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Machine Learning Assignment -1

(Q.1) Here, $f(z) = \log_e(1+z)$

where,

$z = x^T x$, $x \in \mathbb{R}^d$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} \qquad x^T = [x_1, x_2 \cdots\cdots x_d]$$

$$\therefore x^T x = [x_1^2 + x_2^2 + \cdots\cdots + x_d^2]$$

Apply chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}(\log_e(1+z)) \cdot \frac{d}{dx}(x^T x)$$

$$= \frac{d}{dz}(\log_e(1+z)) \cdot \frac{d}{dx}(x_1^2 + x_2^2 + \cdots + x_d^2)$$

$$= \frac{1}{1+z} \cdot \frac{d}{dz}(z) \cdot (2x_1 + 2x_2 + \cdots\cdots + 2x_d)$$

$$= \frac{1}{1+z} \cdot 2(x_1 + x_2 + \dots + x_d)$$

$$= \frac{2}{1+z} \sum_{i=1}^{d} x_i$$

<u>(Q.2)</u>

$$f(z) = e^{-z/2},$$

where

$$z = g(y)$$

$$g(y) = y^T S^{-1} y$$

$$y = h(x) = x - \mu$$

using chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$\therefore \frac{df}{dz} = \frac{d}{dz}(e^{-z/2}) = -\frac{e^{-z/2}}{2}$$

$$\therefore \frac{dz}{dy} = \frac{d}{dy}(y^T S^{-1} y)$$

$$= \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) s^{-1} (y+h) - y^T s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{(y^T s^{-1} + h s^{-1})(y+h) - y^T s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{y^T s^{-1} y} + y^T s^{-1} h + h s^{-1} y + h^2 s^{-1} - \cancel{y^T s^{-1} y}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(y^T s^{-1} + s^{-1} y + h s^{-1})}{\cancel{h}}$$

$$= y^T s^{-1} + s^{-1} y + \lim_{h \to 0} (s^{-1} h)$$

$$= y^T s^{-1} + s^{-1} y$$

$$\therefore \frac{dy}{dx} = (x - \mu) = 1$$

$$\therefore \frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= \frac{e^{-z/2}}{2} \cdot (y^T s^{-1} + s^{-1} y) \cdot 1$$

$$= -\frac{e^{-z/2}}{2} \cdot \frac{1}{s} (y^T + y)$$

(Ans)

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