MANGALORE UNIVERSITY

DEPARTMENT OF ELECTRONICS

PRACTICAL JOURNAL

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CERTIFICATE

This is to certify that Mr/Ms ________________________________ University Reg. No: ________________________ has successfully completed his/her course practical in CSCP 460: Design and Analysis of Algorithms Laboratory prescribed by The Mangalore University for the Second Semester in the laboratory under this department during 2022.

________________ ________________________

Lab in Charge Head of the department

Date:

Examiners:

1. Department Seal

2.

Hash Table

Definition:

Hash table is a data structure that is used to store key-value items. The idea of hash table is to provide a direct access to its items. It calculates 'hash code' of key and uses it to store item, instead of key itself. Only one hash code per key.

Theory:

In hash table, data is stored in array format, where each data value has its own unique index value. Access of data becomes very fast if we know the index of desired data.

Index	0 to n-1
Item	54, 26, 93, 17, 77, 31

- size = 1.3 times the key (large prime number)
- hash item = item % size

Assume the size is 11 then

Key	value
54%11	10
26%11	4
93%11	5
17%11	6
77%11	0
31%11	9
44%11	0

Hash collision: When the hash function generates the same index for multiple keys, there will be a conflict (what value to be stored in that index). This is called a **hash collision.**

Different collision resolution techniques such as:

- Open Hashing (Separate chaining)
- Closed Hashing (Open Addressing)
    - Liner Probing
    - Quadratic probing

1. Separate Chaining-

- This technique creates a linked list to the slot for which collision occurs.
- The new key is then inserted in the linked list.
- These linked lists to the slots appear like chains.
- That is why, this technique is called as separate chaining.

2. Closed Hashing (Open Addressing)

This collision resolution technique requires a hash table with fixed and known size. During insertion, if a collision is encountered, alternative cells are tried until an empty bucket is found. These techniques require the size of the hash table to be supposedly larger than the number of objects to be stored

There are various methods to find these empty buckets:

i) **Linear probing**:

A hash table in which a collision is resolved by putting the item in the next empty place in the array following the occupied place.

Algorithm:

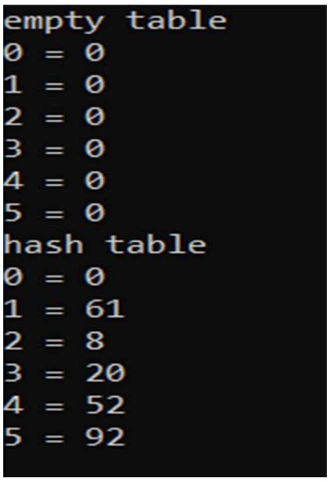
1. START
2. i=0, j=0, h=0,
   k=5, a[k]={8,20,52,61,92}, size=int(1.3*k), b[size];
3. for i=0 to i=6
   b[i]=0
4. while i<k
   h=a[i]%size
   if b[h]==0
   b[h]=a[i]
   else
   while b[h]!=0
   h=(h+1)%size
   b[h]=a[i]
   i=i+1
5. STOP

Program:

```
a=[8,20,52,61,92]
k=len(a)
size=int(1.3*k)
b=[]
for i in range(0,size):
	b.append(0)
i=0
j=0
print("empty table")
for i in b:
	print(j,"=",i)
	j+=1
i=0
```

```
while i<k:
        h=a[i]%size
        if b[h]==0:
                b[h]=a[i]
        else:
                while b[h]!=0:
                        h=(h+1)%size
                b[h]=a[i]
        i=i+1
j=0
print("hash table")
for i in b:
        print(j,"=",i)
        j+=1
```

Output:



```
empty table
0 = 0
1 = 0
2 = 0
3 = 0
4 = 0
5 = 0
hash table
0 = 0
1 = 61
2 = 8
3 = 20
4 = 52
5 = 92
```

ii). Quadratic probing:

Quadratic probing operates by taking the original hash index and adding successive values of an arbitrary quadratic polynomial until an open slot is found.

Algorithm:

1. START
2. i=0, j=0, h=0,k=5,a[k]={8,20,52,61,92},size=int(1.3*k),b[size]
3. for i=0 to i=6
        b[i]=0
4. while i<k
        h=a[i]%size
        if b[h]==0
                b[h]=a[i]
        else
                while b[h]!=0
                        h=(h+i**2))%size
                b[h]=a[i]
        i=i+1
5. STOP

Program:

```
a=[8,20,52,61,92]
k=len(a)
size=int(1.3*k)
b=[]
for i in range(0,size):
        b.append(0)
i=0
j=0
print("empty table:")
for i in b:
        print(j,'=',i)
        j+=1
i=0
while i<k:
        h=a[i]%size
        if b[h]==0:
                b[h]=a[i]
```

```
	else:
		while b[h]!=0:
			h=(h+i**2)%size
		b[h]=a[i]
	i=i+1
j=0
print("hash table:")
for i in b:
	print(j,"=",i)
	j=j+1
```

Output:

```
empty table:
0 = 0
1 = 0
2 = 0
3 = 0
4 = 0
5 = 0
hash table:
0 = 92
1 = 61
2 = 8
3 = 20
4 = 52
5 = 0
```

Greatest Common Divisor (GCD) using Euclidean Algorithm

Definition: To Calculate Greatest Common Divisor (GCD)

Theory:

GCD is the abbreviation for Greatest Common Divisor which is a mathematical equation to find the largest number that can divide both the numbers given by the user. Sometimes this equation is also referred as the greatest common factor. For example, the greatest common factor for the numbers 20 and 15 is 5, since both these numbers can be divided by 5. This concept can easily be extended to a set of more than 2 numbers as well, where the GCD will be the number which divides all the numbers given by the user.

Using Euclidean Algorithm:

The Euclid's algorithm (or Euclidean Algorithm) is a method for efficiently finding the greatest common divisor (GCD) of two numbers. The GCD of two integers X and Y is the largest number that divides both of X and Y (without leaving a remainder).

Pseudo Code of the Algorithm-

1. Let a, b be the two numbers
2. a mod b = R
3. Let a = b and b = R
4. Repeat Steps 2 and 3 until a mod b is greater than 0
5. GCD = b
6. Finish

Python code:

```
# Method to compute gcd (Euclidean algorithm)
def computeGCD(x, y):
```

```
    while(y):
        x, y = y, x % y
    return x
a = int(input("Enter the value for a: "))
b = int(input("Enter the value for b: "))
# prints 12
print ("The gcd of ",a,"and",b," is : ",computeGCD(a,b))
```

Output:

```
root@kali:~/Desktop# python3 euc.py
Enter the value for a: 49
Enter the value for b: 79
The gcd of  49 and 79  is :  1
root@kali:~/Desktop# python3 euc.py
Enter the value for a: 49
Enter the value for b: 21
The gcd of  49 and 21  is :  7
```

```
root@kali:~/Desktop# python3 euc.py
Enter the value for a: 15458
Enter the value for b: 48654
The gcd of  15458 and 48654  is :  2
root@kali:~/Desktop# python3 euc.py
Enter the value for a: 485648
Enter the value for b: 958456
The gcd of  485648 and 958456  is :  8
```

Generating Random Prime Number using Fermat's Little Test

Definition: Given a positive integer number N, Task is to check whether given number prime or not.

Theory: Fermat's little theorem If p is prime, then for every $1 \leq a < p$,

$$a^{N-1} \equiv 1 \pmod{N}$$

If N is prime, then $a^{N-1} \equiv 1 \bmod N$, for all $a < N$.

If N is not prime, then $a^{N-1} \equiv 1 \bmod N$, for at most half the values of $a < N$.

The algorithm of Figure 1.7 therefore has the following probabilistic behavior.

Pr(Algorithm 1.7 returns yes when N is prime) = 1

Pr(Algorithm 1.7 returns yes when N is not prime) $\leq \frac{1}{2}$

We can reduce this one-sided error by repeating the procedure many times, by randomly picking several values of a and testing them all (Figure 1.8).

Pr(Algorithm 1.8 returns yes when N is not prime) $\leq \frac{1}{2^k}$

This probability of error drops exponentially fast, and can be driven arbitrarily low by choosing k large enough. Testing k = 100 values of a makes the probability of failure at most 2^{-100}.

Algorithm:

Input: Positive integer N

Output: YES / NO

Pick positive integers a1, a2, . . . , ak < N at random

if $a_i^{N-1} \equiv 1 \pmod{N}$ { for all i = 1, 2, . . . , k}

return YES

else:

return NO

- Pick a random n-bit number N.
- Run a primality test on N.
- If it passes the test, output N; else repeat the process.

Program:

```
import random as rd
v=2
while(v!=1):
    m=1
    N = rd.randint( 2**2048, 2**2050 )
    while(m<=100):
        a=rd.randint(1,500)
        x = a
        y = N-1
        product = 1
#####implement (x^y) mod N, modular exponentiation#######
        while y > 0:
                if( y & 1 == 1):#i.e, if last bit of y is 1
                        product = (product*x)%N
                y = y>>1;
                x=(x*x)%N;
                v=product
                m=m+1
                if(product!=1):
```

```
            break
#####################################################
if(v==1):
    print("prime number =",N)
```

Output:

```
prime number= 333903416906577490535035460088176371321414038051237102344742913972917266
    4771302063548885486030999055561238013254528402812761418953992556527777599840100746
    4183936850799654392721555752572661286966501730585119405299274361640699473374472448
    0497172517943664758117116256521570305072023381128019599217229870235776185921992028
    2546002411177671581674372825266776079369227786755683752452519124826160546466885060
    3164813509665486624267780033139081254747084237842263958388699677775166744990167435
    1798760067467141255290707888915110977022996305129011566153014645396881471697093836
    55292951334142642345993170580119174952366084369340724
```

```
prime number= 413495499872894763013691774962557077054813238752360223695120342651943513
    1971505394851668716706070815666162854080628724436201208870451952364134483200521251
    2112289654793823961892290005321557079977915748668225440482330775475839920362545538
    2247160172149405776461313551054093431094684058120712976310873605313561613617619104
    6402294591373834210848915683171608236565754855529408827428365048463016481600970943
    43503756647355466742964515154809853878766387864478002866758713888794753746319964911
    9516530991998491041304222226817519431087773325476547596048399621489914786716727924
    47628262188593372390483210464393081867136656063595160
```

```
prime number= 123681129864173618657469474782366350328881696451683162908323200399042163
    2013085101336263527158137561084129119932629419493203381021211162524576720629093240
    5295306878227640853031257850167544602754983319708165173538127519988681485622716229
    4131548089965949272619141344747241781518860282117956776523335840702928863870882151
    7041892085867025651973361320966057221929425884550367984100361467972274286055889435
    7969772158152186766516208645946737240549944193816298824319889920104101301681460154
    4104965819588575677804212737328927557857765010620826837823687797800341293319339108
    5052576327391142550748228772433197458471522558061516669
```

Multiplicative Inverse using Extended-Euclidean Algorithm

Definition: To find the multiplicative inverse of two numbers

Theory:
In math the multiplicative inverse of a number is another number which when multiplied by the original number gives 1 as the product. If ‘ N ’ is a natural number, the multiplicative inverse of ‘ N ’ will be $\frac{1}{N}$ or N^{-1}.

So, the multiplicative inverse of a number is the reciprocal of that number.

For example, for N = 5 the reciprocal will be $\frac{1}{5}$.

Now,

$5 \times \frac{1}{5} = 1$

So, $\frac{1}{5}$ is the multiplicative inverse of 5.

Extended Euclidean algorithm also refers to a very similar algorithm for computing the polynomial greatest common divisor and the coefficients of Bézout's identity of two univariate polynomials.

The extended Euclidean algorithm is particularly useful when a and b are coprime. With that provision, x is the modular multiplicative inverse of a modulo b, and y is the modular multiplicative inverse of b modulo a. Similarly, the polynomial extended Euclidean algorithm allows one to compute the multiplicative inverse in algebraic field extensions and, in particular in finite fields of non-prime order. It follows that both extended Euclidean algorithms are widely used in cryptography. In particular, the computation of the modular multiplicative inverse is an essential step in the derivation of key-pairs in the RSA public-key encryption method.

Algorithm:

Given a & b

Output: gcd(a,b),x,y £ Z such that

gcd(a,b)=ax+by

Initialization: x_0=1, x_1=0, y_0=0, y_1=1, sign=1

1. While b ≠ 0
2. r = a mod b //reminder of the
3. q = a / b
4. a = b
5. b = r
6. xx = x_1
7. yy = y_1
8. $x_1 = q * y_1 + x_0$
9. $y_1 = q * y_1 + y_0$
10. x_0 =xx
11. y_0=yy
12. Sign = -sign
13. End while
14. x = sign * x_0
15. y = sign * y_0
16. gcd = a
17. Return gcd(x,y)

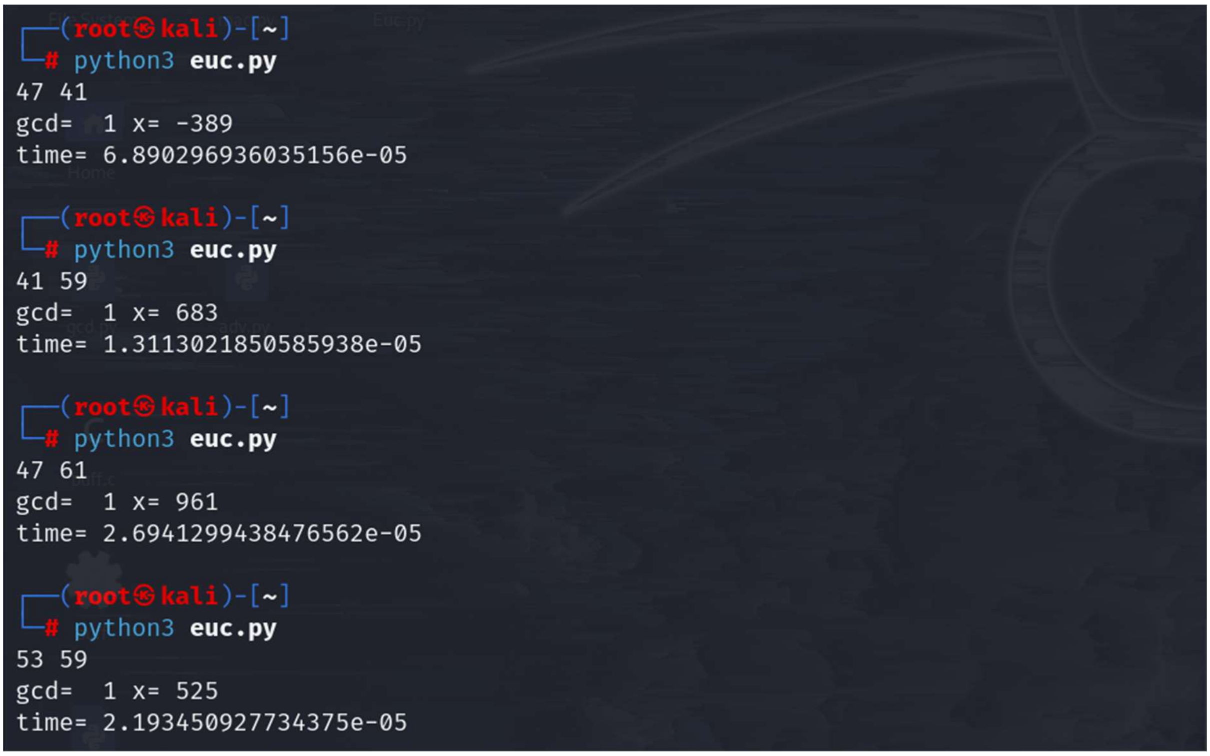
<u>Program code:</u>

```
import random as r
import math
import sympy as s
```

```
m=35
n=65
p=s.randprime(m,n)
q=s.randprime(m,n)
while p==q:
   q=s.randprime(m,n)
print(p, q)
n=p*q
phi=(p-1)*(q-1)
e=r.randint(1,phi)
while math.gcd(e,phi)!=1:
     e=r.randint(1,phi)
a=e
b=phi
xo=1
x1=0
yo=0
y1=1
sign=1
import time
start=time.time()
while(b!=0):
  r=a%b
  q=a//b
  a=b
  b=r
  xx=x1
  yy=y1
  x1=(q*x1)+xo
  y1=(q*y1)+yo
  xo=xx
  yo=yy
  sign=-sign
x=sign*xo
y=-sign*yo
```

gcd=a
print("gcd= ",gcd,"x=",x)
print("time= %s" % ((time.time())-start))

OUTPUT:



```
┌──(root㉿kali)-[~]
└─# python3 euc.py
47 41
gcd=  1 x= -389
time= 6.890296936035156e-05

┌──(root㉿kali)-[~]
└─# python3 euc.py
41 59
gcd=  1 x= 683
time= 1.3113021850585938e-05

┌──(root㉿kali)-[~]
└─# python3 euc.py
47 61
gcd=  1 x= 961
time= 2.6941299438476562e-05

┌──(root㉿kali)-[~]
└─# python3 euc.py
53 59
gcd=  1 x= 525
time= 2.193450927734375e-05
```

```
┌──(root㉿kali)-[~]
└─# python3 euc.py
256994747487543341 464937119869018967
gcd=  1 x= -42674150739939787391390329088298443
time= 7.581710815429688e-05

┌──(root㉿kali)-[~]
└─# python3 euc.py
652768798767342937 415764404745952937
gcd=  1 x= 2410275779363153898099779486140265З
time= 5.1021575927734375e-05

┌──(root㉿kali)-[~]
└─# python3 euc.py
127140049960100591 382767650784861З1
gcd=  1 x= 1624178383501082021641253895522137
time= 8.296966552734375e-05

┌──(root㉿kali)-[~]
└─# python3 euc.py
323683085621291183 174864319823697613
gcd=  1 x= 5557827494717998482480534482063573
time= 7.43865966796875e-05

┌──(root㉿kali)-[~]
└─# python3 euc.py
296947409656671171 262395572918619419
gcd=  1 x= 327268686044447856294356890534750
time= 9.036064147949219e-05
```

Integer Factoring problem using Fermat's method

Definition: Prime Factorization (or integer factorization) is a commonly used mathematical problem often used to **secure public-key encryption systems**. A common practice is to use very large semi-primes (that is, the result of the multiplication of two prime numbers) as the number securing the encryption.

Theory:

In number theory, integer factorization is the decomposition of a composite number into a product of smaller integers. If these factors are further restricted to prime numbers, the process is called prime factorization.

When the numbers are sufficiently large, no efficient non-quantum integer factorization algorithm is known. However, it has not been proven that such an algorithm does not exist. The presumed difficulty of this problem is important for the algorithms used in cryptography such as RSA public-key encryption and the RSA digital signature. Many areas of mathematics and computer science have been brought to bear on the problem, including elliptic curves, algebraic number theory, and quantum computing.

Algorithm:

1. Start
2. Read n
3. a = sqrt(n)
4. i = round(a) + 1
5. b = (i * i) – N
6. c = sqrt(b)
7. if c = int value
    print(N = i * i - c * c)
    p = (i + c)
    q = (i – c)

print(p)
print(q)
else:
i = i + 1
go to step (5)

8. Stop

Program code:

```
import math as m

def fermat(n):
    a =m.sqrt(n)
    i =round(a)+1
    b = i*i - n
    c = m.sqrt(b)
    while c*c != b:
        i = i + 1
        b = i*i - n
        c =m.sqrt(b)
    p=i+c
    q=i-c

    print('p=',p)
    print('q=',q)
    print('pq=',p*q)
    return p, q

n=195559655854585
fermat(n)
```

OUTPUT:

```
In [94]: runfile('C:/Users/Asus/Desktop/if.py', wdir='C:/Users/Asus/Desktop')
p= 15.69041575982343
q= 6.30958424017657
pq= 99.0

In [95]: runfile('C:/Users/Asus/Desktop/if.py', wdir='C:/Users/Asus/Desktop')
p= 55.78404875209022
q= 28.21595124790978
pq= 1574.0

In [96]: runfile('C:/Users/Asus/Desktop/if.py', wdir='C:/Users/Asus/Desktop')
p= 3273.829435256319
q= 3010.170564743681
pq= 9854785.0

In [97]: runfile('C:/Users/Asus/Desktop/if.py', wdir='C:/Users/Asus/Desktop')
p= 46.48999599679679
q= 21.510004003203203
pq= 999.9999999999999

In [98]: runfile('C:/Users/Asus/Desktop/if.py', wdir='C:/Users/Asus/Desktop')
p= 13990567.124582168
q= 13977964.875417832
pq= 195559655854585.0
```

Repeated Squaring Algorithm

Definition: Given 2 positive integer a and b, the task is to find a^b using Repeated Squaring Algorithm.

Theory:

Repeated squaring, or repeated doubling is an algorithm that computes integer powers of a number quickly. The general problem is to compute for an arbitrary integer y. The naive method, doing y multiplications of x, is very slow. It can be sped up by repeatedly squaring x until the current power of x exceeds y, and then collecting the "useful" powers.

Repeated Squaring Method takes the advantage of the fact that $A^x \times A^y = A^{x+y}$.

Any number can be written as the sum of powers of 2. We Just need convert the number to Binary Number System. Now for each position i for which binary number has 1 in it, add 2^i to the sum.

Consider the number 7^{19}, here $(19)_{10}$ can be written as binary number i.e.

$(10011)_2 = (2^4+2^1+2^0)_{10} = (16+2+1)_{10}$

Therefore, in the equation a^b, we can decompose 'b' to the sum of powers of 2.

The main concept for repeated squaring method is decomposing value 'b' to binary, and then for each position i (we start from 0 and loop till the highest position at binary form of 'b') for which binary of 'b' has 1 in i^{th} position, we multiply $a^{\wedge}2^i$ to result. The time complexity of repeated squaring algorithm is (m2 n) or m3.

Consider the number 3^{13}.The binary representation of 13 is $(1101)_2$, so 8+4+1=13

3^1	3
3^2	9 = 3 * 3
3^4	81 = 9 * 9
3^8	6561 = 81 * 81

Here result is the square of the previous result, and hence can be computed in one multiplication.

$$3^1 * 3^4 * 3^8 = 3^{13} = 1594323$$

Algorithm:

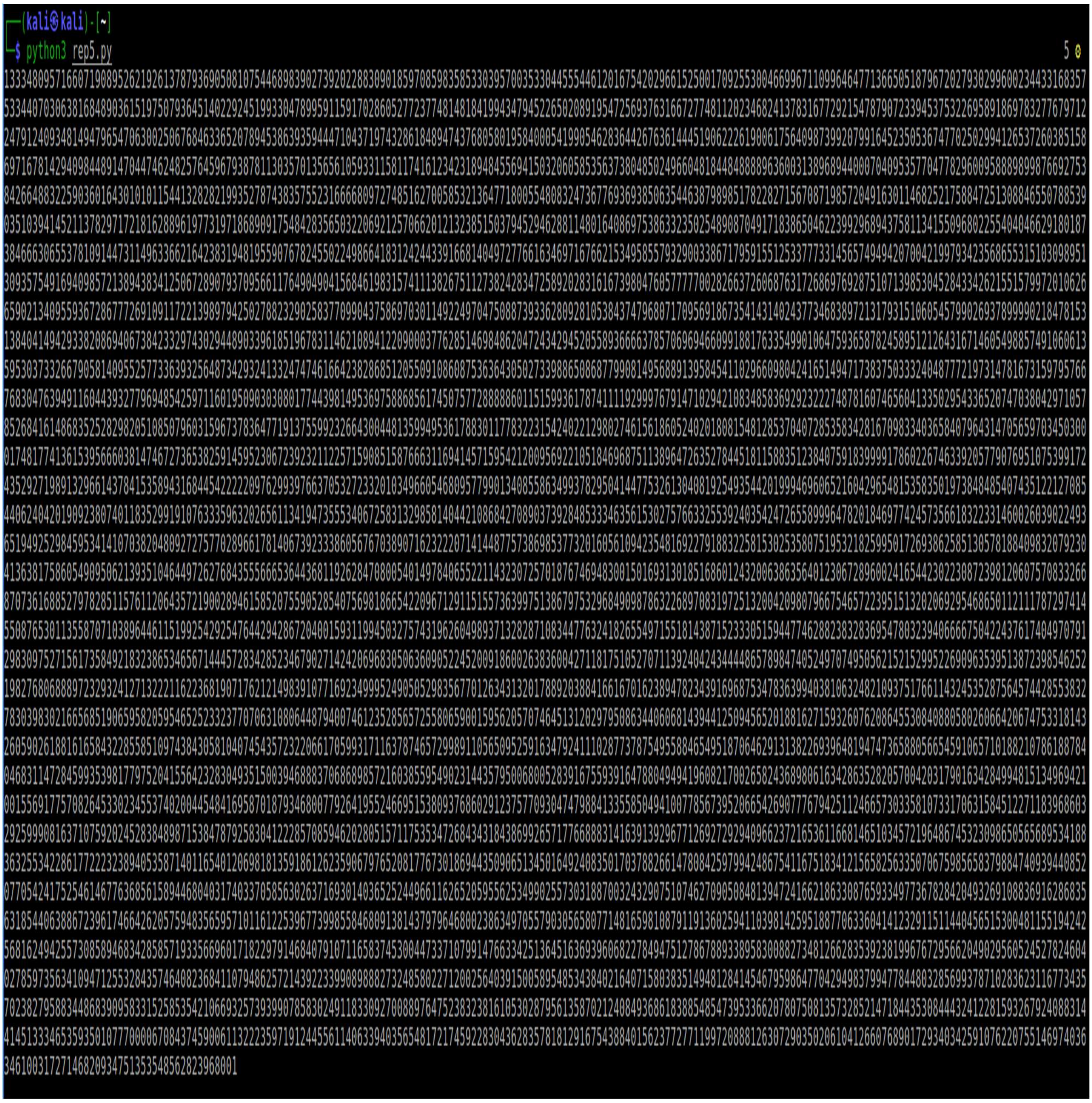
Step 1:
    Start
Step2:
    a=2, b=12, p=1
Step 3:
    if(b==0): return 1{go to step 9}
step 4:
    lsb=b%2
step 5:
    b=b//2
step 6:
    if (lsb==1) {p=p*a)
Step 7:
    A=a*a {go to step 3}
Step 8:
    Print (p)
Step 9:
    Stop

Program:

```
a=2844

n=193
p=1
while (a!=0):
    lsb=a%2
    a=a>>1
    if (lsb==1):
        p=p*n
```

```
    n=n*n
print(p)
```

Result:



RSA ALGORITHM (Rivest -Shamir -Adleman Algorithm)

Definition: Implementing RSA algorithm using Fermat's little test, Fermat's repeated squaring algorithm, Euclid's algorithm for GCD and Extended Euclid's algorithm, to generate encryption and decryption keys.

Theory:
RSA algorithm is asymmetric cryptography algorithm. Asymmetric actually means that it works on two different keys i.e., Public Key and Private Key. The idea of RSA is based on the fact that it is difficult to factorize a large integer. The public key consists of two numbers where one number is multiplication of two large prime numbers. And private key is also derived from the same two prime numbers. So, if somebody can factorize the large number, the private key is compromised. Therefore, encryption strength totally lies on the key size and if we double or triple the key size, the strength of encryption increases exponentially. RSA keys can be typically 1024 or 2048 bits long.

The RSA algorithm holds the following features −

- RSA algorithm is a popular exponentiation in a finite field over integers including prime numbers.
- The integers used by this method are sufficiently large making it difficult to solve.
- There are two sets of keys in this algorithm: private key and public key.

Algorithm:

1. Choose two different random prime numbers p and q
2. Calculate n = p*q
3. Calculate t = (p - 1) *(q - 1)

4. Choose an integer e such that 1 < e < t and gcd (e, t) = 1
5. Choose an integer d such that d = (1 + k.t)/e for some integer k

   Once the parameters n, e, and d are calculated, the RSA key pair is defined as:

- Encryption key — (n, e)
- Decryption key — (n, d)

For a given public key (n, e), its corresponding private key (n, d), and a given message m:

- Encryption

ciphertext c = m^e mod n

- Decryption

plaintext m = c^d mod n

Where, n — Modulus for the public/private keys

e — Public key exponent

d — Private key exponent

Program:

```
import random as r

def gcd_alg(x,y):

    while(y):
```

```
        x,y=y,x%y
    return x
########################################
def randprime():
    v=2
    while (v! =1):
      m=1
      N = r.randint(2*2048,2*2050)
      while(m<=100):
        a=r.randint(1,500)
        x = a
        y = N-1
        product = 1
    ##############################################
        #Implement (x^y) mod N, modular exponentiation
        while y > 0:
          if (y & 1 == 1):#i.e, if last bit of y is 1
            product = (product*x)%N
         y = y>>1;
          x=(x*x) %N
```

```
            v=product
            m=m+1
          if (product! =1):
            break
      ##################################################
      if(v==1):
          return N
###########################################
def ExEuc_alg(a,b):
    x0=1
    x1=0
    y0=0
    y1=1
    sign=1
    while (b!=0):
        r=a%b
        q=a//b
        a=b
        b=r
        xx=x1
```

```
            yy=y1
            x1=(q*x1)+x0
            y1=(q*y1)+y0
            x0=xx
            y0=yy
            sign=-sign
        x=(sign*x0)
        y=(-sign*y0)
        return x
#################################################
p =randprime()
q = randprime()
while (p==q):
        q=randprime()
n = p*q
phi = (p-1) * (q-1)
e=r.randint(1,phi)
while(gcd_alg(e,phi)!=1):
        e=r.randint(1,phi)
a=e
```

```
b=phi

d=ExEuc_alg(a,b)

print("(e,n)=","(",e,",",n,")")

print("(d,n)","(",d,",",n,")")
```

Result:

Find The Median Using Divide And Conquer Method

Definition: Given a list of elements S, Task is to find the median, in the given list of elements.

Theory: The median of a list of numbers is its 50th percentile: half the numbers are bigger than it, and half are smaller. For instance, the median of [45, 1, 10, 30, 25] is 25, since this is the middle element when the numbers are arranged in order. If the list has even length, there are two choices for what the middle element could be, in which case we pick the smaller of the two.

Algorithm:

1. Divide the list into sublists if size n, assume 5.
2. Initialize an empty array M to store medians we obtain from smaller sublists.
3. Loop through the whole list in sizes of 5, assuming our list is divisible by 5.
4. For $\frac{n}{5}$ sublists, use select brute-force subroutine to select a median m, which is in the 3rd rank out of 5 elements.
5. Append medians obtained from the sublists to the array M.
6. Use quickSelect subroutine to find the true median from array M, The median obtained is the viable pivot.
7. Terminate the algorithm once the base case is hit, that is, when the sublist becomes small enough. Use Select brute-force subroutine to find the median.

Program:

```
def median_of_medians(arr):
    if arr is None or len(arr) == 0:
        return None
    return select_pivot(arr, len(arr) // 2)
def select_pivot(arr, k):
    chunks = [arr[i : i+5] for i in range(0, len(arr), 5)]
    sorted_chunks = [sorted(chunk) for chunk in chunks]
    medians = [chunk[len(chunk) // 2] for chunk in sorted_chunks]
    if len(medians) <= 5:
        pivot = sorted(medians)[len(medians) // 2]
    else:
        pivot = select_pivot(len(medians) // 2)
    p = partition(arr, pivot)
    if k == p:
        return pivot
    if k < p:
        return select_pivot(arr[0:p], k)
    else:
        return select_pivot(arr[p+1:len(arr)], k - p - 1)
def partition(arr, pivot):
    left = 0
    right = len(arr) - 1
```

```
    i = 0
    while i <= right:
        if arr[i] == pivot:
            i += 1
        elif arr[i] < pivot:
            arr[left], arr[i] = arr[i], arr[left]
            left += 1
            i += 1
        else:
            arr[right], arr[i] = arr[i], arr[right]
            right -= 1
    return left
arr = [25, 42, 23, 36, 12, 47, 89, 96, 85]
pivot = median_of_medians(arr)
print ("median of the list=",arr,"is",pivot)
```

Output:

```
median of the list= [25, 42, 23, 36, 12, 47, 89, 96, 85] is 42

median of the list= [354, 456, 25, 123, 475, 598, 896, 523] is 475
```

Depth First Search

Definition: Given a graph G(V, E) (directed or undirected), Task is to find reachability from source S to destination D.

Theory: Depth-first search is a surprisingly versatile linear-time procedure that reveals a wealth of

information about a graph. The most basic question it addresses is,

What parts of the graph are reachable from a given vertex?

Undirected graph: A graph with unordered pairs of vertices, so the edge from vertex a to b is identical to the edge from vertex b to a.

Directed graph: A directed graph is a graph that is made up of a set of vertices connected by edges, where the edges have a direction associated with them.

Algorithm:

1. Start

2. N= Total number of vertices; A=1,B=2,C=3,.......,Z=26; E= Edges entries; AL= [] #adjacency list; Stack= [] #empty stack; source= “”; destination= “”;

3. Append edges entries to AL

4. Push source to stack

5. Mark source as visited in AL

6. if(source==destination) {print “reachable”; Goto 14;}

7. prevsource=source

8. Check for available vertex from the source vertex which is not yet visited.

9. If found make source=available_vertex

10. If(source==prevsource){ Pop the source vertex and prevsource from the stack if stack length greater than 1}

11. Push source vertex to stack

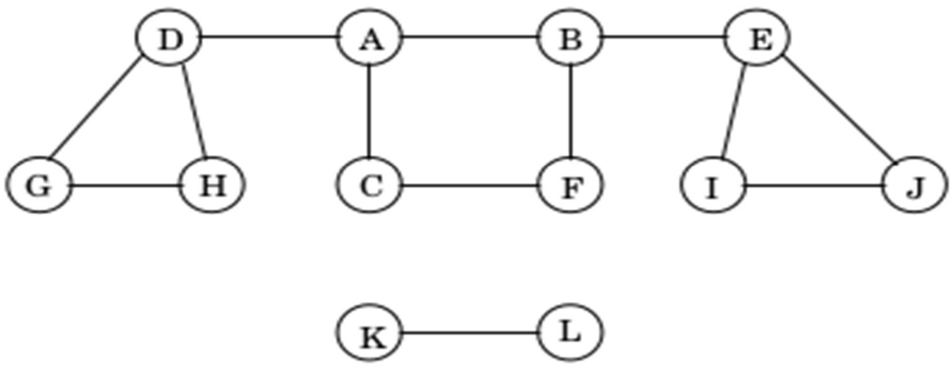
12. If(length of stack is 1){print "not reachable"; Goto 14;}

13. Goto 5

14. Stop

Python code:

Undirected graph:



#Python code to find reachability from source to destination in undirected graphs.

n = 12;

A=1, B=2, C=3, D=4, E=5, F=6, G=7, H=8, I = 9, J=10, K = 11, L = 1

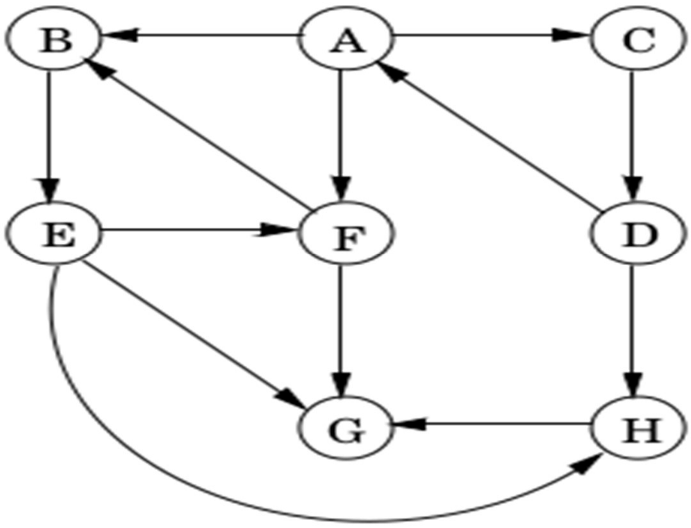
```
E=[[1,2],[1,3],[1,4],[2,1],[2,5],[2,6],[3,1],[3,6],[4,1],[4,8],[4,7],[5,9],[5,10]
,[5,2],[6,3],[6,2],[7,8],[7,4],[8,4],[8,7],[9,10],[9,5],[10,9],[10,5],[11,12],[1
2,11]]
AL = []
for i in range(n):
    AL.append([n+1])
for e in E:
    AL[e[0]-1].append(e[1])
# perform DFS at A (vertex 1)
# n+1 not visited
# n+2 visited
s = “”# source vertex
so = s
d = “” # destination vertex
stack = []
stack.append(s)
while True:
    AL[s-1][0] = (n+2) # mark v=1 for s
    if(s == d):
        print("stack=",stack, "\t", so, " is reachable by", d)
        break
    ss = s; # check the next available vertex for visiting
    for i in range(1, len(AL[s-1])):
        if(AL[AL[s-1][i] - 1][0] == (n+1)):
```

```
                s = AL[s-1][i]
                break
        if(s == ss): # if s is completely explored
            if(len(stack) > 1):
                s = stack.pop()
                s = stack.pop()
        stack.append(s) # push the s which
        if(stack[0] == stack[-1] and len(stack) > 1):
            print("Not Reachable")
            break
```

Output:

```
┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [1, 2, 5, 9, 10]          1  is reachable by 10

┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [1, 2, 5]          1  is reachable by 5

┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
Not Reachable

┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
Not Reachable

┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [11, 12]          11  is reachable by 12

┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [12, 11]          12 is reachable by 11
```

Directed graph:

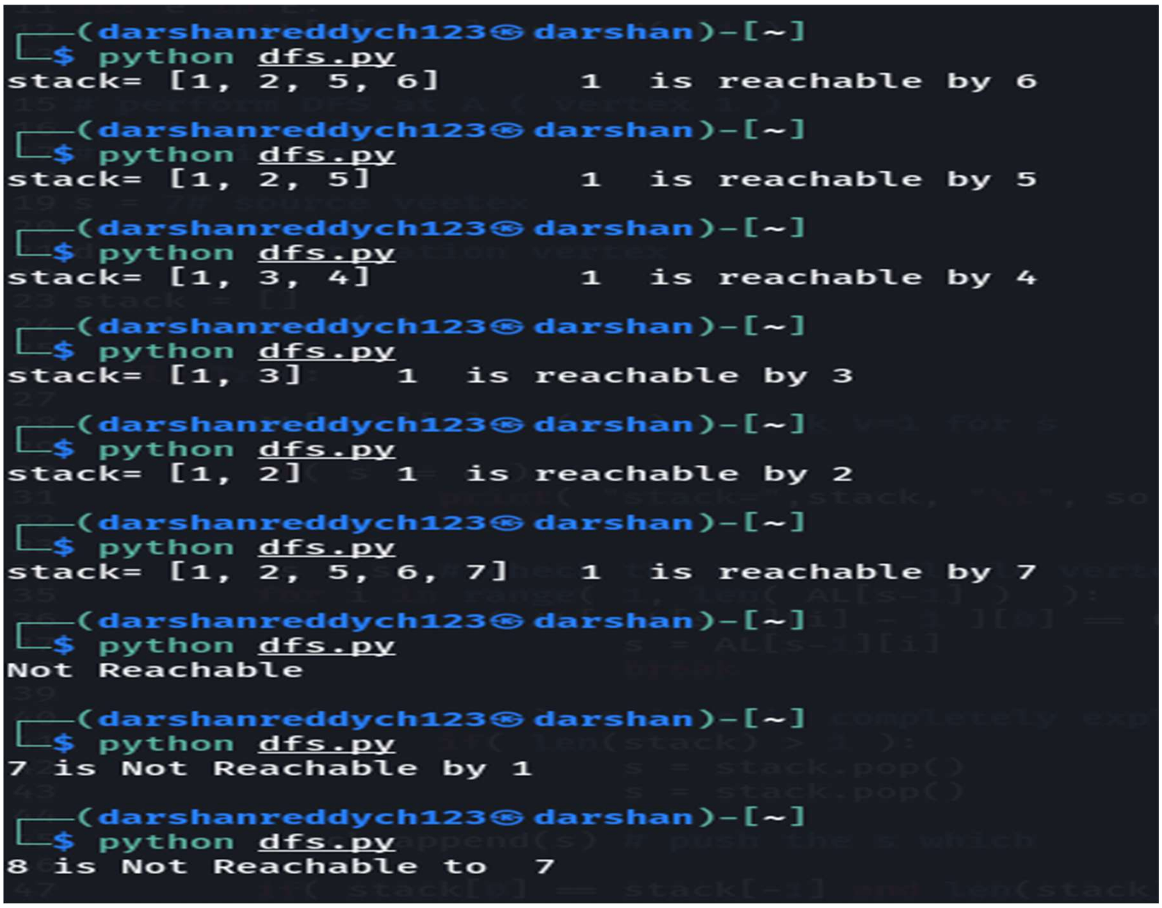


#Python code to find reachability from source to destination in directed graphs.

```
n = 12;
# A=1, B=2, C=3, D=4, E=5, F=6, G=7, H=8, I = 9, J=10, K = 11, L = 12
E=[[1,2],[1,6],[1,3],[2,5],[3,4],[4,1],[4,8],[5,6],[5,7],[5,8],[6,2],[6,7],[8,7]]
AL = []
for i in range(n):
        AL.append([n+1])
for e in E:
        AL[e[0]-1].append(e[1])

# Perform DFS at A (vertex 1)
# n+1 not visited
# n+2 visited
```

```
s = “” # source vertex
so = s
d = “” # destination vertex
stack = []
stack.append(s)
while True:
        AL[s-1][0] = (n+2) # mark v=1 for s
        if(s == d):
                print("stack=",stack, "\t", so, " is reachable by", d)
                break
        ss = s; # check the next available vertex for visiting
        for i in range(1, len(AL[s-1])):
                if(AL[AL[s-1][i] - 1][0] == (n+1)):
                        s = AL[s-1][i]
                        break
        if(s == ss): # if s is completely explored
                if(len(stack) > 1):
                        s = stack.pop()
                        s = stack.pop()
        stack.append(s) # push the s which
        if(stack[0] == stack[-1] and len(stack) > 1):
                print(d,"is Not Reachable to ",ss)
                break
```

Output:


┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [1, 2, 5, 6]   1 is reachable by 6
┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [1, 2, 5]   1 is reachable by 5
┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [1, 3, 4]   1 is reachable by 4
┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [1, 3]   1 is reachable by 3
┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [1, 2]   1 is reachable by 2
┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
stack= [1, 2, 5, 6, 7]   1 is reachable by 7
┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
Not Reachable
┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
7 is Not Reachable by 1
┌──(darshanreddych123㉿darshan)-[~]
└─$ python dfs.py
8 is Not Reachable to 7

Breadth-First Search

Definition: Given a graph G(V, E) (directed or undirected), Task is to find the shortest distance from source S to destination D.

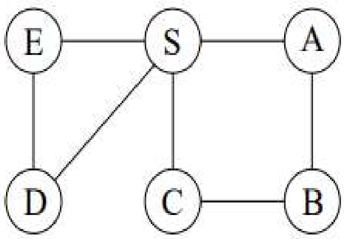
Theory:

Breadth-first search finds shortest paths in any graph whose edges have unit length. We adapt it to a more general graph G = (V, E) whose edge lengths *Ie* are positive integers

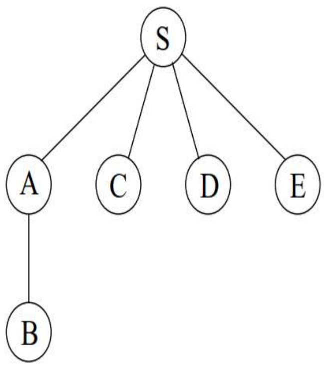
Undirected graph: A graph with unordered pairs of vertices, so the edge from vertex a to b is identical to the edge from vertex b to a.

Directed graph: A directed graph is a graph that is made up of a set of vertices connected by edges, where the edges have a direction associated with them.

(a)



Order of visitation	Queue contents after processing node
	[*S*]
S	[*A C D E*]
A	[*C D E B*]
C	[*D E B*]
D	[*E B*]
E	[*B*]
B	[ ]



Procedure bfs(G, s)

Input: Graph $G = (V, E)$, directed or undirected; vertex $s \in V$
Output: For all vertices u reachable from s, dist(u) is set to the distance from s to u.

```
for all u ∈ V :
        dist(u) = ∞
dist(s) = 0
Q = [s] (queue containing just s)
while Q is not empty:
        u = eject(Q)
        for all edges (u, v) ∈ E:
                if dist(v) = ∞:
                        inject(Q, v)
                        dist(v) = dist(u) + 1
```

Initially the queue Q consists only of s, the one node at distance 0. And for each subsequent distance d = 1, 2, 3, . . ., there is a point in time at which Q contains all the nodes at distance d and nothing else. As these nodes are processed (ejected off the front of the queue), their as-yet-unseen neighbours are injected into the end of the queue.

Algorithm:

1. Pick any node, visit the adjacent unvisited vertex, mark it as visited, display it, and insert it in a queue.
2. If there are no remaining adjacent vertices left, remove the first vertex from the queue.
3. Repeat step 1 and step 2 until the queue is empty or the desired node is found

Program:

```
n=7

E = [
        [1,2],[1,6],
        [2,3],[2,1],
        [3,2],[3,6],[3,7],
        [4,5],[4,6],
        [5,4],[5,6],
        [6,1],[6,3],[6,4],[6,5]
]

AL = []

for i in range(n):
        AL.append([n+1])

for e in E:
        AL[ e[0]-1 ].append(e[1])

for e in AL:
        print(e)
s = 6
Q = [s]
AL[s-1][0] = 0

while ( Q != [] ):

        u = Q.pop(0)

        print(u, AL[u-1][0])

        for v in AL[u-1][1:]:
                if( AL[ v-1 ][0] == n+1 ):
                        Q.append(v)
```

AL[v-1][0] = AL[u-1][0] + 1

Output:

```
[8, 2, 6]
[8, 3, 1]
[8, 2, 6, 7]
[8, 5, 6]
[8, 4, 6]
[8, 1, 3, 4, 5]
[8]
6 0
1 1
3 1
4 1
5 1
2 2
7 2
```

Dijkstra's Algorithm

Definition: Given a graph G(V, E) (directed or undirected) with positive edge lengths {le: e ∈ E}; vertex s ∈ V, task is to find the shortest path between two vertices in graph.

Theory:

Breadth-first search finds shortest paths in any graph whose edges have unit length. Can we

adapt it to a more general graph G = (V, E) whose edge lengths le are positive integers?

Dijkstra's algorithm. The alarm clock algorithm computes distances in any graph with positive integral edge lengths. It is almost ready for use, except that we need to somehow implement the system of alarms. The right data structure for this job is a priority queue (usually implemented via a heap), which maintains a set of elements (nodes) with associated numeric key values (alarm times) and supports the following operations:

Insert. Add a new element to the set.

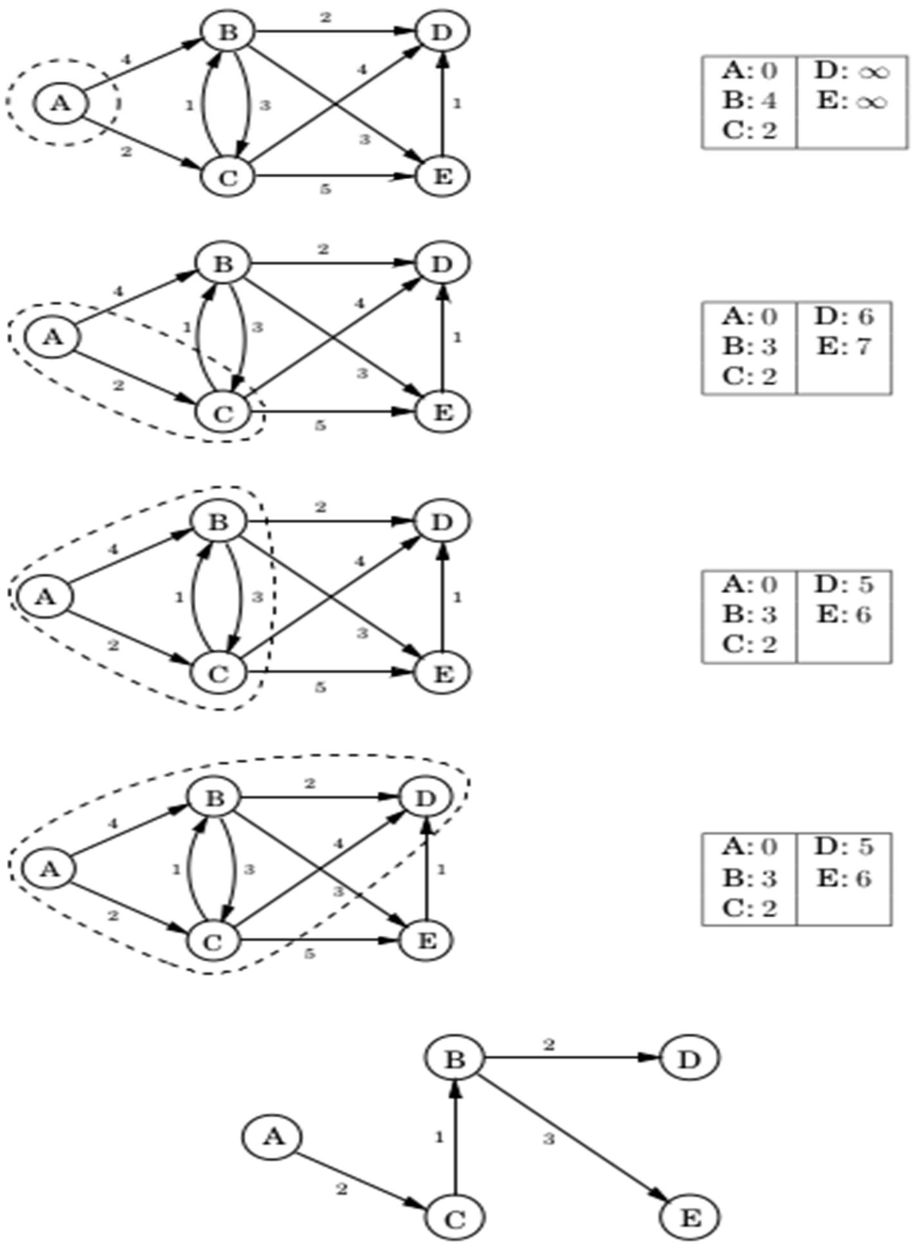
Decrease-key. Accommodate the decrease in key value of a particular element.

Delete-min. Return the element with the smallest key, and remove it from the set.

Make-queue. Build a priority queue out of the given elements, with the given key values. (In many implementations, this is significantly faster than inserting the elements one by one.)

The first two let us set alarms, and the third tells us which alarm is next to go off. Putting this all together, we get Dijkstra’s algorithm.

Figure 4.9 A complete run of Dijkstra's algorithm, with node A as the starting point. Also shown are the associated dist values and the final shortest-path tree.



Algorithm:

1. start

2. n= num of vertices, queue=[], AL=[], E= edges entries with their cost, TSPL=[];

3. for e in E

append edges entries e to AL

4. for i=0 to i=n-1

append [inf,0] to TSPL

5. enqueue source s to queue

6. make cost 0 and visited 1 in TSPL

7. poll the minimum cost vertex from queue, while queue not empty(dequeue)

8. the minimum vertex is the destination vertex where we want to make our survey

9. visit all the neighbours of destination

10. find tentative shortest distance from the source to neighbour of destination reached

11. update in the TSPL if previous cost is greater than present cost

12. enqueue the unvisited neighbours to queue

13. repeat step 9 to step 12 until all Neighbors are visited

14. if queue is empty goto step 15 else make source =destination

goto step 5

15. stop

Python code to implement the Dijkstra's algorithm:

```
#######################################
def poll(PQ):
 minimum = PQ[0][1]
 index = 0
 for i,e in enumerate(PQ[1:]):
     if( minimum > e[1] ):
             minimum = e[1]
             index = i+1
 e = PQ.pop(index)
 return( e )
#######################################

n = 5;

E = [
  [1,2,4],[1,3,2],
  [2,3,3],[2,4,2],[2,5,3],
  [3,2,1],[3,4,4],[3,5,5],
  [5,4,1]
]

AL = []
for i in range(n):
```

```
 AL.append([])

for e in E:
 AL[e[0]-1].append(e[1:])

TSPL = []
for i in range(n):
 TSPL.append([n+100,0]) # n+1 indicates infinity
              # 0 indicates not visited
print(TSPL)
#[[2, 3], [4, 6], [5, 7]]

PQ = []
# implement Dijkstra's algorithm
source = [1,0]
PQ.append(source)
TSPL[source[0]-1][0] = 0
TSPL[source[0]-1][1] = 1

while ( PQ != [] ):   #and source[0] != destination ):

 dest = poll(PQ) # visit the new destination from source, which is the
nesrest neighbour
           # to the destination previosly visited
```

```
dest_index = dest[0]-1
dest_cost = TSPL[dest_index][0]
TSPL[dest_index][1] = 1 # update the visit to dest

for neibhor in AL[dest_index]: # visit all the neighbours of dest

    TSPL_neigh_index = neibhor[0] - 1

    # calculate the tentative shortest distance ( cost ) from source the
    # neihbor of dest reached
    cost = dest_cost + neibhor[1]

    if( cost < TSPL[TSPL_neigh_index][0] ):
        TSPL[TSPL_neigh_index][0] = cost

        found = 0;
        for pq in PQ:
            if( pq[0] == neibhor[0] ):
                pq[1] = cost
                found = 1
                break
        if(found == 0):
            PQ.append( [neibhor[0] , cost] )

print(TSPL,"\n\n", PQ)
```

Output:

```
┌──(darshanreddych123㉿darshan)-[~/Downloads]
└─$ python3 graph013.py
[[0, 1], [3, 1], [2, 1], [5, 1], [6, 1]]

 []
```