DS Midterm LAC

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A1 Let $p$ : Lady is in room A | $q$ : Tiger is in room A

$\neg p$ : Lady is in room B | $\neg q$ : Tiger is in room B.

Sign on room A: $p \wedge \neg q$

Sign on room B: $(p \wedge \neg q) \vee (\neg p \wedge q)$

| $p$ | $q$ | $p \wedge \neg q$ (Sign A) | $\neg p \wedge q$ | $(p \wedge \neg q) \vee (\neg p \wedge q)$ (Sign B) |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| → 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 |

∴ Sign on room A is false.

Sign on room B is true.

∴ Lady is behind door B.

A2. (a) Statement : $\exists x P(x) \longrightarrow \forall y P(y)$

Let $P(x) = x^2 < 0$

The hypothesis of this implication is false and we know that when that is the case then the implication is true regardless of the conclusion.

The hypothesis is false as there does not exist a number in the domain of real numbers for which $x^2 < 0$ where $x$ is the number.

$\exists x P(x)$ is always false for $x \in \mathbb{R}$

Thus $\exists x P(x) \rightarrow \forall y P(y)$ is true.

Hence proved.

(b) Statement : $\exists x P(x) \rightarrow \forall y P(y)$

The statement is <u>NOT</u> true for all predicate.

Let $P(x) = x^2 < 5$

Let the domain be $\{1, 2, 3, 4\} = S$

$\exists x P(x)$ is true as for $x = 1, x \in S$, $x^2 < 5$, Hypothesis is true

$\forall y P(y)$ is false as for $x = 3, 4 \in S$, $x^2 > 5$, Conclusion is false

We know that when hypothesis is true and conclusion is false then the statement of the implication is false.

$\exists x P(x) \rightarrow \forall y P(y)$ is false.

A3. To prove : $P(A) \subseteq P(B)$ iff $A \subseteq B$

(a) Forward proof :

Given : $P(A) \subseteq P(B)$ , To prove $A \subseteq B$

Let $x \in A$

then $\{x\} \in P(A)$

Since $P(A) \subseteq P(B)$

$\{x\} \in P(B)$

then $x \in B$

If the set containing only the element $x$ is a set in the power set of S then the element has to be in S.

$\therefore$ Every element in A also has to be in B.

Thus by the definition of a subset

$A \subseteq B$

Hence proved.

(b) Backward

To prove : $P(A) \subseteq P(B)$ , Given : $A \subseteq B$

If $P(A)$ contains $\phi$ only then all power set contains the empty set, $\phi$.

Else, let ~~if~~ $P(A)$ contains another set of the form $\{x\}$ in $P(A)$

Let $\{x\} \in P(A)$

If a power set contains ~~only~~ a set containing element $x$ only then the element $x$ is present in the set itself.

$\Rightarrow x \in A$

since $A \subseteq B$

$x \in B$

If an element is present in a set then a set containing only that element is ~~also~~ present in its power set.

$\Rightarrow \{x\} \in P(B)$

Every element in $P(A)$ also has to be in $P(B)$.

$\therefore$ By the definition of a subset we get

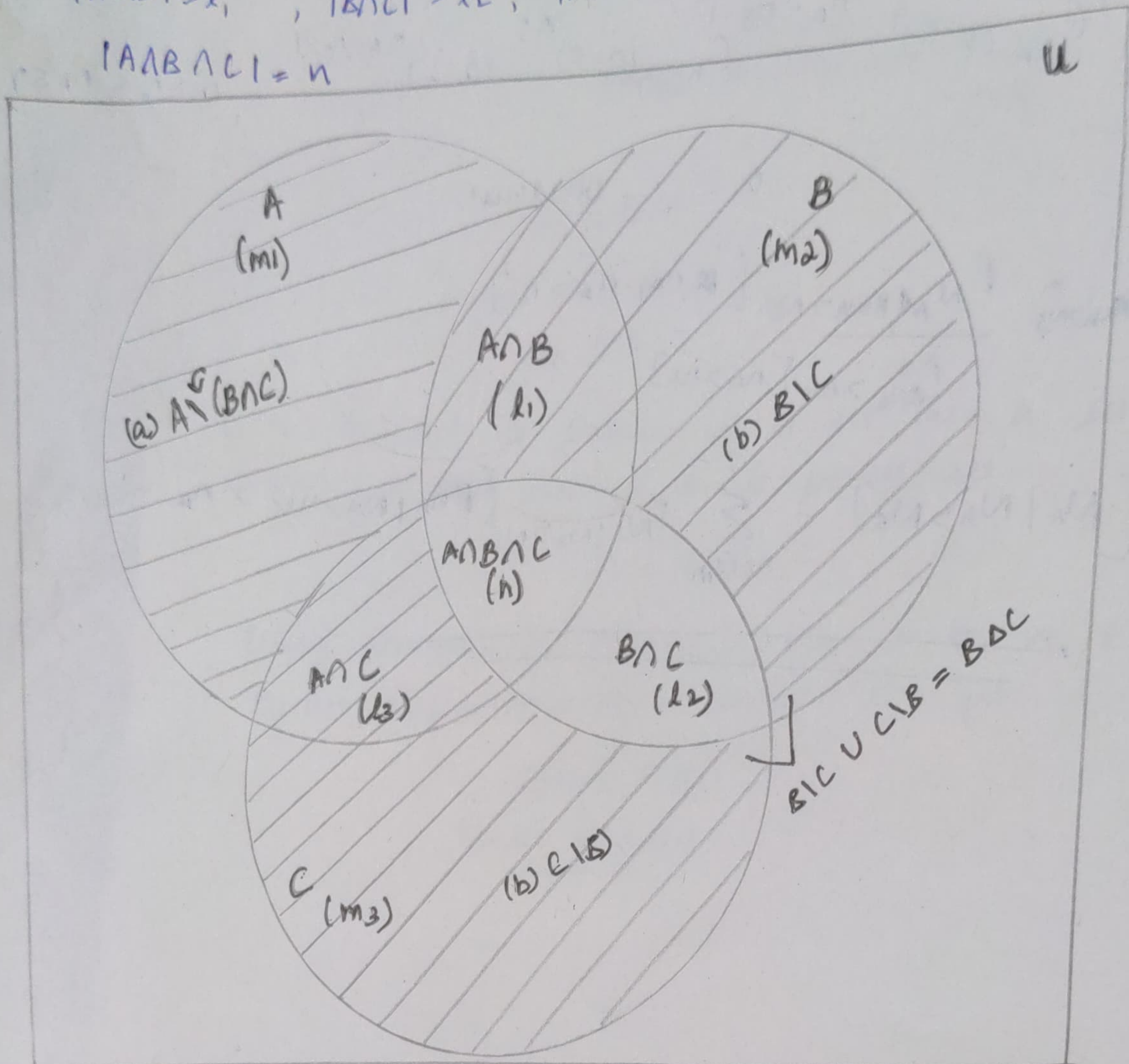
$P(A) \subseteq P(B)$

Hence proved.

A4. $|A| = m_1$, $|B| = m_2$, $|C| = m_3$

$|A \cap B| = l_1$, $|B \cap C| = l_2$, $|A \cap C| = l_3$

$|A \cap B \cap C| = n$



(a) $A \setminus (B \cup C)$

cardinality of $A \setminus (B \cup C) = m_1 + n - (l_1 + l_3)$

$= m_1 + n - l_1 - l_3$

(b) $B \Delta C = (B \setminus C) \cup (C \setminus B)$

cardinality of $B \Delta C = m_3 + m_2 - 2l_2$