12-12-21 Automata Past Paper (2018)

Q1)(a) Define the Formal definition of an FA? Highlight the main differences between FA and TG?

Ans)

Formal Definition of FA:-

A finite automaton is a collection of 5-tuple $(Q, \Sigma, \sigma, q_0, F)$, where:

- $Q$ : Finite set of states
- $\Sigma$ : finite set of the input symbol
- $q_0$: initial state
- $F$ : final state
- $\sigma$ : Transition function

| FA (Finite Automata) | TG (Transition Graph) |
|---|---|
| 1) FA has one start state. | TG can have more than one start state. |
| 2) FA can not change state without an input. | TG can change state without an input (Null transition). |
| 3) Transitions are marked with single letter of alphabet | Transitions can be marked with letters or strings. |
| 4) States transition are shown for all letters of given alphabet. | It does not necessarily shows transition for all letters. |

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Q2)(b) Give a recursive definition for the set of strings of digit 0,1,2,3,....,9 that can not start with the digit 0.

Ans)

**Recursive Definition for the Set of Strings of Digit:-**

Rule 1) 1, 2, 3, 4, 5, 6, 7, 8, 9 are in L.

Rule 2) If Q is any word in L, then Q0, Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9 are also words in L.

L = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, .... }

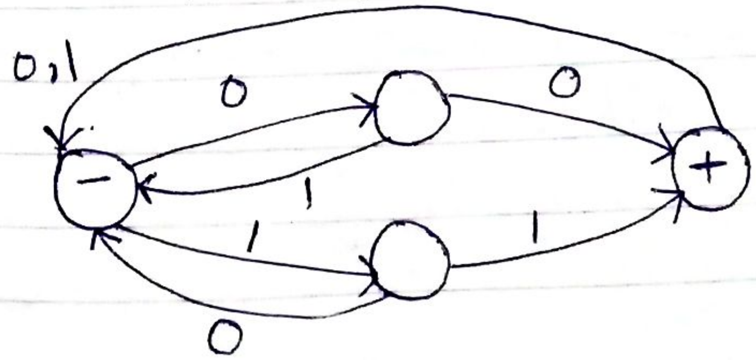
Rule 3) Nothing else are in L.

Q1)(c) Draw a DFA that accepts set of all strings of ~~digits~~ 0 and 1 that end in the last two same symbols.

Ans)

L = { 00, 11, 000, 100, 011, 111, 0111, 1011, .... }

RE = $(0+1)^*(00+11)$



Q1)(d) Write the Regular expressions for the following languages:

i) Write a regular expression that represents the language of zero or more 0's followed by one or more 1's followed by zero or more 0's.

Ans)

$L = \{010, 0010, 00110, 001100, 000111000, 0111000, 001000, \dots\}$

$RE = 0^+ 1^+ 0^+$

(ii) All strings in which the letter y is never tripled. This means that no word contains the substring yyy. $\Sigma = \{x, y\}$

Ans)

$L = \{\Lambda, x, y, xx, yy, xyy, yyx, xyyx, \dots\}$

$RE = (\Lambda + y + yy)(x + xy + xyy)^*$

(iii) Write a RE for the set of all strings not containing bab as substring over input alphabet $\{a, b\}$.

Ans)

$L = \{ \Lambda, a, b, aa, ab, ba, bb, aab, abb, bba, baa, baab, aaab, baaaba, \ldots \}$

$RE = a^*(b^*aaa^*)^* b^* a^*$

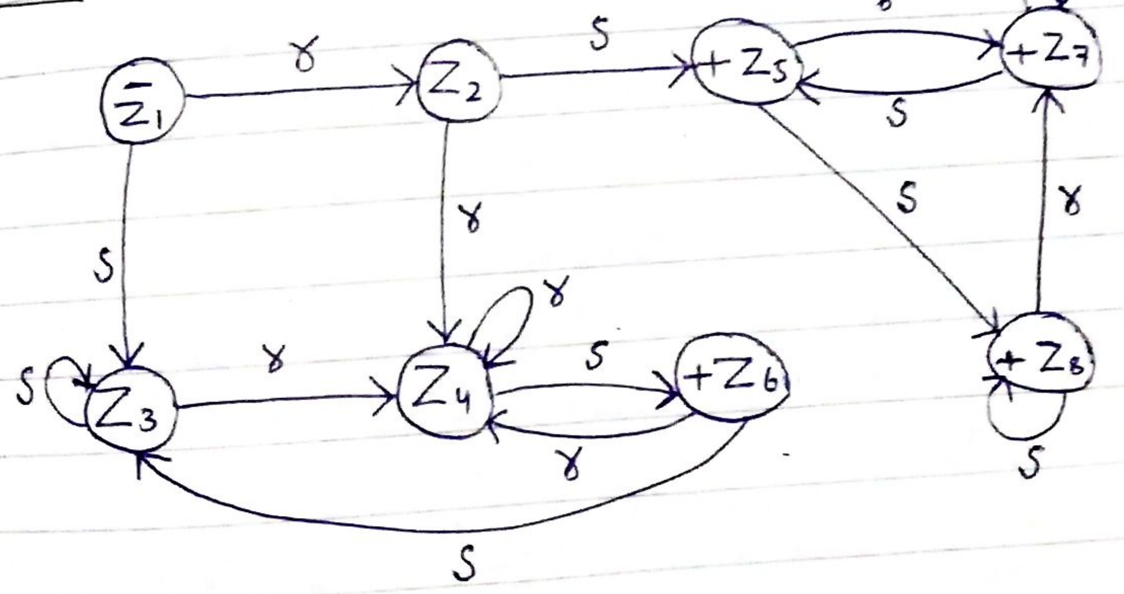
Q2)(a) Design an FA over alphabet $\{r, s\}$, which accepts the set of strings either start with rs or end with rs.

Ans)

$L = \{ rs, rsr, rss, rrs, srs, rsss, rssr, rsrr, rsss, rrrs, ssrs, \ldots \}$

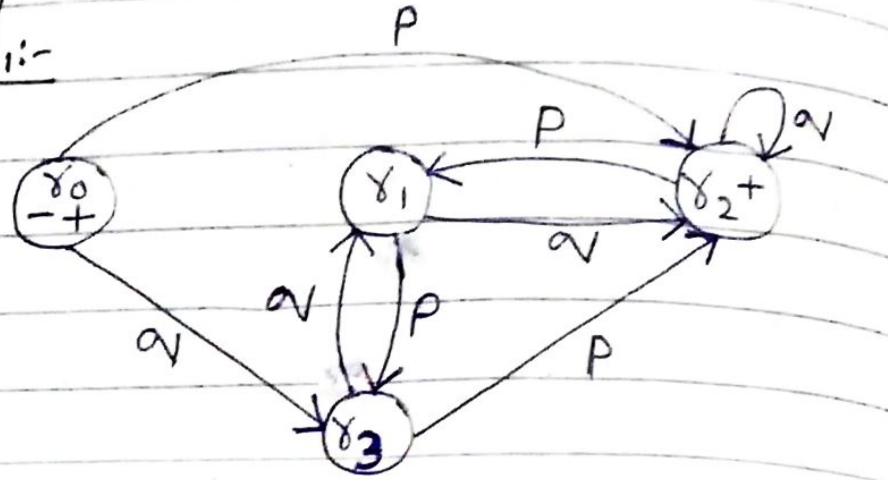
$RE = rs(r+s)^* + (r+s)^* rs$

<u>DFA:-</u>



Q2)(b) Using the Kleen's Theorem part 3, rule 4, construct FA for the (FA)* language.

~~Ans~~) FA₁:-

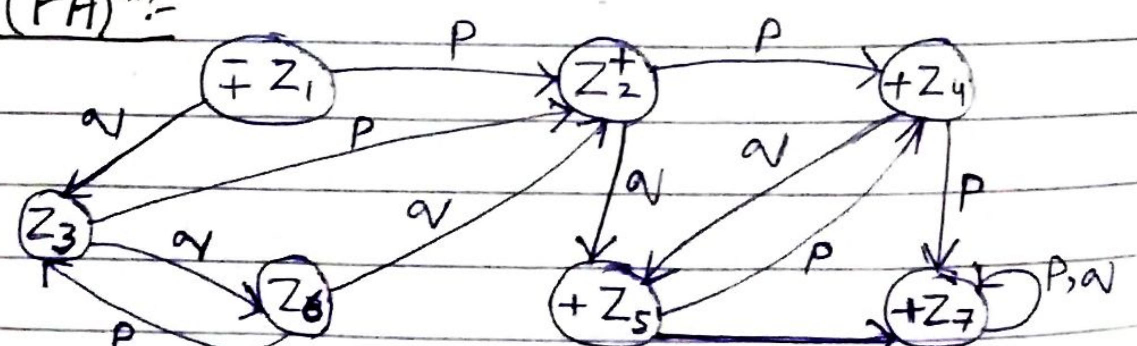



Ans):

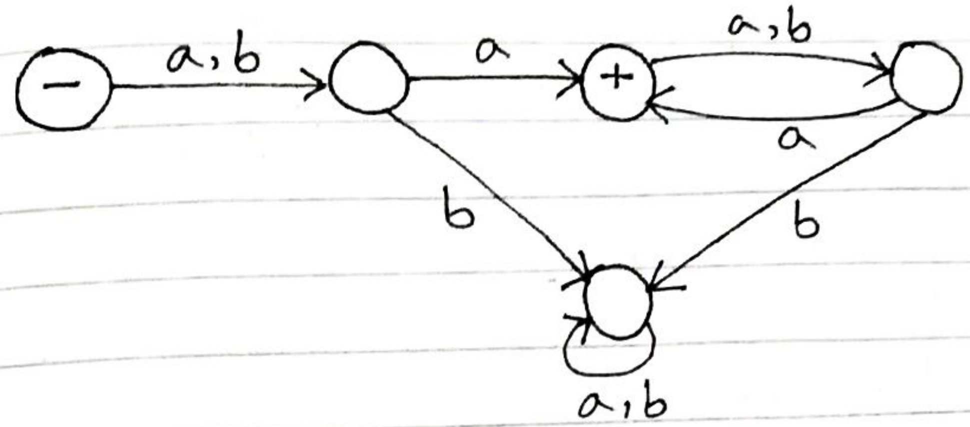
## Kleen's Theorem Part 3, Rule 4:-

| State | p | a |
|---|---|---|
| $\mp\ r_0 = Z_1$ | $r_2 r_0 = Z_2$ | $r_3 = Z_3$ |
| $+\ r_2 r_0 = Z_2$ | $r_1 r_2 r_0 = Z_4$ | $r_2 r_0 r_3 = Z_5$ |
| $r_3 = Z_3$ | $r_2 r_0 = Z_2$ | $r_1 = Z_6$ |
| $+\ r_1 r_2 r_0 = Z_4$ | $r_3 r_1 r_2 r_0 = Z_7$ | $r_2 r_0 r_3 = Z_5$ |
| $+\ r_2 r_0 r_3 = Z_5$ | $r_1 r_2 r_0 = Z_4$ | $r_2 r_0 r_3 r_1 = Z_7$ |
| $r_1 = Z_6$ | $r_3 = Z_3$ | $r_2 r_0 = Z_2$ |
| $+\ r_3 r_1 r_2 r_0 = Z_7$ | $r_2 r_0 r_3 r_1 = Z_7$ | $r_1 r_2 r_0 r_3 = Z_7$ |

(FA)*:-



Q2)(c) Describe in English the language accepted by the following FA and also proved that baaaba is an acceptable word for this FA? Write the RE for this FA?



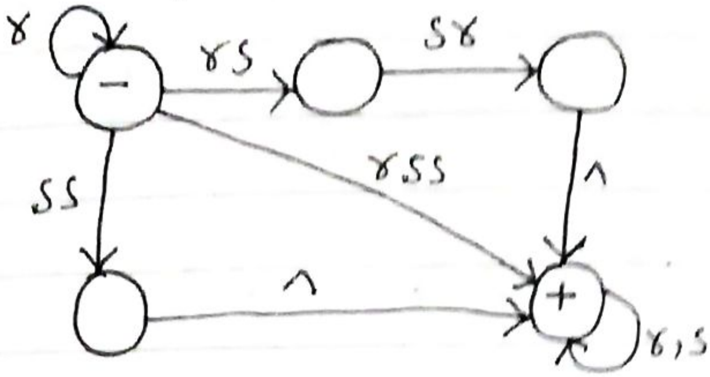

Ans)

i) Draw an FA for the language of strings that have even ~~words~~ and on every even position must be 'a', excluding $\Lambda$. $\Sigma = \{a, b\}$

(ii) Yes, baaaba is an acceptable word for this FA.

(iii) $RE = ((a+b)a)^+$

Q3) (a) Using the Kleen's Theorem convert the following TG into regular expression.

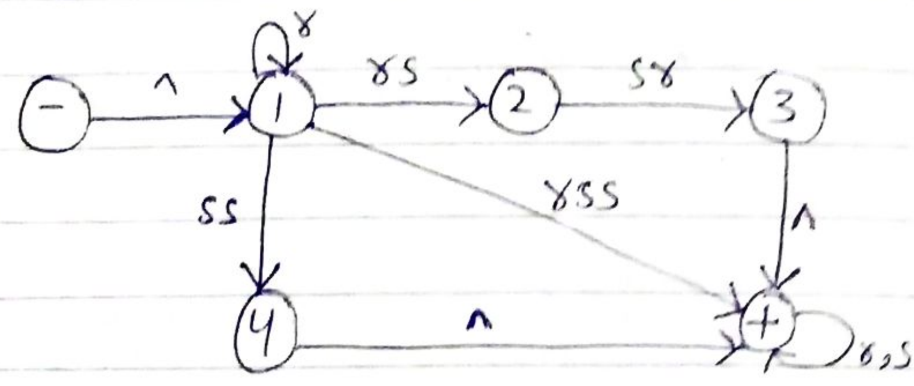



Ans)

Kleen's Theorem Part 2 :-

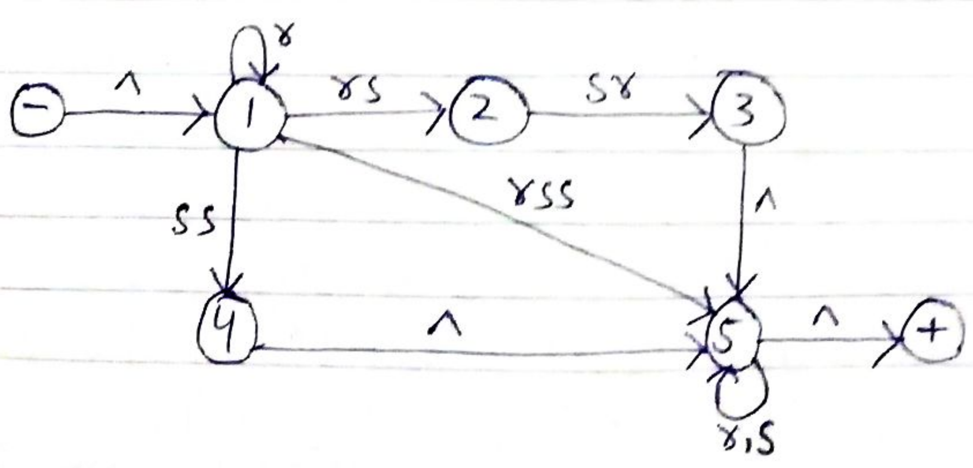
Step 1 :-

Introduce a new initial state



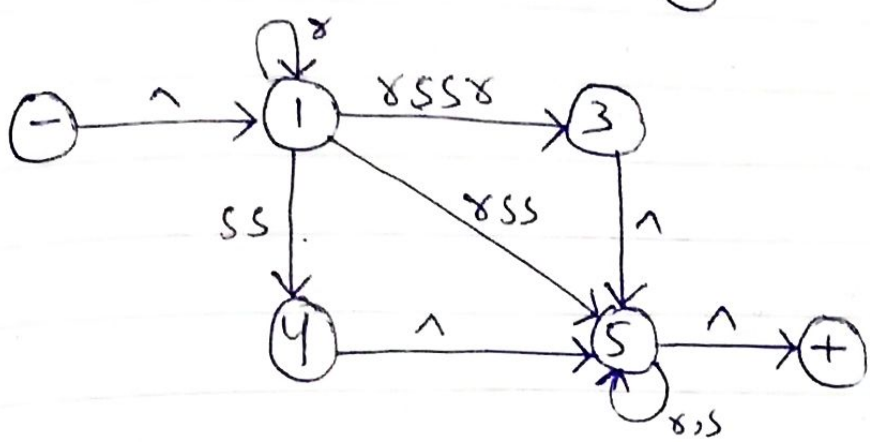

Step 2 :-

Introduce a new final state



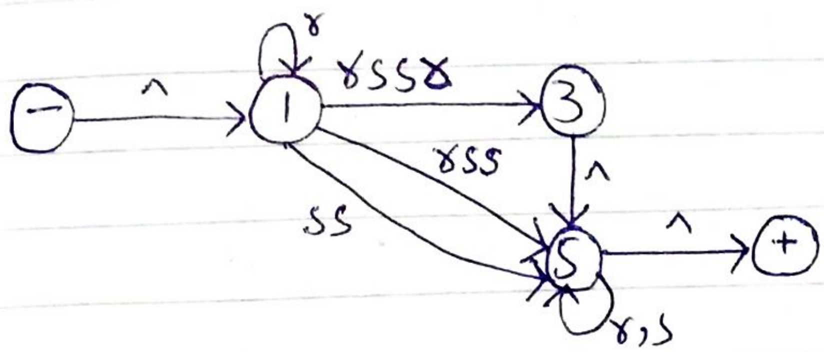
Step 3:-

Eliminate statte 2



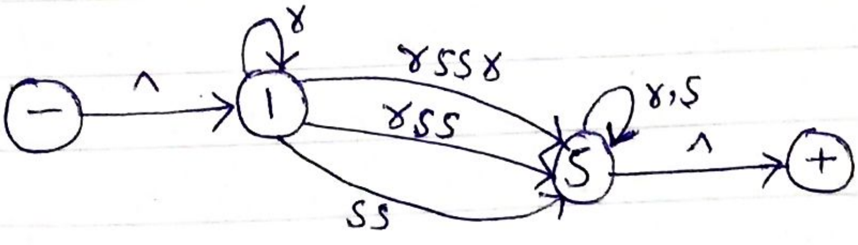

Step 4:-

Eliminate state 4



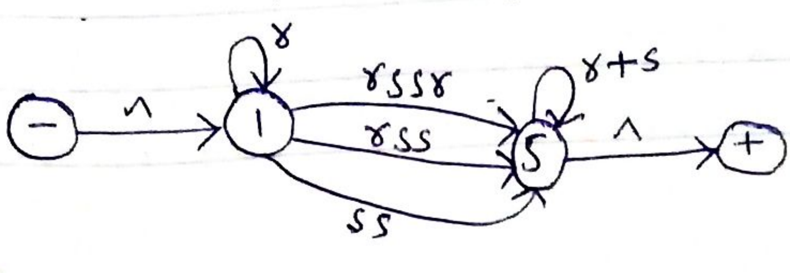

Step 5:-

Eliminate state 3



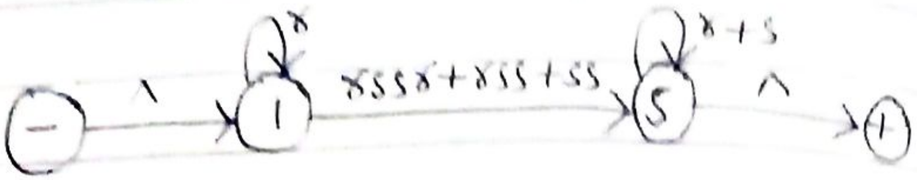

Step 6:-

Replace edges with RE



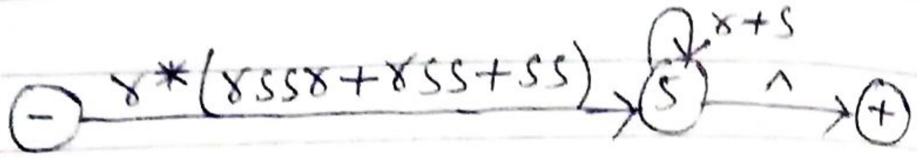
**Step 7:-**

Combine the edges on 5



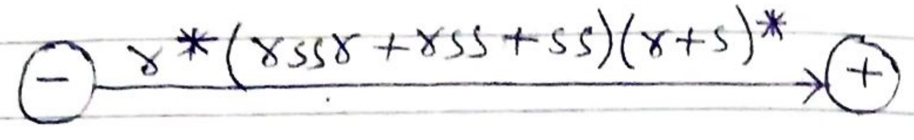

**Step 8:-**

Eliminate state 1




**Step 9:-**

Eliminate state 5



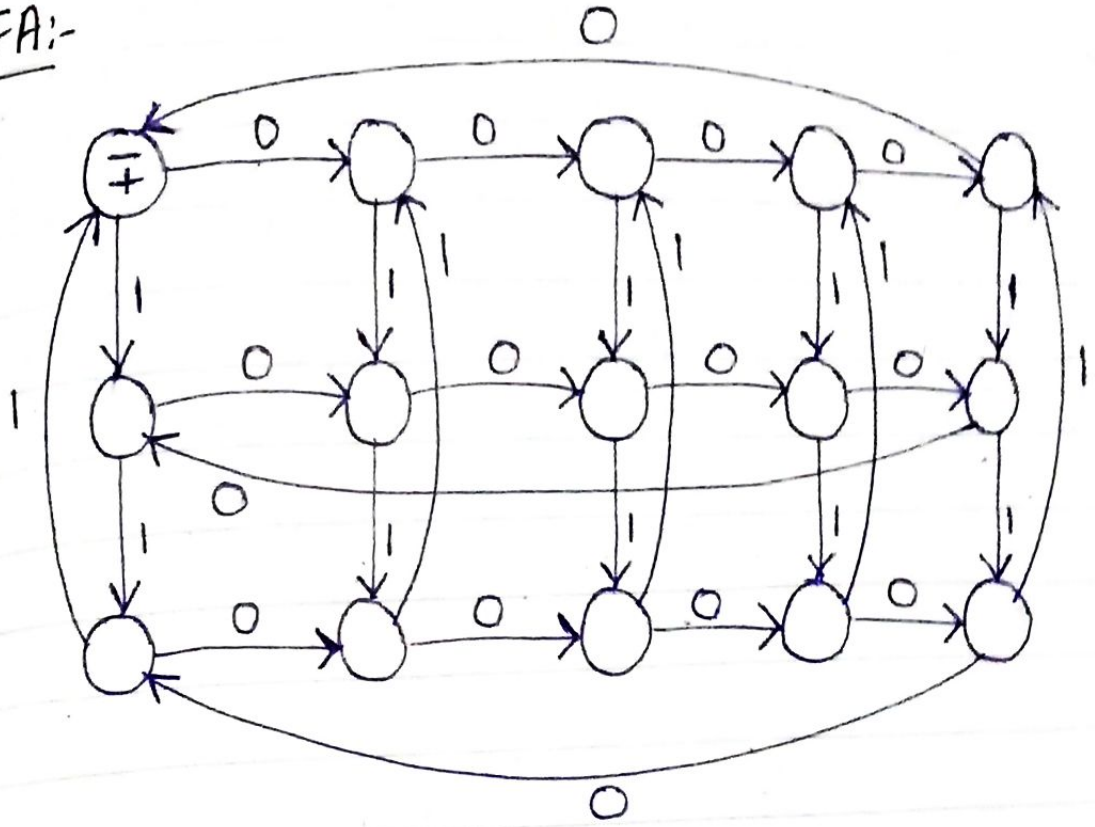

$\therefore$ RE $= r^*(rssr + rss + ss)(r+s)^*$

Q3)(b) Give the DFA accepting the set of strings over alphabets = {0,1}, such that in each string number of 0's is divisible by five and number of 1's is divisible by 3.

Ans)

$L = \{\Lambda, 00000, 111, 00000111, 10101000, 11100000, 1101010100, 010110001100000, \ldots\}$

DFA:-

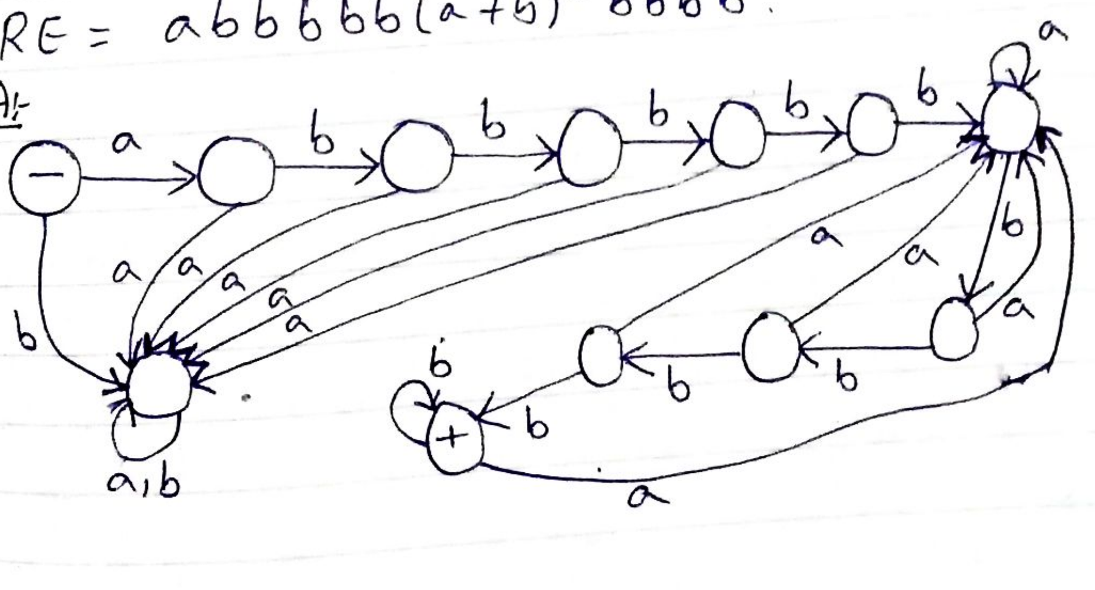


Q3)(d) Design a DFA for the language
$L = \{ab^5wb^4 : w \in \{a,b\}^*\}$

Ans)
$L = \{abbbbbbbbb, abbbbbabbbb,$
$abbbbbabaababbbb, \ldots\}$

$RE = abbbbb(a+b)^*bbbb$.

DFA:-



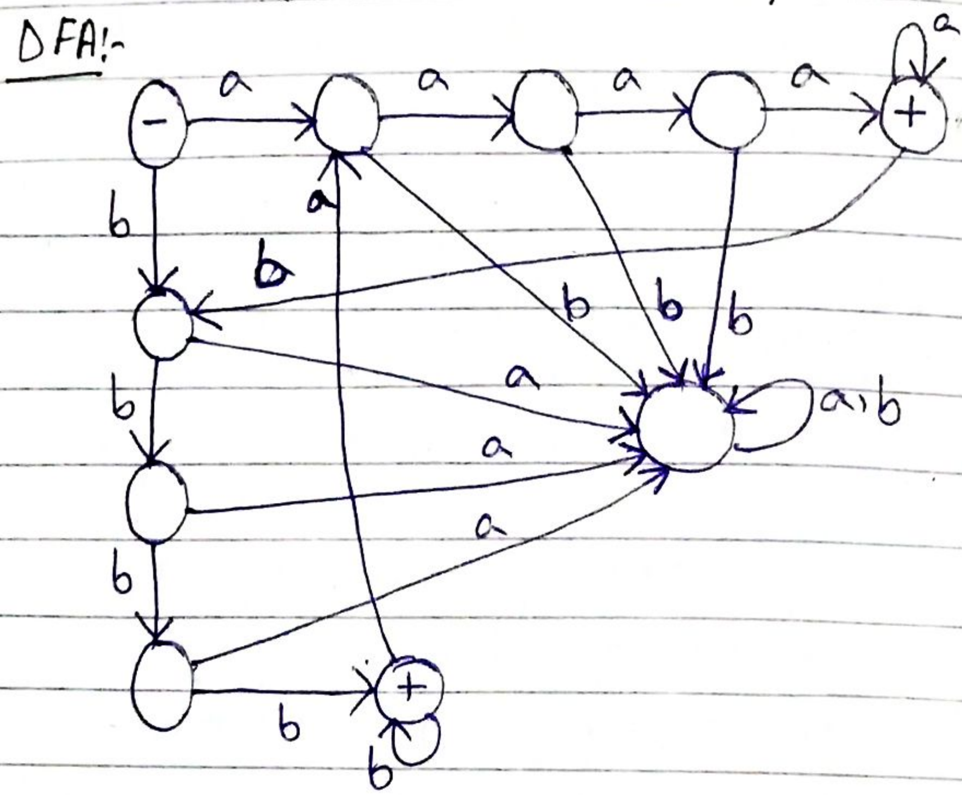
Q4)(a) Design a DFA over alphabet a and b, such that every string accepted by atomaton contains no runs of length less than four.

Ans)

L = { aaaa + bbbb + aaaabbbb, bbbbaaaa, aaaaabbbbaaaaa, bbbbbbaaaa, ....}

$$RE = (aaaa^+ + bbbb^+)^+$$

DFA:-



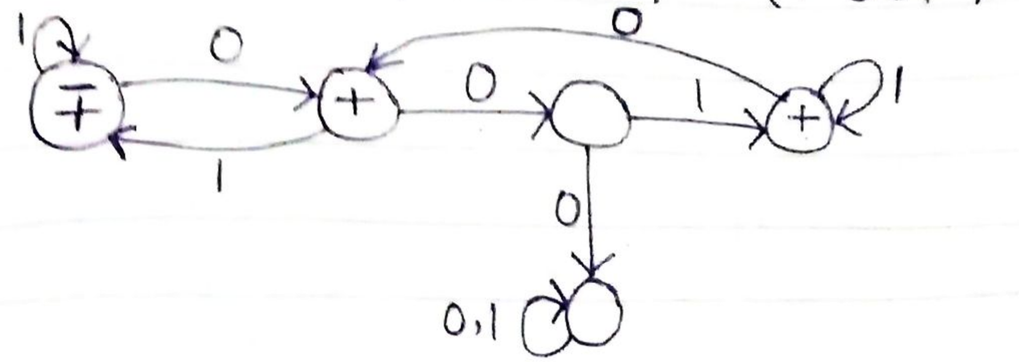

Q4)(b) Design a DFA, which accepts strings in which every 00 is followed immediately by a 1: e.g; 001, 0010, 0010011001 are in the language but 0001 and 00100 are not.

Ans)

$L = \{ \wedge, 0, 1, 01, 10, 11, 011, 110, 101, 111, 001,$
$0011, 1001, 0111, 1101, 1011, \ldots \}$

$RE = 0 + 1^* + (10 + 01 + 1)^* + (1^* 001^+)^*$

DFA:-




Q4)(c) Give the English description for the following regular expressions:

(i) $RE = (aa)^* (bb)^* b$

Ans)
Write the RE for ~~which~~ the language in which even no. of a's followed by odd no. of b's. $\Sigma = \{a, b\}$

(ii) $(0+1)^* 00 (0+1)^*$

Ans)
Write the RE for the language of all strings containing 00 as substring over input alphabet $\{0, 1\}$.

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Q4)(d) Prove that

(i) $(r_1^*)^* = r_1^*$

Ans)

Let $r_1 = \{a\}$

$r_1^* = \{\Lambda, a, aa, aaa, \ldots\}$

$(r_1^*)^* = \{\Lambda, a, aa, aaa, \ldots\}$

Hence,

$(r_1^*)^* = r_1^*$

proved

(ii) $(r_1 + r_2)^* = (r_1^* r_2^*)^*$

Ans)

Let $r_1 = \{a\}$, $r_2 = \{b\}$

$(r_1 + r_2)^* = \{\Lambda, a, b, aa, ab, ba, bb, \ldots\}$

$(r_1^* r_2^*)^* = \{\Lambda, a, b, aa, ab, ba, bb, \ldots\}$

Hence,

$(r_1 + r_2)^* = (r_1^* r_2^*)^*$

proved

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Q3) (c) Design a DFA for the language
$L = \{w : n_a(w) = 2,\ n_b(w) > 2,\ w \in (a,b)^*\}$

Ans/

$L = \{aabbb, bbbaa, bbaba, baabb, bbaabb, babbabb, \ldots\}$

DFA:-