QEA Project 1: Boat Design Technical Report

Hyper Rho Boat

Pranavi Boyalakuntla
Partner: Jack Mao

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Proposed Design

The hull of this boat is defined by $z = \frac{6}{8^4}y^4 + |ln\frac{3-x}{3+x}|$. This is a three-dimensional implicit function that defines the boat as a whole.

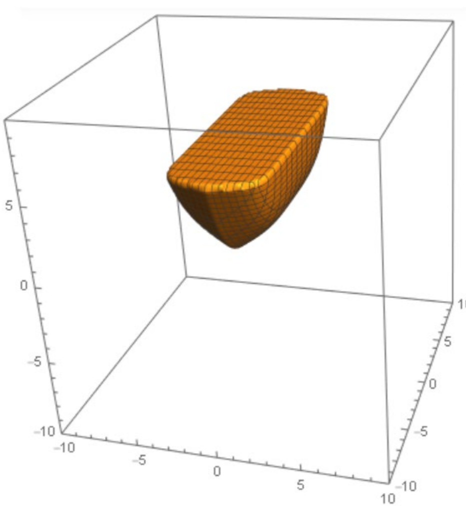

Figure 1: This is a three-dimensional rendering of the function on Mathematica.

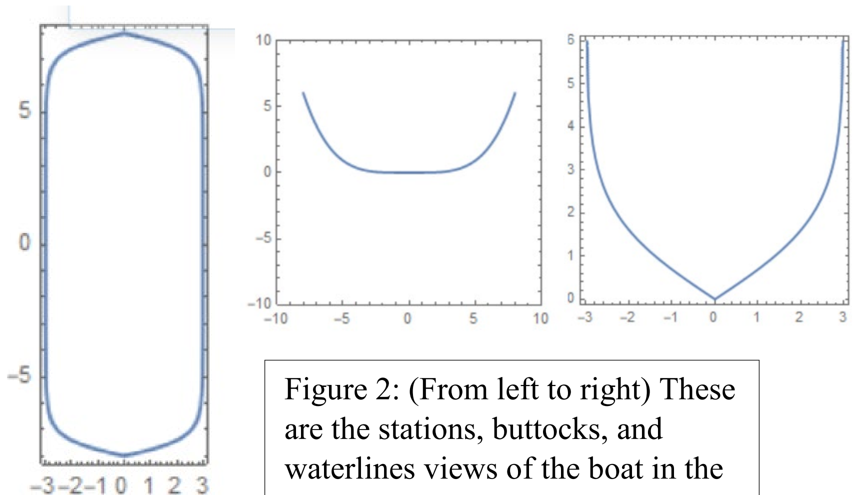

Figure 2: (From left to right) These are the stations, buttocks, and waterlines views of the boat in the x-y plane, y-z plane, and x-z plane, respectively.

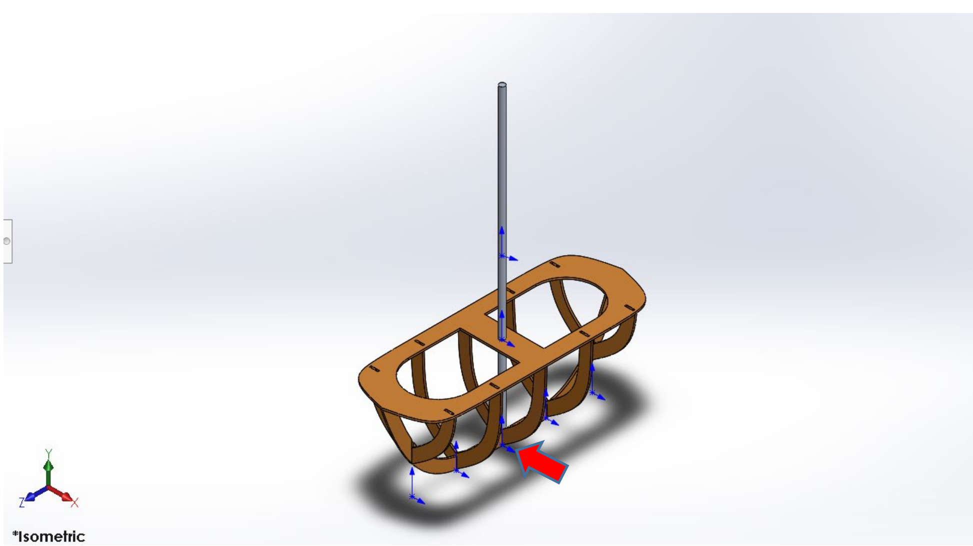

Figure 3: This is the CAD assembly of the boat created by the equations above. The ribs are the equation of the boat at specific chosen y-values. The chosen origin is pointed to by the red arrow.

This is an ideal shape for the boat because the narrower and pointed front and back allows it to encounter less resistance with water, therefore increasing the speed. In addition, the very apparently pointed-at-the-bottom keel allows masses to rest relatively in line with the center axis of the boat which will ensure that the boat floats level with only an error of human assembly.

The boat holds a ballast of .844 grams. When approximating the effect of the ballast on the center of mass of the boat as a whole, it is estimated as a singular point mass lying somewhere on the z-axis. In actuality, the ballast is a combination of washer of different weights. As washers can't be condensed to a singular point very easily because they have a non-negligible volume, the placement of the washers is crucial to maintaining a level floating boat. The figure below explains the chosen placement of the washers.

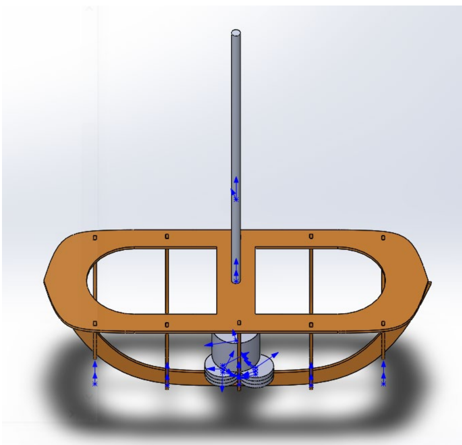

Figure 4: There is a stack of washers weighing .540 grams concentric around the center axis of the mast. Equally, to both the left and right of the center axis of the mast lie washers weighing about 50.6 grams in stacks of 3. They are ideally equidistant, but this is hard to get exactly right in practice. In reality, a leveling tool was used to make sure that the boat would float level.

As the boat design was adjusted, the 'a', 'b', and 'c' of the final hull were fine tuned. They impact the width, height, and length. This ultimately had an effect on the center of mass of the boat. To calculate this particular boat's mass and center of mass, SOLIDWORKS' "mass properties" tool was used. This is the most accurate way to account for the final mass of the boat and COM because it allows one to include the mass densities of the materials, the shape of the mast, and the fact that the assembled boat isn't a solid figure, but rather sections of the curve at different values. To calculate the COM and final mass of the boat and the ballast, the ballast was estimated to be a point mass 1.11 inches above the boat assemblies' origin. The ballast was reduced to this location using a SOLIDWORKS assembly of the washers in a model of the orientation they were put in the boat. The final COM is found using a weighted average of the COM of the boat without the ballast and the COM of the ballast.

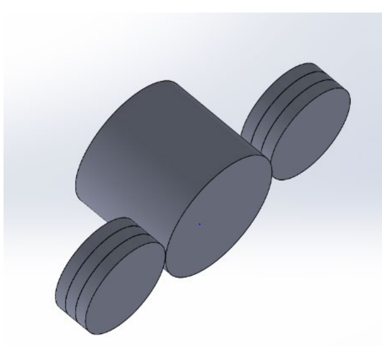

Figure 5: This is the SOLIDWORKS assembly of the ballast to figure out what coordinate points the ballast can be estimated to or, in other words, what the COM of the ballast is.

Design Justification

Objects float if the mass of the maximum volume of water that they can displace is less than their own mass. In other words, an object will float if it is less dense than water. The volume of the boat is 337.06 in^3. The mass of the boat and ballast is 1.07 kg. The density of the boat is 0.0032 $\frac{\mathrm{kg}}{\mathrm{in}^3}$. The density of water is 0.0164 $\frac{\mathrm{kg}}{\mathrm{in}^3}$. Since the density of the boat is less than the density of water, the boat will float.

In order to determine the angle of vanishing stability, or the point at which the righting moment on the boat will no longer be enough to return it back to the vertical position, a curve representing the righting moment at different heel angles is necessary. A righting moment is the torque exerted by the buoyancy force that is applied due to the center of buoyancy and center of mass not being aligned vertically. In order to find the center of buoyancy, we first need to begin by finding the mass of the boat. The mass of the boat can be found through using a SOLIDWORKS model or using the following integral: $M = \iint_R \rho \, dA$, where ρ is material density and the bounds of integration are the bounds of the boat. The mass of this boat was found using a SOLIDWORKS assembly including both the boat and the ballast.

The center of mass needs to be found for the boat so that the distributed force of the righting moment can be approximated to applying to the center of mass. The center of mass in each direction (x, y, and z) can be founds through a SOLIDWORKS assembly or the following integral: $\{x, y, z\}COM = \frac{1}{M} \iint_R \{x, y, z\} \times \rho \, dA$, where ρ is material density, M is the total mass of the boat, and the bounds of integration are the bounds of the boat. For this integral, use x when solving for an x-value, y when solving for a y-value, and z when solving for a z-value. When calculated the COM of this particular boat, the COM of the boat assembly and the ballast assembly were found separately using SOLIDWORKS. Then, they were combined using a weighted average to find the final COM of the boat and ballast.

The forces acting on the boat need to equal zero in order for the boat to be in static equilibrium. This means that the magnitude of the buoyancy force equals the magnitude of the force of gravity on the boat. They are both vectors in opposite directions such that when they are added together, they equal zero. $F_{gravity}$ can be calculated by multiplying the mass calculated earlier by 9.8 $\frac{\mathrm{m}}{\mathrm{s}^2}$ which is the approximated acceleration due to gravity on Earth. $F_{buoyancy}$ is equal to this value, but has a direction opposite to it, according to the principle discussed earlier.

Turning the waterline as the heel angle of the boat changes is much easier than turning the equation of the boat. Therefore, the vector direction for the force of buoyancy changes along with the heel angle. Since the magnitude of the force of buoyancy is equal to the force of gravity, the force of gravity is a scalar multiple of the unit vector specifying direction. The unit vector specifying direction is $\{-\sin\theta, 0, \cos\theta\}$ for the planes that this particular boat is in.

The force of buoyancy acts at its own center: the center of buoyancy. Finding the position of the center of buoyancy essentially uses the same method as finding the center of mass, but only for the area of the boat submerged in water. The mass of the displaced water needs to be found because, then, the draft of the boat can be found. This is because the mass of the displaced water is equal to the mass of the boat. The mass of the displaced water can be found using the following integral: $M = \iint_R \rho\, dA$, where ρ is material density and the bounds of integration are the bounds of the area of the boat submerged underwater. The location of the center of buoyancy can be found using the following integral: $\{x,y,z\}COM = \frac{1}{M} \iint_R \{x,y,z\} \times \rho\, dA$, where ρ is material density, M is the total mass of the boat, and the bounds of integration are the bounds of the area of the boat submerged underwater. For this integral, use x when solving for an x-value, y when solving for a y-value, and z when solving for a z-value.

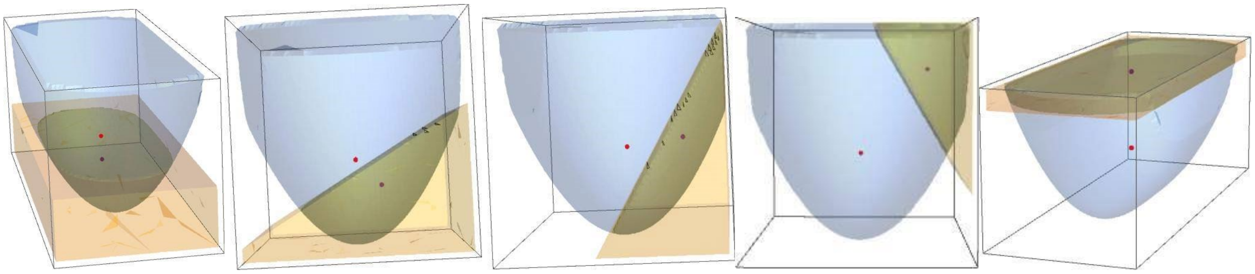

Figure 6: These are diagrams showing the COB and COM at different heel angles in degrees: 0, 30, 60, 120, 180, respectively.

The righting moment is a torque. The equation to find a torque is $\tau = r \times F$, where r is the lever arm between the center of mass of the object the torque is being applied to and the point at which the force is being applied and F is the force being applied. In this case, the vector r can be found by subtracting the position of the center of mass of the whole boat from the center of buoyancy. The vector F is the buoyancy force as that is the force creating the righting moment.

When modeling this boat using Mathematica, the equation initially used to define the bounds of the boat had way too many asymptotes and Mathematica could not integrate it properly with the tools employed. In order to simplify the problem, two methods were considered. The boat could be modeled as a two dimensional extrusion from the mid plane, or the function in the x-z cross section could be approximated to a similar function while keeping the y-z curve the same and modeling it in three-dimensions. The latter, after many trials, seemed to be the better option. The function was approximated to $z = \frac{6}{8^4} y^4 + \frac{x^2}{2.5}$.

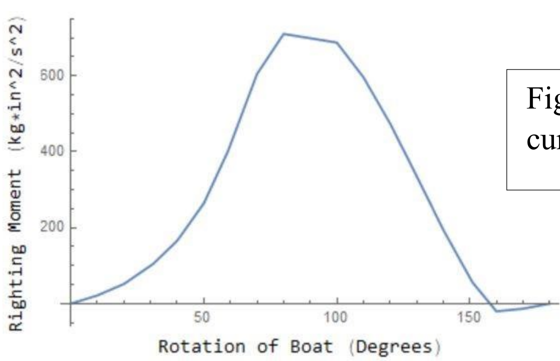

Figure 7: This is the final calculated angle of vanishing stability curve for this boat.

From this AVS curve, the righting moment becomes zero at a little less than 160 degrees. This is the angle at which the boat will no longer right itself. The maximum righting torque is at around 70 degrees. This is when it the force required to continue increasing the heel angle will have to be the greatest.

Expected Performance

If the boat doesn't float flat, the angle of vanishing stability curve will have little to no righting torque until the angle at which the boat leans. This is not the case with this AVS curve. In addition, the masses in the boat are ideally symmetrically distributed.

The angle of vanishing stability for this curve is between 150 degrees and 160 degrees. Initially, the AVS curve predicted an incorrect AVS of around 130 degrees. After finishing the assembly and shrink wrap, the team debugged the AVS curve generation and the correct AVS of between 150 degrees and 160 degrees was proven to be true (*refer to figure 7 for correct AVS curve*).

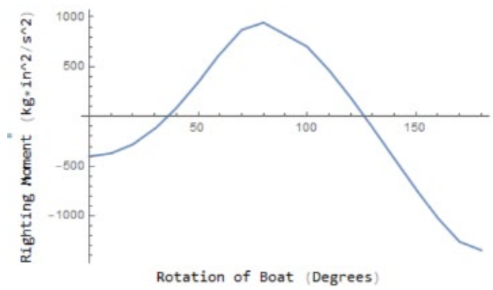

Figure 7: This is the incorrect calculated angle of vanishing stability curve for this boat that the ballast position and amount was reliant on.

This boat is expected to be relatively fast as the hull tapers in to a point and will therefore incur less resistance in water (*refer to figure 1 to visually understand how the hull tapers in*).