e}{p_0}\right)^{\frac{1}{\gamma}} = 11.73\left(\frac{1}{10}\right)^{\frac{1}{1.4}} = 2.26\,\text{kg/m}^3$$

As a check on the results, apply the equation of state at the exit.

$$p_e = \rho_e RTe?$$
$$1.01\times 10^5 = (2.26)(287)(155)$$
$$1.01\times 10^5 = 1.01\times 10^5 \qquad \text{It checks!}$$

4.9 Since the velocity is essentially zero in the reservoir, the energy equation written between the reservoir and the exit is

$$h_0 = h_e + \frac{V_e^2}{2} \quad \text{or,} \quad V_e^2 = 2(h_0 - h_e) \tag{1}$$

However, $h = c_p T$. Thus Eq. (1) becomes

$$V_e^2 = 2c_p(T_0 - T_e)$$

$$V_e^2 = 2c_p\, T_0\left(1 - \frac{T_e}{T_0}\right) \tag{2}$$

However, the flow is isentropic, hence

$$\frac{T_c}{T_0} = \left(\frac{p_c}{p_0}\right)^{\frac{\gamma-1}{\gamma}} \tag{3}$$

Substitute (3) into (1).

$$V_e = \sqrt{2\, c_p\, T_0\left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{4}$$

This is the desired result. Note from Eq. (4) that V_e increases as T_0 increases, and as p_e/p_0 decreases. Equation (4) is a useful formula for rocket engine performance analysis.

4.10 The flow velocity is certainly large enough that the flow must be treated as compressible. From the energy equation,

$$c_p\, T_1 + \frac{V_1^2}{2} = c_p\, T_2 - \frac{V_2^2}{2} \tag{1}$$

At a standard altitude of 5 km, from Appendix A,

$$p_1 = 5.4 \times 10^4 \text{ N/m}^2$$
$$T_1 = 255.7\,\text{K}$$

Also, for air, $c_p = 1005$ joule/(kg)(K). Hence, from Eq. (1) above,

$$T_2 = T_1 + \frac{V_1^2 - V_2^2}{2\, c_p}$$

$$T_2 = 255.7 + \frac{(270)^2 - (330)^2}{2(1005)}$$

$$T_2 = 255.7 - 17.9 = 237.8\,\text{K}$$

Since the flow is also isentropic,

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Thus, $$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}} = 5.4 \times 10^4\left(\frac{237.8}{255.7}\right)^{\frac{1.4}{1.4-1}}$$

$$p_2 = 4.19 \times 10^4 \text{ N/m}^2$$

Please note: This problem and Problem 4.3 ask the same question. However, the flow velocities in the present problem require a compressible analysis. Make certain to examine the solutions of both Problems 4.10 and 4.3 in order to contrast compressible versus incompressible analyses.

4.11 From the energy equation

$$c_p T_0 = c_p T_e + \frac{V_e^2}{2}$$

or, $$T_e = T_0 - \frac{V_e^2}{2\,c_p}$$

$$T_e = 1000 - \frac{1500^2}{2(6000)} = 812.5\,R$$

In the reservoir, the density is

$$\rho_0 = \frac{p_0}{RT_0} = \frac{(7)(2116)}{(1716)(1000)} = 0.0086 \text{ slug/ft}^3$$

From the isentropic relation,

$$\frac{\rho_c}{\rho_0} = \left(\frac{T_e}{T_0}\right)^{\frac{1}{\gamma-1}}$$

$$\rho_e = 0.0086\left(\frac{812.5}{1000}\right)^{\frac{1}{1.4-1}} = 0.0051 \text{ slug/ft}^3$$

From the continuity equation,

$$\dot{m} = \rho_e A_e V_e$$

Thus, $$A_e = \frac{\dot{m}}{\rho_{eV_e}}$$

In consistent units,

$$\dot{m} = \frac{1.5}{32.2} = 0.047 \text{ slug/sec.}$$

Hence,

$$A_e = \frac{\dot{m}}{\rho_c V_e} = \frac{0.047}{(0.0051)(1500)} = 0.061\text{ft}^2$$

4.12 $V_1 = 1500\,\text{mph} = 1500\left(\frac{88}{60}\right) = 2200\,\text{ft/sec}$

$$C_p T_1 + \frac{V_1^2}{2} C_p T_2 \frac{V_2^2}{2}$$

$$V_2^2 = 2C_p(T_1 - T_2) + V_1^2$$

$$V_2^2 = 2(6000)(389.99 - 793.32) + (2200)^2$$

$$V_2 = 6.3\,\text{ft/sec}$$

Note: This is a very small velocity compared to the initial freestream velocity of 2200 ft/sec.

At the point in question, the velocity is very near zero, and hence the point is nearly a stagnation point.

4.13 At the inlet, the mass flow of air is

$$\dot{m}_{\text{air}} = \rho \text{AV} = (3.6391 \times 10^{-4})(20)(2200) = 16.0/\text{slug/sec}$$

$$\dot{m}_{\text{fuel}} = (0.05)(16.01) = 0.8\,\text{slug/sec}$$

Total mass flow at exit $= 16.01 + 0.8 = 16.81\,\text{slug/sec}$

4.14 From Problem 4.11,

$V_e = 1500$ ft/sec

$T_e = 812.5\ R$

Hence,

$$a_e = \sqrt{\gamma R T_e} = \sqrt{(1.4)(1716)(812.5)}$$
$$= 1397\,\text{ft/sec}$$

Thus, $$M_c = \frac{V_e}{a_e} = \frac{1500}{1397} = 1.07$$

Note that the nozzle of Problem 4.11 is just barely supersonic.

4.15 From Appendix A,

$$T_\infty = 216.66\ \text{K}$$

Hence,

$$a_\infty = \sqrt{\gamma R T}$$
$$= \sqrt{(1.4)(287)(216.66)} = 295\,\text{m/sec}$$

Thus, $$M_\infty = \frac{V_\infty}{a_\infty} = \frac{250}{295} = 0.847$$

4.16 At standard sea level, $T_\infty = 518.69\ R$

$$a_\infty = \sqrt{\gamma R T} = \sqrt{(1.4)(1716)(518.69)} = 1116\,\text{ft/sec}$$

$$V_\infty = M_\infty a_\infty = (3)(1116) = 3348\,\text{ft/sec}$$

Since 60 mi/hr - 88 ft/sec., then

$$V_\infty = 3348(60/88) = 2283\,\text{mi/h}$$

4.17 $V = 2200$ ft/sec

$$a = \sqrt{\gamma R T} = \sqrt{(1.4)(1716)(389.99)} = 967.94\,\text{ft/sec}$$

$$M = \frac{V}{a} = \frac{2200}{967.94} = 2.27$$

4.18 The test section density is

$$\rho = \frac{p}{RT} = \frac{1.01 \times 10^5}{(287)(300)} = 1.173\,\text{kg/m}^3$$

Since the flow is low speed, consider it to be incompressible, i.e., with the above density throughout.

$$p_1 - p_2 = \frac{\rho}{2} V_2^2 [1 - (A_2/A_1)^2] \tag{1}$$

In terms of the manometer reading,

$$p_1 - p_2 = \omega \Delta h \tag{2}$$

where $\omega = 1.33 \times 10^5$ N/m^3 for mercury.

Thus, combining Eqs. (1) and (2),

$$\Delta h = \frac{\rho}{2\omega} V_2^2 [1 - (A_2/A_1)^2]$$

$$= \frac{1.173}{(2)(1.33 \times 10^5)} (80)^2 [1 - (1/20)^2]$$

$$\Delta h = 0.028\,\text{m} = 2.8\text{ cm}$$

4.19 $V_2 = 200\,\text{mph} = 300\left(\frac{88}{60}\right) = 293.3$ ft/sec

(a) $p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2}{A_1} V_2$$

$$p_1 + \frac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 V_2^2 = p_2 + \frac{1}{2}\rho V_2^2$$

$$p_1 - p_2 = \frac{1}{2}\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] V_2^2$$

$$p_1 - p_2 = \frac{0.002377}{2}\left[1 - \left(\frac{4}{20}\right)^2\right](293.3)^2$$

$$p_1 - p_2 = 98.15\text{ lb/ft}^2$$

(b) $$p_1 + \frac{1}{2}\rho V_1^2 = p_3 + \frac{1}{2}\rho V_3^3$$

$$A_1V_1 = A_2V_2 \quad : \quad V_1 = \frac{A_2}{A_1}V_2$$

$$A_2V_2 = A_3V_3 \quad : \quad V_3 = \frac{A_2}{A_3}V_2$$

$$p_1 - p_3 = \frac{1}{2}\rho\left[\left(\frac{A_2}{A_3}\right)^2 - \left(\frac{A_2}{A_1}\right)^2\right]V_2^2$$

$$p_1 - p_3 = \frac{0.002377}{2}\left[\left(\frac{4}{18}\right)^2 - \left(\frac{4}{20}\right)^2\right](293.3)^2$$

$$p_1 - p_3 = 0.959\,\text{lb/ft}^2$$

Note: By the addition of a diffuser, the required pressure difference was reduced by an order of magnitude. Since it costs money to produce a pressure difference (say by running compressors or vacuum pumps), then a diffuser, the purpose of which is to improve the aerodynamic efficiency, allows the wind tunnel to be operated more economically.

4.20 In the test section

$$\rho = \frac{p}{RT} = \frac{2116}{(1716)(70 + 460)} = 0.00233\,\text{slug/ft}^3$$

The flow velocity is low enough so that incompressible flow can be assumed. Hence, from Bernoulli's equation,

$$p_0 = p + \frac{1}{2}\rho V^2$$

$$p_0 = 2116 + \frac{1}{2}(0.00233)[150(88/60)]^2$$

(Remember that 88 ft/sec = 60 mi/h.)

$$p_0 = 2116 + \frac{1}{2}(0.00233)(220)^2$$

$$p_0 = 2172\,\text{lb/ft}^2$$

4.21 The altimeter measures pressure altitude. Thus, from Appendix B, $p = 1572$ lb/ft^2. The air density is then

$$\rho = \frac{p}{RT} = \frac{1572}{(1716)(500)} = 0.00183\,\text{slug/ft}^3$$

Hence, from Bernoulli's equation,

$$V_{\text{true}} = \sqrt{\frac{2(p_0 - p)}{\rho}} = \sqrt{\frac{2(1650 - 1572)}{0.00183}}$$

$$V_{\text{true}} = 292\ \text{ft/sec}$$

The equivalent airspeed is

$$V_e = \sqrt{\frac{2(p_0 - p)}{\rho_s}} = \sqrt{\frac{2(1650 - 1572)}{0.002377}}$$

$$V_e = 256\ \text{ft/sec}$$

4.22 The altimeter measures pressure altitude. Thus, from Appendix A,

$$p = 7.95 \times 10^4 \text{ N/m}^2.$$

Hence,

$$\rho = \frac{p}{RT} = \frac{7.95 \times 10^4}{(287)(280)} = 0.989 \text{ kg/m}^3$$

The relation between V_{true} and V_e is

$$V_{\text{true}}/V_e = \sqrt{\rho_s / \rho}$$

Hence,

$$V_{\text{true}} = 50\sqrt{(1.225)/0.989} = 56 \text{ m/sec}$$

4.23 In the test section,

$$a = \sqrt{\gamma RT} = \sqrt{(1.4)(287)(270)} = 329 \text{ m/sec}$$

$$M = V/a = 250/329 = 0.760$$

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0.2(0.760)^2]^{3.5} = 1.47$$

Hence, $p_0 = 1.47p = 1.47(1.01 \times 10^5) = 1.48 \times 10^5 \text{ N/m}^2$

4.24 $p = 1.94 \times 10^4 \text{ N/m}^2$ from Appendix A.

$$M_1^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \frac{2}{1.4 - 1}\left[\left(\frac{2.96 \times 10^4}{1.94 \times 10^4}\right)^{0.286} - 1\right]$$

$$M_1^2 = 0.642$$

$$M_1 = 0.801$$

4.25 $$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_0}{p} = [1 + 0.2(0.65)^2]^{3.5} = 1.328$$

$$p = \frac{p_0}{1.328} = \frac{2339}{1.328} = 1761 \text{ lb/ft}^2$$

From Appendix B, this pressure corresponds to a pressure altitude, hence altimeter reading of 5000 ft.

4.26 At standard sea level,

$T = 518.69\ R$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}M^2 = 1 + 0.2(0.96)^2 = 1.184$$

$T_0 = 1.184T = 1.184\ (518.69)$

$T_0 = 614.3R = 154.3F$

4.27 $a_1 = \sqrt{\gamma RT_1} = \sqrt{(1.4)(287)(220)} = 297\,\text{m/sec}$

$M_1 = V_1 / a_1 = 596/197 = 2.0$

The flow is supersonic. Hence, the Rayleigh Pitot tube formula must be used.

$$\frac{p_{0_2}}{p_1} = \left[\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma - 1)}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1}\right]$$

$$\frac{p_{0_2}}{p_1} = \left[\frac{(2.4)^2(2)^2}{4(1.4)(2)^2 - 2(0.4)}\right]^{3.5} \left[\frac{1 - 1.4 + 2(1.4)(2)^2}{2.4}\right]$$

$$\frac{p_{0_2}}{p_1} = 5.64$$

$p_1 = 2.65 \times 10^4\ \text{N/m}^2$ from Appendix A.

Hence,

$p_{0_2} = 5.64(2.65 \times 10^4) = 1.49 \times 10^5\ \text{N/m}^2$

4.28 $q = \frac{1}{2}\rho V^2 = \frac{1}{2}\left(\frac{\gamma p}{\gamma p}\right)\rho V^2 = \frac{\gamma}{2}p\left(\frac{p}{\gamma p}\right)V^2 = \frac{\gamma}{2}p\frac{V^2}{a^2}$

Hence:

$$q = \frac{\gamma}{2}pM^2$$

4.29 $q_\infty = \frac{\gamma}{2}p_\infty {M_\infty}^2 = 0.7 p_\infty {M_\infty}^2$ (1)

Use Appendix A to obtain the values of p_∞ corresponding to the given values of h. Then use Eq. (1) above to calculate q_∞.

h(km)	60	50	40	30	20
p_∞(N/m^2)	25.6	87.9	299.8	1.19×10^3	5.53×10^3
M	17	9.5	5.5	3	1
q_∞(N/m^2)	5.2×10^3	5.6×10^3	6.3×10^3	7.5×10^3	3.9×10^3

Note that q_∞ progressively increases as the shuttle penetrates deeper into the atmosphere, that it peaks at a slightly supersonic Mach number, and then decreases as the shuttle completes its entry.

4.30 Recall that total pressure is defined as that pressure that would exist if the flow were slowed isentropically to zero velocity. This is a definition; it applies to all flows — subsonic or supersonic. Hence, Eq. (4.74) applies, no matter whether the flow is subsonic or supersonic.

$$\frac{p_0}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\gamma/(\gamma-1)} = [1 + 0.2(2)^2]^{1.4/0.4} = 7.824$$

Hence:

$$p_0 = 7.824\, p_\infty = 7.824(2116) = 1.656 \times 10^4 \frac{\text{lb}}{\text{ft}^2}$$

Note that the above value is not the pressure at a stagnation point at the nose of a blunt body, because in slowing to zero velocity, the flow has to go through a shock wave, which is non-isentropic. The stagnation pressure at the nose of a body in a Mach 2 stream is the total pressure behind a normal shock wave, which is lower than the total pressure of the freestream, as calculated above. This stagnation pressure at the nose of a blunt body is given by Eq. (4.79).

$$\frac{p_{0_2}}{p_1} = \left[\frac{(\gamma + 1)^2 M_\infty^2}{4\gamma M_2^2 - 2(\gamma - 1)}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1 - \gamma + 2\gamma M_1^2}{\gamma + 1}\right] =$$

$$\left[\frac{(2.4)^2 (2)^2}{4(1.4)(2)^2 - 2(0.4)}\right]^{1.4/0.4} \left[\frac{1 - 1.4 + 2(1.4)(2)^2}{2.4}\right] = 5.639$$

Hence,

$$p_{0_2} = 5.639\, p_\infty = 5.639(2116) = 1.193 \times 10^4 \frac{\text{lb}}{\text{ft}^2}$$

If Bernoulli's equation is used, the following wrong result for total pressure is obtained.

$$p_0 = p_\infty + q_\infty = p_\infty + \frac{1}{2}\rho V_\infty{}^2 = p_\infty + \frac{\gamma}{2} p_\infty M_\infty{}^2$$

$$p_0 = 2116 + 0.7(2116)(2)^2 = 0.804 \times 10^4 \frac{\text{lb}}{\text{ft}^2}$$

Compared to the correct result of $1.656 \times 10^4 \frac{\text{lb}}{\text{ft}^2}$, this leads to an error 51%.

4.31 $\dfrac{p_e}{p_0} = \left(1 + \dfrac{\gamma - 1}{2} M_e^2\right)\dfrac{-\gamma}{\gamma - 1}$

$$p_e = 5(1.01 \times 10^5)[1 + 0.2(3)^2]^{-3.5}$$

$$p_e = 1.37 \times 10^4 \text{ N/m}^2$$

$$\frac{T_e}{T_0} = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-1} = [1 + 0.2(3)^2]^{-1}$$

$$T_e = (500)(0.357) = 178.6 \text{ K}$$

$$\rho_e = \frac{p_e}{RT_c} = \frac{1.37 \times 10^4}{(287)(178.6)} = 0.267 \text{ kg/m}^3$$

4.32 $\dfrac{p_e}{p_0} = \left(1 + \dfrac{\gamma - 1}{2} M_e^2\right)^{\frac{-\gamma}{\gamma-1}}$

Hence,

$$M_e{}^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_e}{p_0}\right)^{\frac{1-\gamma}{\gamma}} - 1\right]$$

$$M_e{}^2 = 5[(0.2)^{-0.286} - 1] = 2.92$$

$$M_e = 1.71$$

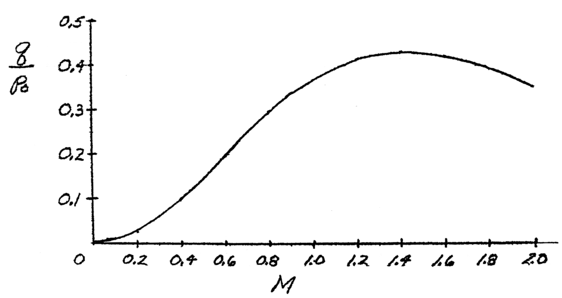
$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e{}^2}\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{\gamma}\right)M^2\right]^{(\gamma+1)/(\gamma-1)}$$

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{(1.71)^2}[(0.833)(1 + 0.2(1.71)^2)]^6$$

$$\frac{A_e}{A_t} = 1.35$$

4.33 $\dfrac{q}{p_0} = \dfrac{\gamma}{2}\dfrac{p}{p_0}M^2 = \dfrac{\gamma}{2}M^2\left[1 + \dfrac{\gamma - 1}{2}M^2\right]^{-\gamma/(\gamma-1)} = 0.7M^2(1 + 0.2M^2)^{-3.5}$

M	M^2	q/p_0
0	0	0
0.2	0.04	0.027
0.4	0.16	0.100
0.6	0.36	0.198
0.8	0.64	0.294
1.0	1.0	0.370
1.2	1.44	0.416
1.4	1.96	0.431
1.6	2.56	0.422
1.8	3.24	0.395
2.0	4.00	0.358



Note that the dynamic pressure increases with Mach number for $M < 1.4$ but decreases with Mach number for $M > 1.4$. I.e., in an isentropic nozzle expansion, there is a peak local dynamic pressure which occurs at $M = 1.4$.

4.34 First, calculate the value of the Reynolds number.

$$\text{Re}_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{(1,225)(200)(3)}{(1.7894 \times 10^{-5})} = 4.10 \times 10^7$$

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty{}^2 = \frac{1}{2}(1.225)(200)^2 = 2.45 \times 10^4 \text{ N/m}^2$$

Hence,

$$\delta_L = \frac{5.2L}{\sqrt{\text{Re}_L}} = \frac{5.2(3)}{\sqrt{4.1 \times 10^7}} = 0.0024 \text{ m} = 0.24 \text{ cm}$$

and

$$C_f = \frac{1.328}{\sqrt{\text{Re}_L}} = \frac{1.328}{\sqrt{4.1 \times 10^7}} = 0.00021$$

The skin friction drag on one side of the plate is:

$$D_f = q_\infty Sc_f = (2.45 \times 10^4)(3)(17.5)(0.00021)$$
$$D_f = 270 \text{ N}$$

The total skin friction drag, accounting for both the top and the bottom of the plate is twice this value, namely

Total D_f = 540 N

4.35 $\delta = \dfrac{0.37L}{(\text{Re}_L)^{0.2}} = \dfrac{0.37(3)}{(4.1 \times 10^7)^{0.2}} = 0.033 \text{ m} = 3.3 \text{ cm}$

From Problem 4.24, we find

$$\delta_{\text{turbulent}}/\delta_{\text{laminar}} = \frac{3.3}{0.24} = 13.75$$

The turbulent boundary layer is more than an order of magnitude thicker than the laminar boundary layer.

$$C_f = \frac{0.074}{(\text{Re}_L)^{0.2}} = \frac{0.074}{(4.1 \times 10^7)^{0.2}} = 0.0022$$

The skin friction drag on one side is then

$$D_f = q_\infty Sc_f = (2/45 \times 10^4)(3)(17.5)(0.0022)$$
$$D_f = 2830 \text{ N}$$

The total, accounting for both top and bottom is

Total D_f = 5660 N

From Problem 4.24, we find

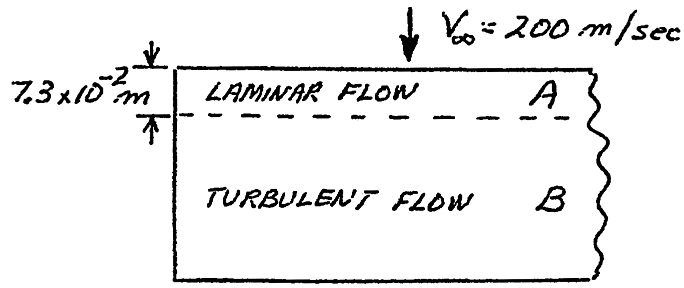
$$(D_{f_{\text{turbulent}}})/(D_{f_{\text{laminar}}}) = \frac{5660}{540} = 10.5$$

The turbulent skin friction drag is an order of magnitude larger than the laminar value.

4.36 $R_{e_{x_{cr}}} = \dfrac{\rho_\infty V_\infty x_{cr}}{\mu_\infty}$

$$x_{cr} = \mathrm{Re}_{x_{cr}} \left(\frac{\mu_\infty}{\rho_\infty V_\infty} \right) = \frac{(10^6)(1.789 \times 10^{-5})}{(1.225)(200)}$$

$$x_{cr} = 7.3 \times 10^{-2} \mathrm{m}$$



The turbulent drag that would exist over the first 7.3×10^{-2} m of chord length from the leading edge (area A) is

$$D_{f_A} = \frac{0.074}{(\mathrm{Re}_{c\tau})^{0.2}} q_\infty S_A \text{ (on one side)}$$

$$D_{f_A} = \frac{0.074}{(10^6)^{0.2}} (2.45 \times 10^4)(7.3 \times 10^{-2})(17.5)$$

$$D_{f_A} = 146 \text{ N} \qquad \text{(on one side)}$$

From Problem 4.25, the turbulent drag on one side, assuming both areas A and B to be turbulent, is 2830N. Hence, the turbulent drag on area B alone is:

$$D_{f_B} = 2830 - 146 - 2684 \text{ N} \quad \text{(turbulent)}$$

The laminar drag on area A is

$$D_{f_A} = \frac{1.328}{(\mathrm{Re}_{cr})^{0.5}} q_\infty S$$

$$D_{f_A} = \frac{1.328}{(10^6)^{0.5}} (2.45 \times 10^4)(7.3 \times 10^{-2})(17.5)$$

$$D_{f_A} = 42 \text{ N} \quad \text{(laminar)}$$

Hence, the skin friction drag on one side, assuming area A to be laminar and area B to be turbulent is

$$D_f = D_{f_A}\text{(laminar)} + D_{f_B}\text{ (turbulent)}$$

$$D_f = 42 + 2684 = 2726 \text{ N}$$

The total drag, accounting for both sides, is

Total $D_f = 5452$ N

Note: By comparing the results of this problem with those of Problem 4.25, we see that the flow over the wing is mostly turbulent, which is usually the case for real airplanes in flight.

4.37 The relation between changes in pressure and velocity at a point in an inviscid flow is given by the Euler equation, Eq. (4.8)

$$dp = -\rho V dV$$

Letting *s* denote distance along the streamline through the point, Eq. (4.8) can be written as

$$\frac{dp}{ds} = -\rho V \frac{dV}{ds}$$

or,

$$\frac{dp}{ds} = -\rho V^2 \frac{(dV/V)}{ds}$$

(a) $\dfrac{(dV/V)}{ds} = 0.02$ per millimeter

Hence,

$$\frac{dp}{ds} = -(1.1)(100)^2(0.02) = 220\frac{\text{N}}{\text{m}^2} \text{ per millimeter}$$

(b) $\dfrac{dp}{ds} = -(1.1)(1000)^2(0.02) = 22{,}000\dfrac{\text{N}}{\text{m}^2}$ per millimeter

Conclusion: At a point in a high-speed flow, it requires a much larger pressure gradient to achieve a given percentage change in velocity than for a low speed flow, everything else being equal.

4.38 We use the fact that total pressure is constant in an isentropic flow. From Eq. (4.74) applied in the freestream.

$$\frac{p_0}{p_\infty} = \left(1 + \frac{\gamma - 1}{\gamma} M_\infty{}^2\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0.2(0.7)^2]^{3.5} = 1.387$$

From Eq. (4.74) applied at the point on the wing,

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = [1 + 0.2(1.1)^2]^{3.5} = 2.135$$

Hence,

$$p = \left[\left(\frac{p_0}{p_\infty}\right) / \left(\frac{p_0}{p}\right)\right] p_\infty = \left(\frac{1.387}{2.135}\right) \text{p}_\infty == 0.65\ p_\infty$$

At a standard altitude of 3 km, from Appendix A, $p_\infty = 7.0121 \times 10^4 \text{ N/m}^2$.

Hence,

$$p = (0.65)(7.0121 \times 10^4) = 4.555 \times 10^4 \text{ N/m}^2$$

4.39 This problem is simply asking what is the equivalent airspeed, as discussed in Section 4.12. Hence,

$$V_e = V\left(\frac{\rho}{\rho_s}\right)^{1/2} = (800)\left(\frac{1.0663 \times 10^{-3}}{2.3769 \times 10^{-3}}\right)^{1/2} = 535.8 \text{ ft.sec}$$

4.40 **(a)** From Eq. (4.88)

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e{}^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e{}^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

$$= \frac{1}{(10)^2}\left\{\frac{2}{2.4}\left[1+0.2\,(10)^2\right]\right\}^6 = 2.87\times 10^5$$

Hence,

$$\frac{A_e}{A_t} = \sqrt{2.87\times 10^5} = 535.9$$

(b) Form Eq. (4.87)

$$\frac{p_0}{p_e} = \left(1+\frac{\gamma-1}{2}M_e{}^2\right)^{\frac{\gamma}{\gamma-1}} = [1+0.2(10)^2]^{3.5} = 4.244\times 10^4$$

At a standard altitude of 55 km, p = 48.373 N/m^2. Hence,

$$p_0 = (4.244\times 10^4)(48.373) = 2.053\times 10^6\ \text{N/m}^2 = 20.3\,\text{atm}$$

(c) From Eq. (4.85)

$$\frac{T_0}{T_e} = 1+\frac{\gamma-1}{2}M_e^2 = 1+0.2\,(10)^2 = 21$$

At a standard altitude of 55 km, T = 275.78 K. Hence,

T_0 = 275.78 (21) = 5791 K

Examining the above results, we note that:

1. The required expansion ratio of 535.9 is huge, but is readily manufactured.

2. The required reservoir pressure of 20.3 atm is large, but can be handled by proper design of the reservoir chamber.

3. The required reservoir temperature of 5791 K is tremendously large, especially when you remember that the surface temperature of the sun is about 6000 K. For a continuous flow hypersonic tunnel, such high reservoir temperatures can not be handled. In practice, a reservoir temperature of about half this value or less is employed, with the sacrifice made that “true temperature” simulation in the test stream is not obtained.

4.41 The speed of sound in the test stream is

$$a_e = \sqrt{\gamma R T_e} = \sqrt{(1.4)(287)(275.78)} = 332.9\,\text{m/sec.}$$

Hence,

$$V_e = M_e a_e = 10(332.9) = 3329\,\text{m/sec}$$

4.42 **(a)** From Eq. 4.88, for $M_e = 20$

$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M_e^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

$$= \frac{1}{(20)^2}\left\{\frac{2}{2.4}[1+0.2\,(20)^2]\right\}^6 = 2.365\times 10^8$$

Hence,

$$\frac{A_e}{A_t} = 15{,}377$$

(b) From Eq. (4.85)

$$\frac{T_0}{T_e} = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{-1} = [1+(0.2)(20)^2]^{-1} = 0.01235$$

Hence,

$$T_e = (5791)(0.01235) = 71.5\,\mathrm{K}$$

$$a_e = \sqrt{\gamma R T_e} = \sqrt{(1.4)(287)(71.5)} = 169.5\,\mathrm{m/sec}$$

$$V_e = M_e a_e = 20(169.5) = 3390\,\mathrm{m/sec}$$

Comments:

1. To obtain Mach 20, i.e., to double the Mach number in this case, the expansion ratio must be increased by a factor of 15,377/535.9 = 28.7. High hypersonic Mach numbers demand wind tunnels with very large exit-to-throat ratios. In practice, this is usually obtained by designing the nozzle with a small throat area.
2. Of particular interest is that the exit velocity is increased by a very small amount, namely by only 61 m/sec, although the exit Mach number has been doubled. The higher Mach number of 20 is achieved not by a large increase in exit velocity but rather by a large decrease in the speed of sound at the exit. This is characteristic of most conventional hypersonic wind tunnels — the higher Mach numbers are not associated with corresponding increases in the test section flow velocities.

4.43 We will use the sketch shown in Figure 4.47, identifying the separate plan form areas A and B.

(a)

$$q_\infty = \tfrac{1}{2}\rho_\infty V_\infty^{\,2} = \tfrac{1}{2}(1.23)(20)^2 = 246\ \text{N/m}^2$$

$$\text{Re}_{x_{\text{cr}}} = \frac{\rho_\infty V_\infty x_{\text{cr}}}{\mu_\infty} = 5 \times 10^5$$

Hence:

$$x_{\text{cr}} = \frac{\mu_\infty(5 \times 10^5)}{\rho_\infty V_\infty} = \frac{(1.789 \times 10^{-5})(5 \times 10^5)}{(1.23)(20)} = 0.3636\ \text{m}$$

Assuming turbulent flow over areas $A + B$,

$$C_{f_{A+B}} = \frac{0.074}{\text{Re}_L^{\,0.2}}$$

Where

$$\text{Re}_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{(1.23)(20)(4)}{(1.788 \times 10^{-5})} = 5.5 \times 10^6$$

$$C_{f_{A+B}} = \frac{0.074}{(5.5 \times 10^6)^{0.2}} = 0.0032$$

$$D_{f_{A+B}} = q_\infty S C_f = (246)(4)(4)(0.00332) = 13.07\ \text{N}$$

The turbulent drag on area A is obtained from

$$C_{f_A} = \frac{0.074}{(\text{Re}_{x_{\text{cr}}})^{0.2}} = \frac{0.074}{(5 \times 10^5)^{0.2}} = 0.00536$$

$$D_{f_A} = q_\infty S\, C_{f_A} = (246)(4)(0.3636)(0.00536) = 1.92\ \text{N}$$

The turbulent drag on area B is

$$D_{f_B} = D_{f_{A+B}} - D_{f_A} = 13.07 - 1.92 = 11.15\ \text{N}$$

The laminar drag on area A is obtained from

$$C_{f_A} = \frac{1.328}{(\text{Re}_{x_{\text{cr}}})^{0.5}} = \frac{1.328}{(5 \times 10^5)^{0.5}} = 0.00188$$

(Laminar) $D_{f_A} = q_\infty S\, C_{f_A} = (246)(4)(0.3636)(0.00188) = 0.67\ \text{N}$

The total friction drag is

$$D_f = D_{f_A} + D_{f_B} = 0.67 + 11.15 = \boxed{11.82\ \text{N}}$$

(b) $V_\infty = 40\,\text{m/sec}$

$$q_\infty = \tfrac{1}{2}\rho_\infty V_\infty^2 = \tfrac{1}{2}(1.23)(40)^2 = 984\,\text{N/m}^2$$

$$\text{Re}_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{(1.23)(40)(4)}{(1.789 \times 10^{-5})} = 11 \times 10^6$$

(Turbulent) $$C_{f_{A+B}} = \frac{0.074}{\text{Re}_L^{0.2}} = \frac{0.074}{(11 \times 10^6)^{0.2}} = 0.00289$$

$$D_{f_{A+B}} = q_\infty S C_{f_{A+B}} = (984)(4)(4)(0.00289) = 45.5\,\text{N}$$

(Turbulent) $$C_{f_A} = \frac{0.074}{(\text{Re}_{x_{cr}})^{0.2}} = \frac{0.074}{(5 \times 10^5)^{0.2}} = 0.00536$$

Note that, since $\text{Re}_{x_{cr}}$ remains the same, but V_∞ is doubled, x_{cr} is half that from part (a)

$$x_{cr} = \frac{0.3636}{2} = 0.1818\,\text{m}$$

$$D_{f_A} = q_\infty S C_{f_A} = (984)(4)(0.1818)(0.00536) = 3.84\,\text{N}$$

Hence,

$$D_{f_A} = D_{f_{A+B}} - D_{f_A} = 45.5 - 3.84 = 41.66\,\text{N}$$

(Laminar) $$C_{f_A} = \frac{1.328}{(\text{Re}_{x_{cr}})^{0.5}} = \frac{1.328}{(5 \times 10^5)^{0.5}} = 0.00188$$

$$D_{f_A} = q_\infty S C_{f_A} = (984)(4)(0.1818)(0.00188) = 1.345\,\text{N}$$

Thus,

$$D_f = D_{f_A} + D_{f_B} = 1.345 + 41.66 = \boxed{43\,\text{N}}$$

(c) $D_f \propto V_\infty^n$

Using the results from parts (a) and (b)

$$\frac{D_{f_a}}{D_{f_b}} = \left(\frac{V_a}{V_b}\right)^n$$

$$\frac{11.82}{43} = \left(\frac{20}{40}\right)^n$$

$$0.275 = (0.5)^n$$

Taking the log of both sides

$$-0.56 = n\,(-0.30)$$

$$n = \frac{0.56}{0.30} = \boxed{1.87}$$

Note: Skin friction drag does not follow the velocity squared law; rather skin friction varies with velocity at a power slightly less than 2.

4.44 **(a)** For turblent flow, $C_f \propto \dfrac{1}{\text{Re}^{0.2}}$. Since, $\text{Re} = \dfrac{\rho_\infty V_\infty L}{\mu}$, then $C_f \propto \dfrac{1}{V_\infty^{0.2}}$

$$D_f = q_\infty S C_f \propto V_\infty{}^2 \left(\frac{1}{V_\infty{}^{0.2}} \right)$$

or,

$$\boxed{Df \propto V_\infty{}^{1.8}}$$

(b) For laminar flow, $C_f \propto \dfrac{1}{\text{Re}^{0.5}} \propto \dfrac{1}{V_\infty{}^{0.5}}$

$$D_f \propto V_\infty^2 \left(\frac{1}{V_\infty^{0.2}} \right)$$

or,

$$\boxed{Df \propto V_\infty{}^{1.5}}$$

4.45 **(a)** $M_\infty = 1$. Since $a_\infty = 340.3$ m/sec at sea level,

$V_\infty = M_\infty a_\infty = (1)(340.3)=340.3$ m/sec

$$q_\infty = \tfrac{1}{2} \rho V_\infty^2 = \tfrac{1}{2}(1.23)(340.3)^2 = 7.12 \times 10^4 \text{ N/m}^2$$

$$\text{Re}_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{(1.23)(340.3)(4)}{(1.789 \times 10^{-5})} = 9.36 \times 10^7$$

$$C_{f_{\text{inc}}} = \frac{0.074}{\text{Re}_L{}^{0.2}} = \frac{0.074}{(9.36 \times 10^7)^{0.2}} = 0.00188$$

From Figure 4.44, for the turbulent case, approximate reading of the graph shows, for $M_\infty = 1$,

$$\frac{C_f}{C_{f_{\text{inc}}}} = 0.91$$

Thus,

$C_f = 0.91(0.00188) = 0.00171$

$D_f = q_\infty S C_f (7.12 \times 10^4)(4)(4)(0.00171)$

$$\boxed{D_f = 1948 \text{ N}}$$

(b) $M_\infty = 3$

$$V_\infty = M_\infty a_\infty = 3(340.3) = 1021\,\text{m/sec}$$

$$q_\infty = \tfrac{1}{2}\rho_\infty V_\infty^{\,2} = \tfrac{1}{2}(1.23)(1021)^2 = 6.41 \times 10^5\ \text{N/m}^2$$

$$\text{Re}_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{(1.23)(1021)(4)}{(1.789 \times 10^{-5})} = 2.81 \times 10^8$$

$$C_{f_\text{inc}} = \frac{0.074}{(2.81 \times 10^8)^{0.2}} = 0.00151$$

From Figure 4.44, for the turbulent case, approximate reading of the graph shows, for $M_\infty = 3$,

$$\frac{C_f}{C_{f_\text{inc}}} = 0.56$$

$$C_f = 0.56(0.00151) = 0.000846$$

$$D_f = q_\infty S\, C_f = (6.41 \times 10^5)(4)(4)(0.000846)$$

$$\boxed{D_f = 8677\,\text{N}}$$

(c)

$$\frac{D_{f_1}}{D_{f_2}} = \left(\frac{V_1}{V_2}\right)^n$$

$$\frac{1948}{8677} = \left(\frac{340.3}{1020.9}\right)^n$$

$$0\,.2245 = (0.333)^n$$

$$-0.6488 = n\,(-0.4772)$$

$$n = \frac{-0.6488}{-0.4772} = 1.36$$

This implies $D_f \propto V_\infty^{1.36}$. This is considerably different than the velocity squared law. In fact, comparing this result, where $n = 1.36$, with the incompressible result in Problem 4.43, where $n = 1.87$, indicates that the added effect of compressibility (higher Mach numbers) continues to drive down the exponent. The comparison is only qualitative, however, because the Reynolds numbers in Problem 4.45 are much larger than those in Problem 4.43.

In any event, the bottom line of Problems 4.43,4.44, and 4.45 is that flows with friction and/or compressibility do not follow the velocity squared law, but rather for such flows, the skin friction drag varies as V_∞^n where n is an exponent less than 2.

4.46 **(a)** The waves travel at the local speed of sound.

$$a = \sqrt{\gamma RT} = \sqrt{(1.4)(287)(288)} = 340.2\,\text{m/sec}.$$

This speed is relative to the gas. Since the gas velocity is zero, then this is also the wave velocity relative to the pipe. One wave travels to the right along the positive x-axis with the velocity 340.2 m/sec, and the other wave travels to the left along the negative x-axis with a velocity of −340.2 m/sec.

(b) The x-location of the right-running wave at $t = 0.2$ sec is

$$x = at = (340.2)(0.2) = 68 \text{ m}$$

The x-location of the left-running wave at $t = 0.2$ sec is

$$x = at = (-340.2)(0.2) = -68 \text{ m}$$

4.47 The waves still travel at the local speed of round <u>relative to the gas</u>. Since the gas itself is moving relative to the pipe, then the wave speed relative to the pipe is the sum of the wave speed relative to the gas and the speed of the gas relative to the pipe. The velocity of right-running wave is therefore

$$V_{\text{wave}} = V_{\text{gas}} + \text{wave speed}$$

(a) When $V_{\text{gas}} = 30$ m/sec,

$$V_{\text{wave}} = 30 + 340.2 = 370.2 \text{ m/sec}$$

The right-running wave travels to the right at 370.2 m/sec relative to the stationary pipe. After 0.2 sec, the x-location of this wave is

$$x = (370.2)(0.2) = 74 \text{ m}$$

The left-running wave travels to the left at the velocity

$$V_{\text{wave}} = 30 - 340.2 = -310.2 \text{ m/sec}$$

After 0.2 sec, the x-location of this left-running wave is

$$x = (-310.2)(0.2) = -62 \text{ m}$$

(b) The flow velocity relative to the pipe is 400 m/sec, which is faster than the speed of sound. The weak pressure distributions, however, continue to propagate to the right and left at the speed of sound <u>relative</u> to the gas. Thus, the right-running wave has a velocity relative to the pipe of

$$V_{\text{wave}} = 400 + 340.2 = 740.2 \text{ m/sec}$$

After 0.2 sec, the location of this wave is:

$$x = (740.2)\ 0.2 = 148.02 \text{ m}$$

The left-running wave has a velocity relative to the pipe of

$$V_{\text{wave}} = 400 - 340.2 = 59.8 \text{ m/sec}$$

After 0.2 sec, the location of this wave is

$$x = (59.8)\ (0.2) = 11.96 \text{ m}$$

Note: The left-running wave, although it is traveling along to the left at the speed of sound relative to the gas, is <u>moving to the right</u> relative to the pipe because the flow velocity in the pipe is higher than the speed of sound.

4.48 The element of air initially has a pressure and density at the standard altitude of 1000 m of, from Appendix A,

$$p_1 = 8.9876 \times 10^4 \text{ N/m}^2$$

and

$$\rho_1 = 1.1117 \text{ kg/m}^3$$

At a standard altitude of 2000 m, $p_2 = 7.9501 \times 10^4$ N/m^2. The density of the element of air, assuming it has experienced an isentropic process, is, from Eq. (4.37)

$$\rho_2 = \rho_1 \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = 1.1117 \left(\frac{7.9501 \times 10^4}{8.9876 \times 10^4}\right)^{\frac{1}{1.4}}$$

$$\rho_2 = 1.0184 \text{ kg/m}^3$$

Comparing with the standard density at 2000 m of 1.0066 kg/m^3, we see that the raised element of air has a higher density and is therefore heavier than its surrounding neighbors. This situation is a stable atmosphere because the isentropically perturbed element will tend to sink back down to its originally lower altitude.

4.49 $A_1 V_1 = A_2 V_2$

$$V_1 = \frac{A_2}{A_1} V_2 = \frac{0.5}{4}(120) = 15 \text{ mph}$$

From the answer to Problem 2.15, 60 mph = 26.82 m/sec.

$$V_1 = \left(\frac{26.82}{60}\right) = (15) = \boxed{6.705 \text{ m/sec}}$$

4.50 $p_1 + 1/2\,\rho\, V_1^2 = p_2 + 1/2\,\rho\, V_2^2$

$$p_2 = p_1 + 1/2\,\rho\left(V_1^2 - V_2^2\right)$$

$$p_1 = 1 \text{ atm} = 1.01 \times 10^5 \text{ N/m}^2$$

$$\rho = 1.23 \text{ kg/m}^3$$

$$V_1 = 6.705 \text{ m/sec}$$

$$V_2 = (120)\left(\frac{26.82}{60}\right) = 53.64 \text{ m/sec}$$

$$p_2 = 1.01 \times 10^5 + 1/2\,(1.23)[(6.705)^2 - (53.64)^2]$$

$$p_2 = \boxed{0.99258 \times 10^5 \text{ N/m}^2}$$

4.51 Let x be the distance along the diffuser, and h_D the height at the diffuser entrance. At any location x, the distance from the top to the bottom wall is $h_D + 2x \tan\theta$ where θ is the angle of the wall. Thus, the cross-sectional area at any location x is

$$A = (2)\,[h_D + 2x \tan\theta]$$

Thus,

$$\frac{dA}{dx} = 4 \tan\theta = 4\tan 15° = \boxed{1.0718 \text{ m}}$$

4.52 The flow velocity at the entrance to the diffuser is the same as in the test section

$$V_2 = 120 \text{ mph} = \boxed{53.64 \text{ m/sec}}$$

At the exit of the diffuser, since

$$V_2 A_2 = V_3 A_3$$

$$V_3 = \left(\frac{A_2}{A_3}\right) V_2 = \left(\frac{0.5}{3.5}\right)(53.64) = \boxed{7.66 \text{ m/sec}}$$

4.53 $AV = \text{const}$

$$A\frac{dv}{dx} + V\frac{dA}{dx} = 0$$

$$\frac{dv}{dx} = -\frac{V}{A}\left(\frac{dA}{dx}\right)$$

From the solution of Problem 4.51,

$$\frac{dA}{dx} = 1.0718 \text{ m}$$

Thus,

$$\frac{dV}{dx} = -1.0718\left(\frac{V}{A}\right)$$

(a) At the entrance, $V_2 = 53.64$ m/sec (from Prob. 4.52)

$$A_2 = (0.5)(2) = 1 \text{ m}^2$$

$$\frac{dV}{dx} = -1.0718\left(\frac{53.64}{1}\right) = \boxed{-57.49 \text{ sec}^{-1}}$$

(b) At the exit, $V_3 = 7.66$ m/sec (from Prob. 4.52)

$$\frac{dV}{dx} = -1.0718\left(\frac{V_3}{A_3}\right) = -1.0718\left(\frac{7.66}{7}\right) = \boxed{-1.173 \text{ sec}^{-1}}$$

4.54 From the Euler equation, Eq. (4.8) in the text,

$$dp = -\rho V dV$$

$$\frac{dp}{dx} = -\rho V \frac{dV}{dx}$$

(a) At the inlet, from the solutions of Problems 4.52 and 4.53,

$$V_2 = 53.64 \text{ m/sec and } \frac{dV}{dx} = -57.49 \text{ sec}^{-1}$$

Thus,

$$\frac{dp}{dx} = -(1.23)(53.64)(-57.49) = \boxed{3.793 \times 10^3 \text{ N/m}^3}$$

(b) At the exit,

$$V_3 = 7.66 \text{ m/sec and } \frac{dV}{dx} = -1.173 \text{ sec}^{-1}$$

Thus,

$$\frac{dp}{dx} = -(1.23)(7.66)(-1.173) = \boxed{11.05 \text{ N/m}^3}$$

4.55 $\tan 15° = \dfrac{(3.5 - 0.5)/2}{x} = \dfrac{1.5}{x}$

$$x = \frac{1.5}{\tan 15°} = \frac{1.5}{0.2679} = \boxed{5.6 \text{ m}}$$

4.56 From the solution of Problem 4.54,

$$\left(\frac{dp}{dx}\right)_2 = 3.793 \times 10^3 \text{ N/m}^3$$

$$\left(\frac{dp}{dx}\right)_3 = 11.05 \text{ N/m}^3$$

$$\left(\frac{dp}{dx}\right)_{\text{ave}} = \frac{3.793 \times 10^3 + 11.05}{2} = 1902 \frac{\text{N}}{\text{m}^2}$$

$$x_s = \frac{183}{\left(\frac{dp}{dx}\right)_{\text{ave}}} = \frac{183}{1902} = \boxed{0.096 \text{ m}}$$

4.57 From the solution of Problem 4.56, $(x_s)_{\text{laminar}} = 0.096$ m.

$$\frac{(x_s)_{\text{turbulent}}}{(x_s)_{\text{laminar}}} = \frac{506\left(\frac{dp}{dx}\right)_{\text{ave}}^{-1}}{183\left(\frac{dp}{dx}\right)_{\text{ave}}^{-1}} = 2.765$$

$$(x_s)_{\text{turbulent}} = (2.765)(0.096) = \boxed{0.265 \text{ m}}$$

4.58 At $h = 7{,}500$ ft, from App. B, $T_\infty = 491.95°R$, $p_\infty = 1602.3$ lb/ft^2, and $\rho_\infty = 1.8975 \times 10^{-3}$ slug/ft^3.

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(491.95} = 1087 \text{ ft/sec}$$

$$V_\infty = 229 \text{ mph } \left(\frac{88}{60}\right) = 335.9 \text{ ft/sec}$$

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{335.9}{1087} = \boxed{0.309}$$

$$p_0 = \rho_\infty + 1/2 \ \rho_\infty V_\infty^2 = 1602.3 + (0.5)(1.8975 \times 10^{-3})(335.9)^2$$

$$= \boxed{1709 \text{ lb/ft}^2}$$

4.59 At $h = 25{,}000$ ft, from App. B, $T_\infty = 429.64°R$, $p_\infty = 786.33$ lb/ft^2, and $\rho_\infty = 1.0663 \times 10^{-3}$ slug/ft^3.

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(429.64)} = 1016 \text{ ft/sec}$$

$$V_\infty = 610 \left(\frac{88}{60}\right) = 894.67 \text{ ft/sec}$$

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{894.67}{1016} = \boxed{0.88}$$

The flow is subsonic, but compressible. From Eq. (4.74),

$$\frac{p_0}{p_\infty} = \left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{p_0}{p_\infty} = [1 + 0.2(0.88)^2]^{3.5} = (1.15488)^{3.5} = 2.303$$

$$\rho_0 = 2.303 \ p_\infty = 2.303 \ (786.33) = \boxed{1811 \frac{\text{lb}}{\text{ft}^2}}$$

4.60 At h = 35,000 ft, from App. B, $T_\infty = 394.08°R$, p_∞ = 499.34 lb/ft^2, and $\rho_\infty = 7.382 \times 10^{-4}$ slug/ft^3.

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(394.08)} = 973 \text{ ft/sec}$$

$$V_\infty = 1328\left(\frac{88}{60}\right) = 1947.7 \text{ ft/sec}$$

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{1947.7}{973} = \boxed{2.00}$$

The flow is supersonic. From Eq. (4.79),

$$\frac{p_{0_2}}{p_\infty} = \left[\frac{(\gamma+1)^2 M_\infty^2}{4\gamma M_\infty^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}} \frac{1-\gamma+2\gamma M_\infty^2}{\gamma+1}$$

$$= \left[\frac{(2.4)^2(2)^2}{4(1.4)(2)^2 - 2(0.4)}\right]^{\frac{1.4}{0.4}} \left[\frac{1-1.4+2(1.4)(2)^2}{2.4}\right] = 5.64$$

$$p_{0_2} = 5.64(499.34) = \boxed{2817 \text{ lb/ft}^2}$$

Chapter 5

5.1 Assume the moment is governed by

$$M = f(V_\infty, \rho_\infty, S, \mu_\infty, a_\infty)$$

More specifically:

$$M = Z(V_\infty{}^a, \rho_\infty{}^b, S^d, a_\infty{}^e, \mu_\infty{}^f)$$

Equating the dimensions of mass, m, length, ℓ, and time t, and considering Z dimensionless,

$$\frac{m\ell^2}{t^2} = \left(\frac{\ell}{t}\right)^a \left(\frac{m}{\ell^3}\right)^b \left(\ell^2\right)^d \left(\frac{\ell}{t}\right)^e \left(\frac{m}{\ell^t}\right)^f$$

$1 = b + f$ (For mass)
$2 = \mathrm{a} - 3b + 2d + e - f$ (For length)
$-2 = -a - e - f$ (for time)

Solving a, b, and d in terms of e and f,

$b = 1 - f$

and, $a = 2 - e - f$

and, $2 = 2 - e - f - 3 + 3f + 2d + e - f$

or $0 = -3 + f + 2d$

$$d = \frac{3 - f}{2}$$

Hence,

$$M = Z V_\infty{}^{2-e-f} \rho_\infty{}^{1-f} S^{(3-f)/2}\, a_\infty{}^e \mu_\infty{}^f$$

$$= Z V_\infty{}^2 \rho_\infty S S^{1/2} \left(\frac{a_\infty}{V_\infty}\right)^e \left(\frac{\mu_\infty}{V_\infty \rho_\infty S^{1/2}}\right)^f$$

Note that $S^{1/2}$ is a characteristic length; denote it by the chord, c.

$$M = \rho_\infty V_\infty^2 S\, c\, Z \left(\frac{a_\infty}{V_\infty}\right)^c \left(\frac{\mu_\infty}{V_\infty \rho_\infty c}\right)^f$$

However, $a_\infty / V_\infty = 1/M_\infty$

and $\dfrac{\mu_\infty}{V_\infty \rho_\infty c} = \dfrac{1}{\mathrm{Re}}$

Let

$$Z\left(\frac{1}{M_\infty}\right)^e \left(\frac{1}{\mathrm{Re}}\right)^f = \frac{c_m}{2}$$

where c_m is the moment coefficient. Then, as was to be derived, we have

$$M = \frac{1}{2} \rho_\infty V_\infty{}^2 c\, c_m$$

or, $M = q_\infty S\, c\, c_m$

5.2 From Appendix D, at 5° angle of attack,

$$c_\ell = 0.67$$
$$c_{m_{c/4}} = -0.025$$

(*Note:* Two sets of lift and moment coefficient data are given for the NACA 1412 airfoil—with and without flap deflection. Make certain to read the code properly, and use only the unflapped data, as given above. Also, note that the scale for $c_{m_{c/4}}$ is different than that for c_ℓ—be careful in reading the data.)

With regard to c_d, first check the Reynolds number,

$$\text{Re} = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(0.002377)(100)(3)}{(3.7373 \times 10^{-7})}$$
$$\text{Re} = 1.9 \times 10^6$$

In the airfoil data, the closest Re is 3×10^6. Use c_d for this value.

$$c_d = 0.007 \quad (\text{for } c_\ell = 0.67)$$

The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(100)^2 = 11.9\,\text{lb/ft}^2$$

The area per unit span is $S = 1(c) = (1)(3) = 3\,\text{ft}^2$

Hence, per unit span,

$$L = q_\infty S c_\ell = (11.9)(3)(0.67) = 23.9\,\text{lb}$$
$$D = q_\infty S c_d = (11.9)(3)(0.007) = 0.25\,\text{lb}$$
$$M_{c/4} = q_\infty S c c_{m_{c/4}} = (11.9)(3)(3)(-0.025) = -2.68\text{ ft.lb}$$

5.3 $$\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{(1.01 \times 10^5)}{(287)(303) = 1.61\,\text{kg/m}^3}$$

From Appendix D,

$$c_\ell = 0.98$$
$$c_{m_{c/4}} = -0.012$$

Checking the Reynolds number, using the viscosity coefficient from the curve given in Chapter 4,

$$\mu_\infty = 1.82 \times 10^{-5} \text{ kg/m sec} \qquad \text{at } T = 303 \text{ K},$$
$$\text{Re} = \frac{\rho_\infty V_\infty c}{\mu_\infty} = \frac{(11.57)(42)(0.3)}{1.82 \times 10^{-5}} = 8 \times 10^5$$

This Reynolds number is considerably less than the lowest value of 3×10^6 for which data is given for the NACA 23012 airfoil in Appendix D. Hence, we can use this data only to give an educated guess; use $c_d \approx 0.01$, which is about 10 percent higher than the value of 0.009 given for Re = 3×10^6 The dynamic pressure is

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.161)(42)^2 = 1024\,\text{N/m}^2$$

The area per unit span is $S = (1)(0.3) = 0.3\,\text{m}^2$. Hence,

$$L = q_\infty S c_\ell = (1024)(0.3)(0.98) = 301\,\text{N}$$
$$D = q_\infty S c_d = (1024)(0.3)(0.01) = 3.07\,\text{N}$$
$$M_{c/4} = q_\infty S c c_m = (1024)(0.3)(0.3)(-0.012) = -1.1\,\text{Nm}$$

5.4 From the previous problem, $q_\infty = 1020\ N/m^2$

$$L = q_\infty S c_\ell$$

Hence,

$$c_\ell = \frac{L}{q_\infty S}$$

The wing area $S = (2)(0.3) = 0.6\,\mathrm{m}^2$

Hence,

$$c_\ell = \frac{200}{(1024)(0.6)} = 0.33$$

From Appendix D, the angle of attack which corresponds to this lift coefficient is

$$\alpha = 2°$$

5.5 From Appendix D, at $\alpha = 4°$,

$$c_\ell = 0.4$$

Also, $$V_\infty = 120\left(\frac{88}{60}\right) = 176\,\mathrm{ft/sec}$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(176)^2 = 36.8\,\mathrm{lb/ft}^2$$

$$L = q_\infty S c_\ell$$

$$S = \frac{L}{q_\infty c_\ell} = \frac{29.5}{(36.8)(0.4)} = 2\ \mathrm{ft}^2$$

5.6 $$L = q_\infty S c_\ell$$

$$D = q_\infty S c_d$$

Hence, $$\frac{L}{D} = \frac{q_\infty S c_\ell}{q_\infty S c_d} = \frac{c_\ell}{c_d}$$

We must tabulate the values of c_ℓ / c_d for various angles of attack, and find where the maximum occurs. For example, from Appendix D, at $\alpha = 0^0$,

$$c_\ell = 0.25$$

$$c_d = 0.006$$

Hence $$\frac{L}{D} = \frac{c_\ell}{c_d} = \frac{0.25}{0.006} = 4.17$$

A tabulation follows.

α	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
c_ℓ	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.05	1.15
c_d	0.006	0.006	0.006	0.0065	0.0072	0.0075	0.008	0.0085	0.0095	0.0105
$\frac{c_\ell}{c_d}$	41.7	58.3	75	84.6	90.3	100	106	112	111	110

From the above tabulation, $$\left(\frac{L}{D}\right)_{\max} \approx 112$$

5.7 At sea level

$$\rho_\infty = 1.225 \text{ kg/m}^3$$
$$p_\infty = 1.01 \times 10^5 \text{ N/m}^2$$

Hence,

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(50)^2 = 1531 \text{ N/m}^2$$

From the definition of pressure coefficient,

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{(0.95 - 1.01) \times 10^5}{1531} = -3.91$$

5.8 The speed is low enough that incompressible flow can be assumed. From Bernoulli's equation,

$$p + \frac{1}{2}\rho V_\infty^2 = p_\infty + \frac{1}{2}\rho_\infty V_\infty^2 = p_\infty + q_\infty$$

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{q_\infty - \frac{1}{2}\rho V^2}{q_\infty} = 1 - \frac{\frac{1}{2}\rho V^2}{\frac{1}{2}\rho_\infty V_\infty^2}$$

Since $\rho = \rho_\infty$(constant density)

$$C_p = \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(\frac{62}{55}\right)^2 = 1 - 1.27 = -0.27$$

5.9 The flow is low speed, hence assumed to be incompressible. From Problem 5.8,

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - \left(\frac{195}{160}\right)^2 = -0.485$$

5.10 The speed of sound is

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(510)} = 1107 \text{ ft/sec}$$

Hence,

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{700}{1107} = 0.63$$

In Problem 5.9, the pressure coefficient at the given point was calculated as −0.485. However, the conditions of Problem 5.9 were low speed, hence we identify

$$C_{p_0} = -0.485$$

At the new, higher free stream velocity, the pressure coefficient must be corrected for compressibility. Using the Prandtl-Glauert Rule, the high speed pressure coefficient is

$$C_p = \frac{C_{p_0}}{\sqrt{1 - M_\infty^2}} = \frac{-0.485}{\sqrt{1 - (0.63)^2}} = -0.625$$

5.11 The formula derived in Problem 5.8, namely

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2,$$

utilized Bernoulli's equation in the derivation. Hence, it is <u>not valid</u> for compressible flow. In the present problem, check the Mach number.

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(505)} = 1101 \text{ ft/sec}$$

$$M_\infty = \frac{780}{1101} = 0.708.$$

The flow is clearly compressible! To obtain the pressure coefficient, first calculate ρ_∞ from the equation of state.

$$\rho_\infty = \frac{p_\infty}{RT_\infty} = \frac{2116}{(1716)(505)} = 0.00244 \text{ slug/ft}^3$$

To find the pressure at the point on the wing where $V = 850$ ft/sec, first find the temperature from the energy equation

$$c_p T + \frac{V^2}{V} = c_p T_\infty + \frac{V_\infty{}^2}{2}$$

$$T = T_\infty + \frac{V_\infty{}^2 - V^2}{2c_p}$$

The specific heat at constant pressure for air is

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{(1.4-1)} = 6006 \frac{\text{ft lb}}{\text{slug } R}$$

Hence,

$$T = 505 + \frac{780^2 - 850^2}{2(6006)} = 505 - 9.5 = 495.5 \, R$$

Assuming isentropic flow

$$\frac{p}{p} = \left(\frac{T}{T_\infty}\right)^{\frac{\gamma}{\gamma-1}}$$

$$p = (2116)\left(\frac{495.5}{505}\right)^{3.5} = 1980 \text{ lb/ft}^2$$

From the definition of C_p

$$C_p = \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty{}^2} = \frac{1980 - 2116}{\frac{1}{2}(0.00244)(780)^2}$$

$$C_p = -0.183$$

5.12 A velocity of 100 ft/sec is low speed. Hence, the desired pressure coefficient is a low speed value, C_{p_0}.

$$C_p = \frac{C_{p_0}}{\sqrt{1 - M_\infty{}^2}}$$

From problem 5.11,

$$C_p = -0.183 \text{ and } M_\infty = 0.708. \text{ Thus, } 0.183 = \frac{C_{p_0}}{\sqrt{1 - (0.708)^2}}$$

$$C_{p_0} = (-0.183)(0.706) = -0.129$$

5.13 Recall that the airfoil data in Appendix D is for low speeds. Hence, at $\alpha = 4°$,

$$c_{\ell_0} = 0.58.$$

Thus, from the Prandtl-Glauert rule,

$$c_\ell = \frac{c_{\ell_0}}{\sqrt{1 - M_\infty{}^2}} = \frac{0.58}{\sqrt{1 - (0.8)^2}} = 0.97$$

5.14 The lift coefficient measured is the high speed value, c_ℓ. Its low speed counterpart is c_{ℓ_0}, where

$$c_\ell = \frac{c_{\ell_0}}{\sqrt{1 - M_\infty^2}}$$

Hence,

$$c_{\ell_0} = (0.85)\sqrt{1 - (0.7)^2} = 0.607$$

For this value, the low speed data in Appendix D yield

$$\alpha = 2°$$

5.15 First, obtain a curve of $C_{p,\text{cr}}$ versus M_∞, from

$$C_{p,\text{cr}} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right)^{\gamma/(\gamma-1)} - 1\right]$$

Some values are tabulated below for $\gamma = 1.4$.

M_∞	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$C_{p,\text{cr}}$	−3.66	−2.13	−1.29	−0.779	−0.435	−0.188	0

Now, obtain the variation of the minimum pressure coefficient, C_p, with M_∞, where $C_{p_0} = -0.90$. From the Prandtl-Glauert rule,

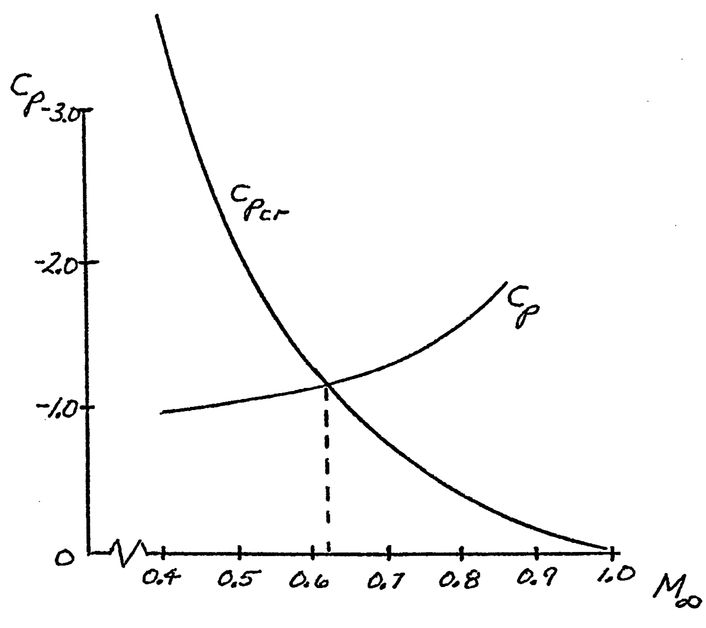
$$C_p = \frac{C_{p_0}}{\sqrt{1-M_\infty{}^2}}$$

$$C_p = \frac{-0.90}{\sqrt{1-M_\infty{}^2}}$$

Some tabulated values are:

M_∞	0.4	0.5	0.6	0.7	0.8	0.9
C_p	−0.98	−1.04	−1.125	−1.26	−1.5	−2.06

A plot of the two curves is given on the next page.



From the intersection point,

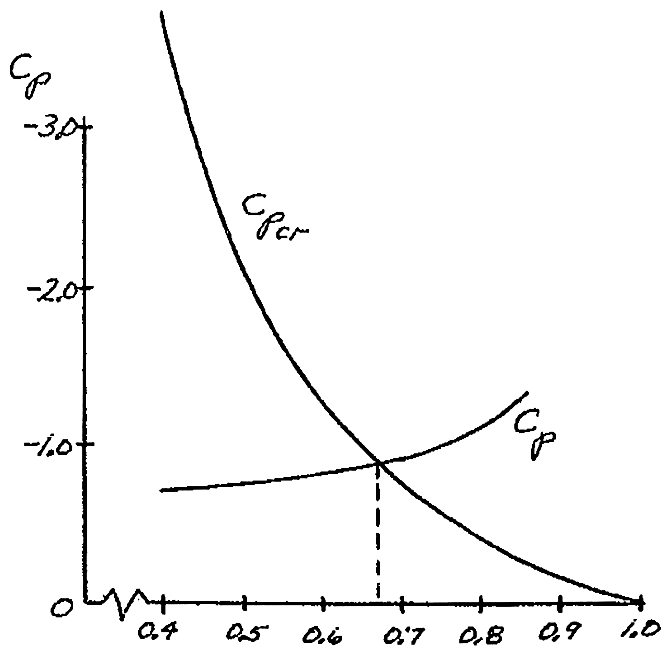
$$M_{\text{cr}} = \boxed{0.62}$$

5.16 The curve of $C_{p,\,\mathrm{cr}}$ versus M_∞ has already been obtained in the previous problem; it is a universal curve, and hence can be used for this and all other problems. We simply have to obtain the variation of C_p with M_∞ from the Prandtl-Glauert rule, as follows:

$$C_p = \frac{C_{p_0}}{\sqrt{1 - M_\infty{}^2}} = \frac{-0.65}{\sqrt{1 - M_\infty{}^2}}$$

M_∞	0.4	0.5	0.6	0.7	0.8	0.9
C_p	−0.71	−0.75	−0.81	−0.91	−1.08	−1.49

The results are plotted below.



From the point of intersection,

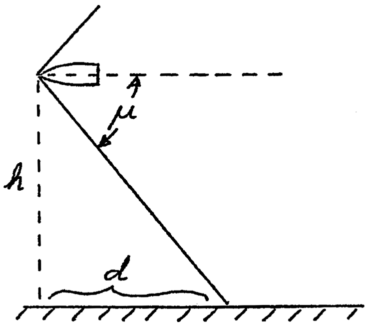
$$M_{\mathrm{cr}} = \boxed{0.68}$$

Please note that, comparing Problems 5.15 and 5.16, the critical Mach number for a given airfoil is somewhat dependent on angle of attack for the simple reason that the value of the minimum pressure coefficient is a function of angle of attack. When a critical Mach number is stated for a given airfoil in the literature, it is usually for a small (cruising) angle of attack.

5.17 Mach angle = μ = arc sin (1/M)

$$\mu = \text{arc sin } (1/2) = 30°$$

5.18



$$\mu = \text{Sin}^{-1}\left(\frac{1}{M}\right) = \text{Sin}^{-1}\left(\frac{1}{25}\right) = 23.6°$$

$$d = h/\text{Tan}\mu = \frac{10\text{km}}{0.436} = 22.9 \text{ km}$$

5.19 At 36,000 ft, from Appendix B,

$$T_\infty = 390.5°R$$

$$\rho_\infty = 7.1 \times 10^{-4} \text{ slug/ft}^3$$

Hence,

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(390.5)} = 969 \text{ ft/sec}$$

$$V_\infty = a_\infty M_\infty = (969)(2.2) = 2132 \text{ ft/sec}$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty{}^2 = \frac{1}{2}(7.1 \times 10^{-4})(2132)^2 = 1614 \text{ lb/ft}^2$$

In level flight, the airplane's lift must balance its weight, hence

$$L = W = 16{,}000 \text{ lb.}$$

From the definition of lift coefficient,

$$C_L = L/q_\infty S = 16{,}000/(1614)(210) = 0.047$$

Assume that all the lift is derived from the wings (this is not really true because the fuselage and horizontal tail also contribute to the airplane lift.) Moreover, assume the wings can be approximated by a thin flat plate. Hence, the lift coefficient is given approximately by

$$C_L = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

Solve for α,

$$\alpha = \frac{1}{4}C_L\sqrt{M_\infty^2 - 1} = \frac{1}{4}(0.047)\sqrt{(2.2)^2 - 1}$$

$$\alpha = 0.023 \text{ \underline{radians} (or 1.2 degrees)}$$

The wave drag coefficient is approximated by

$$C_{D_w} = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} = \frac{4(0.023)^2}{\sqrt{(2.2)^2 - 1}} = 0.00108$$

Hence, $D_w = q_\infty S C_{D_w} = (1614)(210)(0.00108)$

$$D_w = 366 \text{ lb}$$

5.20 **(a)** At 50,000 ft, $\rho_\infty = 3.6391 \times 10^{-4}$ slug/ft^3 and $T_\infty = 390°R$. Hence,

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(390)} = 968 \text{ ft/sec}$$

and $$V_\infty = a_\infty M_\infty = (968)(2.2) = 2130 \text{ ft/sec}$$

The viscosity coefficient at $T_\infty = 390°R = 216.7$ K can be estimated from an extrapolation of the straight line given in Fig. 4.30. The slope of this line is

$$\frac{d\mu}{dT} = \frac{(2.12 - 1.54) \times 10^{-5}}{(350 - 250)} = 5.8 \times 10^{-8} \frac{\text{kg}}{\text{(m)(sec)(K)}}$$

Extrapolating from the sea level value of $\mu = 1.7894 \times 10^{-5}$ kg/(m)(sec), we have at $T_\infty = 216.7$ K.

$$\mu_\infty = 1.7894 \times 10^{-5} - (5.8 \times 10^{-8})(288 - 216.7)$$

$$\mu_\infty = 1.37 \times 10^{-5} \text{ kg/(m)(sec)}$$

Converting to English engineering units, using the information in Chapter 4, we have

$$\mu_\infty = \frac{1.37 \times 10^{-5}}{1.7894 \times 10^{-5}} \left(3.7373 \times 10^{-7} \frac{\text{slug}}{\text{ft sec}} \right) = 2.86 \times 10^{-7} \frac{\text{slug}}{\text{ft sec}}$$

Finally, we can calculate the Reynolds number for the flat plate:

$$\text{Re}_L = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{3.6391 \times 10^{-4}(2130)(202)}{2.86 \times 10^{-7}} = 5.47 \times 10^8$$

Thus, from Eq. (4.100) reduced by 20 percent

$$C_f = (0.8)\frac{0.07}{(\text{Re}_L)^{0.2}} = (0.8)\frac{0.074}{(5.74 \times 10^8)^{0.2}} = 0.00106$$

The wave drag coefficient is estimated from

$$c_{d,w} = \frac{4\alpha^2}{\sqrt{M_\infty{}^2 - 1}}$$

where $\alpha = \dfrac{2}{57.3} = 0.035$ rad.

Thus, $$c_{d,w} = \frac{4(0.035)^2}{\sqrt{(2.2)^2 - 1}} = 0.0025$$

Total drag coefficient = 0.0025 + (2) (0.00106) = 0.00462

Note: In the above, C_f is multiplied by two, because Eq. (4.100) applied to only one side of the flat plate. In flight, both the top and bottom of the plate will experience skin friction, hence that total skin friction coefficient is 2(0.00106) = 0.00212.

(b) If α is increased to 5 degrees, where α = 5/57.3 – 0.00873 rad, then

$$c_{d,w} = \frac{4(0.0873)^2}{\sqrt{(2.2)^2 - 1}} = 0.01556$$

Total drag coefficient = 0.01556 + 2(0.00106) = 0.0177

(c) In case (a) where the angle of attack is 2 degrees, the wave drag coefficient (0.0025) and the skin friction drag coefficient acting on both sides of the plate (2 × 0.00106 = 0.00212) are about the same. However, in case (b) where the angle of attack is higher, the wave drag coefficient (0.0177) is about eight times the total skin friction coefficient. This is because, as α increases, the strength of the leading edge shock increases rapidly. In this case, wave drag dominates the overall drag for the plate.

5.21 $V_\infty = 251\text{ km/h} = \left(251\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1\text{h}}{3600\sec}\right)\left(\dfrac{1000\text{m}}{1\text{km}}\right) = 69.7\,\text{m/sec}$

$$\rho_\infty = 1.225\text{ kg/m}^3$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty{}^2 = \frac{1}{2}(1.225)(69.7)^2 = 2976\,\text{N/m}^2$$

$$C_L = \frac{L}{q_\infty S} = \frac{9800}{(2976)(16.2)} = 0.203$$

$$C_{D_i} = \frac{C_L{}^2}{\pi e\,\text{AR}} = \frac{(0.203)^2}{\pi(0.62)(7.31)} = 0.002894$$

$$D_i = q_\infty S C_{D_i} = (2976)(16.2)(0.002894) = 139.5\,\text{N}$$

5.22 $V_\infty = 85.5\text{ km/h} = 23.75\text{ m/sec}$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty{}^2 = \frac{1}{2}(1.225)(23.75)^2 = 345\,\text{N/m}^2$$

$$C_L = \frac{L}{q_\infty S} = \frac{9800}{(345)(16.2)} = 1.75$$

$$C_{D_i} = \frac{C_L{}^2}{\pi eAR} = \frac{(1.75)^2}{\pi(0.62)(7.31)} = 0.215$$

$$D_i = q_\infty S C_{D_i} = (345)(16.2)(0.215) = 1202\text{ N}$$

Note: The induced drag at low speeds, such as near stalling velocity, is considerably larger than at high speeds, near maximum velocity. Compare the results of Problems 5.20 and 5.21.

5.23 First, obtain the infinite wing lift slope. From Appendix D for a NACA 65-210 airfoil,

$$C_\ell = 1.05 \text{ at } \alpha = 8°$$

$$C_\ell = 0 \text{ at } \alpha_{L=0} = -1.5°$$

Hence, $\quad a_0 = \dfrac{1.05 - 0}{8 - (-1.5)} = 0.11\text{ per degree}$

The lift slope for the finite wing is

$$a = \frac{a_0}{1 + \dfrac{57.3\,a_0}{\pi\, e_i \text{AR}}} = \frac{0.11}{1 + \dfrac{57.3(0.11)}{\pi(9)(5)}} = 0.076\text{ degree}$$

At $\alpha = 6°$, $C_L = a(\alpha - \alpha_{L=0}) = (0.076)[6 - (-1.5)] = 0.57$

The total drag coefficient is

$$C_D = c_d + \frac{C_L{}^2}{\pi\, e\text{AR}} = (0.004) + \frac{(0.57)^2}{\pi(0.9)(5)}$$

$$C_D = 0.004 + 0.023 = 0.027$$

5.24 $q_\infty = \frac{1}{2}\rho_\infty V_\infty{}^2 = \frac{1}{2}(0.002377)(100)^2 = 11.9 \text{ lb/ft}^2$

at $\alpha = 10°$, $L = 17.9$ lb. Hence

$$C_L = \frac{L}{q_\infty S} = \frac{17.9}{(11.9)(1.5)} = 1.0$$

at $\alpha = -2°$, $L = 0$ lb. Hence $\alpha_{L=0} = -2°$

$$a = \frac{dC_L}{d\alpha} = \frac{1.0 - 0}{[10 - (-2)]} = 0.083 \text{ per degree}$$

This is the finite wing lift slope.

$$a = \frac{a_0}{1 + \dfrac{57.3\,a_0}{\pi e\,\text{AR}}}$$

Solve for a_o.

$$a_0 = \frac{a}{1 - \dfrac{57.3\,a}{\pi e\,\text{AR}}} = \frac{0.083}{1 + \dfrac{57.3(0.083)}{\pi\,(0.96)(6)}}$$

$a_0 = 0.11$ per degree

5.25 At $\alpha = -1°$, the lift is zero. Hence, the total drag is simply the profile drag.

$$C_D = c_d + \frac{C_L{}^2}{\pi e \text{AR}} = c_d + 0 = c_d$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(130)^2 = 20.1 \text{ lb/ft}^2$$

Thus, at $\alpha = \alpha_{L=0} = -1°$

$$c_d = \frac{D}{q_\infty S} = \frac{0.181}{(20.1)(1.5)} = 0.006$$

At $\alpha = 2°$, assume that c_d has not materially changed, i.e., the "drag bucket" of the profile drag curve (see Appendix D) extends at least from $-1°$ to $2°$, where c_d is essentially constant. Thus, at $\alpha = 2°$,

$$C_L = \frac{L}{q_\infty S} = \frac{5}{(20.1)(1.5)} = 0.166$$

$$C_D = \frac{D}{q_\infty S} = \frac{0.23}{(20.1)(1.5)} = 0.00763$$

However:

$$C_D = c_d + \frac{C_L{}^2}{\pi e \text{AR}}$$

$$0.00763 = 0.006 + \frac{(0.166)^2}{\pi\, e(6)} = 0.006 + \frac{0.00146}{e}$$

$$e = 0.90$$

To obtain the lift slope of the airfoil (infinite wing), first calculate the finite wing lift slope.

$$a = \frac{(0.166 - 0)}{[2 - (-2)]} = 0.055 \text{ per degree}$$

$$a_0 = \frac{a}{1 - \dfrac{57.3a}{\pi e \text{AR}}} = \frac{0.055}{1 - \dfrac{57.3(0055)}{\pi(0.9)(6)}}$$

$$a_0 = 0.068 \text{ per degree}$$

5.26 $$V_{\text{stall}} = \sqrt{\frac{2W}{\rho_\infty S\, C_{L_{\max}}}} = \sqrt{\frac{2(7780)}{(1.225)(16.6)(2.1)}}$$

$$V_{\text{stall}} = 19.1 \text{ m/sec} = 68.7 \text{ km/h}$$

5.27 **(a)** $\alpha = \dfrac{5}{57.3} = 0.087$ radians

$$c_\ell = 2\pi\alpha = 2\pi(0.087) = 0.548$$

(b) Using the Prandtl-Glauert rule,

$$c_\ell = \frac{c_{\ell_0}}{\sqrt{1 - M_\infty{}^2}} = \frac{0.548}{\sqrt{1 - (0.7)^2}} = 0.767$$

(c) From Eq. (5.50)

$$c_\ell = \frac{4\alpha}{\sqrt{M_\infty{}^2 - 1}} = \frac{4(0.087)}{\sqrt{(2)^2 - 1}} = 0.2$$

5.28 For $V_\infty = 21.8$ ft/sec at sea level

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty{}^2 = \frac{1}{2}(0.002377)(21.8)^2 = 0.565 \text{ lb/ft}^2$$

1 ounce = 1/16 lb = 0.0625 lb.

$$C_L = \frac{L}{q\infty S} = \frac{0.0625}{(0.565)(1)} = 0.11$$

For a flat plate airfoil

$$c_\ell = 2\pi\alpha = 2 \cdot \pi(3/57.3) = 0.329$$

The difference between the higher value predicted by thin airfoil theory and the lower value measured by Cayley is due to the low aspect ratio of Cayley's test wing, and viscous effects at low Reynolds number.

5.29 From Eqs. (5.1) and (5.2), writtten in coefficient form

$$C_L = C_N \cos\alpha - C_A \sin\alpha$$
$$C_D = C_N \sin\alpha + C_A \cos\alpha$$

Hence:

$$C_L = 0.8\cos 6^\circ - 0.06\sin 6^\circ = 0.7956 - 0.00627 = 0.789$$
$$C_D = 0.8\sin\alpha + 0.06\cos\alpha = 0.0836 + 0.0597 = 0.1433$$

Note: At the relatively small angles of attack associated with normal airplane flight, C_L and C_N are essentially the same value, as shown in this example.

5.30 First solve for the angle of attack and the profile drag coefficient, which stay the same in this problem

$$c_L = a\alpha = \frac{a_0\,\alpha}{1 + 57.3\,a_0/(\pi e_1\ \mathrm{AR})}$$

or,
$$\alpha = \frac{C_L}{a_0}[1 + 57.3\,a_0/\pi e_1\,\mathrm{AR}]$$

$$= \frac{0.35}{0.11}\{1 + 57.3(0.11)/[\pi(0.9)(7)]\} = 4.2°$$

The profile drag can be obtained as follows

$$C_D = \frac{C_L}{(C_L/C_D)} = \frac{0.35}{29} = 0.012$$

$$C_D = c_d + \frac{C_L^2}{\pi e\ \mathrm{AR}}$$

or,
$$c_d = C_D - \frac{{C_L}^2}{\pi e\,\mathrm{AR}} = 0.012 - \frac{(0.35)^2}{\pi(.9)(7)} = 0.0062$$

Increasing the aspect ratio at the same angle of attack increases C_L and reduces C_D.
For AR = 10, we have

$$C_L = a\alpha = \frac{a_0\,\alpha}{1 + 57.3\,a_0/(\pi e_1\ \mathrm{AR})}$$

$$= \frac{(0.11)(4.2)}{1 + 57.3(0.11)/[\pi(0.9)(10)]} = 0.3778$$

$$C_D = c_d + \frac{{C_L}^2}{\pi e\ \mathrm{AR}} = 0.062 - \frac{(0.3778)^2}{\pi(.9)(10)} = 0.0062 + 0.005048 = 0.112$$

Hence, the new value of L/D is $\dfrac{C_L}{C_D} = \dfrac{0.3778}{0.0112} = 33.7$

5.31 For incompressible flow, from Bernoulli's equation, the stagnation pressure is

$$p_0 = p_\infty + \tfrac{1}{2}\rho_\infty V_\infty^2$$

Let S be the surface area of each face of the plate. The aerodynamic force exerted on the front face is

$$F_{\text{front}} = p_0 S$$

and the aerodynamic force on the back face is

$$F_{\text{back}} = p_\infty S$$

The net force, which is the drag, is

$$D = F_{\text{front}} - F_{\text{back}} = (p_0 - p_\infty)S$$

From Bernoulli's equation,

$$p_0 - p_\infty = \tfrac{1}{2}\rho_\infty V_\infty^2$$

Thus,

$$D = \tfrac{1}{2}\rho_\infty V_\infty{}^2 S$$

$$C_D = \frac{D}{\frac{1}{2}\rho_\infty V_\infty{}^2 S} = \frac{\frac{1}{2}\rho_\infty V_\infty{}^2 S}{\frac{1}{2}\rho_\infty V_\infty{}^2 S}$$

Hence,

$$\boxed{C_D = 1}$$

5.32 The drag of the airplane is

$$D = q_\infty S C_D \tag{1}$$

The drag of a flat plate at 90° to the flow, with an area f and drag coefficient of 1, is

$$D = q_\infty f C_{D_{\text{pblc}}} = q_\infty f(1) \tag{2}$$

Equating Eqs. (1) and (2)

$$q_\infty S C_D = q_\infty f$$

or,

$$\boxed{f = C_D S}$$

5.33 $f = C_D S = (0.0163)(233) = \boxed{3.8\,\text{ft}^2}$

5.34 From Appendix D, for the NACA 2412 airfoil at zero angle of attack, $C_d = 0.006$. This is the profile drag coefficient, the sum of skin friction drag and pressure drag due to flow separation. To estimate the skin friction drag coefficient, from Eq. (4.101)

$$C_{d,f} = \frac{0.074}{\mathrm{Re}_L^{0.2}} = \frac{0.074}{(8.9 \times 10^6)^{0.2}} = 0.023$$

This is the skin friction coefficient due to flows over only one side of the plate. Including both the top and bottom surfaces of the plate,

$$c_{d,f} = 2(0.003) = 0.006$$

Since from Section 5.12

$$c_d = c_{d,f} + c_{d,p}$$

we have: $c_{d,p} = c_d - c_{d,f} = 0.006 - 0.006 = 0.$ For this case, the pressure drag due to flow separation is negligible.

5.35 From Appendix D, for the NACA 2412 airfoil at an angle of attack of 6°,

$$c_d = 0.0075$$

Assuming, as in Problem 5.34, that the skin friction drag is given by the flat plate result at zero angle of attack, we have

$$c_{d,f} = 0.006$$

Thus,

$$c_{d,p} = c_d - c_{d,f} = 0.0075 - 0.006 = 0.0015$$

For this case, the percentage of drag due to pressure drag is

$$\frac{c_{d,p}}{c_d} = \frac{0.0015}{0.0075} = 0.2, \text{or } 20\%$$

Clearly, as the angle of attack of the airfoil increases, the region of separated flow grows larger, and the pressure drag due to flow separation increases. The large increase in c_d at higher values of c_l, hence a, seen in Appendix D is due almost entirely to an increase in pressure drag due to flow separation.

5.36 In Problem 5.34, the total turbulent skin friction drag coefficient along one surface of the airfoil (top or bottom) was calculated to be 0.003. The transition Reynolds number is 500,000. This implies that the turbulent skin friction coefficient based on the running length of surface from the leading edge to the transition point, x_{cr}, is

$$(c_{d,f})_{x_{cr}} = \frac{0.074}{(\mathrm{Re}_{x,cr})^{0.2}} = \frac{0.074}{(500,000)^{0.2}} = \frac{0.074}{13.8} = 0.00536$$

The contribution of this turbulent skin friction drag coefficient to the total turbulent skin friction drag coefficient for the complete surface calculated to be 0.003 in Problem 5.34 is

$$0.00536\left(\frac{500,000}{8.9 \times 10^6}\right) = 3.01 \times 10^{-4}$$

Hence, the contribution to the overall skin friction drag coefficient for one surface of the airfoil due to the turbulent flow from the transition point to the trailing edge of the airfoil is:

$$(c_{d,f})_{\mathrm{turb}} = 0.003 - 3.01 \times 10^{-4} = 0.0027$$

The laminar skin friction drag coefficient for the flow from the leading edge to the transition point is, from Eq. (4.98)

$$C_{f,\,\mathrm{laminar}} \frac{1.328}{\sqrt{\mathrm{Re}_{x,cr}}} = \frac{1.328}{\sqrt{500,000}} = \frac{1.328}{707.1} = 1.878 \times 10^{-3}$$

Since the laminar skin friction is acting only on that part of the surface from $x = 0$ to $x = x_{cr}$, then its contribution to the total overall skin friction coefficient for the whole top surface is

$$(c_{d,f})_{\mathrm{lam}} = 1.878 \times 10^{-3}\left(\frac{500,000}{8.9 \times 10^6}\right) = 1.055 \times 10^{-4}$$

Therefore, the total skin friction coefficient for one surface, including both the laminar flow from $x = 0$ to x_{cr}, and turbulent flow from x_{cr} to the trailing edge, is

$$c_{d,f} = 1.055 \times 10^{-4} + 0.0027 = 0.0028$$

Including both the upper and lower surfaces, then we have

$$c_{d,f} = 2(0.0028) = 0.0056$$

$$c_d = c_{d,f} + c_{d,p}$$

Thus, $\quad c_{d,p} = c_d - c_{d,f} = 0.006 - 0.0056 = 0.004$

The percentage of the total drag due to pressure drag due to flow separation is then

$$\frac{c_{d,p}}{c_d} = \frac{0.0004}{0.006} = (100) = 6.7\%$$

In contrast to the result obtained in the solution to Problem 5.34, the more realistic result obtained here shows that the pressure dag is not negligible, although it is small compared to the skin friction drag.

Note: This problem is similar to worked Example 4.28 in the text. In Example 4.28, the respective skin friction drag forces on various parts of the surface are calculated and are used to solve the problem. In our solution here for Problem 5.36, we have used only the various skin friction coefficients to solve the problem. We do not need to calculate forces. Indeed, in the statement of the problem there is no velocity, density, or surface area given, so we can not calculate forces; none is needed. However, we have been careful in the present calculation to account for the specific regions of the surface to which the calculated coefficients apply, and to multiply the coefficients obtained over the portion of surface between $x = 0$ and x_{cr} from Eqs. (4.98) and (4.101) by the ratio $x_{cr} / L = 500,000 / 8.9 \times 10^6 = 0.0056$.

It is instructive to work out Example 4.28 in the text using only the various skin friction coefficients as we have used here, and then to calculate total drag D_f at the very end of the calculation. You get the same answer as was obtained in Example 4.28.

5.37 **(a)** From Eq. (4.98),

$$C_f = \frac{1.328}{\sqrt{\text{Re}}} = \frac{1.328}{\sqrt{9\times10^6}} = 4.43\times10^{-4}$$

This is for one side of the flat plate. The total skin friction drag coefficient including both sides of the plate is

$$C_f = 2(4.43\times10^{-4}) = \boxed{0.000885}$$

(b) From Eq. (4.101)

$$C_f = \frac{0.074}{(\text{Re})^{0.2}} = \frac{0.074}{(9\times10^6)^{0.2}} = \frac{0.074}{24.59} = 0.003$$

For both sides of the plate,

$$C_f = 2(0.003) = \boxed{0.006}$$

The section drag coefficient for the NACA 2415 airfoil from App. D is 0.0064.

If the boundary layer were laminar, then the fraction of drag due to friction is (0.000885/0.0064) = 0.138, or 13%. If the boundary layer were turbulent, then the fraction of drag due to friction is 0.006/0.0064 = 0.938, or 93.8%. These results highlight the large effect of a turbulent boundary on the drag of an airfoil as compared to a laminar boundary layer.

5.38 (See Section 4.19 for the solution technique.)

Assuming all turbulent flow,

$$C_f = 0.006 \text{ (from Problem 5.37).}$$

Transition takes place at location

$$\frac{x_{\text{cr}}}{c} = \frac{650{,}000}{9\times10^6} = 0.072$$

The turbulent drag coefficient on the area upstream of transition would be

$$C_f = \left[\frac{0.074}{(650{,}000)^{0.2}}\right]2 = \left(\frac{0.074}{14.54}\right)2 = 0.0102$$

Thus, the relative turbulent drag contribution due to the area downstream of the transition point is

$$0.006-(0.0102)(0.072) = 0.006-0.000734 = 0.005266$$

The <u>laminar</u> drag coefficient on the area upstream of transition is

$$C_f = \left[\frac{1.328}{\sqrt{650{,}000}}\right]2 = \left(\frac{1.328}{806}\right)2 = 0.0033$$

The laminar drag contribution over this area is

$$(0.0033)(0.072) = 0.000238$$

The total skin friction drag coefficient is

laminar contribution + turbulent contribution =

$$0.000238+0.005266 = \boxed{0.0055}$$

The total section drag coefficient is 0.0064 from App. D. Thus,

$$\text{Friction drag percentage} = \left(\frac{0.0055}{0.0064}\right)100 = \boxed{86\%}$$

$$\text{Pressure drag percentage} = \boxed{14\%}$$

5.39 Assuming all turbulent flow,

$$C_f = \frac{0.074}{(\mathrm{Re})^{0.2}} = \frac{0.074}{(3\times 10^6)^{0.2}} = 0.003749$$

Taking into account both sides of the plate

$$C_f = (0.003749)\ 2 = 0.0075$$

Transition takes place at location

$$\frac{x_{\mathrm{cr}}}{c} = \frac{650,000}{3\times 10^6} = 0.2167$$

The turbulent drag coefficient on the area upstream of transition would be

$$C_f = \left[\frac{0.074}{(650,000)^{0.2}}\right] 2 = 0.0102$$

The turbulent drag contribution due to the area downstream of the transition point is

$$0.0075 - (0.0102)(0.2167) = 0.00529$$

The laminar drag coefficient on the area upstream of transition is

$$C_f = \left[\frac{1.328}{\sqrt{650,000}}\right] 2 = 0.0033$$

The laminar drag contribution over this area is

$$(0.0033)(0.2167) = 0.000715$$

The total skin friction drag coefficient is

$$0.000715 + 0.00529 = \boxed{0.0060}$$

The experimental data for the total section drag coefficient from App. D is 0.0068.

$$\text{Friction drag percentage} = \left(\frac{0.006}{0.0068}\right)100 = \boxed{88\%}$$

$$\text{Pressure drag percentage} = \boxed{11.8\%}$$

Chapter 6

6.1 **(a)** $V_\infty = 350\,\text{km/hr} = 97.2\,\text{m/sec}$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(97.2)^2 = 5787\,\text{N/m}^2$$

$$C_L = \frac{W}{q_\infty S} = \frac{38220}{(5787)(27.3)} = 0.242$$

$$C_D = C_{D_0} + \frac{C_L{}^2}{\pi e\text{AR}} = 0.03 + \frac{(0.242)^2}{\pi(0.9)(75)}$$

$$C_D = 0.03 + 0.0028 = 0.0328$$

$$C_L / C_D = 0.242 / 0.0328 = 7.38$$

$$T_R = \frac{W}{C_L/C_D} = \frac{38,220}{7.38} = 5179\,\text{N}$$

(b) $$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.777)(97.2)^2 = 3670\,\text{N/m}^2$$

$$C_L = \frac{W}{q_\infty S} = \frac{38,220}{(3670)(27.3)} = 0.38$$

$$C_d = C_{D_0} + \frac{C_L{}^2}{\pi e\text{AR}} = 0.03 + \frac{(0.38)^2}{\pi(0.9)(7.5)}$$

$$C_D = 0.03 + 0.0068 = 0.0368$$

$$C_L / C_D = 0.38 / 0.0368 = 10.3$$

$$T_R = \frac{W}{C_L/C_D} = \frac{38,220}{10.3} = 3711\,\text{N}$$

6.2 $V_\infty = 200\frac{88}{60} = 293.3\,\text{ft/sec}$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(293.3)^2 = 102.2\,\text{lb/ft}^2$$

$$C_L = \frac{L}{q_\infty S} = \frac{W}{q_\infty S} = \frac{5000}{(102.2)(200)} = 0.245$$

$$C_{D_i} = \frac{C_L{}^2}{\pi e\,\text{AR}} = \frac{(0.245)}{\pi(0.93)(8.5)} = 0.0024$$

Since the airplane is flying at the condition of maximum L/D, hence minimum thrust required, $C_{D_i} = C_{D_e}$. Thus,

$$C_D = C_{D_0} + C_{D_i} = 2C_{D_i} = 2(0.0024) = 0.1048$$
$$D = q_\infty S C_D = (102.2)(200)(0.0048) = 98.1\,\text{lb.}$$

6.3 **(a)** Choose a velocity, say V_∞ =100 m/sec

$$q_\infty \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(100)^2 = 6125\,\text{N/m}^2$$

$$C_L = \frac{W}{q_\infty S} = \frac{103{,}047}{(6125)(47)} = 0.358$$

$$C_D = C_{D_0} + \frac{C_L{}^2}{\pi e \text{AR}} = 0.032 + \frac{(0.358)^2}{\pi(0.87)(6.5)}$$

$$C_D = 0.032 + 0.007 = 0.0392$$

$$T_R = \frac{W}{C_L / C_D} = \frac{103{,}047}{9.13} = 11287\,\text{N}$$

$$P_R = T_R V_\infty = (11{,}287)(100) = 1.129 \times 10^6 \text{ watts}$$

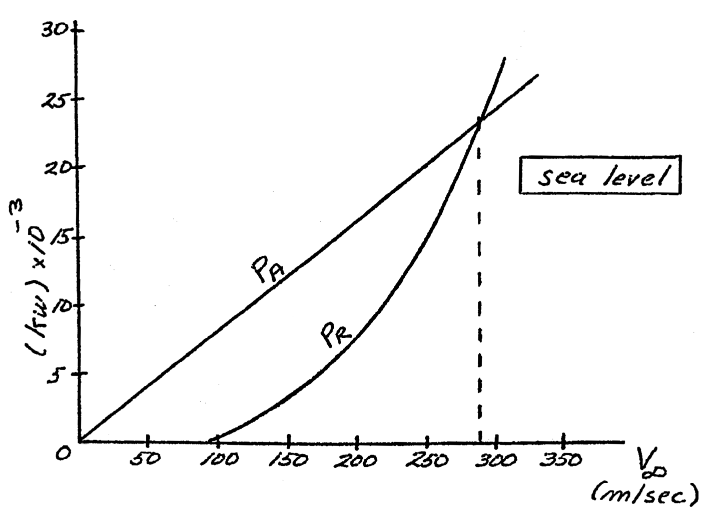
$$P_R = 1129\,\text{kw}$$

A tabulation for other velocities follows on the next page:

V (m/sec)	C_L	C_D	C_L/C_D	P_R (kw)
100	0.358	0.0392	9.13	1129
130	0.212	0.0345	6.14	2182
160	0.140	0.0331	4.23	3898
190	0.099	0.0325	3.05	6419
220	0.074	0.0323	2.29	9900
250	0.057	0.0322	1.77	14,550
280	0.046	0.0321	1.43	20,180
310	0.037	0.0321	1.15	27,780

(b) $P_A = T_A V_\infty = (2)(40298)V_\infty = 80596 V_\infty$

The power required and power available curves are plotted below.



From the intersection of the P_A and P_R curves, we find,

$V_{\max}$ = 295 m/sec at sea level

(c) At 5 km standard altitude, $\rho = 0.7364\,\text{kg/m}^3$

Hence,

$$(\rho_0/\rho)^{1/2} = (1.225/0.7364)^{1/2} = 1.29$$

$$V_{\text{alt}} = (\rho_0/\rho)^{1/2} V_0 = 1.29\,V_0$$

$$P_{R_{\text{alt}}} = (\rho_0/\rho)^{1/2} P_{R_0} = 1.29\,P_{R_0}$$

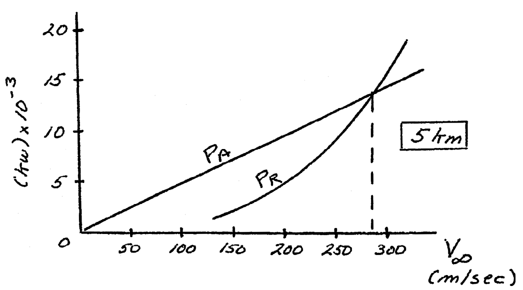
From the results from part (a) above,

V_0 (m/sec)	P_{R_0} (kw)	V_{alt} (m/sec)	$P_{R_{ak}}$ (kw)
100	1129	129	1456
130	2182	168	2815
160	3898	206	5028
190	6419	245	8281
220	9900	284	12,771
250	14,550	323	18,770

(d) $T_{A_{\text{all}}} = T_{A_0}\left(\dfrac{\rho}{\rho_0}\right) = 0.601 T_{A_0}$

Hence, $\quad P_{A_{\text{alt}}} = T_{A_{\text{alt}}} V_\infty = (0.601)(80596)V_\infty = 48438\,V_\infty$

The power required and power available curves are plotted below.



From the intersection of the P_R and P_A curves, we find $V_{max} = 290\,\text{m/sec}$ at 5 km

Comment: The mach numbers corresponding to the maximum velocities in parts (b) and (d) are as follows:

At sea level

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(287)(288.16)} = 340\,\text{m/sec}$$

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{295}{340} = 0.868$$

At 5 km altitude

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(287)(255.69)} = 321\,\text{m/sec}$$

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{290}{321} = 0.90$$

These mach numbers are slightly larger than what might be the actual drag-divergence Mach number for an airplane such as the A-10. Our calculations have not taken the large drag rise at drag-divergence into account. Hence, the maximum velocities calculated above are somewhat higher than reality.

6.4 **(a)** Choose a velocity, say $V_\infty = 100\,\text{ft/sec}$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty{}^2 = \frac{1}{2}(0.002377)(100)^2 = 11.89\,\text{lb/ft}^2$$

$$C_L = \frac{W}{q_\infty S} = \frac{3000}{(11.89)(181)} = 1.39$$

$$C_D = C_{D_0} + \frac{C_L{}^2}{\pi e\,\text{AR}} = 0.027 + \frac{(1.39)^2}{\pi(0.91)(6.2)}$$

$$C_D = 0.027 + 0.109 = 0.136$$

$$T_R = \frac{W}{C_L/C_D} = \frac{3000}{(1.39)/(0.136)} = \frac{3000}{10.22} = 293.5\,\text{lb}$$

$$P_R = T_R \cdot V_\infty = (293.5)(100) = 29350\,\text{ft lb/sec}$$

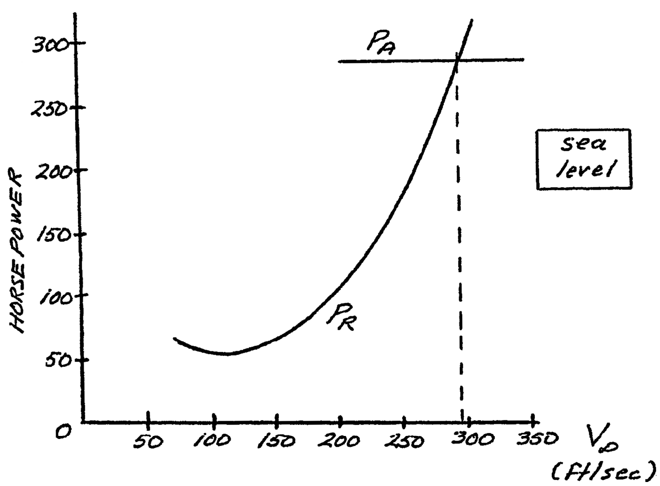
In terms of horsepower,

$$P_R = \frac{29350}{550} = 53.4\,\text{hp}$$

A tabulation for other velocities follows:

V_∞ (ft/sec)	C_L	C_D	C_L/C_D	P_R (hp)
70	2.85	0.485	5.88	64.9
100	1.39	0.136	10.22	53.4
150	0.62	0.0487	12.37	64.3
200	0.349	0.0339	10.29	106
250	0.223	0.0298	7.48	182
300	0.155	0.0284	5.46	300
350	0.114	0.0277	4.12	463

(b) At sea level, maximum $P_A = 0.83\ (345) = 286$ hp. The power required and power available are plotted below.



From the intersection of the P_A and P_R curves,

$$V_{\text{max}} = 295\,\text{ft/sec} = 201\,\text{mph at sea level}$$

(c) At a standard altitude of 12,000 ft,

$$\rho = 1.648 \times 10^{-3}\,\text{slug/ft}^3$$

Hence,

$$(\rho_0/\rho)^{1/2} = (0.002377\,/\,0.001648)^{1/2} = 1.2$$

$$V_{\text{alt}} = (\rho_0/\rho)^{1/2} V_0 = 1.2V_0$$

$$P_{R_{\text{ak}}} = (\rho_0/\rho)^{1/2} P_{R_0} = 1.2P_{R_0}$$

Using the results from part (a) above,

V_0 (ft/sec)	P_{R_0} (hp)	V_{alt} (ft/sec)	$P_{R_{\text{ah}}}$ (hp)
70	64.9	84	77.9
100	53.4	120	64.1
150	64.3	180	76.9
200	106	240	127
250	182	300	218
300	300	360	360

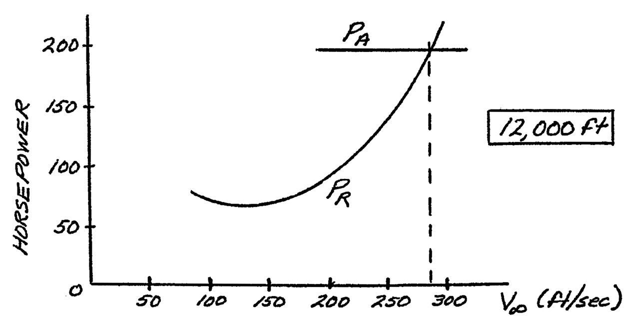
(d) Assuming that the power output of the engine is proportional to p_∞,

$$P_{A_{\text{alt}}} = (\rho/\rho_0)\,P_{A\,0} = \left(\frac{0.001648}{0.002377}\right)P_{A_0} = 0.693\,P_{A_0}$$

At 12,000 ft,

$$P_A = 0.693(286) = 198\,\text{hp}$$

The power required and power available curves are plotted below.



From the intersection of the P_A and P_R curves,

$$\boxed{V_{\max} = 290\,\text{ft/sec} = 198\,\text{mph}}\ \text{at}\,12{,}000\,\text{ft.}$$

6.5 From the P_A and P_B curves generated in Problem 6.3, we find approximately:

excess power = 9000 kw at sea level
excess power = 5000 kw at 5 km

Hence, at sea level

$$\text{R/C} = \frac{\text{excess power}}{W} = \frac{9 \times 10^6\ \text{watts}}{1.0307 \times 10^5\,\text{N}} = 87.3\,\text{m/sec}$$

and at 5 km altitude,

$$\text{R/C} = \frac{5 \times 10^6}{1.03047 \times 10^5} = 48.5\,\text{m/sec}$$

6.6 From the P_A and P_R curves generated in Problem 6.4, we find approximately:

excess power = 232 hp at sea level
excess power = 134 hp at 12,000 ft

Hence, at sea level,

$$\text{R/C} = \frac{\text{excess power}}{W} = \frac{(232)(550)}{3000} = 42.5\,\text{ft/sec}$$

and at 12,000 ft altitude,

$$\text{R/C} = \frac{(134)(550)}{3000} = 24.6\,\text{ft/sec}$$

6.7 Assuming $(\text{R/C})_{\max}$ varies linearly with altitude,

$$(\text{R/C})_{\max} = ah + b$$

From the two $(\text{R/C})_{\max}$ values from Problem 6.5,

$$87.3 = a(\text{o}) + b$$
$$48.5 = a(5000) + b$$

Hence,

$$b = 87.3$$
$$a = -0.00776$$
$$(\text{R/C})_{\max} = -0.00776h + 87.3$$

To obtain the absolute ceiling, set $(\text{R/C})_{\max} = 0$, and solve for h.

$$h = \frac{87.3}{0.00776} = 11{,}250\,\text{m}$$

Hence,

absolute ceiling ≈ 11.3 km

6.8 $(\text{R/C})_{\max} = ah + b$

From the results of Problem 6.6,

$$42.5 = a(0) + b$$
$$24.6 = a(12{,}000) + b$$

Hence,

$$b = 42.5$$
$$a = -0.00149$$
$$(\text{R/C})_{\max} = -0.00149h + 42.5$$

To obtain the absolute ceiling, set $(\text{R/C})_{\max} = 0$, and solve for h.

$$h = \frac{42.5}{0.00149} = 28{,}523\,\text{ft}$$

absolute ceiling ≈ 28,500 ft

COMMENT TO THE INSTRUCTOR

In the previous problems dealing with a performance analysis of the twin-jet and single-engine piston airplanes, the answers will somewhat depend on the precision and number of calculations made by the student. For example, if the P_R curve is constructed from 30 points instead of the six or eight points as above, the subsequent results for rate-of-climb and absolute ceiling will be more accurate than obtained above. Some leeway on the students' answers is therefore advised. In my own experience, I am glad when the students fall within the same ballpark.

6.9 $R = h(L/D)_{\max} = 5000(7.7) = 38{,}500\,\text{ft} = 729\,\text{miles}$

6.10 First, we have to calculate the C_L corresponding to maximum L/D.

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{\sqrt{C_{D,O}\pi e\,\text{AR}}}{2C_{D,0}} = \sqrt{\frac{\pi e\,\text{AR}}{4C_{D,0}}}$$

$$C_{D,0} = \frac{1}{4}\left(\frac{C_L}{C_D}\right)_{\max}^{-2} \pi \text{e}\,\text{AT} = \frac{(0.25)\pi(0.7)(4.11)}{(7.7)^2} = 0.038$$

At maximum $L/D, C_{D,0} = C_{D,\text{i}} = 0.038$

Hence: $C_D = 2(0.038) = 0.076$

$$C_L = C_D\left(\frac{C_L}{C_D}\right) = (0.076)(7.7) = 0.585$$

Also:

$$\theta_{\min} = \text{Arc}\,\text{Tan}\left(\frac{L}{D}\right)_{\max}^{-1} = \text{Arc}\,\text{Tan}\,(0.13)$$

$$\theta_{\min} = 7.4°$$

$$V_\infty = \sqrt{\frac{2\cos\theta}{\rho_\infty C_L}\left(\frac{W}{S}\right)} = \sqrt{\frac{2\cos(7.4°)}{(0.002175)(0.585)}\left(\frac{1400}{231}\right)} = 97.2\,\text{ft/sec}$$

6.11 From Eq. (6.85),

$$\left(\frac{L}{D}\right)_{\max} = \frac{\left(\pi e \text{AR} C_{D_0}\right)^{t/2}}{2C_{D_0}}$$

Putting in the numbers:

$$\left(\frac{L}{D}\right)_{\max} = \frac{\left[\pi(0.9)(6.72)(0.025)\right]^{1/2}}{0.050} = 13.78$$

6.12 Aviation gasoline weighs 5.64 Ib per gallon,
Hence,

$$W_F = (44)(5.64) = 248\,\text{lb}.$$

Thus, the empty weight is

$$W_1 = 3400 - 248 = 3152\,\text{lb}.$$

The specific fuel consumption, in consistent units, is

$$c = 0.42/(550)(3600) = 2.12 \times 10^{-7}\,\text{ft}^{-1}$$

The maximum L/D can be found from Eq. (6.85).

$$\left(\frac{L}{D}\right)_{\max} = \frac{(\pi e \,\text{AR}\, C_{D_c})^{1/2}}{2C_{D_c}} = \frac{\left[\pi(0.91)(6.2)(0.027)\right]^{1/2}}{2(0.027)} = 12.8$$

Thus, the maximum range is:

$$R = \left(\frac{\eta}{c}\right)\left(\frac{C_L}{C_D}\right)_{\max} \ell n\left(\frac{W_0}{W_1}\right)$$

$$R = \left(\frac{0.83}{2.12 \times 10^{-7}}\right)(12.8)\ \ell n\left(\frac{3400}{3152}\right)$$

$$R = 3.8 \times 10^6\,\text{ft}$$

In terms of miles,

$$R = \frac{3.8 \times 10^6}{5280} = 719\,\text{miles}$$

To calculate endurance, we must first obtain the value of $(C_L{}^{3/2}/C_D)_{\max}$ From Eq. (6.87),

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{\max} = \frac{(3C_{D_0}\pi e\,\text{AR})^{3/4}}{4C_{D_c}}$$

Putting in the numbers:

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{\max} = \frac{\left[3(0.027)\,\pi\,(0.91)(6.2)\right]^{1/2}}{4(0.027)} = 11.09$$

Hence, the endurance is

$$E = \left(\frac{\eta}{c}\right)\frac{C_L^{3/2}}{C_D}(2\rho_\infty S)^{1/2}(W_1^{-1/2} - W_0^{-1/2})$$

$$E = \left(\frac{0.83}{2.12 \times 10^{-7}}\right)(11.09)\left[2(0.002377)(181)\right]^{1/2}(3152^{-1/2} - 3400^{-1/2})$$

$$E = 2.67 \times 10^4\,\text{sec} = 7.4\,\text{hr}$$

6.13 One gallon of kerosene weighs 6.67 Ib. Since 1 Ib = 4.448 N, then one gallon of kerosene also weighs 29.67 N. Thus,

$$W_f = (1900)(29.67) = 56{,}370 \text{ N}$$
$$W_t = W_0 - W_f = 136{,}960 - 56{,}370 = 80{,}590 \text{ N}$$

In consistent units,

$$c_t = 1.0 \frac{\text{N}}{(\text{N})(\text{hr})} = 2.777 \times 10^{-4} \text{sec}^{-1}$$

Also, at a standard altitude of 8 km.

$$\rho_\infty = 0.526 \text{ kg/m}^3$$

Since maximum range for a jet air craft depends upon maximum $C_L^{1/2}/C_D$, we must use Eq. (6.86).

$$\left(\frac{C_L^{1/2}}{C_D}\right)_{\max} = \frac{\left(\frac{1}{3} C_{D_c} \pi e \text{AR}\right)^{1/4}}{\frac{4}{3} C_{D_c}}$$

Putting in the numbers,

$$\left(\frac{C_L^{1/2}}{C_D}\right)_{\max} = \frac{\left[\frac{1}{3}(0.032)(0.87)(6.5)\right]^{1/4}}{\frac{4}{3}(0.032)} = 15.46$$

For a jet airplane, the range is

$$R = 2\sqrt{\frac{2}{\rho_\infty S}}\left(\frac{1}{c_t}\right)\left(\frac{C_L^{1/2}}{C_D}\right)(W_0^{1/2} - W_1^{1/2})$$
$$R = 2\left[\frac{2}{(0.526)(47)}\right]^{1/2}\left(\frac{1}{2.777 \times 10^{-4}}\right)(15.46)\left[(136{,}960)^{1/2} - (80{,}590)^{1/2}\right]$$
$$R = 2.73 \times 10^6 \text{m} = 2730 \text{km}$$

The endurance depends on $C_L C_D$. From Eq. (6.85),

$$\left(\frac{L}{D}\right)_{\max} = \frac{(\pi e \text{AR}\, C_{D_c})^{1/2}}{2\, C_{D_c}} = \frac{[\pi\,(0.87)(6.5)(0.032)]^{1/2}}{2\,(0.032)} = 11.78$$

The endurance for a jet aircraft is

$$E = \frac{1}{c_t}\left(\frac{C_L}{C_D}\right)\ell n\left(\frac{W_0}{W_1}\right)$$
$$E = \frac{1}{2.777 \times 10^{-4}}(11.78)\ \ell n\left(\frac{136{,}960}{80{,}590}\right)$$
$$E = 22{,}496 \text{ sec} = 6.25 \text{ hr}$$

6.14 $$\frac{C_L^{3/2}}{C_D} = \frac{C_L^{3/2}}{C_{D_0} + \frac{C_L^2}{\pi e\,\mathrm{AR}}}$$

$$\frac{d(C_L^{3/2}/C_D)}{d\,C_L} = \left[\left(C_{D_0}\frac{C_L^2}{\pi\,e\,\mathrm{AR}}\right)\left(\frac{3}{2}C_L^{\frac{1}{2}}\right) - C_L^{3/2}\left(2\frac{C_L}{\pi\,e\,\mathrm{AR}}\right)\right]\left(C_{D_0} + \frac{C_L^2}{\pi\,e\,\mathrm{AR}}\right)^{-2} = 0$$

$$\frac{3}{2}C_L^{\frac{1}{2}}C_{D_0} - \frac{1}{2}\frac{C_L^{5/2}}{\pi\,e\,\mathrm{AR}} = 0$$

$$3\,C_{D_0} - \frac{C_L^2}{\pi e\,\mathrm{AR}} = 0$$

Hence, $C_{D_0} = \frac{1}{3}C_{D_i}$ *This is Eq. (6.80)*

To obtain Eq. (6.81)

$$\frac{C_L^{1/2}}{C_D} = \frac{C_L^{1/2}}{C_{D_0} + \frac{C_L^2}{\pi e\,\mathrm{AR}}}$$

$$\frac{d(C_L^{1/2}/C_D)}{d\,C_L} = \left[\left(C_{D_0}\frac{C_L^2}{\pi e\,\mathrm{AR}}\right)\frac{1}{2}C_L^{-\frac{1}{2}} - C_L^{-\frac{1}{2}}\left(\frac{2\,C_L}{\pi e\,\mathrm{AR}}\right)\right]\left(C_{D_0} + \frac{C_L^2}{\pi\,e\,\mathrm{AR}}\right)^{-2} = 0$$

$$\frac{1}{2}C_{D_0}C_L^{-\frac{1}{2}} - \frac{3}{2}\frac{C_L^{5/2}}{\pi e\,\mathrm{AR}} = 0$$

$$C_{D_0} - 3\frac{C_L^{3/2}}{\pi e\,\mathrm{AR}} = 0$$

$$C_{D_0} = 3\,C_{D_i}$$

6.15 From Eq. (6.81), $C_{D_0} = 3\,C_{D_i}$

$$C_{D_0} = 3\frac{C_L^2}{\pi e\,\mathrm{AR}}$$

Thus, $C_L = \left(\frac{1}{3}\pi e\,\mathrm{AR}\,C_{D_0}\right)^{\frac{1}{2}}$

$$\left(\frac{C_L^{\frac{1}{2}}}{C_D}\right)_{\max} = \frac{\left(\frac{1}{3}\pi\,e\,\mathrm{AR}\,C_{D_0}\right)^{\frac{1}{4}}}{\frac{4}{3}C_{D_0}}$$ This is Eq. (6.86)

From Eq. (6.80), $C_{D_0} = \frac{1}{3}C_{D_i}$ $\qquad C_{D_0} = \frac{C_L^2}{3\pi e\,\mathrm{AR}}$

Thus, $C_L = (3\pi e\,\mathrm{AR}\,C_{D_0})^{\frac{1}{2}}$

$$\left(\frac{C_L^{\frac{3}{2}}}{C_D}\right)_{\max} = \left(\frac{3\pi e\,\mathrm{AR}\,C_{D_0}}{4\,C_{D_0}}\right)^{\frac{3}{4}}$$ (This is Eq. (6.87)

6.16 AR $= b^2/S$, hence $b = \sqrt{S\,\text{AR}} = \sqrt{(47)(6.5)} = 17.48$ m

$$h = 5 \text{ ft} = (5/3.28)\,\text{m} = 1.524 \text{ m}$$

$$h/b = 1.54/17.48 = 0.08719$$

$$\phi = \frac{(16\,h/b)^2}{1 + (16\,h/b)^2} = \frac{1.946}{2.946} = 0.66$$

$$V_{L0} = 1.2V_{\text{stall}} = 1.2\sqrt{\frac{2W}{\rho_\infty S\, C_{L,\max}}} = 1.2\sqrt{\frac{2(103{,}047)}{(1.255)(47)(0.8)}} = 80.3 \text{ m/sec}$$

Hence, 0.7 $0.7V_{L0} = 56.2$ m/sec. This is the velocity at which the average force is evaluated.

$$q_\infty = \frac{1}{2}\,\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(56.2)^2 = 1935 \text{ N/m}^2$$

$$L = q_\infty\, S\, C_L = (1935)\,(47)(0.8) = 72{,}760 \text{ N}$$

$$D = q_\infty S\, C_D = q_\infty S\left(C_{D_0} + \phi\frac{C_L^2}{\pi e\,\text{AR}}\right)$$

$$D = (1935)(47)[(0.032) + (0.66)(0.8)^2/\pi(0.87)(6.5)]$$

$$D = 5072\,\text{N}$$

From Eq. (6.90)

$$s_{L0} = \frac{1.44\ \text{W}^2}{g\,\rho_\infty C_{L,\max}\left\{\text{T} - \left[\text{D} + \mu_{\text{r}}(W - L)\right]\text{ave}\right\}}$$

$$= \frac{1.44\,(103047)^2}{(9.8)(1.225)(47)(0.8)\left\{80595 - \left[5072 + (0.2)(103047 - 72760)\right]\right\}}$$

$$s_{L0} = 452 \text{ m}$$

6.17 $b = \sqrt{S\,\text{AR}} = \sqrt{(181)(6.2)} = 33.5$ ft.

$h/b = 4/33.5 = 0.1194$

$$\phi = \frac{(16h/b)^2}{1 + (16h/b)^2} = \frac{3.65}{4.65} = 0.785$$

$$V_{L0} = 1.2\sqrt{\frac{2W}{\rho_\infty S\, C_{L,\max}}} = 1.2\sqrt{\frac{2(3000)}{(0.002377)(181)(1.1)}} = 135 \text{ ft/sec}$$

$0.7 = V_{L0} = 0.7(135) = 94.5$ ft/sec.

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.002377)(94.5)^2 = 10.6 \text{ lb/ft}^2$$

$$L = q_\infty S C_L = (10.6)(181)(1.1) = 2110 \text{ lb}$$

$$D = q_\infty S\, C_D = q_\infty S\left(C_{D,0} + \phi\frac{C_L^2}{\pi e\,\text{AR}}\right)$$

$$D = (10.6)(181)\left[(0.027 + \frac{(0.785)(1.1)^2}{\pi(0.91)(6.2)}\right] = 154.6 \text{ lb}$$

$$T = (550\,\text{HP}_A)/V_\infty = (550)(285)/94.5 = 1659 \text{ lb}$$

$$S_{L0} = \frac{1.44\,\text{W}^2}{g\,\rho_\infty S\, C_{L,\max}\{T - [D + \mu_\text{r}(W - L)]_\text{ave}\}}$$

$$= \frac{1.44\,(3000)^2}{(32.2)(0.002377)(181)(1.1)\{1659 - [154.6 + (0.2)(3000 - 2110]\}}$$

$S_{L0} = 572$ ft.

6.18 $$V_T = 1.3\sqrt{\frac{2\text{W}}{\rho_\infty S\, C_{\text{L},\max}}} = 1.3\sqrt{\frac{2(103{,}047)}{(1.23)(47)(2.8)}} = 46.39 \text{ m/sec}$$

$0.7V_T = 32.47$ m/sec.

$$q_\infty \frac{1}{2}\rho_\infty V_\infty^2 = (0.5)(1.23)(32.47)^2 = 648.4 \text{ N/m}^2$$

Since the lift is zero after touchdown, $C_D = C_{D,0}$.

$$D = q_\infty S\, C_{D,0} = (648.4)(47)(0.032) = 975.2 \text{ N}$$

$$S_L = \frac{1.69\,\text{W}^2}{g\,\rho_\infty C_{L,\max}\left[[D + \mu_r(W - L)\right]_{0.7V_T}}$$

$$S_L = \frac{1.69\,(103047)^2}{(9.8)(1.23)(47)(2.8)[975.2 + (0.4)(103{,}047)]} = 268 \text{ m}$$

6.19 $V_T = 1.3\sqrt{\dfrac{2W}{\rho_\infty S\, C_{L,\max}}} = 1.3\sqrt{\dfrac{2(3000)}{(0.002377)(181)(1.8)}} = 114.4 \text{ ft/sec}$

$0.7V_T = 80.08$ ft/sec.

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}0.5(0.002377)(80.08)^2 = 7.62 \text{ lb/ft}^2$$

$$D = q_\infty S\, C_{D,0} = (7.62)(181)(0.027) = 37.2 \text{ lb}$$

$$S_L = \frac{1.69 \text{ W}^2}{g\, \rho_\infty S\, C_{L,\max}\left[[D + \mu_r(W - L)\right]_{0.7V_T}}$$

$$S_L = \frac{1.69\,(3000)^2}{(32.2)(0.002377)(181)(1.8)[37.2 + 0.4\,(3000)]} = 493 \text{ ft}$$

6.20 $V_\infty = 250 \text{ mph} = 250\left(\dfrac{88}{60}\right) \text{ft/sec} = 366.6 \text{ ft/sec}$

$= 366.6\,(0.3048)$ m/sec $= 111.7$ m/sec.

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = (0.5)(1.23)(111.7)^2 = 7673 \text{ N/m}^2$$

$$L = q_\infty S\, C_{L,\max} = (7673)(47)(1.2) = 4.358 \times 10^5 \text{N}$$

$$n = \frac{L}{W} = \frac{4.328 \times 10^5}{103047} = 4.2$$

$$R = \frac{V_\infty^2}{g\sqrt{n^2 - 1}} = \frac{(111.7)^2}{9.8\sqrt{(4.2)^2 - 1}} = 312 \text{ m}$$

$$w = \frac{V_\infty}{R} = \frac{111.7}{312} = 0.358 \text{ rad/sec}$$

6.22 From Eq. (6.13) $$T = D = q_\infty S\, C_D$$

From Eq. (6.1 c) $$C_D = C_{D,0} + \frac{C_L^2}{\pi e\, \mathrm{AR}}$$

Combining (1) and (2) $$T = q_\infty S\left[C_{D,0} + \frac{C_L^2}{\pi e\, \mathrm{AR}}\right]$$

From Eq. (6.14) $$L = W = q_\infty S\, C_L$$

$$C_L = \frac{W}{q_\infty S}$$

Substitute (4) into (3)

$$T = q_\infty S\left[C_{D,0} + \frac{W^2}{q_\infty^2 S \pi e\, \mathrm{AR}}\right] = q_\infty S\, C_{D,0} + \frac{W^2}{q_\infty S \pi e\, \mathrm{AR}}$$

Multiply by q_∞

$$q_\infty T = q_\infty^2 S C_{D,0} + \frac{W^2}{S \pi e\, \mathrm{AR}}$$

or $$q_\infty^2 S C_{D,0} - q_\infty T + \frac{W^2}{S \pi e\, \mathrm{AR}} = 0$$

From the quadratic formula:

$$q_\infty = \frac{T \pm \sqrt{T^2 - \dfrac{4 S C_{D,0} W^2}{S \pi e\, \mathrm{AR}}}}{2 S C_{D,0}}$$

$$q_\infty = \frac{\dfrac{T}{W}\left(\dfrac{W}{S}\right) \pm \dfrac{W}{S}\sqrt{\left(\dfrac{T}{W}\right)^2 - \dfrac{4\, C_{D,0}}{\pi e\, \mathrm{AR}}}}{2\, C_{D,0}} = \frac{1}{2}\rho_\infty V_\infty^2$$

$$V_\infty^2 = \frac{\left(\dfrac{T}{W}\right)\left(\dfrac{W}{S}\right) + \dfrac{W}{S}\sqrt{\left(\dfrac{T}{W}\right)^2 - \dfrac{4 C_{D,0}}{\pi e\, \mathrm{AR}}}}{\rho_\infty C_{D,0}}$$

$$V_\infty = \left[\frac{\left(\dfrac{T}{W}\right)\left(\dfrac{W}{S}\right) + \dfrac{W}{S}\sqrt{\left(\dfrac{T}{W}\right)^2 - \dfrac{4 C_{D,0}}{\pi e\, \mathrm{AR}}}}{\rho_\infty C_{D,0}}\right]^{1/2}$$

For $V_{\max}$: $T = (T_A)_{\max}$

$$(V_\infty)_{\max} = \left[\frac{\left(\dfrac{T_A}{W}\right)_{\max}\left(\dfrac{W}{S}\right) + \dfrac{W}{S}\sqrt{\left(\dfrac{T_A}{W}\right)_{\max}^2 - \dfrac{4 C_{D,0}}{\pi e\, \mathrm{AR}}}}{\rho_\infty C_{D,0}}\right]^{1/2}$$

6.23 From Figure 6.2, $(L/D)_{\max} = 18.5$, and $C_{D,0} = 0.015$. From Eq. (6.85)

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{(C_{D,0}\pi e\,\mathrm{AR})^{1/2}}{2C_{D,0}}$$

or,
$$e = \frac{4C_{D,0}(C_L/C_D)^2_{\max}}{\pi\,\mathrm{AR}}$$

$$e = \frac{4(0.015)(18.5)^2}{\pi(7.0)} = 0.83$$

Please note: Consistent with the derivation of Eq. (6.85) where a parabolic drag polar is assumed with the zero-lift drag coefficient equal to the minimum drag coefficient, for the value of $C_{D,0}$ in this problem we read the minimum drag coefficient from Figure 6.2.

6.24 Drag: $D = q_\infty SC_D$

Power Available: $P_A = \eta\, P_s$

Also, $P_A = T_A V_\infty$

Hence $T_A = \dfrac{\eta P_s}{V_\infty}$

(a) At an altitude of 30,000 ft, $\rho_\infty = 0.00089068$ slug/ft^3 and $T_\infty = 411.86°R$. The speed of sound is

$$a_\infty = \sqrt{\gamma RT_\infty} = \sqrt{(1.4)(1718)(411.86)} = 994.7 \text{ ft/sec}$$

Hence, <u>at Mach one</u>, the flight velocity is $V_\infty = 994.7$ ft/sec

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.00089068)(994.7)^2 = 440.6 \text{ lb/ft}^2$$

The drag coefficient at Mach one, as given in the problem statement is

$$C_{D,0}(\text{at } M_\infty = 1) = 10[C_{D,0}(\text{at low speed})] = 10(0.0211) = 0.211$$

Hence, drag at Mach one is

$$D = q_\infty S\,C_D = (440.6)(334)(0.211) = \underline{31{,}051 \text{ lb}}$$

The thrust available is obtained as follows. The engine produces 1500 horsepower supercharged to 17,500 ft. Above that altitude, we assume that the power decreases directly as the air density. At 17,500 ft, $\rho_\infty = 0.0013781$ slug/ft^3. Hence

$$\text{HP} = \frac{\rho_\infty(\text{at } 30{,}000 \text{ ft})}{\rho_\infty(\text{at } 17{,}500 \text{ ft})}(1500) = \frac{0.00089068}{0.0013781}(1500) = 969 \text{ HP}$$

From Eq. (3), above

$$T_A = \frac{\eta P_s}{V_\infty} = \frac{(0.3)(969)(550)}{994.7} = \underline{161 \text{ lb}}$$

Consider the airplane in a vertical dive at Mach one. The maximum downward vertical force is $W + T_A = 12{,}441 + 161 = 12{,}602$ lb. However, the drag is the retarding force acting vertically upward, and it is 31,051 lb. At Mach one, the drag far exceeds the maximum downward force. Hence, it is <u>not</u> possible for the airplane to achieve Mach one.

(b) At an altitude of 20,000 ft,

$$\rho_\infty = 0.0012673 \text{ slug/ft}^3$$
$$T_\infty = 447.43°R$$

$$a_\infty = V_\infty = \sqrt{(1.4)(1716)(447.43)} = 1036.8 \text{ ft/sec}$$

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(0.0013672)(1036.8)^2 = 681 \text{ lb/ft}^3$$

$$D = q_\infty S C_D = (681)(334)(0.211) = \underline{47{,}993 \text{ lb}}$$

The thrust available is obtained as follows

$$\text{HP} = \frac{\rho_\infty(\text{at } 20{,}000 \text{ ft})}{\rho_\infty(\text{at } 17{,}500 \text{ ft})}(1500) = \frac{0.0012673}{0.0013781}(1500) = 1379 \text{ HP}$$

Hence,

$$T_A = \frac{\eta P_s}{V_\infty} = \frac{(0.3)(1379)(550)}{1036.8} = \underline{219 \text{ lb}}$$

In a vertical, power-on dive at 20,000 ft, the maximum vertical force downward is $W + T_A$ = 12,441 + 219 = 12,660 lb. However, the drag is the retarding force acting vertically upward, and it is 47,993 lb. At Mach one, the drag far exceeds the maximum downward force. Hence it is not possible for the airplane to achieve Mach 1.

6.25 Taking the analytical approach given in Example 6.19, from Eq. (E6.19.1)

$$P_A = \frac{1}{2}\rho_\infty V_\infty^3 S C_{D,0} + \frac{W^2}{\frac{1}{2}\rho V_\infty S \pi e \text{AR}}$$

$$\text{AR} = \frac{b^2}{S} = \frac{(14.45)^2}{11.45} = 19.3$$

$$W = 1020\, kg_f = (1020)(9.8) = 9996 \text{ N}$$

$$P_A = \eta\,(b\text{hp}) = (0.9)(85) = 76.5 \text{ hp}$$

Using consistent units, noting that 1 hp = 746 Watts

$$P_A = (76.5)(746) = 5.71 \times 10^4 \text{Watts}$$

$$\frac{1}{2}\rho_\infty S C_{D,0} = \frac{1}{2}(1.23)(11.45)(0.03) = 0.2113$$

$$\frac{1}{2}\rho_\infty S \pi e \text{ AR} = \frac{1}{2}(1.23)(11.45)\,\pi\,(0.7)(19.3) = 298.9$$

Inserting these numbers into Eq. (1)

$$5.71 \times 10^4 = 0.2113\, V_\infty^3 + \frac{(9996)^2}{298.9\, V_\infty}$$

or,
$$5.71 \times 10^4 = 0.2113 V_\infty^3 + \frac{3.343 \times 10^5}{V_\infty}$$

Solving Eq. (2) for V_∞ $\quad V_\infty = V_{\max} = \boxed{62{,}6 \text{ m/sec.}}$

6.26 From Eq. (6.67)

$$R = \frac{\eta}{c}\frac{C_L}{C_D}\ell n\frac{W_0}{W_1}$$

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{(C_{D,0}\,\pi e\,\mathrm{AR})^{1/2}}{2C_{D,0}} = \frac{\left[(0.03)\pi(0.7)(19.3)\right]^{1/2}}{2(0.03)}18.8$$

Using consistent units,

$$c = 0.2\frac{\mathrm{kg}_f}{(\mathrm{hp})(\mathrm{hr})} = 0.2\left[\frac{\mathrm{kg}_f}{(\mathrm{hp})(\mathrm{hr})}\right]\left(\frac{9.8\mathrm{N}}{1\,\mathrm{kg}_f}\right)\left[\frac{1\text{ hp}}{746\text{ Nm/s}}\right]\left(\frac{1\text{ hr}}{3600\text{ sec}}\right)$$

$$= 7.3\times 10^{-7}_{\mathrm{m}^{-1}}$$

$$W_0 = 1020\text{ kg}_f = 1020(9.8) = 9996\text{ N}$$

$$W_1 = W_0 - W_{\text{fuel}} = 9996 - (295)(9.8) = 7105\text{ N}$$

From Eq. (6.67)

$$R = \frac{\eta}{c}\frac{C_L}{C_D}\ell n\frac{W_0}{W_1} = \left(\frac{0.9}{7.3\times 10^{-7}}\right)(18.8)\,\ell n\left(\frac{9996}{7105}\right)$$

$$R = 3.44\times 10^6\mathrm{m} = \boxed{3440\text{ km}}$$

6.27
$$\left(\frac{C_L^{3/2}}{C_D}\right)_{\max} = \frac{(3C_{D,0}\,\pi e\,\mathrm{AR})^{3/4}}{4C_{D,0}}$$

$$= \frac{\left[3(0.03)\pi(0.7)(19.3)\right]^{3/4}}{4(0.03)} = 22,77$$

From Eq. (6.68)

$$E = \left(\frac{\eta}{c}\right)\left(\frac{C_L^{3/2}}{C_D}\right)(2\rho_\infty S)^{1/2}(W_1^{-1/2} - W_0^{-1/2})$$

Using results from Problem 6.26,

$$E = \left(\frac{0.9}{7.3\times 10^{-7}}\right)22.77[2(1.23)(11.45)]^{1/2}[(7105)^{-1/2} - (9996)^{-1/2}]$$

$$E = 14.9\times 10^7[0.01186 - 0.01000]$$

$$E = 2.77\times 10^5\text{ sec}$$

or, $$E = \frac{2.77\times 10^5}{3600} = \boxed{77\text{ hours}}$$

6.28 $D_i = Q_\infty S\, C_{D,i} = q_\infty S \dfrac{C_L{}^2}{\pi e \,\mathrm{AR}}$

In steady level flight, $L = W$, and we have

$$C_L^2 = \left(\frac{L}{q_\infty S}\right)^2 = \left(\frac{W}{q_\infty S}\right)^2$$

Substitute (2) into (1).

$$D_i = Q_\infty S \left(\frac{W}{Q_\infty S}\right)^2 \frac{1}{\pi e \,\mathrm{AR}}$$

But $\mathrm{AR} = \dfrac{b^2}{S}$, hence Eq. (3) becomes

$$D_i = q_\infty S \left(\frac{W}{d_\infty S}\right)^2 \frac{1}{\pi e}\left(\frac{S}{b^2}\right)$$

$$D_i = \frac{1}{\pi e q_\infty}\left(\frac{W}{b}\right)^2 \; QED$$

6.29 **(a)** $q_\infty = \tfrac{1}{2}\rho_\infty V_\infty^2 = \tfrac{1}{2}(2.0482\times10^{-3})\left[\left(\dfrac{88}{60}\right)(300)^2\right]$

$$q_\infty = 198.26 \text{ lb/ft}^2$$
$$b = 37 \text{ ft}$$
$$W = 10{,}100 \text{ lb}$$

$$D_i = \frac{1}{\pi e q_\infty}\left(\frac{W}{b}\right)^2 = \frac{1}{\pi(0.8)(198.26)}\left(\frac{10{,}100}{37}\right)^2$$

$$D_i = 149.5 \text{ lb}$$

(b) $C_{D_i} = \dfrac{C_L^2}{\pi e \,\mathrm{AR}}$

$$C_L = \frac{W}{q_\infty S} = \frac{10{,}100}{(198.26)(233)} = 0.2186$$

$$\mathrm{AR} = \frac{b^2}{S} = \frac{(37)^2}{233} = 5.87$$

$$C_{D_i} = \frac{C_L^2}{\pi e \mathrm{AR}} = \frac{(0.2186)^2}{\pi(0.8)(5.86)} = 0.003245$$

$$D_i = q_\infty S\, C_{D_i} = (168.26)(233)(0.003245)$$

$$D_i = 149.9 \text{ lb}$$

The results from parts (a) and (b) agree to within 3-place round off error on the author's hand calculator.

6.30 **(a)** From (6.45), copied below, for rate-of-climb at the climb angle θ,

$$T = D + W \sin\theta$$

we have

$$\frac{T}{W} = \frac{D}{W} + \sin\theta$$

Assume $L = W$. Thus,

$$\frac{T}{W} = \frac{D}{L} + \sin\theta$$

or,
$$\frac{T}{W} = \frac{1}{L/D} + \sin\theta$$

(b) At a standard altitude of 31,000 ft, T_∞= 408.3 R.

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(408.3)} = 990 \text{ ft/sec}$$
$$V_\infty = M_\infty a_\infty = 0.85(990) = 841.5 \text{ ft/sec.}$$

The specified rate-of-climb of 300 ft/min, in terms of ft/sec, is 300/60 = 5 ft/sec.
From Eq. (6.48) and Fig. 6.28,

$$\sin\theta = \frac{R/C}{V_\infty} = \frac{5}{841.5} + 5.94\times 10^{-3}$$

Thus,

$$\theta = 0.34° - \text{very small.}$$

Hence,

$$\frac{T}{W} = \frac{1}{L/D} + \sin\theta = \frac{1}{18} + 5.94\times 10^{-3}$$

or

$$\frac{T}{W} = 0.0615$$

Thus

$$T = 0.0615\,W = 0.0615(550{,}000) = 33{,}825 \text{ lb.}$$

This is the required combined thrust from both engines. The thrust of each engine at this flight condition is 33,825/2, or 16,912 Ib.
At 31,000 ft, $\rho_\infty = 8.5841\times 10^{-4}$ slug/ft^3. Assuming the engine thrust is proportional to density, as discussed in Section 6.7, we have for the thrust at sea level,

$$T = \left(\frac{0.002377}{8.5841\times 10^{-4}}\right)33825 = 93{,}666 \text{ lb.}$$

This is comparable to the 84,000 lb sea level static thrust of the particular Trent engine mentioned in the problem.

6.31 To answer this question, we need to compare the thrust with the weight plus drag at Mach 1; in a vertical climb at Mach 1, if T > W + D, then the airplane will accelerate through Mach 1 in a vertical climb. For an airplane in a vertical climb, lift = 0. Hence, the total drag coefficient is that for zero lift.

$$C_D = 0.016(2.3) = 0.368 \text{ at Mach 1.}$$

Let us assume standard sea-level conditions, where $\rho_\infty = 1.23 \text{ kg/m}^3$ and $a_\infty = 340.3$ m/sec. At march $1, V_\infty = a_\infty = 340.3$ m/sec. Thus,

$$q_\infty = \tfrac{1}{2} p_\infty V_\infty = \tfrac{1}{2}(1.23)(340.3)^2 = 7.12 \times 10^4 \text{N/m}^2$$
$$D = q_\infty S\, C_D = (7.12 \times 10^4)(27.84)(0.0368) = 7.3 \times 10^4 \text{N}$$

The weight is given as 8,273 kg_f; in terms of newtons

$$W = (8273)(9.8) = 8.11 \times 10^4 \text{N}$$

Hence, $\quad D + W = (7.3 + 8.11) \times 10^4 = 1.541 \times 10^5 \text{N}$

Compare this with the thrust at sea level,

$$T = 131.6 \text{ k N} = 1.316 \times 10^5 \text{N}$$

For this case, $T < D + W$. The airplane can not accelerate through Mach one going straight up at sea level.

Since $D \propto \rho_\infty$ and $T \propto \rho_\infty$, both D and T will decrease by the same factor with an increase in altitude. Because the weight is fixed,

$$T < D + W \text{ at all altitudes.}$$

Hence, at all altitudes, the airplane can not accelerate through Mach one going straight up.

6.32 At 2000 m, $T_\infty = 275.16$ K and $\rho_\infty = 1.0066 \text{ kg/m}^3$

$$a_\infty = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(287)(275.16)} = 332.5 \text{ m/sec.}$$
$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{100}{332.5} = 0.3$$

Hence, the airplane is flying at a low Mach number, far below the drag-divergence Mach number. Thus, the zero-lift drag coefficient is $C_D = 0.016$.

$$q_\infty = \tfrac{1}{2} \rho_\infty V_\infty^2 = \tfrac{1}{2}(1.0066)(100)^2 = 5033 \text{ N/m}^2$$
$$D = q_\infty S\, C_D = (5033)(27.87)(0.016) = 2243 \text{ N}$$

The acceleration is obtained from Newton's 2nd Law, $F = ma$. The net force acting on the airplane in the upward direction is

$$T - W - D = 131.6 \times 10^3 - (8273)(9.8) - 2.243 \times 10^3 = 131.6 \times 10^3$$
$$- 81.08 \times 10^3 - 2.243 \times 10^3 = 48.28 \times 10^3 \text{N}$$

From the discussion in Section 2.4, the weight in kg_f is the same number as the mass in kg. Hence, the mass of the F-16 is 8273 kg. Thus,

$$a = \frac{F}{m} = \frac{48.28 \times 10^3}{8273} = 5.84 \text{ m/sec}^2$$

6.33 From the definition of TSFC, we have

$$W_f = (\text{TSFC})\,T\,t = (\text{TSFC})\,D\,t = \frac{w}{L/D}(\text{TSFC})\,t \tag{1}$$

Also, the new thrust specific fuel consumption results in a new fuel weight

$$W_{f,\text{new}} = \frac{W_{\text{new}}}{\frac{L}{D}}(\text{TSFC})(1-\varepsilon_f)\,t$$

$$= \frac{W(1+\varepsilon_w)}{L/D}(\text{TSFC})\,(1-\varepsilon_f)\,t \tag{2}$$

Substituting (1) into (2), we have

$$W_{f,\text{new}} = W_f(1+\varepsilon_w)(1-\varepsilon_f) \tag{3}$$

Increase in engine weight $= W_{\text{new}} - W = W(1+\varepsilon_w) - W = \varepsilon_w W$

Decrease in fuel weight $= W_f - W_{f,\text{new}} = W_f - W_f(1+\varepsilon_w)(1-\varepsilon_f)$ (using Eq.(3))

At the break-even point, we have

$$W_{\text{new}} - W = W_f - W_{f,\text{new}}$$

$$\varepsilon_w W = W_f - W_f(1+\varepsilon_w)(1-\varepsilon_f)$$

$$\varepsilon_w \frac{W}{W_f} = 1 - (1+\varepsilon_w)(1-\varepsilon_f)$$

$$\varepsilon_w \frac{W}{W_f} = -\varepsilon_w + \varepsilon_f + \varepsilon_w\varepsilon_f$$

Since ε_w and ε_f are very small fractions, we can ignore the product $\varepsilon_w\varepsilon_f$. Thus,

$$\varepsilon_f = \varepsilon_w\left(1 + \frac{W}{W_f}\right)$$

and from Eq. (1), we have

$$\varepsilon_f = \varepsilon_w\left[1 + \frac{\frac{L}{D}}{(\text{TSFC})\,t}\right]$$

6.34 Weight at beginning of flight $= 1.35 \times 10^6$ lb.

Weight at end of flight $= 1.35 \times 10^6 - 0.5 \times 10^6 = 0.85 \times 10^6$ lb.

Average weight during flight $= \dfrac{1.35 \times 10^6 + 0.85 \times 10^6}{2} = 1.1 \times 10^6$ lb.

From the result of Problem 6.33, we have

$$\varepsilon_w = \frac{\varepsilon_f}{1 + \dfrac{W}{W_f}}$$

where $\varepsilon_f = 0.01, W = 1.1 \times 10^6$ lb, and $W_f = 0.5 \times 10^6$ lb.

$$\varepsilon_w = \frac{0.01}{1 + \dfrac{1.1 \times 10^6}{0.5 \times 10^6}} = \frac{0.01}{3.2} = 0.00313$$

$$W_{\text{new}} = W(1+\varepsilon_w) = 1.1 \times 10^6 (0.00313) = 3438 \text{ lb.}$$

The weight increase allowed for each engine is therefore

$$\frac{3438}{4} = 859 \text{ lb.}$$

6.35 $$T_A = D = q_\infty S\, C_D = q_\infty\ S \left(C_{D,0} + \frac{C_L^2}{\pi eAR} \right)$$

$$P_A = T_A V_\infty = q_\infty\ S\ V_\infty \left(C_{D,0} + \frac{C_L^2}{\pi eAR} \right)$$

$$C_L = \frac{W}{q_\infty S}$$

$$P_A = q_\infty S\ V_\infty \left(C_{D,0} + \frac{W^2}{q_\infty S^2 \pi eAR} \right) = q_\infty S\ V_\infty\ C_{D,0} + \frac{W^2 V_\infty}{q_\infty S \pi eAR}$$

$$\frac{P_A}{W} = 1/2\, \rho_\infty V_\infty^3\ \frac{C_{D,0}}{(W/S)} + \frac{(W/S)}{1/2\ p_\infty V_\infty \pi eAR}$$

$$\frac{P_A}{W} = 1/2\, \rho_\infty V_\infty^3\ \frac{C_{D,0}}{(W/S)} + \frac{2(W/S)}{p_\infty V_\infty \pi eAR} \qquad (1)$$

$P_A = \eta P$ where P is the shaft power from the engine.

At maximum velocity, $P = P_{\max}$. Equation (1) then becomes

$$\boxed{\frac{\eta\ P_{\max}}{W} = \frac{1}{2}\ \rho_\infty\ V_{\max}^3\ \frac{C_{D,0}}{(\text{W/S})} + \frac{2\ (\text{W/S})}{\rho_\infty V_{\max} \pi eAR}}$$

6.36 For the CP-1,

$$P_{max} = 230 \text{ hp} = (230)(550) = 1.265 \times 10^5 \text{ ft lb/sec}$$
$$P_{max}/W = 42.88 \text{ ft/sec}$$
$$\eta = 0.8$$
$$W = 2950 \text{ lb}$$
$$C_{D,0} = 0.025$$
$$W/S = 2950/174 = 16.954 \text{ lb/ft}^2$$
$$e = 0.8$$
$$AR = b^2/S = (35.8)^2/174 = 7.366$$
$$\rho_\infty = 0.002377 \text{ slug/ft}^3 \text{ at sea level}$$

From the equation obtained as the result of Problem 6.35,

$$\frac{\eta P_{max}}{W} = \frac{1}{2}\rho_\infty V_{max}^3 \frac{C_{D,0}}{(W/S)} + \frac{2(W/S)}{\rho_\infty V_{max} \pi e AR}$$

$$0.8(42.88) = 1/2(0.002377)\, V_{max}^3 \frac{(0.025)}{16.954} + \frac{2(16.954)}{(0.002377)V_{max}\pi(0.3)(7.366)}$$

$$\boxed{34.304 = 1.75254 \times 10^{-6} V_{max}^3 + \frac{770.55}{V_{max}}} \qquad (1)$$

Eq. (1) is an equation for V_{max} that yields an analytic solution for V_{max}. But it must be solved by trial and error. We will shortcut the trial and error process for this solutions manual by first trying V_{max} = 265 ft/sec as obtained from the numerical solution.

$$34.304 = 1.75254 \times 10^{-6}(260)^3 + \frac{770.55}{265}$$
$$= 32.614 + 2.9077 = 35.52$$
$$\text{Difference } = 1.216$$

Let us try a slightly smaller value, say V_{max} = 260 ft/sec.

$$34.304 = 1.75254 \times 10^{-6}(260)^3 + \frac{770.55}{260}$$
$$= 32.80 + 2.96 = 33.76$$
$$\text{Difference } = -0.544$$

Try V_{max} = 262 ft/sec.

$$34.304 = 1.75254 \times 10^{-6}(262)^3 + \frac{770.55}{262}$$
$$= 31.519 + 2.941 = 34.46$$
$$\text{Difference } = 0.156$$

Try V_{max} = 261 ft/sec.

$$34.304 = 1.75254 \times 10^{-6}(261)^3 + \frac{770.55}{261}$$
$$= 31.159 + 2.952 = 34.111$$
$$\text{Difference} = -0.192$$

Try $V_{max} = 261.6$ ft/sec.

$$34.304 = 1.75254\times 10^{-6}(26.61)^3 + \frac{770.55}{261.6}$$
$$= 31.3748 + 2.9455 = 34.32$$

Close enough! The analytical value for V_{max} is

$$\boxed{V_{max} = 261.6 \text{ ft/sec}}$$

Compared with the numerical value of 265 ft/sec, a <u>1.3% difference</u>.

6.37 For the CJ-1

$$T_A = (3650)_2 = 7300 \text{ lb}$$
$$W = 19{,}815 \text{ lb}$$
$$\frac{T_A}{W} = \frac{7300}{19{,}815} = 0.368$$
$$S = 318 \text{ ft}^2$$
$$\frac{W}{S} = \frac{19{,}815}{318} = 62.3 \text{ lb/ft}^2$$
$$C_{D,0} = 0.02$$
$$e = 0.81$$
$$AR = (53.3)^2/318 = 8.934$$

From Eq. (6.44)

$$V_{max} = \left[\frac{\left(\frac{T_A}{W}\right)\left(\frac{W}{S}\right) + \left(\frac{W}{S}\right)\sqrt{\left(\frac{T_A}{W}\right)^2 - \frac{4C_{D,0}}{\pi e AR}}}{\rho_\infty C_{D,0}}\right]^{1/2}$$

$$= \left[\frac{(0.368)(62.3) + (62.3)\sqrt{(0.368)^2 - \frac{4(0.02)}{\pi(0.81)(8.934)}}}{(0.002377)(0.02)}\right]^{1/2}$$

$$= \boxed{978.8 \text{ ft/sec}}$$

This is to be compared with the numerical value of 975 ft/sec, a <u>0.3% difference</u>.

6.38 From Eq. 6.53:

$$(R/C)_{\max} = \left(\frac{\eta P}{W}\right)_{\max} - 0.8776 \sqrt{\frac{\frac{W}{S}}{\rho_\infty c_{D,0}}} \frac{1}{(L/D)^{3/2}_{\max}}$$

For the CP-1, at 12,000 ft, hp = 230 $\left(\frac{0.001648}{0.002377}\right) = 159.5$ hp

$$\left(\frac{\eta P}{W}\right) = \frac{(0.8)(1595)(550)}{2950} = 23.79$$

$$\frac{W}{s} = \frac{2950}{174} = 16.954$$

$$\rho_\infty C_{D,0} = (0.001648)(0.025) = 4.12 \times 10^{-5}$$

$$(L/D)_{\max} = \frac{(C_{D,0}\pi eAR)^{1/2}}{2C_{D,0}} = \frac{[(0.025)\ \pi\ (0.8)(7.366)]^{1/2}}{2(0.025)} = 13.606$$

$$(R/C)_{\max} = 23.79 - 0.8776 \sqrt{\frac{16.954}{6.12 \times 10^{-5}}} \frac{1}{(13.606)^{3/2}}$$

$$= 23.79 - 0.8776(641.49)\frac{1}{50.188}$$

$$= 23.79 = 11.217 = 12.573 \text{ ft/sec}$$

$$(R/C)_{\max} = (12.573)(60) = 754.4 \text{ ft/min}$$

The numerical result from Section 6.10 is 755 ft/min. The difference is 0.08%.

6.39 From Eq. (6.52)

$$(R/C)_{\max} = \left[\frac{(W/S)\ z}{3\rho_\infty C_{D,0}}\right]^{1/2} \left(\frac{T}{W}\right)^{3/2}_{\max} \left[1 - \frac{2}{6} - \frac{3}{2(T/W)^2_{\max}\,(L/D)^2_{\max}\,Z}\right]$$

For the CJ-1

$$AR = \frac{(53.3)^2}{318} = 8.934$$

$$W/S = 19{,}815/318 = 62.31$$

$$C_{D,0} = 0.02$$

$$\left(\frac{T}{W}\right)_{\max} = \frac{(3650)(2)}{19.815}\left(\frac{0.0011043}{0.002378}\right) = 0.171$$

$$\left(\frac{L}{D}\right)_{\max} = \frac{(C_{D,0}\pi e AR)^{1/2}}{2\ C_{D,0}} = \frac{[(0.02)\pi(0.81)(8.934)]^{1/2}}{2(0.02)} = \frac{0.673}{0.04} = 16.858$$

$$Z = 1 + \sqrt{1 + \frac{3}{(L/D)^2_{\max}(T/W)^2_{\max}}}$$

$$= 1 + \sqrt{1 + \frac{3}{(16.858)^2(0.1711)^2}}$$

$$= 1 + \sqrt{1 + 0.3606} = 2.166$$

$$(R/C)_{\max} = \left[\frac{(62.31)(2.166)}{3(0.0011043)(0.02)}\right]^{1/2} (0.1711)^{3/2} \times$$

$$\left[1 - \frac{2.166}{6} - \frac{3}{2(0.1711)^2(16.858)^2\,2.166}\right]$$

$$= (2.0369\times 10^6)^{1/2}(0.07077)(0.5558)$$

$$= 56.14\ \text{ft/sec} = (56.14)(60)\ \text{ft/min} = \boxed{3368\ \text{ft/min}}$$

This is compared to the numerical answer of 3369 ft/min given in Section 6.10.

6.40 $T = D = \frac{1}{2}\, \rho_\infty V_\infty^2 S \left(C_{D,0} + \frac{C_L^2}{\pi e AR} \right)$ **(1)**

$$V_{max} = 229 \text{ mph} = 229\left(\frac{88}{60}\right) = 335.87 \text{ ft/sec.}$$

$$P_{max} = P_\eta = (1200)(550)\left(\frac{0.0018975}{0.002378}\right)(0.9) = 9.4795 \times 10^5 \ \frac{\text{ft lb}}{\text{sec}}$$

$$T_{max} = \frac{P_{max}}{V_{max}} = \frac{9.4795 \times 10^5}{335.87} = 2822 \text{ lb}$$

$$S = 987 \text{ ft}^2$$

$$AR = 9.14$$

$$e = 0.8$$

$$1/2\, \rho_\infty V_\infty^2 S = 1/2\,(0.0018975)(335.87)^2(987) = 1.05636 \times 10^5 \text{ lb}$$

$$C_L = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S} = \frac{25{,}000}{1.05636 \times 10^5} = 0.2366$$

$$\frac{C_L^2}{\pi e AR} = \frac{(0.23666)^2}{\pi(0.8)(9.14)} = 0.0024382$$

From EQ. (1),

$$2822 = 1.05636 \times 10^5 (C_{D,0} = 0.0024382)$$

$$C_{D,0} + 0.0024382 = \frac{2822}{1.05636 \times 10^5} = 0.0267$$

$$C_{D,0} = \boxed{0.0243}$$

<u>Note</u>: This answer is consistent with the value of $C_{D,0}$ for the DC-3 given in Figure 6.77.

Chapter 7

7.1 $C_{M_{cg}} = C_{M_{ac}} + C_L(h - h_{ac})$

$C_{M_{ex}} = C_{M_{cg}} - C_L(h - h_{ac}) = 0.005 - 0.05(0.03) = -0.01$

7.2 $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}(1.225)(100)^2 = 6125 \text{ N/m}^2$

At zero lift, the moment coefficient about c.g. is

$$C_{M_{cg}} = \frac{M_{cg}}{q_\infty S\, c} = \frac{-12.4}{(6125)(1.5)(0.45)} = -0.003$$

However, at zero lift, this is also the value of the moment coefficient about the a.c.

$$C_{M_{ac}} = -0.003$$

At the other angle of attack,

$$C_L = \frac{L}{q_\infty S} = \frac{3675}{(6150)(1.5)} = 0.4$$

and $$C_{M_{cg}} = \frac{M_{cg}}{q_\infty S\, c} = \frac{20.67}{(6125)(1.5)(0.45)} = 0.005$$

Thus, from Eq. (7.9) in the text,

$$C_{M_{cg}} = C_{M_{ac}} + C_L(h - h_{ac})$$

Thus,

$$h - h_{ac} = \frac{C_{M_{cg}} - C_{M_{ac}}}{C_L} = \frac{0.005 - (-0.003)}{0.4}$$

$$h - h_{ac} = 0.02$$

The aerodynamic center is two percent of the chord length <u>ahead</u> of the center of gravity.

7.3 From the results of Problem 7.2,

$$q_\infty = 6125 \text{ N/m}^2$$
$$C_{M_{ac}} = -0.003$$
$$h - h_{ac} = 0.02$$

In the present problem, the c.g. has been shifted 0.2c rearward. Hence,

$$h - h_{ac} = 0.02 + 0.2 = 0.22$$

Also, $C_L = \frac{L}{q_\infty S} = \frac{4000}{(6125)(1.5)} = 0.435$

Thus, $C_{M_{cg}} = C_{M_{ac}} + C_L(h - h_{ac})$

$$C_{M_{cg}} = -0.003 + 0.435(0.22) = 0.0927$$

7.4 From Problem 7.2, we know

$$q_\infty = 6125 \text{ N/m}^2$$
$$C_{M_{ac}} = -0.003$$
$$h - h_{ac} = 0.02$$

From information provided in the present problem, at $\alpha_a = 5°$,

$$C_L = \frac{L}{q_\infty S} = \frac{4134}{(6125)(1.5)} = 0.45$$

Hence,

$$a = \frac{dC_L}{d\alpha} = \frac{0.45}{5} = 0.09 \text{ per degree}$$

Also, $V_H = \dfrac{\ell_t S_t}{c\, S} = \dfrac{(1.0)(0.4)}{(0.45)(1.5)} = 0.593$

From Eq. (7.26) in the text,

$$C_{M_{cg}} = C_{M_{ac}} + a\alpha_a\left[(h - h_{ac}) - V_H \frac{a_t}{a}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] + V_H a_t(i_t + \varepsilon_o)$$

$$C_{M_{cg}} = -0.003 + (0.09)(5)\left[(0.02) - (0.593)\left(\frac{0.12}{0.09}\right)(1 - 0.42)\right] + (0.593)(0.12)(2.0)$$

$$C_{M_{cg}} = -0.052$$

Hence, the moment is

$$M_{cg} = q_\infty S\, c\, C_{M_{cg}} = (6125)(1.5)(0.45)(-0.052)$$
$$M_{cg} = -215 \text{ Nm}$$

7.5 $\dfrac{C_{M_{cg}}}{\partial \alpha_a} = a\left[(h - h_{ac}) - V_H \dfrac{a_t}{a}\left(1 - \dfrac{\partial \varepsilon}{\partial \alpha}\right)\right]$

where, from Problems 7.4 and 7.2,

$$\begin{aligned} a &= 0.09 \text{ per degree} \\ h - h_{ac} &= 0.02 \\ C_{M_{ac}} &= -0.003 \\ V_H &= 0.593 \\ a_t &= 0.12 \\ \frac{\partial \varepsilon}{\partial \alpha} &= 0.42 \\ i_t &= 2.08 \end{aligned}$$

Thus, $\dfrac{C_{M_{cg}}}{\partial \alpha_a} = (0.09)\left[(0.02) - (0.593)\left(\dfrac{0.12}{0.09}\right)(1 - 0.42)\right] = -0.039$

The slope of the moment coefficient curve is negative, hence the airplane model is statically stable. To examine whether or not the model is balanced, first calculate C_{M_0}.

$$C_{M_0} = C_{M_{ac}} + V_H a_t (i_t + \varepsilon_0) = -0.003 + (0.593)(0.12)(2.0 + 0) = 0.139$$

The trim angle of attack can be found from

$$C_{M_{cg}} = C_{M_0} + \frac{C_{M_{cg}}}{\partial \alpha}\alpha_e = 0$$

$$\alpha_e = -CM_0/(\partial C_{M_{cg}}/\partial \alpha) = -0.139/(-0.039) = 3.56^\circ$$

This is a reasonable angle of attack, falling within the normal flight range. Hence, the airplane model is also balanced.

7.6 $h_n = h_{ac_{wb}} + V_H \dfrac{a_t}{a}\left(1 - \dfrac{\partial \varepsilon}{\partial \varepsilon}\right)$

From Problem 7.2, $h_n - h_{ac_{wb}} = 0.02$

Hence,

$$h_{ac_{wb}} = h - 0.02 = 0.26 - 0.02 = 0.24$$

$$h_n = 0.24 + 0.593\left(\frac{0.12}{0.09}\right)(1 - 0.42)$$

$$h_n = 0.70$$

By definition,

$$\text{static margin} = h_n - h = 0.70 - 0.26 = 0.44$$

7.7
$$\delta_{\text{trim}} = \frac{C_{M_0} + (\partial C_{M_{\text{cg}}} / \partial \alpha_a)\alpha_n}{V_H(\partial C_{L_1} / \partial \delta_e)}$$

$$C_{M_0} = 0.139 \text{ (from Problem 7.5)}$$
$$\partial C_{M_{\text{cg}}} / \partial \alpha_a = -0.039 \text{ (from Problem 7.5)}$$
$$\alpha_n = 8°$$
$$V_H = 0.593 \text{ (from Problem 7.4)}$$
$$\partial C_{L_t} / \partial \delta_e = 0.04 \text{ (given)}$$

Thus, $\delta_{\text{trim}} = \dfrac{0.139 + (-0.039)(8)}{(0.593)(0.04)} = -7.29°$

7.8
$$F = 1 - \frac{1}{a_t}\left(\frac{\partial C_{L_1}}{\partial \delta_e}\right)\frac{(\partial C_{\text{he}} / \partial \alpha_1)}{(\partial C_{\text{he}} / \partial \delta_e)}$$

$$F = 1 - \frac{1}{0.12}(0.04)\frac{(-0.007)}{(-0.012)} = 0.806$$

$$C'_{M_0} = C_{M_{\text{ac}_{\text{ab}}}} + F\, V_H\, a_t(i_t + \varepsilon_0)$$

$$C'_{M_0} = -0.003 + (0.806)(0.593)(0.12)(2 + 0) = 0.112$$

This is to be compared with $C_{M_0} = 0.139$ from Problem 7.5 for stick-fixed stability.

$$h'_n = h_{\text{ac}_{\text{wb}}} + F\, V_H \frac{a_1}{a}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

$$h'_n = 0.24 + (0.806)(0.593)\frac{(0.12)}{(0.09)}(1 - 0.42) = 0.609$$

This is to be compared with $h_n = 0.70$ from Problem 7.6 for stick-fixed stability.

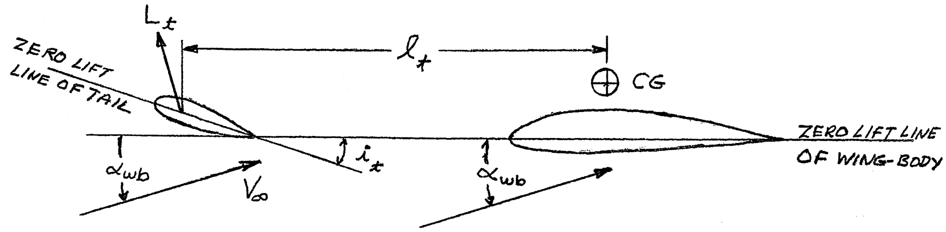
$$h'_n - h = 0.609 - 0.26 = 0.349$$

Note that the static margin for stick-free is 79% of that for stick-fixed.

$$\frac{\partial C'_{M_{\text{cg}}}}{\partial \alpha_a} = -a(h'_n - h) = -(0.09)(0.349) = -0.031$$

This is to be compared with a slope of −0.039 obtained from Problem 7.5 for the stick-fixed case.

7.9



Examining the above figure for a canard, let us trace through the pertinent equations in the text, modifying them appropriately for the canard configuration.

Starting with Eq. (7.12) for the moment generated about the center of gravity due to the tail, the minus sign is replaced by a positive, because the tail is now ahead of the center of gravity, creating a positive moment.

$$M_{\text{cg},t} = \ell_t L_t \tag{1}$$

Thus, Eq. (7.17) becomes

$$C_{M,\text{cg},t} = V_H C_{L,t} \tag{2}$$

Note that the canard tail sees no downwash, and the canard is canted upward relative to the wing-body zero lift link, so Eq. (7.18) for the angle of attack of the tail becomes

$$\alpha_t = \alpha_{\text{wb}} + i_t \tag{3}$$

Therefore, Eq. (7.22) is replaced by

$$C_{M,\text{cg},t} = V_H a_t \alpha_{\text{wb}} + V_H a_t i_t \tag{4}$$

In turn, Eq. (7.24) for the total pitching moment becomes

$$C_{M,\text{cg}} = C_{M,\text{ac}_{\text{wb}}} + C_{L_{\text{wb}}}(h - h_{\text{ac}_{\text{wb}}}) + V_H C_{L,t} \tag{5}$$

Eq. (7.25) becomes

$$C_{M,\text{cg}} = C_{M,\text{ac}_{\text{wb}}} + a_{\text{wb}}\alpha_{\text{wb}}[(h - h_{\text{ac}_{\text{wb}}}) + V_H \frac{a_{\text{t}}}{a_{\text{wb}}}] + V_H a_t i_t \tag{6}$$

Differentiating Eq. (6) with respect to angle of attack, the form that replaces Eq. (7.28) is

$$\frac{\partial C_{M,\text{cg}}}{\partial \alpha_a} = a\left[h - h_{\text{ac}_{\text{wb}}} + V_H \frac{a_t}{a}\right] \tag{7}$$

For static stability, $\dfrac{\partial C_{M,\text{cg}}}{\partial \alpha_a}$ must be negative.

From Eq. (7), therefore,

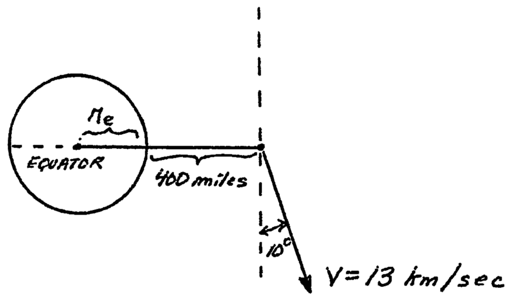
$$(h - h_{\text{ac}_{\text{wb}}}) + V_H \frac{a_t}{a} < 0$$

or, $$(h - h_{\text{ac}_{\text{wb}}}) < V_H \frac{a_t}{a}$$

From inequality (8), for the canard configuration to be statically stable, the center of gravity must be sufficiently far forward of the wing-body aerodynamic center to satisfy the inequality (8). This can readily be done in the design of a canard airplane. Indeed, to help shift the wing-body aerodynamic center rearward, helping to satisfy (8), is the reason that canard designs tend to have the wings shifted farther back on the fuselage compared to conventional rear-tail configurations.

Chapter 8

8.1



$$h_G = 400\,\text{miles} = 0.644 \times 10^6$$

$$r_b = r_e + h_G = 6.4 \times 10^6 + 0.644 \times 10^6 = 7.044 \times 10^6\ \text{m}$$

$$k^2 = 3/986 \times 10^{14}\ \text{m}^3/\text{sec}^2\ \text{from the text}$$

$$h = r^2\theta = rV\theta = r_b V \cos\beta_b = (7.044 \times 10^6)(13 \times 10^3)\cos 10°$$

$$h = 9.018 \times 10^{10}\ \text{m}^2/\text{sec}$$

$$h^2 = 8.133 \times 10^{21}\ \text{m}^4/\text{sec}^2$$

$$p = h^2/k^2 = 8.133 \times 10^{21}/3.986 \times 10^7\ \text{m}$$

This is the numerator of the orbit equation. We now proceed to find the eccentricity. The kinetic energy per unit mass is:

$$T/m = V^2/2 = (13 \times 10^3)^2/2 = 8.45 \times 10^7\ \text{m}^2/\text{sec}^2$$

The potential energy per unit mass is:

$$|\phi/m| = k^2/r_b = 3.98 \times 10^{14}/7.044 \times 10^6 = 5.659 \times 10^7\ \text{m}^2/\text{sec}$$

Hence, $\quad H/m = \left(T - |\phi|\right)/m = (8.45 - 5.659) \times 10^7 = 2.791 \times 10^7\ \text{m}^2/\text{sec}^2$

Thus, the eccentricity is:

$$e = \sqrt{1 + \frac{2h^2}{k^4}\left(\frac{H}{m}\right)} = \sqrt{1 + \frac{2(8.133 \times 10^{21})(2.791 \times 10^7)}{(3.986 \times 10^{14})^2}} = 1.96$$

Obviously, the trajectory is a hyperbola because $e > 1$, and also because $T > |\phi|$. The orbit equation is

$$r = \frac{p}{1 + e\cos(\theta - C)}$$

$$r = \frac{2.04 \times 10^7}{1 + 1.96\cos(\theta - C)}$$

The phase angle, C, is calculated as follows. Substitute the burnout location ($r_b = 7.044 \times 10^6$ m and $\theta = 0°$) into the above equation.

$$7.044 \times 10^6 = \frac{2.04 \times 10^7}{1 + 1.96 \times \cos(-C)}$$

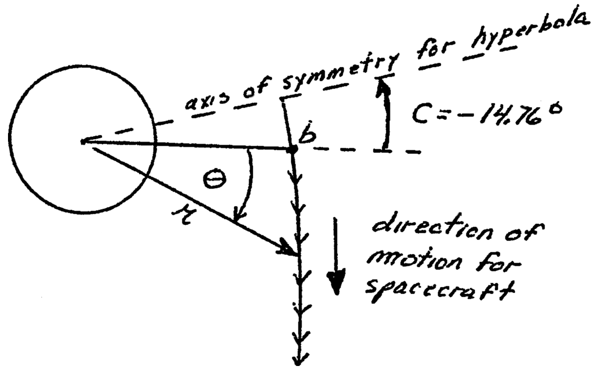
$$\cos(-C) = 0.967$$

Thus $C = -14.76°$.

Hence, the complete equation of the trajectory is

$$\boxed{r = \frac{2.04 \times 10^7}{1 + 1.96 \cos(\theta + 14.76)}}$$

where θ is in degrees and r in meters.



8.2 Escape velocity $= V = \sqrt{2k^2/r}$

$$\text{For Venus: } V = \sqrt{(2)(3.24 \times 10^{14})/6.16 \times 10^6} = 1.03 \times 10^4 \text{ m/sec} = 10.3 \text{ km/sec}$$

$$\text{For Earth: } V = \sqrt{(2)(3.96 \times 10^{13})/6.39 \times 10^6} = 1.11 \times 10^4 \text{ m/sec} = 11.3 \text{ km/sec}$$

$$\text{For Mars: } V = \sqrt{(2)(4.27 \times 10^{13})/10^6} = 5.02 \times 10^3 \text{ m/sec} = 5.02 \text{ km/sec}$$

$$\text{For Jupiter: } V = \sqrt{(2)(1.27 \times 10^{17})/7.14 \times 10^7} = 5.96 \times 10^4 \text{ m/sec} = 56.6 \text{ km/sec}$$

8.3 $G = 6.67 \times 10^{-11}$ m^3/(kg)(sec)^2

$M = 7.35 \times 10^{22}$ kg

$GM = k^2 = (6.67 \times 10^{-11})(7.35 \times 10^{22}) = 4.9 \times 10^{12}$ m^3/sec^2

Orbital velocity is $V = \sqrt{k^2/r}$

$$V_{\text{orbital}} = \sqrt{4.9 \times 10^{12}/1.74 \times 10^6} = 1678 \text{ m/sec} = 1.678 \text{ km/sec}$$

Escape velocity is larger by a factor of $(2)^{1/2}$.

$$V_{\text{escape}} = \sqrt{2}(1.678) = 2.37 \text{ km/sec}$$

8.4 From Kepler's 3rd law,

$$\left(\frac{\tau_1}{\tau_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

$$a_2^3 = (a_1)^3(\tau_2/\tau_1)^2$$

$$a_2 = (a_1)^3(\tau_2/\tau_1)^{2/3}$$

$$a_2 = (1.495 \times 10^{11})(29.7/1.0)^{2/3} = 1.43 \times 10^{12} \text{ m}$$

Please note: The "distant planet" is in reality Saturn.

8.5 In order to remain over the same point on the Earth's equator at all times (assuming the Earth is a perfect sphere), the satellite must have a circular orbit with a period of 24 hours = $8.64.10^4$ sec. As part of the derivation of Kepler's third law in the test, it was shown that

$$\tau^2 = \left(\frac{4\pi^2}{k^2}\right) a^3 = \left(\frac{4\pi^2}{k^2}\right) r^3 \qquad \text{(Remember, the orbit is a circle.)}$$

Hence,

$$r = \left(\frac{k^2}{4\pi^2}\right)^{1/3} \tau^{2/3}$$

$$r = \left(\frac{3.956 \times 10^{14}}{4\pi^2}\right)^{1/3} (8.64 \times 10^4)^{2/3} = 4.21 \times 10^7 \text{ m}$$

The radius of the Earth is 6.4×10^6 m. Hence, the altitude above the surface of the Earth is

$$h_G = 4.21 \times 10^7 - 6.4 \times 10^6 = 3.57 \times 10^7 \text{ m} = 35,700 \text{ km}$$

Circular velocity is $V = \sqrt{\dfrac{k^2}{r}} = \sqrt{\dfrac{3.956 \times 10^{14}}{4.21 \times 10^7}} = 3065 \text{ m/sec}$

8.6 $m = \rho v = \rho\left(\frac{4}{3}\pi r^3\right) = (6963)\left(\frac{4}{3}\pi\right)(0.8)^3 = 1.4933 \times 10^4 \text{ kg}$

$$S = \pi r^2 = \pi(0.8)^2 = 2.01 \text{ m}^2$$

$$\frac{m}{C_D S} = \frac{1.4933 \times 10^4}{(1)(2.01)} = 7429 \text{ km/m}^3$$

$$Z = g_0/RT = 9.8/(287)(29/88) = 0.000118 \text{ m}^{-1}$$

From Eq. (8.97), the density at the altitude for maximum deceleration is

$$\rho = \frac{m}{C_D S} Z \sin\theta = (7429)(0.000118)\sin 30° = 0.4383 \frac{\text{kg}}{\text{m}^3}$$

From Eq. (8.73)

$$h = -\frac{1}{Z}\ell n(\rho/\rho_0) = -\frac{1}{0.000118}\ell n\left(\frac{0.4383}{1.225}\right) = 8710 \text{ m}$$

From Eq. (8.100)

$$\left|\frac{dV}{dt}\right|_{\max} = \frac{V_E^2 Z \sin\theta}{2e} = \frac{(8000)^2(0.000118)(\sin 30°)}{2e} = 694.6 \text{ m/sec}^2$$

In terms of *gs*:

$$\left|\frac{dV}{dt}\right|_{\max} = \frac{694.6}{9.8} = 70.88\ gs$$

From Eq. (8.87)

$$V/V_E = e^{-\left[\frac{(1.225)}{2(7429)\times(0.000118)\sin 30°}\right]} = 0.247$$

$$V = 0.247\, V_E = 0.247(8000) = 1978 \text{ m/sec}$$

8.7 Aerodynamic heating varies as V_∞^3. Hence:

$$\frac{q_2}{q_1} = \left(\frac{36{,}000}{27{,}000}\right)^3 = 2.37$$

$$q_2 = (100)(2.37) = 237 \text{ Btu/(ft}^2\text{)(sec)}$$

8.8 Assume the mean velocity of the earth around the sun is essentially its circular orbital velocity given by

$$V_{\text{earth}} = \sqrt{\frac{k^2}{r}} = 29.77 \times 10^3 \text{ m/sec}$$

where $k^2 = GM$, and M is the mass of the sun. From Eq. (1)

$$k^2 = V^2 r = (29.77 \times 10^3)^2 (147 \times 10^9) = 1.30 \times 10^{20} \text{ m}^3\text{/sec}^2$$

The asteroid is moving at 0.9 times the escape velocity from the sun.

$$V_{\text{asteroid}} = 0.9\sqrt{\frac{2k^2}{r}}$$

To intersect with the earth, the asteroid's value of r in Eq. (2) is the same as far for the earth in Eq. (1). Hence,

$$V_{\text{asteroid}} = 0.9\sqrt{\frac{2(1.3 \times 10^{20})}{(147 \times 10^9)}} = 37.850 \times 10^8 \text{ m/sec}$$

The relative head-on collision velocity, which is the velocity at which the asteroid would enter the earth's atmosphere, is

$$V_{\text{entry}} = V_{\text{earth}} + V_{\text{asteroid}} = 29.77 \times 10^3 + 37.85 \times 10^3 = 67.62 \times 10^3 \text{ m/sec}$$
$$= \boxed{67.62 \text{ km/sec}}$$

8.9 Earth's radius $= R = 6.4 \times 10^6$ m

Following the nomenclature in Figure 8.18,

$r_{\min}$ = R + (altiude above Earth's surface)

$$r_{\min} = 6.4 \times 10^6 + 4.17 \times 10^5 = 6.817 \times 10^6 \text{ m}$$

From Eq. (8.62)

$$r_{\min} = \frac{h^2/k^2}{1+e} \tag{1}$$

and from Eq. (8.61)

$$r_{\max} = \frac{h^2/k^2}{1-e} \tag{2}$$

Combining (1) and (2)

$$\frac{r_{\max}}{r_{\min}} = \frac{1+e}{1-e} = \frac{1.00132}{0.99868} = 1.0026$$

$$r_{\max} = r_{\min}(1.0026) = (6.817 \times 10^6)(1.0026) = 6.8347 \times 10^6 \text{ m}$$

Altitude at apogee $= 6.8347 \times 10^6 - 6.4 \times 10^6 = 0.4347 \times 10^6 \text{ m} = \boxed{434.7 \text{ km}}$

8.10 From Eq. (8.71),

$$\tau^2 = \frac{4\pi^2}{k^2} a^3$$
$$k^2 = 3.956 \times 10^{14} \text{ m}^3/\text{sec}^2$$

From the solution of Problem 8.9,

$$r_{\max} = 6.8347 \times 10^6 \text{ m}$$
$$r_{\min} = 6.817 \times 10^6 \text{ m}$$

$$a = \frac{r_{\max} + r_{\min}}{2} = \frac{(6.8347 + 6.817) \times 10^6}{2} = 6.82585 \times 10^6 \text{ m}$$

Inserting these numbers into Eq. (8.71):

$$\tau^2 = \frac{4\,\pi^2}{k^2} a^3 = \frac{4\pi^2 (6.82585 \times 10^6)^3}{3.956 \times 10^{14}}$$
$$\tau^2 = 3.1738 \times 10^7$$
$$\tau = 5634 \text{ sec}$$

or, $$\tau = \frac{5634}{3600} = \boxed{1.56 \text{ hr}}$$

8.11 The angular momentum per unit mass is $h = r^2\dot{\theta}$. To calculate h, proceed as follows. From Eq. (8.62),

$$r_{\min} = \frac{h^2 / k^2}{1 + e}$$

or,

$$h^2 = r_{\min}(1 + e)k^2$$

From the solution of Problem 8.9, we have $r_{\min} = 6.817 \times 10^6$ m. Thus,

$$h^2 = (6.817 \times 10^6)(1.00132)(3.956 \times 10^{14}) = 2.7 \times 10^{21}$$

or, $h = 5.1965 \times 10^{10}$

Because $h = r^2\theta,$ at perigee

$$\dot{\theta} = \frac{h}{(r_{\min})^2} = \frac{5.1965 \times 10^{10}}{(6.817 \times 10^6)^2} = 1.1182 \times 10^{-3} \text{ rad/sec}$$

$$V_{\text{perigee}} = r_{\min}\dot{\theta} = (6.817 \times 10^6)(1.1182 \times 10^{-3}) = 7623 \text{ m/sec} = \boxed{7.623 \text{ km/sec}}$$

8.12 **(a)** $E_t = \frac{V^2}{2} - \frac{k^2}{r}$

Evaluate this equation at the perigee, where from Example 8.3, $r_p = 7.169 \times 10^6$ m, and $V_p = 9.026$ km/sec

$$E_t = \frac{V_p^2}{2} - \frac{k^2}{r_p} = \frac{(9.026 \times 10^3)^2}{2} - \frac{3.986 \times 10^{14}}{7.169 \times 10^6} =$$

$$4.0734 \times 10^{-7} - 5.56 \times 10^7 = -1.486 \times 10^{-7} \frac{\text{joules}}{\text{kg}}$$

(b) From Example 8.3, $a = 1.341 \times 10^7$ m. Thus

$$E_t = -\frac{k^2}{2a} = -\frac{3.986 \times 10^{14}}{2(1.341 \times 10^7)} = -1.486 \times 10^{-7} \frac{\text{joules}}{\text{kg}}$$

The results are the same, which is to be expected. Eqs. (8.74) and (8.77) are numerically the same.

8.13 From Example 8.6, $V_\theta = 3432$ m/sec.

$$\Delta V = 2V_\theta \sin\left(\frac{v}{2}\right) = 2(3432) \sin 10° = 1191.9 \text{ m/sec}$$

Comparing with the impulse calculated in Example 8.6 for a change in inclination angle of 10°, the present impulse is 1191.9/598.2 = 1.99 larger – almost twice as large for double the inclination angle change. We would expect this because, for relatively small angles (in radians)

$$\sin\left(\frac{v}{2}\right) \approx \frac{v}{2}$$

8.14 From Example 8.7, we already have

$$V_1 = 6794 \text{ m/sec}$$

and

$$r_2 = r_1 = 1.0506 \times 10^7 \text{ m}$$

$$a_2 = \frac{r_{p.2}}{1-e} = \frac{10^7}{1-0.8} = 5 \times 10^7 \text{ m}$$

Thus,

$$V_2 = \sqrt{\frac{2k^2}{r_2} - \frac{k^2}{a^2}} = \sqrt{\frac{2(3.986 \times 10)^{14}}{1.0506 \times 10^7} - \frac{3.986 \times 10^{14}}{5 \times 10^7}}$$

$$V_2 = 8241 \text{ m/sec}$$

$$\text{Tan } \beta_1 = \frac{e_1 \sin \theta_A}{1 + e_1 \cos \theta_A} = \frac{(0.4654) \sin 90°}{1 + 0.4654 \cos 90°} = 0.4654$$

$$\beta_1 = 24.957°$$

For orbit 2, the true anomaly of point A is found from

$$\cos \theta_A = \frac{r_{p,z}(1 + e_2)}{e_2 r_2} - \frac{1}{e_2}$$

$$\cos \theta_A = \frac{1 \times 10^7 (1 + 0.8)}{(0.8)(1.0506 \times 10^7)} - \frac{1}{0.8} = 2.1416 - 1.25 = 0.8916$$

$$\theta_A = 26.93°$$

Thus,

$$\text{Tan } \beta_2 = \frac{e_2 \sin \theta_A}{1 + e_2 \cos \theta_2} = \frac{(0.8)(0.4529)}{1 + (0.8)(0.8916)} = \frac{0.3623}{1.7133} = 0.2115$$

$$\beta_2 = 11.94°$$

$$\alpha = \beta_1 - \beta_2 = 24.957 - 11.94 = 13.017°$$

From Eq. (8.89),

$$(\Delta V)^2 = V_1^2 + V_2^2 - 2V_1 V_2 \cos\alpha$$

$$= (6794)^2 + (8241)^2 - 2(6794)(8241) \cos (13.017°)$$

$$= 14.379 \times 10^7 - 10.91 \times 10^7 = 3.469 \times 10^7$$

Thus,

$$\Delta V = 5890 \text{ m/sec}$$

8.15 From Example 8.8,

$$V_1 = 7771 \text{ m/sec}$$

and

$$r_1 = 6.6 \times 10^6 \text{m}$$

$$r_2 = 6.4 \times 10^6 + 5 \times 10^5 = 6.9 \times 10^6 \text{m}$$

$$a = \frac{r_1 + r_2}{2} = \frac{6.6 \times 10^6 + 6.9 \times 10^6}{2} = \frac{13.5 \times 10^6}{2} = 6.75 \times 10^6 \text{m}$$

From Eq. (8.78),

$$V_{\text{pt}} = \sqrt{\frac{2k^2}{r_1} - \frac{k^2}{a}} = \sqrt{\frac{2(3.986 \times 10)^{14}}{6.6 \times 10^6} - \frac{(3.986 \times 10^{14})}{6.75 \times 10^6}}$$

$$= \sqrt{1.2079 \times 10^8 - 0.5905 \times 10^8} = 7857.5 \text{ m/sec}$$

At point 1, the required impulse to enter the Hohmann transfer orbit is

$$\Delta V_1 = V_{\text{pt}} - V_1 = 7857.5 - 7771 = 86.5 \text{ m/sec.}$$

At point 2 on orbit 2, the required spacecraft velocity is

$$V_2 = \sqrt{\frac{k^2}{r_1}} = \sqrt{\frac{3.986 \times 10^{14}}{6.9 \times 10^6}} = 7600 \text{ m/sec}$$

At point 2 on the Hohmann transfer orbit,

$$V_{\text{at}} = \sqrt{\frac{2k^2}{r_1} - \frac{k^2}{a}} = \sqrt{\frac{2(3.986 \times 10^{14})}{6.9 \times 10^6} - \frac{3.986 \times 10^{14}}{6.75 \times 10^6}}$$

$$= \sqrt{1.1554 \times 10^8 - 0.5905 \times 10^8} = 7516 \text{ m/sec}$$

At point 2,

$$\Delta V_2 = V_2 - V_{\text{at}} = 7600 - 7516 = 84 \text{ m/sec}$$

The total impulse required is

$$\Delta V = \Delta V_1 + \Delta V_2 = 86.5 + 84 = 170.5 \text{m/sec}$$

8.16 From Problem 8.2, for Mars,

$$k^2 = 4.27 \times 10^{13} \text{m}^3/\text{sec}^2$$

$$V_1 = \sqrt{\frac{k^2}{r_1}} = \sqrt{\frac{4.27 \times 10^{13}}{8 \times 10^6}} = 2310 \text{ m/sec}$$

$$a = \frac{r_1 + r_2}{2} = \frac{8 \times 10^6 + 15 \times 10^6}{2} = 11.5 \times 10^6 \text{m}$$

$$V_{\text{pt}} = \sqrt{\frac{2k^2}{r_1} - \frac{k^2}{a}} = \sqrt{\frac{2(4.27 \times 10^{13}}{8 \times 10^6} - \frac{4.27 \times 10^{13}}{11.5 \times 10^6}}$$

$$= \sqrt{1.0675 \times 10^7 - 0.3713 \times 10^7} = 2638.6 \text{ m/sec}$$

At point 1,

$$\Delta V_1 = V_{\text{pt}} - V_1 = 2638.6 - 2310 = 328.6 \text{ m/sec}$$

At point 2 on orbit 2,

$$V_2 = \sqrt{\frac{k^2}{r^2}} = \sqrt{\frac{4.27 \times 10^{13}}{15 \times 10^6}} = 1687 \text{ m/sec}$$

At point 2 on the Hohmann transfer orbit,

$$V_{\text{at}} = \sqrt{\frac{2k^2}{r^2} - \frac{k^2}{a}} = \sqrt{\frac{2(4.27 \times 10^{13})}{15 \times 10^6} - \frac{4.27 \times 10^{13}}{11.5 \times 10^6}}$$

$$= \sqrt{5.693 \times 10^6 - 3.713 \times 10^6} = 1407 \text{ m/sec}$$

$$\Delta V_2 = V_2 - V_{\text{at}} = 1687 - 1407 = 280 \text{ m/sec.}$$

The total required impulse is

$$\Delta V = \Delta V_1 + \Delta V_2 = 328.6 + 280 = 608.6 \text{ m/sec}$$

8.17 Relative to the center of mercury, at periapsis,

$$r_{\min} = 2,440 + 200 = 2640 \text{ km}$$
$$r_{\max} = 2,440 + 15,193 = 17,633 \text{ km}$$

The semi-major axis of the elliptical orbit is

$$a = \frac{r_{\min} + r_{\max}}{2} = \frac{2640 + 17,633}{2} = 10,137 \text{ km}$$
$$= 1.0137 \times 10^7 \text{ m}$$
$$k^2 = GM = \left(6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg sec}^2}\right)(3.3 \times 10^{23} \text{ kg})$$
$$k^2 = 2.20 \times 10^{13} \frac{\text{m}^3}{\text{sec}^2}$$

From Eq. 8.71 in the text:

$$\tau^2 = \frac{4\pi^2 a^3}{k^2} = \frac{4\pi^2 (1.0137 \times 10^7)^3}{2.20 \times 10^{13}} = 1.87 \times 10^9 \text{ sec}^2$$

Hence,

$$\tau = 43243 \text{ sec}$$

Since 3600 sec = 1 hr, we have

$$\tau = \frac{43243}{3600} = \boxed{12.01 \text{ hr}}$$

8.18 From the solution of Problem 8.17, the semi-major axis is $a = 1.0137 \times 10^7$m, and $k^2 = 2.20 \times 10^{13}$ m^3/sec^2. At periapsis, $r_{\min} = 2640$ km $= 2.64 \times 10^6$ m. From the vis-viva equation, Eq. (8.78) in the text,

$$V = \sqrt{\frac{2k^2}{r} - \frac{k^2}{a}}$$

For $r = r_{\min}$, the velocity at periapsis is

$$V = \sqrt{\frac{2(2.20 \times 10^{13})}{2.64 \times 10^6} - \frac{2.20 \times 10^{13}}{1.0137 \times 10^7}} = \sqrt{16.67 \times 10^6 - 2.17 \times 10^6}$$
$$= 3809 \text{ m/sec} = \boxed{3.809 \text{ km/sec}}$$

For $r = r_{\max}$, the velocity at apoapsis is

$$V = \sqrt{\frac{2(2.20 \times 10^{13})}{1.7633 \times 10^7} - 2.17 \times 10^6} = \sqrt{2.495 \times 10^6 - 2.17 \times 10^6}$$
$$= 570 \text{ m/sec} = \boxed{0.57 \text{ km/sec}}$$

8.19 From the solution of Problem 8.18 and 8.19, at periapsis $V = 3809$ m/sec and $r_{min} = 2.64 \times 10^6$m. Thus,

$$\dot{\theta} = \frac{V}{r_{min}} = \frac{3809}{2.64 \times 10^6} = 1.44 \times 10^{-3} \text{ rad/sec}$$

The angular momentum per unit mass is $h = r^2\ \dot{\theta}$. Thus,

$$h = (r_{min})^2 \dot{\theta} = (2.64 \times 10^6)^2\ (1.44 \times 10^{-3})$$

$$h = \boxed{1.00 \times 10^{10}\ \text{m}^2/\text{sec}}$$

The same value is obtained at the apoapsis, where

$$\dot{\theta} = \frac{V}{r_{max}} = \frac{570}{1.7633 \times 10^7} = 3.2326 \times 10^{-5} \text{ rad/sec}$$

$$h = (r_{max})^2 \dot{\theta} = (1.7633 \times 10^7)^2\ (3.2326 \times 10^{-5})$$

$$h = 1.00 \times 10^{10}\ \text{m}^2/\text{sec}.$$

This, of course, is simply verification that the angular momentum is constant for the orbit.

8.20 Eq. (8.63) is

$$a = \frac{h^2/k^2}{1-e^2}$$

or

$$e = \sqrt{1 - \frac{h^2/k^2}{a}}$$

From Problems 8.17 and 8.19,

$$\frac{h^2}{k^2} = \frac{(1.00 \times 10^{10})^2}{2.20 \times 10^{13}} = 0.4545 \times 10^7$$

$$e = \sqrt{1 - \frac{0.4545 \times 10^7}{1.0137 \times 10^7}} = \sqrt{1 - 0.448} = \boxed{0.743}$$

8.21 Notice that the equations for a spacecraft's trajectory in space as derived in Chapter 8 depend only on such quantities as momentum or energy <u>per unit mass</u>. The actual mass of the spacecraft plays a role only in the rocket booster burnout conditions at the end of launch, as shown in Section 9.10. Once the initial burnout conditions are achieved, the subsequent trajectory is governed only by the force of gravity and Newton's second law, for which energy and momentum per unit mass are relevant. So, if you know the complete details of the orbit of a spacecraft, it is still not possible to back out the total mass of the spacecraft.

Chapter 9

9.1 Following the nomenclature in the text;

$$\frac{p_3}{p_2} = \left(\frac{V_2}{V_3}\right)^{\gamma} = 6.75^{1.4} = 13.0$$

$$p_3 = (13.0)(1.0) = 13.0 \text{ atm}$$

$$\frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} = (6.75)^{0.4} = 2.15$$

$$T_3 = (2.15)(285) = 613°\text{K}$$

The heat released per kg of fuel-air mixture is established in the text as 2.43×10^6 joule/kg. Hence,

$$T_4 = \frac{q}{c_v} + T_3 \text{ where } c_v = 720 \text{ joule/kg°K}$$

$$T_4 = \frac{2.43 \times 10^6}{720} + 613 = 3988°\text{K}$$

$$\frac{p_4}{p_3} = \frac{T_4}{T_3}$$

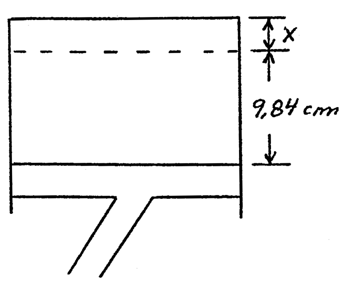
Thus, $p_4 = p_3\left(\frac{T_4}{T_3}\right) = (13.0)\left(\frac{3988}{613}\right) = 84.6 \text{ atm}$

$$\frac{p_5}{p_4} = \left(\frac{V_4}{V_5}\right)^{\gamma} = \left(\frac{1}{6.75}\right)^{1.4} = 0.069$$

$$p_5 = (84.6)(0.069) = 5.84 \text{ atm}$$

$$W_{\text{compression}} = \frac{p_2V_2 - p_3V_3}{1-\gamma}$$

The volumes V_2 and V_3 can be obtained as follows



$$(x + 9.84)/x = 6.75, \text{Thus, } x = 1.71 \text{ cm}$$

Thus,

$$V_2 = \frac{\pi b^2}{4}(9.84 + 1.71) \text{ where } b = \text{bore} = 11.1 \text{ cm}$$

$$V_2 = 1118 \text{ cm}^3 = 1.118 \times 10^{-3} \text{m}^3$$

$$V_3 = V_2/6.75 = 1.118 \times 10^{-3}/6.75 = 1.66 \times 10^{-4} \text{m}^3$$

Thus,

$$W_{\text{compression}} = \frac{p_2V_2 - p_3V_3}{1-\gamma}$$

$$= \frac{\left[(1.0)(1.118 \times 10^{-3}) - (13.0)(1.66 \times 10^{-4})\right]1.01 \times 10^5}{-0.4} = 263 \text{ joule}$$

$$W_{\text{power}} = \frac{p_5V_5 - p_4V_4}{1-\gamma}$$

$$= \frac{\left[(5.84)(1.118 \times 10^{-3}) - (84.6)(1.66 \times 10^{-4})\right]1.01 \times 10^5}{-0.4}$$

$$W_{\text{power}} = 1897 \text{ joule}$$

$$W = W_{\text{power}} - W_{\text{comperssion}} = 1897 - 263 = 1634 \text{ joule}$$

$$P_A = \frac{1}{120}\,\eta\eta_{\text{mech}}\,(\text{RPM})\text{ N W}$$

$$P_A = \frac{(0.85)(0.83)(2800)(4)(1634)}{120} = 1.076 \times 10^5 \text{ watts}$$

or $$P_A = \frac{1.076 \times 10^5 \text{ watts}}{746 \text{ watts/hp}} = 144 \text{ hp}$$

9.2 $$P_A = \frac{1}{120}\,\eta\eta_{\text{mech}}\,(\text{RPM})\,Dpe$$

where $$D = \frac{\pi b^2}{4} s\,N = \frac{\pi(11.1)^2(984)(4)}{4}$$

$$D = 3809 \text{ cm}^3 = 3.809 \times 10^{-3}\text{m}^3$$

$$p_e = \frac{120\,P_A}{\eta\eta_{\text{mech}}(\text{RPM})D}$$

$$p_e = \frac{120(1.076 \times 10^5)}{(0.85)(0.83)(2800)(3.809 \times 10^{-3})}$$

$$p_e = 1.72 \times 10^6 \text{ N/m}^2 = 17 \text{ atm}$$

9.3 First, calculate the mass flow through the engine. For simplicity, we will ignore the mass of the added fuel, and assume the mass flow from inlet to exit is constant. Evaluating conditions at the exit,

$$\rho_e = \frac{p_e}{RT_e} = \frac{1.01 \times 10^5}{287\ (750)} = 0.469 \text{ kg/m}^3$$

$$\dot{m} = \rho e A_e V_e = (0.469)(0.45)(400) = 84 \text{ kg/sec}$$

At the inlet, assume standard sea level conditions.

$$\dot{m} = \rho_\infty A_i V_i = 84 \text{ kg/sec}$$

Hence,

$$V_j = \frac{\dot{m}}{\rho_\infty A_i} = \frac{84}{(1.225)(0.45)} = 152 \text{ m/sec}$$

Note: Even though the engine is stationary, it is sucking air into the inlet at such a rate that the streamtube of air entering the engine is accelerated to a velocity of 152 m/sec at the inlet. The Mach number at the inlet is approximately 0.45. Therefore, by making the assumption of standard sea level density at the inlet, we are making about a 10 percent error in the calculation of V_i. To obtain the thrust,

$$T = \dot{m}\,(V_e - V_j) + (p_e - p_\infty)A_e = 84(400 - 152) + (0)A_e = 20{,}832 \text{ N}$$

Since 1 lb = 4.448 N, we also have

$$T = \frac{20{,}832}{4.448} = 4684 \text{ lb}$$

9.4 At a standard altitude of 40,000 ft,

$$p_\infty = 393.12 \text{ lb/ft}^2$$

$$\rho_\infty = 5.8727 \times 10^{-4} \text{ slug/ft}^3$$

The free stream velocity is

$$V_\infty = 530\left(\frac{88}{60}\right) = 777 \text{ ft/sec}$$

Hence,

$$\dot{m} = \rho_\infty V_\infty A_j = (5.87 \times 10^{-4})(777)(13) = 5.93 \text{ slug/sec}$$

$$T = \dot{m}\,(V_e - V_\infty) + (p_e - p_\infty)\,A_e$$

$$T = (5.93)(1500 - 777) + (450 - 393)(10)$$

$$T = 4287 + 570 = 4587 \text{ lb}$$

9.5 Assume the Mach number at the end of the diffuser (hence at the entrance to the compressor) is close to zero, hence p_2 can be assumed to be total pressure.

$$M_1 = \frac{V_1}{a_1} = \frac{800}{1117} = 0.716$$

$$\frac{p_2}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1{}^2\right)^{\gamma/\gamma-1} = [1 + (0.2)(0.616)^2]^{3.5} = 1.41$$

Hence, $$\frac{p_3}{p_1} = \frac{p_3}{p_2}\frac{p_2}{p_1} = (12.5)(1.41) = 17.6$$

As demonstrated on the pressure-volume diagram for an ideal turbojet, the compression process is isentropic. Hence,

$$\frac{p_3}{p_1} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$t_3 = t_1\left(\frac{p_3}{p_1}\right)^{\frac{\gamma}{\gamma-1}} = 519(17.6)^{0.286} = 1178°$$

Combustion occurs at constant pressure. For each slug of air entering the combustor, 0.05 slug of fuel is added. Each slug of fuel releases a chemical energy (heat) of $(1.4 \times 10^7)(32.3) = 4.51\,\text{s}\,10^8$ ft lb/slug . Hence, the heat released per slug of fuel-air mixture is

$$q = \frac{(4.51 \times 10^8)(0.05)}{1.05} = 2.15 \times 10^7 \text{ ft lb/slug}$$

Because the heat is added at constant pressure,

$$q = c_p(T_4 - T_3)$$

$$T_4 = T_3 + \frac{q}{c_p}$$

For air, $$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4(1716)}{0.4} = 6006\frac{\text{ft lb}}{\text{slug°}R}$$

$$T_4 = 1178 + \frac{2.15 \times 10^7}{6006} = 1178 + 3580 = 4758°R$$

Again, from the *p-v* diagram for an ideal turbojet,

$$\frac{p_6}{p_4} = \frac{p_1}{p_3} = \frac{1}{17.6} = 0.0568$$

Also, $$\frac{p_6}{p_4} = \left(\frac{T_6}{T_4}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_6 = T_4\left(\frac{p_6}{p_4}\right)^{\frac{\gamma}{\gamma-1}} = 4758\,(0.0568)^{0.286} = 2095°R$$

Note: The temperatures calculated in this problem exceed those allowable for structural integrity. However, in real life, the losses due to heat conduction will decrease the temperature. Also, the fuel-air ratio would be decreased in order to lower the temperatures in an actual application.

9.6 $$T = \dot{m}(V_e - V_\infty) + (p_e - p_\infty)A_e$$
$$1000 = \dot{m}(2000 - 950) + 0$$
$$\dot{m} = 0.952 \text{ slug/sec}$$

However, $\dot{m} = \rho_\infty V_\infty A_i$

Thus, $A_i = \dfrac{\dot{m}}{\rho_\infty V_\infty} = \dfrac{0.952}{(0.002377)(950)} = 0.42 \text{ ft}^2$

9.7 At 50 km, $p_\infty = 87.9 \text{ N/m}^2$

$$T = \dot{m}V_e + (p_e - p_\infty)A_e = (25)(4000) + (2 \times 10^4 - 87.9)(2)$$
$$= 100{,}000 + 39{,}824 = 139{,}824 \text{ N}$$

Since $1 \text{ lb} = 4.48 \text{ N}$

$$T = \frac{139824}{4.448} = 31{,}435 \text{ lb}$$

9.8 **(a)** At a standard altitude of 25km,

$$p_\infty = p_e = 2527 \text{ N/m}^2$$
$$I_{sp} = \frac{1}{g_0}\sqrt{\frac{2(1.18)(8314)(3756)}{(0.18)(20)}\left[1 - \left(\frac{2527}{3.03 \times 10^6}\right)^{\frac{0.18}{1.18}}\right]}$$
$$I_{sp} = 375 \text{ sec}$$

(b) From the isentropic relations,

$$\frac{p_e}{p_0} = \left(\frac{T_e}{T_0}\right)^{\frac{\gamma}{\gamma-1}}$$
$$T_e = T_0\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}} = 3756\left(\frac{2527}{3.03 \times 10^6}\right)^{0.1525} = 1274°\text{K}$$
$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.18)(8314)}{(0.18)(20)} = 2725 \text{ joule/kg°K}$$

From the energy equation

$$V_e = \sqrt{2c_p(T_0 - T_e)} = \sqrt{2(2725)(3756 - 1274)} = 3678 \text{ m/sec}$$

(c) $\rho_e = \dfrac{p_e}{RT_e}$ where $R = \dfrac{R}{M} = \dfrac{8314}{20} = 415.7\dfrac{\text{joule}}{\text{kg°K}}$

$$\rho_e = \frac{2527}{(415.7)(1274)} = 0.00477 \text{ kg/m}^3$$
$$\dot{m} = \rho_e A_e V_e = (0.00477)(15)(3678) = 263.5 \text{ kg/sec}$$

(d) From the definition of specific impulse,

$$I_{sp} = \frac{T}{\dot{w}} \quad \text{where } \dot{w} \text{ is the weight flow}$$

$$\dot{w} = \dot{m}g_0 = (263.5)(9.8) = 2582 \text{ N/sec}$$

Hence,

$$T = \dot{w}I_{\text{sp}} = (2582)(375) = 968{,}250\text{N}$$

$$\text{or,} \quad T = \frac{968250}{4.448} = 217682 \text{ lb}$$

(e) $$\dot{m} = \frac{p_0 A^*}{\sqrt{T_\text{o}}} \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$263.5 = \frac{(30(1.01 \times 10^5))A^*}{(3756)^{1/2}} \sqrt{\frac{1.18}{415.7}\left(\frac{2}{2.18}\right)^{12.11}}$$

$$A^* = 0.169 \text{ m}^2$$

9.9 $\dot{m} = 87.6/32.2 = 2.72$ slug/sec

$$\dot{m} = \frac{p_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{\text{R}}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}$$

$$2.72 = \frac{p_0^{(0.5)}}{\sqrt{6000}} \sqrt{\frac{1.21}{2400}\left(\frac{2}{2.21}\right)^{10.52}}$$

$$p_0 = 31{,}736 \text{ lb/ft}^2$$

In terms of atmospheres,

$$p_0 = \frac{31{,}736}{2116} = 15 \,\text{atm}$$

9.10 $V_b = g_0 I_{\text{sp}} \,\ell n \dfrac{M_i}{M_f} = (9.8)(240)\,\ell n\, 5.5 = 4009.6$ m/sec

9.11 $M_f = M_i - M_p$

$$\frac{M_i}{M_f} = \frac{M_i}{M_i - M_p} = \frac{1}{1 - \dfrac{M_p}{M_i}}$$

$$\frac{M_i}{M_f} = e^{V_b / g_0 I_{\text{sp}}} = e^{(11{,}200)/(9.8)(360)} = 23.9$$

where escape velocity is 11.2 km/sec = 11,200 m/sec.

$$23.9 = \frac{1}{1 - \dfrac{M_p}{M_i}}$$

$$\frac{M_p}{M_i} = 0.958$$

9.12 From Eq. (9.43)

$$r = ap_0^n$$

$$\log r = \log a + n \log p_0$$

At $p_0 = 500$ lb/in^2: $\log(0.04) = \log a + n \log(500)$

or: $\log a = -1.3979 - 2.69897n$ (1)

At $p_0 = 1000$ lb/in^2: $\log(0.058) = \log a + n\log(1000)$

or: $\log a = -1.23657 - 3n$ (2)

Solving Eqs. (1) and (1) for a and n, we have

$$0 = -0.16133 + 0.03103n$$

$$n = \frac{0.16133}{0.30103} = 0.5359$$

$$\log a = -1.23657 - 3(0.5359) = -2.84427$$

$$a = 0.0014313$$

Hence, the equation for the linear burning rate is

$$r = 0.0014313\, p_0^{0.5359}$$

For $p_0 = 1500\,\text{lb/in}^2$:

$$r = 0.0014313(1500)^{0.5359} = 0.0721 \text{ in/sec.}$$

In 5 seconds, the total distance receded by the burning surface is 0.0721 (5) = 0.36 in.

9.13

$$V_{b_1} = g_0\, I_{sp}\, \ell n\left[\frac{M_{p_1} + M_{s_1} + M_{p_2} + M_{s_2} + M_L}{M_{s_1} + M_{p_2} + M_{s_2} + M_L}\right]$$

$$= (9.8)(275)\, \ell n \frac{7200 + 800 + 5400 + 600 + 60}{800 + 5400 + 600 + 60}$$

$$= 1934 \text{ m/sec}$$

$$V_{b_2} - V_{b_1} = g_0\, I_{sp}\, \ell n\left[\frac{M_{p2} + M_{s2} + M_L}{M_{s_2} + M_L}\right]$$

$$= (9.8)(275)\, \ell n \frac{5400 + 600 + 60}{600 + 60}$$

$$= 5975 \text{ m/sec}$$

Hence: $V_{b_2} = 5975 + 1934 = 7909$ m/sec

9.14 The velocity of the jet relative to you is $(V_e - V_\infty)$. This is the velocity you feel of the "wind" left behind in the air after the device passes through your space. The energy it has is kinetic space. The energy it has is kinetic energy, which per unit mass is

$$\frac{1}{2}(V_e - V_\infty)^2.$$

Hence, the kinetic energy deposited per unit time is

$$\frac{1}{2}\dot{m}\,(V_e - V_\infty)^2.$$

9.15 Power available $= T\,V_\infty$ (1)

From Eq. (9.24), ignoring the pressure thrust term and assuming that m is essentially $\dot{m}_{\text{air}_\text{a}}$

$$T = \dot{m}\,(V_e - V_\infty) \tag{2}$$

The total power generated by the propulsive device is the useful power plus the wasted power. Hence,

$$\text{Total power generated} = T\,V_\infty + \frac{1}{2}\dot{m}(V_e - V_\infty)^2 \tag{3}$$

Thus,

$$\eta_p = \frac{\text{useful power available}}{\text{total power generated}} = \frac{T\,V_\infty}{T\,V_\infty + \frac{1}{2}\dot{m}\left(V_e - V_\infty\right)^2} \tag{4}$$

Substituting Eq. (2) into (4),

$$\eta_p = \frac{\dot{m}\,(V_e - V_\infty)\,V_\infty}{\dot{m}(V_e - V_\infty)\,V_\infty + \frac{1}{2}\dot{m}\left(V_e - V_\infty\right)^2} \tag{5}$$

Dividing numerator and denominator by m $(V_e - V_\infty)V_\infty$, Eq. (5) becomes

$$\eta_p = \frac{1}{1 + \frac{1}{2}(V_e - V_\infty)/V_\infty} = \frac{1}{\frac{1}{2}\left(1 + \frac{V_e}{V_\infty}\right)}$$

or,

$$\eta_p = \frac{2}{1 + V_e/V_\infty}$$

9.16 From Example 9.3,

$$V_\infty = 500 \text{ miles/hour} = 733 \text{ ft/sec}$$

And

$$V_e = 1600 \text{ ft/sec}$$

$$\eta_p = \frac{2}{1+V_e/V_\infty} = \frac{2}{1 + \left(\frac{1600}{733}\right)} = \boxed{0.63}$$

9.17 Briefly:

(a) The flow velocity increase across the propeller is small, hence V_e is only slightly larger than V_∞, and η_p is high.

(b) The exit velocity of the gas from a rocket engine is supersonic. Hence, V_e is very high compared to the velocity of the rocket, V_∞, and η_p is low.

(c) The exit velocity from a gas turbine jet engine is usually a very high subsonic value, and sometimes a supersonic value just above one. Hence, it is less efficient than a propeller, but considerably more efficient than a rocket engine.

9.18 The temperature entering the compressor is $T_2 = 585°R$. The temperature at the exit of the compressor, T_3, is obtained from the isentropic relation

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = (11.7)^{\frac{0.4}{1.4}} = (11.7)^{0.286} = 2.02$$

$$T_3 = 2.02\ T_2 = 2.02\ (585) = 1182°R$$

Since the compressor does work, per unit mass of gas, w_c, the energy equation (Eq. 4.42) is modified as

$$C_p T_2 + \frac{V_2^2}{2} + W_c = C_p\ T_3 + \frac{V_3^2}{2}$$

For $V_2 = V_3$, this becomes

$$W_c = c_p\ (T_3 - T_2)$$

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{(1.4)(1716)}{0.4} = 6006 \frac{\text{ft lb}}{\text{slug}°R}$$

$$W_c = 6006(1182 - 585) = 3.58 \times 10^6\ \frac{\text{ft lb}}{\text{slug}}$$

The mass flow is $220\ \frac{\text{lb}}{\text{sec}} = \frac{200}{322} = 6.21\ \frac{\text{slug}}{\text{sec}}$.

The total work done per second is

$$\left(6.201 \frac{\text{slug}}{\text{sec}}\right)\left(3.58 \times 10^6 \frac{\text{ft lb}}{\text{slug}}\right) = 2.22 \times 10^7\ \frac{\text{ft lb}}{\text{sec}}$$

Since one HP = $550\ \frac{\text{ft lb}}{\text{sec}}$, the horsepower provided by the compressor is

$$\text{HP} = \frac{2.22 \times 10^7}{550} = \boxed{40{,}364}$$

9.19 From the solution of Problem 9.18, the temperature at the entrance to the combustor is the same as the temperature at the exit of the compressor, namely $T_3 = 1182°R$. The pressure is essentially constant through the combustor, so the total heat added per unit mass to the air (neglecting the small mass of fuel added) is

$$q = c_p (T_4 - T_3) = 6006(2110 - 1182)$$

$$q = 5.57 \times 10^6\ \text{ft lb/slug}$$

or,

$$q = \left(5.57 \times 10^6 \frac{\text{ft lb}}{\text{slug}}\right)\left(\frac{1\ \text{slug}}{32.2\ \text{lb}_\text{m}}\right) = 1.73 \times 10^5\ \frac{\text{ft lb}}{\text{lb}_\text{m}}$$

Since the heat released <u>per unit mass of fuel</u> is given as $1.4 \times 10^7\ \frac{\text{ft lb}}{\text{lb}_\text{m}}$, the amount of fuel mixed with each lb_m of air is $1.73 \times 10^5/1.4 \times 10^7 = 0.0124\ \text{lb}_\text{m}$. The mass flow of air given in Problem 9.18 is 200 lb_m/sec. So the fuel rate is

$$(0.0124)(200) = \boxed{2.48\ \text{lb}_\text{m}/\text{sec}}$$

9.20 In the flow through the turbine, work w_T is extracted from the gas and provided to driving the turbine, and hence to the compressor. From the energy equation,

$$C_p\,T_4 + \frac{V_4^2}{2} - W_T = C_P T_5 + \frac{V_5^2}{2}$$

Since $V_5 = V_4$,

$$c_p\ T_5 = c_p\ T_4 - W_T$$

Since there are no mechanical losses, the work provided by the turbine is the same as the work done by the compressor, $w_T = w_c$. From the solution of Problem 9.18, $w_c = 3.58 \times 10^6\ \frac{\text{ft lb}}{\text{slug}}$. Thus

$$c_p\ T_5 = c_p\ T_4 - 3.58\times10^6$$

or,

$$T_5 = T_4 - \frac{3.58\times10^6}{c_p} = 2110 - \frac{3.58\times10^6}{6006}$$

$$T_5 = \boxed{1514°\,R}$$

9.21 First, calculate the pressure at the entrance to the nozzle, p_5. From Problem 9.18,

$$\frac{p_3}{p_2} = 11.7$$

$$p_3 = 11.7\ p_2 = 11.7(2116) = 24{,}757\ \frac{\text{lb}}{\text{ft}^2}$$

The pressure is constant through the combustor, $p_4 = p_3 = 24{,}757\ \frac{\text{lb}}{\text{ft}^2}$. The flow through the turbine is isentropic. From the solution of Problem 9.20, $T_5 = 1514°R$. Hence

$$\frac{p_5}{p_4} = \left(\frac{T_5}{T_4}\right)^{\frac{y}{y-1}} = \left(\frac{1514}{2110}\right)^{\frac{1.4}{0.4}} = (0.71753)^{3.5} = 0.3129$$

$$p_5 = (0.3129)\,p_4 = (0.3129)(24757) = 7746\frac{\text{lb}}{\text{ft}^2}$$

The expansion through the nozzle is isentropic.

$$\frac{T_6}{T_5} = \left(\frac{p_6}{p_5}\right)^{\frac{y-1}{y}} = \left(\frac{2116}{7746}\right)^{\frac{0.4}{1.4}} = (0.273)^{0.286} = 0.690$$

$$T_6 = 0.690\ T_5 = (0.690)(1514) = 1045°R$$

$$c_p T_5 + \frac{V_5^2}{2} = c_p\,T_6 + \frac{V_6^2}{2}$$

$$V_6^2 = 2c_p(T_5 - T_6) + V_5^2 = 2(6006)(1514 - 1045) + (1500)^2$$

$$= 5.63\times10^6 + 2.25\times10^6 = 7.88\times10^6$$

$$V_6 = (7.88\times10^6)^{1/2} = \boxed{2807\text{ ft/sec}}$$

9.22 From Problem 9.21, $T_6 = 1045^\circ R$ and $V_6 = 2807$ ft/sec

$$a_6 = \sqrt{\gamma R T_6} = \sqrt{(1.4)(1716)(1045)} = 1584 \text{ ft/sec}$$

$$M_6 = \frac{2807}{1584} = \boxed{1.77}$$

Comment: The flow at the exit of the engine is supersonic, and because of this the nozzle must be a convergent-divergent shape. Because the pressure ratio across the compressor given in Problem 9.18 is high, namely 11.7, this is a high performance turbojet engine suitable for use on supersonic airplanes, and a supersonic exit flow is a characteristic of such an engine. However, the result obtained here, namely $M_6 = 1.77$ is a relatively high value, influenced by our assumption of no mechanical or aerodynamic losses that in real life detract from the engine performance.

9.23 From Eq. (9.25), the thrust from the engine is

$$T = \dot{m}\ (V_e - V_\infty) + (p_e - p_\infty)\ A_e.$$

From Problem 9.18, $\dot{m}$ = 200 lb/sec. From the solution of Problem 9.21, V_e = 2807 ft/sec. Also, from Problem 9.21, p_e = 2116 lb/ft^2.
At 36,000 ft, from App. B, $T_\infty = 390.53^\circ R$ and p_∞ = 476 lb/ft^2.

$$a_6 = \sqrt{\gamma R T_\infty} = \sqrt{(1.4)(1716)(390.53)} = 968.6 \text{ ft/sec}$$

At $M_\infty = 2$,

$$V_\infty = 2(968.6) = 1937 \text{ ft/sec}$$

The diameter of the exit is 28 inches = 2.33 ft. Hence

$$A_e = \frac{\pi d^2}{4} = \frac{\pi(2.33)^2}{4} = 4.26 \text{ ft}^2$$

$$\dot{m} = 200 \text{ lb/sec} = \frac{200}{32.2} = 6.21 \text{ slug/sec.}$$

Thus, from Eq. (9.25),

$$T = (6.21)(2807 - 1937) + (2116 - 476)(4.26)$$
$$= 5403 + 6986 = \boxed{12{,}389 \text{ lb}}.$$

Comment: For the engine and the conditions treated in Problems 9.18 – 9.23, we find that the term $(p_e - p_\infty)\ A_e$ in the thrust equation is larger than the term $(V_e - V_\infty)$. This is usually not the case forturbojet engines, but is the case here for the high compression ratio engine treated and the conditions chosen.

9.24 Specific thrust $= \dfrac{T}{\dot{W}}$

From Problem 9.23, T = 12,389 lb and the weight flow through the engine is 200 lb/sec (the same as the mass flow when the mass is expressed in pounds mass). Thus,

$$\text{Specific thrust} = \frac{12{,}389}{200} = \boxed{61.9 \text{ sec}}$$

Chapter 10

10.1 Since $\delta \propto \frac{M^2}{\sqrt{\text{Re}}}$, we have

$$\delta_{M=20} = (\delta_{M=2})\left(\frac{20}{2}\right)^2 = 0.3(100) = 30 \text{ inches} = 2.5\text{ft}$$

This dramatically demonstrates that boundary layers at hypersonic Mach numbers can be very thick.

10.2 From Chapter 4 we have

$$\frac{T_0}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \quad = 1 + \frac{1.4 - 1}{2}(20)^2 = 81.$$

At 59 km, $T_\infty = 258.1$ K

Thus: $T_0 = (81)(258.1) = 20{,}906$ K

Considering that the surface temperature of the sun is about 6000K, the above result is an extremely high temperature. This illustrates that hypersonic flows can be very high temperature flows. At such temperatures, the air becomes highly chemically reacting, and in reality the ratio of specific heats is no longer constant; in turn, the above equation, which assumes constant γ, is no longer valid. Because the dissociation of the air requires energy (essentially "absorbs" energy), the gas temperature at the stagnation point will be much lower than calculated above; it will be approximately 6000K. This is still quite high, and is sufficient to cause massive dissociation of the air.

10.3 Following the nomenclature of Example 10.1,

$$\phi = 57.3(s/R) = 57.3(6.12) = 28.65°$$
$$\theta = 90° - \phi = 61.5°$$

(a) $$\frac{p_{0_2}}{p_\infty} = \left[\frac{(\gamma+1)^2 M_\infty^2}{4\ \gamma\ M_\infty^2 \ - 2(\gamma - 1)}\right]^{\gamma/\gamma-1}\left[\frac{1 - \gamma + 2\gamma M_\infty^2}{\gamma + 1}\right]$$

$$= \left[\frac{(2.4)^2(18)^2}{4(1.4)(18)^2 - 2(0.4)}\right]^{1.4/0.4}\left[\frac{1 - 1.4 + 2(1.4)(18)^2}{2.4}\right] = 417.6$$

$$C_{p,\max} = \frac{2}{\gamma M_\infty^2}\left(\frac{p_{0_2}}{p_\infty} - 1\right) = \frac{2}{(1.4)(18)^2}(417.6 - 1) = 1.836$$

(*Note:* This value of $C_{p,\max}$ is only slightly smaller than the value calculated in Example 10.1, which was 1.838 for $M_\infty = 25$. This is an illustration of the hypersonic Mach number independence principle, which states that pressure coefficient is relatively independent of Mach number at hypersonic speeds.)

(b) From modified Newtonian:

$$C_p = C_{p,\max}\sin^2\theta = (1.836)\sin^2(61.5°) = 1.418$$

10.4 From Eq. (10.11),

$$C_L = 2\sin^2\alpha\cos\alpha$$

$$\frac{dC_L}{d\alpha} = (2\sin^2\alpha)(-\sin\alpha) + 4\cos^2\alpha\sin\alpha = 0$$

$$\sin^2\alpha = 2\cos^2\alpha = 2(1 - \sin^2\alpha)$$

$$\alpha = 54.7°$$

$$C_{L,\max} = 2\sin^2(54.7)\cos(54.7) = 0.77$$

10.5 **(a)** $C_L = 2\sin^2\alpha\cos\alpha$

$C_D = 2\sin^3\alpha + C_{D,0}$

For small α, these become

$$C_L = 2\alpha^2 \qquad (1)$$

$$C_D = 2\alpha^3 + C_{D,0} \qquad (2)$$

$$\frac{C_L}{C_D} = \frac{2\alpha^2}{2\alpha^3 + C_{D,0}} \qquad (3)$$

$$\frac{d\left(\frac{C_L}{C_D}\right)}{d\alpha} = \frac{(2\alpha^3 + C_{D,0})4\alpha - 2\alpha^2(6\alpha^2)}{(2\alpha^3 + C_{D,0})} = 0$$

$$8\alpha^4 + 4\alpha C_{D,0} - 12\alpha^4 = 0$$

$$4\alpha^3 = 4C_{D,0}$$

$$\alpha = (C_{D,0})^{1/3} \qquad (4)$$

Substituting Eq. (4) into Eq. (3):

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{2(C_{D,0})^{2/3}}{2C_{D,0} + C_{D,0}} = \frac{2/3}{(C_{D,0})^{1/3}} = \frac{0.67}{(C_{D,0})^{1.3}}$$

Hence:

$$\left(\frac{L}{D}\right)_{\max} = 0.67/(C_{D,0})^{1/3} \text{ and it occurs at}$$

$$\alpha = (C_{D,0})^{1/3}.$$

(b) Repeating Eq.(2):

$$C_D = 2\alpha^3 + C_{D,0} \tag{2}$$

From the results of part (a), at $(L/D)_{max}$ we have $\alpha = (C,_{D,0})^{1/3}$. Substituting this into Eq.(2),

$$C_D = 2\, C_{D,0} + C_{D,0} = 3\, C_{D,0}$$

Since $C_D = C_{D,0} + C_{D,0}$

where $C_{D,W}$ is the wave drag, we have

$$C_D = C_{D,W} + C_{D,0} = 3C_{D,0}$$

or, $C_{D,W} = 2\, C_{D,0}$

Wave drag = 2 (friction drag) when *L*/*D* is maximum. Or, another way of stating this is that friction drag is one-third the total drag.