CSCE-5380

Data Mining

HW-4

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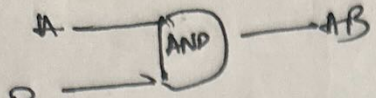
(1) And gate



A	B	out
0	0	0
0	1	0
1	0	0
1	1	1

Let $w_1 = 1.2$ $w_2 = 0.6$. Threshold = 1

L.R = 0.5

(1) A = 0, B = 0, Target = 0

$\Sigma w_i x_i = 0 * 1.2 + 0 * 0.6 = 0$

→ not greater Threshold 1 — So O/p = 0

A = 0, B = 1 Target = 10

$\Sigma w_i x_i = 0 * 1.2 + 1 * 0.6 = 0.6$

→ Again O/p = 0

A = 1 B = 0 Target = 0

$w_i x_i = 1 * 1.2 * 0 * 0.6 = 1.2$

O/p = 1 → greater than threshold

So, new weights are modified based on perceptron rule

$w_i = w_i + \eta (t - o) x_i$

t = Target
o = O/p

$w_1 = 1.2 + 0.5 (0 - 1) = 0.7$

$w_2 = 0.6 + 0.5 (0 - 1) 0 = 0.6$

0.7 0.6 - are modified wts

Restart the process from start

→ $w_1 = 0.7$, $w_2 = 0.6$, Thr = 1, LR (η) = 0.5

$w_i x_i = 0 * 0.7 + 0 * 0.6 = 0$

not greater than Threshold, 'i', o/p = 0

A = 0, B = 1, T = 0

$w_i x_i = 0 * 0.7 * 1 * 0.6 = 0.6$

→ not greater than Threshold → o/p → 0

A = 0, B = 1 Target = 0

$w_i x_i = 0 * 0.7 + 1 * 0.6 = 0.6$

not greater than Threshold, so o/p = 0

A = 1, B = 0 → Target = 0

$1 * 0.7 + 0 * 0.6 = 0.7$

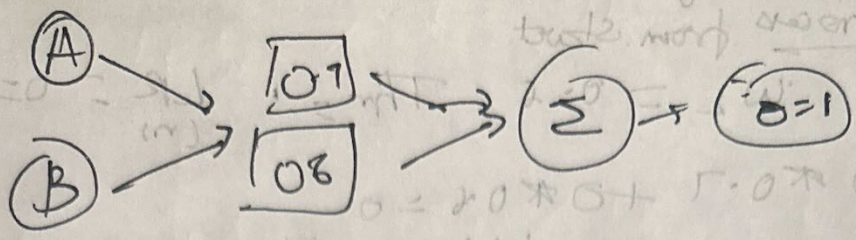
not greater than threshold of 1, so o/p = 0

A = 1, B = 1 → Target = 1

$1 * 0.7 + 1 * 0.6 = 1.3$

greater than 1, so o/p = 1

$w_1 = 0.7$, $w_2 = 0.6$ → weights



Approach 2

∴ we set bias value of -1.5

$$z = w_1 x_1 + w_2 x_2 + b$$

$$z = x_1 + x_2 - 1.5$$

$x_1 = x_2 = 1$

$$1 + 1 - 1.5 = 0.5$$

$$\text{step}(0.5) = 1$$

∴ $x_1 = 1, x_2 = 0$

$$1 + 0 - 1.5 =$$

$$= -0.5$$

$$y = \text{step}(-0.5) = 0$$

$z = x_1 + x_2 - 1.5$ ← proves true for this condition

- AND gate

Approach 3

$w_1 = 0.5 \quad w_2 = 0.5 \quad b = -0.7$

$x_1 = 1, x_2 = 1$

$z = w_1 * x_1 + w_2 * x_2 + b$

$= 0.5 * 1 + 0.5 * 1 - 0.7 = 0.3$

$y = step(z) = 1$

$step(x) = 0$ if $x < 0$
$= 1$ if $x >= 0$

$step(0.3) = 1$

$x_1 + x_2 - 0.7$ → proves true for And function

(3) (a) $x_1 \cdot x_2 \geq x_3 \cdot x_4$

$\rightarrow x_1 \cdot x_2 - x_3 x_4 \geq 0$

can be true for some values

$$F(x) = \sum_{i=1}^{x} w_i \phi(x)$$

$x_1 \; x_2 = x_3 x_4$

x_1

$1 \times x_1 \; x_2 + x_3 x_4 x_1 - 1((x_1 x_2) + (x_3 x_4) x_1)$

$= (1 \; -1) \begin{pmatrix} x_1 x_2 \\ x_3 x_4 \end{pmatrix}$

$w_1 = 1 \quad w_2 = -1$

Feature functions $\phi_1(x) = x_1 \cdot x_2 \quad \phi_2(x) = x_3 \cdot x_4$

Mathematical Expression

$F(x) = x_1 x_2 - x_3 x_4$

(b) Mathematical Expression for Linear classifier of form

$F(x) = W \cdot \phi(x)$

W = weight $\quad \phi(x)$ = Feature function

$F(x) = -1\,(x_1(x_2))$

$W = -1 \qquad \phi(x) = x_1 x_2$

→ Hence we get $\boxed{F(x) = -x_1 \cdot x_2}$

But this case is not valid when $W = 1$

as we get ~~$\phi(x) = x_1 x_2$~~ $\boxed{F(x) = \textcircled{0} x_1 \cdot x_2}$

which can't classify.

(5)

Classifier C_1:

Threshold = 0.15

Pred: + + + + + + + + + +

Actual: − − + − + − + + + +

TP = 5 FN = 0 FP = 5 TN = 0

$$TPR = \frac{TP}{TP+FN} = \frac{5}{5+0} = 1$$

$$FPR = \frac{FP}{FP+TN} = \frac{5}{5+0} = 1$$

Threshold = 0.2

Pred: − + + + + + + + + +

Actual: − − + − + − − + + +

TP = 5 FN = 0 FP = 4 TN = 1

$$TPR = \frac{TP}{TP+FN} = \frac{5}{5+0} = 1$$

$$FPR = \frac{FP}{FP+TN} = \frac{4}{5} = 0.8$$

(iii) Threshold = 0.25

Pred	−	+	+	+	+	+	+	+	+	+
Act	−	−	+	−	+	−	−	+	+	+

TP = 5 TN = 2 FP = 3 FN = 0

$$TPR = \frac{TP}{TP + FN} = \frac{5}{5+0} = \frac{5}{5} = 1$$

$$FPR = \frac{FP}{FP + TN} = \frac{3}{5} = 0.6$$

(iv) Threshold = 0.37

Pred	−	−	−	+	+	+	+	+	+	+
Act	−	−	+	−	+	−	−	+	+	+

TP = 4 FP = 3

FN = 1 TN = 2

$$TPR = \frac{TP}{TP + FN} = \frac{4}{5} = 0.8$$

$$FPR = \frac{FP}{FP + FN} = \frac{3}{3+2} = 0.6$$

(v) Threshold = 0.41

Pred	-	-	-	+	+	+	+	+	+	+
Act	-	-	+	-	+	-	-	+	+	+

TP = 4 FP = 2 FN = 1 TN = 3

$$TPR = \frac{TP}{TP+FN} = \frac{4}{5} = 0.8$$

$$FPR = \frac{FP}{FP+TN} = \frac{2}{5} = 0.4$$

(vi) Th = 0.55

Pred	-	-	-	-	-	+	+	+	+	+
Act	-	-	+	-	+	-	-	+	+	+

TP = 3 FP = 2 FN = 2, TN = 3

$$TPR = \frac{TP}{TP+FN} = \frac{3}{3+2} = 0.6$$

$$FPR = \frac{FP}{FP+TN} = \frac{2}{5} = 0.4$$

(vii) Th = 0.55

Pred	- - - - - - + + + +
Act	- - + - + - - + + +

TP = 3 FP = 1 FN = 2
TN = 4

$TPR = \frac{TP}{TP+FN} = \frac{3}{5} = 0.6$

$FPR = \frac{FP}{FP+TN} = \frac{1}{5} = 0.2$

(viii) Th = 0.9

Pred	- - - - - - - - - +
Act	- + - + - - + + + +

TP = 1 FP = 0
FN = 4 TN = 5

$TPR = \frac{TP}{TP+FN} = \frac{1}{5} = 0.2$

$FPR = \frac{FP}{FP+TN} = \frac{0}{5} = 0$

(viii) Th = 0.8

Pred	- - - - - - - + + +
Act	- - + - + - + + + +

TP = 3 FP = 0, FN = 2, TN = 5

$TPR = \frac{TP}{TP+FN} = \frac{3}{3+2} = 0.5$

$FPR = \frac{FP}{FP+TN} = \frac{0}{0+5} = 0$

(ix) Th = 0.92

Pred	- - - - - - - - + +
Act	- - + - + - - + + +

TP = 2 FP = 0
FN = 3, TN = 5

$TPR = \frac{TP}{TP+FN} = \frac{2}{2+3} = 0.4$

$FPR = \frac{FP}{FP+TN} = \frac{0}{0+5} = 0$

Th = 0.59

TP

Classifier 2

Th = 0.33

TP = 4 FP = 4 TN = 1 FN = 1

TPR = 0.8

FPR = 0.8

Th = 0.22

TP = 4 FP = 5 TN = 0 FN = 1

TPR = 0.8

FPR = 0.1

Th = 0.1

TP = 5 FP = 5 TN = 0 FN = 0

TPR = 1

FPR = 1

Th = 0.41

TP = 4 FP = 3 TN = 2 FN = 1

TPR = 0.8

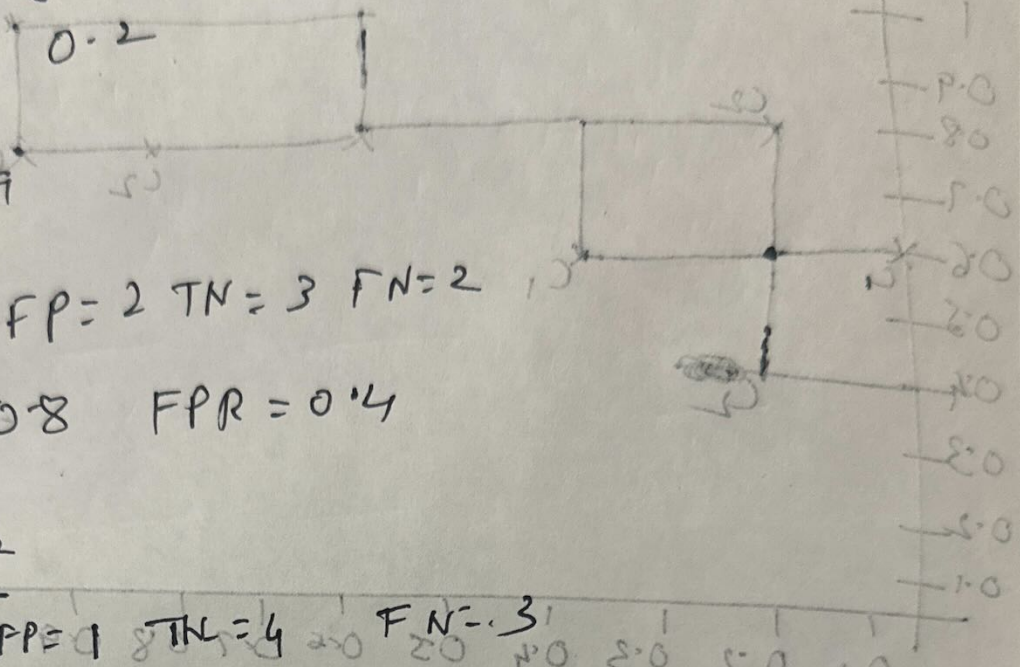
FPR = 0.6

Th = 0.68

TP = 3 FP = 1 TN = 2 FN = 1

TPR = 0.6

FPR = 0.2



Th = 0.59

TP = 3 FP = 2 TN = 3 FN = 2

TPR = 0.8 FPR = 0.4

Th = 0.72

TP = 2 FP = 1 TN = 4 FN = 3

TPR = 0.4 FPR = 0.2

Th = 0.75

TP = 2 FP = 0 TN = 5 FN = 3

TPR = 0.4 FPR = 0

Th = 0.64

TP = 4 FP = 1 TN = 4 FN = 1

TPR = 0.8 FPR = 0.2

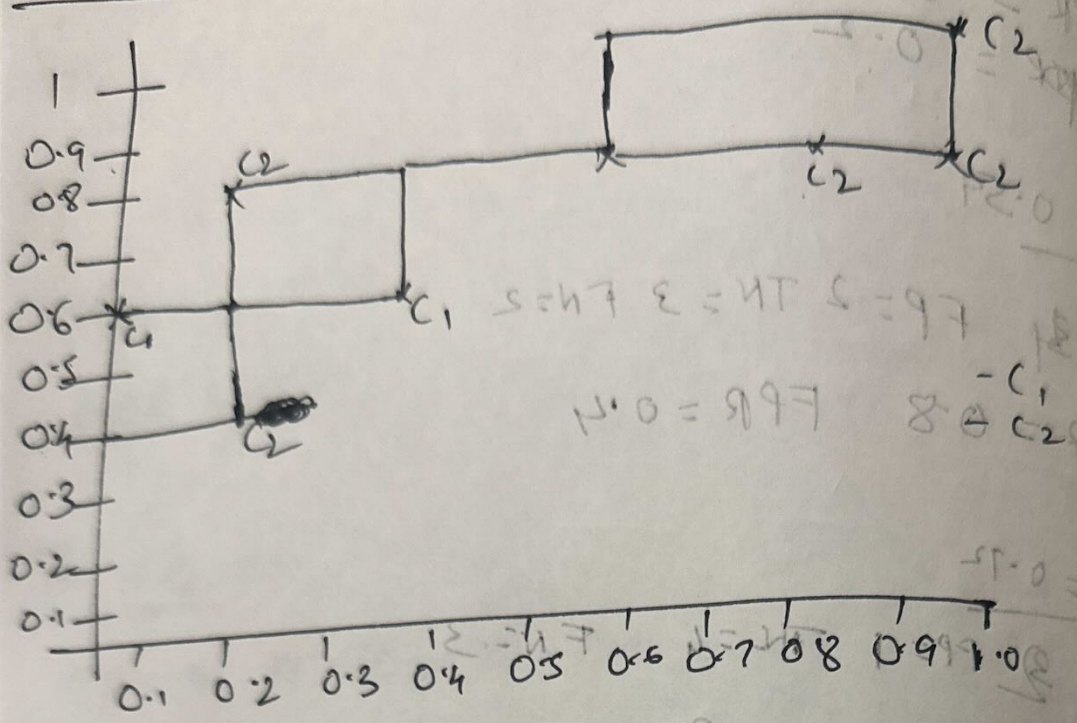
Th = 0.85

TP = 1 FP = 0 TN = 5 FN = 4

TPR = 0.2 FPR = 0

5 (a)

ROC C_1 & C_2



(b) Area under Curve C_1:

$$\frac{1}{2} * \sum_{i=1 \text{ to } n-1} [TPR_{i+1} - TPR_i) * (FPR_{i+1} + FPR_i)$$

TPR_i → TPR at i; FPR_i → FPR at i

$$\Rightarrow 0.6 \times 0.4 + 0.8 \times 0.2 + 1 \times 0.4$$

$$0.24 + 0.16 + 0.40 = 0.80$$

$C_1 \rightarrow 0.80$

Area under C_2:

$$0.4\,(0.2) + 0.8(0.8) = 0.08 + 0.64$$

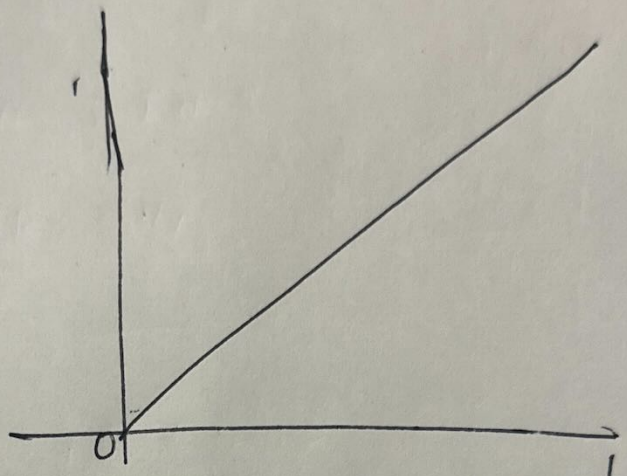
$$= 0.72$$

$C_2 \Rightarrow 0.72$

(c) $C_1 = 0.8$ $C_2 = 0.72$

Classifier 1 has more Area under curve

C_1 provides more accurate results than C_2



It also falls above lower half of plot from point (0,0) to (1,1)

In few cases C_2 can also be considered as ROC curves that C_2 has higher TPR than C_1 for most of FPR values

Overall C_1 (or) C_2 either of them can be Considered