RELATIVE STABLE MAPS IN GENUS ONE VIA RADIAL ALIGNMENTS

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1. Relative space equals closure of the nice locus

Since the log structures only come into play when the source curve is reducible, it follows that the nice locus in the radially aligned setting is the same as the nice locus in the ordinary setting. In particular, it is irreducible.

We now want to show that the relative space in the radially aligned setting is equal to the closure of the nice locus; irreducibility follows immediately. One direction is clear: [WHY?]

It thus remains to show that, given a relative radially aligned map, we can smooth it to one in the nice locus. This is done by considering different cases locally, then glueing.

Case 1: non-contracted genus one internal component. Assume that the curve takes the form

$$C = C_0 \cup C_1 \cup \ldots \cup C_k$$

where all the C_i are smooth, C_0 has genus one, all the other C_i have genus zero, and for $i = 1, \ldots, k$, C_i intersects C_0 at a single node (denoted q_i) and does not intersect any other components.

Suppose furthermore that C_0 is an *internal component*, meaning that it is mapped into H via f, and that C_1,...,C_k are *external components*, meaning that they are not mapped into H via f. Finally, suppose that f is not constant on C_0. The picture is:

[FIGURE]

Suppose that this is a relative stable map. This means that [BLAH]. We claim that it can be smoothed to a relative stable map in the nice locus. The construction depends on choosing an appropriate smoothing of the curve C, so that the map also smooths.

We start with $W = C_0 \times \mathbb{A}^1_t$ (where t denotes a fixed co-ordinate on the affine line). This is a smooth surface, fibred over $\mathbb{A}^1_t$, with fibre equal to the elliptic curve C_0. Consider the points $q_1, \ldots, q_k$ on C_0. We will perform a series of weighted blow-ups at the points $(q_i, 0) \in W$, in order to obtain a surface whose general fibre is smooth (in fact, isomorphic to C_0) and whose central fibre is isomorphic to C.

Fix $i = 1, \ldots, k$ and let m_i be the multiplicity of f with H at $q_i \in C_i$. We define:

$$\beta = \prod_{i=1}^{k} m_i \qquad r_i = \beta / m_i$$

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