Assignment 4

September 15, 2020

1. If $\{a_n\}$ is a bounded sequence, then there exist subsequences $\{a_{n_k}\}$ and $\{a_{m_k}\}$ such that

$$\lim sup\ a_n = \lim a_{n_k} \text{ and } \lim inf a_n = \lim a_{m_k}.$$

2. a) If there exists a subsequence $\{a_{n_k}\}$ of $\{a_n\}$ such that $\lim_{k\to\infty} a_{n_k} = t$. Then $t \le \beta = \lim sup\ a_n$.
   b) If there exists a subsequence $\{a_{m_k}\}$ of $\{a_n\}$ such that $\lim_{k\to\infty} a_{m_k} = s$. Then $s \ge \alpha = \lim inf\ a_n$.

3. If $\{a_n\}$ and $\{b_n\}$ are bounded sequences of real numbers and if $a_n \le b_n$ for all $n \in \mathbb{N}$, then

$$\lim sup\ a_n \le \lim sup\ b_n \text{ and } \lim inf a_n \le \lim inf b_n.$$

4. If $\{a_n\}$ and $\{b_n\}$ are bounded sequences off real numbers. Then

$$\lim sup(a_n + b_n) \le \lim sup a_n + \lim sup b_n$$

   and

$$\lim inf(a_n + b_n) \ge \lim inf a_n + \lim inf b_n.$$

5. Prove that $\lim_{n\to\infty}(1 + \frac{1}{n})^n = e$ assuming that $e = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k!} = \sum_{k=1}^{\infty} \frac{1}{k!}$.