MTH 102: Probability and Statistics

Quiz 2

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Explain your answers. Show your steps. No gadgets allowed. Approximate calculations are fine as long as the approximations are reasonable. You have 45 minutes. Good luck!

Question 1. Valentine Day Bonus 5 **marks** Suppose a quiz is scheduled on any day with probability 0.1 independently of other days. What is the probability that there will be a quiz on Valentine's day, which is the 14^{th} of February? The Valentine week starts on Feb 7 and ends on Feb 14 and includes both the days. What is the probability that there is at least one quiz during a Valentine week? Given that you are giving a quiz on Feb 14, what is the probability that there was at least one quiz during the Valentine week?

Question 2. 45 **marks** A student gives three exams (one after the other) to pass a certification course on Intelligently Dumb Systems. In each exam, the student scores either a 0 or a 1. The probability that the student scores a 1 in the first exam is 0.6. The performance of a student in an exam is impacted by the student's performance in the previous exam. Specifically, if the student scores a 1 in the first exam, the student scores a 1 in the second exam with probability 0.8. Similarly, if a student scores a 1 in the second exam, the student scores a 1 in the third exam with probability 0.8. If the student scores a 0 in the first exam, the student scores a 1 in the second exam with probability 0.3. Similarly, if the student scores a 0 in the second exam, the student scores a 1 in the third exam with probability 0.3. Answer the following questions. [Hint: You may want to draw a tree diagram.]

(A) Calculate the probability that the total score at the end of the three exams is 3.
(B) Calculate the expected value of the score in the first exam.
(C) Calculate the expected value of the score in the second exam.
(D) Calculate the expected value of the total score at the end of the three exams.
(E) Calculate the expected value of the score in the first exam, given that score in the second exam is 1.
(F) Calculate the expected value of the total score, given that score in the second exam is 1.

Question 3. 50 **marks** An autonomous shuttle waits for passengers in five minute intervals. In case, at least 10 passengers arrive, it sets out on its route at the end of the corresponding five minute interval. To exemplify, once the shuttle starts waiting it must wait for at least 5 minutes. If less than 10 passengers arrive in the 5 minutes, it waits for another 5 minutes, and so on, it waits till the end of that five minute interval during which the total number of passengers equals or exceeds 10. Note that the shuttle leaves after waiting for a certain random X minutes, where the range space $S_X = \{5, 10, 15, \ldots\}$. Assume that the passengers that arrive don't leave the shuttle.

The number of passengers that arrive in an interval of T minutes is a Poisson random variable with rate $\lambda = 0.5$ passengers per minute. The number of passengers that arrive in an interval is independent of the number of passengers that arrive in any other disjoint interval of time. To exemplify, the intervals of time $[2, 3]$ and $[3, 4]$ are disjoint. However, $[2, 5]$ and $[3, 6]$ are not. Answer the following questions. [Hint: The PMF of a Poisson RV K with parameter $\alpha = \lambda T$ is $P[K = k] = e^{-\alpha}\alpha^k/k!$, $k \in \{0, 1, 2, \ldots\}$.]

(A) Suppose that 5 customers arrive in the first 10 minutes. Calculate the probability that 1 customer arrived in the first 5 minutes? [Hint: Make judicious use of the property of independence stated in the question.]
(B) Calculate the probability that the shuttle leaves after waiting for 20 minutes.
(C) Calculate the expected value of the customers in the shuttle given that the shuttle departs after 20 minutes.