Assignment 2

September 1, 2020

1. Let A be a non-empty subset of $\mathbb{R}$ and $\alpha \in \mathbb{R}$ be an upper bound of A. Suppose for every $n \in \mathbb{N}$, there exists $a_n \in A$ such that $a_n \geq \alpha - \frac{1}{n}$. Show that α is the supremum of A.

2. Compute Supremum and Infimum of the set $\{\frac{n}{2n+1} : \ n \in \mathbb{N}\}$.

3. Prove that $\lim_{n\to\infty} \frac{1}{n^2} = 0$.

4. Prove that $\lim_{n\to\infty}(\sqrt{n+1} - \sqrt{n}) = 0$.

5. Let $\{s_n\}$ be a sequence of nonnegative real numbers, and suppose that $\lim_{n\to\infty} s_n = 0$. Prove that $\lim_{n\to\infty} \sqrt{s_n} = 0$.