Locus of evader coordinates with same evader velocity and same interception point:

Let evader be at (x_{e_1}, y_{e_1}), target at (x_T, y_T), pursuer at (x_p, y_p)

Let the velocity of the evader to the velocity of the pursuer ratio be $k = \dfrac{v_{e_1}}{v_p}$

The expressions for centre and radius of the corresponding Apollonius circle are

$$x_{c_1} = \frac{x_e - x_p k^2}{1-k^2} \qquad (1) \qquad\qquad y_{c_1} = \frac{y_{e_1} - y_p k^2}{1-k^2} \qquad (2)$$

$$R_1 = \sqrt{x_{c_1}{}^2 + y_{c_1}{}^2 - \frac{(x_{e_1}{}^2 + y_{e_1}{}^2)}{1-k^2} + k^2 \frac{(x_p{}^2 + y_p{}^2)}{1-k^2}} \qquad (3)$$

Equation of Apollonius circle is $(x - x_{c_1})^2 + (y - y_{c_1})^2 = R_1^2$

Let the corresponding interception point be (x_I, y_I).

Objective is to find the set $\{x_e, y_e\}$ that result in (x_I, y_I).

For (x_I, y_I) to be the interception point of any other circle, it has to be the nearest point to the target by definition. So the centres of all Apollonius circles for which (x_I, y_I) is the interception point must lie on the line joining target (x_T, y_T) and (x_I, y_I) $\overline{IT}$. So the following condition will hold.

$$\frac{y_T - y_I}{x_T - x_I} = \frac{y_c - y_{c_1}}{x_c - x_{c_1}} = M \qquad (4)$$

Where (x_c, y_c) is the centre of any Apollonius circle whose interception point is (x_I, y_I).

(x_c, y_c) lies on $\overline{IT}$.

Substituting corresponding expressions of (x_{c1}, y_{c1}), (x_c, y_c) in (4),

$$\frac{y_e - y_p k^2}{1-k^2} - \frac{y_{e_1} - y_p k^2}{1-k^2} = M\left(\frac{x_e - x_p k^2}{1-k^2} - \frac{x_{e_1} - x_p k^2}{1-k^2}\right) \qquad (5)$$

Simplifying,

L: $\quad y_e - y_{e_1} = M(x_e - x_{e_1}) \qquad (6)$

So all the evader initial positions which result in same interception point (x_I, y_I) for constant *k* lie on the line with slope equal to the slope of $\overline{IT}$.

But this equation is also satisfied by all the evader coordinates whose corresponding interception points lie on $\overline{IT}$, as is obvious from (4).

Note that interception point (x_I, y_I) has to lie on all Apollonius circles for which it is interception point.

This means $(x_I - x_c)^2 + (y_I - y_c)^2 = R^2$ (7)

Substituting corresponding (1), (2), (3) in (7)

$$x_I^{\ 2} + y_I^{\ 2} - 2x_I x_e - 2y_I y_e = \frac{k^2(x_p^{\ 2} + y_p^{\ 2})}{1-k^2} - \frac{(x_e^{\ 2} + y_e^{\ 2})}{1-k^2}$$

Solving we get the following circle equation.

$$C: \ (x_e - x_I)^2 + (y_e - y_I)^2 = k^2\left((x_p - x_I)^2 + (y_p - y_I)^2\right) \qquad (8)$$

C is the set of all evader coordinates whose corresponding Apollonius circles pass through (x_I, y_I)

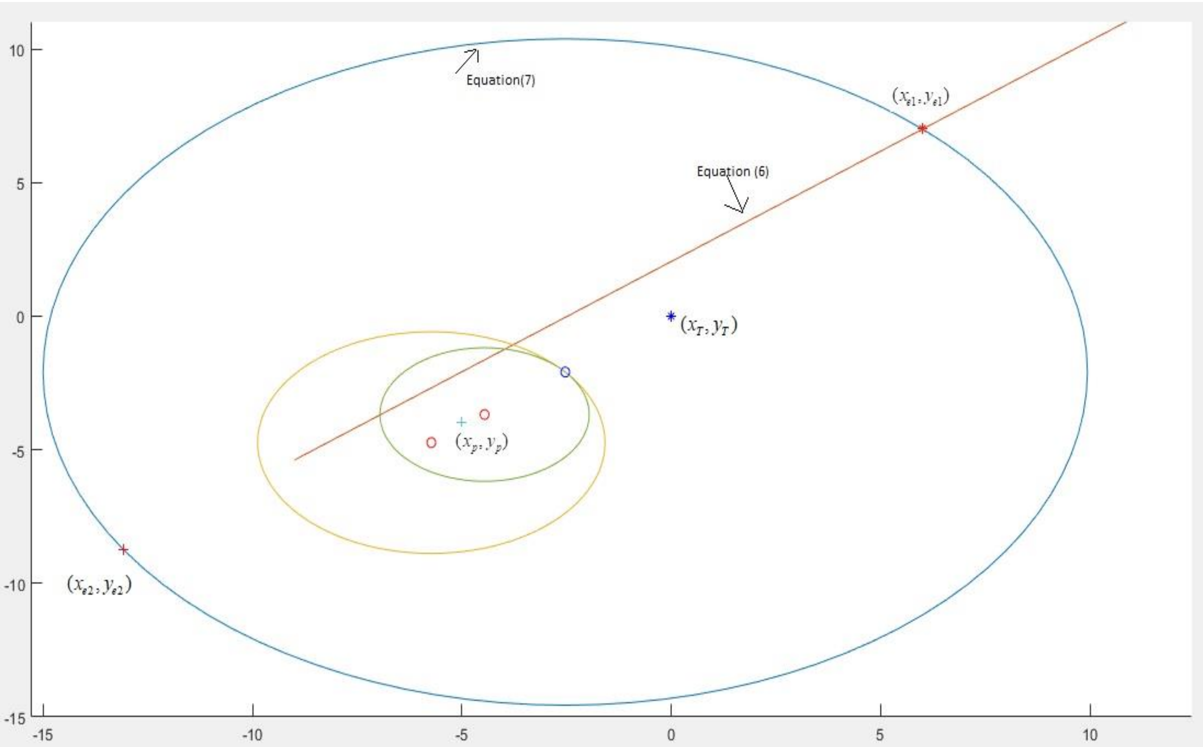

Fig 1. The game geometry with all the possible evader positions and corresponding Apollonius circles

Solving *C* and *L* means that we are finding all the evader coordinates whose corresponding Apollonius circles with their centres lying on $\overline{IT}$, pass through (x_I, y_I). Hence the solution set can also include the cases which are infeasible, cases where the centres of Apollonius circles which pass through (x_I, y_I), lie on $\overline{IT}$, but their interception points lie on the diametrically opposite end as in fig 2.

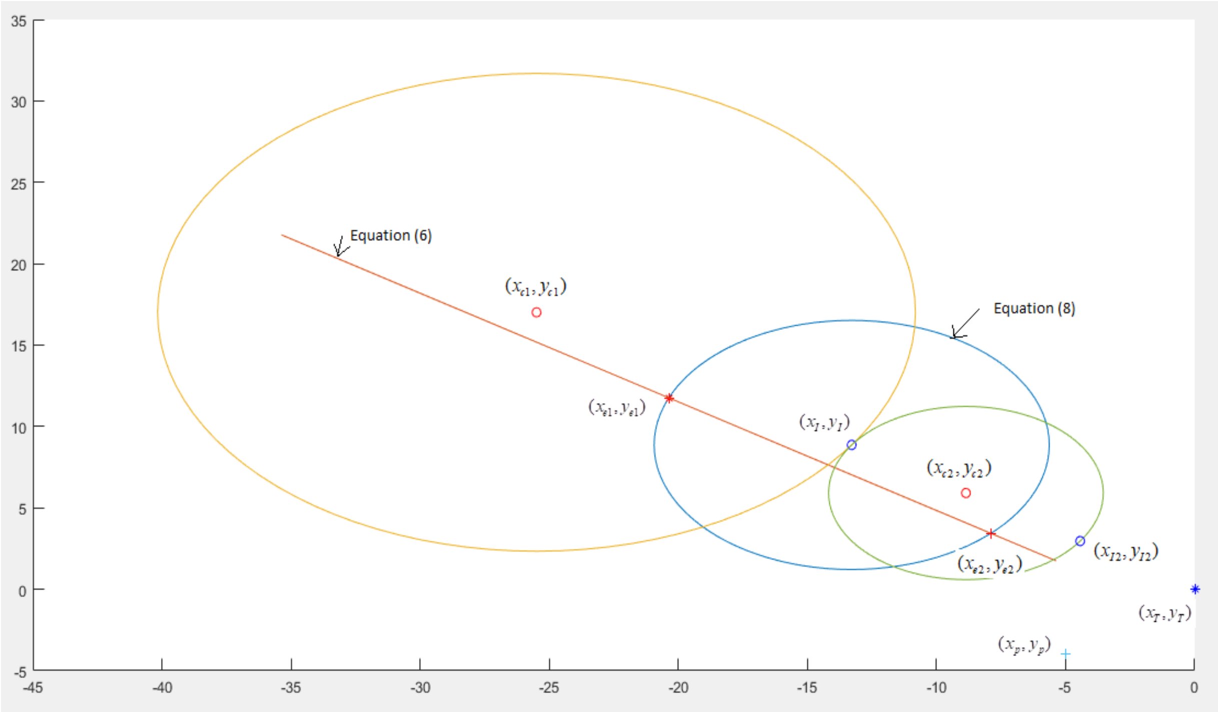

Fig 2. Infeasible case where (x_{e_2}, y_{e_2}) satisfies equations (6) & (8) but still its corresponding interception point is not (x_I, y_I)

Locus of evader coordinates with different evader velocities and same interception point:

Objective is to find the set $\{x_e, y_e, v_e\}$ that result in (x_I, y_I).

Equation (5) changes to

$$\frac{y_e - y_p k_e^2}{1-k_e^2} - \frac{y_{e_1} - y_p k^2}{1-k^2} = M\left(\frac{x_e - x_p k_e^2}{1-k_e^2} - \frac{x_{e_1} - x_p k^2}{1-k^2}\right), \qquad k_e = \frac{v_e}{v_p}$$

$$\Rightarrow \frac{y_e - y_p k_e^2}{1-k_e^2} - M\frac{x_e - x_p k_e^2}{1-k_e^2} = c, \quad \text{Where } c = \frac{y_{e_1} - y_p k^2}{1-k^2} - M\frac{x_{e_1} - x_p k^2}{1-k^2}$$

$$\Rightarrow (y_e - y_p k_e^2) - M(x_e - x_p k_e^2) = c(1-k_e^2)$$

$$\Rightarrow (y_e - Mx_e - c) = (y_p - Mx_p - c)k_e^2$$

$$(y_e - Mx_e - c) = pk_e^2 \qquad (9), \qquad \text{where } p = (y_p - Mx_p - c)$$

And equation (8) becomes $$(x_e - x_I)^2 + (y_e - y_I)^2 = k_e^2\left((x_p - x_I)^2 + (y_p - y_I)^2\right) \qquad (10)$$

Substituting k_e^2 from (9) in (10), $$(x_e - x_I)^2 + (y_e - y_I)^2 = \frac{\left((x_p - x_I)^2 + (y_p - y_I)^2\right)}{p}(y_e - Mx_e - c)$$

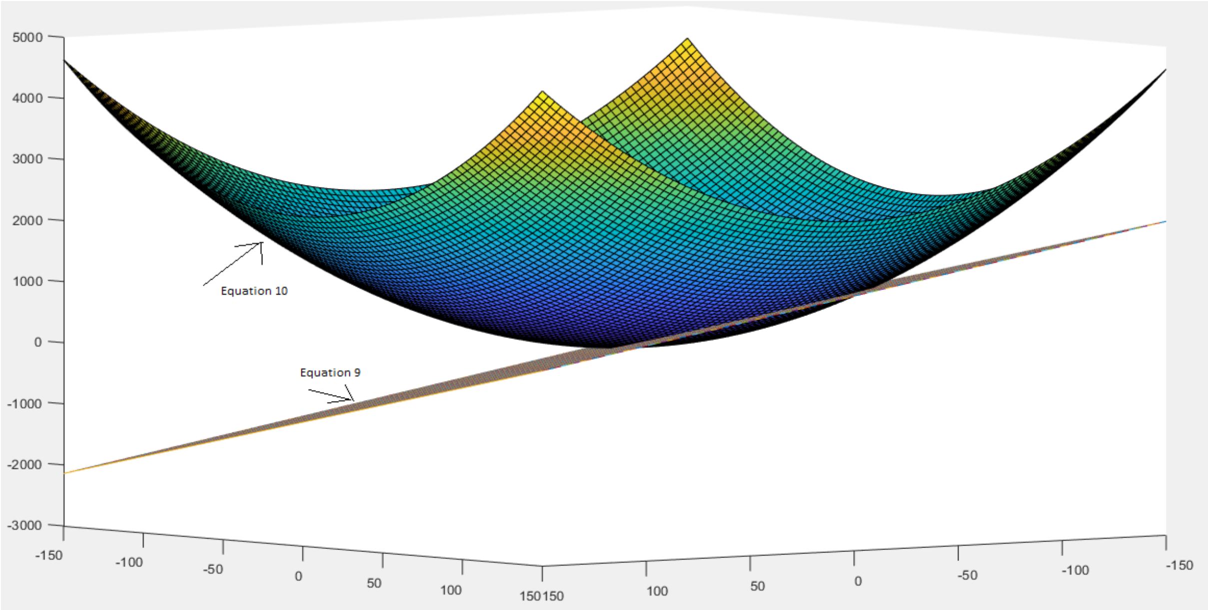

Fig 2. $\{x_e, y_e, v_e\}$ for same interception point will be intersection of paraboloid and a plane (with $k_e{}^2$ considered as z)

Let $\dfrac{\left((x_p - x_I)^2 + (y_p - y_I)^2\right)}{p}$ be L

Rearranging the terms, $(x_e - x_{cm})^2 + (y_e - y_{cm})^2 = \overline{R}^2$ (11)

where $x_{cm} = \left(x_i - \dfrac{LM}{2}\right)$, $y_{cm} = \left(y_i + \dfrac{L}{2}\right)$, $\overline{R}^2 = \dfrac{L^2(M^2+1)}{4} + L(y_I - Mx_I - c)$

So all (x_e, y_e) whose corresponding interception point can be (x_I, y_I) satisfy (11) and corresponding k_e lie on (9)

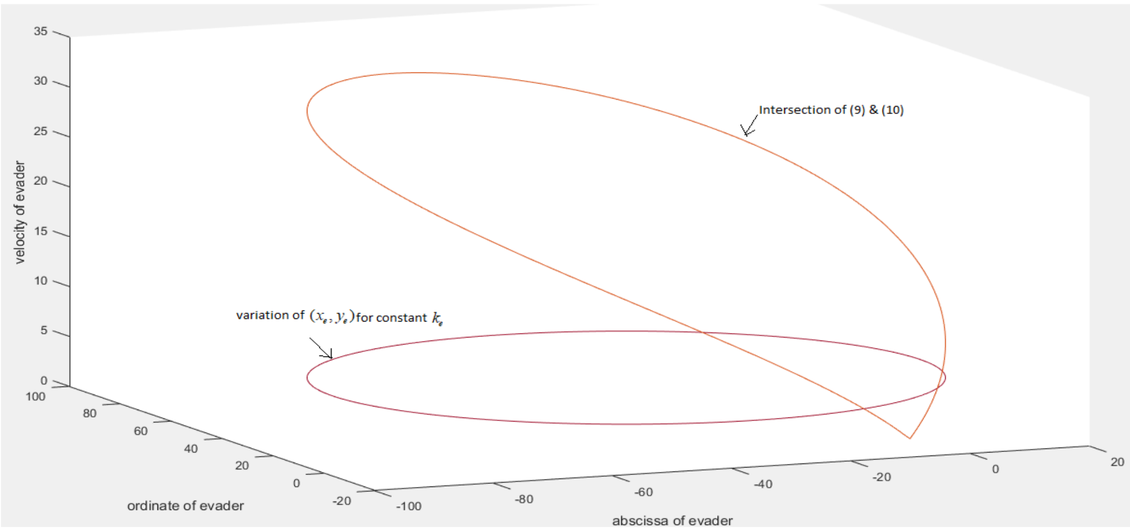

Fig 3. $\{x_e, y_e, v_e\}$