Directed Studies: Assignment 5

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Write a summary of the estimation methods and algorithms, with their key differences, advantageous and disadvantageous in maximum 2 pages, in Overleaf.

I. Gaussian Filters

For Gaussian Filters, the name is self explanatory as they all use multivariate normal distribution. A mean (known as μ) and a covariance matrix (known as $\sum$) is enough to represent the posterior probability as a normal distribution. This representation is called moment represnation as mean and covariance are first and second moments of normal distribution. Gaussian Filters are known as the most popular implementation of Bayes filter in continuous space, despite a number of shortcomings [1]. Gaussians are unimodal as they posses only one maximum. In other words, it creates a posterior around the true state with a small margin of uncertainty which is suitable for local state estimation. On the other hand, when it comes to multi-hypothesis global state estimation, the Gaussian posteriors are no longer a proper choice. Kalman filters and Extended Kalman filters will be disscuessed as the most common Gaussian filters.

A. Kalman Filter (KF)

Kalman Filter uses a few assumptions which is as follows:

- Markov assumption for Bayes Filters.
- Next state probability is a linear function of the current state with added noise.
- Measurement probability is a linear function of current state with added measurement noise.
- The initial guess is a normal distribution (also known as initial belief)

Kalman uses previous state (μ_{t-1} and $\sum_{t-1}$) and updates it using the control variable (u_t) and the current measurement (z_t) to predict a belief from current state which is represented by μ_t and $\sum_t$. KE is **computationally efficient** and incorporates both the measurement uncertainty and the knowledge about the system dynamics to **overcome noisy measurements** and produce a more accurate and robust estimate. On the other hand, it uses a **linearity assumption** for both measurements and dynamics and this is a serious limitation when facing non-linearity. **Gaussian noise assumption** is also a disadvantage when noise exhibits non-Gaussian characteristics or has a non-zero mean. Advanced variants like the unscented Kalman filter (UKF) [2] or particle filters can handle non-Gaussian noise [3].

B. Extended Kalman Filter (EKF)

The Extended Kalman Filter (EKF) is an extension of the Kalman Filter that addresses the limitation of linearity by allowing for nonlinear system dynamics and measurements. The EKF follows the same basic principles as the Kalman Filter, including the Markov assumption for Bayes Filters and the propagation of state estimates using a linear function with added noise. However, in the EKF, instead of assuming a linear relationship, the system dynamics and measurement equations are approximated using Taylor series expansions around the current estimated state.

The EKF is **computationally efficient** compared to more advanced nonlinear filters like the particle filter, but it still suffers from some limitations. EKF uses **linearity approximation** for system dynamics and measurements. The accuracy of the approximation depends on the local linearity of the system, and errors can occur if the non-linearities are significant. EKF still suffers from **Gaussian noise assumption** and **sensitivity to initial conditions**. If the initial guess is far from the true state, or if the covariance matrix is not properly tuned, it can lead to suboptimal or divergent results.

C. Extensions of EKF

There are several extensions and variations of the Extended Kalman Filter (EKF) that have been developed to address specific challenges in nonlinear estimation problems. The **Unscented Kalman Filter** [2] is a popular extension of the EKF that aims to overcome the linearization approximation by using a deterministic sampling technique called the unscented transform. Instead of linearizing the system dynamics and measurements, the UKF captures the nonlinearities by sampling a set of representative points, known as sigma points, and propagating them through the nonlinear functions.

The **Iterated Extended Kalman Filter** [4] is an improvement over the EKF that iteratively refines the state estimate to improve accuracy. In each iteration, the IEKF applies the EKF update step, but instead of using the linearized measurement equations, it linearizes them around the updated state estimate. The **Second-Order Extended Kalman Filter (SOEKF)** [5] enhances the

EKF by incorporating second-order statistics of the state distribution. Instead of relying solely on the mean and covariance, the SOEKF includes the information about the skewness and kurtosis of the distribution. The **Gaussian Mixture Extended Kalman Filter (GMEKF)** [6] extends the EKF by representing the state estimate as a mixture of Gaussian distributions rather than a single Gaussian. It combines the advantages of the EKF with the flexibility of Gaussian mixture models to handle multimodal distributions and complex nonlinearities more effectively.

II. Non-parametric filters

As an alternative for Gaussian techniques, non-parametric methods don't rely on a fixed form of posterior. Instead they approximate the posteriors by a number of values each covering a region in state space [1]. Non-parametric methods are suitable for multi-modal beliefs when robot is facing a global uncertainty. However, having several hypotheses comes at the price of added computational cost. The bright side is that the number of parameters can be modified according to expected complexity.

A. Particle Filters

The key idea is using a set of random state samples drawn from a posterior belief. In comparison with Gaussian Filters that use a parametric (normal) distribution, these filters represent the distribution by a set of samples drawn from the distribution. The number of these samples (known as particles) is the parameter to set. Each particle has a weight that shows the probability of getting current observation assuming the particle as the current state. During the resampling process, particles are resampled randomly, where particles with more weights (or importance factors) have a better chance to be chosen.
Here is a list of advantages for particle filters:

- Particle filters can handle **non-linear and non-Gaussian systems**, making them suitable for a wide range of applications.
- Particle filters can be used in both single-object and **multi-object tracking** scenarios.
- Particle filters can handle a **wide range of** measurement and process noise **distributions**.

As mentioned earlier, particle filters (like other non-parametric methods) suffer from **computational complexity** problem. In particle filters, the particles with low weights tend to have little influence on the estimation, while a few particles with high weights dominate the representation of the posterior distribution which can lead to the problem of **sample degeneracy**. Particle filters are also **sensitive to initial particles** and they tend to collapse toward the mode of normal distribution (which they are generated from) making them **unable to capture multi-modal distributions**.

B. Extensions of Particle Filters

To avoid the above drawbacks, some extensions of particle filter have been introduces. To handle sample degeneracy, **Auxiliary Particle Filter (APF)** [7] is introduced as an extension that addresses the problem of sample degeneracy in particle filters. It introduces auxiliary variables that are sampled jointly with the state variables, leading to improved diversity among particles. The auxiliary variables are used to calculate the weights, and the state variables are resampled based on both the auxiliary and state variable weights. One extension that addresses the complexity of particle filters is the **SMCˆ2 (Sequential Monte Carlo squared) method** [8]. SMCˆ2 combines sequential Monte Carlo (SMC) methods with Markov chain Monte Carlo (MCMC) techniques to reduce the computational burden of particle filters.

III. Handling Outliers

Extremely improbable measurements according to measurement model (outliers) can cause divergence in state estimation if we assume there is a large portion of them. Some methods are needed to reduce or eliminate these outliers. Reference [3] has introduced **Random Sample Consensus (RANSAC)** to handle the outliers. This algorithm randomly selects a subset of data points, fits a model to this subset, and then evaluates the consensus of the model by counting the number of inliers that agree with it. This process is repeated multiple times, and the model with the highest consensus is selected as the final estimate.
RANSAC can **handle a large percentage of outliers** and is **versatile and applicable to different problems**. However, it is **sensitive to the choice of key parameters**, like sample size, threshold value for outliers, maximum iterations, and consensus criteria. Sometimes RANSAC suffers from **slow convergence** when there are too many outliers.
Progressive Sample Consensus (PROSAC) [9] is an extension of RANSAC to address the problem of convergence with large percentage of oultiers. By adaptively selecting data points during the sampling process, PROSAC increases the chances of selecting inliers early on and reduces the influence of outliers. This leads to more accurate model estimation compared to traditional RANSAC, especially when the percentage of outliers is high.

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