Solutions for Chapter 24

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Solution to Exercise 24.1-2

Suppose there's an **acyclic** path from s to v being $\langle v_0, v_1, \ldots, v_k \rangle$, where $v_0 = s$ and $v_k = v$. Similarly with the proof of lemma 24.2, the path has at most $|V| - 1$ edges, and so $k \leq |V| - 1$. Each of the $|V| - 1$ iterations of the **for** loop of lines 2-4 relaxes all E edges. Among the edges relaxed in the ith iteration, for $i = 1, 2, \ldots, k$, is (v_{i-1}, v_i), and $d[v_i]$ becomes finite. Therefore, $d[v_k] = d[v]$ will become finite no later than the kth iteration.

The opposite direction can be proved by appealing to the no-path property (Corollary 24.12).

Solution to Exercise 24.1-3

For any pair of vertices u, $v \in V$, the shortest path between u and v having the minimum number of edges must be acyclic. So we can get the idea from the proof of Lemma 24.2. Consider any vertex v that is reachable from s, and let $p = \langle v_0, v_1, \ldots, v_k \rangle$, where $v_0 = s$ and $v_k = v$, be any acyclic shortest path from s to v. Path p has at most $\min(m, |V| - 1)$ edges, i.e. $k \leq \min(m, |V| - 1)$. Among the edges relaxed in the ith pass, for $i = 1, 2, \ldots, k$, is (v_{i-1}, v_i). By the path-relaxation property, no relaxation will occur on p after the kth pass (with the precondition that G has no negative-weight cycles. So we can make a simple change to the **for** loop of lines 2–4 of Bellman-Ford: if no edge is relaxed during the pass, the loop can terminate. Also, lines 5–8 of Bellman-Ford can be removed because this simple change causes the algorithm to lose the ability to detect negative-weight cycles.