Assignment 6

November 5, 2020

1. Suppose $h : [a, b] \to \mathbb{R}$ be a function. If $h \in \mathcal{R}[a, b]$ exists then so is $h^2 \in \mathcal{R}[a, b]$.

2. Using the above prove that if $f, g \in \mathcal{R}[a, b]$, so is fg.

3. Suppose $\int_a^b f(x)dx$ exists, then prove that $\lim_{t \to a+} \int_a^t f(x)dx = \int_a^b f(t)dt$.

4. In each of the following cases, show that f is integrable using the Riemann criterion.
(a) $f(x) = x$ on $[0, 1]$.
(b) $f(x) = x^3$ on $[3, 7]$.
(c) $f(x) = \frac{1}{X}$ on $[1, 2]$.

5. Evaluate the upper and lower integrals of f and show that f is integrable. Find the integral of f where

$$f(x) = \{ \begin{array}{cc} 0 & 0 \leq x < \frac{1}{2} \\ 10 & x = \frac{1}{2} \\ 1 & \frac{1}{2} < x \leq 1 \end{array}$$