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CMPT-310

ASSIGNMENT - 3

ANSWER - 1 :-

a. $a \to$ Alice , $b \to$ Bob , $c \to$ Christine

$C(x) \to$ member of Alpine club (x)

$S(x) \to$ Skier (x) $M(x) \to$ mountain climber (x)

$L(x, y) \to x$ likes y

$r \to$ rain , $s \to$ snow.

For Knowledge Base (KB)

$KB = \{ C(a), C(b), C(c),$ [a, b, c are members of club]

$\forall x\, (C(x) \land \neg S(x) \supset M(x)),$ [x is not skier ⇒ x is M(x)]

$\forall x\, (M(x) \supset \neg L(x, r)),$ [mountain climbers don't like rain]

$\forall x\, (\neg L(x, s) \supset \neg S(x)),$ [who doesn't like snow ⇒ not skier]

$\forall y\, (L(a, y) \supset \neg L(b, y)),$ [Bob dislikes what Alice likes]

$\forall y\, (\neg L(a, y) \supset L(b, y)),$ [Bob likes what Alice dislikes]

$L(a, r),$ [Alice likes rain]

$L(a, s) \}$ [Alice likes snow]

To prove : there is a member of Alpine club who is a mountain climber but not a skier.

So, Query, $\alpha = \exists x\, (C(x) \land M(x) \land \neg S(x))$

- Using answer extraction Resolution method, firstly writing KB into clauses, we have.

1. $[C(a)]$ 2. $[C(b)]$ 3. $[C(c)]$ 4. $[\sim C(x), S(x), M(x)]$

5. $[\sim M(x), \sim L(x, r)]$ 6. $[L(x, s), \sim S(x)]$

7. $[\sim L(a, y), \sim L(b, y)]$ 8. $[L(a, y), L(b, y)]$

9. $[L(a, r)]$ 10. $[L(a, s)]$

[using the fact that $(a \supset b)$ is $(\neg a \lor b)$]

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$\alpha = \exists x\, [C(x) \land M(x) \land \neg S(x)]$

$\neg\alpha = \forall x\, [\sim C(x) \lor \sim M(x) \lor S(x)]$

Using Answer predicate, [from lecture notes pg. 21]

$[\sim C(x), \sim M(x), S(x), A(x)]$ $\quad$ $[\sim C(x), S(x), M(x)]$

$[C(b)]$

$[\sim C(x), S(x), A(x)]$

x/b

$[L(x, s), \sim S(x)]$

$[S(b), A(b)]$

x/b

$[\sim L(a, y), \sim L(b, y)]$

$[L(b, s), A(b)]$

y/s

$[L(a, s)]$

$[\sim L(a, s), A(b)]$

$[A(b)]$

By this diagram it is certain that Bob is the member of Alpine Club who is a mountain climber but not a skier.

Check : whatever alice likes Bob dislikes.

Alice likes $\rightarrow$ rain $\rightarrow$ not mountain climber
$\quad\quad\quad\;\rightarrow$ snow $\rightarrow$ skier

Bob dislikes $\rightarrow$ rain $\rightarrow$ mountain climber
$\quad\quad\quad\quad\rightarrow$ snow $\rightarrow$ not skier

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b. Now, we know,

Bob likes whatever Alice dislikes

and not

Bob dislikes whatever Alice likes.

So, our updated knowledge base is,

$KB' = \{ C(a), C(b), C(c), \forall x\,((C(x) \wedge \neg S(x)) \supset M(x)),$

$\forall x\,(M(x) \supset \neg L(x,r)),$

$\forall x\,(\neg L(x,s) \supset \neg S(x)),$

$\forall y\,(\neg L(a,y) \supset L(b,y)),$

$L(a,r),\ L(a,s) \}$

$\alpha = \exists x\,(C(x) \wedge M(x) \wedge \neg S(x))$

Interpretation have a domain (universe of discourse) and an interpretation function ϕ, $I = \langle D, \phi \rangle$

Here domain, $D = \{a, b, c, r, s\}$

$\phi(\text{Alice}) = a$ $\phi(\text{Bob}) = b$ $\phi(\text{christine}) = c$

$\phi(\text{rain}) = r$ $\phi(\text{snow}) = s$

Alice likes rain and snow

let Bob likes rain and snow [we are not given Alice's dislikes]

let Christine likes rain and snow.

$\phi(\text{likes}) = \{(a,r), (a,s), (b,r), (b,s), (c,r), (c,s)\}$

$\phi(\text{members}) = \{a, b, c\}$

$\phi(\text{skiers}) = \{a, b, c\}$

$\phi(\text{mountain climbers}) = \{\}$

From the clauses in part a,

1., 2., 3. are true because of $\phi(\text{Alice}), \phi(\text{Bob}), \phi(\text{Christine})$ being true. 4., 5 are true because every member is a skier so there is no mountain climber. 6. is true because all of them are skiers and like snow. 7. is not included (according to question). 8. is given. and Bob likes everything. 9., and 10. are

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true because $(a, s), (a, r) \in \phi(\text{likes})$.

The counterexample here is that ϕ(mountain climbers) is an empty set as there are no mountain climbers. Hence, there doesnot exist a member of Alpine Club who is a mountain climber but not a skier. i.e, $I \models KB'$, but $I \not\models \alpha$.

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ANSWER-2 ⊢

Three Blocks : A, B, C on table T.

Goal : A on B and B ontop of C.

a. As discussed in lecture notes,

Start State :

Op (Action : Start,

Effect : Clear (A) ∧ Clear (B) ∧ Clear (C) ∧

On (A, T) ∧ On (B, T) ∧ On (C, T))

Goal State :

Op (Action : Finish,

Precond : Clear (A) ∧ On(C, T) ∧ On (B, C) ∧ On (A, B))

Actions :

Op (Action : Move (x, y, z),

Precond : Clear (x) ∧ Clear (z) ∧ On (x, y),

Effect : Clear (x) ∧ Clear (y) ∧ ¬ Clear (z) ∧

On (x, z) ∧ ¬ On (x, y))

Op (Action : MoveToTable (x, y),

Precond : Clear (x) ∧ On (x, y),

Effect : Clear (x) ∧ On (x, table) ∧ Clear (y) ∧

¬ On (x, y))

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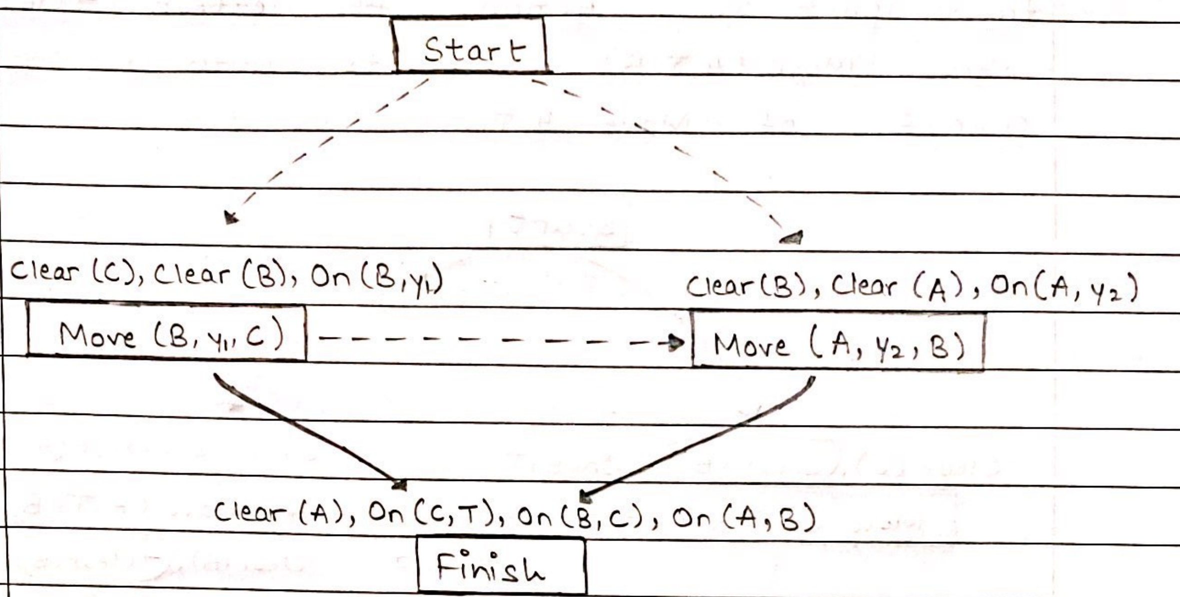
b. Partial plan :

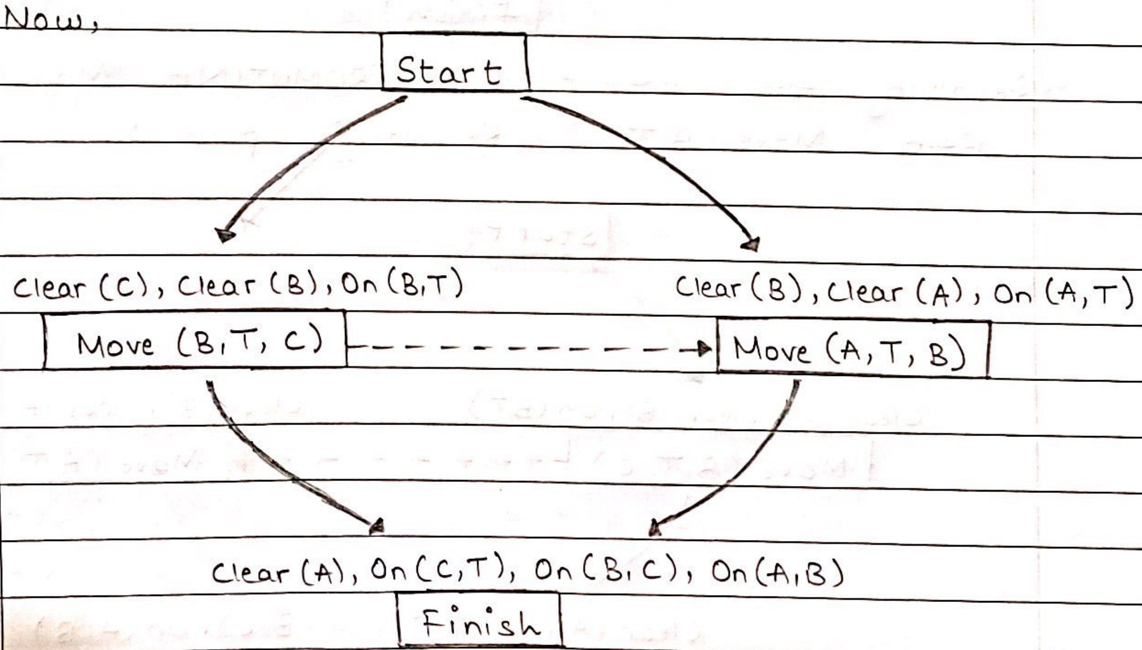
→ causal links

--→ ordering constraints

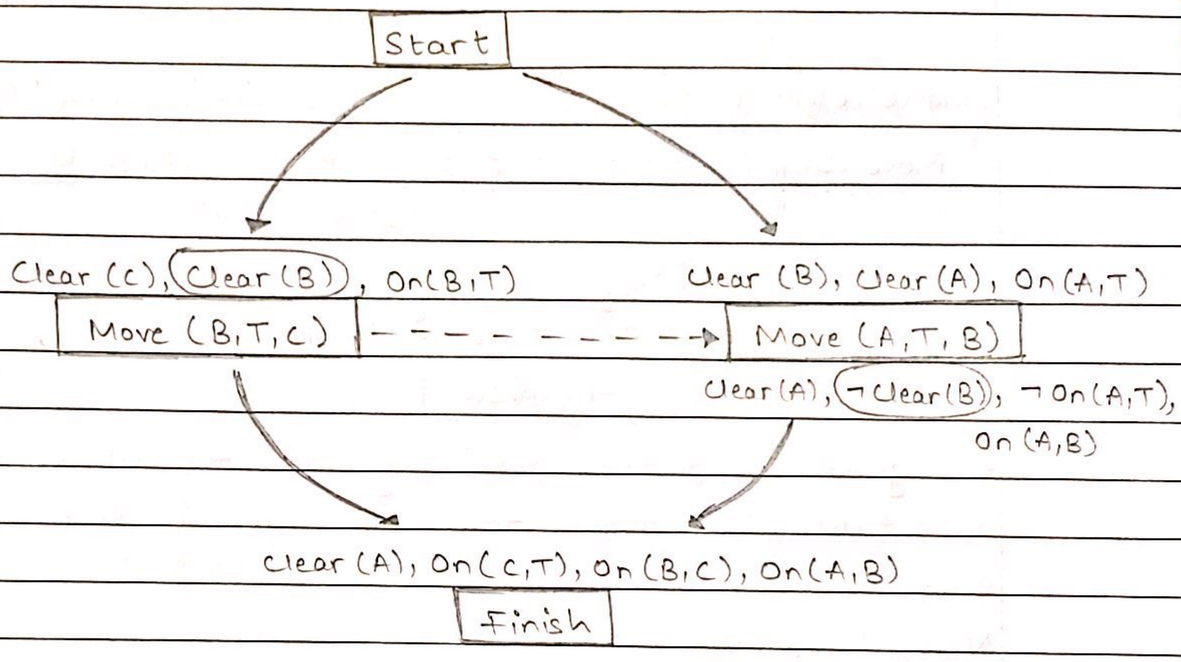


⇒ As given in picture, previously all the blocks are placed on table. In order to make causal links we substitute y_1 and y_2 with T.

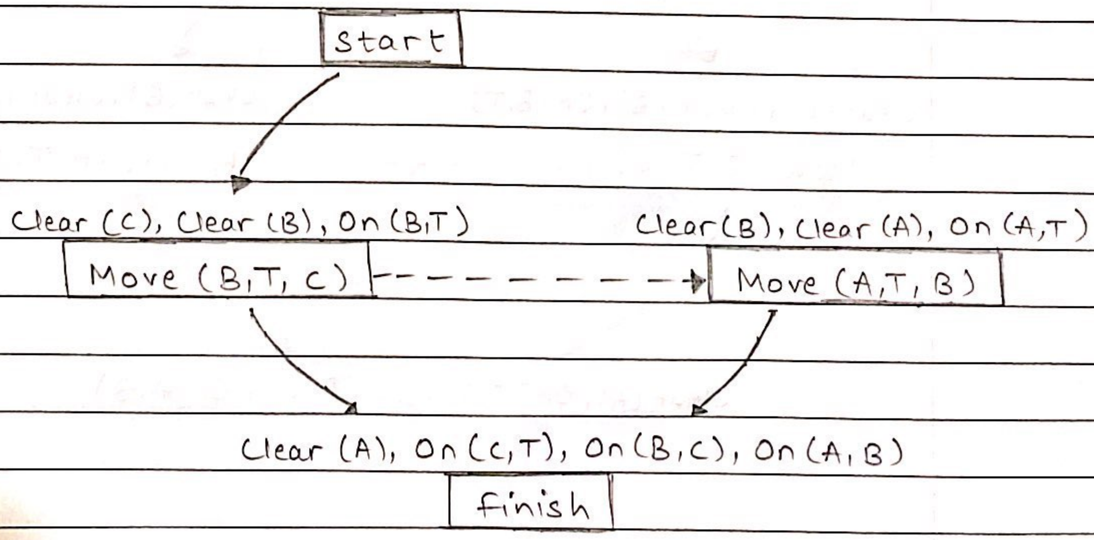
Now,



c. The threat caused by precondition/effect pair is due to Move (B,T,C) and Move (A,T,B). From part a, in Actions, the effect ¬Clear(B) when Move (A,T,B) conflicts with the precondition Clear (B) of Move (B,T,C), i.e,



⇒ Resolving the conflict by PROMOTING Move (A,T,B) after Move (B,T,C). Resulting plan :



⇒ The plan now is CONSISTENT and COMPLETE

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ANSWER 3 :-

We are given,
Let W be well-prepared student, Q_1 be answering first question, Q_2 be answering second question, Q_3 be answering third question.

a. 1. $P(Q_1 | W) = 95\% = 0.95$
2. $P(Q_2 | W) = 95\% = 0.95$
3. $P(Q_3 | W) = 95\% = 0.95$
4. $P(Q_1 | \neg W) = 30\% = 0.30$
5. $P(Q_2 | \neg W) = 50\% = 0.50$
6. $P(Q_3 | \neg W) = 10\% = 0.10$
7. $P(W) = \frac{4}{5} \times 100 = 80\% = 0.80$

b. 1. $P(W | Q_1) = ?$

we know,

$P(W|Q_1) = 1 - P(\neg W | Q_1)$ and — i

$P(W|Q_1) = \frac{P(Q_1|W)P(W)}{P(Q_1)}$ — ii

Comparing i and ii

$$1 - P(\neg W | Q_1) = \frac{P(Q_1|W)\,P(W)}{P(Q_1)}$$

$$1 - \frac{P(Q_1|\neg W)P(\neg W)}{P(Q_1)} = \frac{P(Q_1|W)\,P(W)}{P(Q_1)}$$

$P(Q_1) - P(Q_1|\neg W)P(\neg W) = P(Q_1|W)P(W)$

$P(Q_1) = P(Q_1|W)P(W) + P(Q_1|\neg W)P(\neg W)$

$P(Q_1) = (0.95)(0.80) + (0.30)(1-0.80)$

$= 0.76 + 0.06$

$P(Q_1) = 0.82$

Now, $P(W|Q_1) = \frac{P(Q_1|W)P(W)}{P(Q_1)} = \frac{(0.95)(0.80)}{0.82} = \frac{0.76}{0.82}$

$= 0.927$

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2. $P(Q_1, Q_2 \mid \neg W) = P(Q_1 \mid \neg W)\, P(Q_2 \mid \neg W)$

Since Q_1 and Q_2 are independent of each other. (independent events)

$P(Q_1, Q_2 \mid \neg W) = (0.30)(0.50)$

$= 0.15$

3. $P(Q_3 \mid Q_1, Q_2, W) = ?$

Since $P(Q_3)$ is independent of $P(Q_2)$ and $P(Q_1)$ the above equation simplifies to,

$P(Q_3 \mid W) = 0.95$

c. $P(W \mid Q_1, Q_2, \neg Q_3) = \alpha\, P(W)\, P(Q_1 \mid W)\, P(Q_2 \mid W)\, P(\neg Q_3 \mid W)$

$= \alpha\, (0.80)(0.95)(0.95)(1 - P(Q_3 \mid W))$

$= \alpha\, (0.722)(1 - 0.95)$

$= \alpha\, (0.722)(0.05)$

$= 0.0361\, \alpha$

For α, lets calculate $P(\neg W \mid Q_1, Q_2, \neg Q_3)$

$P(\neg W \mid Q_1, Q_2, \neg Q_3) = \alpha\, P(\neg W)\, P(Q_1 \mid \neg W)\, P(Q_2 \mid \neg W)\, P(\neg Q_3 \mid \neg W)$

$= \alpha\, (0.20)(0.30)(0.50)(1 - 0.10)$

$= \alpha\, (0.03)(0.90)$

$= 0.027\, \alpha$

Now, as we know,

$P(W \mid Q_1, Q_2, \neg Q_3) + P(\neg W \mid Q_1, Q_2, \neg Q_3) = 1$

$0.0361\, \alpha + 0.027\, \alpha = 1$

$0.0631\, \alpha = 1$

$\alpha = \frac{1}{0.0631}$

Now, finally

$P(W \mid Q_1, Q_2, \neg Q_3) = 0.0361\, \alpha = \frac{0.0361}{0.0631}$

$= 0.572$

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d. $P(W \mid Q_1, \neg Q_2, \neg Q_3) = ?$

Same formula as part c,

$$P(W \mid Q_1, \neg Q_2, \neg Q_3) = \alpha\, P(W)\, P(Q_1 \mid W)\, P(\neg Q_2 \mid W)\, P(\neg Q_3 \mid W)$$
$$= \alpha\, (0.80)(0.95)(1 - 0.95)(1 - 0.95)$$
$$= \alpha\, (0.76)(0.05)(0.05)$$
$$= 0.0019\,\alpha$$

For α, lets calculate $P(\neg W \mid Q_1, \neg Q_2, \neg Q_3)$,

$$P(\neg W \mid Q_1, \neg Q_2, \neg Q_3) = \alpha\, P(\neg W)\, P(Q_1 \mid \neg W)\, P(\neg Q_2 \mid \neg W)\, P(\neg Q_3 \mid \neg W)$$
$$= \alpha\, (0.20)(0.30)(1 - 0.50)(1 - 0.10)$$
$$= \alpha\, (0.06)(0.50)(0.90)$$
$$= 0.027\,\alpha$$

As we know,

$$P(W \mid Q_1, Q_2, \neg Q_3) + P(\neg W \mid Q_1, \neg Q_2, \neg Q_3) = 1$$
$$0.0019\,\alpha + 0.027\,\alpha = 1$$
$$0.0289\,\alpha = 1$$
$$\alpha = \frac{1}{0.0289}$$

Now,

$$P(W \mid Q_1, \neg Q_2, \neg Q_3) = \frac{0.0019}{0.0289}$$
$$= 0.0657$$

e. It is important that correct or incorrect answers to some questions do not influence the chance for answering another question correctly. They should be independent of each other because there can be a case when student does not know the answer to either

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one of the question and may get zero due to that and fail the exam. There can be two possibilities here, first, if the student knows the answer to any one question and they are dependent, he/she may know all the answers and get full marks or second, if student doesnot know the answer, he may get all incorrect. Hence, it is crucial that answering some questions correct or incorrect should not influence the chance for answering another question correctly.

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ANSWER 4 :-

a. $P(W \wedge \neg L \wedge R \wedge S)$

$= P(W | \neg L \wedge R) \, P(\neg L | S = spring) \, P(R | S = spring) \, P(S = spring)$

$= (0.95)(1 - L | S = spring)(0.45)(0.25)$

$= (0.017)(1 - 0.15)$

$= (0.017)(0.85)$

$= 0.0909$

$= 0.091$

b. $P(S = winter | \neg R \wedge \neg L)$

$= P(S = winter | \neg R) \cdot \dfrac{P(\neg L | S = winter)}{P(\neg L | \neg R)}$

Firstly

$P(\neg R) = 1 - P(R)$

So,

$P(R) = P(R | S = spring) P(S = spring) + P(R | S = summer) P(S = summer) + P(R | S = autumn) P(S = autumn) + P(R | S = winter) P(S = winter)$

$= (0.45)(0.25) + (0.15)(0.25) + (0.35)(0.25) + (0.20)(0.25)$

$= (0.1125) + (0.0375) + (0.0875) + (0.05)$

$= 0.2875$

$P(\neg R) = 1 - 0.2875 = 0.7125$

Now,

$P(\neg L) = 1 - P(L)$

So,

$P(L) = P(L | S = spring) P(S = spring) + P(L | S = summer) P(S = summer) + P(L | S = autumn) P(S = autumn) + P(L | S = winter) P(S = winter)$

$= (0.15)(0.25) + (0.30)(0.25) + (0.05)(0.25) + (0.00)(0.25)$

$= 0.0375 + 0.075 + 0.0125 = 0.125$

$P(\neg L) = 1 - P(L)$
$= 1 - 0.125$
$= 0.875$

$P(\neg L \mid S=winter) = 1 - P(L \mid S=winter)$
$= 1$

$$P(S=winter \mid \neg R) = \frac{P(\neg R \mid S=winter)\, P(S=winter)}{P(\neg R)}$$
$$= \frac{(1 - P(R \mid S=winter)(0.25)}{0.7125}$$
$$= \frac{(0.80)(0.25)}{0.7125} = 0.281$$

$$P(\neg L \mid \neg R) = \frac{P(\neg L \wedge \neg R)}{P(\neg R)}$$
$$= \frac{(0.875)(0.7125)}{0.7125} = 0.875$$

$$P(S=winter \mid \neg R \wedge \neg L) = \frac{(0.281)(1)}{0.875}$$
$$= 0.321.$$

c. $P(R \mid W \wedge S=summer) = ?$
Since Rain (R) is parent of Wet (W) so, W doesnot affect R so,
$P(R \mid W \wedge S=summer) = P(R \mid S=summer)$
$= 0.15$