Key Equations for Waiting Lines

λ = average number of arrivals per time period (e.g., per hour)
μ = average number of people or items served per time period

(D-1) $P(X) = \dfrac{e^{-\lambda}\lambda^{X}}{X!}$ for $X = 0,1,2,\cdots$

Poisson probability distribution used in describing arrivals.

(D-2) P (service time > t) = $e^{-\mu t}$, for $t \geq 0$

Exponential probability distribution used in describing service times.

Equations D-3 through D-9 describe the operating characteristics in a single-server queuing system that has Poisson arrivals and exponential service times.

(D-3) $p = \lambda/\mu$

Average server utilization in the system.

(D-4) $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)}$

Average number of customers or units waiting in line for service.

(D-5) $L = L_q + \lambda/\mu$

Average number of customers or units in the system.

(D-6) $W_q = \dfrac{L_q}{\lambda} = \dfrac{\lambda}{\mu(\mu - \lambda)}$

Average time a customer or unit spends waiting in line for service.

(D-7) $W = W_q + 1/\mu$

Average time a customer or unit spends in the system.

(D-8) $P_0 = 1 - \lambda/\mu$

Probability that there are zero customers or units in the system.

(D-9) $P_n = (\lambda/\mu)^n P_0$

Probability that there are n customers or units in the system.

(D-10) Total cost = $C_w \times L + C_s \times s$

Total cost is the sum of waiting cost and service cost.

Equations D-11 through D-18 describe the operating characteristics in a multiple-server queuing system that has Poisson arrivals and exponential service times.

(D-11) $p = \lambda/(s\mu)$

Average server utilization in the system.

(D-12) $$P_0 = \frac{1}{\left[\sum_{k=0}^{s-1} \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k\right] + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{(s\mu - \lambda)}}$$

Probability that there are zero customers or units in the system.

(D-13) $$L_q = \frac{(\lambda/\mu)^s \lambda\mu}{(s-1)!(s\mu - \lambda)^2} P_0$$

Average number of customers or units waiting in line for service.

(D-14) $L = L_q + \lambda/\mu$

The average number of customers or units in the system.

(D-15) $W_q = L_q / \lambda$

Average time a customer or unit spends waiting in line for service.

(D-16) $W = W_q + 1/\mu$

Average time a customer or unit spends in the system.

(D-17) $P_n = \frac{(\lambda / \mu)^n}{n!} P_0 \quad \text{for } n \leq s$

Probability that there are n customers or units in the system, for $n \leq s$.

(D-18) $P_n = \frac{(\lambda / \mu)^n}{s!\, s^{(n-s)}} P_0 \quad \text{for } n > s$

Probability that there are n customers or units in the system, for $n > s$.

Equations D-19 through D-24 describe the operating characteristics in a single-server queuing system that has Poisson arrivals and constant service times.

(D-19) $p = \lambda/\mu$

Average server utilization in the system.

(D-20) $L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)}$

Average number of customers or units waiting in line for service.

(D-21) $L = L_q + \lambda/\mu$

Average number of customers or units in the system.

(D-22) $W_q = L_q / \lambda = \frac{\lambda}{2\mu(\mu - \lambda)}$

Average time a customer or unit spends waiting in line for service.

(D-23) $W = W_q + 1/\mu$

Average time a customer or unit spends in the system.

(D-24) $P_0 = 1 - \lambda/\mu$

Probability that there are zero customers or units in the system.

Equations D-25 through D-30 describe the operating characteristics in a single-server queuing system that has Poisson arrivals and general service times.

(D-25) $p = \lambda/\mu$

Average server utilization in the system.

(D-26) $L_q = \dfrac{\lambda^2\sigma^2 + (\lambda/\mu)^2}{2(1-(\lambda/\mu))}$

Average number of customers or units waiting in line for service.

(D-27) $L = L_q + \lambda/\mu$

Average number of customers or units in the system.

(D-28) $W_q = L_q / \lambda$

Average time a customer or unit spends waiting in line for service.

(D-29) $W = W_q + 1/\mu$

Average time a customer or unit spends in the system.

(D-30) $P_0 = 1 - \lambda/\mu$

Probability that there are zero customers or units in the system.

***Equations D-31 through D-38 describe the operating characteristics in a multiple-server queuing system that has Poisson arrivals, exponential service times, and a finite population of size* N.**

(D-31) $$P_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{N!}{(N-n)!n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{N} \frac{N!}{(N-n)!s!s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n}$$

Probability that there are zero customers or units in the system.

(D-32) $$P_n = \frac{N!}{(N-n)!n!}\left(\frac{\lambda}{\mu}\right)^n P_0, \qquad \text{if } 0 \le n \le s$$

Probability that there are exactly n customers in the system, for $0 \le n \le s$.

(D-33) $$P_n = \frac{N!}{(N-n)!s!s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0, \qquad \text{if } s < n \le N$$

Probability that there are exactly n customers in the system, for $s \le n \le N$.

(D-34) $$P_n = 0, \qquad \text{if } n > N$$

Probability that there are exactly n customers in the system, for $n > N$.

(D-35) $$L_q = \sum_{n=s}^{N}(n-s)P_n$$

Average number of customers or units waiting in line for service.

(D-36) $$L = \sum_{n=0}^{s-1} nP_n + L_q + s\left(1 - \sum_{n=0}^{s-1} P_n\right)$$

Average number of customers or units in the system.

(D-37) $W_q = \dfrac{L_q}{\lambda\,(N-L)}$

Average time a customer or unit spends waiting in line for service.

(D-38) $W = \dfrac{L}{\lambda\,(N-L)}$

Average time a customer or unit spends in the system.