Dimensions are independent, 9×9 is just a triple 3×3, so we'll just do one dimension at a time.

- $p = [x, \dot{x}, \ddot{x}]^T$ is some part of the state vector.
- θ is a target specific time constant sampled $\sim\mathcal{U}(\Theta)$
- $\alpha = \frac{1}{\theta}$ is the reciprocal of the time constant.
- σ is a target specific maneuver constant sampled $\sim\mathcal{U}(\Sigma)$

You had jotted down the following in our meeting:

$$p(k) = \begin{pmatrix} 1 & t & \frac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & e^{-\frac{T}{\theta}} \end{pmatrix} p(k-1) + \begin{pmatrix} 0 \\ 0 \\ \sqrt{1 - \left(e^{-\frac{T}{\theta}}\right)^2} \mathcal{N}(0, \sigma^2) \end{pmatrix}$$

I still couldn't quite figure out, in particular, why the process noise looked like that.

The Singer model in `MATLAB` is described as follows:

$$p(k) = \begin{pmatrix} 1 & T & \frac{\alpha T - 1 - e^{-\alpha T}}{\alpha^2} \\ 0 & 1 & \frac{1 - e^{-\alpha T}}{\alpha} \\ 0 & 0 & e^{-\alpha T} \end{pmatrix} p(k-1) + w(k)$$

$w(k)$ is singer process noise, and then after pulling some threads I couldn't figure out how they come up with this. They do reference what I assume is the actual Singer [1] paper.

Singer views the world as constant velocity, with turns, evasive maneuvers, and atmospheric turbulence as perturbations on an otherwise constant velocity trajectory. Acceleration is the "maneuver variable", it is parameterized by variance (magnitude) σ_m^2 and the time constant (duration) θ.

- $A_{\max}$ is the maximum acceleration (symmetric, so $a \in [-A_{\max}, A_{\max}]$).
- $P_{\max}$ is the probability of selecting $A_{\max}$
- P_0 is the probability of selecting 0 acceleration.
- $\sigma_m^2 = \frac{A_{\max}^2}{3}(1 + 4P_{\max} - P_0)$, which I do not follow at all.
- I imagined a pmf triangle or maybe pentagon, but a diagram shows a uniform rectangle everywhere with what appear to be spikes, biggest at 0, smaller ones at $\pm P_{\max}$?
- I don't think I care about this spiky distribution? If it even is? I think maybe it's just uniform?

Singer initializes filters with two measurements exactly how you think $\hat{x} = \begin{pmatrix} y(1) & \frac{y(1)-y(0)}{t} & 0 \end{pmatrix}^T$ and all noise in a, not even a measurement noise in the other two?.

Singer throws out some numbers. $\alpha \approx \frac{1}{60}$ for a lazy turn, $\alpha \approx \frac{1}{20}$ for an evasive turn, and $\alpha \approx 1$ for atmospheric turbulence. I don't think I achieve an evasive turn with this though…?

In this $w(k) =$ white noise driving function with variance $\begin{pmatrix} 0 \\ 0 \\ 2\alpha\sigma^2 \end{pmatrix}$. I don't understand this. Why is there a 2?

My T is always very small. Singer notes that when T is small the model reduces to Newtonian constant acceleration.

Singer is in the $10^{-}2$ to 10^2 of seconds range, and my T is in the $10^{-}3$ to $10^{-}4$ range, and will only get smaller when adding more sensors due to my "fast forward the universe just until the next sensor is idle". I'm only ever moving everything forward by the shortest remaining dwell time. I am pretty sure I break physics, but frankly don't really care here? Due to my microscopic T, I don't think that any "Sample and multiply by magnitude" will ever do what I want. I'm going to try to dump back in my "annoying" targets who. I want to prioritize these programmable targets. Due to how difficult I found it to be to find an interesting scenario, and Peter's feedback that he'd like to see some replayable scenes to evaluate algorithms among, I'm basically going to just implement much more *Videogamey* targets that are programmable. I'm just going to put timers in them and they'll just go from 0 to some A after a set time, then maintain that for some time. That instantaneous acceleration may technically break somebody's back, but I'm integrating over such a short τ that it'll be lost in the measurement noise even if you were staring right at it. Plus this should let me do even more fun targets that do things like count how often they think they've been measured or something. Model drones. Even Singer says this is only appropriate for airships, submarines, and lumbering 1970s airplanes. We care about missiles, tumbling chaff, and quadcopters. I'm giving up on strictly following Singer for now. If doing some stupider noisy acceleration model is sufficient for VKB [2] then it's good enough for me.

Bibliography

[1] R. A. Singer, "Estimating optimal tracking filter performance for manned maneuvering targets," *IEEE Transactions on Aerospace and electronic systems*, no. 4, pp. 473–483, 1970.

[2] G. van Keuk and S. S. Blackman, "On phased-array radar tracking and parameter control," *IEEE Transactions on aerospace and electronic systems*, vol. 29, no. 1, pp. 186–194, 1993.