

Lab - 5

Goal: To generate 10,000 samples from the distribution X such that,

$$\mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N_2(\boldsymbol{\mu}, \Sigma).$$

where,

$$\boldsymbol{\mu} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} 1 & 2a \\ 2a & 4 \end{pmatrix}$$

To generate from this distribution the following steps were taken:

- 1) 10,000 samples from $N(0, 1)$ distribution were generated using Box-Muller Method. These samples are called Z .
- 2) From the given covariance-variance matrix, a matrix A such that $\Sigma = A.A^T$ was determined using Cholesky Factorization.
- 3) The target distribution was determined using

$$\mathbf{X} = \boldsymbol{\mu} + \mathbf{AZ}$$

Observations:

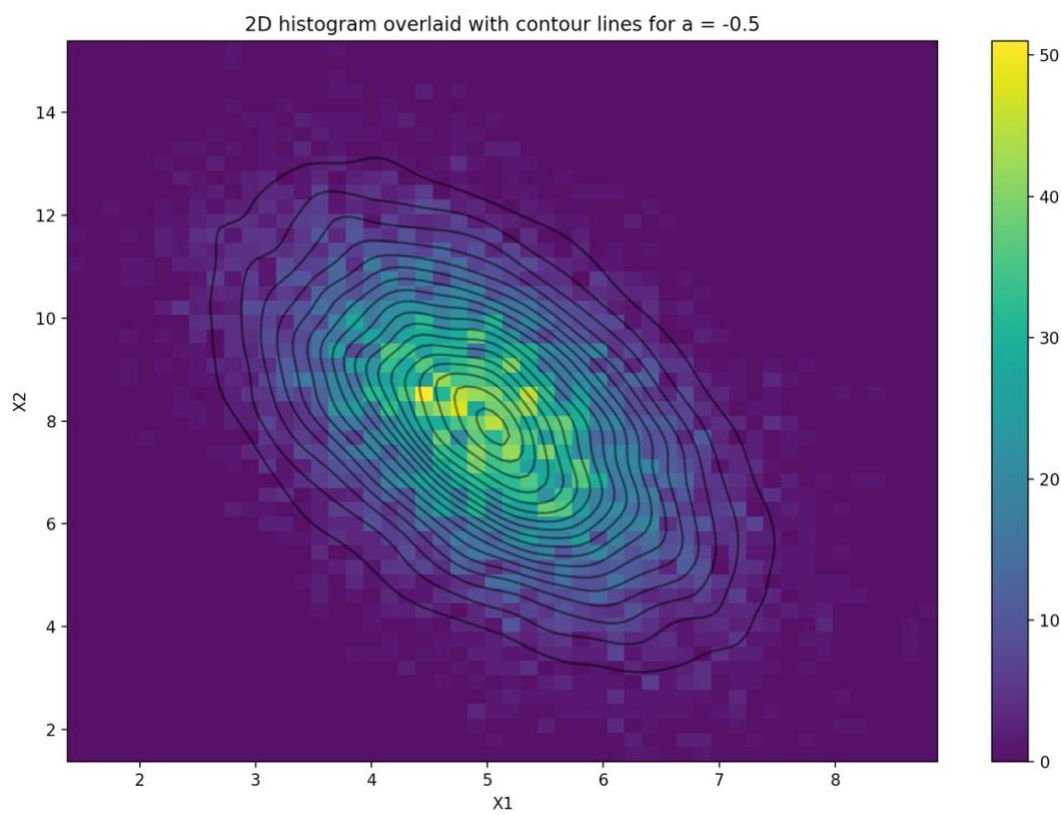
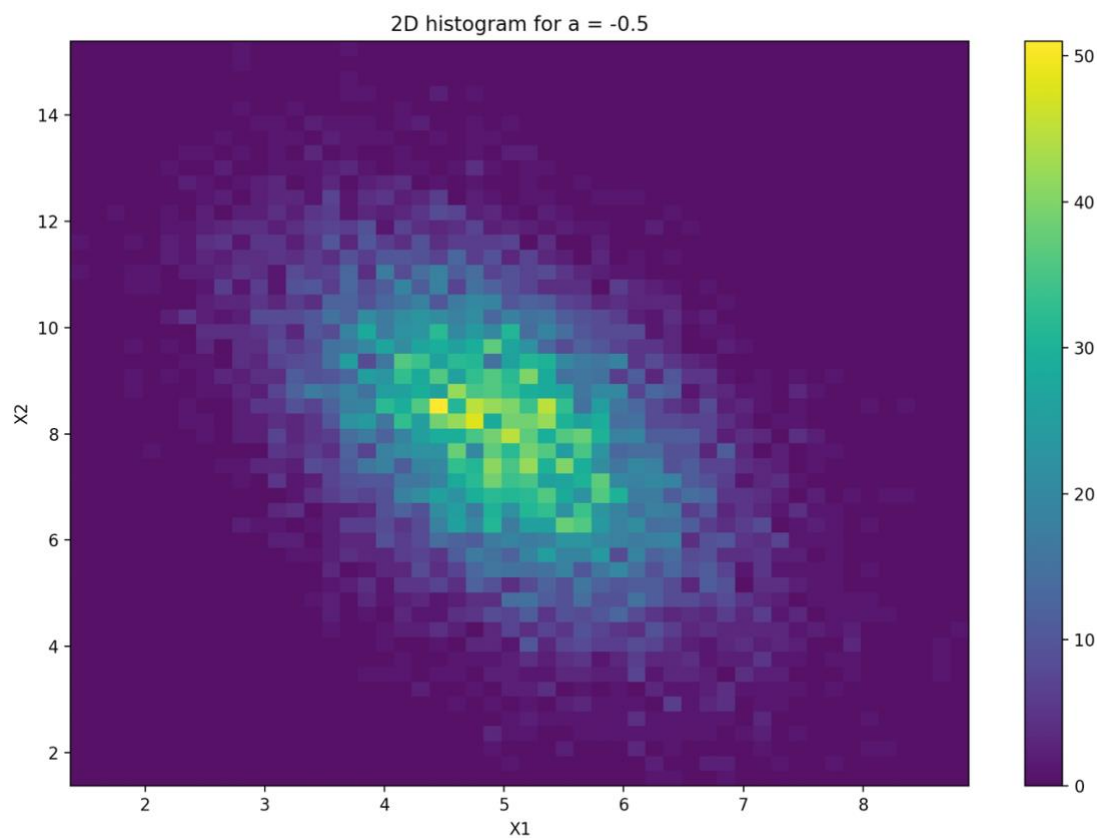
- 1) 1st plot:
The samples from target distribution X using the process were plotted in a 2-D histogram, the color bar on the right side of each image specifies the height of each pixel in the 2-D histogram.
- 2) 2nd plot:
The 2-D histogram in the first plot was overlaid with the contour plot of the actual distribution using inbuilt libraries.

Result:

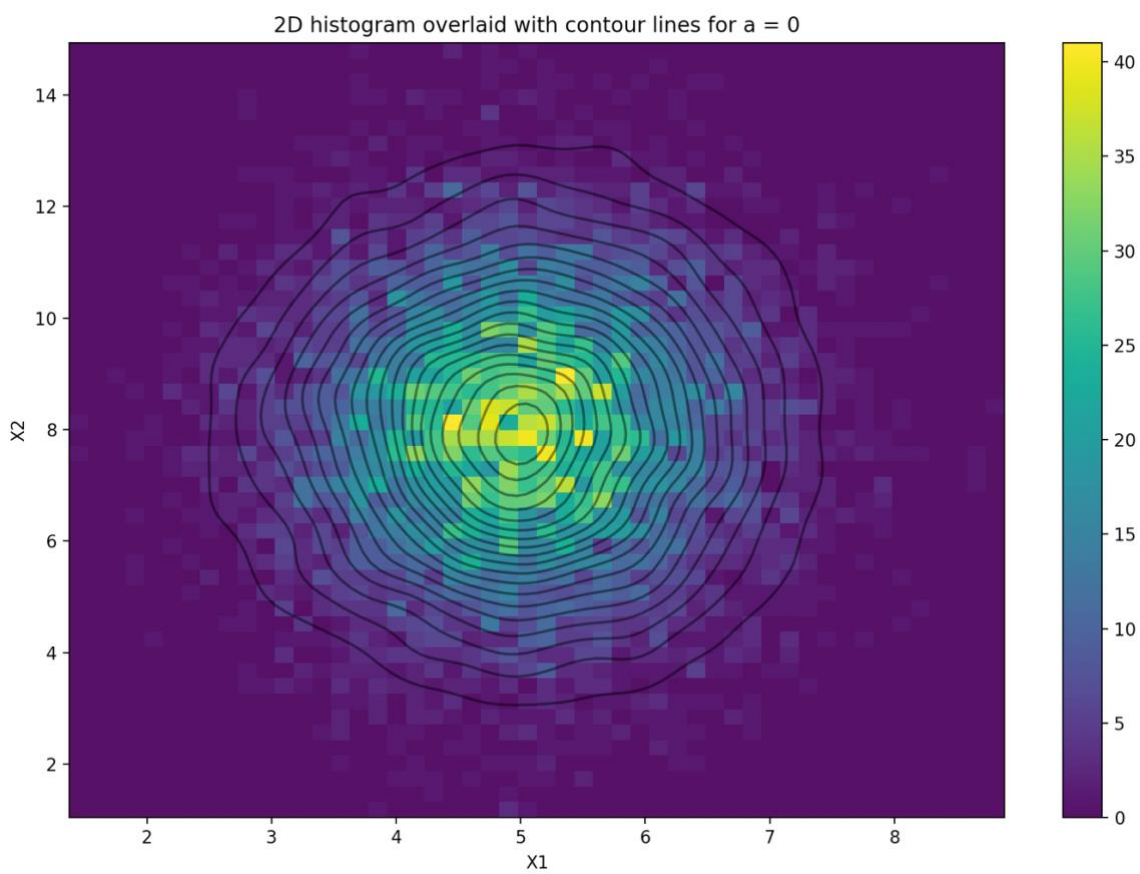
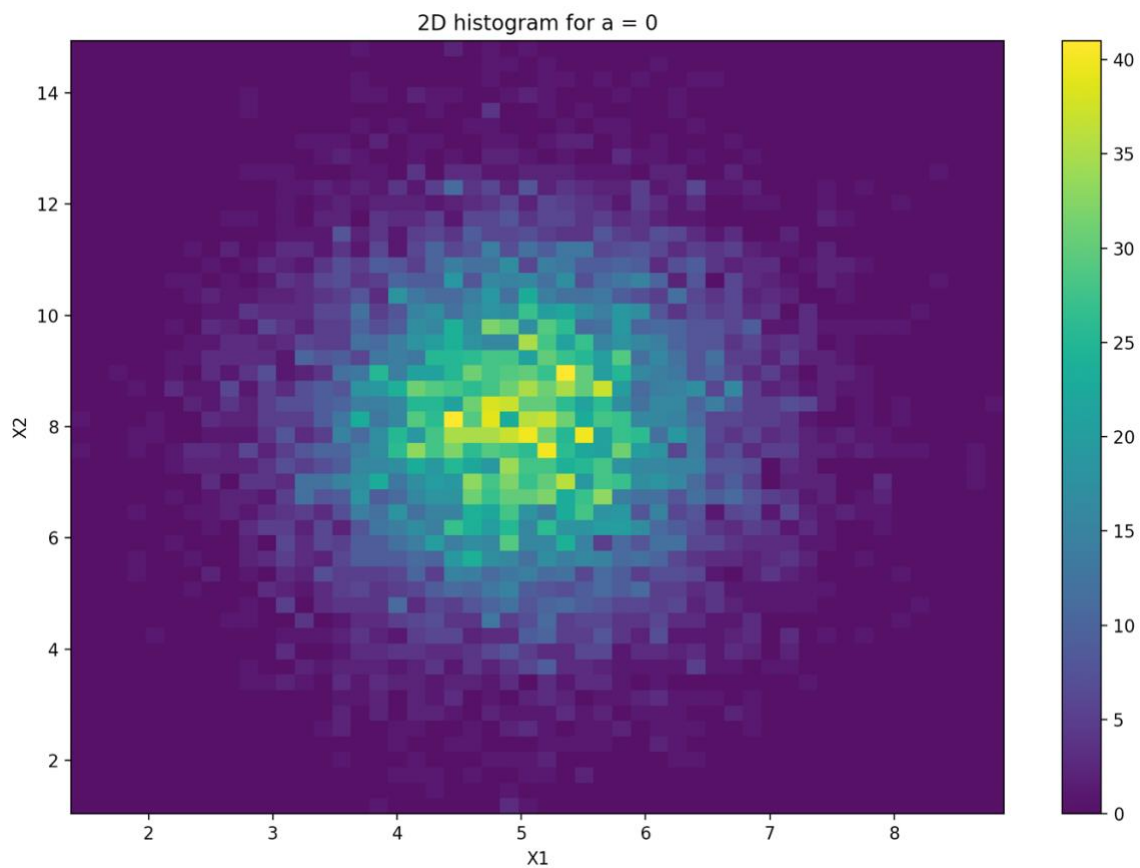
The contour lines generated by the actual distribution are consistent with the 2-D histogram of the samples generated from the target distribution using the above process.

We consider various values of 'a':

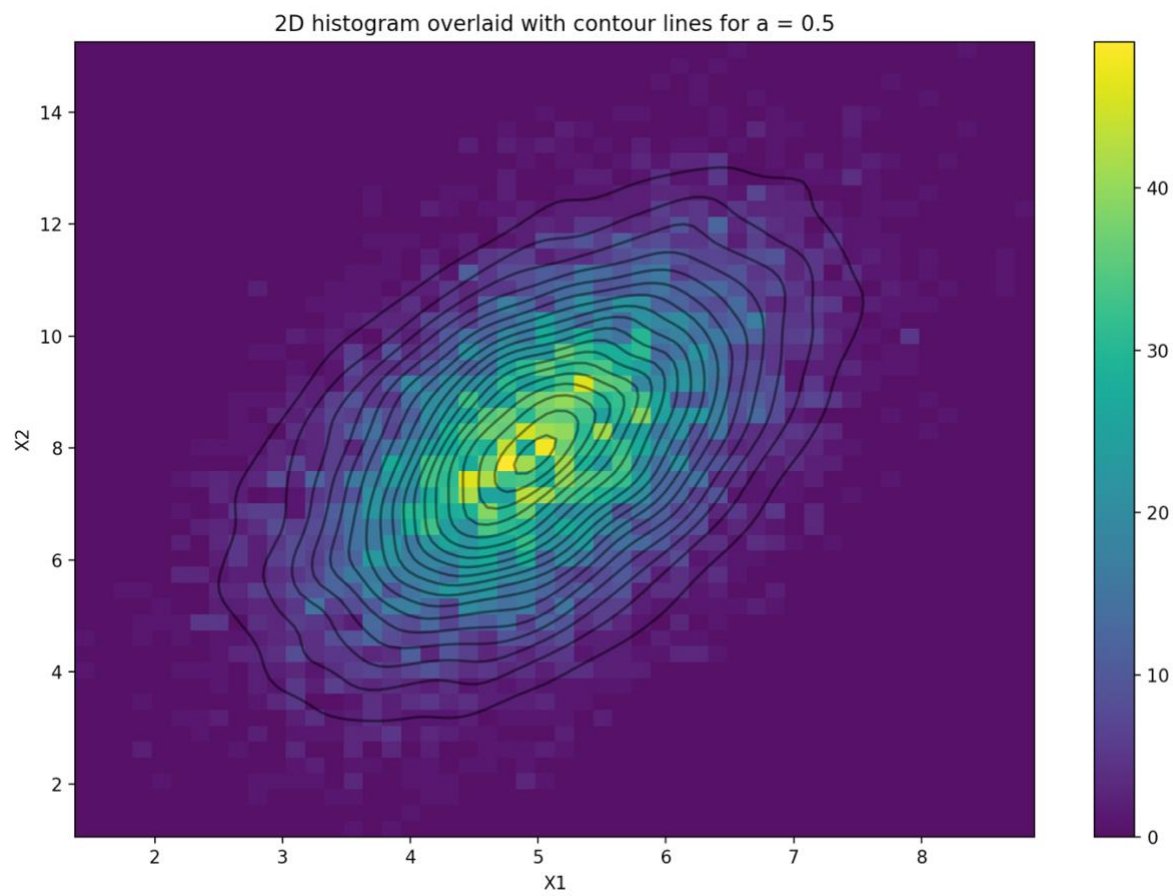
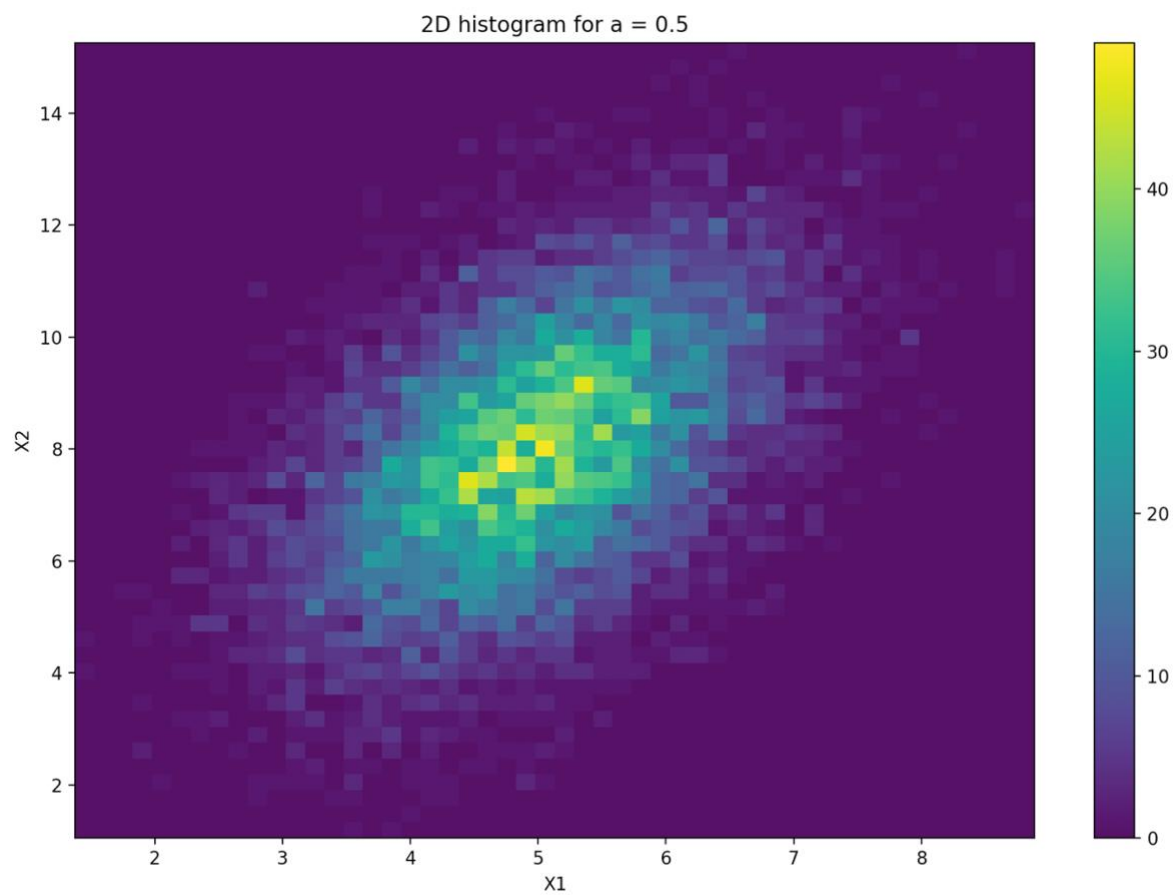
1) $a = -0.5$



2) $a = 0$



3) $a = 0.5$



4) $a = 1$

