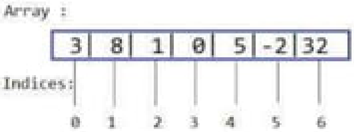
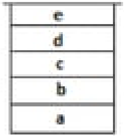
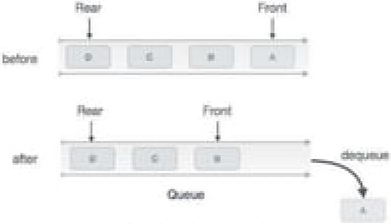
Tree

So far we discussed Linear data structures like





stack



Queue Dequeue

Introduction to trees

- So far we have discussed mainly linear data structures – strings, arrays, lists, stacks and queues

- Now we will discuss a non-linear data structure called tree.

- Trees are mainly used to represent data containing a hierarchical relationship between elements, for example, records, family trees and table of contents.

- Consider a parent-child relationship

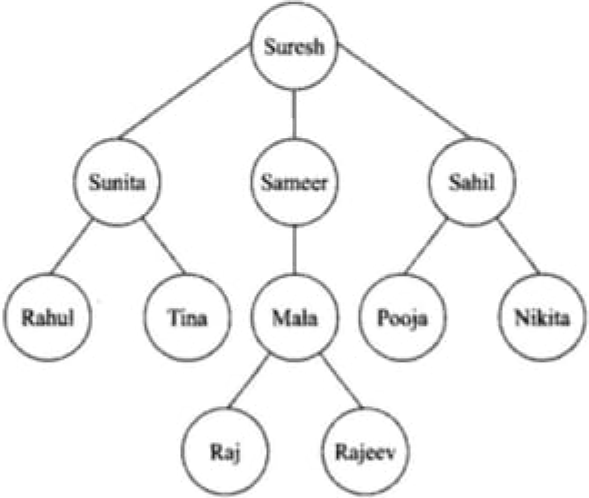
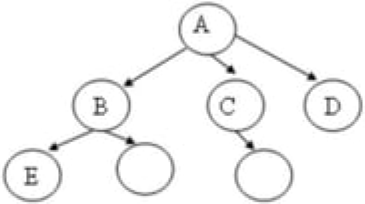


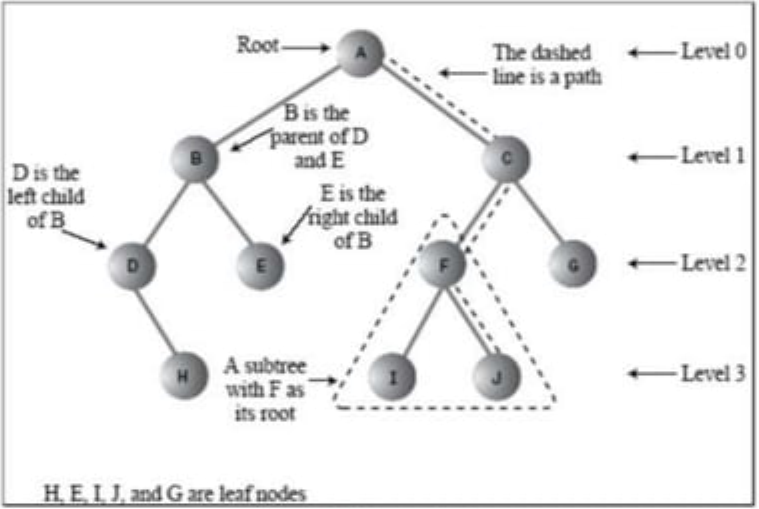
Fig. 8.1 *A Hypothetical Family Tree*

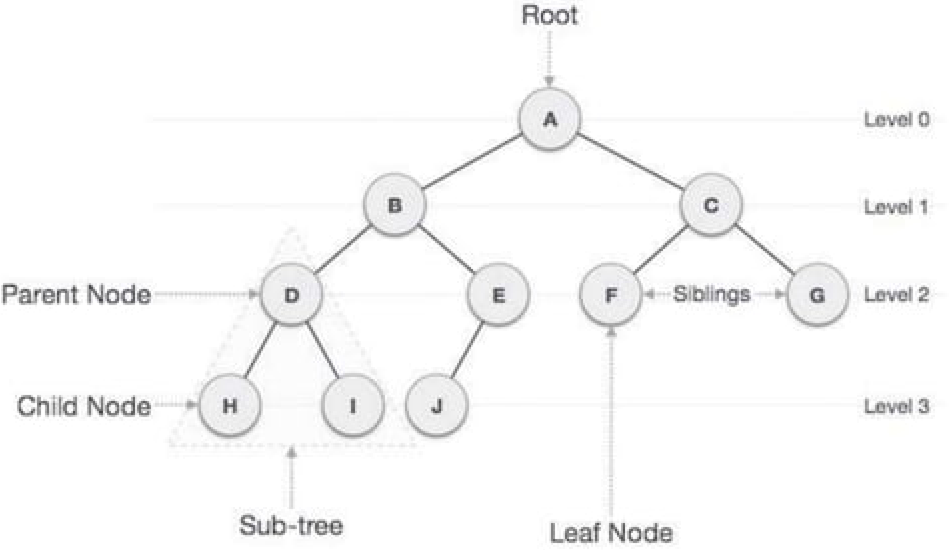
Tree

- A tree is an abstract model of a hierarchical structure that consists of nodes with a parent-child relationship.
  - Tree is a sequence of nodes
  - There is a starting node known as a root node
  - Every node other than the root has a parent node.
  - Nodes may have any number of children



A has 3 children, B, C, D
A is parent of B


Root
A
The dashed line is a path
Level 0
B is the parent of D and E
B
C
Level 1
D is the left child of B
E is the right child of B
D
E
F
G
Level 2
H
A subtree with F as its root
I
J
Level 3
H, E, I, J, and G are leaf nodes



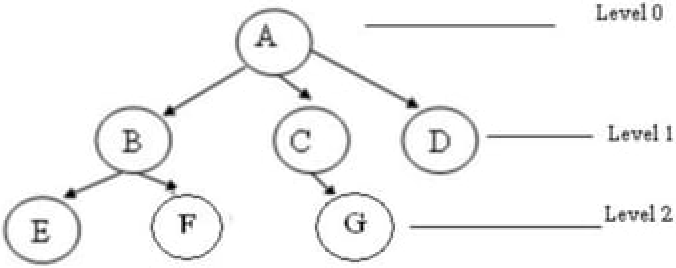
Root
A
Level 0
B
C
Level 1
Parent Node
D
E
F
Siblings
G
Level 2
Child Node
H
I
J
Level 3
Sub-tree
Leaf Node

Some Key Terms:

- **Root** – Node at the top of the tree is called root.
- **Parent** – Any node except root node has one edge upward to a node called parent.
- **Child** – Node below a given node connected by its edge downward is called its child node.
- **Sibling** – Child of same node are called siblings
- **Leaf** – Node which does not have any child node is called leaf node.
- **Sub tree** – Sub tree represents descendants of a node.
- **Levels** – Level of a node represents the generation of a node. If root node is at level 0, then its next child node is at level 1, its grandchild is at level 2 and so on.
- **keys** – Key represents a value of a node based on which a search operation is to be carried out for a node.

Some Key Terms:

- Degree of a node:
  - The degree of a node is the number of children of that node
- Degree of a Tree:
  - The degree of a tree is the maximum degree of nodes in a given tree
- Path:
  - It is the sequence of consecutive edges from source node to destination node.
- Height of a node:
  - The height of a node is the max path length form that node to a leaf node.
- Height of a tree:
  - The height of a tree is the height of the root
- Depth of a tree:
  - Depth of a tree is the max level of any leaf in the tree



- A is the root node
- B is the parent of E and F
- D is the sibling of B and C
- E and F are children of B
- E, F, G, D are external nodes or leaves
- A, B, C are internal nodes
- Depth of F is 2
- the height of tree is 2
- the degree of node A is 3
- The degree of tree is 3

Characteristics of trees

- Non-linear data structure
- Combines advantages of an ordered array
- Searching as fast as in ordered array
- Insertion and deletion as fast as in linked list
- Simple and fast

Application

- Directory structure of a file store
- Structure of an arithmetic expressions
- Used in almost every 3D video game to determine what objects need to be rendered.
- Used in almost every high-bandwidth router for storing router-tables.
- used in compression algorithms, such as those used by the .jpeg and .mp3 file-formats.

Introduction To Binary Trees

- A **binary tree**, is a tree in which no node can have more than two children.

- Consider a binary tree T, here ‘A’ is the root node of the binary tree T.

- ‘B’ is the left child of ‘A’ and ‘C’ is the right child of ‘A’
  - i.e A is a father of B and C.
  - The node B and C are called siblings.

- Nodes D,H,I,F,J are leaf node

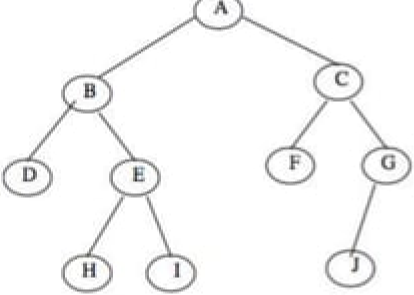
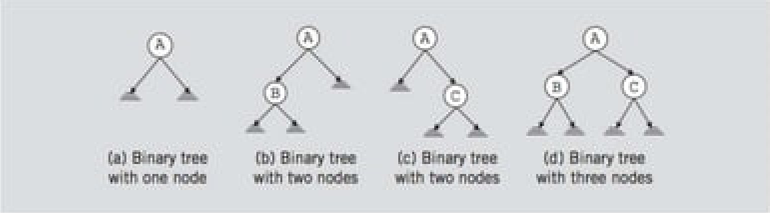


Fig. 8.3. Binary tree

Binary Trees

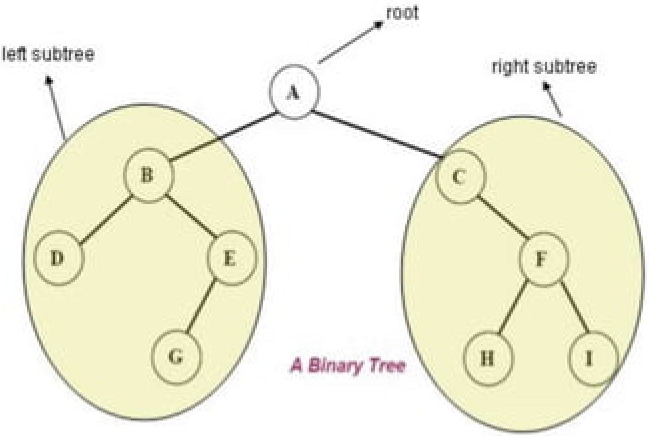
- A **binary tree**, *T*, is either empty or such that
    - I. *T* has a special node called the **root** node
    - II. *T* has two sets of nodes L_T and R_T, called the **left subtree** and **right subtree** of *T*, respectively.
    - III. L_T and R_T are binary trees.



Binary Tree

- A binary tree is a finite set of elements that are either empty or is partitioned into three disjoint subsets.
- The first subset contains a single element called the **root** of the tree.
- The other two subsets are themselves binary trees called the **left** and **right** **sub-trees** of the original tree.
- A left or right sub-tree can be empty.
- Each element of a binary tree is called a **node** of the tree.

The following figure shows a binary tree with 9 nodes where A is the root

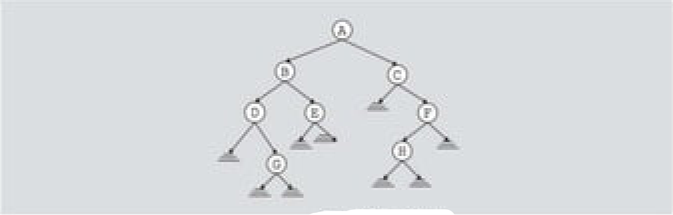


Binary Tree

- The root node of this binary tree is A.

- The left sub tree of the root node, which we denoted by L_A, is the set L_A = {B,D,E,G} and the right sub tree of the root node, R_A is the set R_A={C,F,H}

- The root node of L_A is node B, the root node of R_A is C and so on



Binary Tree Properties

- If a binary tree contains m nodes at level L, it contains at most 2m nodes at level L+1

- Since a binary tree can contain at most 1 node at level 0 (the root), it contains at most 2L nodes at level L.

Types of Binary Tree

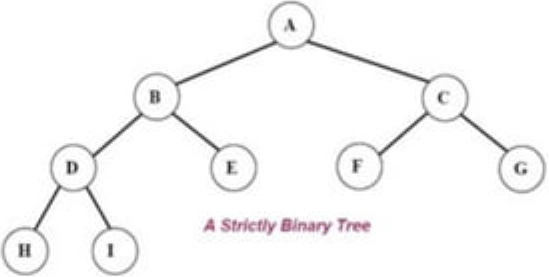
- Complete binary tree
- Strictly binary tree
- Almost complete binary tree

Strictly binary tree

- If every non-leaf node in a binary tree has nonempty left and right sub-trees, then such a tree is called a **strictly binary tree**.

- **Or, to put it another way,** all of the nodes in a strictly binary tree are of degree zero or two, never degree one.

- A strictly binary tree with N leaves always contains 2N – 1 nodes.



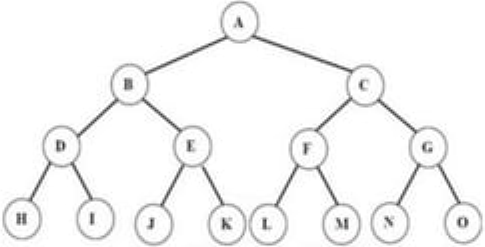
A Strictly Binary Tree

Complete binary tree

- A **complete binary tree** is a **binary tree** in which every level, except possibly the last, is completely filled, and all nodes are as far left as possible.

- A *complete binary tree* of depth **d** is called strictly binary tree if all of whose leaves are at level **d**.

- A complete binary tree has 2^d nodes at every depth **d** and 2^d -1 non leaf nodes



A complete Binary Tree of depth 3

Almost complete binary tree

- An almost complete binary tree is a tree where for a right child, there is always a left child, but for a left child there may not be a right child.

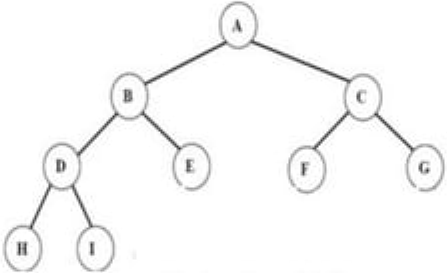


Fig Almost complete binary tree.

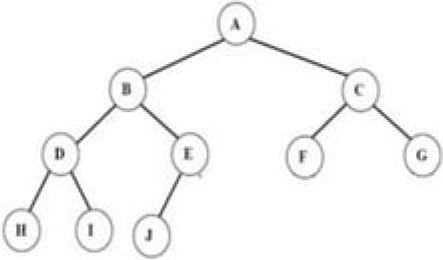


Fig Almost complete binary tree but not strictly binary tree. Since node E has a left son but not a right son.

Operations on Binary tree:

- ✔ **father(n,T):**Return the parent node of the node n in tree T. If n is the root, NULL is returned.
- ✔ **LeftChild(n,T):**Return the left child of node n in tree T. Return NULL if n does not have a left child.
- ✔ **RightChild(n,T)**:Return the right child of node n in tree T. Return NULL if n does not have a right child.
- ✔ **Info(n,T):** Return information stored in node n of tree T (ie. Content of a node).
- ✔ **Sibling(n,T):** return the sibling node of node **n** in tree T. Return NULL if n has no sibling.
- ✔ **Root(T):** Return root node of a tree if and only if the tree is nonempty.
- ✔ **Size(T):** Return the number of nodes in tree T
- ✔ **MakeEmpty(T):** Create an empty tree T
- ✔ **SetLeft(S,T):** Attach the tree S as the left sub-tree of tree T
- ✔ **SetRight(S,T):** Attach the tree S as the right sub-tree of tree T.
- ✔ **Preorder(T):** Traverses all the nodes of tree T in preorder.
- ✔ **postorder(T):** Traverses all the nodes of tree T in postorder
- ✔ **Inorder(T):** Traverses all the nodes of tree T in inorder.

C representation for Binary tree:

```
struct bnode
{
        int info;
        struct bnode  *left;
        struct bnode  *right;
};
struct bnode *root=NUl
```

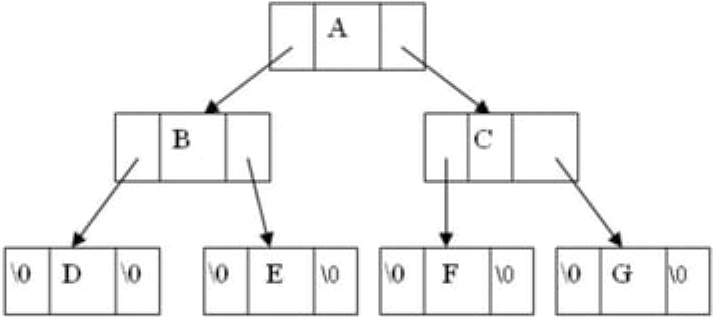


Fig: Structure of Binary tree

Tree traversal

- Traversal is a process to visit all the nodes of a tree and may print their values too.

- All nodes are connected via edges (links) we always start from the root (head) node.

- There are three ways which we use to traverse a tree
  - In-order Traversal
  - Pre-order Traversal
  - Post-order Traversal

- Generally we traverse a tree to search or locate given item or key in the tree or to print all the values it contains.

Pre-order, In-order, Post-order

- Pre-order

<root><left><right>

- In-order

<left><root><right>

- Post-order

<left><right><root>

Pre-order Traversal

- The preorder traversal of a nonempty binary tree is defined as follows:
  - Visit the root node
  - Traverse the left sub-tree in preorder
  - Traverse the right sub-tree in preorder

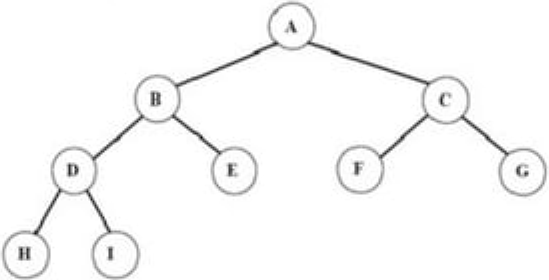


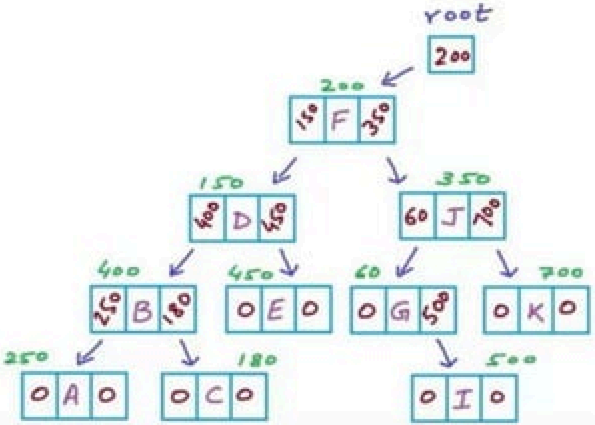
fig Binary tree

The preorder traversal output of the given tree is: A B D H I E C F G

The preorder is also known as depth first order.

Pre-order Pseudocode

```
struct Node{
    char data;
    Node *left;
    Node *right;
}
void Preorder(Node *root)
{
    if (root==NULL) return;
    printf ("%c", root->data);
    Preorder(root->left);
    Preorder(root->right);
}
```



In-order traversal

- The in-order traversal of a nonempty binary tree is defined as follows:
  - Traverse the left sub-tree in in-order
  - Visit the root node
  - Traverse the right sub-tree in inorder

- The in-order traversal outpu of the given tree is H D I B E A F C G

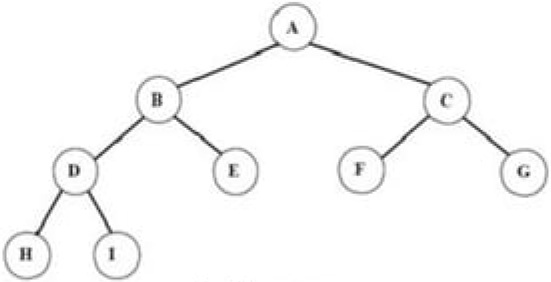
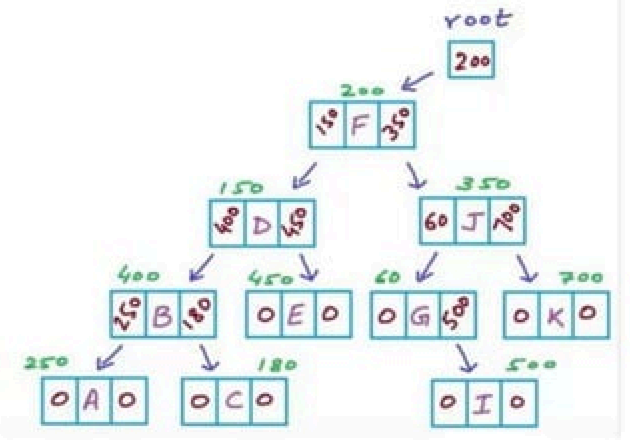


fig Binary tree

In-order Pseudocode

```
struct Node{
    char data;
    Node *left;
    Node *right;
}
void Inorder(Node *root)
{
    if (root==NULL) return;
    Inorder(root->left);
    printf ("%c", root->data);
    Inorder(root->right);
}
```



Post-order traversal

- The in-order traversal of a nonempty binary tree is defined as follows:
  - Traverse the left sub-tree in post-order
  - Traverse the right sub-tree in post-order
  - Visit the root node

- The in-order traversal outpu of the given tree is H I D E B F G C A

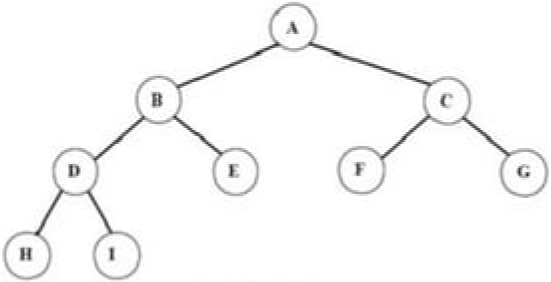
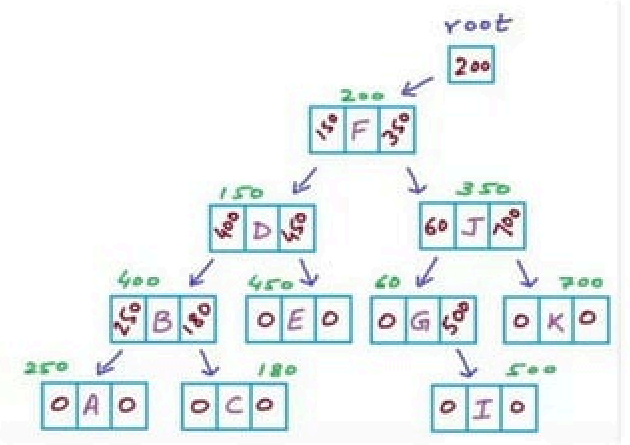


fig Binary tree

Post-order Pseudocode

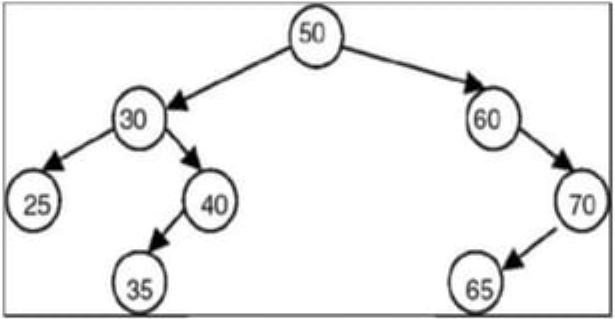
```
struct Node{
    char data;
    Node *left;
    Node *right;
}
void Postorder(Node *root)
{
    if (root==NULL) return;
    Postorder(root->left);
    Postorder(root->right);
    printf ("%c", root->data);
}
```



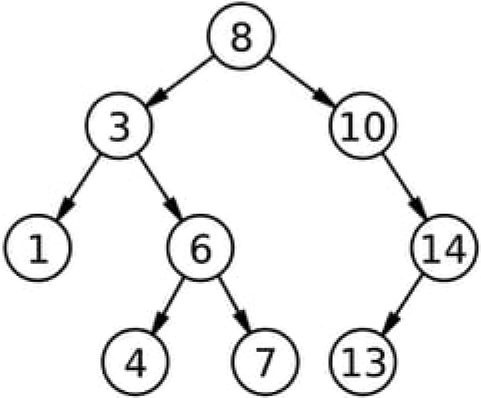
Binary Search Tree(BST)

- A binary search tree (BST) is a binary tree that is either empty or in which every node contains a key (value) and satisfies the following conditions:
  - All keys in the left sub-tree of the root are smaller than the key in the root node
  - All keys in the right sub-tree of the root are greater than the key in the root node
  - The left and right sub-trees of the root are again binary search trees

Binary Search Tree(BST)



The binary search tree.



Binary Search Tree(BST)

- A binary search tree is basically a binary tree, and therefore it can be traversed in inorder, preorder and postorder.

- If we traverse a binary search tree in inorder and print the identifiers contained in the nodes of the tree, we get a sorted list of identifiers in ascending order.

Why Binary Search Tree?

- Let us consider a problem of searching a list.

- If a list is ordered searching becomes faster if we use contiguous list(array).

- But if we need to make changes in the list, such as inserting new entries or deleting old entries, (SLOWER!!!!) because insertion and deletion in a contiguous list requires moving many of the entries every time.

Why Binary Search Tree?

- So we may think of using a linked list because it permits insertion and deletion to be carried out by adjusting only few pointers.

- But in an n-linked list, there is no way to move through the list other than one node at a time, permitting only sequential access.

- Binary trees provide an excellent solution to this problem. By making the entries of an ordered list into the nodes of a binary search tree, we find that we can search for a key in O(logn)

Binary Search Tree(BST)

Time Complexity			
	Array	**Linked List**	**BST**
Search	O(n)	O(n)	O(logn)
Insert	O(1)	O(1)	O(logn)
Remove	O(n)	O(n)	O(logn)

Operations on Binary Search Tree (BST)

- Following operations can be done in BST:

  - Search(k, T): Search for key k in the tree T. If k is found in some node of tree then return true otherwise return false.

  - Insert(k, T): Insert a new node with value k in the info field in the tree T such that the property of BST is maintained.

  - Delete(k, T):Delete a node with value k in the info field from the tree T such that the property of BST is maintained.

  - FindMin(T), FindMax(T): Find minimum and maximum element from the given nonempty BST.

Searching Through The BST

- Compare the target value with the element in the root node
  - ✓ If the target value is **equal**, the search is successful.
  - ✓ If target value is **less**, search the left subtree.
  - ✓ If target value is **greater**, search the right subtree.
  - ✓ If the subtree is **empty**, the search is unsuccessful.

C function for BST searching:

```
void BinSearch(struct bnode *root , int key)
{
        if(root == NULL)
        {
                printf("The number does not exist");
                exit(1);
        }
        else  if (key == root->info)
        {
                printf("The searched item is found"):
        }
        else if(key < root->info)
                return BinSearch(root->left, key);
        else
                return BinSearch(root->right, key);
}
```

Insertion of a node in BST

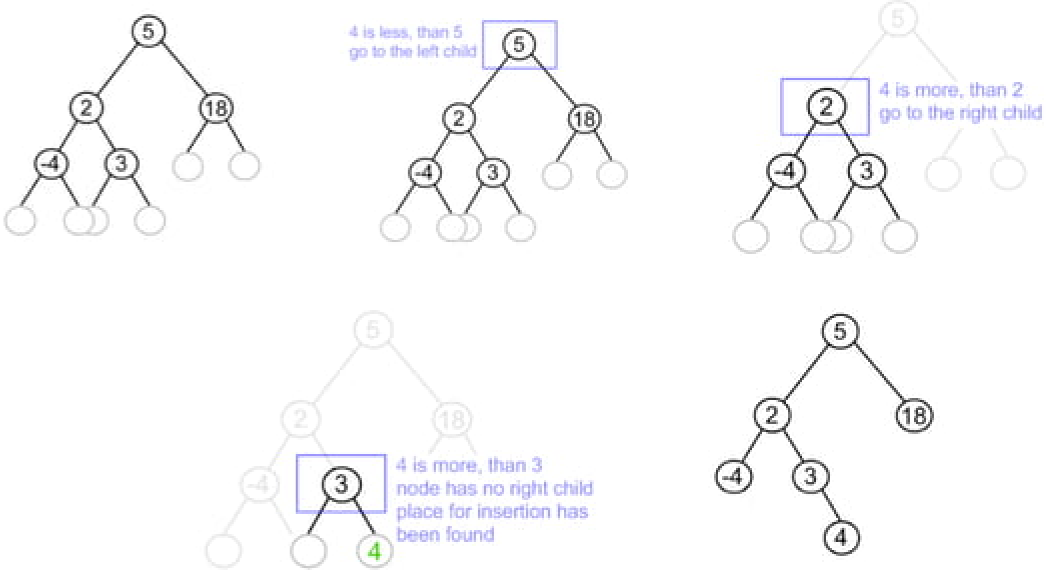
- To insert a new item in a tree, we must first verify that its key is different from those of existing elements.
- If a new value is less, than the current node's value, go to the left subtree, else go to the right subtree.
- Following this simple rule, the algorithm reaches a node, which has no left or right subtree.
- By the moment a place for insertion is found, we can say for sure, that a new value has no duplicate in the tree.

Algorithm for insertion in BST

- Check, whether value in current node and a new value are equal. If so, duplicate is found. Otherwise,

- if a new value is less, than the node's value:
  - if a current node has no left child, place for insertion has been found;
  - otherwise, handle the left child with the same algorithm.

- if a new value is greater, than the node's value:
  - if a current node has no right child, place for insertion has been found;
  - otherwise, handle the right child with the same algorithm.


5
2
18
-4
3
4 is less, than 5
go to the left child
4 is more, than 2
go to the right child
4 is more, than 3
node has no right child
place for insertion has
been found
4

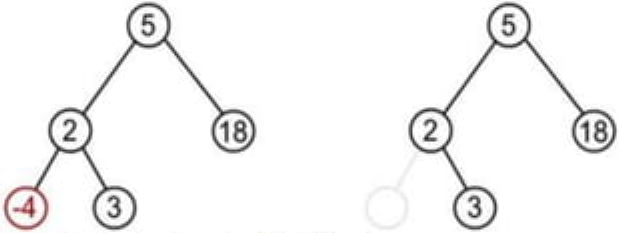
C function for BST insertion:

```
void insert(struct bnode *root, int item)
{
        if(root=NULL)
        {
                root=(struct bnode*)malloc (sizeof(struct bnode));
                root->left=root->right=NULL;
                root->info=item;
        }
        else
        {
                if(item<root->info)
                        root->left=insert(root->left, item);
                else
                        root->right=insert(root->right, item);
        }
}
```

Deleting a node from the BST

- While deleting a node from BST, there may be three cases:

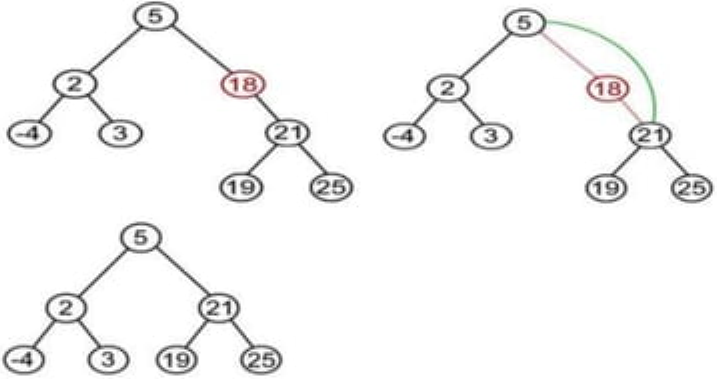
1. The node to be deleted may be a leaf node:
   - In this case simply delete a node and set null pointer to its parents those side at which this deleted node exist.



Suppose node to be deleted is -4

Deleting a node from the BST

2. The node to be deleted has one child
   - In this case the child of the node to be deleted is appended to its parent node. Suppose node to be deleted is 18



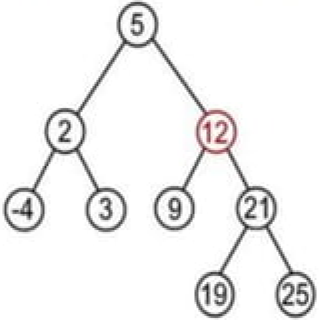
Deleting a node from the BST

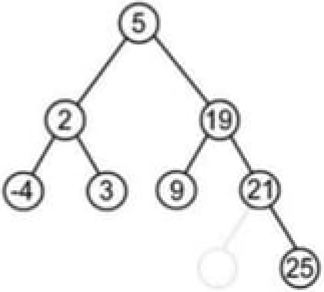
3. the node to be deleted has two children:

In this case node to be deleted is replaced by its in-order successor node.

OR

If the node to be deleted is either replaced by its right sub-trees leftmost node or its left sub-trees rightmost node.





Suppose node to deleted is 12

Find minimum element in the right sub-tree of the node to be removed. In current example it is 19.

General algorithm to delete a node from a BST:

1. start
2. if a node to be deleted is a leaf nod at left side then simply delete and set null pointer to it's parent's left pointer.
3. If a node to be deleted is a leaf node at right side then simply delete and set null pointer to it's parent's right pointer
4. if a node to be deleted has on child then connect it's child pointer with it's parent pointer and delete it from the tree
5. if a node to be deleted has two children then replace the node being deleted either by
   a. right most node of it's left sub-tree or
   b. left most node of it's right sub-tree.
6. End

The deleteBST function:

```
struct bnode *delete(struct bnode *root, int item)
{
    struct bnode *temp;
    if(root==NULL)
    {
        printf("Empty tree");
        return;
    }
    else if(item<root->info)
        root->left=delete(root->left, item);
    else if(item>root->info)
        root->right=delete(root->right, item);
    else if(root->left!=NULL &&root->right!=NULL)  //node has two child
    {
        temp=find_min(root->right);
        root->info=temp->info;
        root->right=delete(root->right, root->info);
    }
    else
    {
        temp=root;
        if(root->left==NULL)
            root=root->right;
        else if(root->right==NULL)
            root=root->left;
        free(temp);
    }
    return(temp);
}
/*********find minimum element function*********/
struct bnode *find_min(struct bnode *root)
{
    if(root==NULL)
        return0;
    else if(root->left==NULL)
        return root;
    else
        return(find_min(root->left));
}
```

Huffman Algorithm

- Huffman algorithm is a method for building an extended binary tree with a minimum weighted path length from a set of given weights.

- This is a method for the construction of minimum redundancy codes.

- Applicable to many forms of data transmission

- multimedia codecs such as JPEG and MP3

Huffman Algorithm

- 1951, David Huffman found the “most efficient method of representing numbers, letters, and other symbols using binary code”. Now standard method used for data compression.

- In Huffman Algorithm, a set of nodes assigned with values if fed to the algorithm. Initially 2 nodes are considered and their sum forms their parent node.

- When a new element is considered, it can be added to the tree.

- Its value and the previously calculated sum of the tree are used to form the new node which in turn becomes their parent.

Huffman Algorithm

- Let us take any four characters and their frequencies, and sort this list by increasing frequency.

- Since to represent 4 characters the 2 bit is sufficient thus take initially two bits for each character this is called fixed length character.

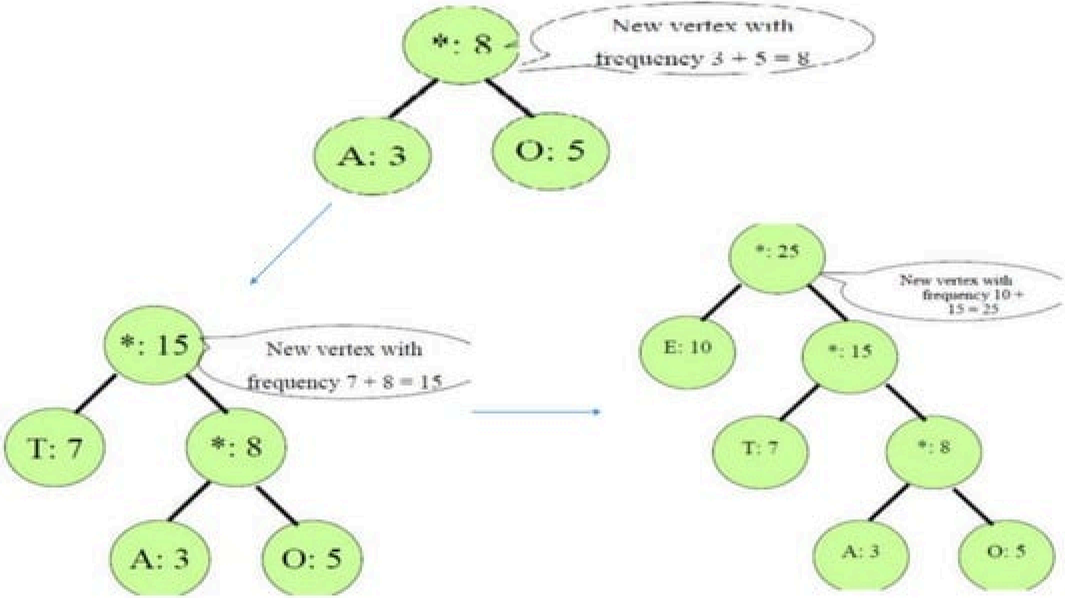
character	frequencies
E	10
T	7
O	5
A	3

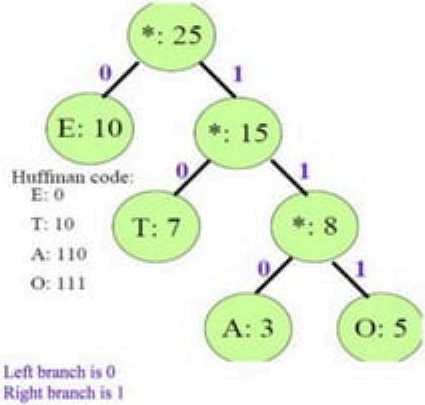
sort

Character	frequencies	code
A	3	00
O	5	01
E	7	10
T	10	11

- Here before using Huffman algorithm the total number of bits required is:

nb=3*2+5*2+7*2+10*2 =06+10+14+20 =50bits


*: 8
New vertex with frequency 3 + 5 = 8
A: 3
O: 5
*: 15
New vertex with frequency 7 + 8 = 15
T: 7
*: 8
A: 3
O: 5
*: 25
New vertex with frequency 10 + 15 = 25
E: 10
*: 15
T: 7
*: 8
A: 3
O: 5



Character	frequencies	code
A	3	110
O	5	111
T	7	10
E	10	0

- Thus after using Huffman algorithm the total number of bits required is

  nb=3*3+5*3+7*2+10*1 =09+15+14+10 =48bits

  i.e

  (50-48)/50*100%=4%

  Since in this small example we save about 4% space by using Huffman algorithm. If we take large example with a lot of characters and their frequencies we can save a lot of space

Huffman Algorithm

- Lets say you have a set of numbers and their frequency of use and want to create a huffman encoding for them

Value	Frequencies
1	5
2	7
3	10
4	15
5	20
6	45

Huffman Algorithm

- Creating a Huffman tree is simple. Sort this list by frequency and make the two-lowest elements into leaves, creating a parent node with a frequency that is the sum of the two lower element's frequencies:

```
      12:*
     /    \
  5:1      7:2
```

- The two elements are removed from the list and the new parent node, with frequency 12, is inserted into the list by frequency. So now the list, sorted by frequency, is:

10:3

12:*

15:4

20:5

45:6

Huffman Algorithm

- You then repeat the loop, combining the two lowest elements. This results in:

```
        22:*
       /    \
    10:3   12:*
           /   \
         5:1    7:2
```

- The two elements are removed from the list and the new parent node, with frequency 12, is inserted into the list by frequency. So now the list, sorted by frequency, is:

15:4

20:5

22: *

45:6

Huffman Algorithm

You repeat until there is only one element left in the list.

```
        35:*
       /    \
   15:4    20:5

        22:*
        35:*
        45:6

                 57:*
           ______/    \______
          /                  \
      22:*                    35:*
     /    \                  /    \
 10:3      12:*          15:4      20:5
          /    \
      5:1      7:2

        45:6
        57:*

                                                 102:*
                      ___________________________/    \___
                     /                                    \
                 57:*                                      45:6
           ______/    \______
          /                  \
      22:*                    35:*
     /    \                  /    \
 10:3      12:*          15:4      20:5
          /    \
      5:1      7:2
```

Now the list is just one element containing 102:*, you are done.

Assignments

- Slides at myblog

http://www.ashimlamichhane.com.np/2016/07/tree-slide-for-data-structure-and-algorithm/

- Assignments at github

https://github.com/ashim888/dataStructureAndAlgorithm/tree/dev/Assignments/assignment_7