

Fundamentals of AI

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Homework 6: Neural Networks & Deep Learning

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Q) To represent the XOR (Exclusive-or) function with a neural network using a sigmoid output activation & a single hidden layer with ReLU activation you can use following structure.

Input Layers: Two neurons, one for each input, representing the two boolean values (0 or 1)

Hidden Layers: You will need a minimum of two neurons in the hidden layers to represent XOR.

→ Simplified diagram of neural network:

Input Layer:

$[x_1]$

$[x_2]$

Hidden Layer:-

$[ReLU]$

$[ReLU]$

$[w_1]$ $[w_2]$

Output Layer:

$[sigmoid]$

$[y]$

→ Now, let's consider how to set the weights to the network to represent XOR, XOR can be represented as a combination of Logical operations, specially using AND, OR & NOT operation.

$$XOR = (x_1 \text{ AND } (\text{NOT } x_2)) \text{ OR } ((\text{NOT } x_1) \text{ AND } x_2)$$

→ To create a neural network to represent this expression, you can use the following weights and biases:

→ For the hidden layer:

• Neuron 1 weights: $[1, -1]$

• Neuron 1 bias: 0

• Neuron 2 weights: $[-1, 1]$

• Neuron 2 bias: 0

→ For the output layer:

• output neuron weights: $[1, 1]$

• output neuron bias: -1

→ with these weights and biases, the network will correctly represent the XOR function as defined by the logical expression above. It will produce the following outputs:

• $(0, 0) \rightarrow \text{output} \approx 0$ (false)

• $(0, 1) \rightarrow \text{output} \approx 1$ (True)

• $(1, 0) \rightarrow \text{output} \approx 1$ (True)

• $(1, 1) \rightarrow \text{output} \approx 0$ (false)

→ These weights & biases create a network that performs the XOR operation using the given logical operations, as specified in the hint.

82) Here's the pseudo code for a one-dimensional convolutional layer (Conv1D) based on the structure provided for the two-dimensional version (Conv2D):

4 Python code.

class Conv1D(K):

create $w[i]$ for each $0 \leq i < K$, initialize randomly

create $d[i]$ for each $0 \leq i < K$, initialize to 0.

method output(input)

Inputs

input is y_d array

create out $[y]$ for each $0 \leq y < y_d - K + 1$

for each $y: 0 \leq y < y_d - K + 1$ do

out $[y] := \text{sigma} (i=0 \text{ to } K-1) \text{ input } [y+i] * w[i]$

return out

method Backprop(error)

Inputs

error is $y_d - K + 1$ array

create ierror $[y]$ for each $0 \leq y < y_d$, init to 0

for each $y: 0 \leq y < y_d - K + 1$ do

for each $i: 0 \leq i < K$ do

$d[i] := d[i] + \text{input} [y+i] * \text{error} [y]$

ierror $[y+i] := \text{ierror} [y+i] + \text{error} [y] * w[i]$

return ierror

method update (learning rate, batch-size)

for each i do

$w[i] := w[i] - \text{learning rate} / \text{batch-size} * d[i]$

$d[i] := 0$

→ This pseudocode defines a one-dimensional convolution layer (Conv1D) with similar operations to the two-dimensional

Hyper-parameters:

① KERNEL SIZE (K): The kernel size (K) is defined at the beginning of the Conv1D class. To extend the pseudocode to allow for variable kernel sizes, we can add a constructor with an initialization method to set the kernel size when creating an instance of the Conv1D layer.

class Conv1D:

def __init__(self, kernel-size):

self.K = kernel-size

self.w = [random-inites for _ in range(self.K)]

self.d = [0] * self.K

Rest of the pseudocode remains the same.

→ with this modification, you can create Conv1D layers with different kernel sizes by passing the desired kernel-size when creating an instance.

2) Activation function: The pseudocode doesn't specify the activation function, which is a crucial hyperparameter in neural network i.e. include an activation function, you can add an activation function variable & apply it to output of convolution operation.

class Conv1D:

def __init__(self, kernel_size, activation):

self.k = kernel_size

self.activation = activation

self.w = [random_init() for _ in range(self.k)]

self.d = [0] * self.k

def output(self, input):

out = [0] * (len(input) - self.k + 1)

for y in range(len(input) - self.k + 1):

convolution_result = sum(input[y+i] * self.w[i]

for i in range(self.k))

out[y] = self.activation(convolution_result)

return out

Rest of the pseudocode remains the same.

→ with this modification, you can specify the activation function when creating a Conv1D layer.

→ These extensions allow you to control the kernel size & the activation function, making our Conv1D layer more versatile & consistent with typical deep learning libraries like keras & pytorch.