Assignment 7

November 18, 2020

1. Let $f : [0, \frac{\pi}{2}] \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \cos^2 x & x \in \mathbb{Q} \\ 0 & otherwise \end{cases}$$

   Find the upper and lower Riemann integrals of f over $[0, \frac{\pi}{2}]$. Is it Riemann integrable?

2. Determine the values of k for which the improper integral $\int_1^\infty \left[\frac{kt}{t^2+1} - \frac{1}{2t}\right] dt$ converges.

3. Find the limit of the improper integral $\int_0^{\frac{\pi}{2}} \log t dt$.

4. Using the above prove the convergence or divergence of the improper integral $\int_0^{\frac{\pi}{2}} \log \sin t dt$.

5. Prove the convergence or divergence of the improper integral $\int_0^1 \frac{e^{t/2}}{\sqrt{1-\cos t}} dt$.