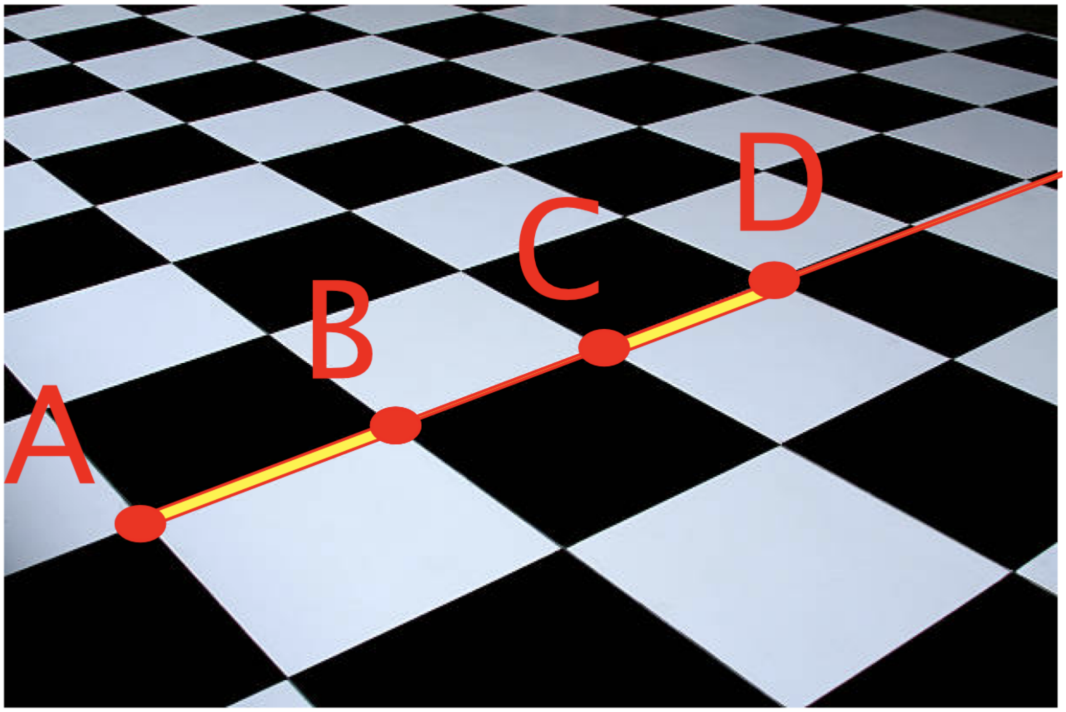
CIS 5800: Machine Perception - Fall 2025
Homework 2 - Written
Due: Wednesday October 8th 2025, 11:59pm ET

Instructions

- This is individual homework and worth 100 points.
- You must submit your solutions on Gradescope (you should already be automatically enrolled via Canvas).
- We recommend that you use LaTeX, but we will accept scanned solutions as well.
- Start early! Please post your questions on Ed Discussion or come to office hours!

Question 1 (10 pts): Cross Ratios

In the following image, the points A, B, C, D are collinear. For the world points we know that $A_W B_W = B_W C_W = C_W D_W$. Given the image distances $AB = 3$ and $CD = 2.5$, compute the image distance BC.



Question 2 (20 pts): Homographies from Pose

1. A camera is initially placed at the origin with its body axes aligned with the world axes. The camera is then rotated about its own x-axis by an angle $\theta = 45°$. Determine the homography H that transforms pixel points before the rotation to pixel points after the rotation.

2. After the rotation from the previous part, the camera is then translated so that its origin lies at the point $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}^\top$ in the global coordinate system. For each of the following planes, determine the 3×3 matrix H that transforms points in world coordinates to points in pixel coordinates:

   (a) The plane $X_W = 0$

   (b) The plane $Y_W = 0$

   (c) The plane $Z_W = 0$

Question 3 (30 pts): Cross Ratios

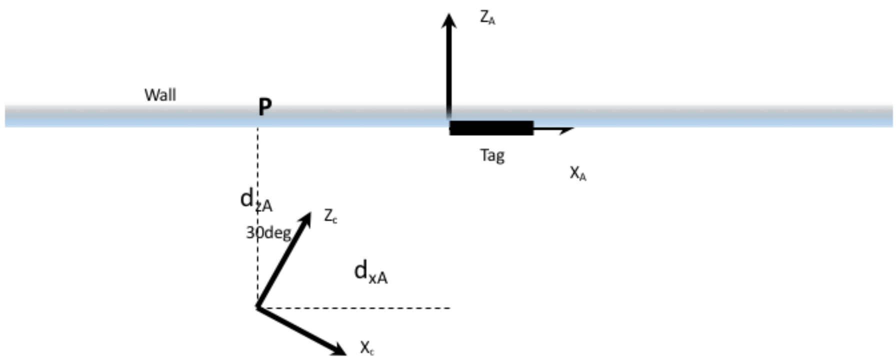
Here we will apply cross-ratios in order to achieve distance transfer. You will need a ruler or a drawing program. You will have to include the final image with all required lines (scanned if you used a ruler) in your solution.

(a) Draw the horizon of the horizontal plane.

(b) Show which point on the red line corresponds to the same height as the white poles.

(c) Assume that the vertical vanishing point is at infinity. Find the height (whole red line segment) of the house if you know that the white poles have height 3m.

Question 4 (40 pts): Collineations with Vanishing Points



In the figure above a camera sees an April tag attached to a wall (all April tag points have $Z_A = 0$). The figure is a bird's eye view. You should assume that the camera is not at the same height as the April tag, meaning that the translation vector starting from the camera and going to the April tag has no coordinate equal to zero.

The projection equation reads

$$\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = K(R \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix} + T)$$

where (u, v) are pixels and (X, Y, Z) are coordinates in the world (e.g. on the April tag). with $R^T R = I$ and

$$K = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Assume that all coordinate systems are right-handed ($\det R = 1$).

1. Write the rotation matrix R that satisifies $\begin{pmatrix} X_C \\ Y_C \\ Z_C \end{pmatrix} = R \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix} + T$.

2. Assuming that the height difference between camera and April tag origins is h, write the translation vector that satisfies $\begin{pmatrix} X_C \\ Y_C \\ Z_C \end{pmatrix} = R \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix} + T$.

3. For a general point $(X_A, Y_A, 0)$ on the april tag write the projection equations for the projection (u, v) in pixels in terms of $f, d_{xA}, d_{zA}, h, X_A, Y_A$, meaning that the right hand side of the equations below should contain only those variables and the (co-)sines of the angle $\theta = 30\,\mathrm{deg}$.

$$
\begin{aligned}
u &= \frac{\cdots}{\cdots} \\
v &= \frac{\cdots}{\cdots}
\end{aligned}
$$

4. Find the projective transformation (invertible 3x3 matrix) from the April tag plane to the image plane.

5. Assume that coordinates of a square on the April tag read $(0, 0), (a, 0), (0, a), (a, a)$. Find their projections in terms of variables.

6. Find the projections in pixels assuming $f = 1000$ pixels, $h = 0, d_{xA} = d_{zA} = 100, a = 50$. Draw the projection of the square, we refer to it as a quadrilateral.

7. Write the four vertices of this quadrilateral in homogenenous coordinates.

8. Find the line coefficients (as in $Au + Bv + Cw = 0$) of the four sides of the quadrilateral.

9. Find the two vanishing points that result from the two pairs of the sides.

10. Find the horizon of the April tag.

11. Find the coordinates of point P in the figure on the wall.

Bonus Question (15 pts): P2P with Gravity

A robot knows the gravity vector with respect to the camera coordinate system. Assume that the Z-axis of the world coordinate system is the same as the gravity direction. There is one unknown orientation θ (we call it the yaw angle) in the unknown orientation matrix between camera and world coordinates. Assume further that we know the projections of two points (x_1, y_1) and (x_2, y_2) in *calibrated* coordinates (no need to multiply with K^{-1}) and the corresponding 3D world coordinates (X_1, Y_1, Z_1) and (X_2, Y_2, Z_2). We have four projection equations with four unknowns (θ, t_x, t_y, t_z).

1. Find the unknowns (θ, t_x, t_y, t_z). Hint: Eliminate the translation parameters. Then express $\cos\theta$ and $\sin\theta$ as a function of one unknown, so that you get a polynomial equation on that unknown.
2. Argue qualitatively why gravity and two points are sufficient to recover the pose of the camera. Which would be a singular case when this cannot happen?