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Queuing Model of Simulation

Type of Queuing Model

A D
$M/M/1\ (\infty - FCFS)$

Markovian server = 1

Arrival = Poisson Distribution

Interarrival = Exponential Distribution

Infinite Object

Service Nature = First Come First Serve

Basic Queuing Formulae:-

Markovian Nature:-

In queuing theory, a decipline within the mathematical theory of probability, a markovian arrival process (MAP) is a mathematical model time between job arrivals to a system. The simplest research process is a poisson process, where time b/w each arrival is exponentially distribution.

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① λ = mean arrival rate (No. of arrivals per unit of time).

② μ = Mean Service Rate per busy server (No. of customers served per unit of time).

③ $P = \lambda/\mu$

④ L_q = Expected avg. no. of customers in the queue: $L_q = L_s - \frac{\lambda}{\mu} = \frac{\lambda}{\mu} \cdot \frac{\lambda}{\mu-\lambda}$

⑤ L_s = Expected avg. no. of customers in the system: $L_s = \frac{\lambda/\mu}{(1-\lambda/\mu)} = \frac{\lambda}{(\mu-\lambda)}$

⑥ W_s = Expected time a customer spends in the system: $W_s = \frac{L_s}{\lambda} = \frac{\lambda}{\lambda(\mu-\lambda)} = \frac{1}{(\mu-\lambda)}$

⑦ W_q = Expected ~~time~~ waiting time per customer in the queue.

$W_q = W_s - \frac{1}{\mu} = W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

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(8) L_n = Avg. length of non-empty queue.

$$L_n = \frac{\mu}{\mu - \lambda}$$

(9) W_n = Avg. ~~length~~ waiting time in non-empty queue.

$$W_n = \frac{1}{\mu - \lambda}$$

Q1) Self service library employee one librarian at its counter. 9 students arrive on an average every 5 minutes. While the librarian can serve 10 customers in 5 minutes. Assuming Poisson Distribution for arrival rate and Exponential Distribution for service rate and interarrival time. Find out the following:

1) Avg. no. of students in the system.
2) Avg. no. of students in the queue.
3) Avg. time student spends in the system.
4) Avg. time student waits before being served.

job rate ka laaz island ho to per specific time se divide kardenge.

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Solution:-

① $\lambda = \frac{9}{5} = \boxed{1.8}$ student/min.

② $\mu = \frac{10}{5} = \boxed{2}$ student/min.

③ $L_s = \frac{\lambda}{\mu - \lambda} = \frac{1.8}{2 - 1.8} = \boxed{9}$ students

④ $L_q = \frac{\lambda}{\mu} \cdot \frac{\lambda}{\mu - \lambda} = \frac{1.8 \times 9}{2} = \boxed{8.1}$ student

⑤ $W_s = \frac{1}{\mu - \lambda} = \frac{1}{2 - 1.8} = \boxed{5}$ minutes

⑥ $W_q = \frac{\lambda}{\mu} \cdot \frac{1}{\mu - \lambda} = \frac{1.8 \times 5}{2} = \boxed{4.5}$ minutes

Ans

Note:-

When the arrival and service times are given in the form of their means then
λ = 1/expected arrival time and
μ = 1/mean service time

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Q2) A mechanic repairing car engines. Finds that the time spent on repairing the engines has exponential distribution within mean 20 minutes. If the engines are repaired in the order in which they come in, and their arrival is approximately poisson within an average rate of 15 for 8-hours day. What is the mechanic's expected idle time each day? How many jobs are ahead of the average engine just brought in.

Solution:-

① $\lambda = \frac{15}{8 \times 60} = \boxed{0.0313}$

② $\mu = \frac{1}{20} = \boxed{0.05}$

③ $L_s = \frac{\lambda}{\mu - \lambda} = \frac{0.0313}{0.05 - 0.0313} = \boxed{1.67}$

for 8 hours

④ $f = 8 \times \frac{\lambda}{\mu} = \frac{8 \times 0.0313}{0.05} = \boxed{5}$

⑤ $\text{Idle} = 8 - f = 8 - 5 = \boxed{3}$

Ans

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Q3) Consider the following single server queue, the interarrival time is exponentially distributed with a mean of 10 minutes and the service time is also exponentially distributed with a mean of 8 minutes.

1) Mean wait in the queue.
2) Mean number in the queue.
3) Mean wait in the system
4) Mean number in the system.
5) Proportion of time the server is idle.

Solution:-

① $\lambda = \frac{1}{10} = \boxed{0.1}$ Customer/min

② $\mu = \frac{1}{8} = \boxed{0.125}$ Customer/min

③ $\rho = \frac{\lambda}{\mu} = \frac{0.1}{0.125} = \boxed{0.8}$ %

④ $L_s = \frac{\lambda}{\mu - \lambda} = \frac{0.1}{0.125 - 0.1} = \boxed{4}$ Customer

⑤ $L_q = \rho \cdot L_s = 0.8 \times 4 = \boxed{3.2}$ Customer

3.2 32 min 40 min .4 min 0.2

$\lambda = \frac{1}{E(I.\cdot)}$

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(6) $W_s = \frac{1}{\mu - \lambda} = \frac{1}{0.125 - 0.1} = \boxed{40}$ min

(7) $W_q = \rho . W_s = 0.8 \times 40 = \boxed{32}$ min

(8) Idle $= 1 - \rho = 1 - 0.8 = \boxed{0.2}$ %

Ans