CIS 5800: Machine Perception - Fall 2025
Homework 1 - Written
Due: Thursday Sep 25 2025, 11:59pm ET

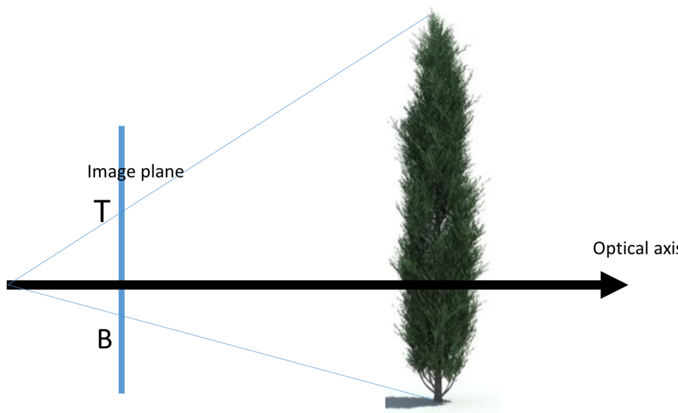
Instructions

- This is individual homework and worth 100 points.
- You must submit your solutions on Gradescope (you should already be automatically enrolled via Canvas).
- We recommend that you use LaTeX, but we will accept scanned solutions as well.
- Start early! Please post your questions on Ed Discussion or come to office hours!

Perspective Projection (no need for homogeneous coordinates) (30pts)

1.1 (10pts) A world point with camera coordinates (300, 600, 1200) is perspectively projected into an image at coordinates (55, 110). Given that the image center is at (0,0) and the aspect ratio (ratio between width and height) of the pixels is 1, what is the focal length f of the camera?

1.2 (20pts) Assume that you see the bottom and the top of a vertical tree in front of you. The image plane is vertical as well with $f = 1$ and you see the bottom and the top of the tree at calibrated coordinates $B = (0, y_1)$ and $T = (0, y_2)$, respectively. Without knowing the tree's height can you tell whether the tree will hit the projection



center if it falls? Prove your answer.

Projective geometry & Homographies (30pts)

2.1 (4pts) Consider homogeneous coordinates $[X, Y, Z]$, what is the point represented when $Z \neq 0$? What about $Z = 0$?

2.2 (4pts) For each of the following pairs of points in $\mathbb{P}^2$, write down an equation for the line that passes through them:

$$(a)\ [-2,5,3] \text{ and } [1,3,4]$$
$$(b)\ [a,0,b] \text{ and } [0,c,b]$$
$$(c)\ [a,0,0] \text{ and } [0,0,a]$$

2.3 (4pts) For each of the following pairs of lines in $\mathbb{P}^2$, determine the point of intersection:

$$(a)\ 3x - y + 2w = 0 \text{ and } x + 5y - w = 0$$
$$(b)\ 2x - 6w = 0 \text{ and } 5x - 2y = 0$$
$$(c)\ 7x + y - w = 0 \text{ and } w = 0$$

2.4 (4pts) Given the two lines $l_1 := w = 0$ and $l_2 := x + 2y + w = 0$ in $\mathbb{P}^2$, for each of the following definitions of l_3, determine if there is a common intersection between l_1, l_2 and l_3 and if so, what that common intersection point is.

$$(a)\ l_3 := x + 2y + 6w = 0$$
$$(b)\ l_3 := -3x - 6y + 6w = 0$$
$$(c)\ l_3 := 7x + y - w = 0$$

2.5 (4pts) Find a projective transformation T that maps the points $[1,0,0]$, $[0,1,0]$ and $[0,0,1]$ to the non-collinear points $[1,-1,1]$, $[1,-2,2]$ and $[-1,2,-1]$, respectively.

2.6 (4pts) A homography H leaves horizontal lines horizontal but maps all vertical lines to lines going through the point $(0,3)$. It leaves the points $(0,0)$ and $(1,1)$ at the same place. Compute H.

2.7 (6pts) Find a condition on the elements of a homography H such that parallel lines remain parallel. Such a transformation is called an affine transformation.

Projective Transformations (30pts)

A projective transformation maps square ABCD to the trapezoid EFGH. Euclidean coordinates of points are marked on the picture. Observe that B coincides with F and C coincides with G.

3.1 (5 pts) Find the coefficients of the lines EF and GH.

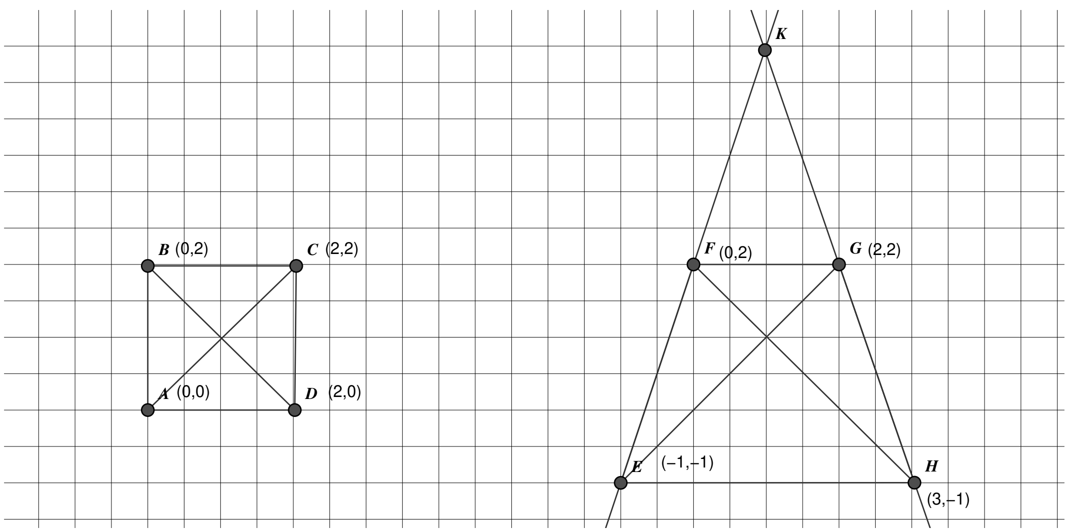
3.2 (5 pts) Find the coordinates of the intersection of the lines EF and GH. Verify that the intersection is at $(1, 5)$. Find the coefficients of the lines EG and FH. Find the coordinates of the intersection of the lines EG and FH. Verify that it is the point $(1, 1)$.

3.3 (15 pts) Find the projective transformation M such that

$$E \sim MA, \qquad F \sim MB, \qquad G \sim MC, \qquad H \sim MD.$$

Hint: You can find it in one step; you do not need to compute two transforms.

3.4 (5 pts) Using the result of the last step verify that $H \sim MD$.



Vanishing Points & Horizon (10pts)

The image of the rectangular facade of a building has two vanishing points, one at $(-b, 0)$ corresponding to horizontal lines and one at $(0, h)$ corresponding to vertical lines. Assume that the origin $(0, 0)$ and the point $(1, 1)$ remain fixed.

4.1 (5 pts) Find the transformation that will map the facade to a rectangle.

4.2 (5 pts) Compute the horizon (the image of the line at infinity).

Bonus Questions (25 pts)

5.1 (15pts) **Affine Transforms**

Prove that, in the case of affine transformations in the projective plane, it is possible to transform a circle into an ellipse, whereas it is impossible to transform an ellipse into a hyperbola or a parabola. Recall the definition of an affine transformation provided in Question 2.7.

5.2 (10 pts) **Vanishing Point Computation**

Consider the one-dimensional projective transformation $y = H_{2\times2}x$ between points on a line of the real world and points on the corresponding line of the image, where the points x and y are parameterized and given in homogeneous coordinates of the form (a, b). Here, $H_{2\times2}$ takes input $x \in \mathbb{P}^1$ to output $y \in \mathbb{P}^1$. Based on the information provided by the homography matrix $H_{2\times2}$ find the vanishing point for the line represented by the said homography.