HomeWork - 7

Reasoning With Uncertainity

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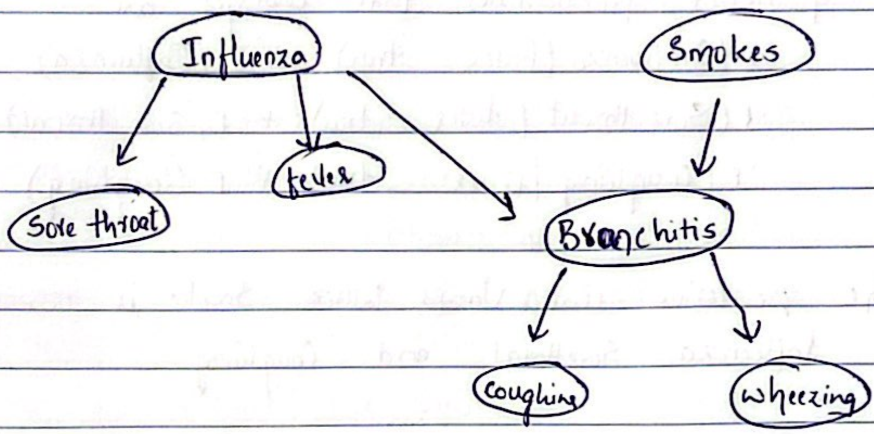
Q.1) Exercise - 2

Consider the belief network of Figure 9.37. This is the "simple diagnostic example" in the AI Space belief network tool



(a) The posterior probabilities of which variable changes when Smoke is observed to be true?
That is for which X is $P(X \mid smoke = true) \neq P(X)$.

Sol: Posibilities:

influenza $(P(influenza \mid Smokes = true) \neq P(influenza))$
Bronchitis $(P(Bronchitis \mid Smokes = true) \neq P(Bronchitis))$
coughing $(P(coughing \mid Smokes = true) \neq P(coughing))$

The belief network tool can be used to verify these probabilities and observe the changes in the posterior probabilities of the variables.

→ The posterior probability of Coughing, influenza and bronchitis after when Smoking is found to be true. Since smoking raises the risk of these respiratory ailments

∴ posterior probabilities that changes are:

$P(\text{Influenza} \mid \text{Fever} = \text{true}) \neq P(\text{Influenza})$,

$P(\text{Sore throat} \mid \text{fever} = \text{true}) \neq P(\text{Sore throat})$ and

$P(\text{Coughing} \mid \text{Fever} = \text{true}) \neq P(\text{Coughing})$

The posterior that varies when Smoke is observed to be true are Influenza, Sorethroat and Coughing

(c) Does the probability of fever change when Wheezing is observed to be true?

i.e., Is $P(\text{Fever} \mid \text{Wheezing} = \text{true}) \neq P(\text{Fever})$? Explain Why

Sol No, the observation of Wheezing does not alter the likelihood of having a fever. Since, Wheezing and Fever are Conditionally independent Variables in our belief network.

$$P(\text{Fever} \mid \text{Wheezing} = \text{true}) = P(\text{Fever})$$

This implies that the presence of lack of Wheezing doesn't give more insight into the likelihood of experiencing a fever and Vice Versa.

→ When Wheezing is observed to be real, the likelihood of having a fever doesn't alter. This is due to fact that in the belief network, fever and Wheezing are Conditionally independent Variable.

→ Finally, We can say that the probability of fever doesn't change when Wheezing is observed to be true.

$$P(\text{Fever} \mid \text{Wheezing} = \text{true}) = P(\text{Fever})$$

(e) What could be observed so that sequentially observing Wheezing doesn't change the probability of Sorethroat that is, Specify a Variable (or) Variables x such that P (Sorethroat | X) = P (Sorethroat | X, Wheezing), or State that there are none. Explain why?

Sol: It is not possible to observe any Variable (x) to guarantee that the chance of sore throat does not alter when Wheezing is seen later on.

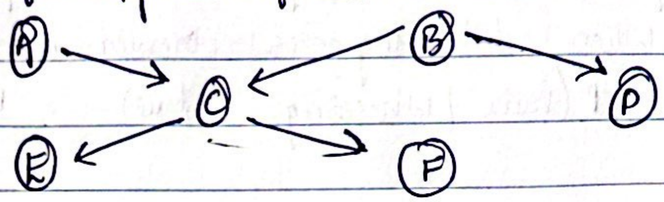
→ This is due to fact that Wheezing is a belief network descendent of Sore throat, and the likelihood of an ancestor may always influenced by viewing a descendent.

→ It is not possible to observe any Variable (x) to guarantee that the chance of sore throat does not alter when Wheezing is seen later on. This is because Wheezing is a Subtype of Sorethroat in the belief network structure.

→ The likelihood of a descendent Variable, like Wheezing may constantly be influenced by observation of it's predecessors. Consequently, the likelihood of Sorethroat will always change based on further Observations.

→ There are no Variables (X), that can be Observed to ensure that Subsequently observing Wheezing doesnot change the probability of Sorethroat.

7) Consider, the following belief n/w



(a) Compute P(e) using Variable Elimination (VE). You show first prune irrelevant Variables. Show the factors that are created for a given elimination ordering.

Sol: We initially wanted to train the beside-the-point variables. In order to compute P(e), the usage of variable elimination (VE)

$$\begin{array}{c} A \rightarrow C \leftarrow B \\ | \\ E, F \end{array}$$

Now, we carry out variable elimination in a certain order. A good way to arrange is to remove any variables from the following order: D, C, B, A.

① Eliminate D.

We have 2 factors involving D: $P(d|b)$ & $P(d|-b)$.

$\therefore \ \phi_1(B) = \sum D_p(D/B) \times P(B)$

$= P(D/B = \text{true}) \times P(B) + P(D/B) = \text{False})$

$= 0.1 \times 0.2 + 0.8 \times 0.2$

$= 0.02 + 0.16 = 0.18,$

$\phi_1(-B) = \sum D_p(D/-B) \times P(-B)$

$= P(D/B = \text{true}) \times P(-B) + P(D/B = \text{False}) \times P(-B)$

$= 0.1 \times 0.8 + 0.8 \times 0.8$

$= 0.08 + 0.64 = 0.72$

② Eliminate Ⓒ

Using $P(C/a, b)$ & $P(C/a, -b)$

$\phi_2(A/B) = \sum C_p(C/A, B) \times \phi_1(B)$

$= P(C/A = \text{true}, B) \times \phi_1(B) + P(C/A = \text{false}, B) \times \phi_1(B)$

$= 0.1 \times 0.18 + 0.8 \times 0.18$

$= 0.018 + 0.144$

$= 0.162$

$\phi_2(A, -B) = \sum C_p(C/A, -B) \times \phi_1(-B)$

$= P(C/A = \text{true}, -B) \times \phi_1(-B) + P(C/A = \text{false}, -B) \times \phi_1(-B)$

$= 0.7 \times 0.72 + 0.4 \times 0.72$

$= 0.504 + 0.288 = 0.792$

③ Eliminate B :

Using $\phi_2(A,B)$ & $\phi_2(A,-B)$

$\phi_3(A) = \Sigma_B \phi_2(A,B) \times P(B) + \phi_2(A,-B) \times P(-B)$

$= \phi_2(A,B) \times P(B) + \phi_2(A,-B) \times P(-B)$

$= 0.162 \times 0.2 + 0.792 \times 0.8$

$= 0.0324 + 0.6336$

$= 0.666$

Finally $P(e) = \Sigma_A \phi_3(A) \times P(A)$

$= \phi_3(A = true) \times P(A = true) + \phi_3(A = false) \times P(A = false)$

$= 0.666 \times 0.9 + 0 \times 0.1$

$= 0.5994$

(b) Suppose you want to compute $P(e/-f)$ using VE. How much of the previous computation is reusable? Show the factors that are different from these impact (a)

Sol: We have $\phi_3(A) = 0.666$

Calculate $\phi_4(C,-F)$

$\phi_4(C,-F) = P(F/C) \times P(-F/C)$

$= (1 - P(F/C)) \times P(C)$

$= (1 - 0.2) \times P(C)$

$= 0.8 \times P(C)$

$\phi_5(A, C, -F) = \phi_3(A) \times \phi_4(C,-F)$

$= 0.666 \times (0.8 \times P(C))$

$= 0.53328 \times P(C)$

$$P(-F) = \sum C \phi_5(A, C, -F)$$

$$= \phi_5(A, C = true, -F) + \phi_5(A, C = False, -F)$$

$$= 0.53328 \times (P(C = true) + 0 \times P(C = False))$$

$$= 0.53328 \times 0.9 + 0$$

$$= 0.479952$$

$$P(e|-f) = \frac{P(e) \times P(-f)}{P}(-f)$$

$$P(e|-f) = P(e) = 0.5994$$

As it is depend on F.