

### Solution to *B.5-3*

Let  $n, n_0, n_2$  be the total number of nodes, the number of leaves, and the number of degree-2 nodes in any binary tree, respectively. If  $n = 1$ , then there's only the root in the binary tree, and we have  $n_0 = 1, n_2 = 0$ . Therefore,  $n_2 = n_0 - 1$ .

Suppose that  $n_2 = n_0 - 1$  holds for any binary tree of size  $n$  where  $n \geq 1$ . Let's add a new node to the tree making the size be  $n' = n + 1$ . If the new node is appended to a leaf in the original tree, then the number of degree-2 nodes is  $n'_2 = n_2$ , the number of leaves is  $n'_0 = n_0$  and of course we have  $n'_2 = n'_0 - 1$  since  $n_2 = n_0 - 1$ . Otherwise if the new node is appended to a degree-1 nodes in the original tree, then we have  $n'_2 = n_2 + 1, n'_0 = n_0 + 1$  and still we have  $n'_2 = n'_0 - 1$ .