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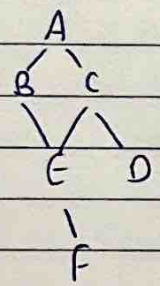
Question 46

Given probabilities
- $P(A)$
- $P(B|A)$
- $P(C|B)$
- $P(D|A,C)$
- $P(E,B)$
- $P(F|E)$

-> we construct the Bayesian network as follows
1. A influences B ($P(B|A)$)
2. B influences C ($P(C|B)$)
3. A and C jointly influence D ($P(D|A,C)$)
4. B influences E ($P(E|B)$)
5. E influences F ($P(F|E)$)

network structure



b) variables that can be pruned to compute $P(d)$
-> To compute $P(d)$. we only need the variables that influence D. According to the Bayesian network
- D depends on A and C
-> C depends on B, which is turn depends on A

we cannot prune A, B, or C However, E and F are irrelevant $P(d)$ and can be pruned

c) Eliminate A to compute $P(d)$

Step 1:- write $P(d)$ in terms of the joint probabilities

$$P(d) = \sum_{a,b,c} P(d|a,c)\,P(c|b)\,P(b|a)\,P(a)$$

Step 2:- Substituting the probabilities

$$P(d) = \sum_{a,c} P(d|a,c) \sum_{b} P(c|b)\,P(b|a)\,P(a)$$

Step 3:- Eliminate A

After elimination A: $\sum_{a} P(d|a,c)\,P(c,b)\,P(b|a)\,P(a)$

1) for b=true, c=true:

~~factor~~

$= P(d|a \wedge c)\,P(c|b)\,P(b|a)\,P(a) + P(d|\neg a \wedge c)\,P(c|b)\,P(b|a)\,P(\neg a)$

$= (0.5)(0.9)(0.9)(0.8) + (0.7)(0.9)(0.3)(0.2)$

$= 0.324 + 0.0378 = 0.3618$

2) b=true, c=false

$= P(d|a \wedge \neg c)\,P(\neg c|b)\,P(b|a)\,P(a) + P(d|\neg a \wedge \neg c)\,P(\neg c|b)\,P(b|\neg a)\,P(\neg a)$

$= (0.6)(0.1)(0.9)(0.8) + (0.2)(0.1)(0.3)(0.2)$

$= 0.0444$

3) b=false, c=true.

$= P(d|a \wedge c)\,P(c|\neg b)\,P(\neg b|a)\,P(a) + P(d|\neg a \wedge c)\,P(c|\neg b)\,P(\neg b|\neg a)\,P(\neg a)$

$= 0.012 + 0.0294 = 0.0414$