14

Recursion

"To iterate is human, to recurse divine..."

Contents

KanNotes

Repetitions

- There are two ways to repeat a set of statements in a function:
  - By using **while** or **for** loop
  - By calling the function from within itself
- The first method is known as iteration, whereas the second is known as recursion.
- The functions that use iteration are called iterative functions and those that use recursion are called recursive functions.

Recursive Function

- A Python function can be called from within its body. When we do so it is called a recursive function.

```
def fun( ) :
    # some statements
    fun( )    # recursive call
```

- Recursive call keeps calling the function again and again, leading to an infinite loop.
- A provision must be made to get outside this infinite recursive loop. This is done by making the recursive call either in if block or in else block as shown below:

```
def fun( ) :
    if condition :
        # some statements
    else
        fun( )    # recursive call
```

```
def fun( ) :
    if condition :
        fun( )
    else
        # some statements
```

- The case when a recursive call is made is called the recursive case, whereas the other case is called the base case.
- If recursive call is made in if block (recursive case), else block should contain the base case logic. If recursive call is made in else block (recursive case), if block should contain the base case logic.

When to use Recursion

- Recursion is useful in 2 scenarios:
    - When a problem can be solved by breaking it down into similar sub-problems.
    - When a problem requires an unknown number of loops.
- Examples of problem as similar sub-problems:
    - Finding factorial value of a number
    - Finding sum of digits of an integer
    - Finding binary equivalent of a number
- Examples of unknown number of nested loops:
    - Finding all combinations of 1 to n, where n is received as input
    - Traversing a binary tree data structure
    - Traversing a graph data structure
- In this book we would cover both sets of problems that can be solved using recursion.

Problem as Similar Sub-problems

- In problem that can be solved by breaking it down into similar sub-problems the computation of a function is described in terms of the function itself.
- For example, suppose we wish to calculate factorial value of **n**. Then

  n! = n * (n - 1) * (n - 2) * (n - 3) * ... * 2 * 1

  We can write this as:

  ```
  n! = 1               if n = 0
     = n * (n -1)!     if n > 0
  ```

- In terms of function this can be written as:

  ```
  factorial(n) = 1                      if n = 0     (base case)
               = n * factorial(n - 1)   if n > 0     (recursive case)
  ```

- If we are to obtain sum of digits of an integer **n**, then the recursive function can be written as

  ```
  sumdig(n) = 0                         if n = 0  (base case)
            = n % 10 + sumdig(n / 10)   if n > 0  (recursive case)
  ```

- Following tips will help you understand recursive functions better:
  - A fresh set of variables are born during each function call—normal call as well as recursive call.
  - Variables created in a function die when control returns from a function.
  - Recursive function may or may not have a return statement.
  - Typically, during execution of a recursive function many recursive calls happen, so several sets of variables get created. This increases the space requirement of the function.
  - Recursive functions are inherently slow since passing value(s) and control to a function and returning value(s) and control will slow down the execution of the function.
  - Recursive calls terminate when the base case condition is satisfied.

Recursive Factorial Function

- A simple program that calculates factorial of a given number using a recursive function is given below, followed by a brief explanation of its working.

```
def refact(n) :
    if n == 0 :
        return 1
    else :
        p = n * refact(n - 1)
    return p

num = int(input('Enter any number: '))
fact = refact(num)
print('Factorial value = ', fact)
```

- Suppose 2 is supplied as input, we should get the output as 2, since 2! evaluates to 2.

- It becomes easier to follow the working of a recursive function if we make copies of the function on paper and then perform a dry run of the program to follow the control flow. In reality multiple copies of function are not created in memory.

- Trace the control flow of the recursive factorial function in Figure 14.1. Assume that we are trying to find factorial value of 2. The solid arrows indicate the call to the function, whereas dashed arrows indicate return from the function.
- Note that **return 1** goes to work only during the last call. All other calls return via **return p**.

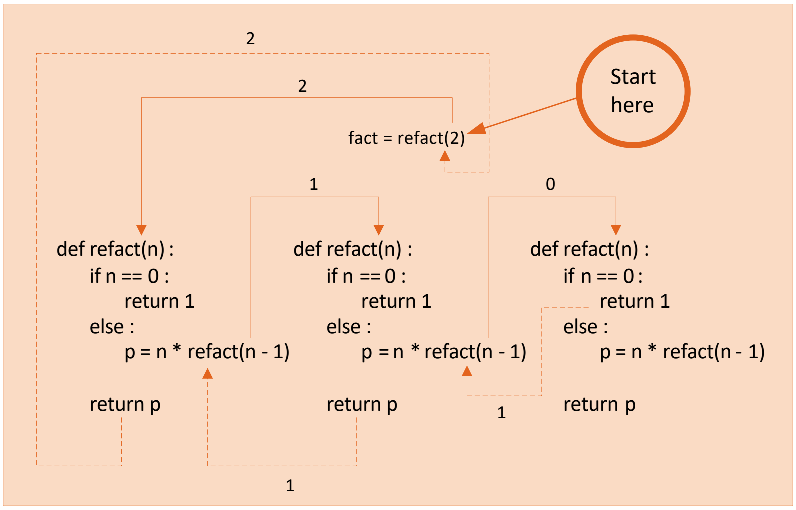


Figure 14.1

Problem with Unknown Loops

- If we are to define a function which generates and returns a list of lists containing all possible combinations of numbers 1, 2 and 3 we can do so through following program:

```
def generate(n) :
    lol = [[ ] for i in range(n ** n)]
    pos = 0
    for i in range(1, n + 1) :
        for j in range(1, n + 1) :
            for k in range(1, n + 1) :
                t = [i, j, k]
                lol[pos] = t
                pos += 1
    return lol
```

```
l = generate(3)
print(l)
```

Since we can generate 27 combinations out of 1, 2, 3 ([1, 1, 1], [1, 1, 2], ... [3, 3, 3]), list of lists **lol** is set up with 27 empty lists. Then through 3 **for** loops we have generated each sub-list and inserted it in **lol**.

- If we are to generate all possible combinations of 1, 2, 3, 4 then we will have to introduce one more **for** loop. If **generate( )** is to remain generic we cannot dynamically add this loop.
- We can make **generate( )** function create the desired combinations by using recursion in place of loops as shown in the following program:

```
def generate(n) :
    t = [ ]
    lol = [[ ] for i in range(n ** n)]
    helper(n, t, lol)
    return(lol)

def helper(n, t, lol) :
    global j
    if len(t) == n :
        lol[ j ] = lol[ j ] + t
        j += 1
        return

    for i in range(1, n + 1) :
        t.append(i)
        helper(n, t, lol);
        t.pop( )

j = 0
l = generate(3)
print(l)
```

In addition to **generate( )** we have defined the **helper( )** function since we wish to build each sub-list incrementally and **generate( )** receives only **n**.

After generating a sub-list like [1, 1, 1], **list** method **pop( )** has been called to remove the last 1 from this sub-list and create the next sub-list [1, 1, 2].

Types of Recursion

- Two types of recursions can exist:
  (a) Head recursion
  (b) Tail recursion

- Head recursion - In this type of recursion the recursive call is made before other processing in the function.

```
def headprint(n) :
    if n == 0 :
        return
    else :
        headprint(n - 1)
        print(n)

headprint(10)
```

Here firstly the recursive calls happen and then the printing takes place. Hence last value of **n,** i.e. 1 gets printed first. So numbers get printed in the order 1 to 10.

- Tail recursion - In this type of recursion processing is done before the recursive call. The tail recursion is similar to a loop—the function executes all the statements before making the recursive call.

```
def tailprint(n) :
    if n == 11 :
        return
    else :
        print(n)
        tailprint(n + 1)

tailprint(1)
```

Here firstly printing takes place and then the recursive call is made. Hence first value of **n,** i.e. 1 gets printed first and then the recursive call is done. So once again numbers get printed in the order 1 to 10.

Recursion Limit

- In head recursion we don't get the result of our calculation until we have returned from every recursive call. So the state (local variables) has to be saved before making the next recursive call. This results in consumption of more memory. Too many recursive calls may result into an error.
- Default recursion limit in Python is usually set to a small value (approximately, 10 ** 4). So if we provide a large input to the recursive function, a **RecursionError** will be raised.
- The **setrecursionlimit( )** function in **sys** module permits us to set the recursion limit. Once set to 10^6 large inputs can be handled without any errors.

Iteration to Recursion

- Given below are the steps that should be followed if we are to convert an iterative function to a recursive function:
  - Use the local variables in the iterative function as parameters of the recursive function.
  - Identify the main loop in the iterative function. This loop typically modifies one or more variables and returns some final value(s).
  - Write the condition in the loop as the base case and the body of the loop as the recursive case.
  - Run to check whether recursive function achieves the desired result.
  - Remove any unnecessary variables and improve the structure of the recursive function.

P</> *Programs*

Problem 14.1

If a positive integer is entered through the keyboard, write a recursive function to obtain the prime factors of the number.

Program

```
def factorize(n, i) :
    if i <= n :
        if n % i == 0 :
            print( i, end =', ' )
            n = n // i
        else :
            i += 1
    factorize(n, i)

num = int(input('Enter a number: '))
print('Prime factors are:')
factorize(num, 2)
```

Output

```
Enter a number: 50
Prime factors are:
2, 5, 5,

Enter a number: 24
Prime factors are:
2, 2, 2, 3,
```

Tips

- In **factorize( )** we keep checking, starting with 2, whether **i** is a factor of **n** (means, can **i** divide **n** exactly). If so, we print that factor, reduce **n** and again call **factorize( )** recursively. If not, we increment **i** and call **factorize( )** to check whether the new **i** is a factor of **n**.

Problem 14.2

A positive integer is entered through the keyboard, write a recursive function to calculate sum of digits of the 5-digit number.

Program

```
def rsum(num) :
    if num != 0 :
        digit = num % 10
        num = int(num / 10)
        sum = digit + rsum(num)
    else :
        return 0
    return sum

n = int(input('Enter number: '))
rs = rsum(n)
print('Sum of digits = ', rs)
```

Output

```
Enter number:
345
Sum of digits = 12
```

Tips

- In the **rsum( )** function, we extract the last digit, reduce the number and call **rsum( )** with reduced value of **num**. Thus if the number entered is 3256, the call becomes **sum = 6 + rsum(325)**.
- During each call additions are kept pending, for example the addition to 6 is kept pending as the program calls **rsum(325)** to obtain sum of digits of 325.
- The recursive calls end when **n** falls to 0, whereupon the function returns a 0, because sum of digits of 0 is 0. The 0 is returned to the previous pending call, i.e. **sum = 3 + rsum (0)**. Now **sum = 3 + 0** is completed and the control reaches **return s**. Now the value of **sum**, i.e. 3 is returned to the previous call made during the pending addition **2 + rsum (3)**. This way all pending calls are completed and finally the sum of 3256 is returned.
- In short, **return 0** goes to work only once (during the last call to **rsum( )**), whereas, for all previous calls **return sum** goes to work.

Problem 14.3

Paper of size A0 has dimensions 1189 mm x 841 mm. Each subsequent size A(n) is defined as A(n-1) cut in half, parallel to its shorter sides. Write a program to calculate and print paper sizes A0, A1, A2, … A8 using recursion.

Program

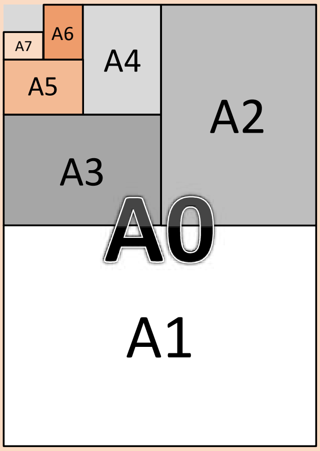
```
def papersizes(i, n, l, b) :
    if n != 0 :
        print(f'A{i}: L = {int(l)} B = {int(b)}')
        newb = l / 2
        newl = b
        n -= 1
        i += 1
        papersizes(i, n, newl, newb)

papersizes(0, 7, 1189, 841)
```

Output

```
A0: L = 1189 B = 841
A1: L = 841 B = 594
A2: L = 594 B = 420
A3: L = 420 B = 297
A4: L = 297 B = 210
A5: L = 210 B = 148
A6: L = 148 B = 105
```

Tips



```
A0: L = 1189 B = 841
A1: L = 841 B = 594
A2: L = 594 B = 420
A3: L = 420 B = 297
A4: L = 297 B = 210
A5: L = 210 B = 148
A6: L = 148 B = 105
```

Figure 14.2

- Figure 14.2 shows different paper sizes are obtained. In function **papersizes( )**, **i** is used to obtain the digit in A0, A1, A2, etc., whereas **n** is used to keep track of number of times the function should be called. The moment **n** falls to 0, the recursive calls are stopped. Alternately, we could have dropped **n** and stopped recursive calls when **i** reaches 7.

Problem 14.4

Write a recursive function to obtain first 15 numbers of a Fibonacci sequence. In a Fibonacci sequence the sum of two successive terms gives the third term. First few terms of the Fibonacci sequence:

1 1 2 3 5 8 13 21 34 55 89....

Program

```
def fibo(old, current, terms) :
    if terms >= 1 :
        new = old + current
        print( f'{new}', end = '\t')
        terms = terms - 1
        fibo(current, new, terms)

old = 1
current = 1
print(f'{old}', end = '\t')
print(f'{current}', end = '\t')
fibo(old, current, 13)
```

Output

```
1       1       2       3       5       8       13      21      34      55
89      144     233     377     610
```

Tips

- This program generates the Fibonacci sequence of numbers using recursion. **terms** is used to keep track of when to stop recursive calls. Since the first two terms are printed before calling **fibo( )**, we have generated only 13 terms through the recursive calls.

Problem 14.5

A positive integer is entered through the keyboard; write a function to find the binary equivalent of this number using recursion.

Program

```
import sys

def dec_to_binary(n) :
    r = n % 2
    n = int(n / 2)
    if n != 0 :
        dec_to_binary(n)
    print(r, end = '')

sys.setrecursionlimit(10 ** 6)
num = int(input('Enter the number: '))
print('The binary equivalent is:')
dec_to_binary(num)
```

Output

```
Enter the number: 32
The binary equivalent is:
100000

Enter the number: 45
The binary equivalent is:
101101
```

Tips

- To obtain binary equivalent of a number, we have to keep dividing the dividend till it doesn’t become 0. Finally, the remainders obtained during each successive division must be written in reverse order to get the binary equivalent.
- Since the remainders are to be written in the reverse order, we start printing only when **n** falls to 0, otherwise we make a call to **dec_to_binary( )** with a reduced dividend value.

___

Problem 14.6

Write a recursive function to obtain the running sum of first 25 natural numbers.

Program

```
def runningSum(n) :
    if n == 0 :
        return 0
    else :
        s = n + runningSum(n - 1)
        return(s)
max = int(input('Enter the positive largest number for running sum: '))
if max > 0 :
    sum = runningSum(max)
    print(f'Running Sum: {sum}')
else :
    print('Entered number is negative')
```

Output

```
Enter the positive largest number for running sum: 25
Running Sum: 325
```

Tips

- We calculate the running sum as we calculate the factorial value, starting from **n** and then go on reducing it moving towards 0. We stop on reaching 0.

Exercises

[A] State whether the following statements are True or False:

(a) A recursive function that calls another execution instance of the same function.

(b) If a recursive function uses three variables **a**, **b** and **c**, then the same set of variables are used during each recursive call.

(c) Multiple copies of the recursive function are created in memory.

(d) A recursive function must contain at least 1 **return** statement.

(e) Every iteration done using a **while** or **for** loop can be replaced with recursion.

(f) Logics expressible in the form of themselves are good candidates for writing recursive functions.

(g) Tail recursion is similar to a loop.

(h) Infinite recursion can occur if the base case is not properly defined.

(i) A recursive function is easy to write, understand and maintain as compared to a one that uses a loop.

[B] Answer the following questions:

(a) Following program calculates sum of first 5 natural numbers using tail recursion and head recursion.

(b) There are three pegs labeled A, B and C. Four disks are placed on peg A. The bottom-most disk is largest, and disks go on decreasing in size with the topmost disk being smallest. The objective of the game is to move the disks from peg A to peg C, using peg B as an auxiliary peg. The rules of the game are as follows:

- Only one disk may be moved at a time, and it must be the top disk on one of the pegs.
- A larger disk should never be placed on the top of a smaller disk.

Write a program to print out the sequence in which the disks should be moved such that all disks on peg A are finally transferred to peg C.

(c) A string is entered through the keyboard. Write a recursive function that counts the number of vowels in this string.

(d) A string is entered through the keyboard. Write a recursive function removes any tabs present in this string.

(e) A string is entered through the keyboard. Write a recursive function that checks whether the string is a palindrome or not.

(f) Two numbers are received through the keyboard into variables **a** and **b**. Write a recursive function that calculate the value of $\mathbf{a}^{\mathbf{b}}$.

(g) Write a recursive function that reverses the list of numbers that it receives.

(h) A list contains some negative and some positive numbers. Write a recursive function that sanitizes the list by replacing all negative numbers with 0.

(i) Write a recursive function to obtain average of all numbers present in a given list.

(j) Write a recursive function to obtain length of a given string.

(k) Write a recursive function that receives a number as input and returns the square of the number. Use the mathematical identity $(n - 1)^2 = n^2 - 2n + 1$.

[C] What will be the output of the following programs?

(a)
```
def fun(x, y) :
    if x == 0 :
        return y
    else :
        return fun(x - 1, x * y)
print(fun(4, 2))
```

(b)
```
def fun(num) :
    if num > 100 :
        return num - 10
    return fun(fun(num + 11))
print(fun(75))
```

(c)
```
def fun(num) :
    if num == 0 :
        print("False")
    if num == 1 :
        print("True")
    if num % 2 == 0 :
        fun(num / 2)
fun(256)
```