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OSAA Assignment 1

(1) Given

$x[n] = \begin{cases} 1, & \text{if } 0 \le n \le 9 \\ 0, & \text{elsewhere} \end{cases}$ where $N \le 9$

$h[n] = \begin{cases} 1, & \text{if } 0 \le n \le N \\ 0, & \text{elsewhere} \end{cases}$

To find:

N value.

Given that

$y[n] = x[n] * h[n]$

$y[4] = 5$, $y[14] = 0$

$$y[n] = x[n] * h[n] = \sum_{K=-\infty}^{\infty} x[K] \cdot h[n-K]$$

$N \le 9$

$$\boxed{y[n] = \sum_{K=0}^{9} h[n-K]}$$

$$y[4] = \sum_{K=0}^{9} h[4-K] = 5$$

$$= h[4] + h[3] + h[2] + h[1] + \underbrace{h[0]}_{1} + \underbrace{h[-1] + h[-2] + h[-3] + h[-4]}_{0} \text{ (from given)}$$

$$= h[4] + h[3] + h[2] + h[1] + 1$$

$$h[4] + h[3] + h[2] + h[1] = 4$$

$$\Rightarrow N \ge 4 \rightarrow (1) \; (\because h[n])$$

The value of $h[n]$ can either be 1 (or) 0

$\Rightarrow h[4] + h[3] + h[2] + h[1] \ge 4$

$N = 4$.

$$y[14] = \sum_{k=0}^{9} h[14-k] = 0$$

$$\Rightarrow h[14] + h[13] + h[12] + h[11] + h[10] + h[9] + h[8] + h[7] + h[6] + h[5] = 0$$

$h[n] = 1$ for $0 \le n \le N$

$\Rightarrow$ $N < 5 \rightarrow$ ②

From ① and ②

$N \ge 4 \quad N < 5$

$\Rightarrow \boxed{N = 4}$

③ $x[n] = [0\ 0\ 1\ 1\ 1\ 0\ 0]$

$y[n] = [0\ 1\ 2\ 3\ 2\ 1\ 0]$

Let $h[n] = y[n]$

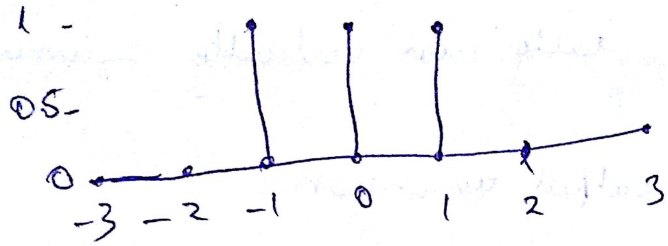
$z[n]$ is the output.

$$z[n] = x[n] * y[n]$$
$$= x[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

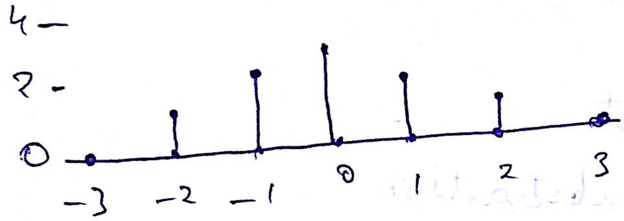
	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0
1	0	0	1	1	1	0	0
2	0	0	2	2	2	0	0
3	0	0	3	3	3	0	0
2	0	0	2	2	2	0	0
1	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0

$z[n] = [0\ 0\ 0\ 1\ 3\ 6\ 7\ 6\ 3\ 1\ 0\ 0\ 0]$

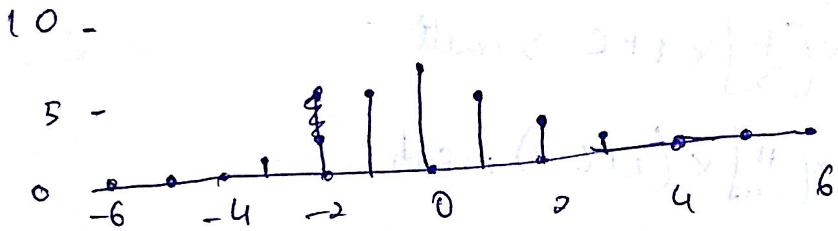
x[n]



y[n]



$h[n] = x[n] + y[n]$



5. Size of the image : (W, H, C)

No. of filters = N.

Size : (F, F, C)

Step size : 8

zero padding : Z

The number of channels are equal.

The filter only moves across images in 2D, which is along width and height.

i) Each step in convolution gives rise to 1 entry in output

if the step size is 1

each cell of image gives rise to 1 entry in output.

and if the step size is equal to S, each cell ~~of image gives rise~~ to at a distance of S horizontally and vertically becomes center.

$$\left(\left\lceil \frac{W}{S} \right\rceil, \left\lceil \frac{H}{S} \right\rceil, 1 \right) \Rightarrow \text{output dimension.}$$

ii) in convolution.

$F \times F \times C \rightarrow$ mult

$(F \times F \times C) - 1 \rightarrow$ substraction.

for whole image

$\left\lceil \frac{W}{S} \right\rceil \times \left\lceil \frac{H}{S} \right\rceil \times FFC \Rightarrow$ mult

$\left\lceil \frac{W}{S} \right\rceil \times \left\lceil \frac{H}{S} \right\rceil \times (FFC - 1) \Rightarrow$ sub

if there are N filters.

Multiplication is $N \times \left\lceil \frac{W}{S} \right\rceil \times \left\lceil \frac{H}{S} \right\rceil \times FFC$

Substraction is $N \times \left\lceil \frac{W}{S} \right\rceil \times \left\lceil \frac{H}{S} \right\rceil \times (FFC - 1)$

(4) Given :-

$y[n] = x[n] - y^2[n-1] + y[n-1]$.

where $x[n] = \alpha u[n] \quad 0 \leq \alpha \leq 1$.

For Linear System :-

$(x_1 \rightarrow y_1)$ and $(x_2 \rightarrow y_2) \Rightarrow x_1 + x_2 \rightarrow y_1 + y_2$.

$y_1[n] = x_1[n] - y_1^2[n-1] - y_1[n-1]$

$y_2[n] = x_2[n] - y_2^2[n-1] + y_2[n-1]$

$(x_1+x_2) \Rightarrow n$

$x_1[n] + x_2[n] - y_1^2[n-1] - y_2^2[n-1]$

$y_3 = y_1+y_2 = x_1[n]+x_2[n] - y_1^2[n-1] - y_2^2[n-1] - 2y_1[n-1]y_2[n-1]$
$+ y_1[n-1] + y_2[n-1]$

if linear system

$y_1[n-1]\, y_2[n-1] = 0$

it is not confirmed

so non linear.

for Time invariant:

if $x[n] \to y[n] \Rightarrow x[n-k] \to y[n-k]$

$z[n] = x[n-k]$

$y[n] = x[n-k] - y^2[n-k-1] + y[n-k-1]$

assuming $z[n] = x[n-k]$.

$y[n] = z[n] - y_2^2[n-1] + y_2[n-1]$

$z[n] = x[n-k]$

$y[n-k] = x[n-k] - y^2[n-k-1] + y[n-k-1]$

It is Time invariant.

Given:

$y[-1] \to \alpha$

$y[n] = x[n] - y^2[n-1] + y[n-1]$

if $n=0$

$$y[0]=x[0]-y^2[-1]+y[-1]$$

$$y[0]=\alpha-(\sqrt{\alpha})^2+\sqrt{\alpha}$$

$$y[0]=\sqrt{\alpha}.$$

assume $y[n]=\sqrt{\alpha}$

$$y[n+1]=x[n+1]-y^2[n]+y[n]$$

$$=\cancel{\alpha}-(\cancel{\sqrt{\alpha}})^2+\sqrt{\alpha}$$

$$=\sqrt{\alpha}.$$

if $n\to\infty$ $\quad y[n]=\sqrt{\alpha}$.

Proved.

(5) The output of figure 1 and matrix should leave exactly one white line (white: 0) b/w 2 black blocks. meaning the centre row should be 255's all others should be 0's. This can be done through a simple matrix. With the idea that every white except centre. So every white element in Figure 1 x 1 - Top element to that gives result

Matrix $\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$

Scanned by CamScanner