Seminar 7

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Multi-qubit Circuits

Week 6 | October 30th, 2022

Lecture Overview

- Amplitudes of states and Born's rule
- The Z gate
- Multi-Qubit Circuits
- The CX gate

Amplitudes & Born's Rule

The amplitude of a state is the contribution it makes to a superposition. Mathematically, it is the number in front of a state.

|0> broken down into: 1 * |0> + 0 * |1>

|1> broken down into: 0 * |0> + 1 * |1>

Classically, these are the only two options. Quantum mechanically, there are many possible amplitudes.

|state> = root(0.7) |0> + root(0.3) |1> (O will have a higher probability being the collapsed state)

|state> = root(0.5 |0> + root(0.5) |1> (Equal probability of 0 and 1 being the collapsed state)

The above is equivalent to: |+> = (1/root(2))|0> + (1/root(2))|1>

Born's Rule:

Born's rule says that the square of these amplitudes give the probability of measuring a state as 0 or 1:

$$|\mathrm{state}\rangle = \mathbf{a}|\mathbf{0}\rangle + \mathbf{b}|\mathbf{0}\rangle$$

$$\mathrm{prob}(\mathbf{0}) = \mathbf{a}^{\mathbf{2}}$$

$$\mathrm{prob}(\mathbf{1}) = \mathbf{b}^{\mathbf{2}}$$

We can use Born's rule to determine the probability of measuring 0 or 1 for any state!

a^2 + b^2 = 1 **(implied derivation)**

The Z Gate

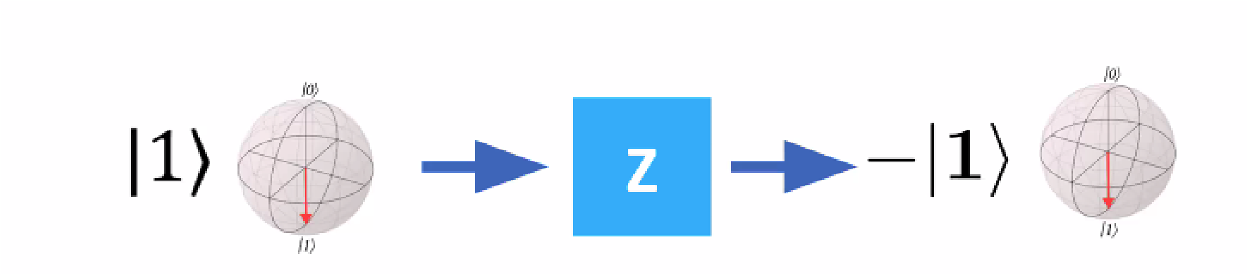
Two New Gates:

Z Gate & Controlled X Gate

Z Gate

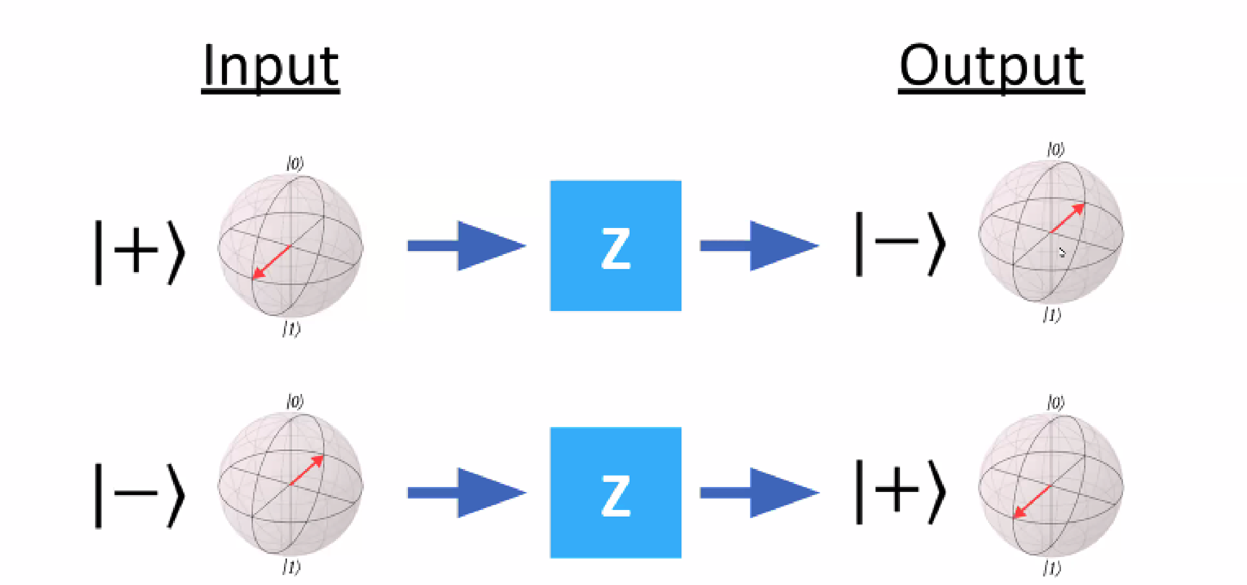
The Z gate is a 180 degree rotation around the z-axis (when applied to the "0" state, nothing will change).

When applied to the “1” state:



The minus sign is an example of phase. The sign between the 0 and 1 is called phase. Phase is responsible for interference.

The Z gate rotates the + and - states:



Process of applying the Z Gate to a Specific State:

Z acting on the + state	$\mathbf{Z}\|+\rangle = \mathbf{Z}(\frac{1}{\sqrt{2}}\|\mathbf{0}\rangle + \frac{1}{\sqrt{2}}\|\mathbf{1}\rangle)$
Distribute Z to each state (0 and 1)	$\mathbf{Z}\|+\rangle = \frac{1}{\sqrt{2}}\mathbf{Z}\|\mathbf{0}\rangle + \frac{1}{\sqrt{2}}\mathbf{Z}\|\mathbf{1}\rangle$
Apply Z to 0 and 1 states	$\mathbf{Z}\|+\rangle = \frac{1}{\sqrt{2}}\|\mathbf{0}\rangle + -\frac{1}{\sqrt{2}}\|\mathbf{1}\rangle$
Addition of negative = subtraction	$\mathbf{Z}\|+\rangle = \frac{1}{\sqrt{2}}\|\mathbf{0}\rangle - \frac{1}{\sqrt{2}}\|\mathbf{1}\rangle$

Thus: **Z |+> = |->**

Coin Toss vs. Superposition

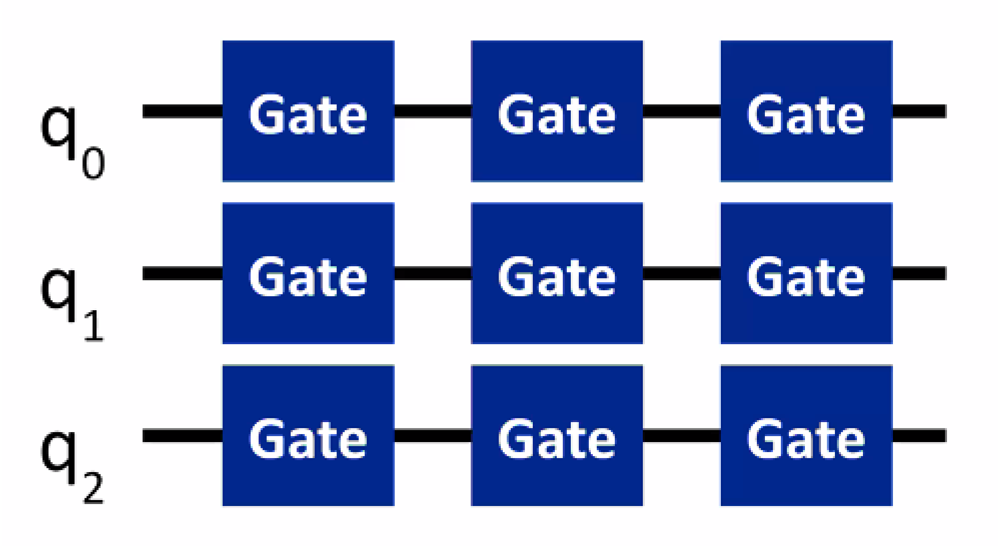
Applying a Hadamard gate similar to flipping a coin. Applying two continuous Hadamard gates will revert the action of the Hadamard gate. This is the result of quantum interference with quantum gates and thus the actions cancel each other out resulting in a 100% probability of getting the 0 state. This does not happen in the classical situation of flipping a coin.

Multi-Qubit Circuits & Gates

- Most quantum algorithms and protocols operate using multiple qubits, not just one
- What we learn this week can be used for several powerful applications that we'll explore soon!

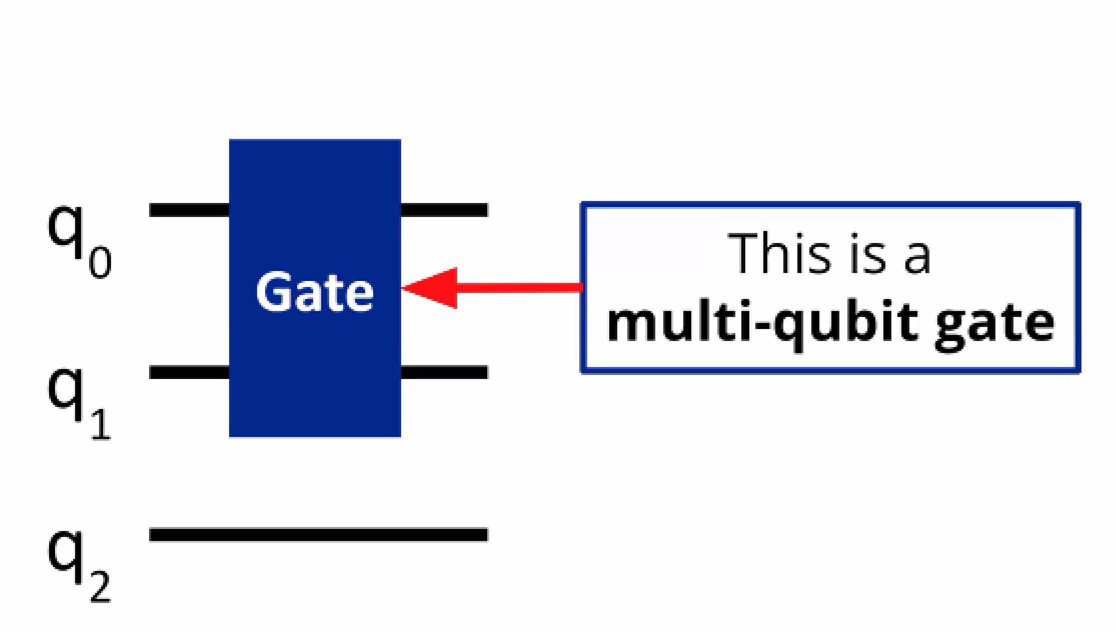
Quantum Circuits

So far, we've seen single qubit circuits with multiple gates. But we can also have as many qubits as we want, by adding more qubits with their own linea



Measurements become really important now!

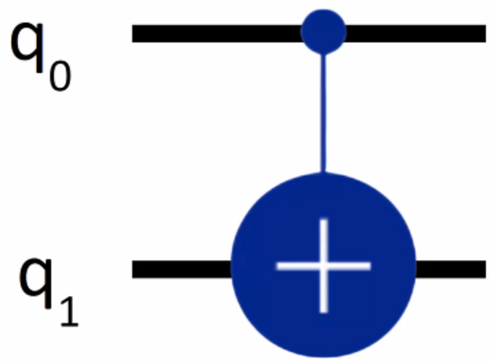
We can have gates that act on multiple qubits at once. These are called multi-qubit gates.



Controlled X Gate (Multi-Qubit Gate)

The CX Gate or CNOT.

There are two qubit where the top qubit is the control qubit and the bottom qubit is the target qubit



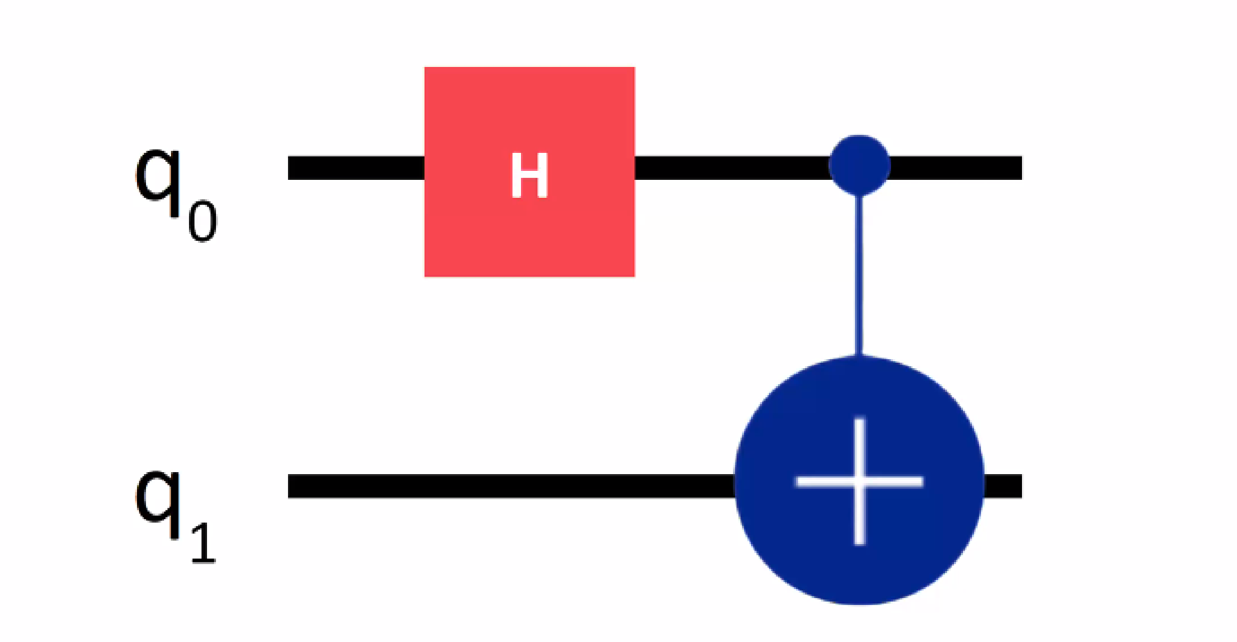
Input		Output	
Control bit	Target bit	Control bit	Target bit
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

The state of your work depends on the project leader's state of satisfaction with it. The gate that gets applied to the target qubit depends on the control qubit - **This is the controlled gate**

- The control qubit never changes.
- If the control is 0, do nothing to the target.
- If the control is 1, apply an X to the target.

The state of one qubit would be controlled by a superposition of states in another qubit. In other words, one system would be deeply related to another through quantum mechanical properties.

This is circuit creates entanglement.



After applying the CNOT gate the quantum system becomes entangled!

Hadamard + CNOT gate together will create an entangled state in a quantum system!

If you measure the state of one qubit, the other qubit will collapse and will be the same state as the other qubit.

Questions:

What is a phase?

What is the symbol used in the control gate diagram?

How does entanglement work with CNOT gate and |+> state?