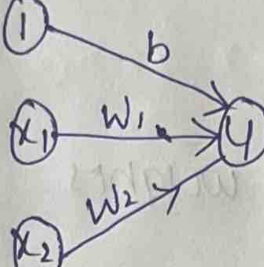
Q1. Demonstrate how the perception model can be used to represent the AND functions b/w a pair of Boolean variables.

⇒ Truth table for AND for pair of boolean variables

x_1	x_2	t
1	1	1
1	-1	-1
-1	1	-1
-1	-1	-1

perception network:



$$y = f(y_{in}) = \begin{cases} 1 & y_{in} > 0 \\ 0 & y_{in} = 0 \\ -1 & y_{in} < 0 \end{cases}$$

let initial weights and constant $b = 0$

i.e $\omega_1 = \omega_2 = b = 0$

second input $\rightarrow$ $\begin{array}{ccc} x_1 & x_2 & t \\ \end{array}$ $[\,1 \quad -1 \quad -1\,]$

$\Rightarrow y_{in} = 1 + (1)(1) + (1)(-1) = 1$

$y = f(y_{in}) = 1$

∴ Again weight change is required

∴ $\omega_1(new) = 1 + (1)(-1)(1)$

$= 0$

$\omega_2(new) = 1 + (1)(-1)(-1) = 2$

$b(new) = 1 + (1)(-1) = 0$

$\therefore$ New weights $\Rightarrow w_1 = b = 0$; $w_2 = 2$

	x_1	x_2	t
Third input $\rightarrow$	-1	1	-1

$\Rightarrow y_{in} = 0 + (0)(-1) + 2(1) = 2$

$\therefore y = f(y_{in}) = 1 \neq t$

$\therefore$ $w_1(new) = 0 + (1)(-1)(-1) = +1$

$w_2(new) = 2 + (1)(-1)(1) = 1$

$b(new) = 0 + (1)(-1) = -1$

first input perception $\rightarrow$ $\begin{bmatrix} x_1 & x_2 & t \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore y_{in} = b + w_1 x_1 + w_2 x_2$

$= 0 + 0(1) + 0(1) = 0$

$\therefore y = f(y_{in}) = 0$

since $y \neq t$

$\therefore$ we need update weights

$w_i(new) = w_i(old) + \alpha t x_i$

where α is learning rate & here

$\alpha = 1$

$\therefore w_1(new) = 0 + (1)(1)(1)$

$= 1$

$w_2(new) = 0 + (1)(1)(1)$

$= 1$

$b(new) = b(old) + \alpha t$

$= 0 + (1)(1)$

$= 1$

$\therefore$ New weights $\Rightarrow$

$w_1 = w_2 = b = 1$

forth Input $\rightarrow$ $\begin{array}{ccc} x_1 & x_2 & t \\ \end{array}$ $\begin{bmatrix} -1 & -1 & -1 \end{bmatrix}$

$\Rightarrow y_{in} = (-1) + 1(-1) + 1(-1)$

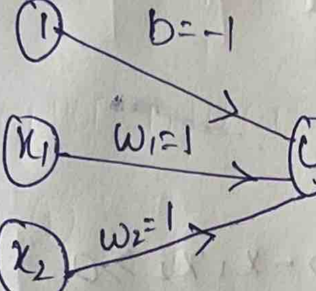
$= -3$

$\therefore y = f(y_{in}) = -1 = t$

$\therefore$ therefore final weights $\Rightarrow$ (after first Epoch)

$$\boxed{\begin{array}{l} w_1 = 1 \\ w_2 = 1 \\ b = -1 \end{array}}$$

$\therefore$ perception network becomes



$\rightarrow y = f(y_{in}) = \begin{cases} 1 & y_{in} > 0 \\ 0 & y_{in} = 0 \\ -1 & y_{in} < 0 \end{cases}$

Q3: (a) for the given instance, there are four continuous values

x_1, x_2, x_3 & x_4

the final $y=1$ if product of x_1 & x_2 is greater than or equals to x_3 & x_4

i.e

$$x_1 x_2 \geq x_3 x_4$$

$$\Rightarrow x_1 x_2 - x_3 x_4 \geq 0$$

Also

$$f(x) = w_1 \phi_1 + w_2 \phi_2$$

$\therefore w_1 = 1$ & $w_2 = -1$

$\phi_1 = x_1 x_2$ & $\phi_2 = x_3 x_4$

$\therefore$ final $y = \begin{cases} 1 & \text{if } (x_1 \cdot x_2 - x_3 x_4 \geq 0) \\ -1 & \text{otherwise} \end{cases}$

(b)

x_1	x_2	y
1	1	-1
1	-1	1
-1	1	1
-1	-1	-1

for this, we have x_1 & x_2 as variables & According to the table

$$f(x) = -x_1 x_2$$

$$w_1 = -1 \text{ \& } \phi_1 = x_1 x_2$$

$$y = \begin{cases} 1 & \text{if } f(x) \Rightarrow -x_1 x_2 \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

Q5 :

$P(y=+/c_1)$	0.15	0.2	0.25	0.37	0.41	0.55	0.65	0.8	0.92	0.99
$P(y=+/c_2)$	0.33	0.22	0.1	0.41	0.68	0.59	0.72	0.75	0.64	0.95
y	-	-	+	-	+	-	-	+	+	+

a) ⇒ ROC curve for C_1

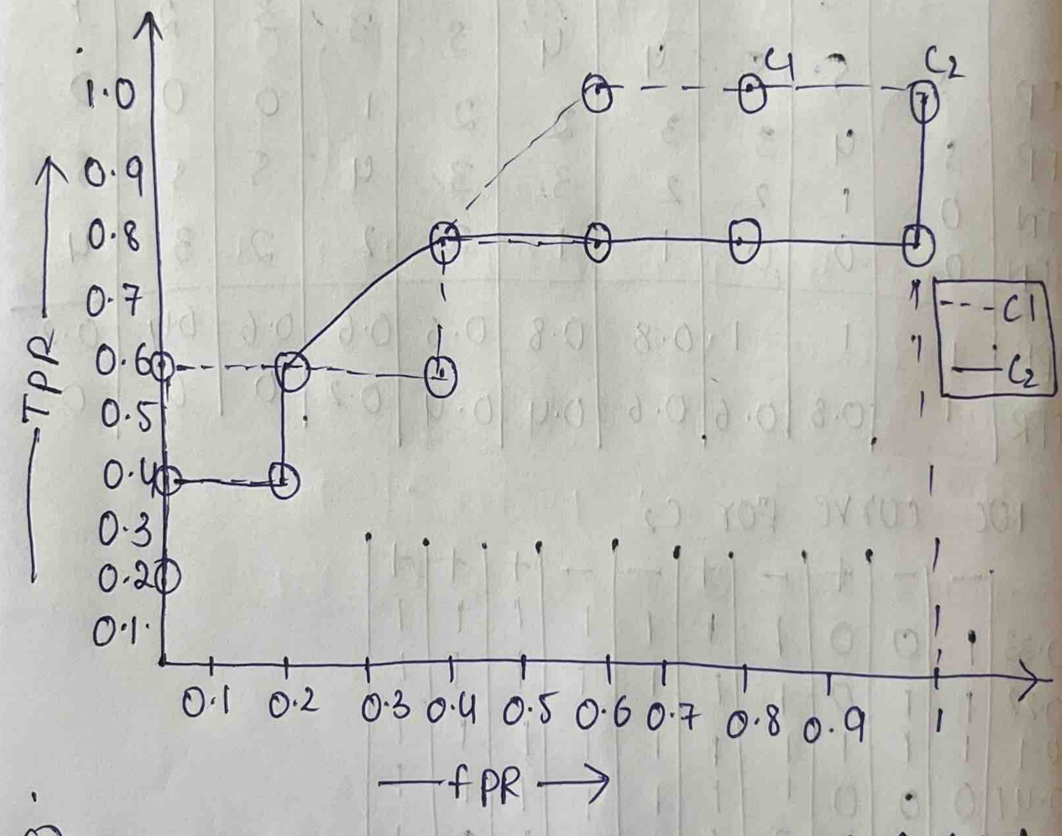
	-	-	+	-	+	-	-	+	+	+
0.15	1	1	1	1	1	1	1	1	1	1
0.2	0	1	1	1	1	1	1	1	1	1
0.25	0	0	1	1	1	1	1	1	1	1
0.37	0	0	0	1	1	1	1	1	1	1
0.41	0	0	0	0	1	1	1	1	1	1
0.55	0	0	0	0	0	1	1	1	1	1
0.65	0	0	0	0	0	0	1	1	1	1
0.8	0	0	0	0	0	0	0	1	1	1
0.92	0	0	0	0	0	0	0	0	1	1
0.99	0	0	0	0	0	0	0	0	0	1

	0.15	0.2	0.25	0.37	0.41	0.55	0.65	0.8	0.92	0.99
TP	5	5	5	4	4	3	3	3	2	1
FP	5	4	3	3	2	2	1	0	0	0
TN	0	1	2	2	3	3	4	5	5	5
FN	0	0	0	1	1	2	2	2	3	4
TPR	1	1	1	0.8	0.8	0.6	0.6	0.6	0.4	0.2
FPR	1	0.8	0.6	0.6	0.4	0.4	0.2	0	0	0

ROC curve for C_2

	-	-	+	-	+	-	-	+	+	+
0.33	1	0	0	1	1	1	1	1	1	1
0.22	1	1	0	1	1	1	1	1	1	1
0.1	1	1	1	1	1	1	1	1	1	1
0.41	0	0	0	1	1	1	1	1	1	1
0.68	0	0	0	0	1	0	1	1	0	1
0.59	0	0	0	0	1	1	1	1	1	1
0.72	0	0	0	0	0	0	1	1	0	1
0.75	0	0	0	0	0	0	0	1	0	1
0.64	0	0	0	0	0	0	1	1	1	1
0.95	0	0	0	0	0	0	0	0	0	1

	0.1	0.22	0.33	0.41	0.59	0.64	0.68	0.72	0.75	0.95
TP	5	4	4	4	4	3	3	2	2	1
FP	5	5	4	3	2	1	1	1	0	0
TN	0	0	1	2	3	4	4	4	5	5
FN	0	1	1	1	1	2	2	3	3	4
TPR	1	0.8	0.8	0.8	0.8	0.6	0.6	0.4	0.4	0.25
FPR	1	1	0.8	0.6	0.4	0.2	0.2	0.2	0	0



(b) AUC for $C_1 \Rightarrow (0.6 \times 0.4) + (0.8 \times 0.2) + (0.4 \times 1)$

$= 0.24 + 0.16 + 0.4$

$= 0.8$

AUC for $C_2 \Rightarrow (0.4 * 0.2) + \{(0.6 * 0.2) + (0.5 \times 0.2 \times 0.2)\}$

$+ (0.6 * 0.8)$

$= 0.08 + 0.12 + 0.020 + 0.48$

$= 0.7$

classifier 1 has bigger area under the curve

(c) classifier 1 will be preferred over classifier 2 as area under curve (AUC) of classifier 1 is greater than classifier 2