SOLUTION 1

For $t<0$: (1) L_1, L_2 are short, and (2) no current flows through any of the 4 resistors, $\Rightarrow$

$$i_1(0^-) = -8\text{ A},\ i_2(0^-) = -4\text{ A},\ i_3(0^-) = 0,$$

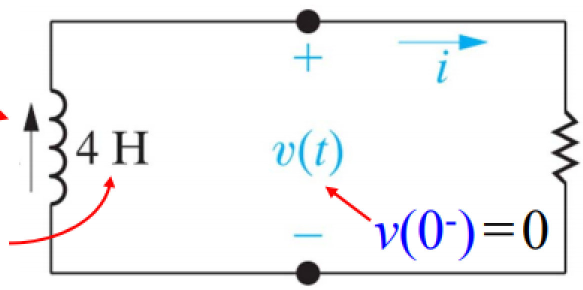
$$w_1(0^-) = (5\text{ H})(8\text{ A})^2/2 = 160\text{ J}, w_2(0^-) = (20)(4)^2/2 = 160\text{ J}.$$

For $t>0$, switch is open, the initial energy stored in the 2 inductors is dissipated via the 4 resistors.

The equivalent circuit becomes:

$I_0 = (8+4) = 12$ A

$L_{eq} = (5\text{H}//20\text{H}) = 4$ H

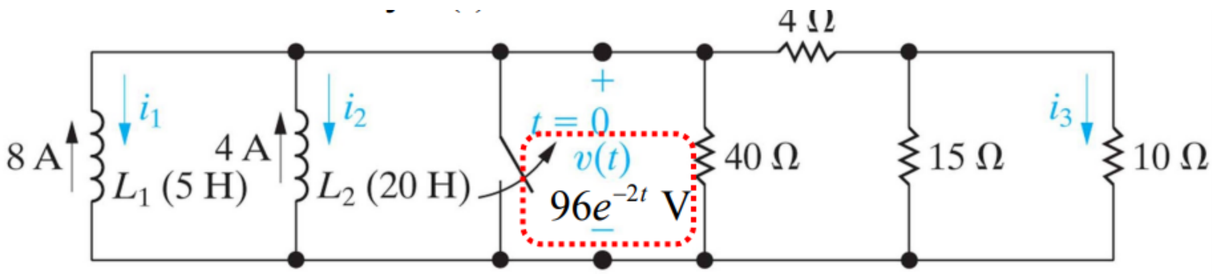


$R_{eq} =$ 8 Ω

The solutions to $i(t)$, $v(t)$ are:

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{4}{8} = 0.5\text{ s},\ \Rightarrow \begin{cases} i(t) = I_0 e^{-(t/\tau)} = 12e^{-2t}\text{ A}, \\ v(t) = Ri(t) = 96e^{-2t}\text{ V}. \end{cases}$$

The two inductor currents $i_1(t)$, $i_2(t)$ can be calculated by $v(t)$:



$$i_1(t) = i_1(0) + \frac{1}{L_1}\int_0^t v(t')dt' = -8 + \frac{1}{5}\int_0^t 96e^{-2t}dt' = 1.6 - 9.6e^{-2t} \text{ A.}$$

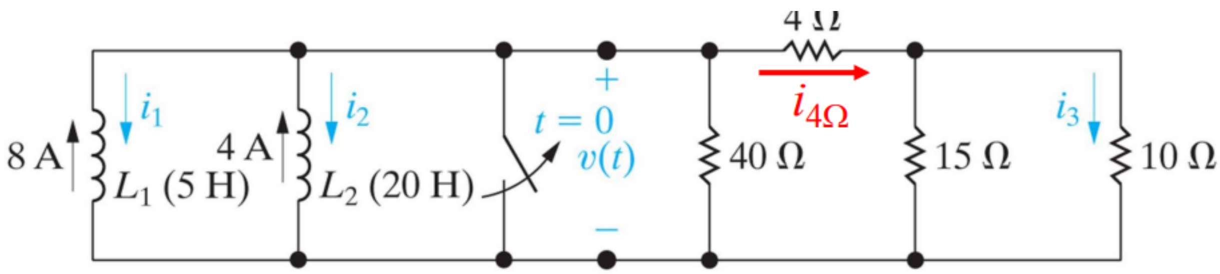
$$i_2(t) = i_2(0) + \frac{1}{L_2}\int_0^t v(t')dt' = -4 + \frac{1}{20}\int_0^t 96e^{-2t}dt' = -1.6 - 2.4e^{-2t} \text{ A.}$$

The energies stored in the two inductors are:

$$w_1(t \to \infty) = \frac{1}{2}(5\text{ H})(1.6\text{ A})^2 = 6.4\text{ J},$$

$$w_2(t \to \infty) = \frac{1}{2}(20)(-1.6)^2 = 25.6\text{ J},$$

By current division, $i_3(t) = 0.6i_{4\Omega}(t)$, while $i_{4\Omega}(t)$ can be calculated by $v(t)$:



$$i_{4\Omega}(t) = \frac{v(t)}{4\Omega + (15\Omega // 10\Omega)} = \frac{96e^{-2t}}{10} = 9.6e^{-2t} \text{ A.}$$

$$\Rightarrow i_3(t) = \frac{3}{5} i_{4\Omega}(t) = 5.76e^{-2t} \text{ A.}$$

SOLUTION 2

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. 4.40(a), from which we find R_N. Thus,

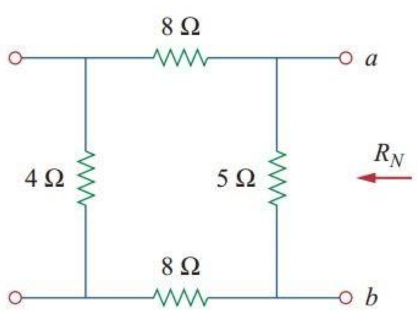
$$R_N = 5 \,\|\, (8 + 4 + 8) = 5 \,\|\, 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N, we short-circuit terminals a and b, as shown in Fig. 4.40(b). We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

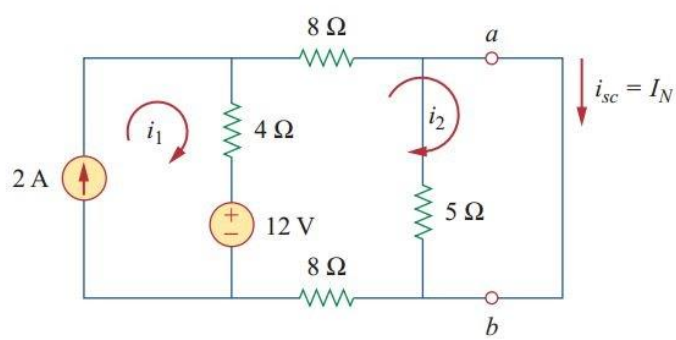
$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

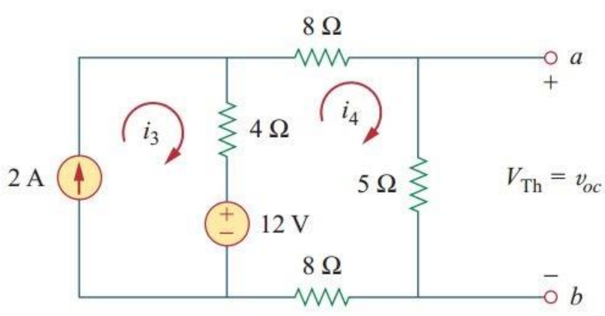
$$i_2 = 1\text{ A} = i_{sc} = I_N$$



(a)



(b)



(c)

Alternatively, we may determine I_N from V_{Th}/R_{Th}. We obtain V_{Th} as the open-circuit voltage across terminals a and b in Fig. 4.40(c). Using mesh analysis, we obtain

$$i_3 = 2 \text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8 \text{ A}$$

and

$$v_{oc} = V_{Th} = 5i_4 = 4 \text{ V}$$

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{ A}$$

as obtained previously. This also serves to confirm Eq. (4.12c) that $R_{Th} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega$. Thus, the Norton equivalent circuit is as shown in Fig. 4.41.

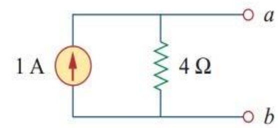
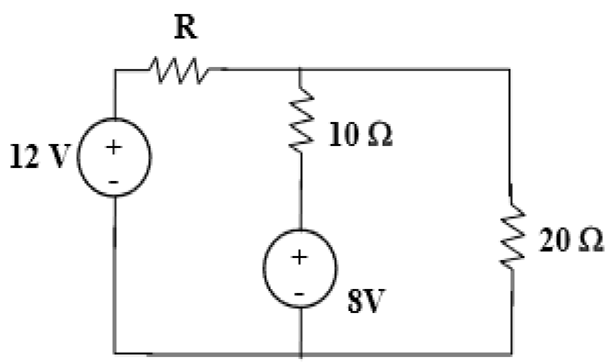


Figure 4.41
Norton equivalent of the circuit in Fig. 4.39.

Solution 3



Removing the 10 ohm resistor and solving for the Thevenin Circuit results in:

R_{Th} = (Rx20/(R+20)) and a $V_{oc} = V_{Th}$ = 12x(20/(R +20)) + (-8)

As R goes to zero, R_{Th} goes to zero and V_{Th} goes to 4 volts, which produces the maximum power delivered to the 10-ohm resistor.

$$P = vi = v^2/R = 4x4/10 = 1.6 \text{ watts}$$

Notice that if R = 20 ohms which gives an R_{Th} = 10 ohms, then V_{Th} becomes -2 volts and the power delivered to the load becomes 0.1 watts, much less that the 1.6 watts.

Solution 4

(a) $$\tau = RC = \frac{1}{4}$$

$$-i = C\frac{dv}{dt}$$

$$-0.2e^{-4t} = C(10)(-4)e^{-4t} \longrightarrow C = \underline{\mathbf{5\ mF}}$$

$$R = \frac{1}{4C} = \underline{\mathbf{50\ \Omega}}$$

(b) $$\tau = RC = \frac{1}{4} = \underline{\mathbf{0.25\ s}}$$

(c) $$w_C(0) = \frac{1}{2}CV_0^2 = \frac{1}{2}(5\times 10^{-3})(100) = \underline{\mathbf{250\ mJ}}$$

(d) $$w_R = \frac{1}{2}\times\frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2\left(1 - e^{-2t_0/\tau}\right)$$

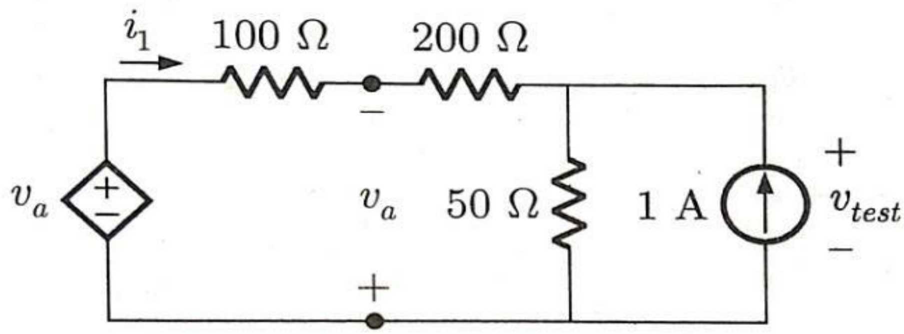
$$0.5 = 1 - e^{-8t_0} \longrightarrow e^{-8t_0} = \frac{1}{2}$$

or $$e^{8t_0} = 2$$

$$t_0 = \frac{1}{8}\ln(2) = \underline{\mathbf{86.6\ ms}}$$

Solution 5

Using source transform



$$v_a = 100i_1 + 200i_1 + 50\,(i_1 + 1)$$

$$\begin{aligned} v_a &= 100i_1 - v_a \\ \Rightarrow \qquad v_a &= 50i_1 \\ 50i_1 &= 300i_1 + 50i_1 + 50 \\ \Rightarrow \qquad i_1 &= -\frac{1}{6}\ \mathrm{A} \\ v_{test} &= 50\left(1 - \frac{1}{6}\right) = \frac{125}{3}\,\Omega \end{aligned}$$