Date [illegible] March-2022

OR (OPERATION RESEARCH):-

It is also known as quantitative Analysis for management or quantitive methodology for management.

Always quantitative numerical not qualitative

APPLICATIONS OF OR:-

- Any sort of forecasting
- Scheduling Production
- Inventory control management
- Transportation

BUILDING BLOCKS OF OR:-

- Statistics
- Probability Theory

[Q] Is OR an ART or a Science?

[A] OR is both art and science but respective meaning in both fields.

- According to T.L. Saaty "OR is an art of giving bad answers to problems which otherwise have have worse answers".
- According to P.M. Moorse and G.E. Kimbal: "OR is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control".

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CHARACTERISTICS OF OR:

- Apply scientific method to problem solving.

For example: certain algorithms,
Predefined steps used for reaching to an optimal solution.

- It uncovers or exhibit new problems.

For example: While looking at a problem and finding the solution, one can find some new problem for instance, while customer spends more time comparitively in our shopping mart, we may look at some resources are being wasted. (For example: few more point of sales (POS.) are free in 2 shifts) while the main problem was goods were not organized. in the mart not placed nicely.
So, we can realized that upon other problem we can optimized.

- It improves quality of decision because we use numerical methods, numerical algorithms and quantitive methods. Since, we are using numerical methods and quantitat methods, then it will use ~~co~~ computers extensively.

OPERATIONS RESEARCH:

It allows us to take up or deals with real world problem or put it into a mathematical problem.
For example:
I have certain amount and I have to perform 5 to 6 tasks which require these amount within a month. Now, this is an art to pick up some basic tasks more and more to accomplish in a month using the same ~~month~~ amount / funds. That is how best I can optimize upon the amount I have.

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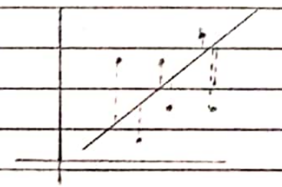
Q. What is Model?

A. The model implies with that deals with mathemical and statistics.

- Mathematical Model: $y = mx + b$
- Statistical Model: $y = b_0 + b_1x + E$

Basically, the model (equation) is an approximation of real data.

For example:
Mathematical Method: $J = \int^{b} f(x)\, dx$
Approximation Method: Trapezoidal Rule.

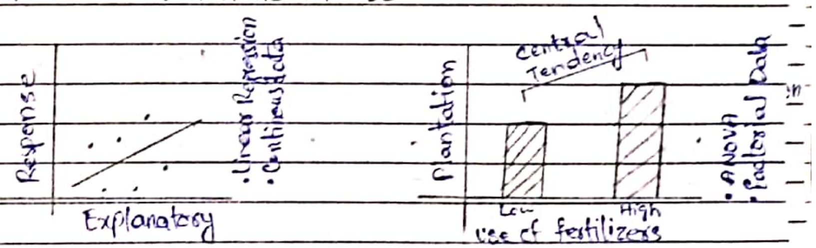


Here dotted lines show the distance (error) to the actual points.

① We use model to explore relationships.
for eg.: I might be interested in relationship between hight and age of students.

② We use statistics to determine how useful and how reliable our model is.

TYPES OF STATISTICAL MODEL:



Regression Analysis: Relationship b/w two quantoitative data is called Regression Analysis.

ANOVA: To identify the variance b/w the sample data.

ANOVA :-

Analysis of variance (ANOVA) by sir RA Fisher. "The ~~and~~ analysis of variance which separate the ~~variable~~ variance to one set of causes from the varieties ~~of~~ to other sets.
For eg. We are intersted in testing the null hypothesis that the three variety of wheats produce equal yields on the average.

$H_0 : \mu_A = \mu_B = \mu_C$
$H_a : \mu_A \neq \mu_B \neq \mu_C$

We conduct experiments by planting diffrent varieties of wheats on plots of land, there might be difference. in the means of various varieties due to experiment errors. Also these might be variation due to experimental error + any variation due to the different varieties of wheat. The ANOVA is a method of splitting the total variance of our data into one constituent parts which measures different sources of variations.

The total variation is split up into the following meaningful component.
1- Variance within the sub-group of variance
2- Variance b/w the sub-group of variance.

Variance: Difference of Random variable (x) from its mean ($\bar{x}$)"
[Variance = $x - \bar{x}$]

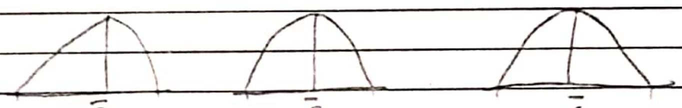
after this, the two variances are tested for their significance by the variance ratio or the F-test.

"Also analysts use the ANOVA test to determine the influence that independent variables have on the dependent variables in a regression strategy."

Variation: The variance reflects the variability of the given data set by the taking the average of squared deviation from its mean.

Basically, we can use ANOVA for compare more than two populations or population having more than two samples.

Consider the given figure



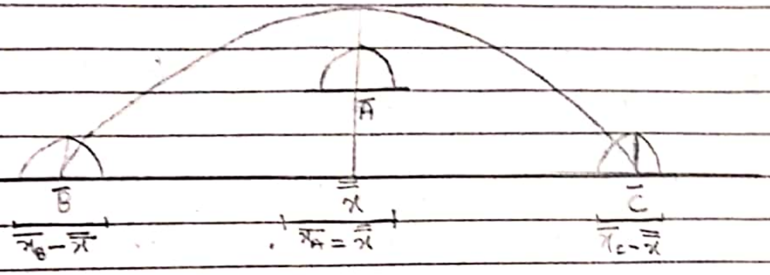
$\bar{A}$ $\bar{B}$ $\bar{C}$

If we moved to known the above three means are comming from the same population when apply ANOVA test?

Because, ANOVA = $\dfrac{\text{Variability b/w the means}}{\text{variability within the ~~distrib~~ distribution}}$

then we have total variation as equal to
$= \dfrac{\text{variability b/w the means}}{\text{variability within the distribution}}$

then graphically we can show:



ASSUMPTIONS:

-1- Each population is having distribution

-2- The population from which samples are drawn have the equal variance. that is $S_1^2 = S_2^2 = S_3^2 = \dots = S_K^2$ for k samples.

-3- Each sample is drawn randomly and they are independent

HYPOTHESIS TEST FOR ANOVA

We define ANOVA technique in two classes:

1) One way ANOVA
2) Two way ANOVA

ASSIGNMENT #01

"It is classified according to only one factor."

Q. There are three varieties of wheats ABC shown in 4 plots. Each of the following yields in quintles per acre were obtained.
Setup a table of ANOVA and find out whether there is a significance between the yields of the three varieties.

PLOTS	SAMPLES		
	A	B	C
1	10	9	4
2	6	7	7
3	7	7	7
4	9	5	6
TOTAL	32	28	24

Solution:-

Step ①:- State the null hypothesis

$H_0: \bar{x}_A = \bar{x}_B = \bar{x}_C$

$H_A: \bar{x}_A \neq \bar{x}_B \neq \bar{x}_C$

Step ②:- Calculate the variance b/w the samples

(a) Calculate the means of each sample

$\bar{x}_A = \frac{32}{4} = 8, \quad \bar{x}_B = \frac{28}{4} = 7, \quad \bar{x}_C = \frac{24}{4} = 6$

(b) Calculate the grand mean:

$$\bar{\bar{x}} = \frac{\bar{x}_A + \bar{x}_B + \bar{x}_C}{3} = \frac{21}{3} = 7$$

$\boxed{\bar{\bar{x}} = 7}$

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(c) Take the difference b/w the means of various sample and ground mean:

$\bar{x}_A - \bar{\bar{x}}$	$(\bar{x}_A - \bar{\bar{x}})^2$	$\bar{x}_B - \bar{\bar{x}}$	$(\bar{x}_B - \bar{\bar{x}})^2$	$\bar{x}_C - \bar{\bar{x}}$	$(\bar{x}_C - \bar{\bar{x}})^2$
8-7=1	$(1)^2 = 1$	7-7=0	0	6-7=-1	1
8-7=1	1	7-7=0	0	6-7=-1	1
8-7=1	1	7-7=0	0	6-7=-1	1
8-7=1	1	7-7=0	0	6-7=-1	1
	4		0		4

Sume of squares = $\Sigma(\bar{x} - \bar{\bar{x}}) = 4 + 0 + 4 = 8$

Step 3:- Calculate the variance within the sample

(a) Calculate the mean of each sample

already did in step 2(a)

(b) Take the deviations from various items in a sample from the mean values of the respective sample and square it.

$A - \bar{x}_A$	$(A - \bar{x}_A)^2$	$B - \bar{x}_B$	$(B - \bar{x}_B)^2$	$C - \bar{x}_C$	$(C - \bar{x}_C)^2$
10-8=2	$2^2 = 4$	9-7=2	$2^2 = 4$	4-6=-2	$-2^2 = 4$
6-8=-2	$-2^2 = 4$	7-7=0	0	7-6=1	1
7-8=-1	$-1^2 = 1$	7-7=0	0	7-6=1	1
9-8=1	$1^2 = 1$	5-7=-2	4	6-6=0	0
	10		8		6

Sum of sq. with the samples:

$\Sigma(x - \bar{x}_i)^2 = 10 + 8 + 6 = 24$

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Step 4:- Calculate the value of F-ratio

Name of variation	Sum of squares	Degree of freedom	Mean sum of squares	F-test
b/w the samples	SSC = 8	$\nu_1 = c-1 = 3-1$ $\nu_1 = 2$	MSC = SSC/ν_1 = 8/2 = 4	$F = \frac{MSC}{MSE}$
within the samples	SSE = 24	$\nu_2 = n-c = 12-3$ $\nu_2 = 9$	MSE = SSE/ν_2 = 24/9 = 2.66	= 4/2.66 F = 1.5

$\boxed{F_{cal} = 1.5}$

$F_{tab} = F_{(0.01, (n-c), (c-1))} = F_{(0.01, 9, 2)} = 8.0215$

$F_{tab} = 8.0215$

$F_{cal} < F_{tab}$

Result/conclusion:

We accept null hypothesis and prove that there is a significance difference b/w the means of varieties of wheat.

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Q. A rifle club performed an experiment on a randomly selected group of first time shooters. The purpose of the experiment was to determine whether shooting accuracy is affecting by the method of sighting used only the right eye open, only the left eye open or both eyes open. Fifteen begining shooters were selected and split into three groups. Each group experienced the same training and practising procedures with one exception: the method of sighting used. After completing training each student was given the same number rounds and asked to shoot at a target. Their scores appear in the table below:

Method of Sighting

Right Eye	Left Eye	Both Eyes
12	10	16
10	17	14
18	16	16
12	13	11
14		20
		21

no. of rows = replicates
Right eye has 5 replicates
Left eye has 4 replicates
Both eyes has 6 replicates

$SS_{total} = \Sigma x^2 - (\Sigma(x))^2$
SS_{method} = variance b/w the factor
SS_{error} = variance with in the factor

Date

at 0.05 level of significance, is there sufficient to reject the claim that these methods of sightings are equally effective.

Replicates	Right Eye	Left Eye	Both Eyes
K=1	12	10	16
K=2	10	17	14
K=3	18	16	16
K=4	12	13	11
K=5	14		20
K=6			21
Total	C_R	C_L	C_B
Σ_C	66	56	98

Steps:-

1) (a) Describe the population parameter of interest.
$H_0: \mu_R = \mu_L = \mu_B$
(b) $H_A: \mu_R \neq \mu_L \neq \mu_B$

2) ~~(a)~~ Hypothesis Test Criteria
(a) Check the assumptions

$SS_{total} = \Sigma x^2 - (\Sigma x)^2$

$\Sigma X = 66 + 56 + 98 = 220$
$\Sigma x^2 = 12^2 + 10^2 + 18^2 + 12^2 + 14^2 + 10^2 + 17^2 + 16^2 + 13^2 + 16^2$
$+ 14^2 + 16^2 + 11^2 + 20^2 + 21^2$
$\Sigma x^2 = 3392$

$SS_{total} = 3392 - (220)^2$
$SS_{total} = 165.33$

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$$SS_{method} = \left(\frac{C1^2}{K1} + \frac{C2^2}{K2} + \frac{C3^2}{K3} + \ldots\right) - \frac{\Sigma X^2}{n}$$

$= 3255.87 - 3226.67$

$= 29.2$

$$SS_{error} = \Sigma X^2 - \left(\frac{C1^2}{K1} + \frac{C2^2}{K2} + \frac{C3^2}{K3} + \ldots\right)$$

$= 3392 - 3255.87$

$= 136.13$

$MS_{method} = SS_{method} / df_{method}$

$df_{method} = c-1 = 3-1 = 2$

$df_{error} = n-1 = 15-1 = 14$

$df_{total} = n-c = 15-3 = 12$

$MS_{method} = 29.2 \div 2 = 14.6$

$MS_{error} = SS_{error} / df_{error}$

$= 136.13/14$

$= 11.34$

(b) Identify the probability distribution

$$F = \frac{MS_{method}}{MS_{error}} = \frac{14.6}{11.34} = 1.287$$

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(C) Determine the level of significance

$\alpha = 0.05$

Step 3) Sample Evidence:

(a) Collect the sample information

Source	D.F	SS	MS
Method	2	29.2	14.6
Error	12	136.13	11.34
Total	14	165.33	

(b) Calculate the values of test statistics:

$F_{tabulated\ value} = F_{(2, n, 0.05)} = 3.89$

$\because F_{cal} = 1.287$

$F_{cal} < F_{tab}$

Result: Since, calculated value is less than the tabulated value we therefore accept null hypothesis and conclude that the three methods are equally effective.

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Q: Show that there is a significance difference ~~of~~ evidence in the three population means μ_F, μ_G and μ_H

Factor level

	μ_F	μ_G	μ_H
	3	5	8
	2	6	7
	3	5	7
	4	5	8
Total	12	21	30

Solution:-

① State the null hypothesis

$H_0: \mu_F = \mu_G = \mu_H$

$H_A: \mu_F \neq \mu_G \neq \mu_H$

② Calculate the variance b/w the samples

(a) Calculate means of each sample

$$\mu_F = \frac{12}{4} = 3, \quad \mu_G = \frac{21}{4} = 5.25, \quad \mu_H = \frac{30}{4} = 7.5$$

(b) Calculate the grand mean.

$$\bar{\mu} = \frac{\mu_F + \mu_G + \mu_H}{3} = \frac{3 + 5.25 + 7.5}{3} = 5.25$$

(c) Take the difference b/w the means of variance sample and grand mean

Date ________

Source	SS	DF	MS	Fcal
b/w the samples	SSC = 40.5	$\mu_1 = c-1$ $3-1=2$	$MSC = SSC/\mu_1$ $40.5/2 = 20.25$	$F = \frac{MSC}{MSE}$
within the samples	SSE = 3.75	$\mu_2 = n-c$ $12-3=9$	$MSE = SSE/\mu_2$ $3.75/9 = 0.4166$	$\frac{20.25}{0.4166}$ $= 48.6$

$\boxed{F_{cal} = 48.6}$

$F_{tab} = F_{(0.01, 9, 2)} = 8.0215$

$F_{cal} > F_{tab}$

Result: Therefor we reject the null hypothesis and accept the alternative and conclude that means are different.

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TWO WAY ANOVA

Note: It is classified according to two factors or two criterias.

Q.

Medical field	Pain Killer1	Pain Killer2	Pain Killer3	Total	G-Total
	10	7	4	21	
Medicine	12	9	5	26	98
	11	8	6	25	
	9	12	5	26	
	12	13	6	31	
Surgery	13	15	6	34	120
	10	12	4	26	
	13	12	4	29	
Total	$P_1 = 90$	$P_2 = 88$	$P_3 = 40$		

Step #01: $H_0: \mu_{P_1} = \mu_{P_2} = \mu_{P_3}$

$H_A: \mu_{P_1} \neq \mu_{P_2} \neq \mu_{P_3}$

$H_0: Med_{P_1} = Med_{P_2} = Med_{P_3}$

$H_A: Med_{P_1} \neq Med_{P_2} \neq Med_{P_3}$

① Correction Term: $\frac{(\Sigma X)^2}{N} = 1980$

② Sum of Sq. Total : SST

$SST = \Sigma X^2 - C_x = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$

$SST = 2254 - 1980 = 274$

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$\mu_F - \bar{\mu}$	$(\mu_F - \bar{\mu})^2$	$\mu_G - \bar{\mu}$	$(\mu_G - \bar{\mu})^2$	$\mu_H - \bar{\mu}$	$(\mu_H - \bar{\mu})^2$
3−5.25=−2.25	$(-2.25)^2$=5.0625	0	0	2.25	5.0625
−2.25	5.0625	0	0	2.25	5.0625
−2.25	5.0625	0	0	2.25	5.0625
−2.25	5.0625	0	0	2.25	5.0625
−9	20.25	0	0	9	20.25

Sum of Squares $= \Sigma(\mu_i - \bar{\mu})^2 = 20.25 + 0 + 20.25$
$= 40.5$

③ Calculate the variance within the sample.
(a) Calculate the mean of each sample
already found in step 2 (a)

(b) Take the deviation from various items in a sample from the mean values of the respective sample and square it.

$F-\mu_F$	$(F-\mu_F)^2$	$G-\mu_G$	$(G-\mu_G)^2$	$H-\mu_H$	$(H-\mu_H)^2$
3−3=0	0	−0.25	0.0625	0.5	0.25
−1	1	0.75	0.5625	−0.5	0.25
0	0	−0.25	0.0625	−0.5	0.25
1	1	−0.25	0.0625	0.5	0.25
	2		0.75		1

Sum of squares within sample $= 2 + 0.75 + 1$
$\Sigma(x_i - \mu_x)^2 = 3.75$

④ Calculate the value of f-ratio

∴ In Two way ANOVA, we show one more calculation of interaction b/w row and column.

Step#02: For variation in Pain Killer: SSC

$$SSC = \frac{\Sigma C_1^2}{8} + \frac{\Sigma C_2^2}{8} + \frac{\Sigma C_3^2}{8} - \frac{(\Sigma x)^2}{N}$$

$$= \frac{90^2}{8} + \frac{88^2}{8} + \frac{40^2}{8} - 1980$$

SSC = 200

Step#03: For Variation in medical fields: SSR

$$SSR = \frac{R_1^2}{R_{N_1}} + \frac{R_2^2}{R_{N_2}} = \frac{98^2}{12} + \frac{120^2}{12}$$

SSR = 20

Step#04: Sum of Sq. with in group: SSG

$$SSG = \frac{\Sigma\Sigma(CR)^2}{8} = \frac{\Sigma\Sigma(PMed)^2}{8} = Cx - SSC - SSR$$

$$SSG = \frac{[10+12+11+9]^2}{4} + \frac{[7+9+8+12]^2}{4} + \frac{[4+5+6+5]^2}{4}$$

$$+ \frac{[12+13+10+13]^2}{4} + \frac{[13+15+12+12]^2}{4} + \frac{[6+6+4+4]^2}{4}$$

SSG = 1980 − 200 − 20 = 17

Step#05: Residual Sum of Sq.: SSE

SSE = SST − SSC − SSR − SSG
= 274 − 200 − 20 − 17
SSE = 37

Date

Degree of freedom: F-distribution

$F = MSS_r / MSE$

Source	D.F	F-ratio	F-critical $\alpha = 0.05$	Hypothesis
Pain Killer	$c-1 = 2$	48.73	$F_{(2,18)} = 3.55$	reject H_0
Medical Field	$r-1 = 1$	9.81	$F_{(1,18)} = 4.41$	reject H_0
interaction	$(c-1)(r-1) = 2$	3.97	$F_{(2,18)} = 3.55$	reject H_0
residual	$c \cdot r \cdot (n-1) = 18$			
Total	$N-1 = 23$			

Result:

Since, $F_{cal} > F_{tab}$ so we reject H_0 in each and every case

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Q] For the following data find SSE and show that

$SSE = [(K-1)S_1^2 + (K-1)S_2^2 + (K-1)S_3^2]$

where S_i^2 is the variance for the ith factor level.

Solution:

Factor Level

A	B	C
8	6	10
4	6	12
2	4	14
14	16	36

$\Sigma x^2 = 8^2 + 4^2 + 2^2 + 6^2 + \dots + 14^2 = 612$

$\Sigma C_i = 14 + 16 + 36 = 66$

$$SSE = \Sigma x^2 - \left(\frac{\Sigma C_1^2}{K} + \frac{\Sigma C_2^2}{K} + \frac{\Sigma C_3^2}{K} \right)$$

$$= 612 - \left(\frac{14^2}{3} + \frac{16^2}{3} + \frac{36^2}{3} \right)$$

$SSE = 29.333$

$SS_{method} = 98.666$

$SST = 127.99$

$K = 3$

$(K-1)S_1^2 + (K-1)S_2^2 + (K-1)S_3^2 = SSE$

$(3-1)(3.055)^2 + (3-1)(1.154)^2 + (3-1)(2)^2 = 29.333$

$2(3.055)^2 + 2(1.154)^2 + 2(2)^2$

$= 29.32 \cong 29.333$

proved

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ANOVA (Without Replication)

Q/

Location	A	B	C	Total	Mean
1	18	13	12	43	14.33
2	20	23	21	64	21.33
3	14	12	9	35	11.67
4	11	17	10	38	12.67
Total	63	65	52	180	60
Mean	15.75	16.25	13	45	15

① Calculate row wise and colum-wise means

$\bar{X}_{R1} = 14.23$, $\bar{X}_{R2} = 21.33$, $\bar{X}_{R3} = 11.67$, $\bar{X}_{R4} = 12.67$

$\bar{X}_{CA} = 15.75$, $\bar{X}_{CB} = 16.25$, $\bar{X}_{CC} = 13$

② Variance of Rows mean S_R^2

$S_R^2 = \text{Variance}(14.333, 21.33, 11.667, 12.667)$

$S_R^2 = 19.037$

Variance of column means S_C^2

$S_C^2 = \text{Variance}(15.75, 16.25, 13)$

$S_C^2 = 3.0625$

③ Mean Sum of squares for Row $= MS_A = CS_R^2$

Mean Sum of Squares for Column $= MS_B = RS_C^2$

Here,

C = no. of colums $= 3$

R = no. of Rows $= 4$

Now,

$MS_A = [illegible] \times [illegible] = 3 \times 11.0315$

$MS_A = [illegible] 1.111$

$MS_B = K \times [illegible] = 4 \times [illegible] .0625$

$MS_{E} = 12.25$

ANOVA Table					
Source	d.f	SS	MS	F-ratio	F tab
A	[illegible]	[illegible]	[illegible]	8.12	$F_{(0.05)}$ = 4.7511
B	[illegible]	[illegible]	[illegible]	1.14	$F_{(0.05)}$ = 5.1433
Residual	[illegible]	42.17	7.03	-	
Total	12-1=11	238	~~[illegible]~~	~~[illegible]~~	

Calculate to fill out ANOVA table

MS x df = SS

Variance (Y) [illegible] = 21.63
11

<u>For A</u>

$F_{cal} > F_{tab}$, 8.12 > 4.7511

We reject H_0

<u>For B</u>

$F_{cal} < F_{tab}$

1.14 < 5.1433

We accept H_0

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Q| A new blend has develop to increase manufacturing cost of different types of fabric and the makers of the blends want to better understands which of the three formulation of blends are most effective for cotton, lawn, silk and ~~[illegible]~~ jacquard fabrics. It has three blends of one sample on each of the four types of fabrics. The fabric ~~formulation~~ manufacturing for the 12 formulations are as follows:

Solutions:-

~~Fabric\Blends~~

	Blends\Fabrics	Cotton	Lawn	Silk	Jacquard
~~Cotton~~	X	123	138	110	151
~~Lawn~~	Y	145	165	140	522
~~Silk~~	Z	156	176	185	617
	Total	424	479	435	692

Summation of Rows:

$\Sigma X = 522$

$\Sigma Y = 617$

$\Sigma Z = 692$

$\Sigma G = 1831$ ← Grand Total

Step #01:-

(a) H_0': $\mu_1 = \mu_2 = \mu_3 = \mu_4$

H_1': $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$

(b) H_0'': $\mu_1 = \mu_2 = \mu_3 = \mu_4$

H_1'': $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$

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Step #02 :-

$\sum x_{ij} = (123)^2 + (145)^2 + \dots + (175)^2 = 284975$

Step #03 :-

Correction Term $= C_x = \frac{(\sum x)^2}{N} = \frac{(1831)^2}{12}$

$\boxed{C_x = 279380}$

Step #04 :-

$SST = \sum x^2 - C_x = 284975 - 279380$

$\boxed{SST = 5595}$

Step 5 :-

$SSC = \left(\frac{C_1^2}{m_1} + \frac{C_2^2}{m_2} + \frac{C_3^2}{m_3} + \frac{C_4^2}{m_4}\right) - C_x$

$= \left(\frac{(424)^2}{3} + \frac{(479)^2}{3} + \frac{(435)^2}{3} + \frac{(493)^2}{3}\right) - 279380$

$\boxed{SSC = 1117}$

Step 6 :-

$SSR = \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3}\right) - C_x$

$= \left(\frac{(522)^2}{4} + \frac{(617)^2}{4} + \frac{(692)^2}{4}\right) - 279380$

$\boxed{SSR = 3629}$

Step 7 :-

$SSE = SST - SSC - SSR = 5595 - 1117 - 3629$

$\boxed{SSE = 849}$

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Step 8 :- Degree of Freedom

D.F (Total) $= N-1 = 12-1 = 11$

D.F (fabrics b/w columns) $= c-1 = 4-1 = 3$

D.F (Blends b/w row) $= r-1 = 3-1 = 2$

D.F (Error) $= (c-1)(r-1) = 3 \times 2 = 6$

Step 9 :- ANOVA Table :-

Source	D.F	SS	MSS = SS/D.F	F-ratio
Fabrics	3	1117	1117/3 = 372.33	$F = \frac{MSS_x}{MSSE}$
Blends	2	3629	3629/3 = 1814.5	$F_{C_1} = 2.631$
Error	6	849	849/3 = 141.5	$F_{C_2} = 12.823$
Total	11	5595		

Step 10 :-

$F_{tab_1} = F_{tab_1}(0.05, 3, 6) = 4.76$

$F_{C_1} < F_{tab_1}$

Step 11 :-

$F_{tab_2} = F_{tab_2}(0.05, 2, 6) = 5.14$

$F_{C_2} > F_{tab_2}$

Step 12 :- Result

Since, the calculated value of F_{C_1} is less than tabulated value of F_{tab_1} therefore we accept H_0' and conclude that all means of fabrics are equal. And, since, the calculated value of F_{C_2} is greater than the tabulated value of F_{tab_2}, we reject H_0'' and conclude that all means of blends are not equal.

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SYSTEM MODELS AND SIMULATIONS

Models

- Statistical
- Mathematical
- Stochastic (Randomness)

System is a collection of entities for example person and machine which interact together to accomplish a task.

We define the state of the system for the connection of variable necessary to a system at a particular time, relative to the objective to the time.

In a study of bank, the possible state variables are: the no. of busy teller, no. of customers in bank and no. of arrival of each customer.

① System:-

System is a group of object that are joined together in some regular interaction toward the accomplishment of some purpose.

② System Environment :-

A system is often effected by changes ~~or~~ occuring outside the system. For example: factory: arrival of order, bank: arrival of ~~a~~ customer, hospital: arrival of patient.

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③ Components:

- An entity is an object of interest in a system
- An attribute denotes the property of attribute e.g. ~~ratine of~~ machines; quantity of each order.
- Any purpose of causing changes in a system is called state change of the system.
- The state of the system is defined as the list a collection of variables.

④ Event: An event is defined an instantaneous occurrence that may change state of the system.

⑤ Types of systems:-

(i) Endogenous Systems:-

For The term endogenous means to describe the process e.g. withdrawing cash in bank.

(ii) Exogenous System:-

The term exogenous is used to describe activities and events in the environment that affect the system.

e.g. arrival of customer.

(iii) Closed System:

A system in which there is no exogenous activity is said to be closed system.

we e.g. water in an isolated flask how it keeps of the water inside for longer time. system at a particular time, relative to the objective to the time.

(iv) Open system: [illegible] the possible state [illegible] system [illegible] of [illegible] called open system.

e.g. Banking System:-

① Customer arrives and bank performs their [illegible] object that are joined together.

[A] [illegible] system and its components [illegible]

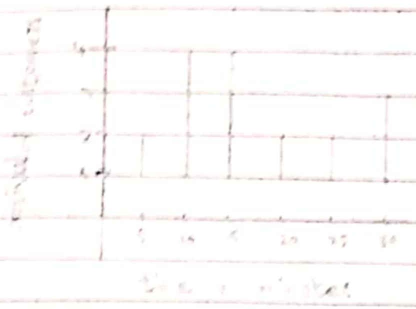
[A] System [illegible] purpose.
Attributes: account

② System [illegible] arrival of customer
A [illegible] customer [illegible] by change according [illegible] variables, no. of customers [illegible] arrival of order, bank: arrival of customer, hospital: arrival

STOCHASTIC MODEL:-

① It is a greek word which means "random" or "change" [illegible] analysis [illegible] with system [illegible] which involve [illegible] probability and [illegible]. e.g. for example [illegible]

* A random variable [illegible] that assigned a real number to each outcome of a random experiment by the random process.
* The state of the system is defined as the collection. A stochastic or random process is defined as the sequence of random variables: mathematically
$X(t_n): \; n = 1, 2, 3, \ldots, k$



X is a random variable observation at time t, the no. of states n may be finite or infinite.

For example, the poisson distribution

$$P(n) = \frac{e^{-\lambda t} \; \lambda^{t}}{t-1}$$

Thus, the states of the system are given by $1, 2, 3, \ldots, n$.

Queuing Model of Simulation

Types of Queuing model:

$$\overset{A}{M} / \overset{B}{M} / 1 \quad (\infty - \text{FCFS})$$

Markovian Server = 1
Arrival = Poisson Distribution
Inter arrival = Exponential Distribution
Infinite Object (∞)
Service Nature = FCFS

Basic Queuing Formula:
Markovian Nature:-

In Queuing theory, a discipline within the mathematical theory of probability, a Markovian Arrival Process (MAP) is a mathematical model time b/w job arrivals ~~and~~ to a system. The simplest research process is a poisson process, where time between each arrival is exponentially distributed.

① λ = mean arrival rate (# of arrivals per unit of time).

② μ = Mean service rate per busy server. (no. of customers served per unit of time).

③ $P = \lambda / \mu$

④ L_q = Expected avg. no. of customers in the queue : $L_q = L_s - \frac{\lambda}{\mu} = \frac{\lambda}{\mu} \cdot \frac{\lambda}{\mu - \lambda}$

⑤ L_s = Expected avg. no. of customers in the system: $L_s = \frac{\lambda/\mu}{1-\lambda/\mu} = \frac{\lambda}{\mu - \lambda}$

⑥ W_s = Expected waiting time a customer spends in the system = $W_s = \frac{L_s}{\lambda} = \frac{\lambda}{\lambda(\mu-\lambda)} = \frac{1}{\mu - \lambda}$

⑦ W_q = Expected waiting time per customer in the queue = $W_q = W_s - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}$

⑧ L_n = Avg. length of non-empty queue
$L_n = \frac{\mu}{\mu - \lambda}$

⑨ W_n = Avg. waiting time of non-empty queue.
$W_n = \frac{1}{\mu - \lambda}$

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Q1 Self service library employee one Librarian at its counter. 9 students arrive on an average every 5 minutes. While the librarian can serve 10 students in 5 minutes. Assuming poisson distribution for arrival rate and exponential distribution for service rate and inter-arrival time. Find out the following:

1) Avg. no. of students in the system.
2) Avg. no. of students in the queue.
3) Avg. time student spends in the system.
4) Avg. time student waits before being served.

Solution:-

① $\lambda = \frac{9}{5} = 1.8$ student/minute

② $\mu = \frac{10}{5} = 2$ student/min.

③ $L_s = \frac{\lambda}{\mu-\lambda} = \frac{1.8}{2-1.8} = 9$ students

④ $L_q = \frac{\lambda}{\mu} \cdot \frac{\lambda}{\mu-\lambda} = \frac{\lambda}{\mu} \cdot L_s = \frac{1.8}{2}(9) = 8.1$ students

⑤ $W_s = \frac{1}{\mu-\lambda} = \frac{1}{2-1.8} = 5$ minutes

⑥ $W_q = \frac{\lambda}{\mu} \cdot \frac{1}{\mu-\lambda} = \frac{\lambda}{\mu} \cdot W_s = \frac{1.8}{2} \times 5 = 4.5$ min.

Ans.

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Note:-

When the arrival time and service time are given in the form of their means then

$\lambda = 1/$ expected arrival time and

$\mu = 1/$ mean service time.

Q2 A mechanic repairing car engines. Find that the time spent on repairing the engines has exponential distribution with mean 20 min. If the engines are repaired in the order in which they come in, and their arrival is approximately poisson with an avg. rate of 15 for 8-hours day. What is the mechanic's expected ~~arrival~~ idle time each day? How many jobs are ahead of the average engine just brought in.

Solution:-

① $\lambda = \frac{15}{8 \times 60} = 0.0313$

② $\mu = \frac{1}{20} = 0.05$

③ $L_s = \frac{\lambda}{\mu-\lambda} = \frac{0.0313}{0.05-0.0313} = 1.67$

For 8-hours

④ $f = 8 \times \frac{\lambda}{\mu} = 8 \times \frac{0.0313}{0.05} = 5$

⑤ $\text{Idle} = 8 - f = 8 - 5 = 3$

Ans.

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Q3 Consider the following single server queue. the inter arrival time is exponentially distributed with a mean of 10 minutes and the service time is also exponentially distributed with a mean of 8 minutes. Find:

1) Mean wait in the queue.
2) Mean number in the queue.
3) Mean wait in the system.
4) Mean number in the system.
5) Proportion of time the server is idle.

Solution :-

① $\lambda = \frac{1}{10} = 0.1$ customer/min.

② $\mu = \frac{1}{8} = 0.125$ customer/min

③ $\rho = \frac{\lambda}{\mu} = \frac{0.1}{0.125} = 0.8\ \%$

④ $L_s = \frac{\lambda}{\mu - \lambda} = \frac{0.1}{0.125 - 0.1} = 4$ customers.

⑤ $L_q = \rho \cdot L_s = (0.8)(4) = 3.2$ customers.

⑥ $W_s = 1/\mu - \lambda = 1/0.125 - 0.1 = 40$ min.

⑦ $W_q = \rho \cdot W_s = 0.8 \times 40 = 32$ min.

⑧ Idle $= 1 - \rho = 1 - 0.8 = 0.2\ \%$

Ans.