MA323 - Monte Carlo Simulation

# Lab - 1

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1. **Generate the sequence of numbers $x_i$ for a = 6, b = 0, m = 11, and $x_0$ ranging from 0 to 10. Also, generate the sequence of numbers $x_i$ for a = 3, b = 0, m = 11, and $x_0$ ranging from 0 to 10. Observe the sequence of numbers generated and observe the repetition of values. Tabulate these for each group of values. How many distinct values appear before repetitions? Which, in your opinion, are the best choices and why?**

   i) a = 6, b = 0, m = 11, and $x_0$ ranging from 0 to 10

   Observations for 11 samples:

| i/x_0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **1** | 0 | 6 | 1 | 7 | 2 | 8 | 3 | 9 | 4 | 10 | 5 |
| **2** | 0 | 3 | 6 | 9 | 1 | 4 | 7 | 10 | 2 | 5 | 8 |
| **3** | 0 | 7 | 3 | 10 | 6 | 2 | 9 | 5 | 1 | 8 | 4 |
| **4** | 0 | 9 | 7 | 5 | 3 | 1 | 10 | 8 | 6 | 4 | 2 |
| **5** | 0 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| **6** | 0 | 5 | 10 | 4 | 9 | 3 | 8 | 2 | 7 | 1 | 6 |
| **7** | 0 | 8 | 5 | 2 | 10 | 7 | 4 | 1 | 9 | 6 | 3 |
| **8** | 0 | 4 | 8 | 1 | 5 | 9 | 2 | 6 | 10 | 3 | 7 |
| **9** | 0 | 2 | 4 | 6 | 8 | 10 | 1 | 3 | 5 | 7 | 9 |
| **10** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **11** | 0 | 6 | 1 | 7 | 2 | 8 | 3 | 9 | 4 | 10 | 5 |

   Number of distinct values before repetition:

| repetitions/x_0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 1 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |

**Observation:** Only 1 distinct value appears before repeating itself when $x_0$ equals 0. But, for $x_0$ values from 1 to 10, 10 distinct values from 1 to 10 appear before the sequence repeats itself. It should be noted that the period length equals the maximum possible period length i.e. m – 1.

ii) a = 3, b = 0, m = 11, and $x_0$ ranging from 0 to 10

Observations for 11 samples:

| i/x_0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **1** | 0 | 3 | 6 | 9 | 1 | 4 | 7 | 10 | 2 | 5 | 8 |
| **2** | 0 | 9 | 7 | 5 | 3 | 1 | 10 | 8 | 6 | 4 | 2 |
| **3** | 0 | 5 | 10 | 4 | 9 | 3 | 8 | 2 | 7 | 1 | 6 |
| **4** | 0 | 4 | 8 | 1 | 5 | 9 | 2 | 6 | 10 | 3 | 7 |
| **5** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **6** | 0 | 3 | 6 | 9 | 1 | 4 | 7 | 10 | 2 | 5 | 8 |
| **7** | 0 | 9 | 7 | 5 | 3 | 1 | 10 | 8 | 6 | 4 | 2 |
| **8** | 0 | 5 | 10 | 4 | 9 | 3 | 8 | 2 | 7 | 1 | 6 |
| **9** | 0 | 4 | 8 | 1 | 5 | 9 | 2 | 6 | 10 | 3 | 7 |
| **10** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **11** | 0 | 3 | 6 | 9 | 1 | 4 | 7 | 10 | 2 | 5 | 8 |

Number of distinct values before repetition:

| repetitions/x_0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | **0** | 1 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |

**Observation:** Only 1 distinct value appears before repeating itself when $x_0$ equals 0. But, for $x_0$ values from 1 to 10, 5 distinct values appear before the sequence repeats itself. It should be noted that the period length is less than the maximum possible period length i.e. m – 1.

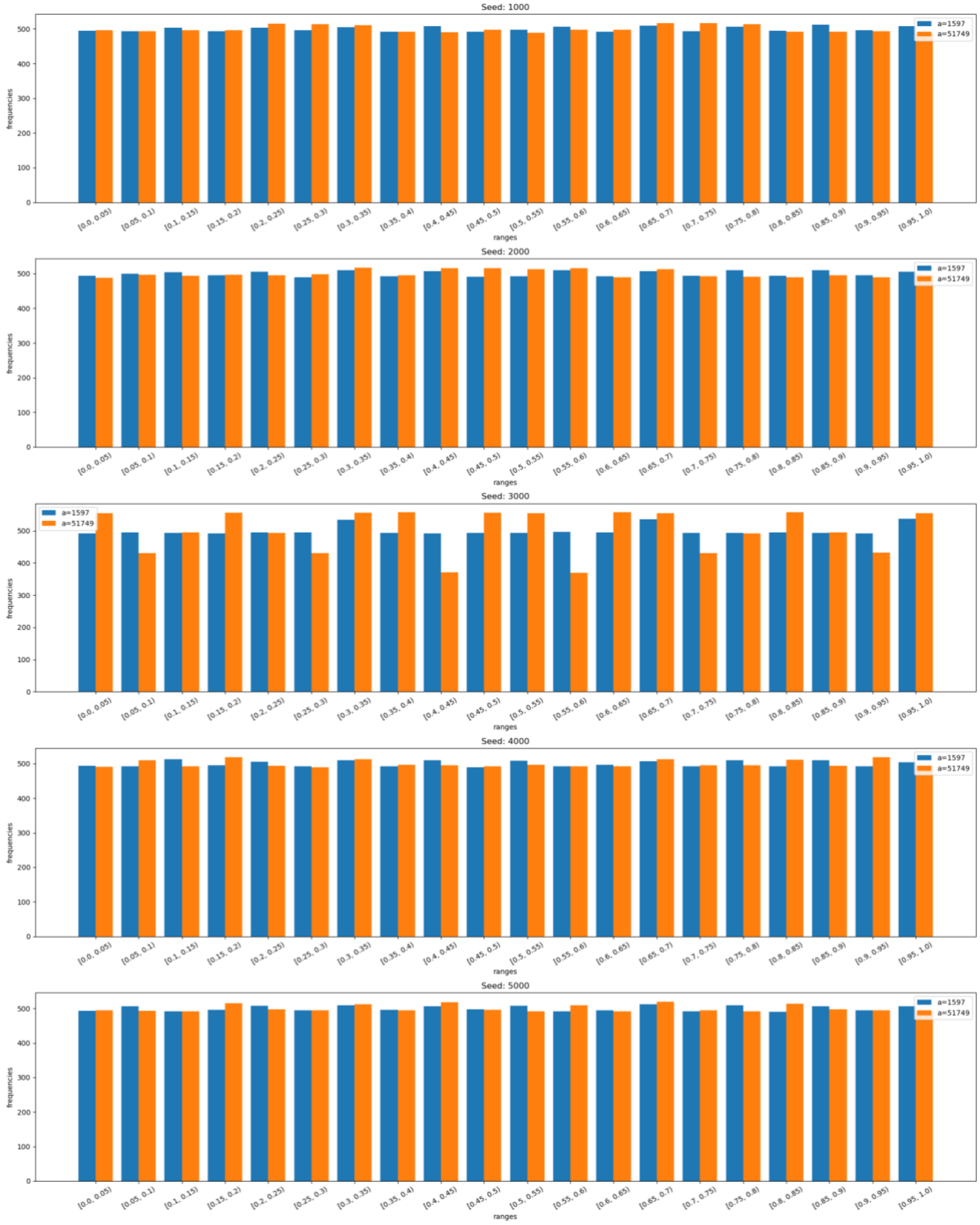
**Inference:** Part (i) has period length 10 which is maximum for the given value of m, while part (ii) has period length 5 which is lesser than the maximum achievable period length 10 for the given value of m. Part (i) is preferred over part (ii) as there will be an increase in the randomness with more numbers. **Hence part (i) i.e. a = 6, b = 0, m = 11 with $x_0$ from 1 to 10 will be the best choice.**

2. Generate a sequence $u_i$, i = 1, 2, . . . , 10000 with m = 244944, a = 1597, 51749 (choosing $x_0$ as per your choice). Then group the values in the ranges [0, 0.05), [0.05, 0.10), [0.10, 0.15), . . . , [0.95, 1) and observe their frequencies (i.e., the number of values falling in each group). For 5 different $x_0$ values, tabulate the frequencies in each case, draw the bar diagrams for these data and put in your observations.

Observations for $x_0$ = {1000, 2000, 3000, 4000, 5000}

| | a = 1597 | | | | | a = 51749 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1000 | 2000 | 3000 | 4000 | 5000 | 1000 | 2000 | 3000 | 4000 | 5000 |
| range/x_0 | | | | | | | | | | |
| [0.0, 0.05) | 495 | 494 | 492 | 494 | 493 | 496 | 488 | 555 | 491 | 495 |
| [0.05, 0.1) | 494 | 499 | 495 | 493 | 506 | 494 | 497 | 431 | 510 | 493 |
| [0.1, 0.15) | 503 | 504 | 494 | 513 | 492 | 496 | 494 | 495 | 493 | 491 |
| [0.15, 0.2) | 493 | 496 | 492 | 496 | 496 | 496 | 497 | 556 | 519 | 515 |
| [0.2, 0.25) | 504 | 505 | 495 | 505 | 507 | 515 | 495 | 494 | 494 | 497 |
| [0.25, 0.3) | 496 | 490 | 495 | 492 | 495 | 514 | 498 | 431 | 490 | 495 |
| [0.3, 0.35) | 505 | 510 | 534 | 510 | 509 | 511 | 517 | 556 | 513 | 512 |
| [0.35, 0.4) | 492 | 493 | 493 | 493 | 496 | 492 | 496 | 557 | 497 | 494 |
| [0.4, 0.45) | 508 | 507 | 492 | 510 | 506 | 491 | 516 | 371 | 495 | 518 |
| [0.45, 0.5) | 492 | 491 | 493 | 489 | 498 | 498 | 516 | 556 | 493 | 496 |
| [0.5, 0.55) | 498 | 493 | 493 | 508 | 508 | 489 | 513 | 554 | 497 | 491 |
| [0.55, 0.6) | 506 | 510 | 497 | 492 | 491 | 497 | 515 | 370 | 492 | 509 |
| [0.6, 0.65) | 492 | 493 | 495 | 497 | 494 | 497 | 490 | 557 | 493 | 492 |
| [0.65, 0.7) | 510 | 507 | 535 | 507 | 512 | 517 | 513 | 555 | 513 | 519 |
| [0.7, 0.75) | 494 | 494 | 494 | 493 | 491 | 517 | 493 | 431 | 496 | 495 |
| [0.75, 0.8) | 507 | 510 | 493 | 510 | 509 | 514 | 491 | 492 | 495 | 491 |
| [0.8, 0.85) | 495 | 494 | 495 | 492 | 490 | 492 | 489 | 557 | 512 | 513 |
| [0.85, 0.9) | 512 | 510 | 494 | 510 | 506 | 492 | 495 | 495 | 494 | 497 |
| [0.9, 0.95) | 496 | 495 | 492 | 492 | 495 | 493 | 490 | 432 | 518 | 495 |
| [0.95, 1.0) | 508 | 505 | 537 | 504 | 506 | 489 | 497 | 555 | 495 | 492 |

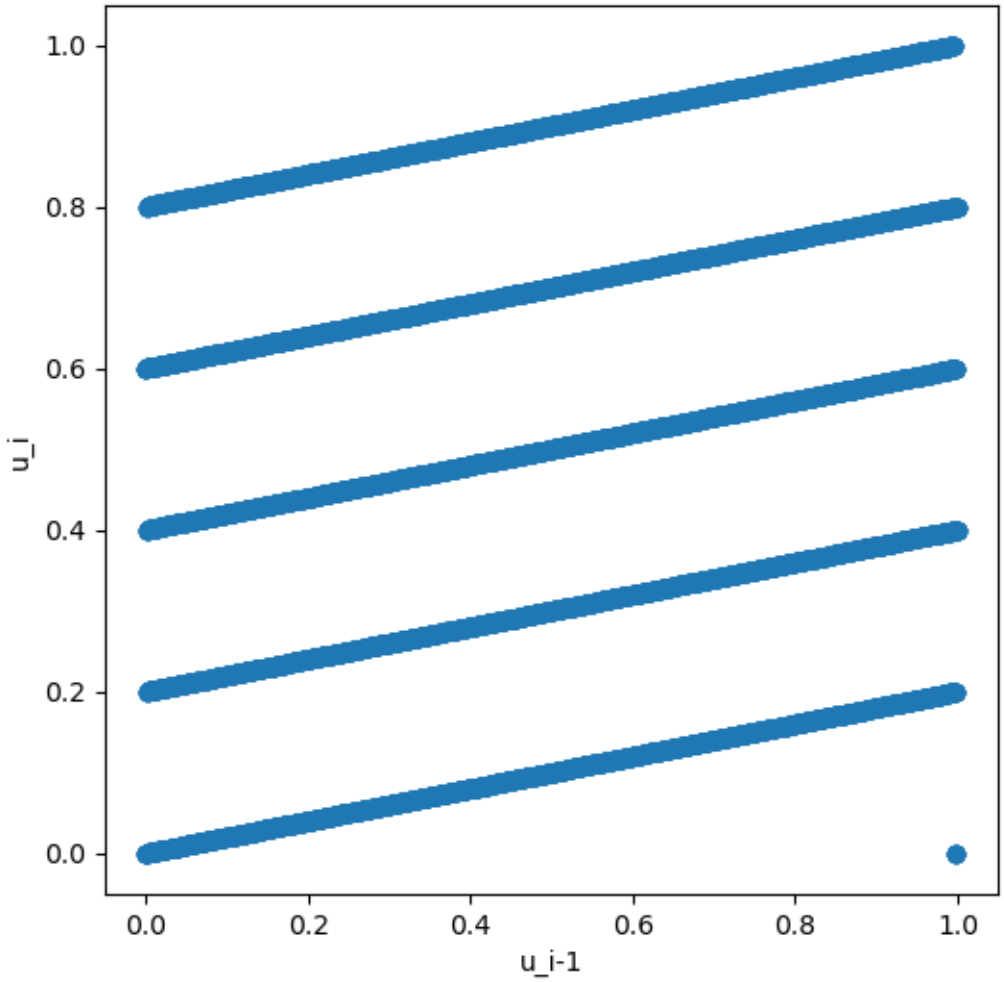
Bar graph showing distribution of samples among intervals:




**Observation:** The generated numbers are uniformly distributed between 0 and 1. Frequency distribution of the generated numbers for different seed values $x_0$ in the same interval length for both values of a is similar. Therefore, the random number generator follows a uniform distribution.

3. **Generate a sequence $u_i$, i = 1, 2, ..., 10000 with a = 1229, b = 1, m = 2048. Plot in a two-dimensional graph the points $(u_{i-1}, u_i)$, i.e., the points $(u_1, u_2)$, $(u_2, u_3)$, $(u_3, u_4)$, . . .**

Scatterplot for $(u_{i-1}, u_i)$ for $x_0 = 1000$:




**Observation:** There are 5 parallel lines initiating from different y-coordinates in this scatterplot. There is an outlier at x = 1.0. This plot is a spectral test for evaluating the performance of random number generators. The further apart these lines are, worse the generator is.