

Assignment 1

August 25, 2020

1. Let x and $y \geq 0$ be two real numbers. Prove that $|x| \leq y$ iff $-y \leq x \leq y$.
2. If r is rational ($r \neq 0$) and x is irrational. Prove that $r + x$ and rx are irrational.
3. Let E be a non empty set of real numbers. Suppose α is a lower bound of E and β is an upper bound of E . Prove that $\alpha \leq \beta$.
4. Let S and T be nonempty bounded subsets of \mathbb{R} .
Prove if $S \subseteq T$, then $\inf T \leq \inf S \leq \sup S \leq \sup T$.
5. Prove $\sup(S \cup T) = \max\{\sup S, \sup T\}$.
6. Let E be a non empty set of real numbers which is bounded below. Let $-E$ be the set of all numbers $-x$, where $x \in E$. Prove that
$$\inf E = -\sup(-E).$$
7. $x \in \mathbb{R}$. Then $|x| < \varepsilon$ for every $\varepsilon > 0$ iff $x = 0$.
8. The set of integers \mathbb{Z} is a countable set.
9. The set of rational numbers \mathbb{Q} is countable.
10. The set of irrationals is uncountable.