Righting Moment of Aquatic Vessels

To choose the shape of the boat, we took into consideration of 2 factors as design requirements: how fast the boat will sail across the water and if the AVS (angle of vanishing stability) of the boat is between 120 - 140 degrees.

Orientation of the boat

Before designing the equation of the boat, we made several definitions. The boat will be created on a x-y-z plane. The x-z cross section of the boat will determine the hull shape. The y-z cross section determines the cross sections that are parallel to the keel. The x-y cross sections define the cross sections that are parallel to the deck.

Functional Representation of the Boat

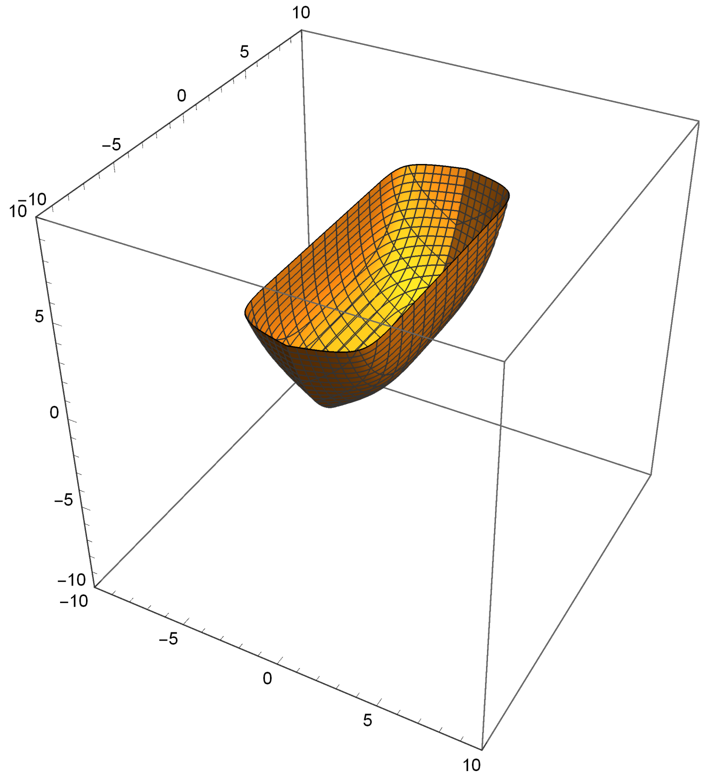
The two graphs below are two different ways of representing the boat in 3-dimensions. This is the boat equation:

In[108]:= `z == 6 / 8^4 * y^4 + Abs[Log[(3 - (x)) / (3 + (x))]]`

Out[108]= $z == \frac{3\,y^4}{2048} + \mathrm{Abs}\left[\mathrm{Log}\left[\frac{3-x}{3+x}\right]\right]$

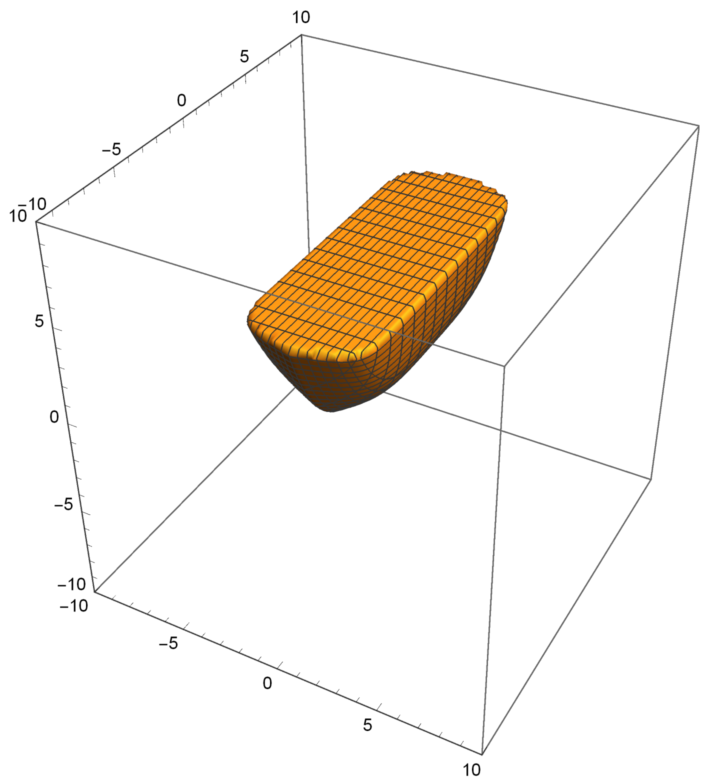
In[86]:=
```
With[{a = 3.01, b = 8, c = 6, range = 10},
 ContourPlot3D[z == c / b^4 * y^4 + Abs[Log[(a - (x)) / (a + (x))]], {x, -a, a}, {y, -b, b},
  {z, 0, c}, PlotRange → {{-range, range}, {-range, range}, {-range, range}}]]
```

Out[86]=



In[87]:=
```
With[{a = 3.001, b = 8, c = 6, range = 10},
 RegionPlot3D[c / b^4 * y^4 + Abs[Log[(a - x) / (a + x)]] < z < c && -a < x < a && -b < y < b,
  {x, -3, 3}, {y, -b, b}, {z, -c, c},
  PlotRange → {{-range, range}, {-range, range}, {-range, range}}, PlotPoints → 50]]
```

Out[87]=



Below, we are defining the implicit regions of this three-dimensional shape, the water line, and the overlapping area labeled “under”.

In[89]:=
```
boat = With[{a = 3, b = 8, c = 6}, ImplicitRegion[
    c / b^4 * y^4 + Abs[Log[(a - x) / (a + x)]] < z < c && -a < x < a && -b < y < b, {x, y, z}]];
```

In[90]:= ImplicitRegion$\left[\text{Abs}\left[\text{Log}\left[\frac{3-x}{3+x}\right]\right] + \frac{3\,y^4}{2048} < z < 6 \;\&\&\; -3 < x < 3 \;\&\&\; -8 < y < 8,\ \{x, y, z\}\right]$;

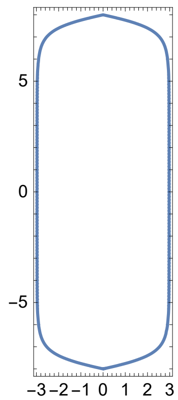
In[91]:=
```
water = ImplicitRegion[0 < z < Tan[θ Degree] * x + d && -3 < x < 3 && -8 < y < 8, {x, y, z}];
```

In[92]:=
```
under = RegionIntersection[boat, water];
```

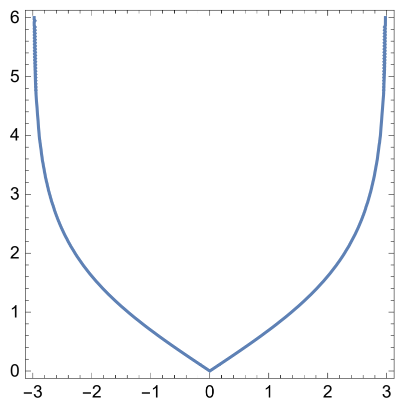
Renderings of the Cross Sections of the Boat

Below the renderings of the sections from the three directions (stations, buttocks, and waterlines) are shown.

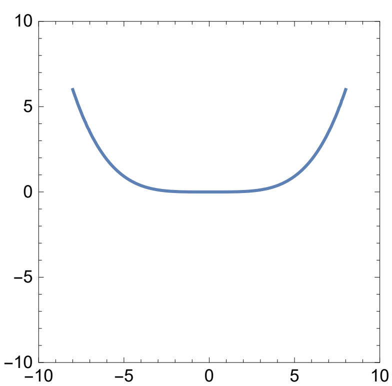
In[93]:=
```
ContourPlot[6 == 6 / 8^4 y^4  + Abs[Log[(3.001 - x) / (x + 3.001)]],
 {x, -3, 3}, {y, -8, 8}, AspectRatio → Automatic]
```

Out[93]=


In[94]:=
```
ContourPlot[z == Abs[Log[(3.001 - x) / (x + 3.001)]], {x, -3, 3}, {z, 0, 6}]
```

Out[94]=


In[95]:=
```
ContourPlot[z == 6 / 8^4 y^4 , {y, -8, 8}, {z, 0, 6},
 AspectRatio → Automatic, PlotRange → {{-10, 10}, {-10, 10}}]
```

Out[95]=


Plot of Righting Moment as a Function of Heel Angle

The initial equation defining the boat has way too many asymptotes and doesn't allow for analytical solutions at some points using the tools learned. Therefore, we had two different approaches we could take. The first one was modeling the boat as a two dimensional extrusion from the mid plane. The second one was approximating the function to be a more easily integrated function. The new function is:

```
In[109]:= z == 6 / 8^4 * y^4 + x^2 / 2.5
```

$$Out[109]= z == 0.4\, x^2 + \frac{3\, y^4}{2048}$$

Below is the Mathematica function that calculates the AVS curve for the approximation function.

```
In[96]:= Submerged[theta_] := Module[{}, angle = theta °;
  ballast = .844;
  mass = 0.23 + ballast;
  com = {0, 0, (7.05 * .23 + 1.11 * ballast) / (mass)};
  boat =
   ImplicitRegion[x^2/2.5 + 6. * y^4 / 8.^4 < z < 6. && -8. < y < 8. && -2.9 < x < 2.9, {x, y, z}];
  water = ImplicitRegion[If[angle < 90 °, z < Tan[angle] x + d, z > Tan[angle] x + d],
    {{x, -2.9, 2.9}, {y, -8., 8.}, {z, 0, 6.}}];
  under = RegionIntersection[boat, water];
  disp = Quiet[1000. * (0.0254^3)
     NIntegrate[1., {x, y, z} ∈ under, AccuracyGoal → 3, PrecisionGoal → 3]];
  draft = Quiet[N[d /. FindRoot[disp == mass, {d, 0, 10},
       AccuracyGoal → 3, PrecisionGoal → 3]]];
  cob = RegionCentroid[under /. {d → draft}];
  cob = {cob[[1.]], cob[[2.]], cob[[3.]]};
  buoyancy = -1/10 (mass 98 / 0.0254) {-Sin[angle], 0, Cos[angle]};
  torque = (cob - com) × buoyancy;
  torque[[2.]]]
```

In[97]:=
```
Labeled[ListLinePlot[Table[{theta, Submerged[theta]},
   {theta, {0, 10, 20, 31, 40, 50, 59, 70, 80, 100, 110, 120, 130, 140, 151, 160, 170, 180}}]],
 {"Rotation of Boat (Degrees)", "Righting Moment (kg*in^2/s^2)"},
 {Bottom, Left}, RotateLabel → True]
```

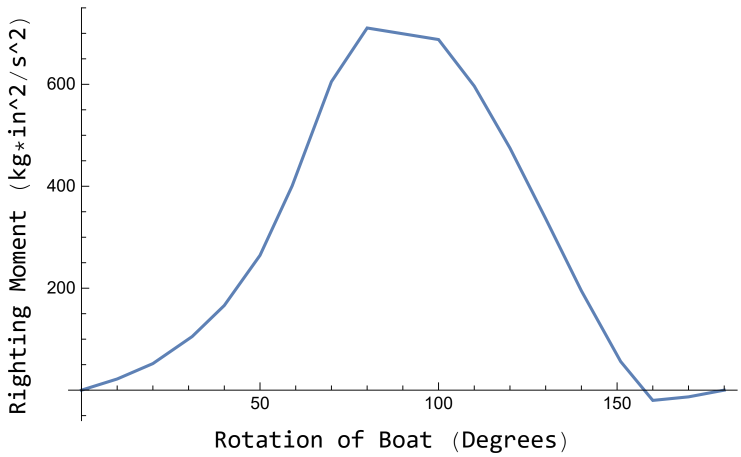
... DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

... DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

... DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

... General: Further output of DiscretizeRegion::drf will be suppressed during this calculation.

Out[97]=



This AVS curve shows an AVS of somewhere between 150 and 160 degrees. The reason for this is because we realized there was something wrong with our AVS curve after we finished sealing the boat. After fixing the AVS curve, the new AVS angle was way off from what we predicted.

Positions of COM and COB at Different Heel Angles

At 0 Degrees:

In[132]:=
```
Submerged[0]
```

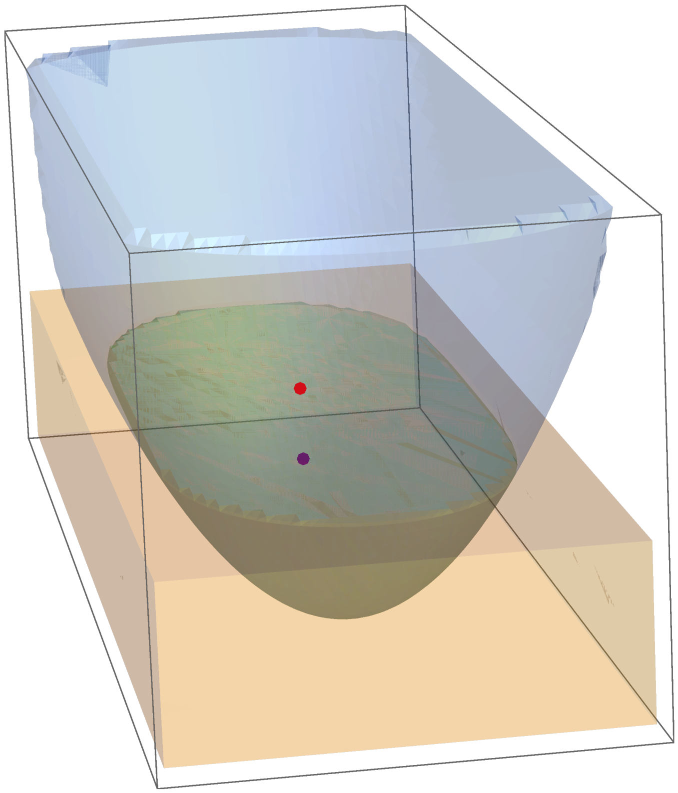
... DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

... DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

Out[132]= -5.80532×10^{-15}

In[117]:=
```
With[{angle = 0, dd = draft},
 Show[RegionPlot3D[{water /. {d → dd, θ → angle}, boat, under /. {d → dd, θ → angle}},
   AspectRatio → Automatic, PlotPoints → 40, PlotStyle → Opacity[0.2]], ListPointPlot3D[
   {cob}, PlotStyle → {Purple}], ListPointPlot3D[{com}, PlotStyle → {Red}]]]
```

Out[117]=



In[133]:=
```
com
cob
```

Out[133]= {0, 0, 2.38207}

Out[134]= $\{-1.40097 \times 10^{-17}, -1.34804 \times 10^{-12}, 1.41701\}$

At 60 Degrees:

In[135]:=
```
Submerged[60]
```

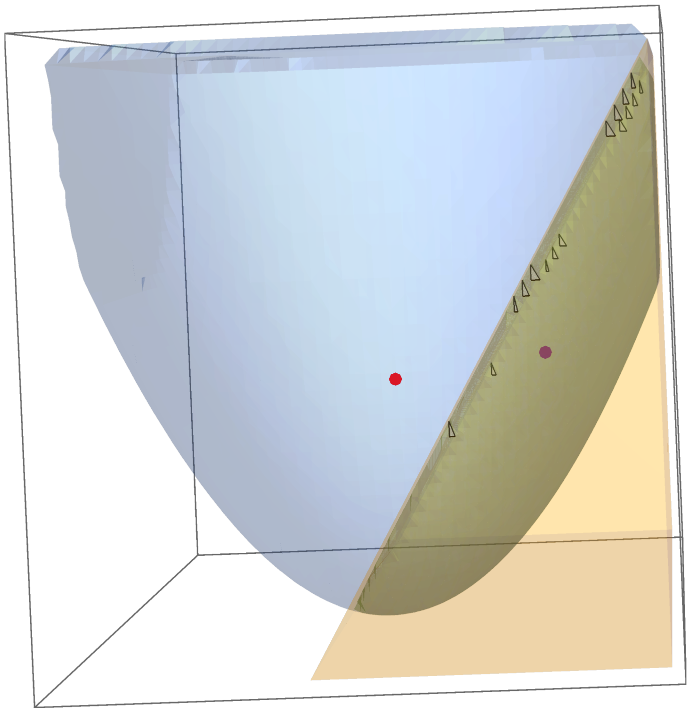
DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

Out[135]= 419.933

In[121]:=
```
With[{angle = 60, dd = draft},
 Show[RegionPlot3D[{water /. {d → dd, θ → angle}, boat, under /. {d → dd, θ → angle}},
   AspectRatio → Automatic, PlotPoints → 40, PlotStyle → Opacity[0.2]], ListPointPlot3D[
   {cob}, PlotStyle → {Purple}], ListPointPlot3D[{com}, PlotStyle → {Red}]]]
```

Out[121]=



In[138]:=
```
com
cob
```

Out[138]= {0, 0, 2.38207}

Out[139]= {1.63409, 0., 2.60881}

At 120 degrees:

In[140]:=
```
Submerged[120]
```

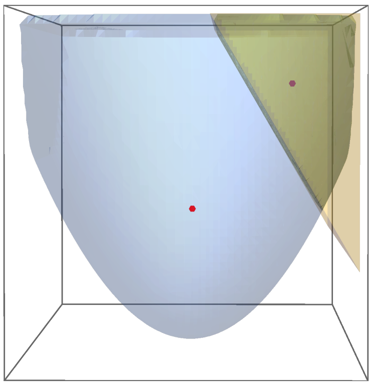
... DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

... DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

Out[140]= 474.917

In[125]:=
```
With[{angle = 120, dd = draft},
 Show[RegionPlot3D[{water /. {d → dd, θ → angle}, boat, under /. {d → dd, θ → angle}},
   AspectRatio → Automatic, PlotPoints → 40, PlotStyle → Opacity[0.2]], ListPointPlot3D[
   {cob}, PlotStyle → {Purple}], ListPointPlot3D[{com}, PlotStyle → {Red}]]]
```

Out[125]=



In[141]:=
```
com
cob
```

Out[141]= {0, 0, 2.38207}

Out[142]= $\{1.9436, 2.12792 \times 10^{-16}, 4.8276\}$

At 180 Degrees:

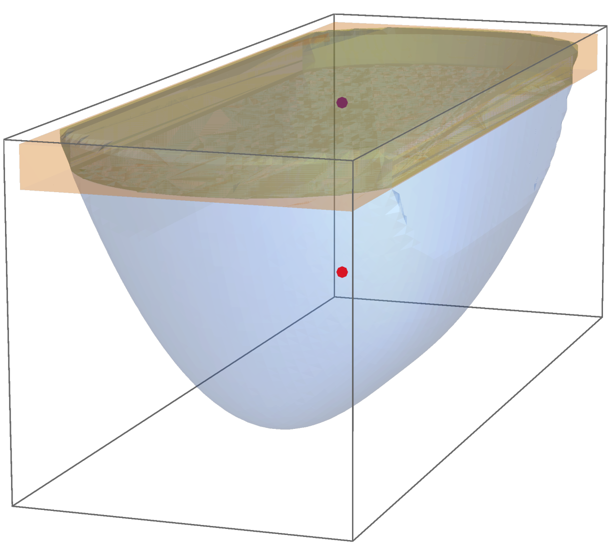
In[143]:=
```
Submerged[180]
With[{angle = 180, dd = draft},
 Show[RegionPlot3D[{water /. {d → dd, θ → angle}, boat, under /. {d → dd, θ → angle}},
   AspectRatio → Automatic, PlotPoints → 40, PlotStyle → Opacity[0.2]], ListPointPlot3D[
   {cob}, PlotStyle → {Purple}], ListPointPlot3D[{com}, PlotStyle → {Red}]]]
```

... DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

... DiscretizeRegion: DiscretizeRegion was unable to discretize the region ImplicitRegion[≪2≫].

Out[143]= 4.33934×10^{-7}

Out[144]=



In[145]:=
```
com
cob
```

Out[145]= {0, 0, 2.38207}

Out[146]= $\{-1.04719 \times 10^{-9}, 4.97959 \times 10^{-9}, 5.62118\}$