MSO202A COMPLEX ANALYSIS
Assignment 6

Exercise Problems:

1. If $0 < |z| < 4$, show that $\dfrac{1}{4z - z^2} = \dfrac{1}{4z} + \displaystyle\sum_{n=0}^{\infty} \dfrac{z^n}{4^{n+2}}$.

2. Write the two Laurent series in powers of z that represent the function $f(z) = \frac{1}{z(1+z^2)}$ in different domains.

3. Which of the following singularities are removable/pole:
(a) $\dfrac{\sin z}{z^2 - \pi^2}$ at $z = \pi$, (b) $\dfrac{\sin z}{(z-\pi)^2}$ at $z = \pi$ (c) $\dfrac{z \cos z}{1 - \sin z}$ at $z = \pi/2$.

4. Find the residue at $z = 0$ of the following functions and indicate the type of singularity they have at 0. (a) $\dfrac{1}{z + z^2}$ (b) $z \cos \dfrac{1}{z}$ (c) $\dfrac{z - \sin z}{z}$ (d) $\dfrac{\cot z}{z^4}$.

5. Use Cauchy's residue theorem to evaluate the integral of each of the following functions around the circle $|z| = 3$. (a) $\dfrac{e^{-z}}{z^2}$, (b) $\dfrac{e^{-z}}{(z-1)^2}$, (c) $z^2 e^{\frac{1}{z}}$ and $(d)\dfrac{z+1}{z^2 - 2z}$.

Problem for Tutorial:

6. Prove Jordan's inequality: Given any $R > 0$, $\int_0^{\pi} e^{-R \sin\theta} d\theta < \frac{\pi}{R}$.

7. Find the Laurent series of the function $f(z) = \frac{6z+8}{(2z+3)(4z+5)}$ in the regions $\{z : \frac{5}{4} < |z| < \frac{3}{2}\}$, $\{z \in \mathbb{C} : |z| > \frac{3}{2}\}$, $\{z : |z| < \frac{5}{4}\}$.

8. Find the isolated singularities and compute the residue of f: (a) $\frac{e^z}{z^2-1}$ (b) $\frac{3z}{z^2+iz+2}$ (c) $\cot \pi z$.

9. Let $f(z) = \frac{\pi \cot \pi z}{(z+\frac{1}{2})^2}$. Compute the residue of f at the isolated singularities.