Financial Engineering Laboratory (MA 374)

Lab 03

Naveen Kumar A G 210123075

$$S(0) = 100; K = 100; T = 1; M = 100; r = 8\%; \sigma = 30\%.$$

These are the default values used otherwise, mentioned values are used. Initial prices of put and call options were computed using the following configuration:

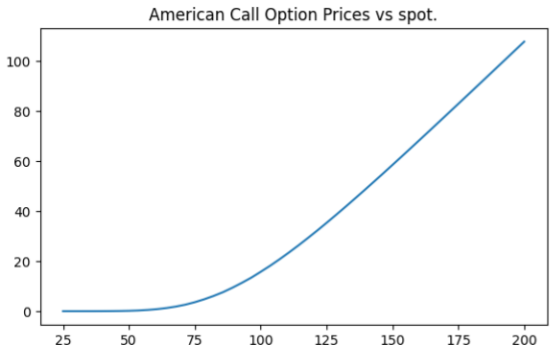
$$u = e^{\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t}\ ;\ d = e^{-\sigma\sqrt{\Delta t}+\left(r-\frac{1}{2}\sigma^2\right)\Delta t}$$

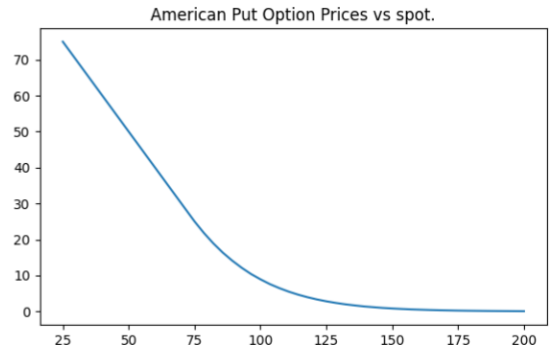
American call option price: 15.736778626185727

American put option price: 8.923113287677717

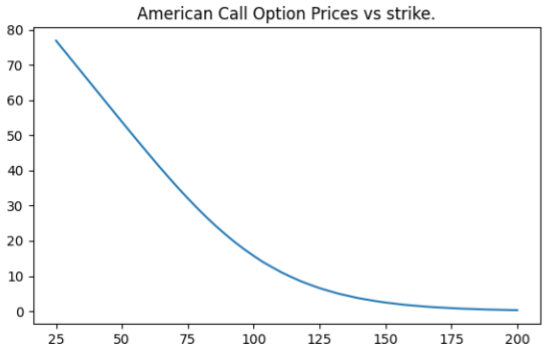
Q1: Sensitivity analysis of initial prices of American put and call options.

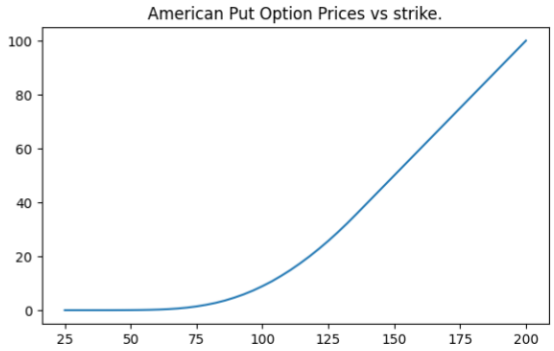
a) Varying spot price at T = 0.



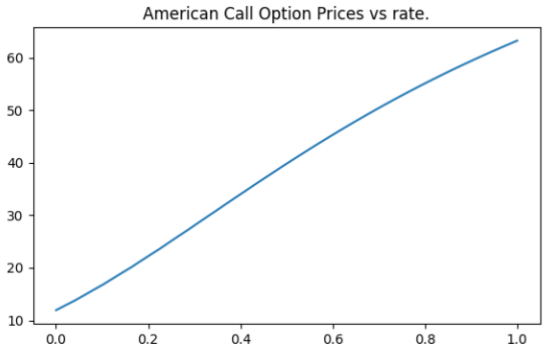


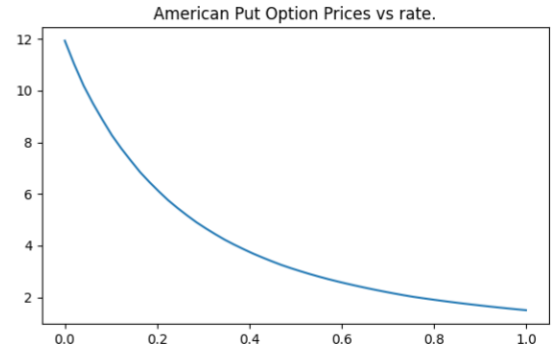
b) Varying strike price of the option.



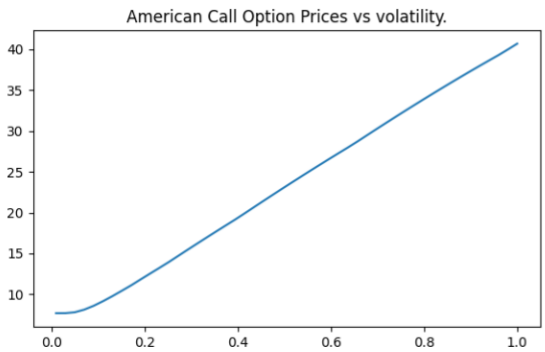


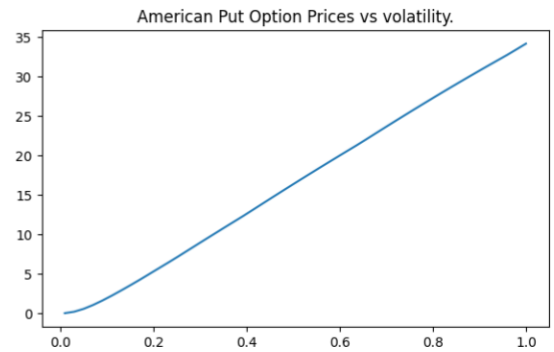
c) Varying interest rate over the life of the option.





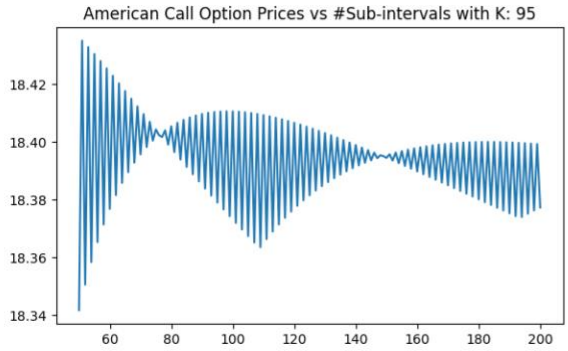
d) Varying volatility over the life of the option.

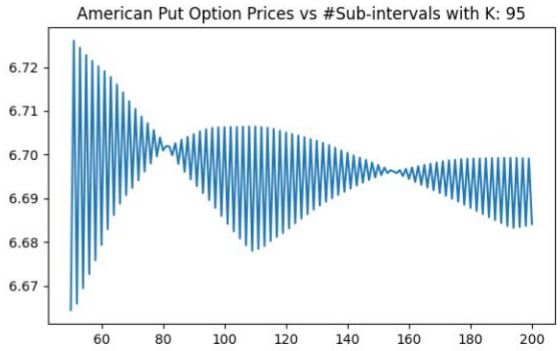




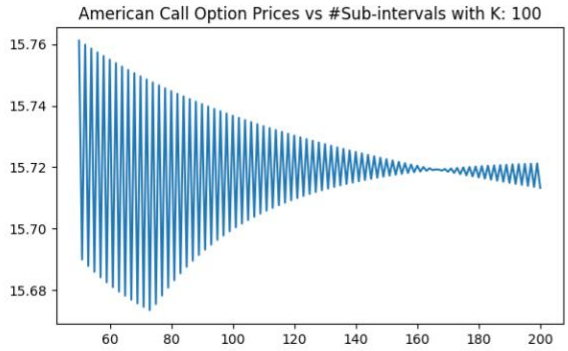
e) Varying number of sub-intervals in the time interval [0, T].

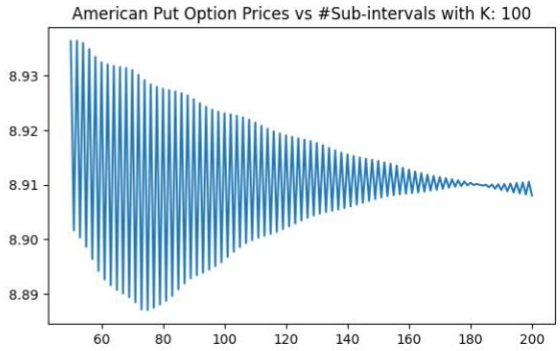
i) K: 95



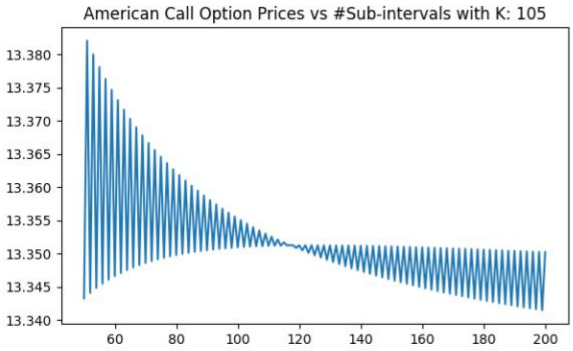


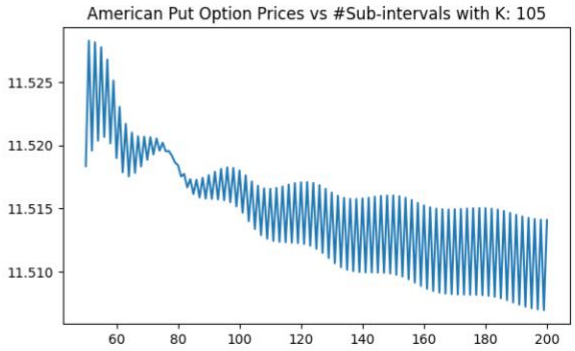
ii) K: 100





iii) K: 105





Q2: Lookback European Call Option Pricing with binomial model.

$$S(0) = 100; T = 1; r = 8\%; \sigma = 30\%.$$

Payoff of lookback European call option:

$$V = \max_{0 \le i \le M} S(i) - S(M),$$

a) Lookback Call Option Prices for M=5: 15.372952215663785
Time taken: 0.0001010894775390625
Lookback Call Option Prices for M=10: 16.950340491777677
Time taken: 0.0019259452819824219
Lookback Call Option Prices for M=25: 18.53378150009417
Time taken: 109.73245692253113

Lookback Call option price for M = 50 takes a lot of time using the binomial model. It is determined in the next question using Markov based computationally efficient binomial algorithm.

b) The lookback call option price seems to increase with the number of subintervals and converges. More observations of the underlying security can be made with more subintervals. Hence, given these subtleties the option can be priced higher.

c) Option prices at all intermediate points for M = 5.

```
At step 5
S_0.u^5.d^0: 0
S_0.u^4.d^1: 21.002491662264447
S_0.u^4.d^1: 0
S_0.u^3.d^2: 34.29714522948986
S_0.u^4.d^1: 0
S_0.u^3.d^2: 16.05969832296735
S_0.u^3.d^2: 14.189941164644068
S_0.u^2.d^3: 42.06197481701972
S_0.u^4.d^1: 0
S_0.u^3.d^2: 16.05969832296735
S_0.u^3.d^2: 0
S_0.u^2.d^3: 26.225545739139193
S_0.u^3.d^2: 0
S_0.u^2.d^3: 24.601948051238253
S_0.u^2.d^3: 24.601948051238267
S_0.u^1.d^4: 45.914488453717624
S_0.u^4.d^1: 0
S_0.u^3.d^2: 16.05969832296735
S_0.u^3.d^2: 0
S_0.u^2.d^3: 26.225545739139207
S_0.u^3.d^2: 0
S_0.u^2.d^3: 12.280157724719814
S_0.u^2.d^3: 10.850435176426544
S_0.u^1.d^4: 32.162975578905915
S_0.u^3.d^2: 0
S_0.u^2.d^3: 12.280157724719814
S_0.u^2.d^3: 9.440589282577335
S_0.u^1.d^4: 30.75312968505669
S_0.u^2.d^3: 9.440589282577307
S_0.u^1.d^4: 30.753129685056678
S_0.u^1.d^4: 30.753129685056678
S_0.u^0.d^5: 47.04990888934698
```

```
At step 4
S_0.u^5.d^0: 10.332480622856941
S_0.u^4.d^1: 16.872978416162187
S_0.u^4.d^1: 7.900801695311675
S_0.u^3.d^2: 27.67676028588705
S_0.u^4.d^1: 7.900801695311675
S_0.u^3.d^2: 12.902037888217313
S_0.u^3.d^2: 12.103285439254643
S_0.u^2.d^3: 34.696462804744556
S_0.u^4.d^1: 7.900801695311675
S_0.u^3.d^2: 12.90203788821732
S_0.u^3.d^2: 6.0414018382528445
S_0.u^2.d^3: 21.163223292550345
S_0.u^3.d^2: 6.0414018382528445
S_0.u^2.d^3: 19.775755431345573
S_0.u^2.d^3: 19.77575543134555
S_0.u^1.d^4: 38.28243217635734

At step 3
S_0.u^5.d^0: 13.386169289151377
S_0.u^4.d^1: 17.504464673898433
S_0.u^4.d^1: 10.235825536366997
S_0.u^3.d^2: 23.026215406441327
S_0.u^4.d^1: 10.235825536367
S_0.u^3.d^2: 13.384908157013324
S_0.u^3.d^2: 12.702323203700724
S_0.u^2.d^3: 28.566489442465265
```

```
At step 2
S_0.u^5.d^0: 15.199750099616733
S_0.u^4.d^1: 16.365773501799982
S_0.u^4.d^1: 11.622592457585522
S_0.u^3.d^2: 20.305310141288484

At step 1
S_0.u^5.d^0: 15.532131468492961
S_0.u^4.d^1: 15.709699760878115

At step 0
S_0.u^5.d^0: 15.372952215663785
```

Q3: Lookback European Call Option Pricing with Markov Based model.

$$S(0) = 100; T = 1; r = 8\%; \sigma = 30\%.$$

Payoff of lookback European call option:

$$V = \max_{0 \le i \le M} S(i) - S(M),$$

a) Lookback Call Option Prices for M=5: 15.372952215663785
Time taken: 0.00015306472778320312
Lookback Call Option Prices for M=10: 16.950340491777677
Time taken: 0.0005440711975097656
Lookback Call Option Prices for M=25: 18.53378150009417
Time taken: 0.03839111328125
Lookback Call Option Prices for M=50: 19.39046523552243
Time taken: 3.7967309951782227

b) The lookback call option price seems to increase with the number of subintervals and converges. More observations of the underlying security can be made with more subintervals. Hence, given these subtleties the option can be priced higher.

c) Option prices at all intermediate points for M = 5.

```
At step 5                       At step 4                       At step 2
S_0.u^5.d^0: 0                  S_0.u^4.d^0: 10.332480622856941 S_0.u^2.d^0: 15.199750099616733
S_0.u^4.d^1: 21.002491662264447 S_0.u^3.d^1: 16.872978416162187 S_0.u^1.d^1: 16.365773501799982
S_0.u^4.d^1: 0                  S_0.u^3.d^1: 7.900801695311675  S_0.u^1.d^1: 11.622592457585522
S_0.u^3.d^2: 34.29714522948986  S_0.u^2.d^2: 27.67676028588705  S_0.u^0.d^2: 20.305310141288484
S_0.u^3.d^2: 16.05969832296735  S_0.u^3.d^1: 7.900801695311675
S_0.u^3.d^2: 14.189941164644068 S_0.u^2.d^2: 12.902037888217313 At step 1
S_0.u^2.d^3: 42.06197481701972  S_0.u^2.d^2: 12.103285439254643 S_0.u^1.d^0: 15.532131468492961
S_0.u^4.d^1: 0                  S_0.u^1.d^3: 34.696462804744556 S_0.u^0.d^1: 15.709699760878115
S_0.u^3.d^2: 16.05969832296735  S_0.u^2.d^2: 12.90203788821732
S_0.u^3.d^2: 0                  S_0.u^2.d^2: 6.0414018382528445 At step 0
S_0.u^2.d^3: 26.225545739139193 S_0.u^1.d^3: 21.163223292550345 S_0.u^0.d^0: 15.372952215663785
S_0.u^2.d^3: 24.601948051238253 S_0.u^2.d^2: 6.0414018382528445
S_0.u^2.d^3: 24.601948051238267 S_0.u^1.d^3: 19.775755431345573
S_0.u^1.d^4: 45.914488453717624 S_0.u^1.d^3: 19.77575543134555
S_0.u^2.d^3: 26.225545739139207 S_0.u^0.d^4: 38.28243217635734
S_0.u^3.d^2: 0
S_0.u^2.d^3: 12.280157724719814 At step 3
S_0.u^2.d^3: 10.850435176426544 S_0.u^3.d^0: 13.386169289151377
S_0.u^1.d^4: 32.162975578905915 S_0.u^2.d^1: 17.504464673898433
S_0.u^2.d^3: 12.280157724719814 S_0.u^2.d^1: 10.235825536366997
S_0.u^2.d^3: 9.440589282577335  S_0.u^1.d^2: 23.026215406441327
S_0.u^1.d^4: 30.75312968505669  S_0.u^2.d^1: 10.235825536367
S_0.u^2.d^3: 9.440589282577307  S_0.u^1.d^2: 13.384908157013324
S_0.u^1.d^4: 30.753129685056678 S_0.u^1.d^2: 12.702323203700724
S_0.u^0.d^5: 47.04990888934698  S_0.u^0.d^3: 28.566489442465265
```

Q4: Comparison of European Call option (binomial) with European Call option (Markov based).

a) Binomial method.

European Call Option Prices for M=5: 16.200135785709474
Time taken: 7.915496826171875e-05
European Call Option Prices for M=10: 15.749706920472518
Time taken: 0.0012862682342529297
European Call Option Prices for M=25: 15.746918255600486
Time taken: 59.001708984375

Efficient method.

European Call Option Prices for M=5: 16.200135785709474
Time taken: 0.00012111663818359375
European Call Option Prices for M=10: 15.749706920472518
Time taken: 0.0005576610565185547
European Call Option Prices for M=25: 15.746918255600486
Time taken: 0.0014958381652832031
European Call Option Prices for M=50: 15.761196879829424
Time taken: 0.013704061508178711

b) The European call option price seems to increase with the number of subintervals and converges. More observations of the underlying security can be made with more subintervals. Hence, given these subtleties the option can be priced higher.

c) Option prices at all intermediate points for M = 5.

```
At step 5
S_0.u^5.d^0: 102.55077163378829
S_0.u^4.d^1: 54.881827348487604
S_0.u^3.d^2: 18.431444369798342
S_0.u^3.d^2: 18.431444369798328
S_0.u^2.d^3: 0
S_0.u^4.d^1: 54.881827348487576
S_0.u^3.d^2: 18.431444369798314
S_0.u^2.d^3: 0
S_0.u^1.d^4: 0
S_0.u^2.d^3: 0
S_0.u^1.d^4: 0
S_0.u^0.d^5: 0

At step 4
S_0.u^4.d^0: 77.47158700522355
S_0.u^3.d^1: 36.07841068723719
S_0.u^2.d^2: 9.071271363629885
S_0.u^3.d^1: 36.07841068723716
S_0.u^1.d^3: 0.0
S_0.u^2.d^2: 9.071271363629892
S_0.u^1.d^3: 0.0
S_0.u^0.d^4: 0.0
```

```
At step 3
S_0.u^3.d^0: 55.87793139136847
S_0.u^2.d^1: 22.21919542461549
S_0.u^2.d^1: 22.219195424615474
S_0.u^1.d^2: 4.464542360415781
S_0.u^1.d^2: 4.464542360415784
S_0.u^0.d^3: 0.0

At step 2
S_0.u^2.d^0: 38.43209515752477
S_0.u^1.d^1: 13.131857964608423
S_0.u^1.d^1: 13.131857964608432
S_0.u^0.d^2: 2.1972816917221

At step 1
S_0.u^1.d^0: 25.375255893366365
S_0.u^0.d^1: 7.543996674048179

At step 0
S_0.u^0.d^0: 16.200135785709474
```