(0000CJA100119033) Test Pattern

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2019 - 2020)

JEE(Advanced)
ALLEN COMMON TEST
29-09-2019

JEE(Main+Advanced) : NURTURE COURSE [PHASE : I, I(A), II & III]

ANSWER KEY PAPER-1

PART-1 : PHYSICS

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	D	A	B	A,D	A,B,C	A,B,C	A,B	B,C	A,B,C
	Q.	11	12								
	A.	A,D	A,C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	8.80	6.12 to 6.13	2.00	10.23 to 10.39	0.00	30.00				

PART-2 : CHEMISTRY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	C	B	C	A,B,C,D	A,B,C,D	C,D	A,B,D	A,B,C,D	A,B
	Q.	11	12								
	A.	B	A,B,C,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	3100.00	0.10	5.00	–38.00	6.00	6.00				

PART-3 : MATHEMATICS

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	D	C	B	A	A,C,D	A,B,C	A,C	A,D	A,C	A,C
	Q.	11	12								
	A.	A,B	A,D								
SECTION-II	Q.	1	2	3	4	5	6				
	A.	0.40	2.80	0.50	1.25	0.60	9.00				

(0000CJA100119034) Test Pattern

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(Academic Session : 2019 - 2020)

JEE(Main)
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JEE(Main+Advanced) : NURTURE COURSE [PHASE : I, I(A), II & III]

ANSWER KEY PAPER-2

PART-1 : PHYSICS

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A	D	C	A	D	B	C	B	A	A
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	C	C	A	C	B	D	B	D	B
SECTION-II	Q.	1	2	3	4	5					
	A.	0.75	63.00	64.40	1.50	0.67					

PART-2 : CHEMISTRY

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A	B	B	C	B	D	C	A	C	D
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	B	B	A	D	B	D	A	D	B
SECTION-II	Q.	1	2	3	4	5					
	A.	1.00	2.00	8.00	14.00	4.00					

PART-3 : MATHEMATICS

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	C	D	A	D	D	C	C	C	C	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	D	C	D	D	C	C	C	A	C
SECTION-II	Q.	1	2	3	4	5					
	A.	0.75	6.33	9.00	6.00	0.60					

PART-1 : PHYSICS — SOLUTION

SECTION-I

1. Ans. (C)

Sol. $F_x = -\frac{dU}{dx} = -2x$. $F_y = -\frac{dU}{dx} = -4y$

$N_x = 4$ $\qquad N_z = 0.3 \times 10 = 3$

$N = 5$

2. Ans. (D)

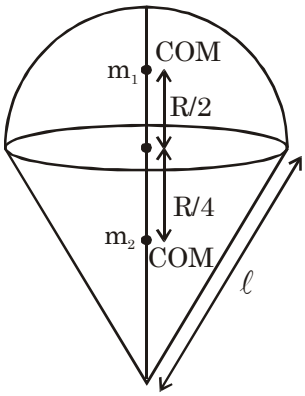
Sol. Mass of shell $(m_1) = \sigma(2\pi R^2)$

Mass of cone $(m_2) = \sigma\pi R\ell$

$$= \sigma\pi R\sqrt{R^2 + \left(\frac{3R}{4}\right)^2}$$

$$m_2 = \sigma\pi R^2\sqrt{\frac{16+9}{16}}$$

$$m_2 = \frac{5}{4}\pi\sigma R^2$$

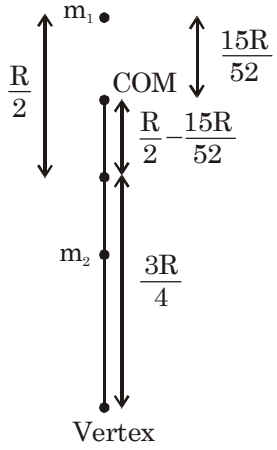


x is distance of $COM_{(System)}$ from m_1

$$x = \frac{m_2\left(\frac{3R}{4}\right)}{(m_1+m_2)} = \frac{\left(\frac{5}{4}\pi\sigma R^2\right)\left(\frac{3R}{4}\right)}{\left(\frac{5}{4}\pi\sigma R^2 + 2\pi\sigma R^2\right)}$$

$$x = \frac{\left(\frac{5}{4}\times\frac{3R}{4}\right)}{\left(\frac{5}{4}+2\right)} = \frac{\frac{15}{16}R}{\left(\frac{5+8}{4}\right)} = \frac{15}{16}\times\frac{4R}{13} = \frac{15}{52}R$$

Distance of $COM_{(System)}$ from vertex of cone

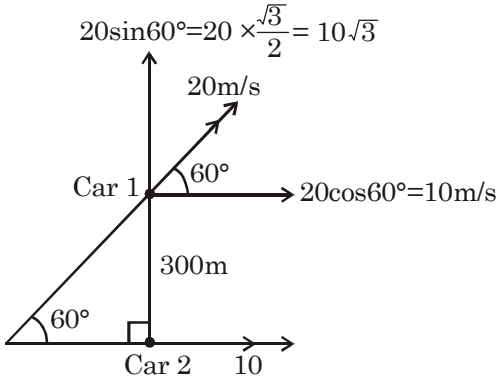


$$= \frac{3R}{4} + \left(\frac{R}{2} - \frac{15R}{52}\right) \quad = \frac{3R}{4} + \frac{(26R-15R)}{52}$$

$$= \frac{3R}{4} + \frac{11R}{52} = \frac{(39+11)}{52} \quad = \frac{50}{52}R = \frac{25}{26}R$$

3. Ans. (A)

Sol.



$\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$

$= \left(10\hat{i} + 10\sqrt{3}\,\hat{j}\right) - \left(10\hat{i}\right)$

$= 10\sqrt{3}\,\hat{j}$

To double the distance car 1 has to move 300 m more. So time taken

$$t = \frac{300}{10\sqrt{3}}$$

$t = 10\sqrt{3}$ sec.

4. Ans. (B)

Sol. $F_{avg} = \frac{\Delta P}{\Delta t} = \frac{P_f - P_i}{\Delta t}$

30 shots in 60 sec.

P of each bullet = $\left(\frac{10}{1000} \times 100\right) = 1 kg\, m/s$

$P_i = 0$

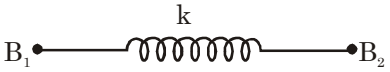
$P_f = 30 \times 1$

$F_{avg} = \frac{30-0}{60} = \frac{1}{2} N$

$F_{avg} = 0.5N$

5. Ans. (A,D)

Sol.



After displacement kx = 30

Total x = 10 cm + 20 cm = 30 cm = $\frac{30}{100} m$

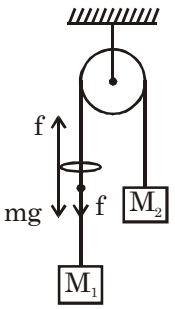
$k\frac{(30m)}{100} = 30$

k = 100 N/m

Simultaneously both are moving so work done can not be calculated.

6. Ans. (A,B,C)

Sol.



$a_R = \frac{f}{M_1 + M_2}$

$a_R = \frac{f}{4}$

$a_b = \frac{mg - f}{m}$

$a_b = g - \frac{f}{m}$

$a = \frac{mg}{5.5} = \frac{1.5(10)}{5.5} = \frac{30}{11}$

$\frac{30}{11} = \frac{15 - f}{1.5}$

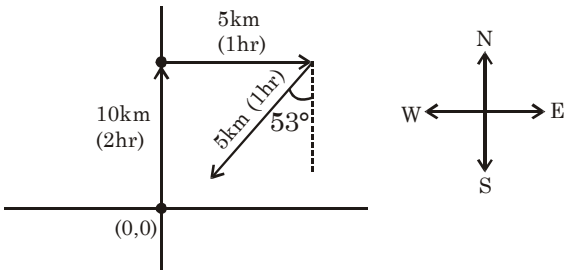
$15 - \frac{30}{11} \times 1.5 = f \approx 11$

To make system to move together limiting friction 11N is required

When limiting friction is 14 N, 11 N static friction will act and system will move together.

7. Ans. (A,B,C)

Sol.



Distance travelled is 20 km.

Average speed = $\frac{20km}{4hr} = 5km/hr$

Displacement

$= 10\hat{j} + 5\hat{i} + \left(5\sin 37°\left(-\hat{i}\right) + 5\cos 37° - \hat{j}\right)$

$5 \times \frac{3}{5}$ $\qquad$ $5 \times \frac{4}{5}$

$= 10\hat{j} + 5\hat{i} + \left(-3\hat{i} - 4\hat{j}\right)$

$= 2\hat{i} + 6\hat{j}$

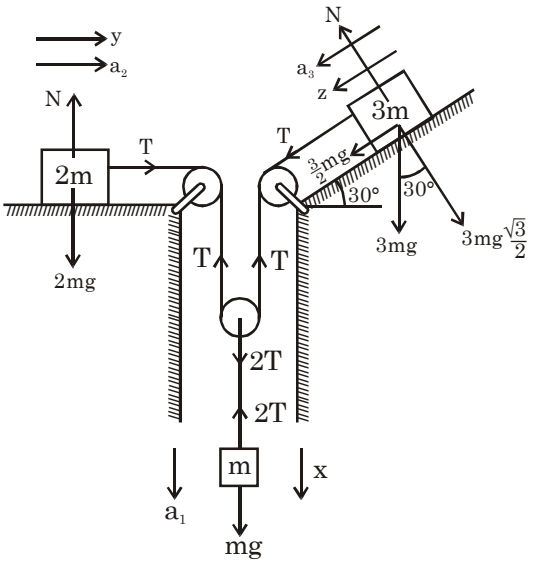
$|Displacement| = \sqrt{4 + 36} = \sqrt{40} = 6.32km$

Avg velocity = $\frac{6.32km}{4hr} = 1.58km/hr$

≈ 1.6 km/hr

8. Ans. (A,B)

Sol.



$T = 2ma_2$ ... (i)

$mg - 2T = ma_1$ ... (ii)

$$T + \frac{3}{2}mg = 3ma_3 \quad \text{... (iii)}$$

$$x = \frac{y+z}{2}$$

$$a_1 = \frac{a_2 + a_3}{2}$$

$2a_1 = a_2 + a_3$ ... (iv)

$$mg - 2T = m\frac{11g}{29}$$

$$2T = mg - \frac{11}{29}mg$$

$$2T = \frac{(29-11)}{29}mg$$

$$2T = \frac{18}{29}mg$$

$$T = \frac{9mg}{29}$$

$2T = 4ma_2$

$mg - 2T = ma_1$

$mg = 4ma_2 + ma_1$

$g = 4a_2 + a_1$ ... (v)

$$a_2 = \frac{g - a_1}{4}$$

$2T + 3mg = 6ma_3$

$mg - 2T = ma_1$

$4mg = ma_1 + 6ma_3$

$4g = a_1 + 6a_3$ ... (vi)

$$a_3 = \frac{4g - a_1}{6}$$

$$2a_1 = \frac{g - a_1}{4} + \frac{4g - a_1}{6}$$

$$2a_1 + \frac{a_1}{4} + \frac{a_1}{6} = \frac{g}{4} + \frac{2g}{3}$$

$$\Rightarrow \frac{(24+3+2)}{12}a_1 = \frac{3g + 8g}{12}$$

$$a_1 = \frac{11g}{29}$$

9. Ans. (B,C)

Sol. $R = 9.6$ m $\qquad H_{max} = 1.8$ m

$$\frac{\mu^2 \sin 2\theta}{g} = 9.6 \qquad \frac{\mu^2 \sin^2\theta}{2g} = 1.8$$

$$\frac{\mu^2 2\sin\theta\cos\theta}{g} = 9.6 \qquad \frac{\mu^2 \sin^2\theta}{2g} = 1.8$$

$$\frac{2\cos\theta \times 2}{\sin\theta} = \frac{9.6}{1.8}$$

$$\frac{1}{\tan\theta} = \frac{96}{18\times 4} = \frac{4}{3}$$

$$\tan\theta = \frac{3}{4} \qquad \theta = 37°$$

$$\frac{\mu^2}{2g}\left(\frac{3}{5}\right)^2 = \frac{18}{10}$$

$$\mu^2 = \frac{18}{10} \times \frac{20\times 25}{9}$$

$\mu^2 = 4 \times 25$

$\mu = 2 \times 5$

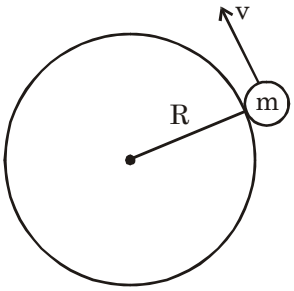
$\mu = 10$ m/s

10. Ans. (A,B,C)

11. Ans. (A,D)

12. Ans. (A,C,D)

Sol.



Time to complete full circle = $\frac{2\pi R}{v}$

$v_{avg} = \frac{2R}{(\pi R)} v = \frac{2v}{\pi}$

$v_{ins} = v_{avg.\ speed}$

SECTION-II

1. Ans. 8.80

Sol. $(20 - \ell_0)k - (16 - \ell_0)k$

$= m \times 400 \times 20$

$k \times (16 - \ell_0) = m \times 400 \times 36$

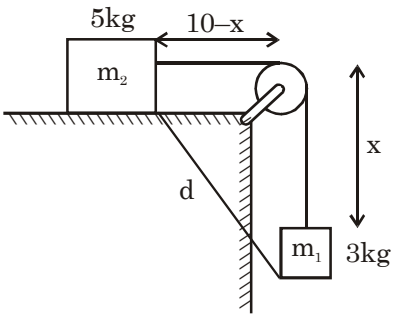
$4k = 8000\ m$

$2000\ m\ (16 - \ell_0) = 14400m$

$\ell_0 = 16.0 - 7.2 = 8.80$

2. Ans. 6.12 to 6.13

Sol.



Distance between them is (d)

$d = \sqrt{x^2 + (10 - x)^2}$

$d = \sqrt{x^2 + 100 - 20x + x^2}$

$d = \sqrt{2x^2 - 20x + 100}$

for d to be minimum

$\frac{d(d)}{dx} = \frac{1}{2} \frac{(4x - 20)}{\sqrt{2x^2 - 20x + 100}} = 0$

$4x = 20$

$x = 5$

$a = \frac{m_1 g}{m_1 + m_2} = \frac{3(10)}{8} = \frac{30}{8} = \frac{15}{4} m/s^2$

$v = 0 + \frac{15}{4} \times t$

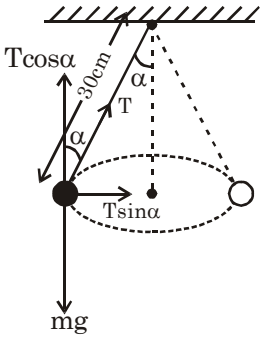
$v^2 = 2\left(\frac{15}{4}\right)5$

$v = \sqrt{75} = 6.12$

3. Ans. 2.00

4. Ans. 10.23 to 10.39

Sol.



$\alpha = 30°$

$T\cos 30 = mg$

$T\sin 30 = m\omega^2 r$

$\varepsilon = 30 \sin 30$

$= 30\left(\frac{1}{2}\right) = 15cm$

$\tan 30 = \frac{\omega^2 r}{g}$

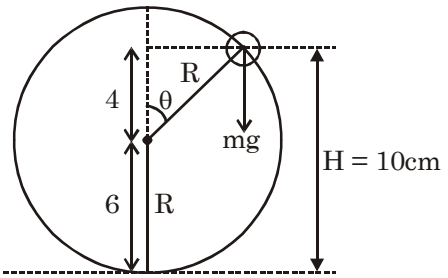
$g = \frac{4\pi^2}{1/\sqrt{3}}\left(\frac{15}{100}\right)$

$g = 4^2 \times 10 \times \sqrt{3}\frac{15}{100}$

$g = 6\sqrt{3} = 6(1.72) = 10.32$

5. Ans. 0.00

Sol.



$m = 10gm$

$R = 6cm$

$R\cos\theta = 4$

$\cos\theta = \frac{4}{6} = \frac{2}{3}$

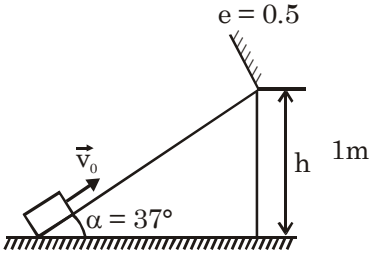
at $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

$mg\cos\theta = \frac{mv^2}{R}$

So N = 0

6. Ans. 30.00

Sol.



$e = \frac{v_{sep.}}{v_{app.}}$

$\mu = 0.3$

$\sin 37° = \frac{1}{\ell}$

$\ell = \frac{5}{3}$

Before collision velocity of particle is

$v^2 = v_0^2 - 2\left\{10\left(\frac{3}{5}\right) + 0.310\frac{4}{5}\right\}\frac{5}{3}$

$v^2 = 100 - \frac{10}{3}(6 + 2.4) = 100 - \frac{84}{3} = 100 - 28 = 72$

$v = \sqrt{72}$

Velocity after collision = ev

$= \frac{1}{2}\sqrt{72}$

Velocity of particle at bottom when it reaches again

$v^2 = \mu^2 + 2as$

$v^2 = \frac{1}{4}(72) - 2(6 - 2.4)\frac{5}{3}$

$v^2 = \frac{72}{4} + \frac{10}{3}(3.6)$

$= 18 + \frac{36}{3}$

= 18 + 12

$v^2 = 30$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans(C)

2. Ans.(C)

3. Ans.(B)

4. Ans.(C)

5. Ans.(A,B,C,D)

6. Ans.(A,B,C,D)

7. Ans(C,D)

8. Ans.(A,B,D)

$\frac{m_{L_1}}{m_{L_2}} = 2 \Rightarrow \frac{\frac{n_1R}{P_1}}{\frac{n_2R}{P_2}} = 2 = \left(\frac{n_1}{P_1}\right) = 2\left(\frac{n_2}{P_2}\right)$

Hence, if $n_1 = 2n_2 \Rightarrow P_1 = P_2$

if $n_1 < 2n_2 \Rightarrow P_1 < P_2$

if $n_1 > 2n_2 \Rightarrow P_1 > P_2$

if $n_1 = n_2$ & T is same at pt.1 & pt2.

Hence, Boyle's law is applicable.

9. Ans.(A,B,C,D)

10. Ans.(A,B)

11. Ans.(B)

12. Ans.(A,B,C,D)

SECTION-II

1. Ans.(3100.00)

(A) The balanced equation is -

$Ca_3(PO_4)_2 + 3SiO_2 + C + O_2 + 3H_2O \rightarrow 3CaSiO_3 + CO_2 + 2H_3PO_4$

Since, equal masses of $Ca_3(PO_4)_2$ and SiO_2 are used, we can easily say that $Ca_3(PO_4)_2$ is a limiting reagent.

Moles of phosphoric acid produced

$= \frac{1.96\times10^3\times1000}{98}$

$\therefore$ Moles of $Ca_3(PO_4)_2$ consumed

$= \frac{1}{2}\left[\frac{1.96\times10^6}{98}\right]$

Mass of $Ca_3(PO)_2$ consumed in kg

$= \left[\frac{1.96}{2}\times\frac{10^6}{98}\times310\frac{1}{1000}\right]$kg

$= 3100.00$

2. Ans.(0.10)

Sol $Al + 3HCl \rightarrow AlCl_3 + \frac{3}{2}H_2$

$\frac{1.5}{18}$mmol $\quad \frac{1.5}{6}$mmol

wt. of Al = $\frac{\frac{1.5}{18}\times 27}{1000}$gm $= 2.25\times10^{-3}$gm

Volume of Al removed

$= \frac{2.25\times10^{-3}}{2.25} = 10^{-3}cm^3$

$10^{-3} = \pi\times r^2\times\left(\frac{0.7}{22}\right)$

r = 0.1 cm

3. Ans.(5.00)

4. Ans.(–38.00)

5. Ans.(6.00)

6. Ans.(6.00)

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (D)

Sol. $\sec^2\theta\,\text{cosec}^2\theta + 2\text{cosec}^2\theta = 8$

$(1+\tan^2\theta)\left(1+\frac{1}{\tan^2\theta}\right)+\frac{2}{\tan^2\theta} = 8-2$

$\tan^4\theta - 4\tan^2\theta + 3 = 0 \Rightarrow \tan^2\theta = 1, 3$

$\theta \in \left\{\frac{\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{3}, \frac{-\pi}{3}\right\} \Rightarrow \lambda = 4$

$-\log_5(2x+5) + \log_5(16-x^2) = 1$

$\Rightarrow \frac{16-x^2}{2x+5} = 5$

$\Rightarrow x^2 + 10x + 9 = 0 \Rightarrow x = -1, -9$

only x = –1 satisfy, t = 1

2. Ans. (C)

Sol. $ar^{n-1} = 192 \Rightarrow r^{n-1} = 64$

$381 = \frac{a(r^n-1)}{r-1} \Rightarrow 127 = \frac{64r-1}{r-1}$

$\Rightarrow$ r = 2 and n = 7

cosx.siny = 1 possible when

cosx = siny = 1 or cosx = siny = –1

$x \in \{0, 2\pi\}$, or $x \in \{\pi, 3\pi\}$,

$y \in \left\{\frac{\pi}{2}, \frac{5\pi}{2}\right\}$ $\qquad$ $y \in \left\{\frac{3\pi}{2}\right\}$

Total of ordered pair = 6

3. Ans. (B)

Sol. $(a-b^2)(a+b^2) = 19 = 19\times1$

only possible when $a + b^2 = 19$ and $a - b^2 = 1$

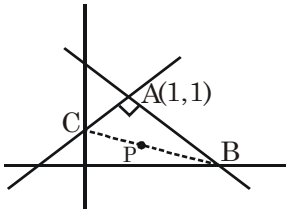
(a, b) = (10, 3), $m = \left(\frac{a}{2}-b\right) \Rightarrow m = 2$

$\alpha+\beta = \alpha^2+\beta^2 \Rightarrow 2k = 4k^2 - 2(2k-1)$

$\Rightarrow k = \frac{1}{2}, 1$, n = 2

4. Ans. (A)

Sol.



Let P ≡ (h, k), P is the circumcentre of ΔABC

B ≡ (2h, 0), C ≡ (0, 2k)

Now $M_{AC}.M_{AB} = -1$

$\left(\frac{1-2k}{1}\right)\left(\frac{1}{1-2h}\right) = -1$

$1 - 2k = 2h - 1 \Rightarrow h + k = 1 \Rightarrow x + y = 1$

5. Ans. (A,C,D)

Sol. $2x^2 + 5xy + 3y^2 + 6x + 7y + 4 = 0$

$(2x + 3y + \lambda_1)(x + y + \lambda_2) = 0$

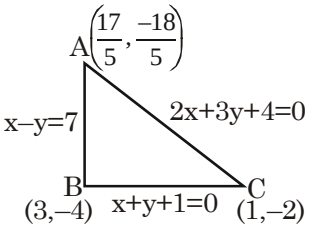
$2\lambda_2 + \lambda_1 = 6$ and $3\lambda_2 + \lambda_1 = 7$

$\lambda_2 = 1, \lambda_1 = 4$

Pair of lines are $2x + 3y + 4 = 0$ and $x + y + 1 = 0$

Point of intersection of the pair of lines is $(1, -2)$

$$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right| = \frac{1}{5}$$



orthocentre : B(3, –4)

circumcentre of $\Delta ABC = \left(\frac{11}{5}, \frac{-14}{5}\right)$

6. Ans. (A,B,C)

Sol. $\frac{\tan x \tan y + 1}{\tan y} = p$ ...(1)

and $\frac{\tan x \tan y + 1}{\tan x} = q$ ...(2)

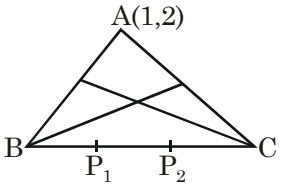
from (1) and (2) we get $\frac{\tan x}{\tan y} = \frac{p}{q}$

from (1) $\frac{\cos(x-y)}{\sin y \cos x} = p$

from (2) $\frac{\cos(x-y)}{\sin x \cos y} = q$

7. Ans. (A,C)

Sol. Image of A in lines L_1 and L_2 well lie on BC



$P_1 \equiv (2, 1)$, $P_2 \equiv \left(\frac{-11}{5}, \frac{2}{5}\right)$

Equation of BC is $x - 7y + 5 = 0$

Incentre = (0, 0)

r = perpendicular distance from (0,0) to

BC = $\frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$

8. Ans. (A,D)

Sol. $\cos x - \sin\alpha\cot\beta\sin x = \cos\alpha$

$$\left(1-\tan^2\frac{x}{2}\right) - 2\tan\frac{x}{2}\sin\alpha\cot\beta = \cos\alpha\left(1+\tan^2\frac{x}{2}\right)$$

$$\Rightarrow \quad (\cos\alpha+1)\tan^2\frac{x}{2} - 2\tan\frac{x}{2}\sin\alpha\cot\beta - (1-\cos\alpha)$$

$$\tan^2\frac{x}{2} - 2\tan\frac{x}{2}.\cot\beta\left(\frac{\sin\alpha}{1+\cos\alpha}\right) - \tan^2\frac{\alpha}{2} = 0$$

$$\tan^2\frac{x}{2} - \frac{2\tan\frac{x}{2}}{2\tan\frac{\beta}{2}}\left(1-\tan^2\frac{\beta}{2}\right)\times\tan\frac{\alpha}{2} - \tan^2\frac{\alpha}{2} = 0$$

$$\tan^2\frac{x}{2} - \tan\frac{x}{2}\left[\tan\frac{\alpha}{2}\cot\frac{\beta}{2} - \tan\frac{\beta}{2}\tan\frac{\alpha}{2}\right]$$

$$+\left[-\tan\frac{\alpha}{2}\cot\frac{\beta}{2}\times\tan\frac{\beta}{2}\tan\frac{\alpha}{2}\right] = 0$$

$$\tan\frac{x}{2} = -\tan\frac{\alpha}{2}\cot\frac{\beta}{2}, \quad \tan\frac{\alpha}{2}\tan\frac{\beta}{2}$$

9. Ans. (A,C)

Sol. $P(x) = (x^4 - x^3 - x^2 - 1)(x^2 + 1) + (x^2 - x + 1)$

$P(a) + P(b) + P(c) + P(d) = \Sigma a^2 - \Sigma a + 4$

$\Sigma a = 1$, $\Sigma ab = -1$, $\Sigma a^2 = 3$

$x^4 - x^3 - x^2 - 1 = (x - a)(x - b)(x - c)(x - d)$

$(1 + a)(1 + b)(1 + c)(1 + d) = 0$

10. Ans. (A,C)

Sol. $\tan(2\pi|\sin\theta|) = \tan\left(\frac{\pi}{2} - 2\pi|\cos\theta|\right)$

$$2\pi|\sin\theta| = n\pi + \frac{\pi}{2} - 2\pi|\cos\theta|$$

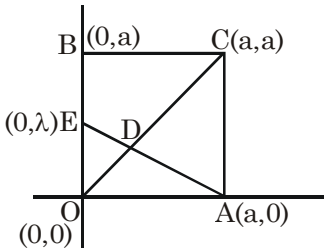
$$|\sin\theta| + |\cos\theta| = \frac{2n+1}{4} \in \left[1, \sqrt{2}\right]$$

$$\Rightarrow |\sin\theta| + |\cos\theta| = \frac{5}{4}$$

$$y = \left(\frac{5}{4}\right)^x, \quad x \geq 1$$

11. Ans. (A,B)

Sol.



ΔODE is similar to ΔADC

$$\frac{3}{4}=\frac{\lambda}{a} \Rightarrow \lambda=\frac{3a}{4}$$

$$E\equiv\left(0,\frac{3a}{4}\right),\ AE=\sqrt{a^2+\frac{9a^2}{16}}=\frac{5a}{4}$$

$$\text{Now } AD=\frac{4}{7}AE \Rightarrow 5=\frac{4}{7}\times\frac{5a}{4}$$

$\Rightarrow a = 7$

area of square OACB = 49

Equation of AE is $3x + 4y = 21$

$$O'\equiv\left(\frac{126}{25},\frac{168}{25}\right)$$

$\Delta AO'E$ will be right angle at O' so circumcentre of $\Delta AO'E$ is $\left(\frac{7}{2},\frac{21}{8}\right)$

12. Ans. (A,D)

Sol. $$\cot\frac{A}{2}=\frac{s(s-a)}{\Delta}=\frac{s-a}{r}=x,$$

$$\cot\frac{B}{2}=\frac{s-b}{r}=y,$$

$$\cot\frac{C}{2}=\frac{s-c}{r}=z$$

$$\text{Now } x+y+z=\frac{s}{r}$$

$$\left(\cot\frac{A}{2}\right)^2+\left(2\cot\frac{B}{2}\right)^2+\left(3\cot\frac{C}{2}\right)^2=\left(\frac{6s}{7r}\right)^2$$

$$x^2+4y^2+9z^2=\frac{36}{49}(x+y+z)^2$$

$$13x^2+160y^2+405z^2-72xy-72yz-72zx=0$$

$$(2x-18z)^2+(3x-12y)^2+(4y-9z)^2=0$$

$$x=4y=9z$$

$$\Rightarrow \frac{s-a}{36}=\frac{s-b}{9}=\frac{s-c}{4}=\frac{s}{49}$$

$$a:b:c=13:40:45$$

$$\sum\frac{\cot\frac{A}{2}}{\cot\frac{B}{2}}=\frac{x}{y}+\frac{y}{z}+\frac{z}{x}$$

$$=4+\frac{9}{4}+\frac{1}{9}=\frac{229}{36}$$

SECTION-II

1. Ans. 0.40

Sol. $$\cos^2x-\cos x.\cos^2 5x+\frac{\cos^4 5x}{4}+\frac{\cos^2 5x}{4}-\frac{\cos^4 5x}{4}=0$$

$$\left(\cos x-\frac{\cos^2 5x}{2}\right)^2+\frac{\cos^2 5x.\sin^2 5x}{4}=0$$

$$\text{Now } \cos x=\frac{\cos^2 5x}{2} \text{ and } \sin 10x=0$$

$$\Rightarrow x=\frac{n\pi}{10} \text{ and } \cos x=\frac{1}{2}\cos^2\frac{n\pi}{2}$$

Case-I : If n = odd

$$\cos x=0 \Rightarrow x=\frac{\pi}{2},\frac{3\pi}{2}$$

Case-II : If n = even

$$\cos x=\frac{1}{2} \Rightarrow x=\frac{\pi}{3},\frac{5\pi}{3} \text{ (Not possible)}$$

2. Ans. 2.80

Sol. $f(x) = 10\cos^2x - 6\sin x\cos x + 2\sin^2x$

$f(x) = 4\cos 2x - 3\sin 2x + 6$

$f(x) \in [1, 11]$, $\lambda = 16$

$t = 8\sec 39°\sin 17°\sin 43°\cos 13°$

$$t=\frac{8\sin 17°\sin 43°\sin 77°}{\cos 39°}=\frac{2\sin 51°}{\cos 39°}=2$$

3. Ans. 0.50

Sol. $a_n = 30a_{n-1} - 70a_{n-2}$

$$\frac{a_n}{50^n}=\frac{30}{50}.\frac{a_{n-1}}{50^{n-1}}-\frac{70}{50^2}.\frac{a_{n-2}}{50^{n-2}}$$

$$\sum_{n=2}^{\infty}\frac{a_n}{50^n}=\frac{3}{5}\sum_{n=2}^{\infty}\frac{a_{n-1}}{50^{n-1}}-\frac{7}{250}\sum_{n=2}^{\infty}\frac{a_{n-2}}{50^{n-2}}$$

$$\left(S-1-\frac{1}{50}\right)=\frac{3}{5}(S-1)-\frac{7}{250}S$$

$$S\left(1-\frac{3}{5}+\frac{7}{250}\right)=\left(1-\frac{3}{5}+\frac{1}{50}\right)$$

$$\Rightarrow S\left(\frac{107}{250}\right)=\frac{21}{50}\Rightarrow S=\frac{105}{107}$$

4. Ans. 1.25

Sol. $$\frac{\frac{n(n+1)}{2}-(3n-3)}{n-3}\le\frac{43}{4}\le\frac{\frac{n(n+1)}{2}-6}{n-3}$$

$$\frac{35}{2}\le n\le\frac{47}{2}\Rightarrow n=18,\ 19,\ 20,\ 21,\ 22,\ 23$$

let three numbers be {a – d, a, a + d}

$$\frac{\frac{n(n+1)}{2}-3a}{(n-3)}=\frac{43}{4}$$

$$\Rightarrow n-3=\frac{4\left[\frac{n(n+1)}{2}-3a\right]}{43}$$

n – 3 should be 4λ
⇒ only possible n = 19 and 23
C-1 n = 19

$$\frac{19\times20}{2}-3a=16\times\frac{43}{4}\Rightarrow a=6$$

numbers = {6 – d, 6, 6 + d}
Five set possible for d = 1, 2, 3, 4, 5
C-2 n = 23

$$\frac{23\times4}{2}-3a=20\times\frac{43}{4}\Rightarrow a=\frac{61}{3}\ \text{(rejected)}$$

5. Ans. 0.60

Sol. (a + 2)(b + 2)(c + 2) = 210
= 5 × 6 × 7 = 3 × 5 × 14 = 3 × 7 × 10
(a, b, c) = (3, 4, 5) or (1, 3, 12) or (1, 5, 8)
a + b + c = 12 or 16 or 14

6. Ans. 9.00

Sol. $$\sum_{i=1}^{17}(i+3)^2x_i=\sum_{i=1}^{17}i^2x_i+3\left(\sum_{i=1}^{17}(2i+3)x_i\right)$$

= 1 + 3(111) = 334

Now $$\left(\sum_{i=1}^{17}(i+2)^2x_i-\sum_{i=1}^{17}(i+1)^2x_i=\sum_{i=1}^{17}(2i+3)x_i=111\right)$$

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JEE(Main+Advanced) : NURTURE COURSE [PHASE : I, I(A), II & III]

PAPER-2

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (A)

Sol. Man should travel due North.

2. Ans. (D)

Sol. $\text{Area} = v_f - v_i \Rightarrow \frac{1}{2}(m)(60) = v_f - 1$

$\Rightarrow v_f = 421$ m/s

3. Ans. (C)

Sol. $a_i = -\frac{v^2}{R}$; $a_f = -\frac{3v^2}{R}$

$$\Delta a = a_f - a_i = \frac{3v^2}{R} + \frac{v^2}{R} = \frac{4v^2}{R}$$

4. Ans. (A)

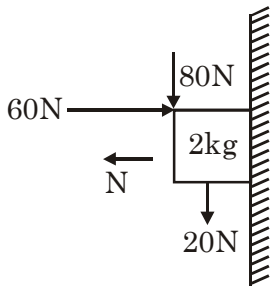
Sol. $\vec{F} = -(2xy+3y)\hat{i} - (x^2+3x)\hat{j}$

5. Ans. (D)

Sol. Self explanatory

6. Ans. (B)

Sol.



$N = 60N$

$(f_s)_{max} = \mu_s N = (0.1)(160) = 6N$

$$a = \frac{20+80-6}{2} = \frac{94}{2} = 47 \text{ m/s}^2$$

7. Ans. (C)

Sol. $mg - N = \frac{mv^2}{r}$

but $N = 0$

So, $mg = \frac{mv^2}{r} \Rightarrow v = \sqrt{gr}$

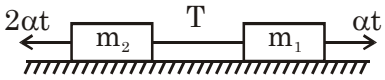
8. Ans. (B)

Sol. $v_1^{\,1} = (1+e)v_2 - ev_1$

$$= \left(\frac{3}{2}\right)(-1) - \frac{1}{2}(3) = -\frac{3}{2} - \frac{3}{2} = -3 \text{ m/s}$$

9. Ans. (A)

Sol.



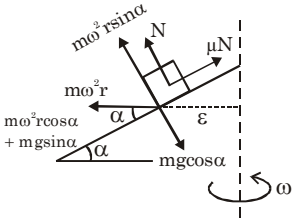
$$a = \frac{\alpha t}{2m}$$

$$2\alpha t - T = m\left(\frac{\alpha t}{2m}\right)$$

$$\frac{2\alpha t}{2} = T \Rightarrow t = \frac{2T}{3x}$$

10. Ans. (A)

Sol.



$N = mg\cos\alpha - m\omega^2 r\sin\alpha$

$m\omega^2 r + mg\sin\alpha = \mu N$

$\Rightarrow m\omega^2 r + mg\sin\alpha = \mu(mg\cos\alpha - m\omega^2 r\sin\alpha)$

putting values and solving

$\omega = 4.5$ rad/s

11. Ans. (D)

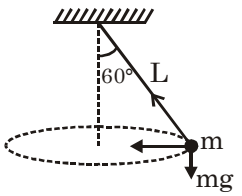
Sol. $Mx = m(\ell - \ell_1 - x)$

$\Rightarrow (M+m)x = m(\ell - \ell_1)$

$$x = \frac{m(\ell - \ell_1)}{M+m}$$

12. Ans. (C)

Sol.



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$T\sin 60° = \frac{mv^2}{L}$

$T\cos 60° = mg$

$\Rightarrow \frac{T}{2}mg \Rightarrow T = 2mg$

Putting value

$\frac{(2mg)\sin 60° \times L}{2} = \frac{mv^2}{2}$

$\Rightarrow KE = \left(\frac{40}{100}\right)\left(\frac{\sqrt{3}}{2}\right) = 0.3J$

13. Ans. (C)

Sol. $F = ax$

$\Rightarrow mv\frac{dv}{dx} = ax$

$\Rightarrow \int_0^v mvdv = \int_0^x axdx$

$\Rightarrow \frac{mv^2}{2} = \frac{\alpha x^2}{2}$

$\Rightarrow KE = \frac{ax^2}{2}$

14. Ans. (A)

Sol. $F = -kv$

$mv\frac{dv}{dx} = -Rv$

$\Rightarrow \int_{10}^{0} mdv = -k\int_0^x dx$

$\Rightarrow x = 1m$

15. Ans. (C)

Sol. $k_i + v_i = k_f + v_f$

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_f^2 + 0$

$\Rightarrow v_f = \sqrt{v^2 + 2gh}$

16. Ans. (B)

Sol. $u > \sqrt{5Rg}$

so 2R

17. Ans. (D)

Sol. Let speed of ball after collision be u.

$2\sqrt{\frac{2h}{g}} \cdot u = \ell \qquad \Rightarrow U = \frac{\ell}{2}\sqrt{\frac{g}{2h}}$

Now, U = 2v

$\Rightarrow v = \frac{U}{2} = \frac{\ell}{4}\sqrt{\frac{g}{2h}}$

18. Ans. (B)

Sol. $\vec{F}_{ext} \neq 0$

$\Rightarrow \vec{a}_{com} \neq 0$

$\Rightarrow \vec{v}_{com}$ will change

19. Ans. (D)

Sol. $I = \left(\frac{58}{1000}\right)(64) = \text{Area}$

$\frac{(58)(64)}{1000} = \frac{1}{2}\frac{(8)}{1000}(F_{max})$

$\Rightarrow F_{max} = 928N$

20. Ans. (B)

Sol. $N = 400 - \frac{3F}{5}$

$\frac{4F}{5} = (0.4)\left(400 - \frac{3F}{5}\right)$

$\Rightarrow \frac{4F}{5} = \left(\frac{4}{16}\right)\left(400 - \frac{3F}{5}\right)$

$\Rightarrow 2F + \frac{3F}{5} = 400$

$\Rightarrow \frac{13F}{5} = 400 \qquad \Rightarrow F = \frac{2000}{13} \approx 153N$

SECTION-II

1. Ans. 0.75

Sol. <v> = 3/4 = 0.75 m/s

2. Ans. 63.00

Sol. $x = [(31.5)\cos 37°]t = [(31.5)\cos 68°]t$

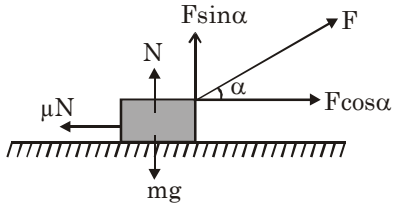
and $y = [(31.5)\sin 37°]t - \frac{1}{2}gt^2$

$= [(31.5)\sin 68°]t - \frac{1}{2}gt^2$

On solving, x = 63m

3. Ans. 64.40

Sol.



$N = mg - F\sin\alpha$

$F\cos\alpha = \mu N$

$F\cos\alpha = \mu(mg - F\sin\alpha)$

On solving, $F = 64.4N$

4. Ans. 1.50

Sol. $W_f = \Delta KE = \frac{1}{2}\mu v_{red}^2 = \frac{1}{2}\left(\frac{0.5\times1}{1.5}\right)(9) = \frac{3}{2}J$

5. Ans. 0.67

Sol. $$x_{com} = \frac{(1)(1)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)+.....}{1+\frac{1}{2}+\frac{1}{4}}$$

$$= \frac{1+\frac{1}{4}+\frac{1}{16}+.....}{1+\frac{1}{2}+\frac{1}{4}}$$

$$\Rightarrow x_{com} = \frac{\frac{1}{1-\frac{1}{4}}}{\frac{1}{1-\frac{1}{2}}} = \frac{2}{3}$$

PART-2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (A)

2. Ans. (B)

3. Ans. (B)

4. Ans. (C)

5. Ans. (B)

6. Ans. (D)

Balanced chemical reaction is

$2MnO_4^- + 3CN^- + H_2O \rightarrow 2MnO_2 + 3CNO^- + 2OH^-$

7. Ans. (C)

8. Ans.(A)

Sol HCl + H_2SO_4

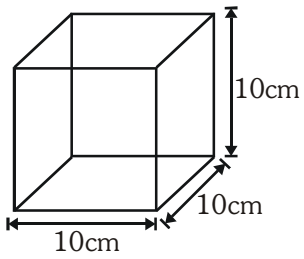
x m. mole y m. mole

4m eq

$x + 2y = 4$ (i)

$$x = \frac{0.287}{143.5}\times1000$$

9. Ans. (C)



Volume of Cube = $(\text{length})^3$

$= (10\text{ cm})^3 = 1000$ cc

Mass of cube = Volume × density

= 1000 cc × 8 g/cc = 8000 g

$$\text{No. of moles} = \frac{\text{mass}}{\text{atomic mass}} = \frac{8000}{M}$$

$$\text{No. of atoms} = \frac{8000}{M}\times6\times10^{23}$$

$$\Rightarrow \quad 6\times10^{25} = \frac{8000\times6\times10^{23}}{M}$$

$$\Rightarrow \quad \boxed{M = 80}$$

10. Ans. (D)

11. Ans. (D)

For given % of elements

Relative moles of given elements

Ce	N	O
$\frac{50.4}{140}$	$\frac{15.1}{14}$	$\frac{34.5}{16}$
0.36	1.07	2.16
1	3	6

so the emperical formula is $Ce(NO_2)_3$

12. Ans. (B)

13. Ans. (B)

Let Molarity = M

So 1 L of solution = M mole of solute

= M × 40 g of NaOH

$\because$ M % (w/w) means

In 100 g solution = M g of NaOH

So 40 M g solute will be in = $\frac{100}{M}\times 40M$

= 4000 g of solution

So $d = \frac{w_{solution}}{V_{solution}}$

$= \frac{4000g}{1000ml}$

= 4 g/ml

14. Ans. (A)

$NaOH + NaH_2PO_3 \longrightarrow Na_2HPO_3 + H_2O$

2 20×0.1

$NaOH + NaHCO_3 \longrightarrow Na_2CO_3 + H_2O$

6 60×0.1

15. Ans. (D)
16. Ans. (B)
17. Ans. (D)
18. Ans. (A)
19. Ans. (D)
20. Ans. (B)

SECTION-II

1. Ans. (1.00)
2. Ans. (2.00)
Sol. P = 1, Q =1
3. Ans. (8.00)

Sol. $C_4H_x + \left(4+\frac{x}{4}\right) O_2 (g) \rightarrow 4 CO_2 (g) + \frac{x}{2} H_2O(g)$

$10\ cm^3$ y

0 $y - 10\left(4+\frac{x}{4}\right)$ 40 5x

expansion–

$V_f - V_i = 10$

$y - 10\left(4+\frac{x}{4}\right) + 40 + 5x - (10 + y) = 10$

$-2.5x + 5x = 20$

$2.5x = 20$

$25x = 200$

$x = 8$

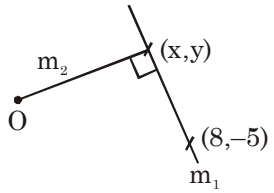
4. Ans. (14.00)
5. Ans. (4.00)

PART-3 : MATHEMATICS — SOLUTION

SECTION–I

1. Ans. (C)

Sol. $\left.\begin{matrix} x+y-3=0 \\ 2x+3y-1=0 \end{matrix}\right\}(8,-5)$



$m_1m_2 = -1$

$\Rightarrow \left(\frac{y+5}{x-8}\right)\left(\frac{y}{x}\right) = -1$

$\Rightarrow x^2 + y^2 - 8x + 5y = 0$

2. Ans. (D)

Sol. $3200 = 100.r^{10} \Rightarrow r^{10} = 32 \Rightarrow r = 2^{1/2}$

General term = $100.(2^{1/2})^{n-1}$

For rational term, n = 1,3,5,7,9,11

$\therefore$ 6 terms.

3. Ans. (A)

Sol. $\frac{b^2\left(1-2\sin^2 A\right) - a^2\left(1-2\sin^2 B\right)}{b^2 - a^2}$

$= \frac{b^2 - a^2 - 2\left(b^2\sin^2 A - a^2\sin^2 B\right)}{b^2 - a^2}$

$= 1 - \frac{2\left(\left(b\sin A\right)^2 - \left(a\sin B\right)^2\right)}{b^2 - a^2} = 1$

4. Ans. (D)

Sol. $x^2 + 4 = 4x\cos x \Rightarrow \cos x = \underbrace{\frac{x}{4} + \frac{1}{x}}_{\geq 1}$

$\therefore$ no solution.

5. Ans. (D)

Sol. $S = 1 + 2^2 + 3^2 + ...+20^2$

$\Rightarrow S = \frac{20.21.41}{6} \Rightarrow S = 2870 = 41\times 7\times 10$

6. Ans. (C)

Sol. $\cos^2\alpha - 2\cos\alpha = -(\cos^2\beta - 2\cos\beta)$

$(\cos\alpha - 1)^2 - 1 = -(\cos\beta - 1)^2 + 1$

$\Rightarrow (\cos\alpha - 1)^2 + (\cos\beta - 1)^2 = 2$

$\because \alpha, \beta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow \cos\alpha = 1; \cos\beta = 1$ (only possibility)

$\Rightarrow \alpha = \frac{\pi}{2}, -\frac{\pi}{2}\ ; \beta = \frac{\pi}{2}, -\frac{\pi}{2}$

7. Ans. (C)

Sol. Product of roots = 8 – a

$f(8-a) < 0 \Rightarrow (8-a)^2 - 2a(8-a) + 8 - a < 0$

$\Rightarrow (8-a)(8-a-2a+1) < 0$

$\Rightarrow (8-a)(9-3a) < 0$

$\Rightarrow (a-8)(a-3) < 0 \Rightarrow a \in (3,8)$

8. Ans. (C)

Sol. $\because$ Coefficients are rational

$\therefore 2 - \sqrt{3}$ is also a root of given equation.

$\therefore x^2 - \left((2+\sqrt{3}) + (2-\sqrt{3})\right)x + (2+\sqrt{3})(2-\sqrt{3})$

is a factor

$\Rightarrow x^2 - 4x + 1$ is a factor

$\therefore$ other factor : $x^2 + 6x + 7$

$\therefore$ other roots : $-3 \pm \sqrt{2}$

9. Ans. (C)

Sol. $s^2\left(\frac{s-a}{\Delta}\right)\left(\frac{s-b}{\Delta}\right) = \frac{s^2(s-a)(s-b)}{s(s-a)(s-b)(s-c)}$

$$= \frac{s}{s-c} = \frac{\frac{a+b+c}{2}}{\frac{a+b+c}{2} - c}$$

$$= \frac{a+b+c}{a+b-c} = \frac{5c}{3c} = \frac{5}{3}$$

10. Ans. (B)

Sol. $a + ar + ar^2 = 70$

$4a, 5ar, 4ar^2 \longrightarrow$ AP

$10ar = 4a + 4ar^2$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow r = 2; r = \frac{1}{2}$

$\therefore$ possible a = 10 at r = 2

and a = 40 at $r = \frac{1}{2}$

$\therefore$ Middle term = 20

11. Ans. (B)

Sol. Sequence of common terms

5,11,17....

n^{th} term of common sequence, $t_n = 5 + 6(n-1)$

$\Rightarrow t_n = 6n - 1$

100^{th} term of first sequence = 2 + (100 – 1)3

= 299

100^{th} term of second sequence

=3+(100 – 1)2

= 201

$\therefore t_n \le 201 \Rightarrow 6n - 1 \le 201 \Rightarrow n \le 33\frac{2}{3}$

$\Rightarrow n = 33$

12. Ans. (D)

Sol. Line parallel to x – 3y + 5 = 0, passing through (2,3)

$$y - 3 = \frac{1}{3}(x-2) \Rightarrow x - 3y + 7 = 0$$

point of intersection with x – 2y + 5 = 0 is (–1,2)

Required distance $= \sqrt{3^2 + 1^2} = \sqrt{10}$

13. Ans. (C)

Sol. $\left(\frac{\sqrt{3}a}{2}\right)^2 + b^2 + \left(\frac{a}{2}\right)^2 + c^2 = ab\sqrt{3} + ac$

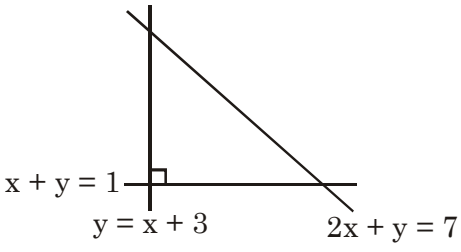
$$\Rightarrow \left(\frac{\sqrt{3}a}{2} - b\right)^2 + \left(\frac{a}{2} - c\right)^2 = 0$$

$$\Rightarrow b = \frac{\sqrt{3}a}{2}; c = \frac{a}{2}$$

$\Rightarrow b^2 + c^2 = a^2$

14. Ans. (D)

Sol.



Orthocenter will be at right angle vertex (–1, 2)

15. Ans. (D)

Sol. $4\left(\frac{y}{x}\right)^3-\left(\frac{y}{x}\right)^2-9\left(\frac{y}{x}\right)+a=0$

$\Rightarrow 4m^3-m^2-9m+a=0 \begin{cases} m_1 \\ m_2 \\ m_3 \end{cases}$

$m_1m_2=-1$

$m_1m_2m_3=-\frac{a}{4}\Rightarrow m_3=\frac{a}{4}$

$\because$ m_3 is a root

$\therefore\ 4.\frac{a^3}{64}-\frac{a^2}{16}-\frac{9a}{4}+a=0$

$\Rightarrow \frac{a^3}{16}-\frac{a^2}{16}=\frac{5a}{4}\Rightarrow a^2-a=20$

16. Ans. (C)

Sol. We can find possible fourth vertex using the fact that diagonals of parallelogram bisect each other

$\therefore$ (–1, 0); (–11, –8); (7, 10) are possible

17. Ans. (C)

Sol. $4\text{cosec}^2x-10=\cot^2x$

$4\text{cosec}^2x-10=\text{cosec}^2x-1$

$\Rightarrow \text{cosec}^2x=3$

$\Rightarrow \sin^2x=\frac{1}{3}\Rightarrow \sin x=\frac{1}{\sqrt{3}}\text{ or }-\frac{1}{\sqrt{3}}$

2 solution in $[0,2\pi]$ for which cotx > 0.

18. Ans. (C)

Sol. If A + B = 45°, then (1 + tanA) (1 + tanB) = 2

AB = (1 + tan1°).(1 + tan11°).(1 + tan20°).

(1 + tan25°).(1 + tan34°)(1 + tan44°)

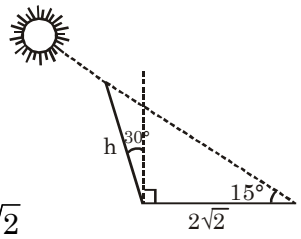
= 2.2.2 = 8

19. Ans. (A)

Sol. $\left.\begin{matrix}\sin 2x=1\\ \sin 4x=1\end{matrix}\right\}$ Not possible simultaneously in $[0,2\pi]$

20. Ans. (C)

Sol. $\frac{\sin 15^\circ}{h}=\frac{\sin 45^\circ}{2\sqrt{2}}$



$h=\frac{\sqrt{3}-1}{2\sqrt{2}}\cdot 2\sqrt{2}\cdot\sqrt{2}$

$h=\sqrt{6}-\sqrt{2}$

SECTION–II

1. Ans. 0.75

Sol. $bm^2+2hm+a=0$, where $m=y/x$

roots of this equation are m_1 & $3m_1$

$3m_1+m_1=\frac{-2h}{b}\Rightarrow m_1=\frac{-h}{2b}$

$3m_1\,.\,m_1=\frac{a}{b}\Rightarrow 3\left(\frac{h}{2b}\right)^2=\frac{a}{b}$

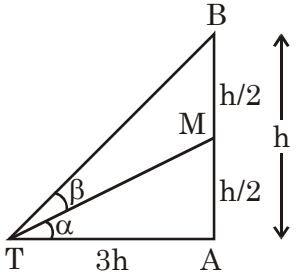
$\Rightarrow 3h^2=4ab$

2. Ans. 6.33

Sol. $\tan\alpha=\frac{h/2}{3h}\Rightarrow\tan\alpha=\frac{1}{6}$

$\tan(\alpha+\beta)=\frac{h}{3h}\Rightarrow\tan(\alpha+\beta)=\frac{1}{3}$

$\Rightarrow \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{1}{3}$



$\Rightarrow \frac{\frac{1}{6}+\tan\beta}{1-\frac{1}{6}.\tan\beta}=\frac{1}{3}$

$\Rightarrow \tan\beta=\frac{3}{19}$

$\Rightarrow \cot\beta=\frac{19}{3}$

3. Ans. 9.00

Sol. $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$

$\Rightarrow 2\sin 2x\cos x + \sin 2x = 2\cos 2x\cos x + \cos 2x$

$\Rightarrow (\sin 2x)(2\cos x + 1) = (\cos 2x)(2\cos x + 1)$

$$\Rightarrow \underbrace{\cos x = -\frac{1}{2}}_{3\text{solutions}} \text{ or } \underbrace{\tan 2x = 1}_{6\text{solutions}}$$

$\therefore$ Total 9 solutions.

4. Ans. 6.00

Sol. $x^2 + ax + b = 0 \begin{cases} c \\ d \end{cases}$

$x^2 + cx + d = 0 \begin{cases} a \\ b \end{cases}$

$\therefore c^2 + ac + b = 0$ ...(i)

$a^2 + ac + d = 0$ ...(ii)

$\Rightarrow c^2 - a^2 + b - d = 0$ ...(iii)

Also $c + d = -a$ ...(iv)

and $a + b = -c$ ...(v)

subtract : $c - a + d - b = c - a$

$\Rightarrow d = b$ ...(vi)

Also $cd = b$ ...(vii)

$ab = d$ ...(viii)

by (iv), (v), (vi) : $c = a = 1$

by (iv), (v) : $b = d = -2$

$\therefore |a| + |b| + |c| + |d| = 6$

5. Ans. 0.60

Sol. $x^2 + 3x + 4 = 0$ has both roots imaginary

$\therefore$ both roots of both equation are equal

$$\therefore \frac{a}{1} = \frac{b}{3} = \frac{c}{4} = \lambda \Rightarrow \frac{a+c}{b} = \frac{\lambda + 4\lambda}{3\lambda} = \frac{5}{3}$$

$$\Rightarrow \frac{b}{a+c} = \frac{3}{5}$$