Assignment 3

September 8, 2020

1. Let $\{a_n\}$ and $\{b_n\}$ be two sequences. Suppose $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$, and $s_n = \frac{a_n^3+4a_n}{b_n^2+1}$. Evaluate $\lim_{n\to\infty} s_n$.

2. Let $t_1 = 1$ and $t_{n+1} = \frac{t_n^2+2}{2t_n}$ for $n \geq 1$. First prove $t_n \neq 0$ for all $n \geq 1$. Assume $\{t_n\}$ converges and find the limit. ??

3. Let $\{t_n\}$ be a bounded sequence, i.e., there exists M such that $|t_n| \leq M$ for all $n \in \mathbb{N}$, and let $\{s_n\}$ be a sequence such that $\lim_{n\to\infty} s_n = 0$. Prove $\lim_{n\to\infty}(s_n t_n) = 0$.

4. Let $\{s_n\}$ be a sequence such that $|s_{n+1} - s_n| < 2^{-n}$ for all$n \geq 1$. Prove $\{s_n\}$ is a Cauchy sequence.
   Is the result true if we only assume $|s_{n+1} - s_n| < \frac{1}{n}$ for all $n \geq 1$.

5. Let $s_1 = 1$ and $s_{n+1} = \frac{1}{3}(s_n + 1)$ for $n \geq 1$.
   (a) Find s_2, s_3 and s_4.
   (b) Use induction to show $s_n > \frac{1}{2}$ for all $n \geq 1$.
   (c) Show (s_n) is a decreasing sequence.
   (d) Show that $\lim_{n\to\infty} s_n$ exists and find the limit.