

$$\begin{aligned}
G_{12} &= \langle \mathcal{T} \psi_2 \psi_1^\dagger \rangle_0 - V_{34} \langle \mathcal{T} \psi_2 \overbrace{\psi_3^\dagger \psi_4^\dagger} \overbrace{\psi_4 \psi_3} \psi_1^\dagger \rangle_0 - V_{34} \langle \mathcal{T} \psi_2 \overbrace{\psi_3^\dagger \psi_4^\dagger} \overbrace{\psi_4 \psi_3} \psi_1^\dagger \rangle_0 \\
&\quad + V_{34} V_{56} \langle \mathcal{T} \psi_2 \overbrace{\psi_3^\dagger \psi_4^\dagger} \overbrace{\psi_4 \psi_3} \overbrace{\psi_5^\dagger \psi_6^\dagger} \overbrace{\psi_6 \psi_5} \psi_1^\dagger \rangle_0 + V_{34} V_{56} \langle \mathcal{T} \psi_2 \overbrace{\psi_3^\dagger \psi_4^\dagger} \overbrace{\psi_4 \psi_3} \overbrace{\psi_5^\dagger \psi_6^\dagger} \overbrace{\psi_6 \psi_5} \psi_1^\dagger \rangle_0 + \cdots \\
&= g_{12} + (-1)^2 V_{34} g_{13} g_{32} g_{44} + (-1) V_{34} g_{13} g_{34} g_{42} \\
&\quad + (-1)^3 V_{34} V_{56} g_{13} g_{35} g_{52} g_{46} g_{64} + (-1)^2 V_{34} V_{56} g_{13} g_{35} g_{54} g_{46} g_{62} + \cdots
\end{aligned}$$

$$\begin{aligned}
&= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \\
&\quad + \text{Diagram 4} + \text{Diagram 5} + \cdots
\end{aligned}$$

The diagrams represent Feynman diagrams for the Green's function G_{12} . Each diagram consists of a horizontal line with arrows pointing from left to right, representing fermion propagation. The vertices are labeled with numbers 1, 2, 3, 4, 5, 6. Diagram 1 is a simple line from 1 to 2. Diagram 2 shows a loop at vertex 3, with a wavy line connecting 3 to 4 and a fermion line from 4 back to 3. Diagram 3 shows a loop between vertices 3 and 4, with wavy lines connecting 3 to 4 and 4 to 3. Diagram 4 shows a loop between vertices 4 and 6, with wavy lines connecting 4 to 6 and 6 to 4. Diagram 5 shows a more complex loop structure involving vertices 3, 5, 4, and 6.