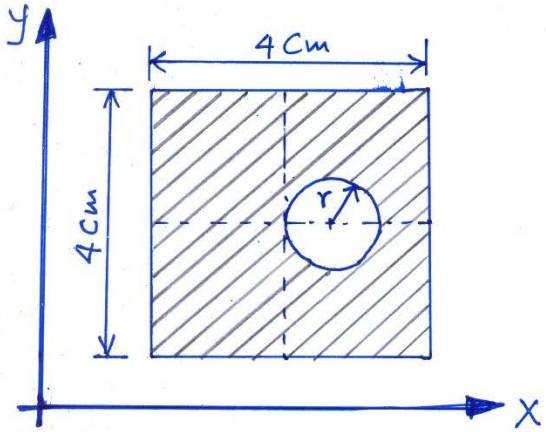
Dynamics Problems

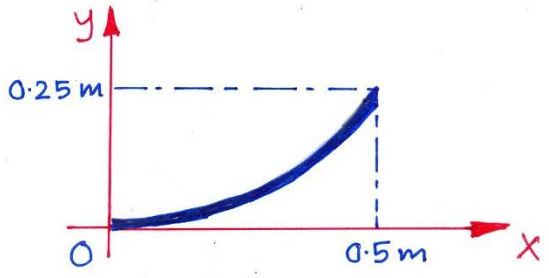
1.) Moment of Inertia Tensor



A thin square plate lies in the x-y plane. Its mass is 'm'. Its edges are 4cm each. A circular hole of radius r=0.7cm is cut out as shown.

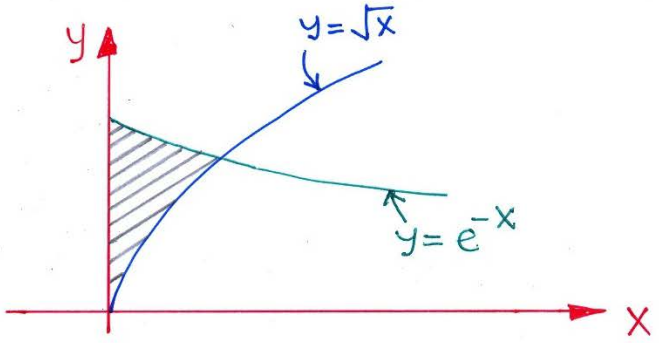
Find $[\underline{\underline{I}}_{cm}]_{xyz}$.

2.) A thin wire lies in the x-y plane along the curve $y = x^2$ as shown. Its mass is 'm'.

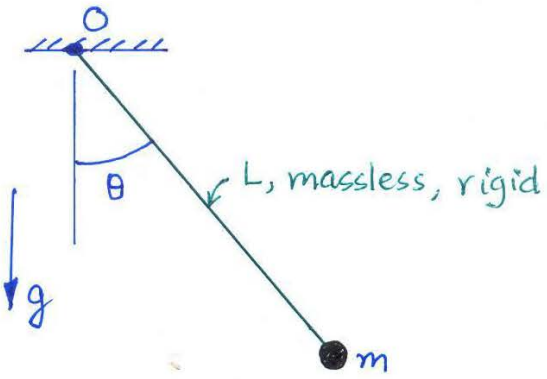


Find $[\underline{\underline{I}}_{cm}]_{xyz}$

3.) Find the shaded area.



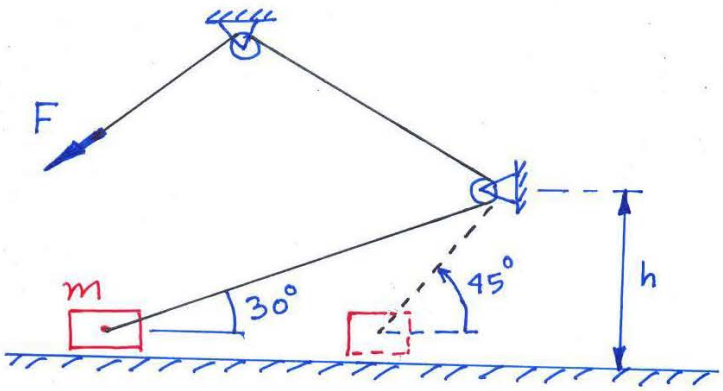
4.) A pendulum of length $L = 1\,m$, mass $m = 1\,kg$ is released from rest at $\theta = 40^\circ$ as shown. There is gravity ($g = 9.8\,m/s^2$).



The pendulum experiences a constant retarding moment $M = 0.1\,Nm$ during its swing. Find the angle it swings through before its angular velocity first becomes zero again.

Use your calculator, and trial and error, if needed.

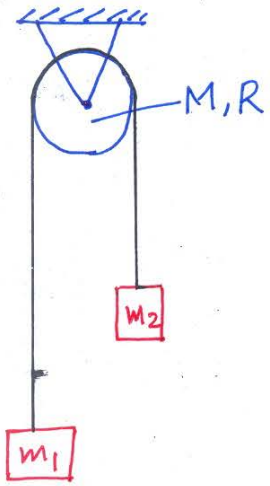
5.)



A mass m starts at rest on a horizontal, frictionless surface. It is pulled by a string that passes over two light, frictionless pulleys as shown. The Force F is constant. Take $m = 3\,kg$, $h = 0.5\,m$, $F = 4\,N$.

What is the velocity of the mass when the string's angle with the horizontal has increased from 30° to 45°?

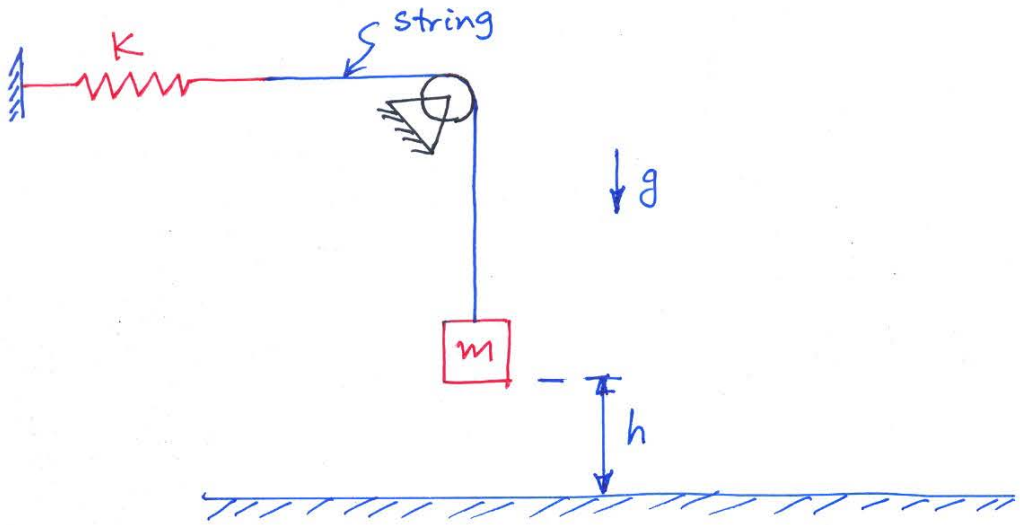
6.)



Two masses, $m_1 = 1\,kg$ and $m_2 = 2\,kg$, connected by a string, are released from rest as shown. The pulley is frictionless and may be treated as a uniform disk of mass $M = 2\,kg$ and radius $R = 20\,cm$.

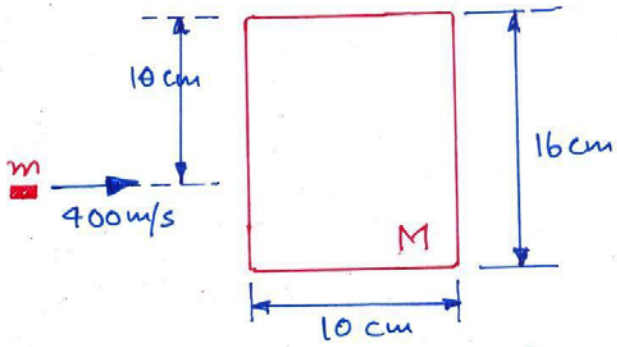
What is the angular velocity of the pulley at the instant when m_2 has traveled down a distance of 1 m?

7.)



A mass m is released from rest at a height h above the ground as shown. At the instant of release, the massless spring is unstretched. Take $m = 1\,kg$, $h = 0.3\,m$, $g = 9.8\,m/s^2$, $K = 100\,N/m$. The string breaks instantly when its tension exceeds 11 N. With what velocity does the mass hit the ground?

8.)



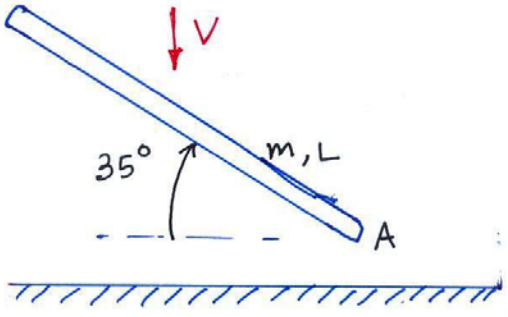
A bullet of mass m = 15g travelling at 400 m/s embeds itself in an initially stationary block of mass M = 3Kg and dimensions 10 cm X 16 cm (inplane). The path of the bullet's travel prior to impact is 10cm from the block's top edge, as shown. The penetration into the block is small. All motion remain planar.

— After impact, what is the angular velocity of the block?

— How much energy is lost in the impact?

9.)

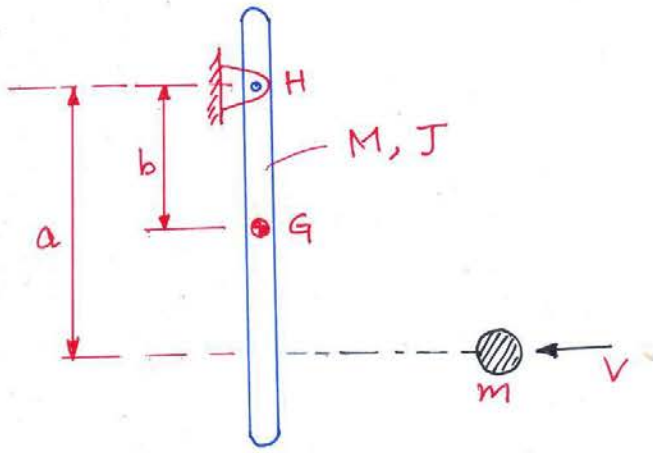
A rigid uniform rod of mass m and length L is dropped with no angular velocity and hits a rigid horizontal frictionless floor with



downward speed V. The angle made by the rod with the horizontal is 35° at the instant of impact. The post-impact velocity of A has a vertical component of $\frac{V}{2}$.

— What percentange of the rod's kinetic energy is dissipated?

10.)



A rigid long rod hangs from a rigid frictionless hinge at H. The rod hass mass M, central moment of inertia J, and its center of mass G is located at a distance 'b' from H. A projectile of mass m, velocity V, hits at a distance 'a' from H as shown.

① What is the radius of gyration about H of the rod?

② For what 'a' is the impulse at H equal to zero?