Date:

OPERATION RESEARCH:-

The course deals use of mathmetical model for corporate & governmental activities. Most of the planning problem consists of an economic objective. which we want to maximize under scare resource OR consist of.

- Limiting & defining the correct problem.
- Formulating a mathmetical.
- Calculating an optimized solution of problem
- Interpreting & Implement the find solution.
- Deterministic problem. (Mathematical problem)
- Statistic / Stochastic problem.
- Linear & non-linear programming.
- Integer programming.
- Network model.
- Simple queuing theory +
- Simulation.

OR is also called quantities techniques of managment or quantities analysis of managment. It is probability nature.

APPLICATION OF O.R:-

Forecasting
Product scheduling
Inventary contrd managment
Budgeting
Transportation.

Date

Building block or core of O.R.
Statistics & probability theory.

Note:-

Advance spred sheet will be used to find numerical solution for some of analyzed problem.

Characteristics Of O.R:-

1. Applied scientific method to solve problem.
For e.g: certain algo, predefine steps used for reaching to an optimal research.

2. It is uncores or exhibit problem.
For e.g: while looking a problem finding a solution, one take find some new problem. for instance, while coristumer spend more time comparitivly other, we will look at some resources are.
For e.g: for more shops are free.

3. It improve quality of decision biggest we used numerical method, analysis & quantitative method. Since we using numerical & quantitative method it we used computer extensively.

Date

4. Since quatitative method, we have only quatitative solution, no quantitative solution.

5. It allows to take a real world problem & focus or put this into a mathmetical model
For e.g: I have certain amount & I have to perform five task which required this amount in this month. Now it is art to pick some basic task more & more to be accomplished using a same amount in a month. That is how best I can optimized upon a required amount I have.

Q What is model?

A The model implies to deals with math and statistic.

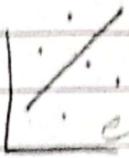
Mathmatical Model:- $Y = mx + b$

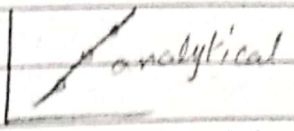
Statistics Model:- $Y = b_0 + b_1 x_1 + \epsilon$

Basically the model (equation) is explain of real data.
For e.g.
Mathematical model: Integration $\int_a^b f x$.
Approximation method (Trapezoidal)
Statistical

Date

Here the denoted lines shows the distance (error) from the model of actual data.





We used model to explore relationship for e.g.

I might be interested relationship height & age of student.

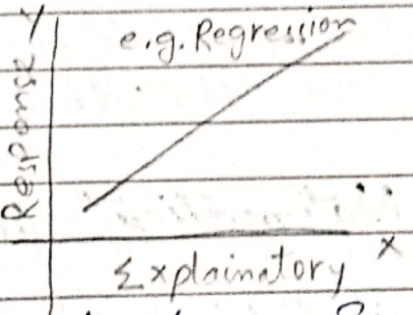
We use statistic to determine how useful & how reliable our model is.

Types Of Statistics Model:-

1- Continous Data:-

It depends on continous data.

e.g Regression,



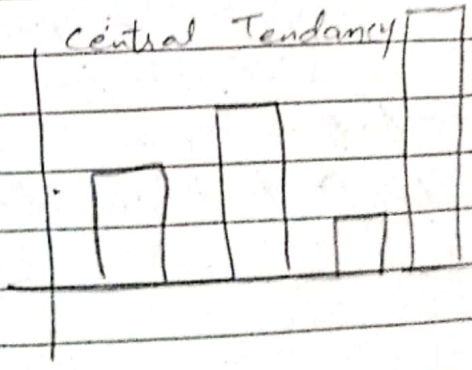
Explainatory: how many unit change & effect on response.

e.g

2. Integer Data:-

It depends on discrete.

e.g. ANOVA.



Date:

Test of Equality of Means of 2 samples or more

ANALYSIS OF VARIANCE (ANOVA):-

originated by

The technique ANOVA is given by R.A. Fisher. The analysis of method which separate the variation refer able to one set of causes from the variation to other set.

For e.g. We are interested in testing the null hypothesis that the three variety of wheat produce equal yeild on the average.

H_0: $\mu_A = \mu_B = \mu_C$

H_A: $\mu_A \neq \mu_B \neq \mu_C$

We conduct experiment by planting different varities of wheat on plotting on land. there might be difference in the mean of various variety due to experimental error also there might be variation dueto experimental error + any variation due to the different variety of wheat.

The analysis of variance is a method of splitting the total variation of our data into constituent which measure different sources of variation. The total variation is split up into the following meaningful component.

1. Variance with in the sub group of sample.

Teacher's Signature

Date

2. Variance between the subgroup of sample.

After this, the two variances are tested for their significant by the variance ratio or the f-test.

"Also analysists use the ANOVA test to determine the influence that independent variables have on the dependent variable in a regression study".

VARIANCE:-

The distance between the random variable to the mean. $(x - \bar{x})$

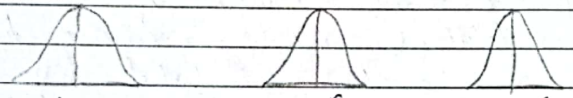
OR

It is define as the expectation of S.D. of the random variable from its mean.

$(x - \bar{x})^2$

Basically, we do anova for comparision of more than two population or population having more than two sub group.

Consider the example.



between the column

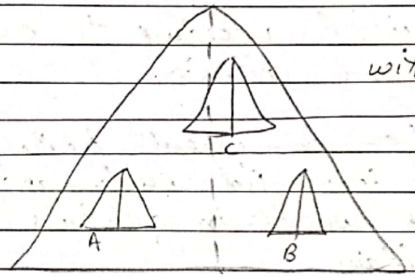
Date

If we want to know all above three mean are coming from the same populations than we can apply ANOVA test. because

∴ ANOVE = variability b/w the means / variability within the distribution

than we have total variance = variability b/w the means + variability within distribution.

than graphically concept says.



within column

Assumption:-

Each population having normal dist.
The population from which the sample are drawn have the equal variance that is

$S_1^2 = S_2^2 = S_3^2 \ldots\ldots S_K^2$

Each sample is drawn randomly and they are independent.

Date:

Hypothesis test for F-Distribution:-

$H_0: \mu_1 = \mu_2 = \mu_3 \ldots\ldots$

$H_A \quad \mu_1 \neq \mu_2 \neq \mu_3 \ldots\ldots$

We define ANOVA into two classes.

One way ANOVA (one factor)
Two way ANOVA (two factor).

Q. It is classified according to one factor or one criteria.

Three varieties of wheat A, B, C sown in (4) four plot each and the following wheat in quintals per acre ~~one~~ were obtain

Plot	Vasities		
	A	B	C
1	10	9	4
2	6	7	7
3	7	7	7
4	9	5	6
Total	32	28	24

Set of table of ANOVA and find out whether there is a significant of

Page No.

Teacher's Signature

Date:

mean of yield of the three variety.

Solution:-

Step 1:-

H_0 : $\bar{X}_A = \bar{X}_B = \bar{X}_C$

H_A : $\bar{X}_A \neq \bar{X}_B \neq \bar{X}_C$

Step 2:

Calculate the variance b/w the sample.

a) Calculate mean of each sample.

$\bar{X}_A = 32/4 = 8$

$\bar{X}_B = 28/4 = 7$

$\bar{X}_C = 24/4 = 6$

b) Calculate the grand mean.

$$\bar{\bar{X}} = \frac{\bar{X}_A + \bar{X}_B + \bar{X}_C}{3} = \frac{32 + 28 + 24}{3}$$

$$\boxed{\bar{\bar{X}} = 7}$$

c) The difference b/w the mean of variance sample and grand mean. and square it

$\bar{X}_A - \bar{\bar{X}}$	$(\bar{X}_A - \bar{\bar{X}})^2$	$(\bar{X}_B - \bar{\bar{X}})$	$(\bar{X}_B - \bar{\bar{X}})^2$	$(\bar{X}_C - \bar{\bar{X}})$	$(\bar{X}_C - \bar{\bar{X}})^2$
8 - 7	1	7 - 7	0	6 - 7	-1
8 - 7	1	7 - 7	0	6 - 6	-1
8 - 7	1	7 - 7	0	6 - 7	-1
8 - 7	1	7 - 7	0	6 - 7	-1
	4		0		4

Page No.

Teacher's Signature

minitab

Sum of square b/w samples

$\Sigma(\bar{x} - \bar{\bar{x}})^2 = 4+0+4 = 8$

Step 3:-

Calculate the variance within the sample.

a) Calculate the mean for each sample.

b) Take the deviation from variance item in a sample from the mean value of respective sample & square it.

$(A-\bar{X}_A)$	$(A-\bar{X}_A)^2$	$(B-\bar{X}_B)$	$(B-\bar{X}_B)^2$	$(C-\bar{X}_C)$	$(C-\bar{X}_C)^2$
10-8	4	9-7	4	4-6	4
6-8	4	7-7	0	7-6	1
7-8	1	7-7	0	7-6	1
9-8	1	5-7	4	6-6	0
Total	10		8		6

Sum of square within the sample

$(\Sigma(x-\bar{x})^2) = 10+8+6 = 24$

Step 4:-

Calculate the value of F-ratio:-

Sum of variance	sum of square	D.F	Mean sum of square	F-Test
Between the sample	SSC = 8	$u_1 = c-1$ 3-1 = 2	MSC = SSC/c-1 8/2 = 4	F = MSC/MSE
Within the sample	SSE = 24	$u_2 = n-c$ 12-3 = 9	MSE = SSE/n-c 24/9 = 2.66	F = 4/2.66 F = 1.50

M.S.E = error.

$F_{cal} = 1.50$, $F_{tab} = 8.0215$

Step 5:-

$F_{cal} < F_{tab}$

$1.50 < 8.0215$

Result:-

We accept null hypothesis and, Hence, prove that there is a significant difference b/w the mean of varities of wheat.

Date:

Replicates:- Rows in ANOVA table

Factors:- Column in ANOVA table. also called method.

Within OR Within the Column:-
Difference between the rows

Between OR Between the Column:-
Difference between the column.

SS (method):- Sum of square between the column. Also called "ANOVA test". Difference by the "mean"
column.

SS (error):- Sum of square within the column.
row.

Date:

Q A rifle club perform an experiment on a randomly selected group of first time shooter. The purpose of the exp was to determine whether shooting accuracy is affected by the method of sighting used:
Only right eye open, only left eye open or both eyes open. 15 shooter were selected & split into three group. Each group experience the same training and practising procedures with one exception; the method of sighting used. After completing training, each student was given the same. $\therefore \alpha = 0.05$

Method Of Sighting

Right Eye	Left Eye	Both Eyes
12	10	16
10	17	14
18	16	16
12	13	11
14		20
		21

Solution:-

Step 1:-

$H_0 : \mu_R = \mu_L = \mu_B$
$H_0 : \mu_R \neq \mu_L \neq \mu_B$

Teacher's Signature

Date:

Replicant (K)	Right Eye	Left Eye	Both Eyes
1	12	10	16
2	10	17	14
3	18	16	16
4	12	13	11
5	14		20
6			21
Total	$C_R = 66$	$C_L = 56$	$C_B = 98$

$\Sigma x = C_R + C_L + C_B = 66 + 56 + 98 = 220.$

$\Sigma x^2 = 12^2 + 10^2 + 16^2 + 10^2 + 17^2 + \ldots = 3392$

Step 2:-

Sum Of Square Among/Between The Factors:-

$$SS(\text{method}) = \left(\frac{C_R^2}{K_1} + \frac{C_L^2}{K_2} + \frac{C_B^2}{K_3}\right) - \frac{(\Sigma x)^2}{N}$$

$$SS(\text{method}) = \left(\frac{66^2}{5} + \frac{56^2}{4} + \frac{98^2}{6}\right) - \frac{(220)^2}{15}$$

$SS(\text{method}) = 3255.87 - 3226.67$

$SS(\text{method}) = 29.2.$

Date:

Step 3:-

Sum Of Square Within The Factors:-

$$SS(\text{error}) = \Sigma x^2 - \left(\frac{C_R^2}{K_1} + \frac{C_L^2}{K_2} + \frac{C_B^2}{K_3}\right)$$

$SS(\text{error}) = 3392 - 3255.87$

$SS(\text{error}) = 136.13.$

Step 4:-

Sum Of Square Total:-

$$SS(\text{total}) = \Sigma x^2 - \frac{(\Sigma x)^2}{N}$$

$$SS(\text{total}) = 3392 - \frac{(220)^2}{15}$$

$SS(\text{total}) = 165.33.$

Verification:-

$SS(\text{method}) + SS(\text{error}) = SS(\text{total})$

$29.20 + 136.13 = 165.33$

Date:

Step 5:-

MSSC:- Mean Sum Of Square Between the factor:-

$$MSSC = \frac{SS(method)}{c-1} = \frac{29.20}{2} \quad \because df(method) = c-1$$

$$MSSC = 14.6$$

Step 6:-

MSSE:- Mean Sum Of Square Within the factor:-

$$MSSE = \frac{SS(error)}{n-c} = \frac{136.13}{15-3}$$

$\because df(error) = n-c.$

$$MSSE = 11.33$$

Step 7:-

F-Ratio:-

$$F_{cal} = \frac{MSSC}{MSSE} = \frac{14.6}{1.22} = 1.288$$

Step 8:-

$$F_{tab} = F_{(2,12,0.05)} = 3.89$$

Step 9:-

$F_{cal} < F_{tab}$

$1.288 < 3.89$

Conclusion:-

Accept null hypothesis. There is a significant difference b/w the mean of method of sighting.

Page No.

Teacher's Signature

Date: _____

Two Way Anova

Note:- It is classified according to two factors or two criteria.
It is with replication (rows works in it)

Given:-

Medical Field	Pain Killer 1	Pain Killer 2	Pain Killer 3	Total
Medicine	10	7	4	
	12	9	5	
	11	8	6	$R_1 = 98$
	9	12	5	
Surgery	12	13	6	
	13	15	6	$R_2 = 12$
	10	12	4	
	13	12	4	
Total	$P_1 = 90$	$P_2 = 88$	$P_3 = 40$	

Step 1:-

$H_0 : \mu_{P_1} = \mu_{P_2} = \mu_{P_3}$

$H_A : \mu_{P_1} \neq \mu_{P_2} \neq \mu_{P_3}$

Date

Step 2:-

correlation term $= \frac{(\Sigma x)^2}{N} = \frac{10^2+7^2+4^2\ldots+4^2}{24}$

$C_x = \frac{47524}{24} = 1980.$

Sum Of Square Total:-

$SS_{(total)} = \Sigma x^2 - \frac{(\Sigma x)^2}{N}$

$= 2254 - 1980$

$SST = 274$

SSC:-

For variation in painkiller programme.

$= \frac{C_1^2}{8} + \frac{C_2^2}{8} + \frac{C_3^2}{8} - \frac{(\Sigma x)^2}{N}$

$= \frac{90^2}{8} + \frac{88^2}{8} + \frac{40^2}{8} - 1980$

$SSC = 200$

Date

SSR:-

For variation in medical field, Sum of square rows.

$SSR = \frac{R_1^2}{RN_1} + \frac{R_2^2}{RN_2}$

$= \frac{98^2}{12} + \frac{120^2}{12}$

$SSR = 20$

SSG:-

Sum of square of groups SSG The interaction between rows and column.

$SSG = \frac{\Sigma\Sigma (PM)^2}{r} = C_M - SSC - SSR$

$SSG = \left[\frac{(10+12+11+9)^2}{4} + \frac{(7+9+8+12)^2}{4} + \frac{(4+5+6+5)^2}{4}\right.$

$\left.\frac{(12+13+10+13)^2}{4} + \frac{(13+15+12+12)^2}{4} + \frac{(6+6+4+4)^2}{4}\right]$

$- 1980 - 200 - 20$

$SSG = 17$

Date:

SSE:-

Residual sum of square.

$SSE = SST - SSC - SSR - SSG$

$SSE = 274 - 200 - 20 - 17$

$SSE = 37$

Step 3:-

	D.F	F-ratio $F = MSSR/MSE$	F-criteria $\alpha = 0.05$
Pain Killer	$c-1=2$	48.73	$F_{(2,18)} = 3.55$
Medical Field	$R-1$ $2-1=1$	9.81	$F_{(1,18)} = 4.41$
Interaction	$(c-1)\times(R-1)$ $2\times1=2$	3.97	$F_{(2,18)} = 3.55$
Residual	$C.R\times(n-1)$ $3.2\times(4-1)=18$		
Total	$N-1 = 24-1 = 23$		

Assignment

Date:

For the following data, find SS(error) & show that $SSE = [(K-1)S_1^2 + (K-1)S_2^2 + (K-1)S_3^2]$ where S_i^2 is variance for factor level.

Factors level

8	6	10
4	6	12
2	4	14
S_i = 14	16	36

$\Sigma x^2 = 8^2 + 4^2 + \dots 14^2 = 612$

$C = 14 + 16 + 36 = 66.$

$$SSE = 612 - \left(\frac{14^2}{3} + \frac{16^2}{3} + \frac{36^2}{3}\right)$$

$$612 - 582.66$$

$$\boxed{SSE = 29.333}$$

$SS(method) = 98.666$

$SST = 127.99$

$K = 3$ ∴ K is replicates

$(K-1)S_1^2 + (K-1)S_2 + (K-1)S_3^2)$ ∴ S is variace of each column.

$2(3.055)^2 + 2(1.154)^2 + 2(2)^2$

$$\boxed{= 29.32}$$

Teacher's Signature

Date:

Anova (Without Replication)

Potatoes

Location	A	B	C	Mean
1	18	13	12	14.33
2	20	23	21	21.33
3	14	12	9	11.66
4	11	17	10	12.66
Mean	15.75	16.25	13	15

① Calculate Raw & column wise mean.

② Variance of raw mean S_R^2

= variance (14.33 + 21.33 + 11.66 + 12.66)

$S_R^2 = 19.051$

③ Variance of column mean S_c^2.

= variance (15.75 + 16.25 + 13).

$S_c^2 = 3.0625.$

④ Mean sum of square for Row.

$MS_A = C \times S_R^2$

Mean sum of square for column

$MS_B = R S_c^2$

Teacher's Signature

Date:

Here,

C = no. of column = 3

R = no. of rows = 4

Now,

$MS_A = C * S_R^2 = 3 \times 19.051$

$MS_A = 57.153$

$MS_B = R * S_c^2 = 4 \times 3.0625$

$MS_B = 12.25$.

ANOVA TABLE

Sample	D.f	S.S	M.S	F
(row) A	4-1 = 3	171.33	57.111	$8.12 = 57.11/7.03$
(column) B	3-1 = 2.	24.5	12.25	$1.72 = 12.25/7.03$
Residual	R.C = 3x2 = 6	42.17	$7.03 = \frac{171.3}{24.5}$	
Total	11	238		

$MS \times D.f = S.S.$

$Var(y) = 238/11 = SS_{total}/n-1$

$SS_{total} = 238$

$Var(y) = 21.636.$

Date:

$F_{cal} = 8.12$

$F_{cal} = 1.72$

$F_{tab} = F_{(3,2,0.05)} =$

F(tabulated):

$F \quad F_{(3,2,0.05)} = 19.164$

$F_{(cal)}$ by row $= 8.12$

$F_{cal} < F_{(tab)}$

$8.12 < 19.164$.

$F_{(cal)}$ by column $= 1.72$.

$1.72 < 19.164$

Conclusion:

Accept null hypothesis.

Date:

Q A new brand increase the manufacturing of different types of fabric & makers of the brands want to better understand which of three formulation of brand are most effective for cotton, lawn, silk, jaquard fabric. It has 3 blend of one sample on each fabric. of 4 types of fabric. The fabric has 12 formulation as follows.

Fabric	x	Y	Z	Mean
Cotton	123	145	156	141.333
Lawn	158	185	176	173
Silk	110	140	185	145
Jaquard	151	167	175	164.333
Mean:	135.5	159.25	173.	156.

① Calculate row & column wise mean.

② Variance of row mean
= variance (141.33+173+145+164.33)

$S_R^2 = 231.5$.

Varian of column mean.

$S_c^2 = 6317.4$

Date:

(3) Mean sum of square for row

$MS_A = C \times S_i^2$

$MS_A = 3 \times 231.5$

694.5

Mean sum of square for column.

$MS_B = R \times S_i^2 = 4 \times 6317.4$

$MS_B = 25269.6.$

ANOVA TABLE

	D.f	S.S	M.S	F
(row) A	4-1=3	2083.5	694.5	16939.02
(column) B	3-1=2.	50539.2	25269.6	616331.7
	3x2=6	0.246	0.041	
	11	52622.9		

$SS(total) = 52622.9.$

$Var(Y) = 52622.9/11$

$Var(Y) = 4783.9$

Page No.

Teacher's Signature

Date

Variance of Row		Variance of Column	
Cotton	282.3	X	517.5
lawn	187	Y	432.25
Silk	1425	Z	148.6
Jaqvard.	149.3		

$F_{(tab)} = F_{(3,2,0.05)} = 19.164.$

$F_{(cal)}$ by row = 16939.02.

$F_{cal} > F_{tab}$

$16939.02 > 19.164$

$F_{(cal)}$ by column = 6.16331.7

$6.16331.7 > 19.164.$

Conclusion:-

Reject ~~Accept~~ null hypothesis.

Page No.

Teacher's Signature

System, Model & Simulation

System
Event
Activities
States
Types of system

System:-

System is a collection of entity for e.g. person & machine which interact whether to accomplaish a particular task.

~~A system is defined~~

In pratice what is meant by system? depend upon the objective of particular entity.

The collection of entities that surprise of system for study might be only a subset of the overall system for another.

For e.g. If one wants to study a bank to determine the number of tailor & needed to provide it just want to take chaque.

A system can define as a portion of bank consisting of tailors & coustomer waiting in line or being served.

OR

System is group of object that are joint together in sum regular interaction toward the accomplisment the same purpose.

randomness appears on the behalf on time (stochastic model)

System Environment:

A system is often effected by changing occuring outside the system for e.g bakery! environment arrival of orders

Components Of System:-

- **Entity:-**
  An entity is an object of interest of system.
- **Attribute:-**
  An attribute the property of entity.
- **Activity:-**
  Any purpose of causing chaning
  For e.g. Manufacturing process.

State of system:-

The state of system, collection of variable necessary describe a system & at any time, relative to the objective of study.
How does the system
It is also known as description of all the entities, attribute, In activity as they insist at one point of time.

Teacher's Signature

Event:-

An event as define instantaneous occurance that the change state of system.
Sometime is like entity.

Types Of System:-

Endogenous System:-

It is used to describe activities and event occurance, with in a system.
For e.g. joindrawingcash in a valuebank.

Exogenous System:-

It is used to describe activities & event in a environment that effect the system
For e.g. Arrival of customer.

Closed System:-

A system, there is no exogenous activity & event is said to be a closed system.
For e.g. water in a insulated ~~glass~~ flask.
that is mechanism how keep the water in a flask for a long time.

Teacher's Signature

Date:

Open System:-

A system for which, there is both exogenous & endogenous activity and event, is a open system.
For e.g. bank system. Customer arrival and bank perform's there internal duties & task's.
Find out the system & component.

1. System : Bank.
2. Entities : Customer
3. Attributes : Account
4. Activities : making deposit's with drawing cash
5. Event : Arrival, Departure
6. State : No. of customer waiting/ arrive.

Stochastic Model / Process:-

(Randomness with time).
It is Greek word means randomness, or chance, Stochastics analysis deals with model which involve uncertainities or randomness.
For e.g. Tossing of coin. (H, T)
A random variable is a rule ~~of~~ or function that assign a real number every outcome of random experiment. (Probability)
While a random process is a rule of function that assign is time function to every random experiment. (Stochastic)

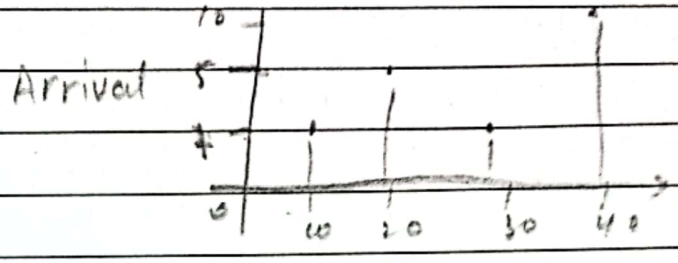
Date

Stochastic OR Random Process:-

It define as collection sequence of random variable, Mathmetically.

$X(t_n)$: $n = 1, 2, 3, \ldots$

Randomness in function of time



$X(t)$ stands for the observation of time t.

n : ~~no. of~~ ~~arrival~~. It is infinite depend on time range.

For e.g. Poisson Distribution.

$$P_n(t) = \frac{e^{-\lambda t} \cdot \lambda t^n}{n!} \quad \therefore n = 1, 2, 3 \ldots$$

represent stochastic OR random process infinite number of state. There the random variable "n" denotes the no. of occurance repeats the time "0" to "t",

Assuming the system starts zero" (0) time.

Does the state of system "t" are given by $n = 1, 2, 3, \ldots$

Teacher's Signature.