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CSC E 5380 - Homework 2

1. a)

Attributes	Yes Count	No Count
1. Tobacco Smoking	5	5
2. Radon Exposure	2	8
3. Chronic Cough	7	3
4. Weight Loss	5	5
5. Lung Cancer	5	5

Calculate Initial Entropy:

We have 5 title & 5-ve example of lung cancer, where $N=10$

$$\begin{aligned}
 E(S) &= -\left(\frac{5}{10} - \sum_{i=1}^2 P_i \log_2(P_i)\right) \\
 &= -\frac{5}{10} \log_2 \frac{5}{10} - \frac{5}{10} \log_2 \frac{5}{10} \\
 &= 1
 \end{aligned}$$

Attributes:

Tobacco Smoking:

Values $\rightarrow [Yes, No]$

$$\begin{aligned}
 S_{Yes} [+1, -1] \quad E(S_{Yes}) &= -\frac{4}{5} \log_2 \frac{4}{5} - \frac{1}{5} \log_2 \frac{1}{5} \\
 &= 0.7219
 \end{aligned}$$

$$\begin{aligned}
 S_{No} \rightarrow [+1, -4] \quad \therefore E(S_{No}) &= -\frac{1}{5} \log_2 \frac{1}{5} - \frac{4}{5} \log_2 \frac{4}{5} \\
 &= 0.7219
 \end{aligned}$$

$$\therefore \text{Gain}(s) = E - \sum_{v \in V} \frac{|S_v|}{|S|} E(s_v)$$

$$= E - \frac{5}{10} E(s_{yes}) - \frac{5}{10} E(s_{no})$$

$$= 1 - \frac{5}{10} (0.7219) - \frac{5}{10} (0.7219)$$

$$\text{Gain}(s, \text{Tobacco Smoking}) = \underline{\underline{0.2781}}$$

Attribute: Random Exposure: Value \rightarrow [yes, no]

$$s_{yes} \leftarrow [+2, -0] \quad E(s_{yes}) = -\frac{2}{2} \log_2 \frac{2}{2} - \frac{0}{2} \log_2 \frac{0}{2}$$

$$= -1 \times (0) - 0$$

$$= 0 - 0$$

$$s_{no} \rightarrow [+3, -5] \quad E(s_{no}) = -\frac{3}{8} \log_2 \frac{3}{8} - \frac{5}{8} \log_2 \frac{5}{8}$$

$$= 0.954$$

$$\text{Gain}(s, \text{Random}) = E - \sum_{v \in \{yes, no\}} \frac{|S_v|}{|S|} E(s_v)$$

$$= 1 - \frac{2}{10} E(s_{yes}) - \frac{8}{10} E(s_{no})$$

$$= 1 - \frac{2}{10} (0) - \frac{8}{10} (0.954)$$

$$\text{Gain}(s, \text{Random}) = \underline{\underline{0.2368}}$$

Attribute: Chronic Cough

$$S_{Yes} \rightarrow [+4, -3] \quad E(S_{Yes}) = -\frac{4}{7} \log_2 \frac{4}{7} - \frac{3}{7} \log_2 \frac{3}{7}$$

$$= 0.9852$$

$$S_{No} \rightarrow [+1, -2]$$

$$E(S_{No}) = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3}$$

$$= 0.9182$$

$$\text{Gain}(S, \text{Chronic}) = E - \sum_{v \in \{y_1, y_2\}} \frac{|S_v|}{s} E(S_v)$$

$$= 1 - \frac{7}{10} E(S_{Yes}) - \frac{3}{10} E(S_{No})$$

$$= 1 - \frac{7}{10} (0.9852) - \frac{3}{10} (0.9182)$$

$$\text{Gain}(S, \text{Chronic}) = 0.0349$$

Attribute: Weight loss

$$S_{Yes} [+3, -2] \quad E(S_{Yes}) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5}$$

$$= 0.9209$$

$$S_{No} [+2, -3]$$

$$E(S_{No}) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5}$$

$$= 0.9709$$

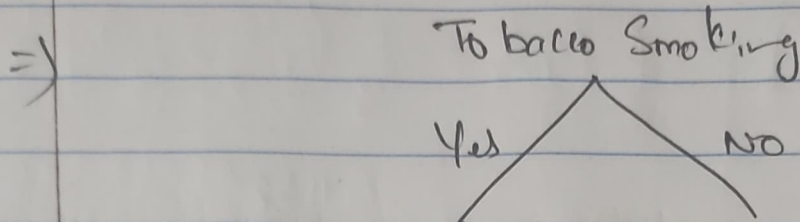
$$\text{Gain}(S, \text{Weight}) = E - \frac{5}{10} E(S_{Yes}) - \frac{5}{10} E(S_{No})$$

$$= 1 - \frac{5}{10} (0.9209) - \frac{5}{10} (0.9709)$$

$$\text{Gain}(S, \text{Weight}) = 0.0291$$

From the information, we will consider Tobacco Smoking as a root has it have max. information gain.

As there is one 'No' for 'Yes' in Cancer. we will repeat process



Exposure	Chronic	Weightless	Cancer	Exposure	Chronic	Weightless	Cancer
Yes	Yes	No	Yes	Yes	No	Yes	Yes
No	Yes	No	Yes	No	Yes	No	No
No	Yes	Yes	Yes	No	Yes	Yes	No
No	Yes	Yes	Yes	No	Yes	No	No
No	No	No	No	No	No	Yes	No

In 'Yes' $E = -\frac{4}{5} \log_2 \frac{4}{5} - \frac{1}{5} \log_2 \frac{1}{5} = 0.7219$

Attribute Exposure:-

$S_{Yes} \rightarrow [+1, 0] \quad E(S_{Yes}) = -\frac{1}{1} \log_2 \frac{1}{1} - 0 = 0$

$S_{No} \rightarrow [+3, -1] \quad E(S_{No}) = -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4}$
 $= 0.8112$

$\text{Gain}(S, \text{Exposure}) = 0.7219 - \frac{1}{5}(0) - \frac{4}{5}(0.8112) = 0.07294$

Attribute Cough:-

$S_{Yes} \rightarrow [+4, 0] \quad E(S_{Yes}) = -\frac{4}{4} \log_2 \frac{4}{4} - 0 = 0$

$S_{No} \rightarrow [0, -1] \quad E(S_{No}) = 0 - \frac{1}{1} \log_2 \frac{1}{1} = 0$

$$\therefore \text{Gain}(S, \text{rough}) = 0.7219 - 0 - 0 = 0.7219$$

Attribute: weight loss:

$$S_{\text{Yes}} [+2, -0] \therefore E(S_{\text{Yes}}) = -\frac{2}{2} \log_2 \frac{2}{2} - 0 = 0$$

$$S_{\text{No}} [+2, -1] \therefore E(S_{\text{No}}) = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} \\ = 0.9182$$

$$\text{Gain}(S, \text{Weight}) = 0.7219 - \frac{2}{5}(0) - \frac{3}{5}(0.9182) = \underline{\underline{0.1709}}$$

$$\Rightarrow \text{Max Gain} = \text{Gain}(S, \text{Weight}) = \underline{\underline{0.1709}} \\ \text{from all three.}$$

In NO:

$$E = -\frac{1}{5} \log_2 \frac{1}{5} - \frac{4}{5} \log_2 \frac{4}{5} = 0.7219$$

Attribute Exposure:

$$S_{\text{Yes}} [+1, -0] \quad E(S_{\text{Yes}}) = 0$$

$$S_{\text{No}} [+0, -4] \quad E(S_{\text{No}}) = 0$$

$$\text{Gain}(S, \text{Radon Exposure}) = 0.7219$$

Attribute Chronic:

$$S_{\text{Yes}} [+6, -3] \quad E(S_{\text{Yes}}) = 0$$

$$S_{\text{No}} [+1, -1] \quad E(S_{\text{No}}) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2}$$

$$= 1$$

$$Gain(S, Chronic) = 0.7219 - 0 - \frac{2}{5} (1) = 0.3219$$

Attribute - Weight loss.

$$S_{Yes} [+1, -2] \quad E(S_{Yes}) = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} \\ = 0.9182$$

$$S_{No} [+0, -2] \quad E(S_{No}) = 0$$

$$Gain(S, weight) = 0.7219 - \frac{3}{5} (0.9182) = 0 \\ = 0.1709$$

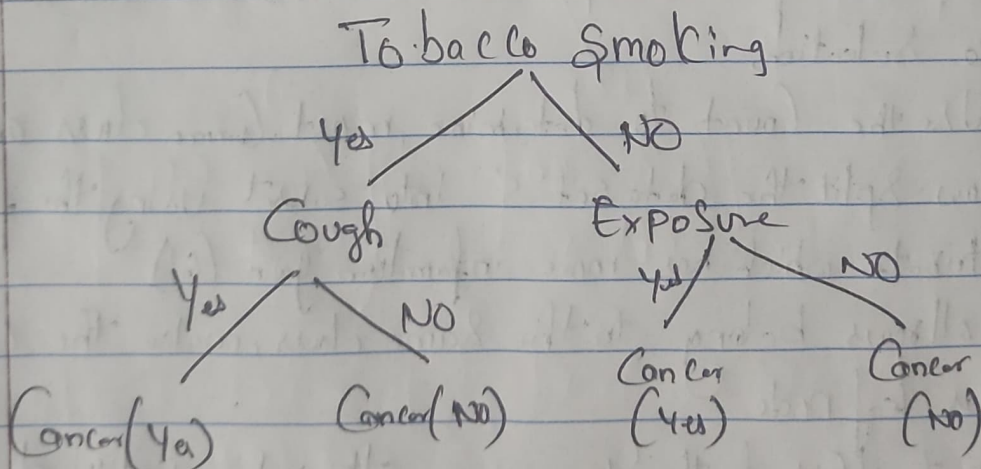
$$\therefore Gain(S, Random) = 0.7219$$

$$Gain(S, Chronic) = 0.3219$$

$$Gain(S, weight) = 0.1709$$

(\therefore Max. Value of Gain)

\therefore The tree will be



(b) To Calculate training error of decision Tree, we are considering the Confusion matrix

	NO Cancer	Cancer
NO Cancer	5	0
lung Cancer	0	5

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

$$= \frac{5 + 5}{5 + 5 + 0 + 0}$$

$$= \frac{10}{10} = 1$$

$$= \frac{5 + 5}{5 + 5 + 0 + 0}$$

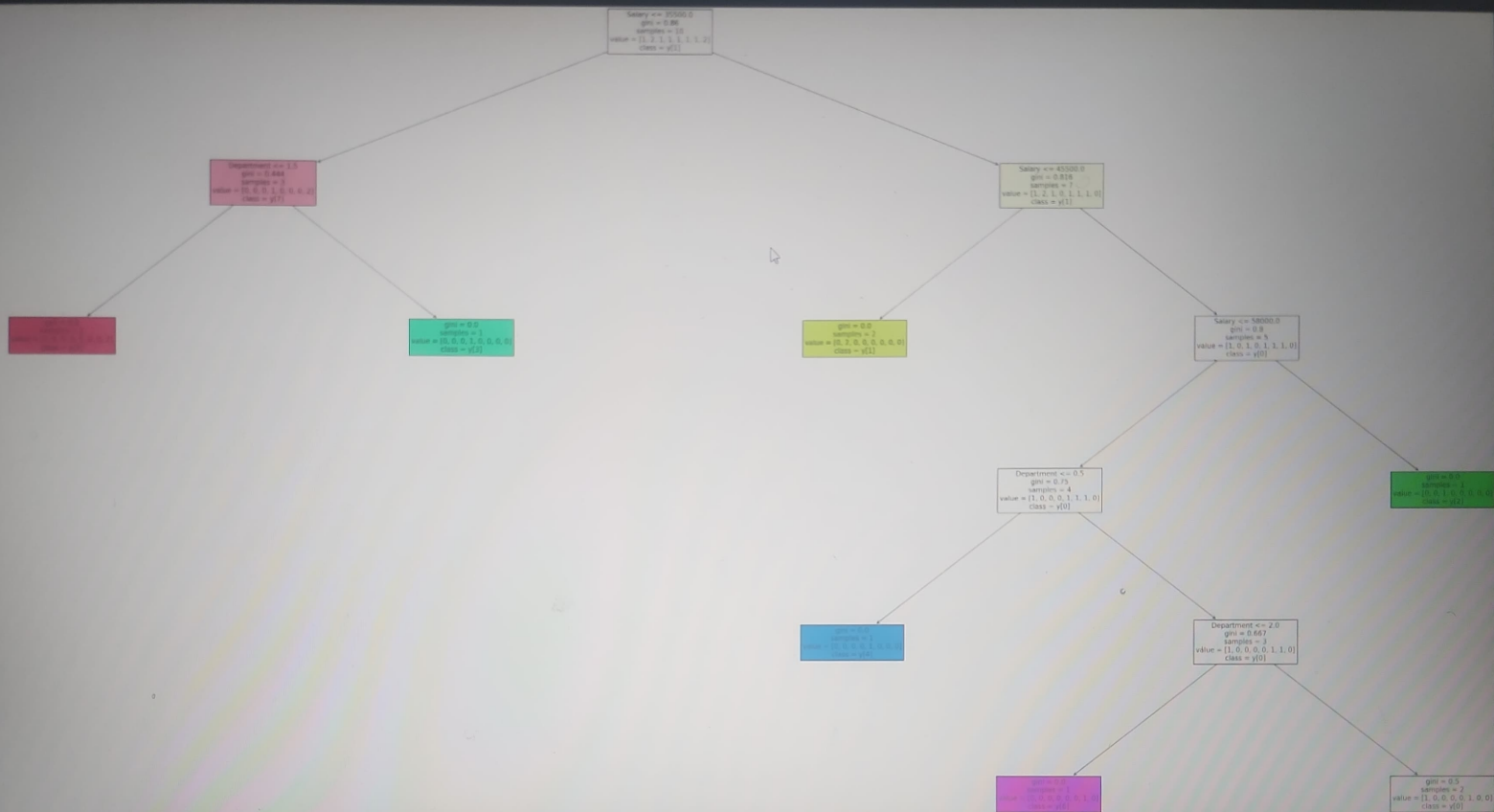
$$= \frac{10}{10} = 1$$

Training error =

$$\frac{FP + FN}{TP + TN + FP + FN} = \frac{0 + 0}{5 + 5 + 0 + 0} = \frac{0}{10} = 0$$

2 a) The basic decision tree algorithm should be modified as follows to take into consideration the Count of each Generalised data tuple.

- The Count of each tuple must be included into calculation of attribute selection.
- Use the Count to determine most common class among tuples.
- Now split the data set 'S' into subset using the feature / attribute which has more information gain (G).
- If all rows belong to the same class, make the current node as leaf node.
- Repeat this for the remaining attributes, until decision tree has all the leaf nodes.



2c) We first estimate prior probabilities for "Status" class labels

$$\therefore P(\text{Senior}) = 5/11 \quad \& \quad P(\text{Junior}) = 6/11$$

\Rightarrow Now conditional probabilities

$P(\text{department} / \text{status})$

Class	Sales	Systems	Marketing	Secondary
Senior	$1/5$	$2/5$	$1/5$	$1/5$
Junior	$1/3$	$1/3$	$1/6$	$1/6$

$P(\text{age} / \text{status})$

Class	21-25	26-30	31-35	36-40	41-45	46-50
Senior	0	0	$2/5$	$1/5$	$1/5$	$1/5$
Junior	$1/6$	$1/2$	$1/3$	0	0	0

$P(\text{Salary} / \text{status})$

Class	26K-30K	31-35K	36-40K	41-45K	46-50K	56K-70K
Senior	0	0	$1/5$	0	$2/5$	$2/5$
Junior	$1/3$	0	0	$1/6$	$1/3$	0

\therefore So for test instance

$$V_{NB} = \arg \max_j P(V_j) \prod_i P(q_i | V_j)$$

V_j (Yes, No)

\therefore (system, "26-30", "46-50k")

Two labels $\begin{cases} \rightarrow \text{Senior} \\ \rightarrow \text{Junior} \end{cases}$

$$P(\text{Senior}/a) = P(\text{Senior}) \times P(\text{system} | \text{Senior}) \times P(26-30 | \text{Senior}) \\ \times P(46K-50K | \text{Senior})$$

$$= \frac{5}{11} \times \frac{2}{5} \times 0 \times \frac{2}{5}$$

$= 0$

$$\therefore \boxed{P(\text{Senior}/a) = 0}$$

$$P(\text{Junior}/a) = P(\text{Junior}) \times P(\text{system} | \text{Junior}) \times P(26-30 | \text{Junior}) \times \\ P(46K-50K | \text{Junior})$$

$$= \frac{6}{11} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{33}$$

$$\therefore \boxed{P(\text{Junior}/a) = 0.030}$$

∴ By Comparing the two values & the max value is junior.

→ Hence, the label for this instance (person) is junior

$$\Rightarrow P(\text{senior} | a) < P(\text{junior} | a)$$