On the dynamics underlying the emergence of coherent structures in barotropic turbulence

Bakas N. A.[1*], Constantinou N. C.[2], Ioannou P. J.[3]

1 Laboratory of Meteorology, Department of Physics, University of Ioannina

2 Scripps Institution of Oceanography, University of California, San Diego, USA

3 Department of Physics, National and Kapodistrian University of Athens

*corresponding author e-mail: nbakas@uoi.gr

Abstract Planetary turbulent flows are observed to self-organize into large scale structures such as zonal jets and coherent vortices. In this work, the eddy-mean flow dynamics underlying the formation of both zonal and nonzonal coherent structures in a barotropic turbulent flow is investigated within the statistical framework of stochastic structural stability theory (S3T). Previous studies have shown that the coherent structures emerge due to the instability of the homogeneous turbulent flow in the statistical dynamical S3T system and that the statistical predictions of S3T are reflected in direct numerical simulations. In this work, the dynamics underlying the structure forming S3T instability are studied. It is shown that, for weak planetary vorticity gradient beta, both zonal jets and non-zonal large-scale structures form from upgradient momentum fluxes due to shearing of the eddies by the emerging flow. For large beta, the dynamics of the S3T instability differs for zonal and non-zonal flows. Shearing of the eddies by the mean flow continues to be the mechanism for the emergence of zonal jets while non-zonal large-scale flows emerge from resonant and near-resonant triad interactions between the large-scale flow and the stochastically forced eddies.

1 Introduction

Atmospheric turbulence is commonly observed to be organized into large scale zonal jets with coherent waves embedded in them. The jets control the transport of heat in the atmosphere, while the coherent waves produce significant spatiotemporal variability. It is therefore important to understand the mechanisms for the emergence, equilibration, and maintenance of these coherent structures.

A simple model that exhibits many aspects of turbulent self-organization into coherent structures, is a barotropic flow on a β-plane with turbulence sustained by random stirring. Numerical simulations of this model have shown that robust, large scale zonal jets and coherent westward propagating waves emerge in the flow and are sustained at finite amplitude (Galperin et al., 2010). Recently, it was shown that the formation of these coherent structures occurs as a bifurcation phenomenon. As the energy input of the stochastic forcing is increased, the flow bifurcates from a turbulent, spatially homogeneous state to a state in which zonal jets and/or nonzonal coherent structures emerge and are maintained by turbulence (Bakas and Ioannou 2014). In this work, we address the eddy–mean flow dynamics underlying the emergence of both zonal and nonzonal structures.

Since organization of turbulence into coherent structures involves complex nonlinear interactions among disparate scales, an attractive approach is to study the statistical state dynamics of the flow rather than single realizations of the turbulent field, an approach that is followed in stochastic structural stability theory (S3T; Farrell and Ioannou 2003). Recent studies employing S3T addressed the bifurcation from a homogeneous turbulent regime to a structure-forming regime and identified the emerging structures as linearly unstable modes of the homogeneous turbulent state equilibrium. Comparisons of the structure characteristics predicted by S3T with direct numerical simulations have shown that the predictions of S3T are reflected in direct numerical simulations. In this work we investigate the eddy-mean flow dynamics underlying this cooperative eddy–mean flow instability.

2 Formulation of Stochastic Structural Stability Theory

Consider a non-divergent barotropic flow with velocity field $\mathbf{u}=(u,v)$ on a β-plane with cartesian coordinates $\mathbf{x}=(x,y)$. Relative vorticity $\zeta = \partial_x v - \partial_y u$, evolves according to:

$$(\partial_t + \mathbf{u}\cdot\nabla)\zeta + \beta v = -r\zeta + f_e \qquad (1)$$

where β is the gradient of planetary vorticity. Linear dissipation at the rate r parameterizes Ekman drag at the surface. Turbulence is supported by the random stirring f_e that models processes vorticity sources from convection and baroclinic instability, that are absent in barotropic dynamics. We assume that f_e is temporally delta correlated, spatially homogeneous and isotropic, injecting energy at a rate ε in a narrow ring of wavenumbers with radius K_f.

S3T describes the statistical dynamics of the first two same time moments of (1). The first moment is the ensemble mean of the vorticity $Z(\mathbf{x}, t) \equiv \langle\zeta\rangle$, where the brackets denote an ensemble average over forcing realizations. The second moment $C(\mathbf{x}_1, \mathbf{x}_2, t) \equiv\langle\zeta'_1\zeta'_2\rangle$, is the two point correlation function of the vorticity deviation from the mean $\zeta'_i \equiv \zeta_i - Z_i$, where the subscript $i = 1, 2$ refers to the value of relative vorticity at $\mathbf{x}_i$. We adopt the general interpretation that the ensemble average is a Reynolds average over the fast turbulent motions (Bakas and Ioannou, 2014). With this definition of the ensemble mean, we seek to obtain the statistical dynamics of the interaction of the coarse-grained ensemble average field Z, which can be zonal or non-zonal coherent structures, with the fine-grained incoherent field represented by the vorticity covariance *C*. The equations governing the evolution of the first two moments are:

$$(\partial_t + \mathbf{U}\cdot\nabla)Z + \beta V + rZ = -\nabla\cdot\langle\mathbf{u}'\zeta'\rangle = G(C) \quad (2)$$

$$\partial_t C + (A_1 + A_2)C = \Xi \quad (3)$$

where $\mathbf{U}$, $\mathbf{u}'$ are the ensemble mean and the eddy velocity fields respectively, $\langle\mathbf{u}'\zeta'\rangle$ is the ensemble mean vorticity flux, whose divergence can be expressed as a function of the vorticity covariance $\nabla\cdot\langle\mathbf{u}'\zeta'\rangle = G(C)$,

$$A = -\mathbf{U}\cdot\nabla - (\beta + \partial_y Z)\partial_x \Delta^{-1} + \partial_x Z \partial_y \Delta^{-1} - r$$

governs the dynamics of linear perturbations about the instantaneous mean flow $\mathbf{U}$ and Ξ is the spatial correlation function of the external forcing. In obtaining (3), we have ignored the eddy-eddy interactions or equivalently the third cumulant, so that (2)-(3) form a closed deterministic system.

3 Dynamics underlying the structure forming instability

The S3T system (2), (3) has the statistical equilibrium $Z^E=0$, $C^E=\Xi/2r$, that has zero large scale flow and a homogeneous eddy field with the spatial covariance of the forcing. The stability of the homogeneous equilibrium is assessed by introducing perturbations $\delta Z = e^{i\mathbf{n}\mathbf{x}+\sigma t}$and $\delta C = C_h(\mathbf{x}_a - \mathbf{x}_b)e^{i\mathbf{n}(\mathbf{x}_a+\mathbf{x}_b)/2+\sigma t}$, where $\mathbf{n}$ is the perturbation wavevector, linearizing (2)-(3) about the equilibrium and calculating the eigenvalues σ. The resulting equation for σ is (Bakas et al 2015):

$$\sigma + 1 = \varepsilon f(\sigma|\delta U, C^E)$$

where *f* is the vorticity flux induced by the distortion of the incoherent homogeneous eddy field with covariance C^E by the mean flow δU. We will call this induced flux as the vorticity flux feedback on δU. If the real part of the feedback is positive, it has the tendency to reinforce the preexisting jet perturbation and therefore destabilizes it. In order to illuminate the eddy-mean flow dynamics underlying the instability, we will study the vorticity flux feedback at the stability boundary (real(σ)=0), at which the growing eigenfunctions follow the Rossby wave dispersion $\sigma_i = \mathrm{imag}(\sigma) = \beta n_x/(n_x^2 + n_y^2)$. At the stability boundary the real part of the vorticity flux feedback can be written as:

$$f_r = real[f(\sigma = \sigma_i|\delta U, C^E)] = \int_0^\pi F d\theta$$

where $F(\theta,n)$ is the contribution to f_r from the stochastically forced waves with wavevectors $\mathbf{k}$ and $-\mathbf{k}$ that are characterized only by the angle θ between their phase lines and the direction perpendicular to the mean flow wavevector $\mathbf{n} = (n_x, n_y) = (n\cos\varphi, n\sin\varphi)$ (cf Fig. 1). We will now determine the contribution of the various waves to the vorticity flux feedback and identify the angle θ that produces the most significant contribution to this feedback for both zonal jet ($\varphi = 0$) and non-zonal wave ($\varphi \neq 0$) perturbations.

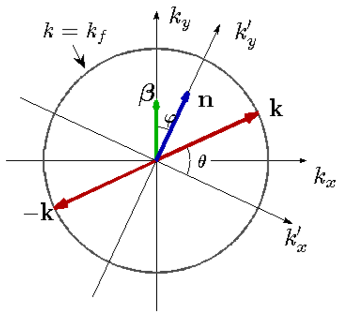

Fig. 1. A non-zonal perturbation with wavevector **n** at an angle ϕ to the northward direction interacts with the stochastically forced waves with wavevector **k** at an angle θ with respect to the direction of **n**. The vorticity flux feedback f_r arises from the contribution of the eddies with wavevectors **k** and -**k**.

Consider first the limit of small non-dimensional planetary vorticity gradient $\tilde{\beta} = \beta/K_f r \ll 1$, in which we can expand the integrand F in powers of $\tilde{\beta}$: $F = F_0 + \tilde{\beta}^2 F_2 + \cdots$. The leading-order term F_0 is shown in Fig. 2a. For β=0, the dynamics are rotationally symmetric and for isotropic forcing f_r, is independent of φ. Therefore all zonal and non-zonal eigenfunctions with the same wavenumber n grow at the same rate. Upgradient fluxes ($F_0 > 0$) to a mean flow with wavenumber n are induced by waves with phase lines inclined at angles $|\theta| \leq 30^o$. The eddy–mean flow dynamics was investigated in the limit of $\tilde{n} = n/K_f \ll 1$ by Bakas and Ioannou (2013). It was shown that the vorticity fluxes can be calculated by time averaging the fluxes over the life cycle of an ensemble of localized stochastically forced wavepackets. The wavepackets evolve under the influence of the infinitesimal local shear and are rapidly dissipated before they shear over. As a result, their effect on the mean flow is dictated by the instantaneous change in their momentum fluxes. Any pair of wavepackets having a central wavevector with phase lines forming angles $|\theta| \leq 30^o$ surrender instantaneously momentum to the mean flow and reinforce it, whereas pairs with $|\theta| \geq 30^o$ gain instantaneously momentum and oppose jet formation. For isotropic forcing, the net vorticity flux produced when integrated over all angles vanishes. However, a net feedback is produced at order $\tilde{\beta}^2$.

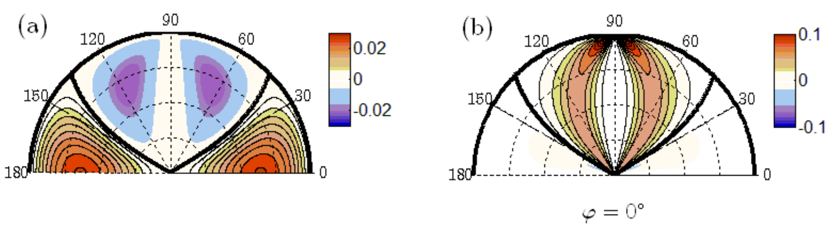

Fig. 2. Contours of (a) $F_0(\theta,\tilde{n})$ and (b) $F_2(\theta,\tilde{n})/\tilde{n}^4$ for a zonal jet perturbation in a polar plot ($\tilde{n}$ radial and θ azimuthal). The thick line is the zero contour and the radial grid interval is δn=0.25.

To understand the contribution of β to the vorticity flux feedback, we plot $F_2/\tilde{n}^4$ for a zonal jet (Fig. 2b) as a function of the mean-flow wavenumber $\tilde{n}$ and wave angle θ. It can be seen that at every point, F_2 has the opposite sign to F_0, implying that β tempers the fluxes of β=0. However, in the sector $|\theta| \geq 30^o$ the values of F_2 are much larger than in the sector $|\theta| \leq 30^o$ and the net fluxes integrated over all angles are upgradient. This behavior can be explained as follows. Any pair of wavepackets with wavevectors at angles $|\theta| \leq 30^o$ apart from the instantaneous momentum gain described above, have their group velocity increased (decreased) while propagating northward (southward), as shearing changes their meridional wavenumber. The instantaneous change in the momentum fluxes resulting from this speed up (slow down) of the wavepackets is positive in the region of excitation leading to upgradient fluxes. The opposite happens for pairs with $|\theta| \geq 30^o$. In the case of a nonzonal perturbation (not shown), the angles for which the waves have significant positive or negative contributions to the flux feedback are roughly the same as in the case of zonal jets. These results therefore show that, for $\tilde{\beta} \ll 1$, the instability mechanism for nonzonal structures is the same as the mechanism for zonal jet formation, which is shearing of the eddies by the mean flow.

Consider now the emergence of coherent structures in the limit $\tilde{\beta} \gg 1$. The contribution F of each wave to the vorticity flux feedback f_r for the case of nonzonal structures at $\tilde{\beta} = 100$ is shown in Fig. 3. In contrast to the case with $\tilde{\beta} \ll 1$, there is only a small band of waves that contribute significantly to the flux feedback, as indicated with the narrow tongues in Fig. 3a. The reason for this selectivity in the response is that for $\tilde{\beta} \gg 1$ the components that produce appreciable fluxes are concentrated on the (θ, n) curves that satisfy the resonant condition $\omega_{\mathbf{k+n}} = \omega_{\mathbf{k}} + \omega_{\mathbf{n}}$. This is the resonant condition satisfied when a Rossby wave with wavevector $\mathbf{k}$ and frequency $\omega_{\mathbf{k}}$ forms a resonant triad with the nonzonal structure with wavevector $\mathbf{n}$ and frequency $\omega_{\mathbf{n}}$. It is found that the largest destabilizing feedback occurs when two positively contributing resonances are near coalescence. The reason is that when the resonances are apart, the significant contributions come from near-resonant waves with angles within a band of $O(1/\tilde{\beta})$ around the resonant angles and the net contribution to the vorticity flux are $O(1/\tilde{\beta})$. When the resonances are near coalescence, the band of near-resonant waves increases ($O(1/\sqrt{\tilde{\beta}})$) and the flux feedback increases proportionally.

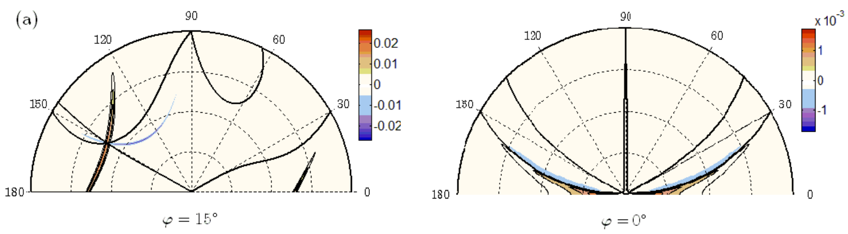

Fig. 3. Contours of $F\ (\theta,\tilde{n})$ for (a) a non-zonal jet perturbation (ϕ=15^o) and (b) a zonal jet perturbation (ϕ=0^o) in a $(\tilde{n},\theta)$ polar plot ($\tilde{n}$ radial and θ azimuthal) for $\tilde{\beta}=100$. The thick line is the zero contour and the radial grid interval is δn=0.25.

An interesting exception to the results discussed above occurs for zonal jet perturbations. In that case, the resonant contribution is exactly zero and the positive vorticity flux feedback is obtained from a broad band of the non-resonant waves with $\theta\approx 0$ (cf.Fig. 3b). The reason is that the stochastically forced eddies for $\tilde{\beta}\gg 1$ propagate with $O(\beta)$ group velocities. Therefore, in contrast to the limit of $\tilde{\beta}\ll 1$ in which they evolve according to their local shear, the forced waves will respond to the integrated shear of the sinusoidal perturbation over their large propagation extent, which will be very weak. As a result only waves with small group velocities can have a significant contribution.

4 Conclusions

We examined the eddy-mean flow dynamics underlying the instability that gives rise to large scale structures in barotropic turbulence within the statistical framework of S3T. In the limit of weak planetary vorticity gradient, the dynamics are similar for both emerging zonal jets and non-zonal structures. In this limit, shearing of the forced eddies by the infinitesimal mean flow changes at leading order their momentum fluxes and at second order their group velocity. For an isotropic forcing the instability is controlled by the second order effect that produces upgradient fluxes. In the limit of strong planetary vorticity gradient, the eddy–mean flow dynamics are different for zonal and non-zonal perturbations. Zonal jets continue to induce upgradient fluxes through wave shearing. The dynamics underlying the emergence of non-zonal structures are dominated by near resonant interactions. Resonance occurs between the emerging structure, which close to the stability boundary satisfies the Rossby wave dispersion, and the stochastically forced waves satisfying the Rossby wave frequency resonant condition.

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