OPERATION RESEARCH:- The course deals use of mathematical model for corporate & govermental activities. Most of the planning problem consists of an economic objective which we want to maximize under scare resource OR consist of.

- Limiting & defining the correct problem.
- Formulating a mathematical.
- Calculating an optimized solution of problem
- Interpreting & Implement the find solution.
- Deterministic problem. (Mathematical problem)
- Statistic / Stochastic problem.
- Linear & non-linear programming.
- Integer programming.
- Network model.
- Simple queving theory +
- Simulation.

OR is also called quantities techniques of managment or quantities analysis of managment. It is probability nature.

APPLICATION OF O.R:-

Forecasting
Product scheduling
Inventary control managment
Budgeting
Transportation.

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Building block or core of O.R.
Statistics & probability theory.

Note:-
Advance spred sheet will be used to find numerical solution for some of analyzed problem.

Characteristics Of O.R:-

1. Applied scientific method to solve problem.
For e.g. certain algo, predefine steps used for reaching to an optimal research.

2. It is uncover or exhibit problem.
For e.g: while looking a problem finding a solution, one take find some new problem. for instance, while costumer spend more time comparitivly other, we will look at some resources are.
For e.g: for more shops are free.

3. It improve quality of decision biggest we used numerical method, analysis & quantitative method, Since we using numerical & quantitative method it we used computer extencively.

4. Since qualitative method, we have only qualitative solution, no quantitative solution.

5. It allows to take a real world problem & focus or put this into a mathematical model
For e.g: I have certain amount & I have to perform five task which required this amount in this month. Now it is art to pick some basic task more & more to be accomplished using a same amount in a month. That is how best I can optimized upon a required amount I have.

Q What is model?

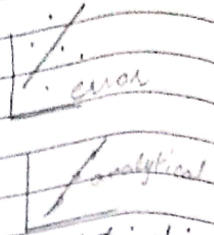
A The model implies to deals with math and statistic.

Mathmatical Model:- $Y = mx+b$

Statistics Model:- $Y = b_0 + b_1x_1 + \epsilon$

Basically the model (equation) is explain of real data.
For e.g.
Mathematical model: Integration $\int_{\alpha}^{L} f^x$.
Approximation method (Trapoziodal)
↓
Statistiscal

Here the denoted lines shows the distance (error) from the model of actual data.

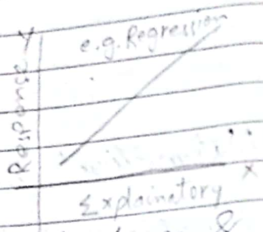


We used model to explore relationship
For e.g.
I might be interested relationship height & age of student.
We use statistic to determine how useful & how relaible our model is.

Types Of Statistics Model:-

1- **Continous Data:-**

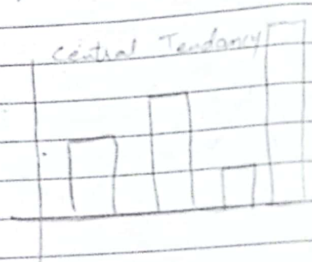
It depends on continous data.
e.g Regression.
Explainatory: how many unit change & effect on response.



Integer Data:-

It depends on discrete.

e.g. ANOVA.



Analysis Of Variance (ANOVA):-

The technique ANOVA originated by sir r.a.fisher. The analysis of method which seprate the variation refer able to one set of causes from the variation to other set.

For e.g: We are interested in testing the null hypothesis that the three variety of wheat produce equal yeild on the average.

$H_0: \mu_A = \mu_B = \mu_C$
$H_A: \mu_A \neq \mu_B \neq \mu_C$

We conduct experiment by planting different varities of wheat on plotting on land. there might be difference in the mean of various variety due to experimental error. also there might be variation due to experimental error + any variation due to the different variety of wheat.

The analysis of variance is a method of splitting the total variation of our data into constitutent which measure different sources of variation. The total variation is split up into the following meaningful component.

1. Variance with in the sub group of sample.

2. Variance between the subgroup of sample.

After this, the two variances are tested for their significant by the variace ratio or the f-test.

"Also analysts use the ANOVA test to determine the influence that independent variables have on the dependent variable in a regression study."

VARIANCE:-

The distance between the random variable to the mean. $(x-\bar{x})$

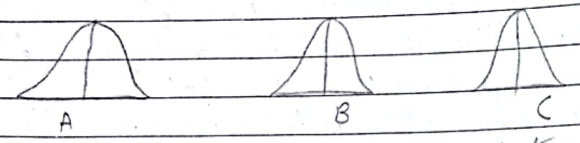
OR

It is define as the expectation of S.D of the random variable from its mean.

$\boxed{\bar{x}-\bar{x}}$ $(x-\bar{x})^2$

Basically, we do anova for comparision of more than two population or population having more than two sub group.

Consider the example.

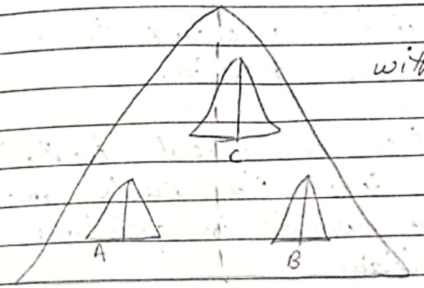


If we want to know all above three mean are coming from the same population than we can apply ANOVA test. because

$\therefore$ ANOVE $= \dfrac{\text{variability b/w the means}}{\text{variability within the distribution}}$

than we have total variance = variability b/w the means + variability within distribution.

than graphically concept says.



Assumption:-

Each population having normal dist.

The population from which the sample are drawn have the equal variance that is

$S_1^2 = S_2^2 = S_3^2 \ldots\ldots S_k^2$

Each sample is drawn randomly and they are independent.

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Hypothesis test for F-Distribution:-

$H_0: \mu_1 = \mu_2 = \mu_3 \ldots$

$H_A: \mu_1 \neq \mu_2 \neq \mu_3 \ldots$

We define ANOVA into two classes.

One way ANOVA (one factor)
Two way ANOVA (two factor).

Q1 It is classified according to one factor or one criteria.

Three varieties of wheat A, B, C sown in (4) four plot each and the following wheat in quintals per acre ~~one~~ were obtain

Plot	Varities		
	A	B	C
1	10	9	4
2	6	7	7
3	7	7	7
4	9	5	6
Total	32	28	24

Set of table of ANOVA and find out whether there is a significant of

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mean of yield of the three variety.

Solution:-

Step 1:-

$H_0: \bar{X}_A = \bar{X}_B = \bar{X}_c$

$H_A: \bar{X}_A \neq \bar{X}_B \neq \bar{X}_c$

Step 2:

Calculate the variance b/w the sample.

a) Calculate mean of each sample.

$\bar{X}_A = 32/4 = 8$

$\bar{X}_B = 28/4 = 7$

$\bar{X}_c = 24/4 = 6$

b) Calculate the grand mean.

$$\bar{\bar{X}} = \frac{\Sigma\bar{X}_A + \Sigma\bar{X}_B + \Sigma\bar{X}_c}{3} = \frac{32 + 28 + 24}{3}$$

or $\frac{\Sigma x}{N}$

$$\boxed{\bar{\bar{X}} = 7}$$

c) The difference b/w the mean of variance sample and grand mean. and square it

$\bar{x}_A - \bar{\bar{x}}$	$(\bar{x}_A - \bar{\bar{x}})^2$	$(\bar{x}_B - \bar{\bar{x}})$	$(\bar{x}_B - \bar{\bar{x}})^2$	$(\bar{x}_c - \bar{\bar{x}})$	$(\bar{x}_c - \bar{\bar{x}})^2$
8 − 7	1	7 − 7	0	6 − 7	−1
8 − 7	1	7 − 7	0	6 − 6	−1
8 − 7	1	7 − 7	0	6 − 7	−1
8 − 7	1	7 − 7	0	6 − 7	−1
	4		0		4

Minitab

Sum of square b/w samples

$$(\Sigma(\bar{x} - \bar{\bar{x}}))^2 = 4+0+4 = 8$$

or $n_1(\bar{x}_1 - \bar{\bar{x}})^2 + n_2(\bar{x}_2 - \bar{\bar{x}})^2 + n_3(\bar{x}_3 - \bar{\bar{x}})^2$

Step 3:-

Calculate the variance within the sample.

a) Calculate the mean for each sample.

b) Take the deviation from variance item in a sample from the mean value of respective sample & square it.

$(A-\bar{x}_A)$	$(A-\bar{x}_A)^2$	$(B-\bar{x}_B)$	$(B-\bar{x}_B)^2$	$(C-\bar{x}_C)$	$(C-\bar{x}_C)^2$
10 - 8	4	9 - 7	4	4 - 6	4
6 - 8	4	7 - 7	0	7 - 6	1
7 - 8	1	7 - 7	0	7 - 6	1
9 - 8	1	5 - 7	4	6 - 6	0
Total	10		8		6

Sum of square within the sample.

$$(\Sigma(x - \bar{x})^2) = 10+8+6 = 24$$

$$S.S.T = 8 + 24$$

$$= 32$$

Step 4:-

Calculate the value of F-ratio:-

Sum of variance	Sum of square	D.F	Mean sum of square	F-Test
Between the sample	SSC = 8	$\nu_1 = c-1$ $3-1=2$	$MSC = SSC/\nu_1$ $8/2 = 4$	$F = \frac{MSC}{MSE}$
Within the sample	SSE = 24	$\nu_2 = n-c$ $12-3=9$	$MSE = SSE/n-c$ $24/9 = 2.66$	$F = 4/2.66$ $F = 1.50$

M.S.E = error.

$F_{cal} = 1.50$, $F_{tab} = \cancel{8.0215}$ 4.26

Step 5:-

$F_{cal} < F_{tab}$

$1.50 < \cancel{8.0215}$ 4.26

Result:-

We accept null hypothesis and Hence, prove that there is a significant difference b/w the mean of varities of wheat.

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Replicates:- Rows in ANOVA table

Factors:- Column in ANOVA table. also called method.

Within OR Within the Column:- Difference between the rows

Between OR Between the Column:- Difference between the column.

SS(method):- Sum of square between the column. Also called "ANOVA test". Difference by the "mean"

column

SS(error):- Sum of square within the column.

row

Date.

Q A rifle club perform an experiment on a randomly selected group of first time shooter. The purpose of the exp was to determine whether shooting accuracy is affected by the method of sighting used: Only right eye open, only left eye open or both eyes open. 15 shooter were selected & split into three group. Each group experience the same training and practising procedures with one exception; the method of sighting used. After completing training, each student was given the same. $\therefore \alpha = 0.05$

Method Of Sighting

Right Eye	Left Eye	Both Eyes.
12	10	16
10	17	14
18	16	16
12	13	11
14		20
		21

Solution:-

Step 1:-

$H_0 : \mu_r = \mu_L = \mu_B$

$H_0 : \mu_r \neq \mu_L \neq \mu_B$

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Replicant (K)	Right Eye	Left Eye	Both Eyes
1	12	10	16
2	10	17	14
3	18	16	16
4	12	13	11
5	14		20
6			21
Total	C_r: 66	C_l: 56	C_b: 98

$\Sigma x = C_r + C_l + C_b = 66 + 56 + 98 = 220.$

$\Sigma x^2 = 12^2 + 10^2 + 16^2 + 10^2 + 17^2 + \dots = 3392$

Step 2:-

Sum Of Square Among /Between The Factors:-

$$SS(method) = \left(\frac{C_r^2}{K_1} + \frac{C_l^2}{K_2} + \frac{C_b^2}{K_3}\right) - \frac{(\Sigma x)^2}{N}$$

$$SS(method) = \left(\frac{66^2}{5} + \frac{56^2}{4} + \frac{98^2}{6}\right) - \frac{(220)^2}{15}$$

$$SS(method) = 3255.87 - 3226.67$$

$$SS(method) = 29.2.$$

Correlation factor: $C_x = \frac{(\Sigma x)^2}{N} = \frac{220}{15}$

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Step 3:-

Sum Of Square Within The Factors:-

$$SS(error) = \Sigma x^2 - \left(\frac{C_r^2}{K_1} + \frac{C_l^2}{K_2} + \frac{C_b^2}{K_3}\right)$$

$$SS(error) = 3392 - 3255.87$$

$$SS(error) = 136.13.$$

Step 4:-

Sum Of Square Total:-

$$SS(total) = \Sigma x^2 - \frac{(\Sigma x)^2}{N}$$

$$SS(total) = 3392 - \frac{(220)^2}{15}$$

$$SS(total) = 165.33.$$

Verification:-

$$SS(method) + SS(error) = SS(total)$$

$$29.20 + 136.13 = 165.33$$

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Step 5:-

MSSC:- Mean Sum Of Square Between the factor:-

$$MSSC = \frac{SS(method)}{c-1} = \frac{29.20}{2}$$

$\therefore df(method) = c-1$

$$MSSC = 14.6$$

Step 6:-

MSSE:- Mean Sum Of Square Within the factor:-

$$MSSE = \frac{SS(error)}{n-c} = \frac{136.13}{15-3}$$

$\therefore df(error) = n-c$.

$$MSSE = 11.33$$

Step 7:-

F-Ratio:-

$$F_{cal} = \frac{MSSC}{MSSE} = \frac{14.6}{1.22} = 1.288$$

Step 8:-

$$F_{tab} = F_{(2,12,0.05)} = 3.89$$

Step 9:-

$F_{cal} < F_{tab}$

$1.288 < 3.89$

Conclusion:-

Accept null hypothesis. There is a significant difference b/w the mean of method of sighting.

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Two Way Anova

Note:- It is classified according to two factors or two criteria.
It is with replication (rows works in it)

Given:-

Medical Field	Pain Killer 1	Pain Killer 2	Pain Killer 3	Total
	10	7	4	
Medicine	12	9	5	
	11	8	6	$R_1 = 98$
	9	12	5	
	12	13	6	
Surgery	13	15	6	$R_2 = 120$
	10	12	4	
	13	12	4	
Total	$P_1 = 90$	$P_2 = 88$	$P_3 = 40$	

Step 1:-

$H_0 : \mu_{P_1} = \mu_{P_2} = \mu_{P_3}$

$H_A : \mu_{P_1} \neq \mu_{P_2} \neq \mu_{P_3}$.

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Step 2:-

correction term $= \frac{(\Sigma x)^2}{N} = \frac{10^2+7^2+4^2 \ldots +4^2}{24}$

$C_x = \frac{47524}{24} = 1980.$

Sum Of Square Total:-

$SS(total) = \Sigma x^2 - \frac{(\Sigma x)^2}{N}$

$= 2254 - 1980$

$SST = 274$

SSC:-

For variation in painkiller programme.

$= \frac{C_1^2}{8} + \frac{C_2^2}{8} + \frac{C_3^2}{8} - \frac{(\Sigma x)^2}{N}$

$= \frac{90^2}{8} + \frac{88^2}{8} + \frac{40^2}{8} - 1980$

$SSC = 200$

SSR:-

For variation in medical field.

Sum of square rows.

$SSR = \left(\frac{R_1^2}{RN_1} + \frac{R_2^2}{RN_2}\right) - C_x$

$= \left(\frac{98^2}{12} + \frac{120^2}{12}\right) - 1980$

$SSR = 20$

SSG:-

Sum of square of groups SSG

The interaction between rows and column

$SSG = \frac{\Sigma\Sigma (PM)^2}{r} = C_x - SSC - SSR$

$SSG = \left[\frac{(10+12+11+9)^2}{4} + \frac{(7+9+8+12)^2}{4} + \frac{(4+5+6+\ldots}{4}\right.$

$\frac{(12+13+10+13)^2}{4} + \frac{(13+15+12+12)^2}{4} + \frac{(6+6+4+\ldots}{4}$

$- \cancel{1980} - 200 - 20$

$SSG = 17$

1541

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SSE:-

Residual sum of square.

$SSE = \Sigma x^2 - (\frac{c_1}{4} + \dots + \frac{c_4}{4})$

$SSE = SST - SSC - SSR - SSG$

$SSE = 274 - 200 - 20 - 17$

$SSE = 37$

Step 3:-

	D.F	F-ratio $F = MSSA/MSE$	F-criteria $\alpha = 0.05$
Pain Killer	$c-1=2$	48.73	$F_{(2,18)} = 3.55$
Medical Field	$R-1$ $2-1=1$	9.81	$F_{(1,18)} = 4.41$
Interaction	$(c-1)\times(R-1)$ $2\times1=2$	3.97	$F_{(2,18)} = 3.55$
Residual	$C\cdot R\times(n-1)$ $3\cdot2\times(4-1)=18$		
Total	$N-1=24-1=23$		

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Assignment

Date

For the following data, find SS(error) & show that $SSE = [(K-1)S_1^2 + (K-1)S_2^2 + (K-1)S_3^2]$ where S_i^2 is variance for factor level.

Factors level

	8	6	10
	4	6	12
	2	4	14
$\Sigma x =$	14	16	36

$\Sigma x^2 = 8^2 + 4^2 + \dots + 14^2 = 612$

$C = 14 + 16 + 36 = 66$

$SSE = 612 - \left(\frac{14^2}{3} + \frac{16^2}{3} + \frac{36^2}{3}\right)$

$612 - 582.66$

$\boxed{SSE = 29.33}$

$SS(\text{method}) = 98.666$

$SST = 127.99$

$\because K$ is replicates

$\because S$ is variance of each column

$K = 3$

$[(K-1)S_1^2 + (K-1)S_2^2 + (K-1)S_3^2]$

$2(3.055)^2 + 2(1.154)^2 + 2(2)^2$

$\boxed{= 29.32}$

$L.H.S = R.H.S$

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Anova (Without Replication)

Potatoes

Location	A	B	C	Mean
1	18	13	12	14.33
2	20	23	21	21.33
3	14	12	9	11.66
4	11	17	10	12.66
Mean	15.75	16.25	13	$\boxed{15}$

① Calculate Raw & column wise mean.

② Variance of raw mean S_R^2

$= \text{variance}(14.33 + 21.33 + 11.66 + 12.66)$

$S_R^2 = 19.051$

④ Variance of column mean S_c^2.

$= \text{variance}(15.75 + 16.25 + 13)$

$S_c^2 = 3.0625.$

③ Mean sum of square for Row.

$MS_A = C \times S_R^2$

Mean sum of square for column

$MS_B = R\, S_c^2$

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Here,

C = no. of column = 3

R = no. of rows = 4

Now,

$MS_A = C * S_R^2 = 3 \times 19.051$

$MS_A = 57.153$

$MS_B = R * S_c^2 = 4 \times 3.0625$

$MS_B = 12.25.$

ANOVA TABLE

Sample	D.f	S.S	M.S	F
(row) A	4-1=3	57.1113 171.33	57.111	$8.12 = 57.11/7.03$
(column) B	3-1=2	24.5	12.25	$1.72 = 12.25/7.03$
Residual	R.C = 3x2 = 6	42.17	$7.03 = \frac{171.3}{24.5}$	
Total	11	238		

$MS \times D.f = S.S.$

$Var(Y) = 238/11 = SS_{total}/n-1$

$SS_{total} = 238$

$Var(Y) = 21.636.$

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$F_{cal} = 8.12$ $\quad F_{tab} = F_{(3,2,0.05)}$

$F_{cal} = 1.72$

F(tabulated):

$F_{(3,2,0.05)} = 19.164$

$F_{(cal)}$ by row $= 8.12$

$F_{cal} < F_{(tab)}$

$8.12 < 19.164$.

$F_{(cal)}$ by column $= 1.72$.

$1.72 < 19.164$

Conclusion:-

Accept null hypothesis.

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Q A new brand increase the manufacturing of different types of fabric & makers of the brands want to better understand which of three formulation of brand are most effective for cotton, lawn, silk, jaquard fabric. It has 3 blend of one sample on each fabric. of 4 types of fabric. The fabric has 12 formulation as follows.

Fabric	X	Y	Z	Mean
Cotton	123	145	156	141.333
Lawn	158	185	176	173
Silk	110	140	185	145
Jaquard	151	167	175	164.333
Mean	135.5	159.25	173.	$\boxed{156}$

① Calculate row & column wise mean.

② Variance of row mean

= variance (141.33+173+145+164.33)

$S_R^2 = 231.5$.

Varian of column mean.

$S_C^2 = 6317.4$

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(3) Mean sum of square for row

$MS_A = C \times S_r^2$

$MS_A = 3 \times 231.5$

694.5

Mean sum of square for column.

$MS_B = R \times S_c^2 = 4 \times 6317.4$

$MS_B = 25269.6.$

ANOVA TABLE

	D.f	S.S	M.S	F
(row) A	4-1=3	2083.5	694.5	16939.02
(column) B	3-1=2	50539.2	25269.6	616331.7
	3×2=6	0.246	0.041	
	11	52622.9		

$SS_{(total)} = 52622.9.$

$Var(Y) = 52622.9/11$

$Var(Y) = 4783.9$

Variance of Row

Cotton	282.3
lawn	187
Silk	1425
Jaquard.	149.3

Variance of Column

X	517.5
Y	432.25
Z	148.6

$F_{(tab)} = F_{(3, 2, 0.05)} = 19.164.$

$F_{(cal)}$ by row = 16939.02.

$F_{cal} > F_{tab}$

$16939.02 > 19.164$

$F_{(cal)}$ by column = 6.16331.7

6.16331.7 > 19.164.

Conclusion:

Reject ~~Accept~~ null hypothesis.

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Fabric	x	y	z	Total.
Cotton	123	145	156	424
Lawn	135	165	176	479
Silk	110	140	185	435
Jaquard	151	167	175	493
Total	522	617	692	1831

Solution!

$H_0 = \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_A = \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$

$H_0' = \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_A' = \mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$

$\Sigma x^2_{ij} = (123)^2 + \ldots\ldots + (175)^2$

$\Sigma x^2_{ij} = 284975.$

Correction factor $= \frac{(\Sigma m)^2}{N} = \frac{(1931)^2}{12}$.

$Cx = 279380$.

SST:-

$SST = \Sigma x^2 - Cx = 284975 - 279380$

$SST = 5595$.

SSC:-

$$SSC = \left(\frac{C_1^2}{n_1} + \frac{C_2^2}{n_2} + \frac{C_3^2}{n_3}\right) - Cx.$$

$$SSC = \left\{\frac{(522)^2}{4} + \frac{(617)^2}{4} + \frac{(692)^2}{4}\right\} - 279380$$

$SSC = 3629.25$.

SSR:-

$$SSR = \left\{\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} + \frac{R_4^2}{n_4}\right\} - Cx$$

$$SSR = \left\{\frac{(424)^2}{3} + \frac{(479)^2}{3} + \frac{(435)^2}{3} + \frac{(493)^2}{3}\right\} - Cx$$

$SSR = 1117$

SSE:-

$SSE = SST - SSC - SSR$

$SSE = 5595 - 3629.25 - 1117$

$SSE = 848.75$.

Degree Of Freedom:-

D.F (Total) $= N-1 = 12-1 = 11$

D.F (Blend b/w Column) $= C-1 = 3-1 = 2$

D.F (fabric b/w Row) $= R-1 = 4-1 = 3$

D.F (Error) $= (C-1)(R-1) = (2)(3) = 6$.

Source	D.F	SS	M.S = SS/D.F	F-ratio MSSx/MSSe
Fabric	3	1117	372.3	2.632
Blend	2	3629.25	1814.6	12.832
Error	6	848.75	141.45	
Total	11	5595		

$F_{(tab)} = F_{(0.05, 3, 6)} = 4.76$

$F_{cal} < F_{tab}$

$2.631 < 4.76$

$F_{(tab)} = F_{(0.05, 2, 6)} = 5.14$

$F_{cal} > F_{tab}$

$12.8 > 5.14$

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Conclusion:

H_0 is accepted

H_1 is rejected

SYSTEM, MODEL & SIMULATION

System ∴ randomness appears
Event — acc on the behalf on
Activities — time (stochastic model)
Status
Types of system

System:-

System is a collection of entity for e.g. person & machine which interact whether to accomplaish a particular task. ~~A system is define~~

In practice what is meant by system? depend upon the objective of particular entity.

The collection of entities that surprise of system for study might be only a subset of the overall system for another.

For e.g. If one wants to study a bank to determine the number of tailor needed to provide it just want to take chaque. A system can define as a portion of bank consisting of tailors & coustomer waiting in line or being served.

OR

System is group of object that are joint together in sum regular interaction toward the accomplisment the same purpose.

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System Enviroment:-

A system is often effected by chaning occuring outside the system
For e.g Factory! environment arrival of orders

Components Of System:-

1- **Entity:-** An entity is an object of interest of system.

2- **Attribute:-** An attribute the property of entity.

3- **Activity:-** Any purpose of causing chaning
For e.g. Manufacturing process.

State of system:-

The state of system, collection of variable necessary describe a system & at any line, relative to the objective of study.
How does the system
It is also known as description of all the entities, attribute, In activity as they insist at one point at time.

Event:-

An event as define instantanious occurance that the change state of system.
Sometime is like entity.

Types Of System:-

Endogenous System:-

It is used to describe activities and event occurance with in a system.
For e.g. ~~join~~ drawing cash in a ~~water~~ bank.

Exogenous System:-

It is used to describe activities & event in a enviroment that effect the system
For e.g. Arrival of customer.

Closed System:-

A system, there is no exogenous activity & event is said to be a closed system.
For e.g. water in a insulated ~~glass~~ flask that is mechanism how keep the water in a flask for a long time.

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Open System:-

A system for which, there is both exogenous & endogenous activity and event, is a open system.
For e.g. bank system. Customer arrival and bank perform's there internal duties & tasks.
Find out the system & component.

1. System : Bank
2. Entities : Customer
3. Attributes : Account
4. Activities : making deposit's with drawing cash
5. Event : Arrival, Departure
6. State : No. of customer waiting, arrive.

Stochastic Model / Process:-

(Randomness with time).
It is Greek word means randomness, or chance. Stochastics analysis deals with model which involve uncertainities or randomness.
For e.g. Tossing of coin. (H, T)
A random variable is a rule ~~of~~ or function that assign a real number every outcome of random experimant. (Probability)
While a random process is a rule of function that assign is time function to every random experimint. (Stochastic)

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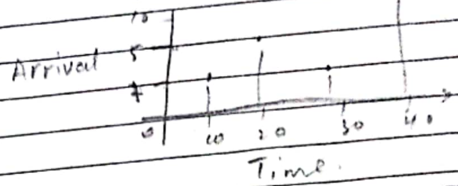
Stochastic OR Random Process:-

It define as collection sequence of random variable.
Mathmetically, $X(t_n)$: n = 1, 2, 3, ...
Randomness in function of time



$X(t)$ stands for the observation of time t.
n : ~~no. of~~ s ~~arrival~~. It is infinite depend on time range.

For e.g. Poisson Distribution.

$$P_n(t) = \frac{e^{-\lambda t} \cdot \lambda t^n}{n!} \quad ; n = 1, 2, 3 \ldots$$

represent stochastic OR random process infinite number of state. There the random variable "n" denotes the no. of occurance repeats the time "0" to "t",

Assuming the system starts zero" (0) time.
Does the state of system "t" are given by n = 1, 2, 3, ...

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Queuing Model Of Simulation:-

Types Of Queueing Model

$M/M/1$ (∞ - FCFS)
A D S

Markavian Server = S = 1
Arrival = A = Poisson Distibution
Inter arrival = D = Exponential Distribution.
Infinite Object (∞).
Service Nature = FCFS

Markavian Nature:

Discipline within the mathematical theory of probability, a Markavian Arrival Process (MAP) is a mathematical model time b/w job arrival to the system.
The simplest research process is a poisson process where time b/w each arrival is exponential distribution.

① λ = mean arrival rate (# of arrivals per unit of time)

② μ = mean service rate per busy server (no of customers served per unit of time)

③ $P = \lambda / \mu$

④ L_q = Expected avg no of customers in queue: $L_q = L_s * \frac{\lambda}{\mu} = \frac{\lambda}{\mu} \cdot \frac{\lambda}{\mu - \lambda}$

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(5) L_s = Expected avg no of customers in the system: $L_s = \frac{\lambda/\mu}{1-\lambda/\mu} = \frac{\lambda}{\mu-\lambda}$

(6) W_s = Expected waiting time a customer spends in the system: $W_s = \frac{L_s}{\lambda} = \frac{\lambda}{\lambda(\mu-\lambda)} = \frac{1}{\mu-\lambda}$

(7) W_q = Expected waiting time per customer in the queue: $W_q = W_s - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}$

(8) L_n = Avg length of non empty queue

$L_n = \frac{\mu}{\mu-\lambda}$

(9) W_n = Avg. waiting of non empty queue

$W_n = \frac{1}{\mu-\lambda}$

Date:

Q.11 Self service library employee one librarian at its counter. 9 students arrive on an average every 5 minutes. Assuming poisson distribution for arrival rate & exponential distribution for service rate & inter arrival time. Find out the following.

1) Avg. no of students in the system.
2) Avg. no of students in the queue.
3) Avg. time students spends in the system.
4) Avg. time students waits before being served.

Solution:-

① $\lambda = \frac{9}{5} = 1.8$ student/min

② $\mu = \frac{10}{5} = 2$ student/min

③ i) $L_s = \frac{\lambda}{\mu-\lambda} = \frac{1.8}{2-1.8} = 9$ Students

④ ii) $L_q = \frac{\lambda}{\mu} \cdot \frac{\lambda}{\mu-\lambda} = \frac{\lambda}{\mu} \cdot L_s = \frac{1.8}{2}(9) = 8.1$ Students

⑤ iii) $W_s = \frac{1}{\mu-\lambda} = \frac{1}{2-1.8} = 5$ minutes

⑥ iv) $W_q = \frac{\lambda}{\mu} \cdot \frac{1}{\mu-\lambda} = \frac{\lambda}{\mu} \cdot W_s = \frac{1.8}{2} \times 5 = 4.5$ min

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Note:-
When the arrival time and service time are given in the form of their means then
$\lambda = 1/$ expected arrival time and
$\mu = 1/$ mean service time

Q1: A mechanic repairing car engines. Find that the time spent on repairing the engine has exponential distribution with mean 20 min. If the engines are repaired in the order in which they come in and their arrival is approximately poisson with an avg. rate of 15 for 8-hours day. What is the mechanic's expected idle time each day? How many jobs are a head of the average engine just brought in?

Solution:

① $\lambda = \frac{15}{8\times60} = 0.0313$

② $\mu = \frac{1}{20} = 0.05$

③ $L_s = \frac{\lambda}{\mu-\lambda} = \frac{0.0313}{0.05-0.0313} = 1.67$

For 8-hours

④ $\rho = \frac{8\times\lambda}{\mu} = \frac{8\times0.0313}{0.05} = 5$

⑤ Idle $= 8-\rho = 8-5 = 3$

Ans

Date:

Q3: Consider the following single server queue, the inter arrival time is exponentially distributed with a mean of 10 minutes & the service time is also exponentially distributed with a mean of 8 minutes. Find:
① Mean wait in the queue
② Mean number in the queue
③ Mean wait in the system
④ Mean number in the system
⑤ Proportion of time the server is idle.

Solution:-

① $\lambda = \frac{1}{10} = 0.1$ customer/min

② $\mu = \frac{1}{8} = 0.125$ customer/min

③ $\rho = \frac{\lambda}{\mu} = \frac{0.1}{0.125} = 0.8\%$

④ $L_s = \frac{\lambda}{\mu-\lambda} = \frac{0.1}{0.125-0.1} = 4$ customers

⑤ $L_q = \rho . L_s = (0.8)(4) = 3.2$ customers.

⑥ $W_s = 1/\mu-\lambda = 1/0.125-0.1 = 40$ min

⑦ $W_q = \rho . W_s = 0.8\times40 = 32$ min

⑧ Idle $= 1-\rho = 1-0.8 = 0.2\%$

Ans

M/G/1 (∞ - FCFS)

Q Consider the following single server queue. The interarrival time is exponentially distributed with a mean of 10 min and the service time has the uniform distribution with a maximum of 9 min & a minimum of 7 min. Find the following.

1. Mean wait in the queue
2. Mean number in the queue
3. Mean wait in the system
4. Mean number in the system
5. Proportion of time the server is idle.

Solution:-

① $\lambda = \frac{1}{10} = 0.1$ customer/min

② $\mu = \frac{1}{\frac{a+b}{2}} = \frac{1}{\frac{7+9}{2}} = \frac{1}{\frac{16}{2}} = \frac{1}{8} = 0.125$ customer/min

③ $\sigma^2 = \frac{(b-a)^2}{12} = \frac{(9-7)^2}{12} = \frac{4}{12} = \frac{1}{3} = 0.33$

④ $\rho = \frac{\lambda}{\mu} = \frac{0.1}{0.125} = 0.8$

⑤ $L_q = \frac{\lambda^2\sigma^2 + \rho^2}{2(1+\rho)} = \frac{(0.1)^2(0.33) + (0.8)^2}{2(1+0.8)} = 1.608$ Customers

⑥ $W_q = \frac{L_q}{\lambda} = \frac{1.608}{0.1} = 16.08$ min

⑦ $W_s = W_q + \frac{1}{\mu} = 16.08 + \frac{1}{0.125} = 24.08$ min

⑧ $L_s = \lambda . W_s = (0.1)(24.08) = 2.408$ customers

⑨ Idle $= 1 - \rho = 1 - 0.8 = 0.2$ min

Ans.

Simulation Of Random Number By Poisson Process.

$\lambda = 2.65$ (arrival mean).

S.No.	Commulative $(e^{-\lambda} \cdot \lambda^x) \div x!$	C. lookup	Min b/w arrival (x)	Interarrival R.N (IAT)	Poisson Arrival (AT)
1	0.0706	0	0	0	0
2	0.2578	0.0706	1	4	4
3	0.5059	0.2578	2	3	7
4	0.7250	0.5059	3	6	13
5	0.8702	0.7250	4	0	13
6	0.9472	0.8702	5	4	17
7	0.98118	0.9472	6	0	17
8	0.9940	0.98118	7	0	17
9	0.9983	0.9940	8	5	22
10	0.9995	0.9983	9	3	25
				$\Sigma = 25$	$\Sigma = 135$

$\theta = 2.45$

S.No.	Service time (ST) $(-\theta * \ln(Rand))$ Round of	Time Service Begin (TSB) TSE ≥ AT or AT > TSE	Time Service End (TSE) {ST + TSB}	Cust wait time (W_q) {TSB − AT}	Cust time in system (SS) {TSE − AT}	Idle time Server {AT − TSE}
1	2	0 always	2	0 always	2	0 always
2	4	4	8	0	4	2
3	7	8	15	1	8	0
4	0	15	15	2	2	0
5	1	15	16	2	3	0
6	1	17	18	0	1	1
7	0	18	18	1	1	0
8	0	18	18	1	1	0
9	1	22	23	0	1	4
10	3	25	28	0	3	2
	$\Sigma = 19$			$\Sigma = 7$	$\Sigma = 26$	$\Sigma = 9$

Output Statistics:-

$\therefore t = \text{total}$

$\therefore t = 10$

$W_q = \frac{W_t}{t} = \frac{07}{10} = 0.7$

Probability of custome waiting $= 5/10 = 0.5$

Average service time $= \bar{M} = St/t = 19/10 = 1.9$

Time B/w arrival $= \bar{\lambda} = IAT/t = 25/09 = 2.7$

Waiting time customer spend in system $= W_s = SS/t = 26/10 = 2.6$

Probability of Idle time $= \frac{\text{Idle}}{\text{TSE(last)}} = \frac{9}{28} = 0.32$

Percentage Of Utilization:-

$\frac{\text{Total idle time}}{\text{Total busy time(SS)}} \times 100 \Rightarrow \frac{9}{26} \times 100$

utilization = 34.6 %

Queuing System

Date:

G mean 2 parameter dist

G/G/1 :-

M / M / 1

Inter arrival (expo) service time (expo)

G / G / 1 System :-

gamma normal

Consider the following single A / B / 1 serve Q, IAT has gamma distribution with the mean of 10 min, variance of 20 min square. The service has normal dist 8 min & vari of 25 min square.

InterA (anyDist) service (Any Dis

M / G / 1

M / M / C

(expo) (expo) multi serv

Find:

i) Mean wait in queue (W_q)
ii) Mean number in queue (L_q)
iii) Mean wait in system (W_s)
iv) Mean number in system (L)
v) Idle time ($1-\rho$).

Solution :-

when IAT mean $\lambda = 1/\text{mean}$.

$\lambda = 1/10 = 0.1$

$\sigma_a^2 = 20 =$

Lq change when dist has 2 param

$\mu = 1/8 = 0.125$

$\sigma_s^2 = 25$

Lq: error

$\rho \cancel{\phi} = \lambda/\mu = 4/5 = 0.8.$

Error estimation in Lq in paper
ready made value = Observe
simulate = True value
efficient = lo error.

Date:

Date:

C_a^2 = Square coefficient variation of interarrival.

C_s^2 = Square coefficient variation of service time

$$C_a^2 = \frac{\sigma_a^2}{(1/\lambda)^2} \quad ; \quad C_s^2 = \frac{\sigma_s^2}{(1/\mu)^2}$$

$$C_a^2 = \frac{20}{(0.1)^2} = 0.2\cancel{000}$$

$$C_s^2 = \frac{25}{(0.125)^2} = \cancel{200}\ \cancel{1600}\ 0.3125$$

For G/G/1 (Marchel approx)

$$L_q \approx \frac{\rho^2 (1+C_s^2)(C_a^2 + \rho^2 C_s^2)}{2 \cdot (1-\rho)(1+\rho^2 C_s^2)}$$

$$L_q \approx \frac{(0.8)^2 [(1+\overset{0.3125}{\cancel{1600}})(\overset{0.2}{\cancel{2000}}) + (0.8^2)(\overset{0.3125}{\cancel{1600}})]}{2(1-0.8)(1+0.8^2 \times \underset{0.3125}{\cancel{1600}})}$$

$L_q \approx 0.801$ approx to original

In the above, mean rate arrival is λ
& σ_s^2

$$W_q = \frac{L_q}{\lambda} = \frac{0.801}{0.1} = 8.01 \text{ approx}$$

which is the simulation estimate

Wait in queue is 8 min.

Mean wait in System :-

$$W_s = \frac{1}{\mu - \lambda} = 40 \text{ min}$$

Mean Number in System

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{0.1}{0.125 - 0.1} = 4 \text{ min}$$

Idle Time :-

$(1-\rho) = 1 - 0.8$

0.2 min.
0.2×100

20% idle.

Date:

Multi Server Queues:-

We will only consider the identical (homogenous) server case in which there are 'C' (identical server in parallel). & there is just one waiting line.
i.e. The queue is a single ~~server~~ channel
Here, the no. of identical server is represent by the letter C. So, $\rho = \frac{\lambda}{C\mu}$

For M/M/C queue,

$$L_q = \frac{P_0 (\lambda/\mu)^c \rho}{C!(1-\rho)^2} \quad -(1)$$

We have,

$$P_0 = \frac{1}{\sum_{m=0}^{c-1} \left[\frac{(C\rho)^m}{m!} + \frac{(C\rho)^c}{C!(1-\rho)} \right]} \quad -(2)$$

when P_0 represent zero customer in the system. Hence W_q can be obtain as $W_q = L_q/\lambda$

Then for G/G/C queue, the following approx is used

G/G/C M/M/C

$$W_q \approx W_q * \frac{C_a^2 + C_s^2}{2} \quad -(3)$$

Date:

When W_q M/M/C denotes the waiting time in queue for the A/B/C queue. The above equation -(3) works well M/G/C queue.

Q Consider the following senerio the IAT has an exponential dist with a mean of 10 min. There are 2 server, & the service time of each server has the uniform dist. with the maximum of 20 & minimum of 10 minutes.

Find the following:-

i) Mean wait in queue (W_q)
ii) Mean number in queue (L_q).
iii) Mean wait time in system (W_s)
iv) Mean number time in system (L_s).
v) Proportion time senerio is idle

While using Discrete event simulation on the same dataset, it is OBSERVE shown that the mean waiting time in queue is 9.5693 minute. Also compute the error in G/G/C approx.

Note:-
The above problem is based on M/G/1 queueing system.
We have $\lambda = 1/10 = 0.1$
where as $C_a^2 = 1$ that is $\sigma_a^2 = 100$

Date:

- The mean service time will be

$\frac{max+min}{2} = \frac{10+20}{2} = 15$, so, $\mu = \frac{1}{15}$

The variance of service time be

$\frac{(max-min)^2}{12} = \frac{(20-10)^2}{12} = 8.33$ so, $\sigma_s^2 = 8.33$

Also $\rho = \frac{\lambda}{c\mu} = \frac{1/10}{2 \times \frac{1}{15}} = 0.75$

$C_s^2 = \frac{\sigma_s^2}{(1/\mu)^2} = \frac{8.33}{(1/\frac{1}{15})^2} = 0.03$

Compute the error G/G/C approx, first assume the queue to be M/M/C queue & compute its Lq. We known the Lq is multiserver queue is

$$L_q = \frac{P_o (\lambda/\mu)^c \rho}{c!(1-\rho)^2}$$

we need to calculate P_o by eq (2).

$P_o = 0.1453$

Then $L_q = \frac{0.1453 (\frac{1/10}{1/15})^2 * 0.75}{2!(1-0.75)^2}$

$L_q = 1.961$

Date:

Then, $W_q = \frac{L_q}{\lambda} = \frac{1.961}{1/10}$

$W_q = 19.61$

Now, we can transform to a G/G/2 queue using approx in eq (3)

$$W_q^{G/G/c} \approx W_q^{M/M/c} * \frac{C_a^2 + C_s^2}{2}$$

$$W_q^{G/G/c} \approx (19.61) \frac{(1+0.03)}{2}$$

$W_q^{G/G/c} = 10.09$ min

Then,

$L_q^{G/G/c} = W_q^{G/G/c} * \lambda$

$= 10.09 \times 1/10$

$L_q = 1.009$

Error Approximation:-

$$|E_a| = \left| \frac{10.09 - 9.5693}{10.09} \right| \times 100$$

$|E_a| = 5.16\%$

Wait in System (W):-

$W = W_q + 1/\mu = 10.09 + 1/\frac{1}{15}$

$W = 25.09$ min.

Number In System:-

$L = \lambda W \Rightarrow (1/10)(25.09)$

$L = 2.509$.