

$$\begin{aligned}
G_{12} &= \langle \psi_2 \bar{\psi}_1 \rangle_0 - V_{34} \langle \psi_2 \bar{\psi}_3 \bar{\psi}_4 \psi_4 \psi_3 \bar{\psi}_1 \rangle_0 - V_{34} \langle \psi_2 \bar{\psi}_3 \bar{\psi}_4 \psi_4 \psi_3 \bar{\psi}_1 \rangle_0 \\
&\quad + V_{34} V_{56} \langle \psi_2 \bar{\psi}_3 \bar{\psi}_4 \psi_4 \psi_3 \bar{\psi}_5 \bar{\psi}_6 \psi_6 \psi_5 \bar{\psi}_1 \rangle_0 + V_{34} V_{56} \langle \psi_2 \bar{\psi}_3 \bar{\psi}_4 \psi_4 \psi_3 \bar{\psi}_5 \bar{\psi}_6 \psi_6 \psi_5 \bar{\psi}_1 \rangle_0 + \cdots \\
&= g_{12} + (-1) \xi V_{34} g_{13} g_{32} g_{44} + (-1) V_{34} g_{13} g_{34} g_{42} \\
&\quad + (-1)^2 \xi V_{34} V_{56} g_{13} g_{35} g_{52} g_{46} g_{64} + (-1)^2 V_{34} V_{56} g_{13} g_{35} g_{54} g_{46} g_{62} + \cdots
\end{aligned}$$

$$\begin{aligned}
&= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} \\
&\quad + \text{Diagram 4} + \text{Diagram 5} + \cdots
\end{aligned}$$

The diagrams represent Feynman-like diagrams for the expansion of  $G_{12}$ . Each diagram consists of a horizontal line with arrows pointing from left to right, representing fermion propagation. Vertices are marked with dots and labeled with indices 1, 2, 3, 4, 5, 6. Wavy lines represent interactions between vertices. Diagram 1 is a simple line from 1 to 2. Diagram 2 has a loop at vertex 3. Diagram 3 has a loop between vertices 3 and 4. Diagram 4 has a loop between vertices 4 and 6. Diagram 5 has two loops between vertices 3, 5, 4, and 6. Ellipses indicate higher-order terms in the expansion.