Six Monthly Progress Report of Research Scholars
Period: Jan to May, 2017
Department of Electrical Engineering

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2.	Roll Number	:	EE15D004
3.	Program	:	Ph.D
4.	Specialization	:	Control and Instrumentation
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7.	Date of Registration	:	
8.	Date of GTC/DC Meetings	:	Not Held
9.	Area of Research	:	Pursuit Evasion Games
10.	Date of Comprehensive exams	:	02/08/16, 03/08/16, 04/08/16, 05/08/16

Courses

Course	Course Name	Type	Semester	Grade
EE6417	Allied Topics in Control Systems	Core	Jul-Nov 2015	S
EE6415	Non Linear Control Systems	Core	Jan-May 2016	S
EE5411	Synthesis of Control Systems	Core	Jul-Nov 2016	B
EE5121	Optimization Methods in Signal Processing and Comm	Elective	Jan-May 2016	B
MA5330	Real Analysis	Elective	Jul-Nov 2015	D
ID6020	Introduction to Research	Compulsory	Jan-May 2016	Pass

Date : June 15, 2017
IIT Madras, Chennai

Jitesh

Signature of Scholar

Bh

Signature of Guide
(15/06/17)

Towards Real-Time Autonomous Target Area Protection in the Presence of Obstacles

1 Problem Definition

The research problem here investigates optimal strategies for the agents in the Target Guarding Problem, in the presence of obstacles. The problem chosen involves two mobile agents: the *pursuer* and the *evader*. Both the pursuer P and the evader E move on a two dimensional planar surface, refer to Figure 1. The game also features a target zone, T, which is guarded by the pursuer P. The target may or may not be mobile. The goal of the evader is to capture the target. The goal of the pursuer is to guard the target against an attack by the evader. Thus, the pursuer aims to capture the evader before the target is attacked. It is usually desired that the pursuer intercepts the evader as far away from the target as possible. The play area may include obstacles which might affect the strategies of the agents, refer Figure. The aim is to derive an optimal strategy (guidance law) for the pursuer to intercept the evader in *real-time*.

Here, a geometrical approach is taken for determining an optimal guidance law for the pursuer. This optimal guidance law is then reworked to enable a real time implementation. In cases with obstacles, the solution has to be found out by determining the dominance regions of the agents. The dominance region of the pursuer constitutes the set of points in the play area that can be reached by the pursuer before the evader. Alternately, the dominance region of the evader is the set of points in the play area that can be reached by the evader before the pursuer. If the target is present in the dominance region of the pursuer, the game is won by the pursuer, by using the optimal strategy.

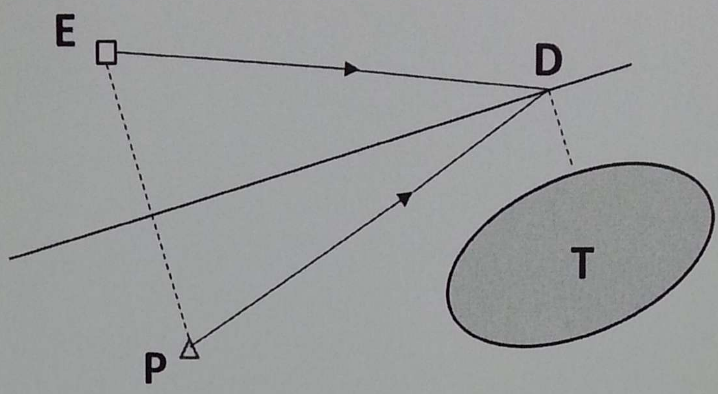
For effective implementation of the strategies in real time, both the guidance law as well as the dominance regions are to be coded as table look up. The case of the target lying in the evader's dominance region is to be investigated and the optimal strategy of the pursuer has to be derived in such a case. Thereafter the game with obstacles is to be analyzed using dominance regions and a technique has to be developed to compute these dominance regions in *real-time*.

This work addresses three problems that form part of the *real-time* solution for the Target Guarding Problem:

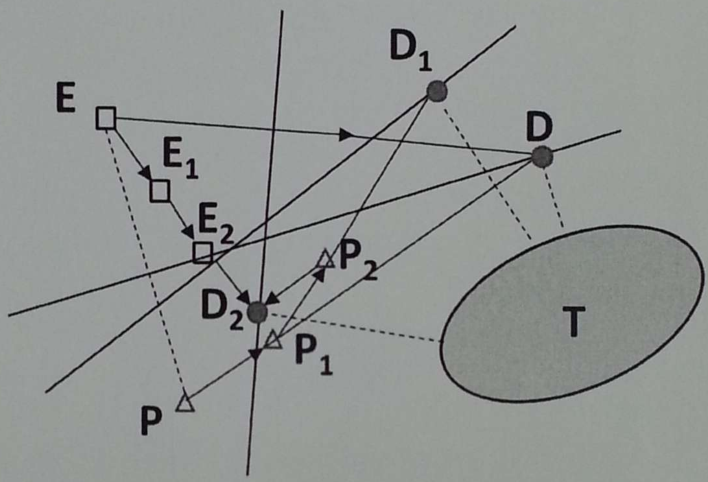
- Find the optimal heading direction or the optimal guidance law (COIP) for the pursuer to intercept the evader before the evader captures the target. The COIP should be found out in real time.

- Find out the optimal strategy for the pursuer when the target is located in the evader's dominance region.

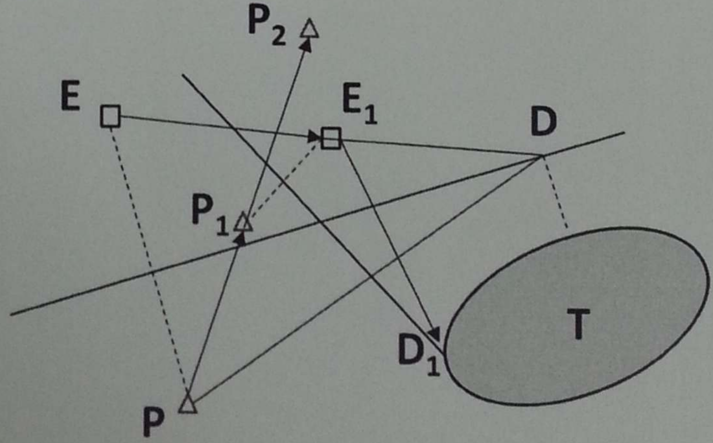
- Derive the dominance region of the agents in real-time, in the presence of obstacles on the play area.



(a) P and E play optimally



(b) P plays optimally, but E does not



(c) E plays optimally, but P does not

Figure 1: Target guarding problem from [1].

2 Work Done till date

A new real time algorithm for deriving the instantaneous optimal path for the pursuer has already been presented in the last report. This algorithm is an improvement to the previously proposed COIP guidance law, because it is at least two times faster, and requires only the ratio of the distance between the pursuer and the target to the distance between the pursuer and the evader. It's worth noting that the guidance laws reported in prior literature required the actual co-ordinates of the agents with respect to a static reference frame. Thus the proposed algorithm is useful in scenarios where a localization system is not available to provide the coordinates of the individual players. Also, it facilitates the tabulation of optimal heading values for different locations of the target and the evader, which when used for computing optimal heading in real-time is about seven times faster than the original COIP algorithm.

Most of the literature on TGP, starting with the pioneering work in [1], only consider the case of T being in P's dominance region. This is because if T were to be in E's dominance region (Figure 2), E can reach T before P could capture E, and thus, ending the game.

However, this is true only if E plays optimally throughout the game. If T were to be located in E's dominance region (within the circle), and if E wrongly estimates T to be outside it, P's strategy should capitalize on this mistake. This scenario is shown in Figure 3. If E operates based on this misinterpretation for sufficiently long, it might be possible for P to protect T, despite starting at a disadvantage. It is in view of this that the following section analyze what should P's strategy be, in order to take maximal advantage of E's mistakes, when T is in E's dominance region.

The analysis proceeds as follows: At any instant, let the positions of the players P, E and T be $(x_p, y_p), (x_e, y_e)$ and (x_t, y_t) respectively. Ideally, P would want to move in a direction that makes the target T lie inside his dominance region. In other words, the objective of P is to minimize

$$\alpha = R - |\overrightarrow{CT}| \tag{1}$$

Let

$$\overrightarrow{\Delta P} = \Delta r \left(\cos\beta\hat{i} + \sin\beta\hat{j} \right) \tag{2}$$

be the vector that results out of P's instantaneous strategy.

Let $\frac{d}{d\beta}$ be denoted by "$'$". The objective for P is to choose β to minimize (1), which means β should be chosen such that:

$$\alpha' = R' - \frac{\overrightarrow{CT}.\overrightarrow{CT}'}{|\overrightarrow{CT}|} = 0 \tag{3}$$

This results in P choosing a heading angle β, where

$$\tan\beta = \frac{y_p - y_i}{x_p - x_i} \tag{4}$$

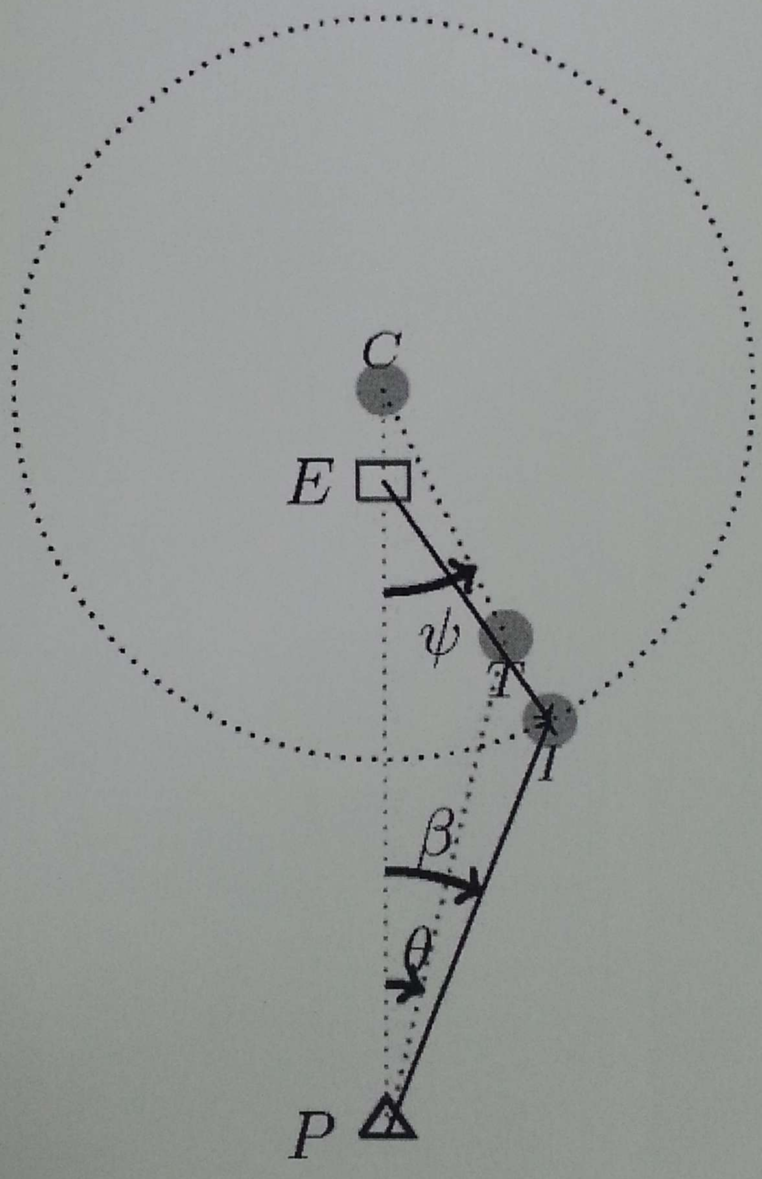


Figure 2: E misinterprets the target to lie outside his dominance region

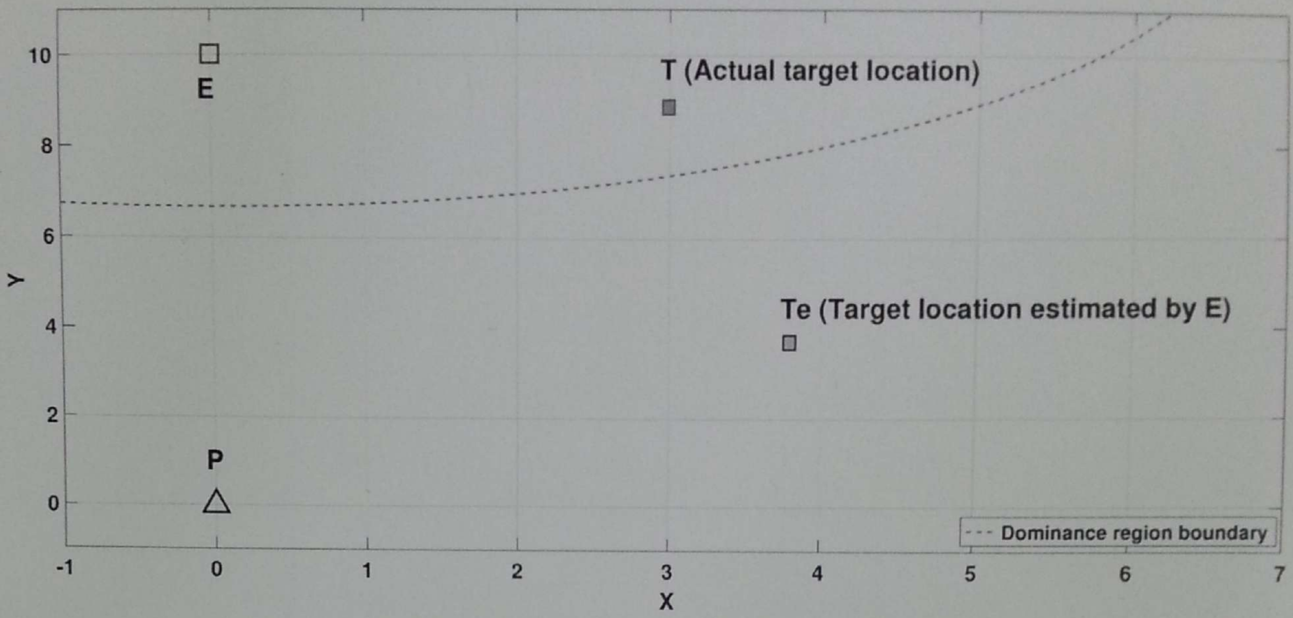


Figure 3: E misinterprets the target to lie outside his dominance region

Derivation of (4) involves a fairly long series of algebraic manipulations, and hence is not presented here.

Equation (4) implies that P should move towards the point I, which is the same strategy as the case when T is in P's dominance region. Note that, instead of pursuing T, E is now effectively pursuing I, because of the miscalculation of T's position on E's part. The value of β given by (4) also ensures minimization of α given in (1). This is because, geometrically, minimizing α is equivalent to ensuring that I is as close to T as possible. This is indeed the case, since C, T and I are collinear (Figure 2), and CI is the diameter of a circle (the actual dominance region of E). So an interesting result is obtained that the optimal strategy for the pursuer remains the same, irrespective of whether T is located inside or outside P's dominance region.

3 Future Plans

The work done till date solves two parts of the real time implementation of player strategies in the Target Guarding Problem. Future research will focus on solving the next step:

- Derive the dominance region of the agents in real-time, in the presence of obstacles on the play area.

4 Visible Research Output

-NA-

References

[1] Rufus Isaacs. *Differential games: a mathematical theory with applications to warfare and pursuit, control and optimization*. John Wiley and Sons, Inc., New York, 1965.