

## Part IV

### 8 Multiple sequence alignment

#### 8.1 Multiple sequence alignment

A multiple sequence alignment is an effective tool to understand the characteristics of genes by comparing multiple sequences of different species at the same time.

#### Multiple Sequence Alignment (MSA) for protein sequences



Figure 8.1: An MSA of insulin proteins of seven sequences

#### Notation of MSA

- $\mathcal{A}$  : Alignment
- $m$  : Number of sequences in  $\mathcal{A}$
- $s_j^i$  : An amino acid or a nucleotide of sequence  $i$  and position  $j$  (without gaps)
- $\bar{s}_j^i$  : An amino acid or a nucleotide of sequence  $i$  and column  $j$  (with gaps)

#### Example of MSA notation

HUMAN: TP-K  
MOUSE: TLSK  
RAT : TPSK

- $m$  : 3
- $s_1^1$  : T (1st position of HUMAN)
- $s_2^2$  : L (2nd position of MOUSE)
- $s_4^3$  : K (4th position of RAT)
- $\bar{s}_3^1$  : - (3rd position of HUMAN)

#### Making an optimal MSA

- Insert gaps to the sequences in  $\mathcal{A}$
- Maximize the score of  $\mathcal{A}$

### All combinations of elements per column

The number of all possible combinations of elements per column can be calculated as follows.

$$\sum_{i=0}^{m-1} \binom{m}{i} = 2^m - 1$$

### Example of the number of combinations

$$\begin{array}{cccccc} s_1^1 & - & s_3^1 & s_4^1 & - & - & s_7^1 \\ s_1^2 & s_2^2 & - & s_4^2 & - & s_6^2 & - \\ s_1^3 & s_2^3 & s_3^3 & - & s_5^3 & - & - \end{array}$$

- $m$ : 3
- $2 \times 2 \times 2 - 1 = 7$

### Alignment methods

- Dynamic programming with  $m$ -dimensional array (deterministic)
- Progressive alignment (heuristics)

### SP score

One of the common methods to calculate the score of an alignment is using SP (sum-of-pairs) scores. SP uses pair-wise scores on all possible paired sequences to obtain the final score for the alignment. SP is defined as below.

$$S(\mathcal{A}) = \sum_{i=1}^{m-1} \sum_{j=i+1}^m S(\bar{s}^i, \bar{s}^j)$$

**N.B.** The score of  $S(\bar{s}^i, \bar{s}^j)$  is 0 when both elements are gaps.

### Example of SP score

Use the simple scoring scheme and calculate the SP score. Simple scoring scheme: Match: 1, Mismatch: 0, and Gap penalty: 1

Seq1 A-GC  
Seq2 ACG-  
Seq3 A-TC

$$\begin{aligned} S(\bar{s}^1, \bar{s}^2) &= 1 - 1 + 1 - 1 = 0 \\ S(\bar{s}^1, \bar{s}^3) &= 1 + 0 + 0 + 1 = 2 \\ S(\bar{s}^2, \bar{s}^3) &= 1 - 1 + 0 - 1 = -1 \end{aligned}$$

$$S(\mathcal{A}) = S(\bar{s}^1, \bar{s}^2) + S(\bar{s}^1, \bar{s}^3) + S(\bar{s}^2, \bar{s}^3) = 0 + 2 - 1 = 1$$

### Exercise 8.1

Use the simple scoring scheme and calculate the SP score.

Seq1 A-CC

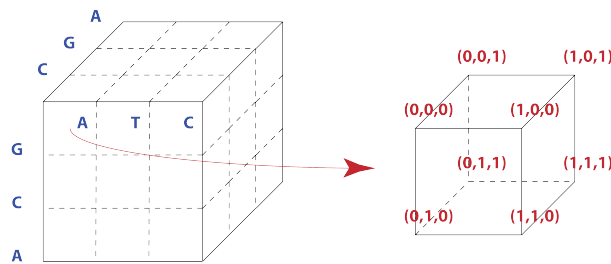
Seq2 C-TC

Seq3 CAG-

## 8.2 Dynamic programming with $m$ -dimensional array

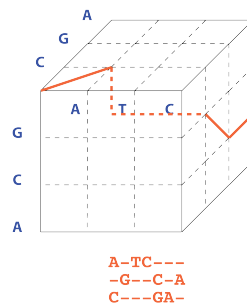
Dynamic programming (DP) can be extended to handle multiple alignments.

### Multi-dimensional array for dynamic programming



**Figure 8.2:** A three-dimensional DP array

### Example of alignment representation



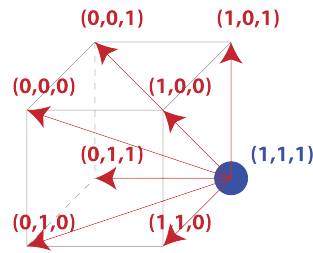
**Figure 8.3:** An alignment with a three-dimensional DP array

### The number of candidate scores for a vertex

The number of the inbound neighboring vertices is defined as follows.

$$\sum_{i=0}^{m-1} \binom{m}{i} = 2^m - 1$$

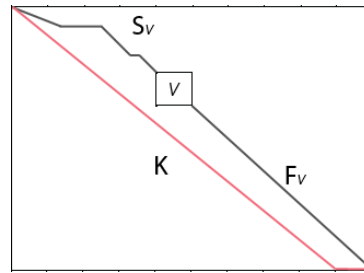
## Example of edges of 3-dimensional cell



**Figure 8.4:** An example of seven different edges to one vertex when  $m = 3$

## A pruning method

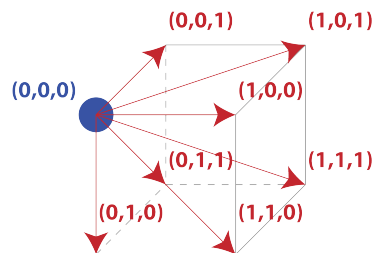
- $K$  : a score of an MSA (it does not need to be the optimal)
- $\nu$  : current vertex
- $S_\nu$  : best score from the start vertex to  $\nu$  (by DP)
- $F_\nu$  : best score from the end vertex to  $\nu$  (by non-DP)
- if  $S_\nu + F_\nu < K$  then  $\nu$  does not lie on the optimal path



**Figure 8.5:** Score estimation

## Forward-recursion DP for MSA

Instead of looking up inbound neighboring vertices, the forward recursion DP sends the calculated score to all outbound neighboring vertices.



**Figure 8.6:** Values are forwarded to all outgoing neighbors