Assignment 5

September 18, 2020

1. Prove that if $\sum_{n=1}^{\infty} a_n$ is a convergent series of non negative real numbers, and $p > 1$, then $\sum_{n=1}^{\infty} a_n^p$ converges.
2. Prove if $\{a_n\}$ is a decreasing sequence of real numbers and if $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} na_n = 0$.
3. Show $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges if and only if $p > 1$.
4. Test the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(3n)!+4^{n+1}}{(3n+1)!}$.
5. Suppose $\{a_n\}$ and $\{b_n\}$ are sequences of non negative real numbers, such that $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} b_n$ both converge, then prove that $\sum_{n=1}^{\infty} a_n b_n$ converges.