

MTH 102: Probability and Statistics

Quiz 4

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Question 1. A radio receives three independent signals W , X , and Y . The signal W is Gaussian distributed with mean 0 and variance 1. The signal X is Gaussian distributed with mean 1 and variance 4. The signal Y is a Bernoulli random variable with parameter $p = 0.5$. The total power is approximated by $Z = W^2 + X^2 + Y^2$. We are interested in $E[Z]$, $E[Z^2]$, and $\text{Var}[Z]$. Do the following.

- 1) Using information given about W , X , Y and Z calculate as many of the three above stated moments of Z as you can. State the moments you can calculate and your answers for the same in the provided text box. Provide detailed steps that led to the answer later in the upload.
- 2) Use the moments you were able to calculate to find as good an upper bound as you can for the probability $P[Z > 2]$. In the text box state the inequality you used and the value of the upper bound. Provide detailed steps that led to the answer later in the upload.

Question 2. You would like to come up with a relative frequency estimate R for whether a randomly selected person is infected by the Novel Coronavirus. You would like to design an experiment such that with probability at least as large as $1 - 10^{-6}$ the relative frequency estimate is within $\pm 10^{-6}$ of the true unknown probability r that a person is infected by the virus. Do the following.

- 1) You must decide on the number of tests n of randomly chosen people you plan to conduct. Provide the range of values that n may take to satisfy the above mentioned conditions. In the text box state the range. Provide detailed steps that led to the answer later in the upload.
- 2) For any given value of n chosen from the desired range calculate the resulting mean squared error. In the text box state the choice of n and the mean squared error. Provide detailed steps that led to the answer later in the upload.