Topographic beta-plane turbulence and form stress

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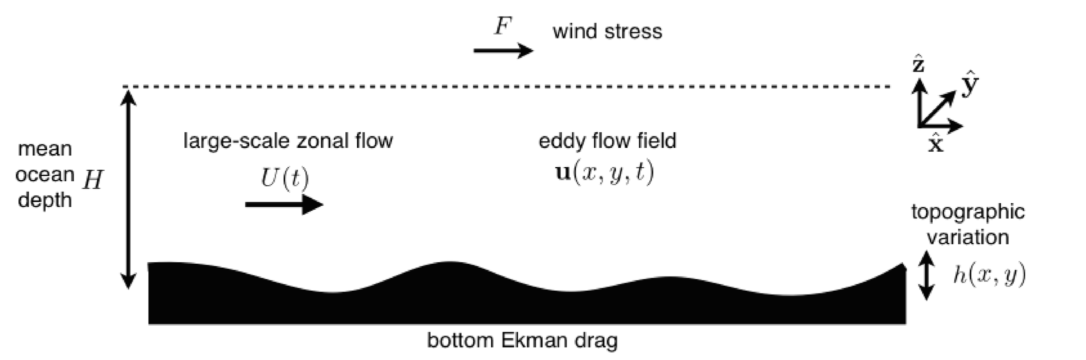
Objectives

Study the regimes of geostrophic turbulence above topography when forced by steady wind stress.

- How much momentum input by the wind stress is balanced by bottom drag, and how much by topographic form stress?
- What topographic features affect the large-scale flow?
- How does the mass transport depend on wind stress?

Model



The simplest model of topographic form stress:
Single-layer quasi-geostrophic setting, forced by a steady zonal mean wind stress in a doubly periodic domain of size $2\pi L \times 2\pi L$.
Flow consists of:

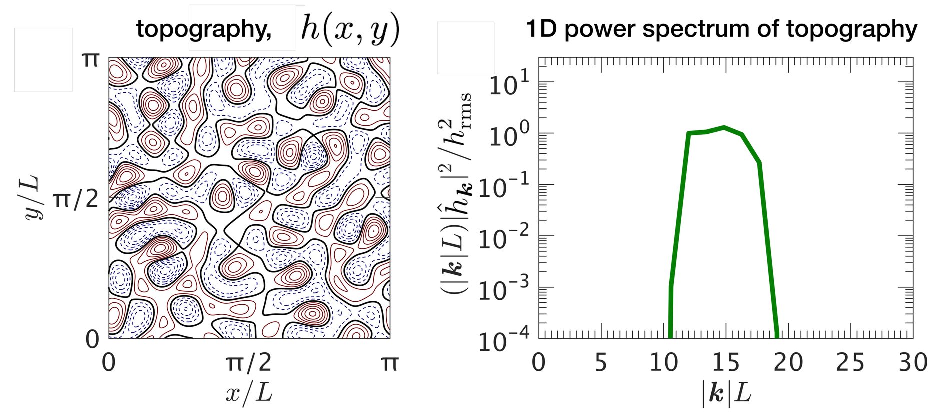
- a large-scale zonal mean flow: $U(t)$,
- an eddy flow field: $\psi(x,y,t)$ ($u = -\psi_y$, $v = \psi_x$).

Evolution:

$$\nabla^2\psi_t + \mathsf{J}(\psi - Uy\,, \nabla^2\psi + \beta y + \eta) = -\mu\nabla^2\psi, \qquad (1)$$
$$U_t = F - \mu U - \langle \psi\eta_x\rangle. \qquad (2)$$

- $\mathsf{J}(a,b) \stackrel{\text{def}}{=} a_x b_y - a_y b_x$: Jacobian,
- μ: Ekman drag coefficient,
- β: planetary vorticity gradient,
- $\langle\psi\eta_x\rangle$: topographic form stress ($\langle\ \cdot\ \rangle$ is domain average),
- $F = \tau/(\rho_0 H)$: zonal mean wind stress forcing,
- $\nabla^2\psi + \beta y + \eta$: quasigeostrophic PV,
- $\eta(x,y) = f_0 h(x,y)/H$: topographic PV,

Flow field can be decomposed into ***standing*** (time-mean) and ***transients***, e.g. $\psi(x,y,t) = \bar\psi(x,y) + \psi'(x,y,t)$.



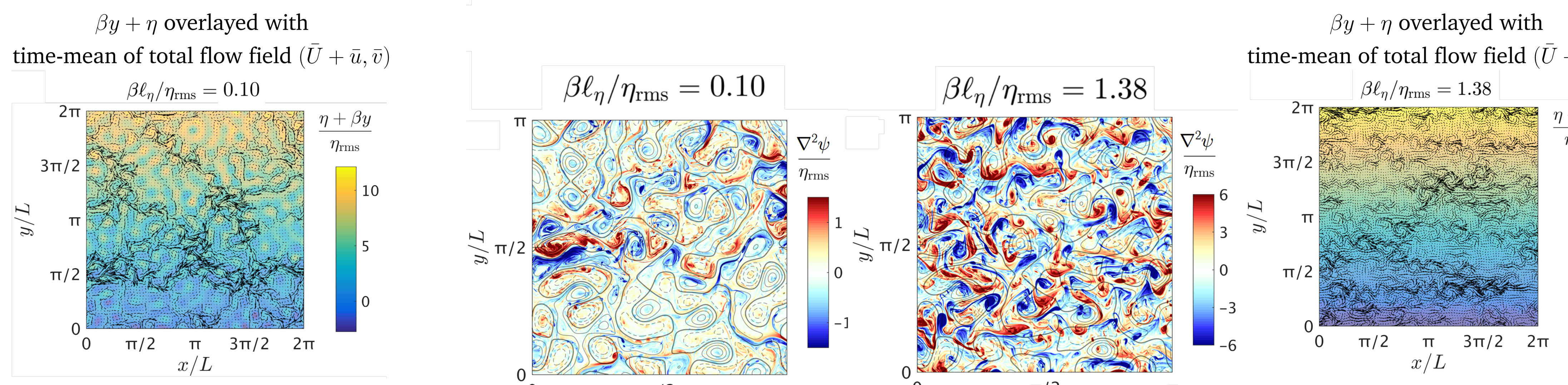
Properties of topography: homogeneous, isotropic, monoscale,

$$\eta_{\rm rms} = \sqrt{\langle\eta^2\rangle}\ ,\quad \ell_\eta = \sqrt{\frac{\langle\eta^2\rangle}{\langle|\nabla\eta|^2\rangle}}\ ,\quad \frac{L}{\ell_\eta} \approx 14.5.$$

Three main non-dimensional parameters:

$\beta\ell_\eta/\eta_{\rm rms}$	$F/(\mu\eta_{\rm rms}\ell_\eta)$	$\mu/\eta_{\rm rms} = 10^{-2}$
planetary vorticity gradient	wind stress forcing	dissipation

Two Flow Examples

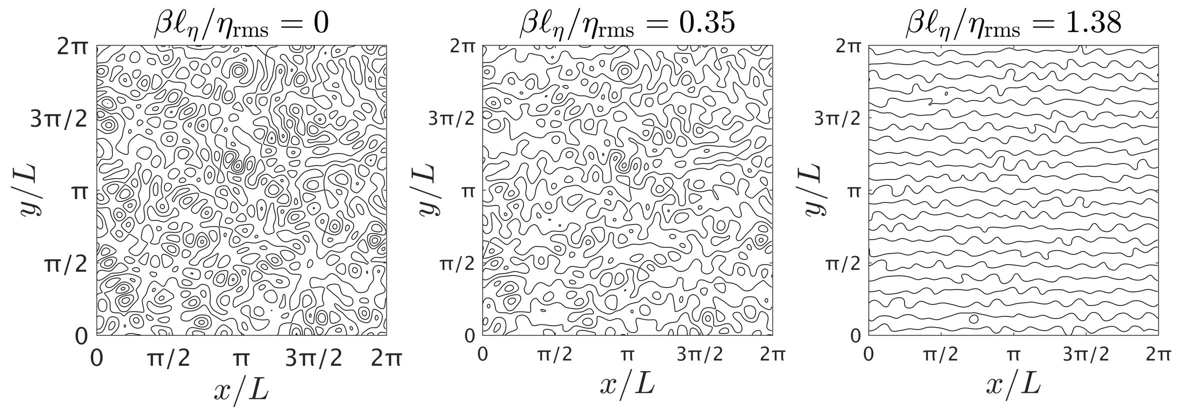


flow is "channelled" in some part of the domain

flow is evenly spread throughout the domain

Open vs. closed geostrophic contours

The ratio $\beta\ell_\eta/\eta_{\rm rms}$ $(=\beta/\sqrt{\langle|\nabla\eta|^2\rangle})$ controls whether there exist closed geostrophic contours, i.e. level sets of $\beta y + \eta$.

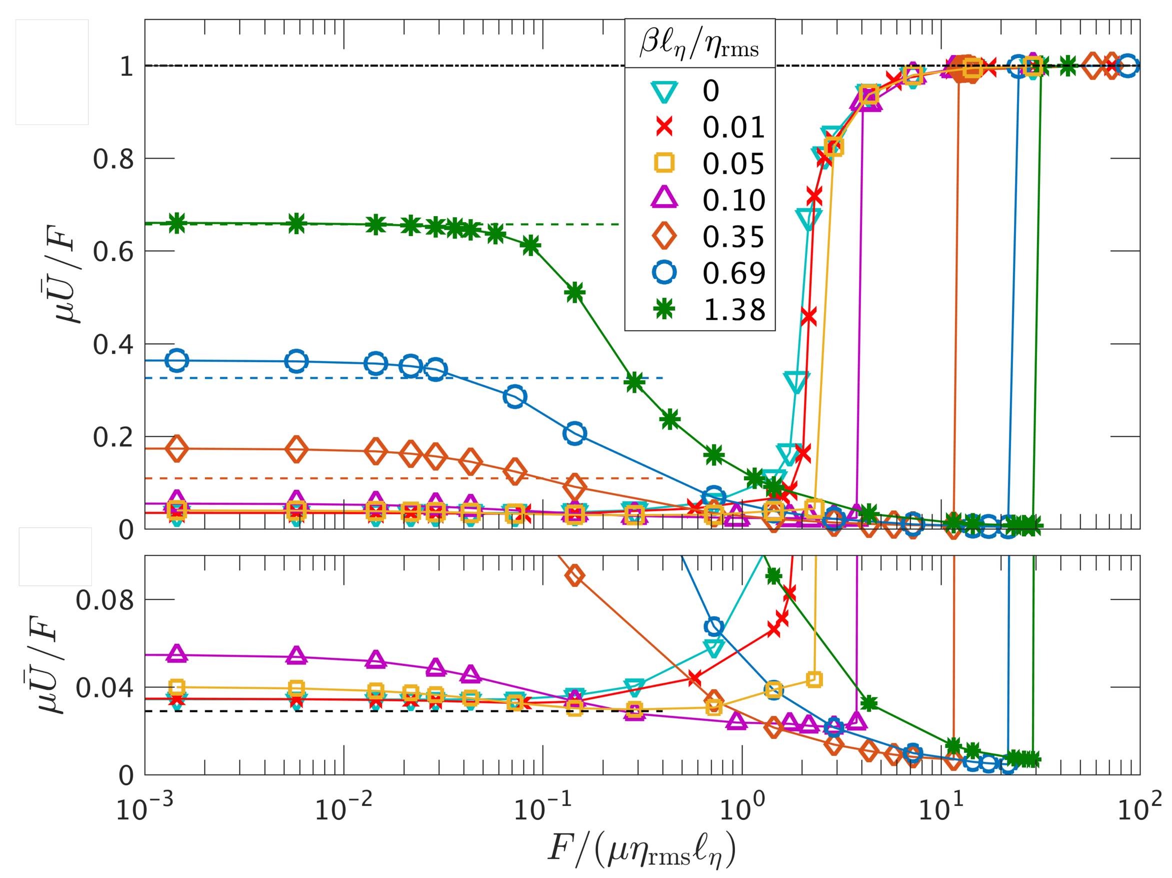


Structure of geostrophic contours (level sets of $\beta y + \eta$).

What balances the wind stress?

Time-average (denoted with bar) of U equation (2):

$$1 = \underbrace{\frac{\mu\bar U}{F}}_{\substack{\text{percentage}\\\text{balanced}\\\text{by bottom drag}}} + \underbrace{\frac{\overline{\langle\psi\eta_x\rangle}}{F}}_{\substack{\text{percentage}\\\text{balanced}\\\text{by form stress}}}$$



Weakly and strongly forced solutions (quasilinear – QL)

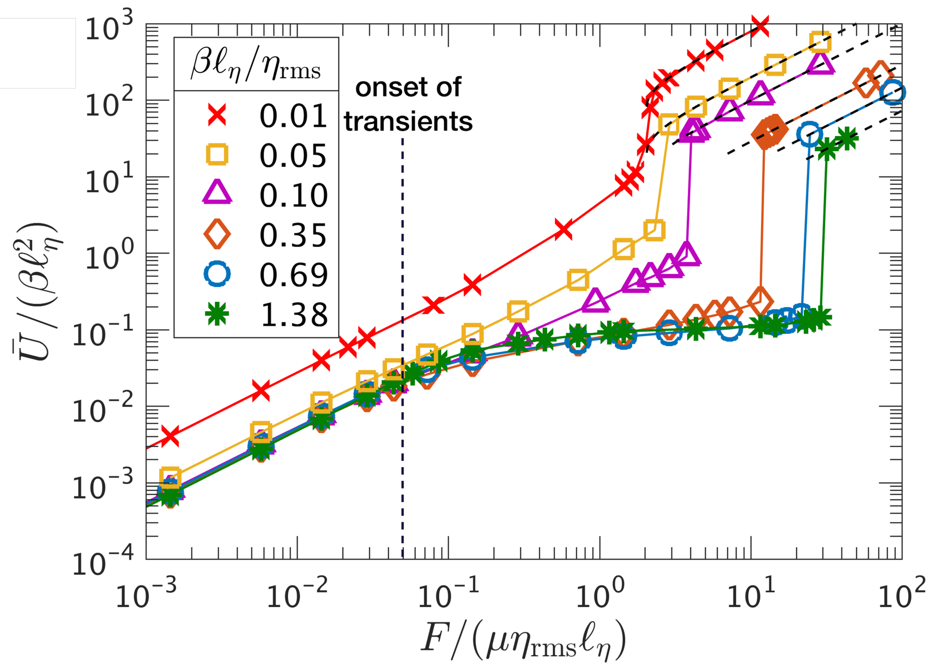
Assume steady flow. Then (2):

$$F - \mu U - \langle\psi\eta_x\rangle = 0$$

- Weakly forced: neglect quadratic terms $\mathsf{J}(\psi - Uy, \nabla^2\psi)$ in (1):
$$\mathsf{J}(\psi - Uy, \eta + \beta y) + \mu\nabla^2\psi = 0.$$
- Strongly forced: neglect $\mathsf{J}(\psi, \nabla^2\psi + \eta)$ in (1):
$$\beta\psi_x + U\nabla^2\psi_x + U\eta_x + \mu\nabla^2\psi = 0.$$

$\implies$ obtain scalings for $\bar U$ with F. Good agreement with numerics.

Eddy saturation regime



Large-enough $\beta\ell_\eta/\eta_{\rm rms} \implies \bar U$ is $\approx$independent of F.

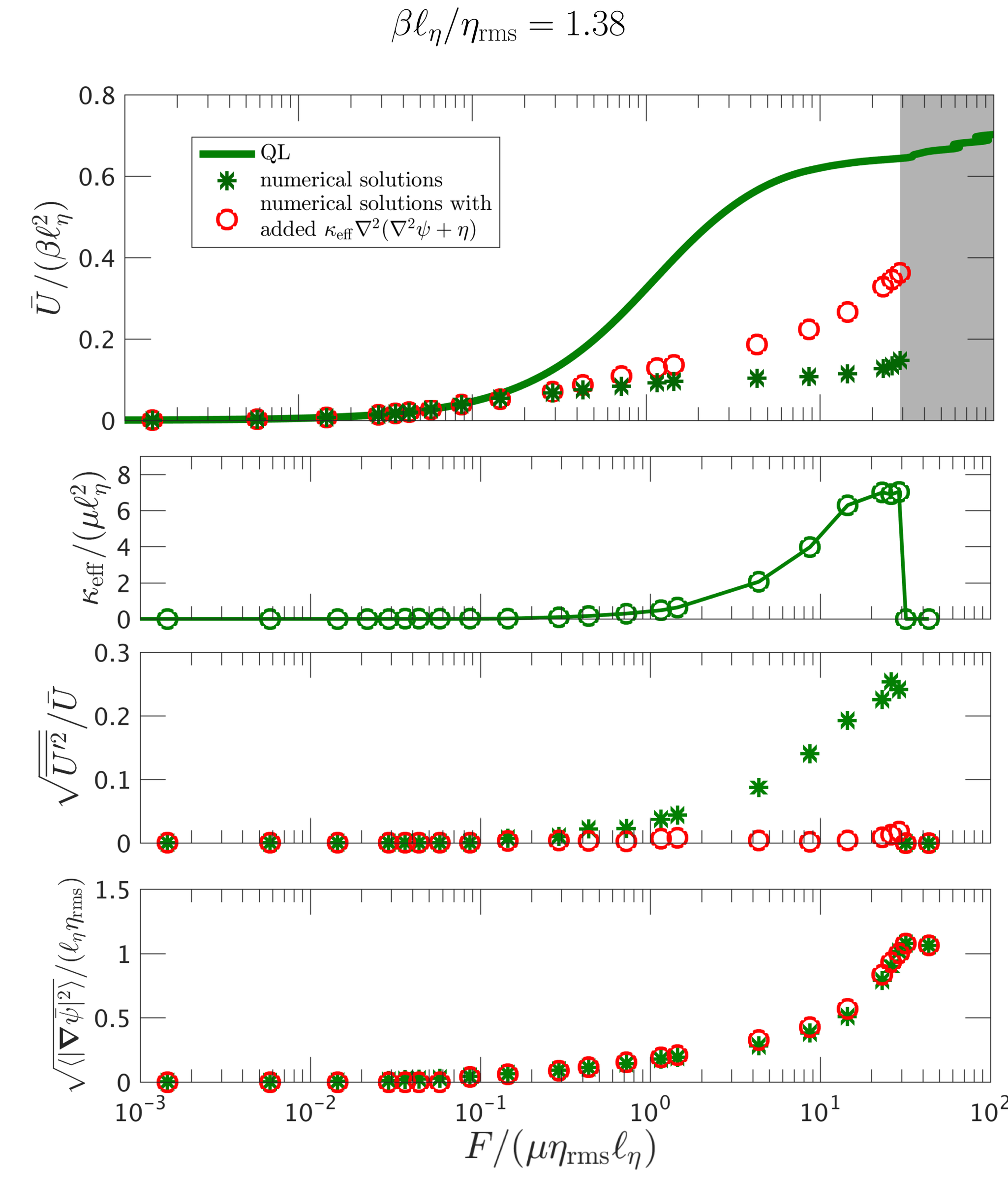
A theory for eddy saturation

Approximate the transient eddies as effective PV diffusion:

$$\nabla\cdot\underbrace{\overline{(U'+u',v')\,\nabla^2\psi'}}_{\text{eddy PV flux}} = -\kappa_{\rm eff}\nabla^2(\nabla^2\bar\psi + \eta).$$

Determine $\kappa_{\rm eff}$ from the eddy energy power integral.

The effective PV diffusion approximation is good for $\beta\ell_\eta/\eta_{\rm rms}$ larger than ≈ 0.5.



Effectively parametrize transients without reducing the amplitude of standing eddies.

Conclusions

- There exists flow regimes with flow depending:
  – only on statistical properties of topography (large $\beta\ell_\eta/\eta_{\rm rms}$),
  – on the geometrical structure of topography (small $\beta\ell_\eta/\eta_{\rm rms}$).
- Existence of eddy saturation regime in this barotropic, doubly periodic model.
  explanation: effective PV homogenization theory.
- Large zonal transport ensues as wind increases.
  explanation: enstrophy power integral imposes the need for such transition as wind stress crosses a threshold (see paper).

Constantinou, N. C. and Young, W. R. (2016) Beta-plane turbulence above monoscale topography, ***J. Fluid Mech.*** (submitted), arXiv:

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