MTH 201: Probability and Statistics

Quiz 1

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Explain your answers. Show your steps. No gadgets allowed. Approximate calculations are fine as long as the approximations are reasonable. You have 45 minutes. Keep Calm and Finish the Quiz. Good luck!

I. Two Coin Tosses (30 marks)

You perform an experiment that involves tossing a coin twice and noting the outcome of each toss. A coin toss leads to an outcome from the set $\{H, T\}$, where outcome H denotes heads and outcome T denotes tails. The coin tosses are independent and the probability that a toss gives heads is p. Answer the following questions.

(a) (4 marks) In terms of the given set of outcomes, write down the event that the first toss gives heads. Express the probability of the event in terms of probabilities of the outcomes it contains.

(b) (4 marks) Similar to above, write down the event that at least one toss gives heads. Express the probability of the event in terms of the probabilities of the outcomes it contains.

(c) (4 marks) You pick the outcome of any coin toss. What is the probability that the outcome is H?

(d) (4 marks) Express the probability of heads in the second toss conditioned on the knowledge that the first toss gave heads using standard notation. What is this probability as a function of p?

(e) (4 marks) Consider the probability of two heads conditioned on heads in the first toss. Express it using notation. What is this probability as a function of p?

(f) (4 marks) What is the probability that at least one heads was observed, given that the second toss gave heads?

(g) (6 marks) What is the probability of heads on the first toss, given that at least one heads was observed?

II. Catching the Copycat (70 marks)

There are two coins A and C, of which C is a copycat. You are heading an effort to catch the copycat coin C. Coin A has a mind of its own and tossing it gives heads with probability $2/3$. The behavior of coin C is dependent on whether it is tossed first or after coin A. If coin C is tossed first, the toss gives heads with probability $1/2$. On the other hand, if C is tossed after coin A, coin C tries to copy the outcome of the toss of coin A. Specifically, if the toss of A gave heads, the toss of C gives heads with probability $3/4$. If the toss of A gave tails, the toss of C gives heads with probability $1/3$.

You are scheduled to see the coins perform at an exhibition. The performance will proceed as follows. The two coins (identities unknown) will be brought in a jar. One of the coins will be picked randomly from the jar and tossed. Following this, the other coin in the jar will be tossed. You decide that you will choose the coin that is tossed first to be coin C in case the outcomes of the two tosses are different. Else, you will choose the second coin to be tossed to be coin C. Answer the following questions. [Hint: Tree diagrams may come in handy.]

(a) (4 marks) What is the probability that coin A is chosen for the first toss?

(b) (12 marks) What is the probability that the first coin toss gives heads and the second gives tails?

(c) (12 marks) What is the probability that the first coin toss gives tails and the second gives heads?

(d) (12 marks) What is the probability that both coin tosses give heads?

(e) (12 marks) What is the probability that both coin tosses give tails?

(f) (18 marks) What is the probability that you will choose the correct coin C?