# Statistical Inference Course Project: an Exploration of Applying the Central Limit Theorem to a Exponential Distribution

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## Overview

In this class project we are exploring the Central Limit Theorem. Specifically, we will simulate an exponential distribution with $\lambda$ of 0.2. We will take 40 random exponentials and calculate the mean over 1000 simulations. The sample mean and sample variance should be close to the theoretical mean and variance.

## Simulations

We will simulate the exponential distribution 1000 times for 40 random exponentials.

```
lambda <- 0.2
set.seed(42)
mns = NULL
for (i in 1 : 1000) mns = c(mns, mean(rexp(40,lambda)))
```

## Sample mean versus theoretical mean

The sample mean is 4.9865083 compared to the theoretical mean of 5. See the figure below with the R output comparing the mean across the 1000 simulations to the theoretical mean of $\frac{1}{\lambda}$.

## Sample variance versus theoretical variance

The sample variance is the population variance divided by the sample size. In this case the sample variance is 0.6344405 compared to the theoretical sample variance of 0.625. See the figure below with the R output comparing the variance of the means of the 1000 simulations to the theoretical variance of $\frac{(\frac{1}{\lambda})^2}{40}$.

```
knitr::kable(data.frame(c("Mean","Variance"),c(mean(mns),var(mns)),c(1/lambda,(1/lambda)^2/40)),col.nam
```

Table 1: Figure: Comparison of Sample and Theoretical Statistics

| Statistic | Sample | Theoretical |
|---|---:|---:|
| Mean | 4.9865083 | 5.000 |
| Variance | 0.6344405 | 0.625 |

## Distribution

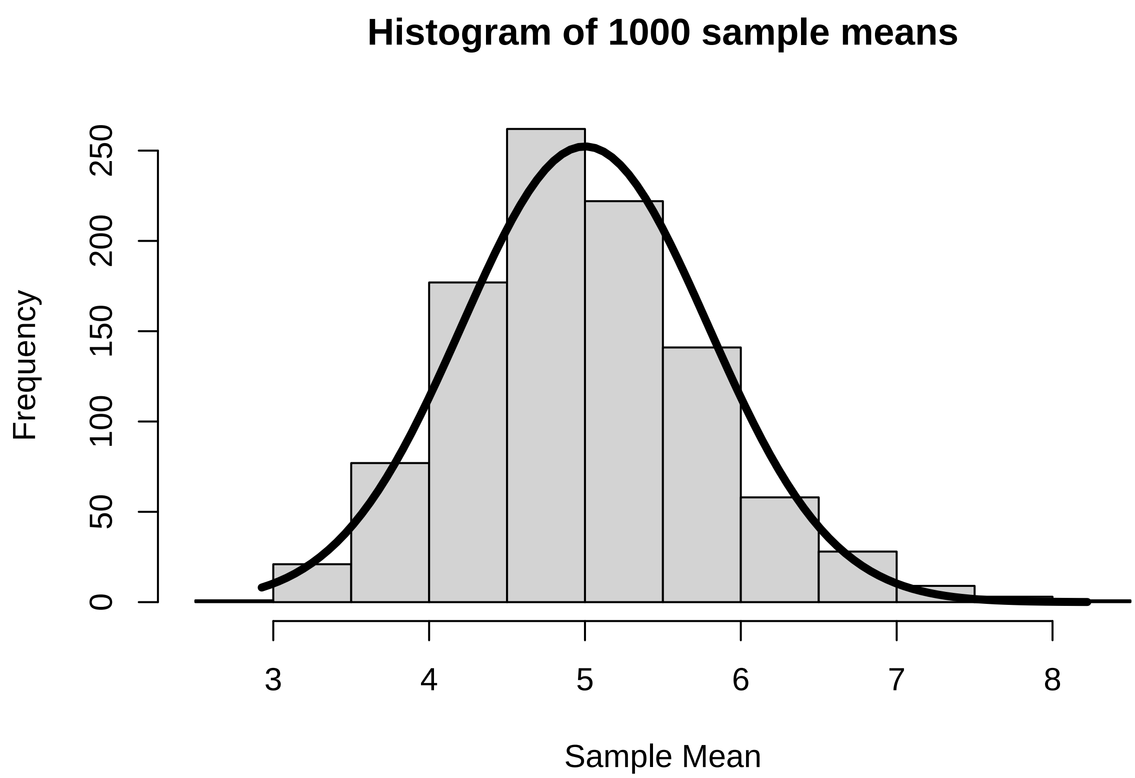
Per the CLT, the distribution of means should be normally distributed with a mean equal to the population mean and a variance equal to the population variance divided by the sample size.

I will overlay a normal distribution with these parameters over a histogram of the sample means.

```
hist_data <- hist(mns,main="Histogram of 1000 sample means",xlab="Sample Mean")
x_values <- seq(min(mns), max(mns), length = 100)
y_values <- dnorm(x_values, mean = 1/lambda, sd =(1/lambda)/sqrt(40))
y_values <- y_values * diff(hist_data$mids[1:2]) * length(mns)
hist_data
```

```
## $breaks
##  [1] 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5
##
## $counts
##  [1]   1  21  77 177 262 222 141  58  28   9   3   1
##
## $density
##  [1] 0.002 0.042 0.154 0.354 0.524 0.444 0.282 0.116 0.056 0.018 0.006 0.002
##
## $mids
##  [1] 2.75 3.25 3.75 4.25 4.75 5.25 5.75 6.25 6.75 7.25 7.75 8.25
##
## $xname
## [1] "mns"
##
## $equidist
## [1] TRUE
##
## attr(,"class")
## [1] "histogram"
```

```
lines(x_values, y_values, lwd = 4)
```




### Appendix

Here is the data for the 1000 sample means we simulated

```
mns
```

```
##    [1] 4.915756 6.941835 4.775331 5.310784 7.002644 5.320620 5.349217 5.584491
##    [9] 5.675150 4.689581 4.344556 6.312625 4.435001 6.017505 5.252852 4.822530
##   [17] 5.797813 5.428165 5.087272 4.369322 7.300695 4.386380 4.639684 4.476738
##   [25] 6.506295 5.038921 4.149865 4.697668 5.464270 4.818283 5.075756 4.766316
##   [33] 5.729564 4.454204 4.371153 4.273316 4.561490 4.990427 5.883218 4.178961
##   [41] 5.761507 4.204386 5.294196 5.948492 5.147837 6.561700 5.116437 5.324370
##   [49] 4.846137 4.809980 5.006344 4.578631 4.522309 4.269751 3.785409 4.787385
##   [57] 4.939340 4.210792 4.887253 5.932574 4.076066 4.111665 5.604068 4.318169
##   [65] 4.535208 5.113398 6.221628 4.187519 3.700945 4.082352 4.710370 4.154324
##   [73] 4.843434 5.771539 5.350361 5.122669 5.432817 5.368854 5.213403 5.640059
##   [81] 4.731423 5.544904 3.662853 3.984775 5.026290 7.448004 5.682902 4.611124
##   [89] 5.602797 3.718097 4.294280 5.084669 5.283177 5.997779 6.873398 5.608256
##   [97] 5.772798 5.656365 4.470182 4.637363 4.724106 5.991009 3.056635 3.999526
##  [105] 4.831447 4.439854 4.808950 4.823020 5.584983 5.533404 5.732691 5.899998
##  [113] 4.221977 6.217919 4.242958 5.965948 5.242062 5.400401 5.667402 6.676996
##  [121] 7.015775 5.933641 4.680437 6.017428 4.544207 4.921813 5.778801 5.619104
##  [129] 4.758067 3.948897 6.511689 4.988435 3.867518 4.751703 3.986589 5.290251
##  [137] 3.624395 4.773450 5.831041 5.149339 5.530634 4.531821 4.428804 5.130249
##  [145] 4.905561 5.028805 4.617091 4.670742 5.300123 5.501352 4.475126 6.465954
```

```
## [153] 3.417665 3.852407 5.173052 5.020559 5.058966 5.543747 5.267862 5.245204
## [161] 5.496944 4.827385 4.421643 4.876625 5.026827 4.708724 4.942708 4.643749
## [169] 4.232580 6.543792 4.734965 4.471166 4.932080 5.353826 4.355795 5.086370
## [177] 5.667930 5.148468 4.792966 4.368271 4.140657 4.711379 4.771578 5.737934
## [185] 6.638920 4.174885 4.202981 4.178064 4.233754 5.490864 4.086635 5.093257
## [193] 4.197759 5.625519 5.155310 4.632150 5.004563 5.064709 5.475907 4.359322
## [201] 6.074658 5.672906 4.768945 3.808490 5.822045 4.547892 4.258085 3.135229
## [209] 5.503591 6.418503 4.506215 4.013546 7.110412 4.251880 5.746799 4.131984
## [217] 4.638362 4.995474 5.147958 4.653784 5.685575 5.905634 4.540658 5.693368
## [225] 4.462193 6.852030 6.187128 4.788021 3.962237 4.886237 6.526365 6.295928
## [233] 4.626991 4.935316 4.763555 5.470588 4.931726 5.243696 5.424871 4.470334
## [241] 3.169814 4.576302 4.976655 3.728954 4.809820 4.980153 4.974588 5.403972
## [249] 5.117583 3.331514 6.164374 4.779412 3.843928 5.834848 6.033658 4.769412
## [257] 4.313376 4.337710 4.277889 5.783758 4.868482 5.557935 5.373938 5.478153
## [265] 4.749706 4.969267 5.353649 5.560902 4.815529 6.003435 3.945543 4.664663
## [273] 4.255389 5.973042 4.743625 4.572341 4.119918 4.545718 5.320238 4.361557
## [281] 5.149023 4.607146 5.512567 7.159310 4.708853 5.766581 3.973060 5.107286
## [289] 4.941676 4.526376 3.996117 3.706957 5.803645 6.710208 4.449503 6.097619
## [297] 4.119427 4.973244 6.520525 3.993599 6.004850 4.880851 5.028459 5.732645
## [305] 5.323530 4.896905 5.425620 5.959925 4.300767 6.943361 5.134960 5.144402
## [313] 5.758911 3.977708 4.527564 4.932252 6.598244 4.991401 4.926490 4.609646
## [321] 6.194482 5.556951 5.161114 5.374715 5.817495 6.004689 4.855497 4.237895
## [329] 4.333901 4.277759 4.285118 4.790203 4.845251 4.717081 5.384720 4.827864
## [337] 4.410998 4.195175 4.418513 5.817084 4.605029 4.935724 5.476899 4.252706
## [345] 6.407113 4.541196 4.698877 4.664226 4.906632 4.089411 5.846572 5.110334
## [353] 3.578683 4.572824 4.870849 5.489600 8.223341 5.108174 4.387259 4.798789
## [361] 5.894355 5.111630 5.418691 5.277027 3.774977 4.552924 4.866045 4.461923
## [369] 4.225222 3.738491 5.258628 5.306549 4.275429 4.160323 4.680782 4.680001
## [377] 6.624414 5.650474 5.988097 4.678466 4.693476 4.439550 4.479561 5.015243
## [385] 4.757679 5.541558 5.257470 6.564261 5.294789 5.552021 5.865167 4.349706
## [393] 5.234916 4.958731 5.416636 4.564673 5.260985 5.536876 5.756691 4.673094
## [401] 5.740954 5.902294 5.067953 4.898041 5.068787 4.275915 5.181244 3.934156
## [409] 4.973692 5.318670 5.147939 4.952885 4.882525 5.111610 5.317891 5.143965
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## [441] 4.507622 5.748503 4.171726 4.463563 4.246308 4.849914 4.722449 5.998961
## [449] 4.809349 5.046230 4.276738 4.575236 3.759441 4.172257 5.955561 3.137861
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## [465] 4.377511 4.763760 5.412415 3.870747 4.595041 6.045203 5.403028 5.866557
## [473] 5.915340 4.981027 7.664141 6.210521 4.854781 5.242338 5.406747 5.412211
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## [489] 5.365768 7.339530 5.112742 5.176088 5.865780 4.947288 6.517913 3.224670
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## [545] 4.643178 6.114941 4.998424 4.137286 6.215577 5.504636 5.222118 4.734742
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## [569] 3.756097 4.146440 4.785660 4.904586 3.332412 4.609839 4.689479 4.909495
## [577] 5.463818 5.498240 4.397801 5.937759 5.809900 6.220728 4.269433 5.057811
```

```
## [585] 4.170965 5.482087 6.404871 4.832364 5.075772 4.257099 4.031169 4.443036
## [593] 5.783058 4.232320 5.456138 4.864045 4.898611 5.233520 6.315769 4.980684
## [601] 5.125886 5.210692 5.285610 5.429541 4.559863 5.046168 4.882238 5.409397
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## [641] 4.003827 4.697193 4.906316 4.603712 4.980704 5.819176 5.613945 6.324420
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## [737] 3.879443 5.316387 5.835422 4.197931 5.928988 4.408651 3.885829 4.844980
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## [809] 4.664267 4.244389 4.696897 5.750923 4.238334 5.500431 4.252293 5.964225
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## [937] 5.693662 6.194623 5.550189 5.002328 3.672463 3.415815 4.811159 4.928336
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## [953] 5.368004 4.593793 3.105067 4.001292 3.628199 3.642212 6.223690 4.845930
## [961] 6.200128 5.207410 4.455917 5.324984 4.728490 4.876876 4.265723 3.694574
## [969] 5.436649 4.359431 5.618001 5.491524 5.743520 6.214857 4.875003 4.185701
## [977] 5.505449 4.644221 4.841398 5.081574 5.313217 4.303297 6.077329 3.565459
## [985] 4.973874 4.931630 4.131254 4.284371 6.177968 4.955817 4.967086 4.863433
## [993] 4.737361 5.213368 4.459940 6.987129 4.766258 5.030009 5.280228 5.423759
```