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M200363CS

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① (a) Fourier transform = $\sin(x)$

$$f(x) = \sin(x)$$

we know that $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$

$$F(x) = \int_{-\infty}^{+\infty} f(x) \cdot e^{-i2\pi u x}\, dx$$

we are taking sin range from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$

$$F(x) = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{e^{ix} - e^{-ix}}{2i} \cdot e^{i2\pi u x}\, dx$$

$$= \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{e^{ix} \cdot e^{-i2\pi u x}}{2i}\, dx - \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{e^{-ix} * e^{-i2\pi u x}}{2i}\, dx$$

$$= \frac{1}{2i} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} e^{i(1-2\pi u)dx}\, dx - \frac{1}{2i} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} e^{-i(1+2\pi u)x}\, dx$$

$$= \frac{1}{2i} \left[\frac{e^{i(1-2\pi u)}}{i(1-2\pi u} \right]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} - \frac{1}{2i} \left[\frac{e^{-i(1+2\pi u)}}{-i(1+2\pi u)} \right]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}$$

$$= \frac{1}{2i} \frac{e^{i(1-2\pi u)\frac{\pi}{2}} - e^{i(1-2\pi u)\frac{\pi}{2}}}{i(1-2\pi u)} - \frac{1}{2i} \frac{e^{i(1+2\pi u)\frac{\pi}{2}} - e^{i(1+2\pi u)}}{-i(1+2\pi u)}$$

$$= \frac{1}{i} \frac{\sin(1-2\pi u)\pi}{(1-2\pi u)} - \frac{1}{i} \frac{\sin(1+2\pi u)\pi}{(1+2\pi u)}$$

let $\omega = 2\pi u$ then

$$= \frac{1}{i} \frac{\sin(1-\omega)\frac{\pi}{2}}{i(1-\omega)} - \frac{1}{i} \frac{\sin(1+\omega)\frac{\pi}{2}}{i(1+\omega)}$$

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(b) Fourier Transform = $\cos(x)$

$$f(x) = \cos(x)$$

we know that $\cos x = \frac{e^{ix} + e^{-ix}}{2}$

$$F(x) = \int_{-\infty}^{+\infty} f(x) \cdot e^{-i2\pi u x}\, dx$$

we are taking cos range from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$

$$F(x) = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{e^{ix} + e^{-ix}}{2} \cdot e^{-i2\pi u x}\, dx$$

$$= \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{e^{ix} * e^{-i2\pi u x}}{2}\, dx + \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{e^{-ix} * e^{-i2\pi u x}}{2}\, dx$$

$$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} e^{i(1-2\pi u)x}\, dx + \frac{1}{2}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} e^{-i(1+2\pi u)x}\, dx$$

$$= \frac{1}{2}\left[\frac{e^{i(1-2\pi u)}}{i(1-2\pi u)}\right]_{+\frac{\pi}{2}}^{-\frac{\pi}{2}} + \frac{1}{2}\left[\frac{e^{-i(1+2\pi u)}}{-i(1+2\pi u)}\right]_{+\frac{\pi}{2}}^{-\frac{\pi}{2}}$$

$$= \frac{1}{2} \frac{e^{i(1-2\pi u)\frac{\pi}{2}} - e^{i(1-2\pi u)\frac{\pi}{2}}}{i(1-2\pi u)} + \frac{1}{2} \frac{e^{-i(1+2\pi u)\frac{\pi}{2}} - e^{i(1+2\pi u)\frac{\pi}{2}}}{-i(1+2\pi u)}$$

$$= \frac{\sin(1-2\pi u)\pi}{(1-2\pi u)} + \frac{\sin(1+2\pi u)\pi}{(1+2\pi u)}$$

let $w = 2\pi u$ then

$$\frac{\sin(1-w)\frac{\pi}{2}}{(1-w)} + \frac{\sin(1+w)\frac{\pi}{2}}{(1+w)}$$

(c) $f(x) = \cos(4\pi x)$

$$= \frac{e^{i4\pi x} + e^{-i4\pi x}}{2}$$

$$FT(\cos 4\pi x) = \left[\frac{e^{i4\pi x} + e^{-i4\pi x}}{2}\right]$$

$$= \frac{1}{2}\left[2\pi\delta(\omega-4\pi) + 2\pi\delta(\omega+4\pi)\right]$$

$$= \pi\delta(\omega-4\pi) + \pi\delta(\omega+4\pi)$$

(d) $f(x) = e^{-j8\pi x}$

$$F(x) = \int_{-\infty}^{+\infty} f(x) \cdot e^{-j2\pi ux}\, dx$$

$$= \int_{-\infty}^{+\infty} e^{-j8\pi x} \cdot e^{-j2\pi ux}\, dx$$

$$= \int_{-\infty}^{0} e^{-j8\pi x} \cdot e^{-j2\pi ux}\, dx + \int_{0}^{+\infty} e^{-j8\pi x} \cdot e^{-j2\pi ux}\, dx$$

$$= \int_{-\infty}^{0} e^{2\pi xj(4-u)}\, dx + \int_{0}^{+\infty} e^{-2\pi xj(4+u)}\, dx$$

$$= \left[\frac{e^{2\pi xj(4-u)}}{2\pi j(u-4)}\right]_{-\infty}^{0} + \left[\frac{e^{-2\pi xj(4+u)}}{-2\pi j(u+4)}\right]_{0}^{+\infty}$$

$$= \left[\frac{1}{2\pi j(4-u)} - 0\right] + \left[0 + \frac{1}{2\pi j(4+u)}\right]$$

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$$= \frac{1}{2\pi j(4-u)} + \frac{1}{2\pi j(4+u)}$$

$$= \frac{2\pi j(4+u) + 2\pi j(4-u)}{2\pi j(16-u^2)}$$

$$= \frac{2\pi j * 8}{2\pi j(16-u^2)}$$

$$= \frac{8}{(16-u^2)}$$

(e) $f(x) = 2 + \cos(2\pi x) - j\sin(2\pi x) + \cos(3\pi x) - j\sin(3\pi x)$

we can solve this problem using Linearity property.

we know that $\int_{-\infty}^{+\infty} 1 \cdot e^{-i2\pi x} dx = \delta(x)$

$f(x) = 2$ $\quad$ $FT[f_1(x)] = 2 \cdot \delta(\omega)$

$f_2(x) = \cos(2\pi x) = \frac{e^{i2\pi} + e^{-i2\pi x}}{2}$

$FT[f_2(x)] = \pi[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$

$f_3(x) = -j\sin(2\pi x)$

$\sin 2\pi x = \frac{e^{i2\pi x} - e^{j2\pi x}}{2j}$

$FT[f_3(x)] = -j \cdot j\pi[\delta(\omega+2\pi) - \delta(\omega-2\pi)]$

$= \pi[\delta(\omega+2\pi) - \delta(\omega-2\pi)]$

$f_4(x) = \cos 3\pi x$, $\frac{e^{i3\pi x} + e^{-i3\pi x}}{2}$

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$FT[f_4(x)] = \pi[\delta(\omega - 3\pi)] + \delta(\omega + 3\pi)]$

$f_5(x) = -j\sin(3\pi x)$

$$\sin 3\pi x = \frac{e^{j3\pi x} - e^{-j3\pi x}}{2j}$$

$F[\cancel{6}\ f_5(x)) = j \cdot j\pi[\delta(\omega + 3\pi) - \delta(\omega - 3\pi)$

$= \pi[\delta(\omega + 3\pi) - \delta(\cancel{8}\omega - 3\pi)]$

$FT[f(x)] = FT[f_1(x)] + FT[f_2(x)] + FT[f_3(x)]$
$+ FT[f_4(x)] + FT[f_5(x)]$

$= 2\cdot\delta(\omega) + \pi[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$
$+ \pi[\delta(\omega + 2\pi) - \delta(\omega - 2\pi)]$
$+ \pi[\delta(\omega - 3\pi) + \delta(\omega + 3\pi)] +$
$\pi(\delta(\omega + 3\pi) - \delta(\omega - 3\pi)]$

$= 2.\delta(\omega) + 2\pi[\delta(\omega + 2\pi) + \delta(\omega + 3\pi)]$ Ans

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(2) Linearity Property of 1D continuous fourier transform

$$y(t) = \alpha \cdot x_1(t) + \beta \cdot x_2(t)$$

$$Y(\omega) = \int_{-\infty}^{+\infty} (\alpha \cdot x_1(t) + \beta \cdot x_2(t)) e^{-j\omega t} dt$$

$$= \alpha \int_{-\infty}^{+\infty} x_1(t) e^{-j\omega t} dt + \beta \int_{-\infty}^{+\infty} x_2(t) e^{-j\omega t} dt$$

$$= \alpha \cdot X_1(\omega) + \beta \cdot X_2(\omega)$$

$$\alpha \cdot x_1(t) + \beta \cdot x_2(t) \xleftrightarrow{F} \alpha \cdot X_1(\omega) + \beta \cdot X_2(\omega)$$

(3) $$f(x) = \begin{cases} 1, & -\frac{a}{2} \leq x \leq \frac{a}{2} \\ 0, & \text{otherwise} \end{cases}$$

Time Shift property for $f(x-2)$

$$f(x-2) = \begin{cases} 1, & -\frac{a}{2} + 2 \leq x \leq +\frac{a}{2} + 2 \\ 0, & \text{if otherwise} \end{cases}$$

$$F(x) = \int_{-\infty}^{+\infty} f(x) \cdot e^{-i 2\pi \omega x} dx$$

We are taking range from $-\frac{a}{2} + 2$ to $+\frac{a}{2} + 2$

$$F(u) = \int_{-\frac{a}{2}+2}^{+\frac{a}{2}+2} 1 \cdot e^{-i2\pi ux} \, dx$$

$$= \left[\frac{e^{-i2\pi ux}}{-i2\pi u} \right]_{-\frac{a}{2}+2}^{+\frac{a}{2}+2}$$

$$= \left[\frac{e^{-i2\pi u\frac{a}{2}+2}}{-i2\pi u} - \frac{e^{i2\pi u\frac{a}{2}+2}}{-i2\pi u} \right]$$

$$= \left[\frac{e^{-i\pi ua} \cdot e^{-i2\pi u^2} - e^{i\pi ua} \cdot e^{-i2\pi u}}{-i2\pi u} \right]$$

$$= e^{-i2\pi u \cdot 2} \left[\frac{e^{i\pi ua} - e^{-i\pi ua}}{i2\pi u} \right]$$

$$= e^{-i2\pi u2} \frac{\sin(\pi ua)}{\pi u}$$ proved