



Seminar 5

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Qubits to Quantum Circuits

Week 4 | October 16th, 2022

Qubits, Gates, Circuits

- Representing Qubits
 - Ket Notation
 - The Bloch Sphere
- Quantum Gates: X, H, and Z
- Quantum Circuits

Representing Qubits

There are different types of Qubits: Superconducting (currently the most popular way), Trapped Ions, Diamond NV Centers, Photonics. Each way involves a different type of physics. How do we represent in an abstract form to think about quantum computing from a theoretical perspective?

Notation

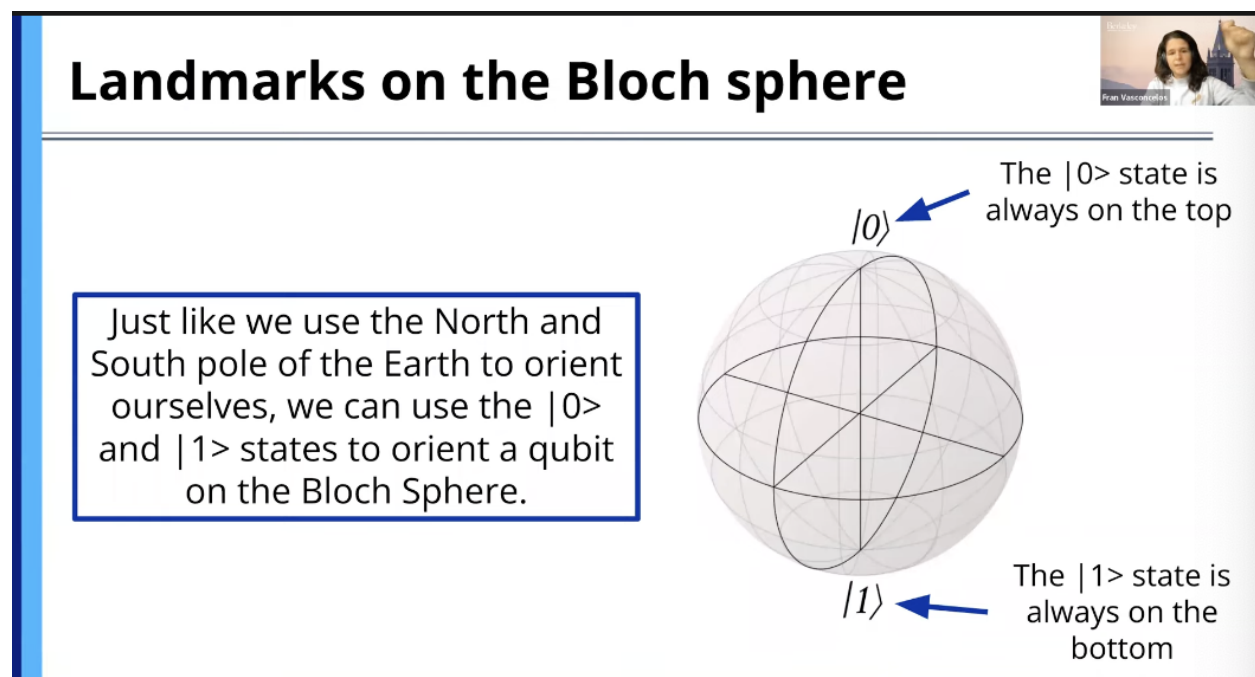
$|0\rangle$, $|1\rangle$ is called “ket” notation. This is a mathematical way to represent the state of a qubit. A “Bloch Sphere” is a more geometric, visual aid for qubits. Ket notation is used by physicists to indicate a quantum state. Up arrow is the “0” qubit, while Down arrow is the “1” qubit. Will learn about “bra-ket” or “Dirac” notation eventually throughout the class

Schrodinger’s “Ket”:

The state is represented as: $|(\text{dead cat})\rangle + |(\text{alive cat})\rangle$. When the box is opened, the sum will change to just one of the two “ket” states of the equation above.

Visualizing Superpositions:

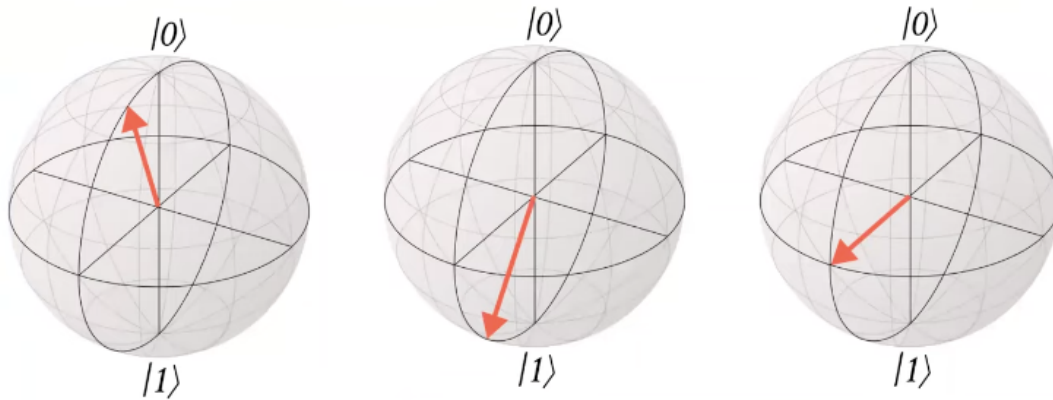
Arrows are not that helpful when displaying superpositions since there can be many different types of superposition values. For example: 50-50 vs 10-90 superpositions. “Ket” notations will help with displaying these differences mathematically. “Bloch Sphere” allows us to represent all possible qubit states (similar to a globe of earth).



Locating a qubit on the Bloch sphere:

Draw an arrow starting from the centroid to the specific state/location on the sphere. These arrows are “vectors” mathematically. Anything that is not $|0\rangle$ or $|1\rangle$ is a

superposition. There are an infinite amount of superposition states since there are an infinite amount of surface points



Equal Superposition:

50% $|0\rangle$ and 50% $|1\rangle$. This is visually represented by an arrow orthogonal to the “0” and “1” state. If there are two arrow orthogonal and opposite to each other, they would be “+” (pointed left) and “-” (pointed right) states.

Unequal Superpositions:

Arrows closer to one particular state distance-wise indicates that the superposition has more contribution of that particular state meaning when it collapses it is likely that it collapses to that particular state. “They’re more likely to give us a certain value at measurement (kind of like an unfair coin)”.

Random:

Radius of Bloch Sphere = 1

“Pure States” lie on the sphere, and there are cases where there can be states that lie inside the sphere (norm < 1). These are statistical ensembles that have uncertainty on what quantum state you actually are in.

“Bloch” sphere is used to represent the state of a **single** qubit, not **multiple**

“+”/“-” states are two equal superposition states between 0 and 1. The “+” points in the positive x-axis direction whereas the “-” points in the negative x-axis direction.

the 3-D nature of the Bloch Sphere rises from the fact that the q-states are complex representations, not real values (which would only need a 2-D circle)

Quantum Gates:

Q-gate are able to manipulate qubits through transformations on inputs. Quantum Gates = rotations on the Bloch Sphere. Every gates perform a different kind of rotation on the Bloch Sphere.

There are many quantum gates. Every one changes qubit states differently.

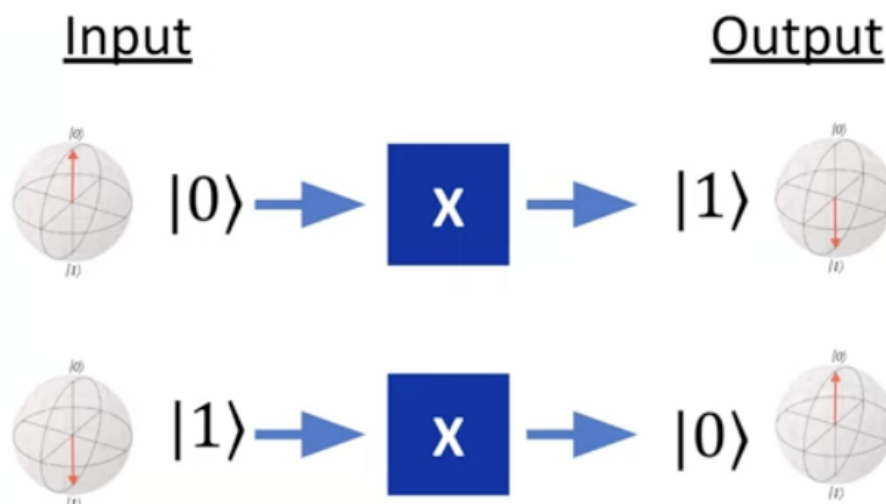


Two Types of Gates: X-Gate, H-gates

X-Gate:

The X-Gate gives the opposite of the input

Rule: the X gate gives the opposite of the input



Bloch Sphere Coordinates: X, Y, Z

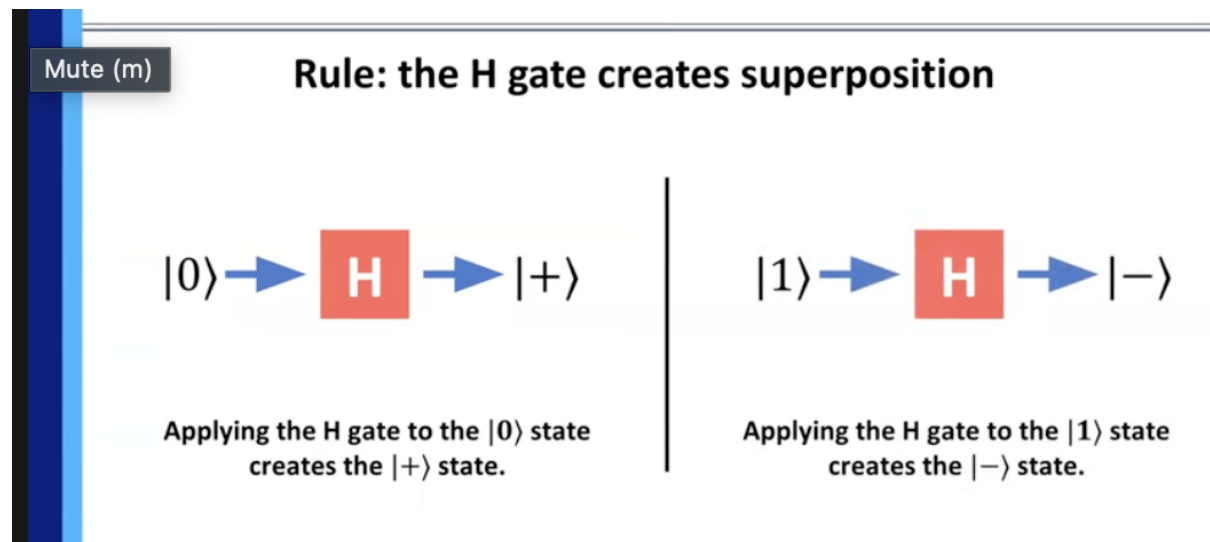
We can place the Bloch Sphere on a coordinate system: X, Y, Z. Any point can be described by 3 numbers: (x, y, z). The z-axis runs vertically. These coordinates can be

leveraged to understand what exactly these quantum gates are doing. “0” and “+” state lie on different axis. **The “X” gate performs a 180 degree rotation around the X-axis.** The Y and Z gates similarly apply rotations around their respective axis (y and z-axis)

- X-Gates do not affect any movements for + state or - state since they already exist on the x-axis.

H Gates: Hadamard Gate

Rule: The H gate creates superposition



The reason this gate is important is because it does something that is **Quantum**. This gate takes a specific state and maps it to a quantum state, something only possible in the quantum realm. **The H-gate has its own unique rotation: It performs a combination of rotations around the X and Z-axis simultaneously.**

- What would be the result of applying the X gate to a qubit in the $|+\rangle$ state?
- How can we relate the $|0\rangle$ and $|1\rangle$ states with the $|+\rangle$ and $|-\rangle$ states?
- Does the order in which gates are applied matter? Is the final state the same if we first apply an X gate and then an H gate, and if we reverse the order of the gates?

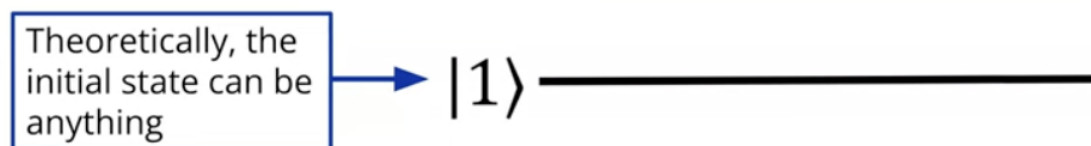
Quantum Circuits:

At the start of every circuit you have inputs (q-states or qubits). Visually you place the ket notations and then a straight line is written after this ket notation where the q-gates are placed. *In Qiskit, qubits always start as $|0\rangle$.* This is because we can apply arbitrary q-gates to the input to map it to any q-state on the Bloch Sphere.

Quantum Circuits



Here is how we draw a **quantum circuit**:

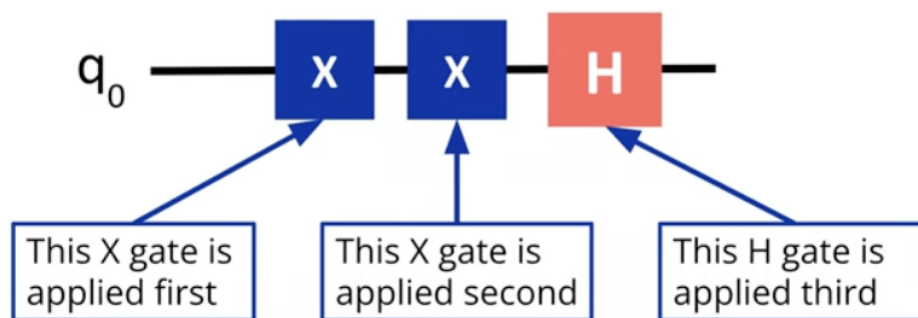


- Applying q-gates corresponds to putting energy into system (system initially starts at "0" state which lacks energy)

In general, we want q-circuits that act on multiple qubits. We denote these qubits using notations like q_0 (first qubit), q_1 , q_n , where $n-1$ is the total number of qubits in our system.

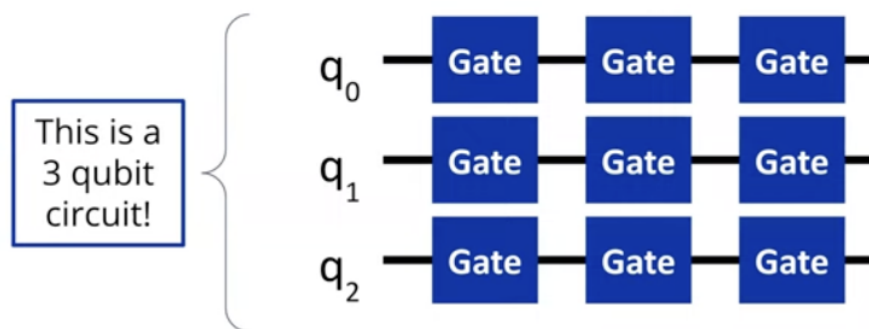
We can have as many gates as we want in a circuit and we apply them one at a time from left to right.

We can have as **many gates** as we want. We apply them one at a time from left to right:

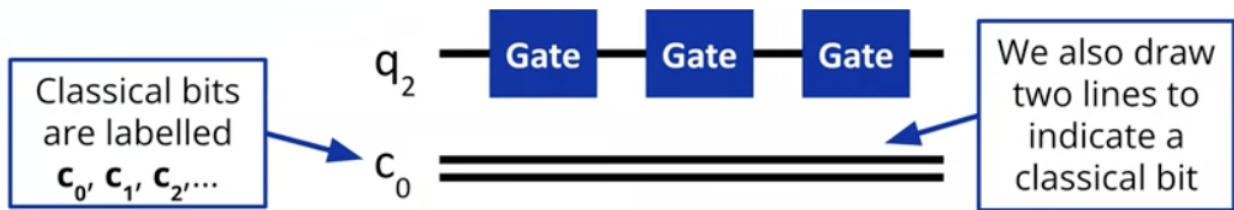


We can have Q-circuits that have as many qubits as we want, by adding more qubits with their own lines:

We can also have as **many qubits** as we want, by adding more qubits with their own lines below:



Quantum circuits can have classical bits (c_0 , c_1 , ...) inside them. They can be incorporated as well but are a advanced topic:



Key Questions: How do we design interesting q-circuits to perform certain types of computations.