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HW - 3

CSCE 5380

① $P(S|UG) = 0.15\%$

$P(S|G) = 0.25\%$

$P(G) = 1/5 = 0.25$

$P(UG) = 0.8$

$P(G|S) = P(G \cap S) / P(S)$

and $P(G \cap S) = P(G) \cdot P(S)$

$= 0.2 \times 0.25 = 0.05$

$P(S) = (0.2 \times 0.25) + (0.8) \times 0.15 = 0.17$

$P(G|S) = 0.05 / 0.17 = 0.294$

$\rightarrow 29.4\%$.

independent.

(2)

x_1	x_2	x_3	+ve	-ve
1	1	1	20	8
1	0	0	20	17
0	1	0	5	8
0	0	0	5	17
			50	50

Consider x_1 & x_2 only

x_1	x_2	+ve - case
1	1	
1	0	
0	1	
0	0	

	1	0	
1	20/50	20/50	
0	5/20	5/50	

$P(x_1 = 1 \cap x_2 = 1) = \frac{20}{50} = 0.4$

$P(x_1 = 1 \cap x_2 = 0) = 20/50 = 0.4$

$P(x_1 = 0 \cap x_2 = 1) = 5/50 = 0.1$

$P(x_1 = 0 \cap x_2 = 0) = 5/50 = 0.1$

$P(X_1 = 1) = \frac{20+20}{50} = \frac{40}{50} = 0.8$

$P(X_1 = 0) = \frac{5+5}{50} = 0.2$

$P(X_2 = 1) = \frac{20+5}{50} = 0.5$

$P(X_2 = 0) = \frac{17+8+5+17}{50} = 0.5$

-ve Case

	1	0
1	8/50	17/50
0	8/50	17/50

$P(X_1 = 1, X_2 = 1) = 8/50 = 0.16$

$P(X_1 = 1, X_2 = 0) = 17/50 = 0.34$

$P(X_1 = 0, X_2 = 1) = 8/50 = 0.16$

$P(X_1 = 0, X_2 = 0) = 17/50 = 0.34$

Calculation ~~$P(X_1 \cap X_2)$~~ $= P(X_1=1)\ P(X_2=1)$

$\rightarrow \frac{40}{50} \times \frac{25}{50} = 2/5$

$\rightarrow P(X_1=1)\ P(X_2=0)$

$= \frac{40}{50} \times \frac{25}{50} = 2/5$

$P(X_1=0) \cdot P(X_2=1) = \frac{10}{50} \times \frac{25}{50} = 1/10$

$P(X_1=0) \cdot P(X_2=0) = \frac{10}{50} \times \frac{25}{50} = 1/10$

X_1 and X_2 Each other

In case +ve

$\rightarrow P(X_1 \cap X_2) = P(X_1) \cdot P(X_2)$ for any value 0 or 1

Case -ve

$P(X_1=1)\ P(X_2=1) = \frac{25}{50} \times \frac{16}{50} = 8/50$

$P(X_1=1)\ P(X_2=0) = \frac{25}{50} + 34/50 = 17/50$

$P(X_1=0)\ P(X_2=1) = \frac{16}{50} \times 25/50 = 9/50$

$P(X_1=0)\ P(X_2=0) = \frac{25}{50} + 34/50 = 17/50$

$P(X_1 \cap X_2) = P(X_1) \cdot P(X_2)$

X_1 and X_2 are Independent of Eachother

③

x_1	x_2	x_3	+ve	-ve
1	1	1	20	8
1	0	0	20	17
0	1	0	5	8
0	0	0	5	17

$P(x_1=1|+) = 40/50 = 0.8$

$P(x_1=1|-) = 25/50 = 0.5$

$P(x_2=1|+) = 25/50 = 0.5$

$P(x_2=1|-) = 16/50 = 0.32$

$P(x_3=1|+) = 20/50 = 0.4$

$P(x_3=1|-) = 9/50 = 0.16$

① $x_1=1 \quad x_2=1 \quad x_3=1$

$P(x|+) =$

$P(x_1=1|+) \times P(x_2=1|+) \times P(x_3=1|+)$

$0.8 \times 0.5 \times 0.4$

$= 0.16$

$P(x_1=0|+) = 10/50 = 0.2$

$P(x_1=0|-) = 25/50 = 0.5$

$P(x_2=0|+) = 25/50 = 0.5$

$P(x_2=0|-) = 34/50 = 0.68$

$P(x_3=0|+) = 30/50 = 0.6$

$P(x_3=0|-) = 42/50 = 0.84$

$P(x|-) =$

$P(x_1=1|-) \times P(x_2=1|-) \times P(x_3=1|-)$

$0.5 \times 0.32 \times 0.16$

$= 0.0256$

$P(+|x) = P(+) \cdot P(x|+) \rightarrow 0.5 \times 0.16 = 0.08$

$P(-|x) = P(-) \cdot P(x|-) \rightarrow 0.5 \times 0.025 = 0.0128$

$\therefore P(+|x) > P(-|x)$

Class is positive +

(1, 1, 1) → +

(ii) $x_1 = 1 \quad x_2 = 0 \quad x_3 = 0$

$P(x|+) = P(x_1 = 1|+) \cdot P(x_2 = 0|+) \cdot P(x_3 = 0|+)$

$= 0.8 \times 0.5 \times 0.6 = 0.24$

$P(x|+) = 0.24$

$P(+|x) = P(+) \cdot P(x|+)$

$= 0.5 \times 0.24 = 0.12$

$P(x|-) = P(x_1 = 1|-) \times P(x_2 = 0|-) \times P(x_3 = 0|-)$

$= 0.5 \times 0.68 \times 0.84 = 0.2856$

$P(-|x) = P(-) P(x|-) \Rightarrow 0.5 \times 0.2856$

$= 0.1428$

$P(-|x) > P(+|x)$

class = −

$1 \div 0.20 \rightarrow$ class −

(iii) $x_1 = 0, \quad x_2 = 1, \quad x_3 =$

$P(x|+) = P(x_1 =$

$= 0.2 \times 0$

$P(+|x) = P(+$

$P(x|-) = P($

$=$

$P(-|x) = P(-$

$P(-|x) >$

$0,$

(iv) $x_1 = 0, \quad x_2$

$P(x|+) = P(x_1 =$

$= 0.$

$P(+|x) = P(-$

$P(x|-) = P($

$=$

$P(-|x) = P(-$

$P(-|x) > P$

(iii) $x_1 = 0, x_2 = 1, x_3 = 0$

$P(x|+) = P(x_1 = 0|+) \times P(x_2 = 1|+) \times P(x_3 = 0|+)$

$= 0.2 \times 0.5 \times 0.6 = 0.06$

$P(+|x) = P(+) \cdot P(x|+) = 0.5 \times 0.6 = 0.03$

$P(x|-) = P(x_1 = 0|-) \times P(x_2 = 1|-) \times P(x_3 = 0|-)$

$= 0.5 \times 0.32 \times 0.84 = 0.1344$

$P(-|x) = P(-) \cdot P(x|-) = 0.5 \times 0.1344 = 0.067$

$P(-|x) > P(+|x) =$ class −

0, 1, 0 → class −

(iv) $x_1 = 0, x_2 = 0, x_3 = 0$

$P(x|+) = P(x_1 = 0|+) + P(x_2 = 0|+) \times P(x_3 = 0|+)$

$= 0.2 \times 0.5 \times 0.6 = 0.06$

$P(+|x) = P(+) \cdot P(x|+) = 0.5 \times 0.06 = 0.03$

$P(x|-) = P(x_1 = 0|-) \; P(x_2 = 0|-) \; P(x_3 = 0|-)$

$= 0.5 \times 0.6 \times 0.84 = 0.2856$

$P(-|x) = P(-) \cdot P(x|-) = 0.5 \times 0.2856 = 0.1428$

$P(-|x) > P(+|x)$ ∴ class −

x_1	x_2	x_3	+	−	predicted	error
			20	8		
1	1	1				
1	0	0	20			20
0	1	0		8		5
0	0	1	5			5

Error rate $= \frac{8+20+5+5}{100} \Rightarrow \frac{38}{100} = 0.38\%$

error rate → 0.38 or 38%.

(4)

(a)

Accident	Weather	Constr	+ve	-ve
	good	no	5	30
No	good	yes	10	20
No		no	10	5
Yes.	good			
Yes	bad	no	10	5
Yes	bad	yes	5	0

$$P(\text{Accident} = \text{yes} \mid +) = \frac{25}{40} = 0.625$$

$$P(\text{Accident} = \text{yes} \mid -) = \frac{10}{60} = \frac{1}{6} = 0.16$$

$$P(\text{weather} = \text{good} \mid +) = \frac{10+10+5}{40} = \frac{25}{40} = 0.625$$

$$P(\text{weather} = \text{good} \mid -) = \frac{30+20+5}{60} = \frac{55}{60} = 0.91$$

$$P(\text{Construction} = \text{yes} \mid +) = \frac{15}{40} = 0.37$$

$$P(\text{Construction} = \text{yes} \mid -) = \frac{20}{60} = \frac{1}{3} = 0.33$$

(b)

$P(\text{Accident} = \text{no} \mid \text{weather} = \text{bad}, \text{Construction} = \text{Yes})$

40% = 0.4 (Congestation)

60% = 0.6 (no congestation)

(i) Congestation case

$P(\text{Accident} = \text{No} \mid +ve)$

$= \frac{15}{40} = 0.375$

$P(\text{weather} = \text{bad} \mid +) = \frac{15}{40} = 0.375$

$P(\text{Construction} = \text{Yes} \mid +) = \frac{15}{40} = 0.375$

Using Bayes Theoram

$P(+ve \mid \text{no}, \text{bad}, \text{Yes}) = 0.375 \times 0.375 \times 0.375 \times 0.4$

$= 0.021$

(ii) No Congestation Case

$P(\text{Accident} = \text{no} \mid -ve)$

$= \frac{50}{60} = \frac{5}{6}$

$P(\text{weather} = \text{bad} \mid -ve)$

$= \frac{5}{60}$

$P(\text{Construction} = \text{Yes} \mid +)$

$= \frac{20}{60}$

Bayes theoram

$P(-ve \mid \text{no}, \text{bad}, \text{Yes})$

$= 0.83 \times 0.083$

$= 0$

$0.021 >$

→ There exists
highway for pa

(ii) No Congestation Case

$$P(\text{Accident} = \text{no} \mid -\text{ve}) = \frac{50}{60} = \frac{5}{6} = 0.83$$

$$P(\text{weather} = \text{bad} \mid -\text{ve}) = \frac{5}{60} = 0.083$$

$$P(\text{Construction} = \text{Yes} \mid +) = \frac{20}{60} = 0.33$$

Bayes theorem

$$P(-\text{ve} \mid \text{no}, \text{bad}, \text{Yes}) = 0.83 \times 0.083 \times 0.33 \times 0.6 = 0.0136$$

$$0.0217 > 0.0136$$

→ There exists traffic Congestation on highway for particular day