MTH 102: Probability and Statistics

Quiz 3 As Homework

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Try and work this out on your own. This is important to work through on your own. You may discuss to clarify what is needed but you must work out everything by yourself.

Question 1. Runners Tom and Jerry have running speeds that belong to the set $\{1, 1.25, 1.5\}$ strides per second. Let the speed of runner A be given by the random variable X and that of B be given by Y. Do the following.

(a) Provide an example marginal distribution each for the speeds of Tom and Jerry. It is known that their distributions are non-identical and not uniform. Also, all probabilities are in the open interval $(0, 1)$.

(b) We are also interested in their speeds when they run together. Provide three example unique joint distributions corresponding to your choice of marginal distributions. At least one of the joint distributions must correspond to when the running speeds are independent. All probabilities must be in the open interval $(0, 1)$.

(c) From the three joint distributions you crafted above, choose one that corresponds to running speeds being dependent. Mention your choice clearly.

(d) For the chosen joint distribution, calculate

(1) All possible conditional distributions of X given Y.

(2) $P[X = Y]$,

(3) $P[X > Y|Y = 1]$,

(4) Var$[X - Y]$.

Question 2. We start two independent clock timers, namely Analog and Digital. Let random variable X denote the expiry time of Analog and Y denote the expiry time of Digital. X is uniformly distributed with PDF $f_X(x) = 1/T,\ 0 \leq x \leq T$, and Y is exponentially distributed with PDF $f_Y(y) = \mu e^{-\mu y},\ y \geq 0$. Let Z be the random variable that gives the time at which the first of the two timers expires. Do the following.

1) Calculate the complementary CDF $P[Z > z|X = x]$ for $z \in (-\infty, \infty)$. Sketch the CCDF. [Hint: Use the fact that X and Y are independent. Note X is set to x.]

2) Calculate $E[Z|X = x]$

3) Calculate $E[Z]$ using the above result and iterated expectations.

Question 3. Repeat Question 2 for when X is exponentially distributed with PDF $f_X(x) = \mu e^{-\mu x},\ x \geq 0$ instead of being uniformly distributed.