MA323 - Monte Carlo Simulation

Lab - 2

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1. **Consider the recursion:**

$$\mathbf{U_{i+1} = (U_{i-17} - U_{i-5})}$$

In the event that $U_i < 0$, set $U_i = U_i + 1$.
(a) Use linear congruence generator to generate the first 17 values of U_i.

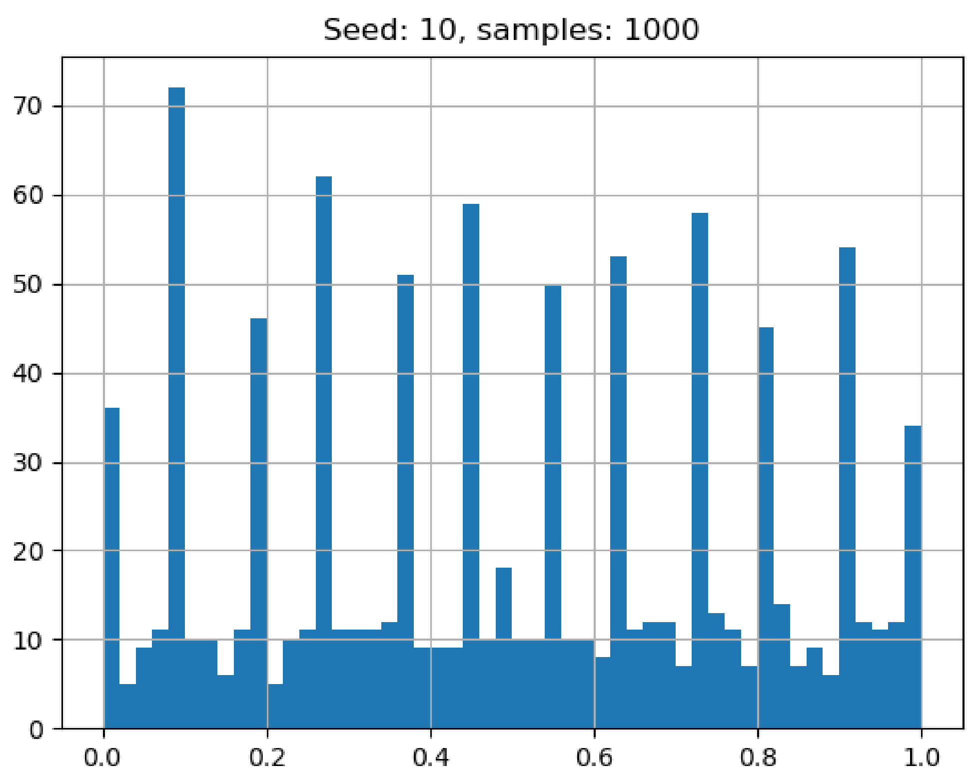
For a = 6, b = 0 and $x_0 = 10$

x_0/i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
10	10	5	8	4	2	1	6	3	7	9	10	5	8	4	2	1	6	3

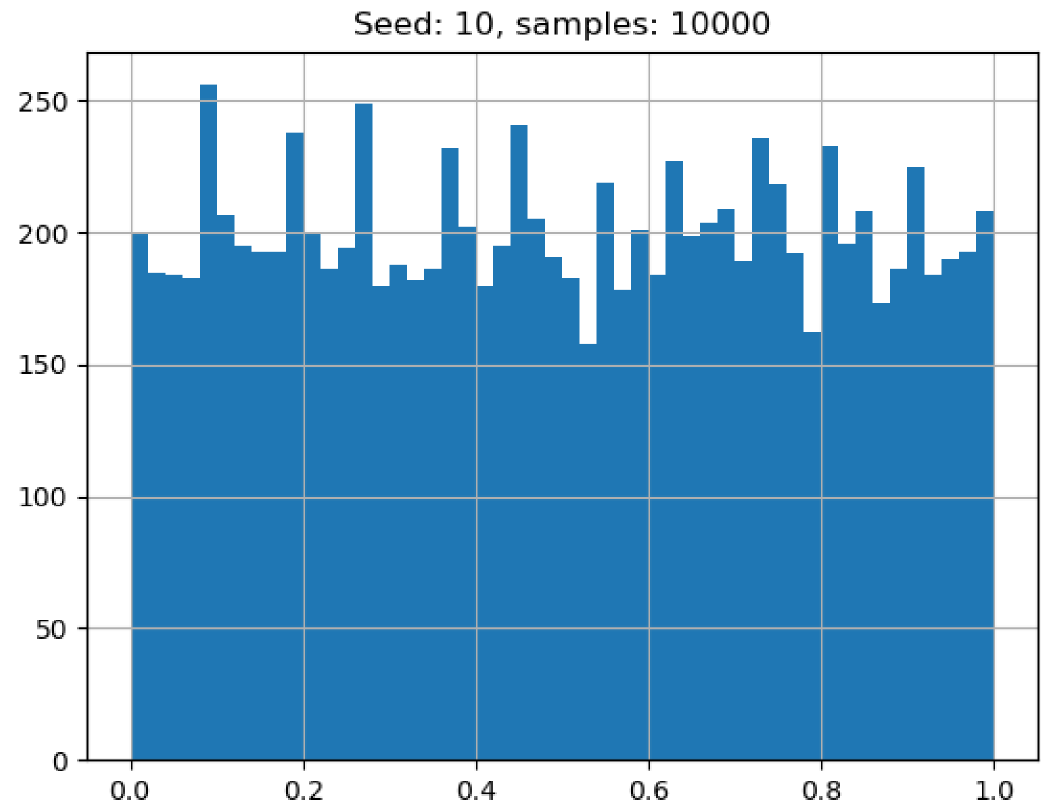
(b) Then generate the values of U_{18}, U_{19}, ..., U_N for N = 1,000, 10,000, and 100,000 based on the recursion above.

(c) For each N, plot histogram. What are your observations?

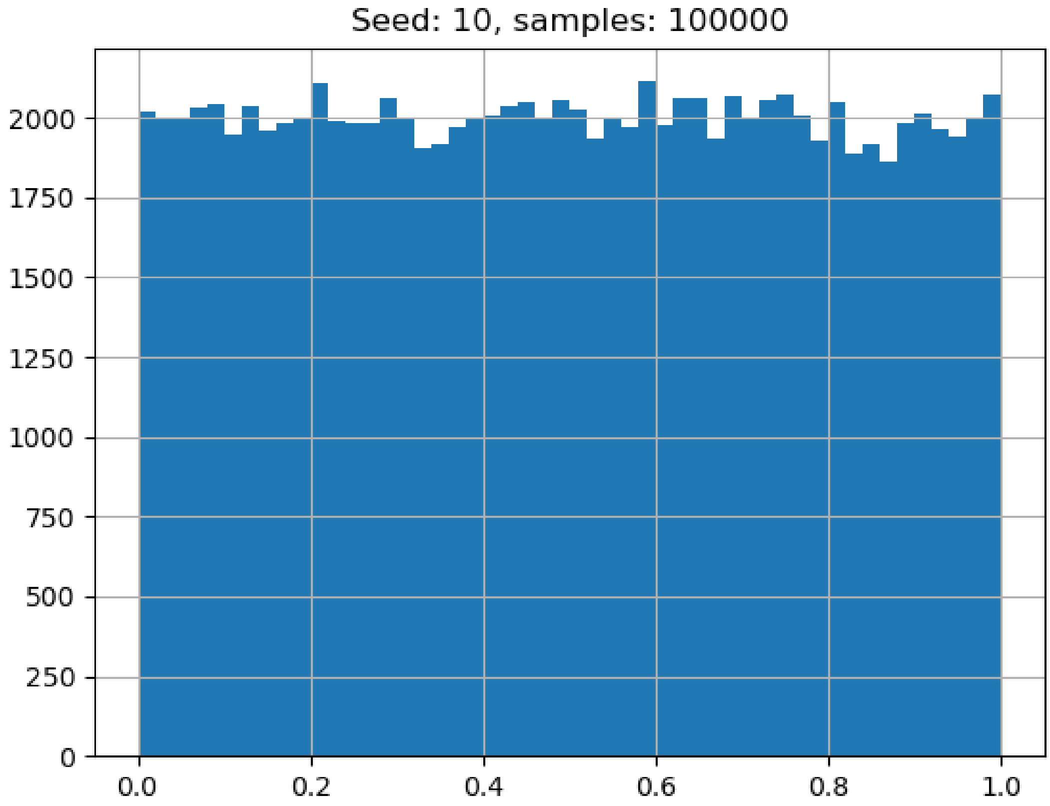
N = 1,000



N = 10,000

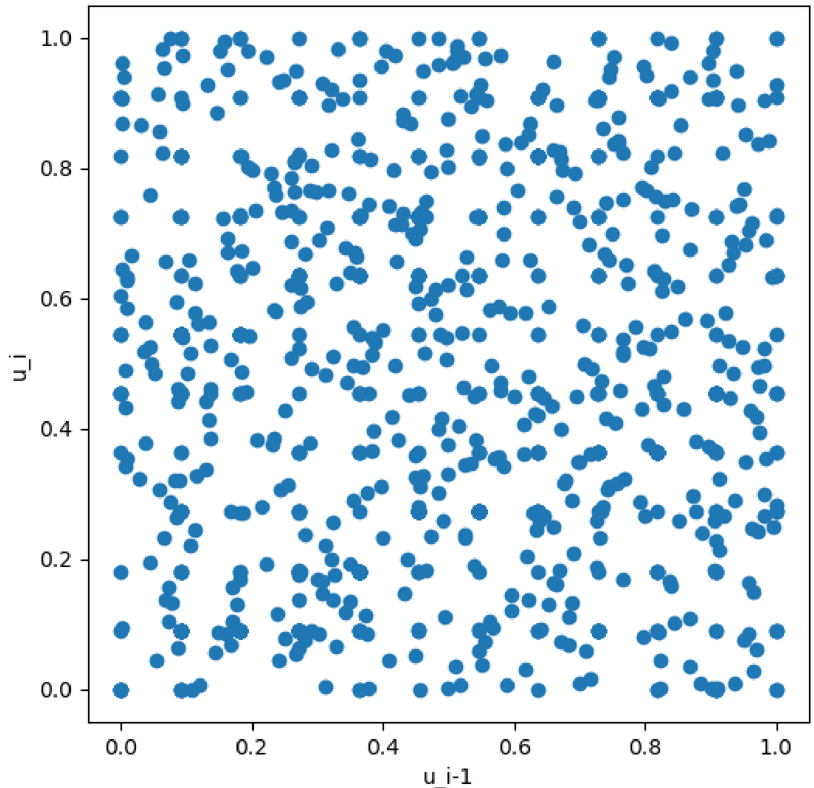


N = 100,000

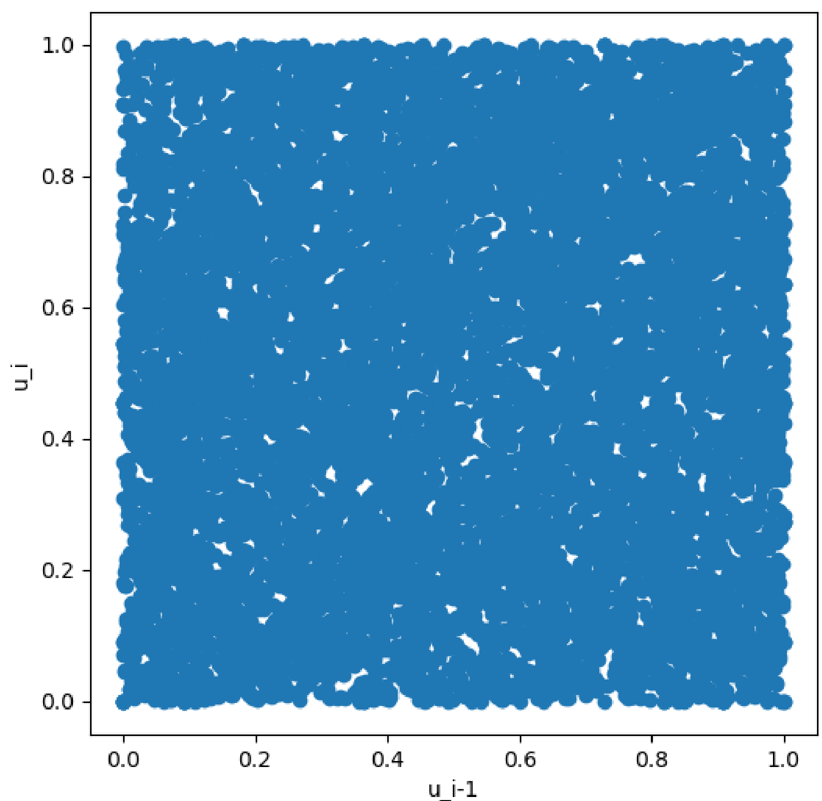


(d) For each N, plot (U_i, U_{i+1}). What are your observations?

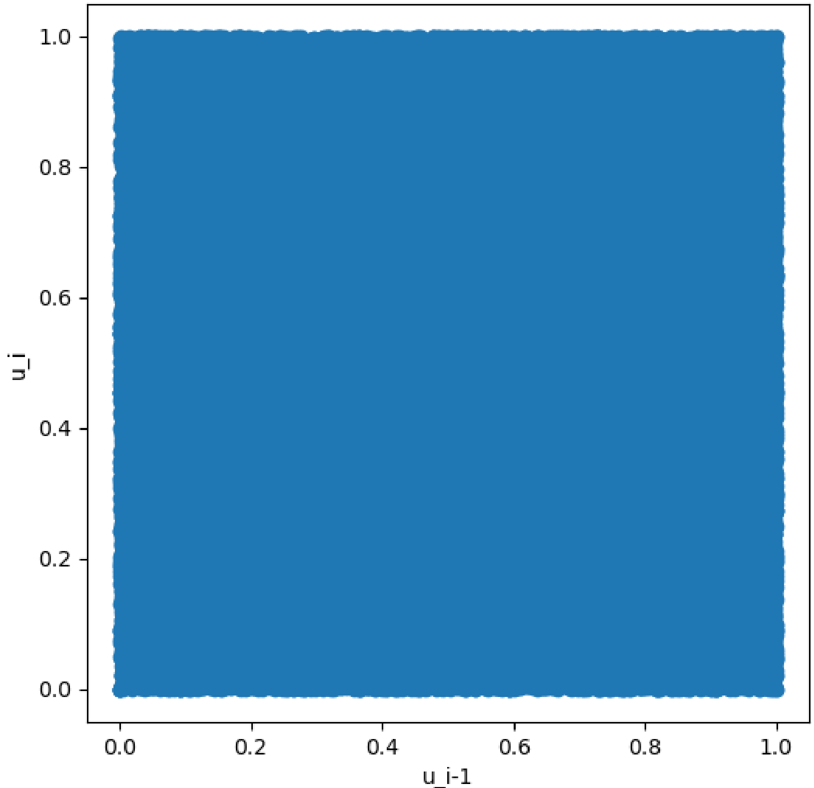
N = 1,000



N = 10,000



N = 100,000



Observations: Two plots each having 1,000, 10,000, and 100,000 samples. The values used for generating the first 17 numbers using Linear Congruence Generator (LCG) are : a = 6, b = 0, m = 11, x0 = 10. Next, Lagged Fibonacci Generator (LFG) is used to generate further sequences of numbers.

Histograms:

1) It can be concluded that the numbers generated by LFG are not as uniform as in [0, 1) as the numbers generated by LCG.
2) As the sample size is increased, the distribution of the ‘randomly generated numbers’ becomes more uniform.

Scatterplots:

1) It is observed that the (U_i, U_{i+1}) plot has not definite pattern as there was with LCG. This implies that the sequence of numbers generated by LFG is more random than LCG.
2) The density of the plots on increasing the sample size implies that the uniformity of LFG increases.

2. **Consider the exponential distribution with CDF**

$$F(x) = 1 - e^{-x/\theta},\ x \geq 0,$$

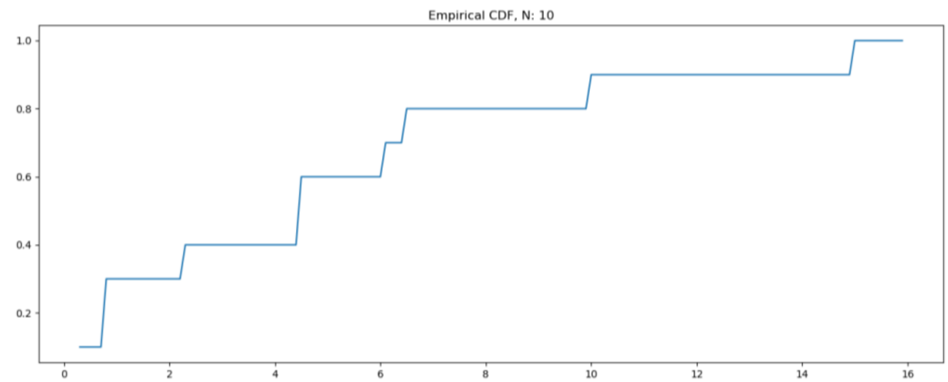
where $\theta > 0$.

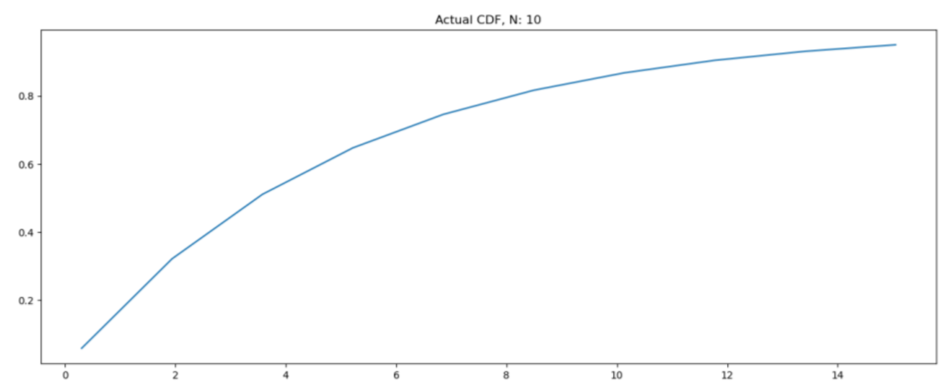
(a) Generate X_1, X_1, . . . , X_N from the above distribution for N = 10, 100, 1000, 10000, 100000.

(b) For each value of N, plot the empirical CDF of these generated values, and the actual distribution function (using the above formula).

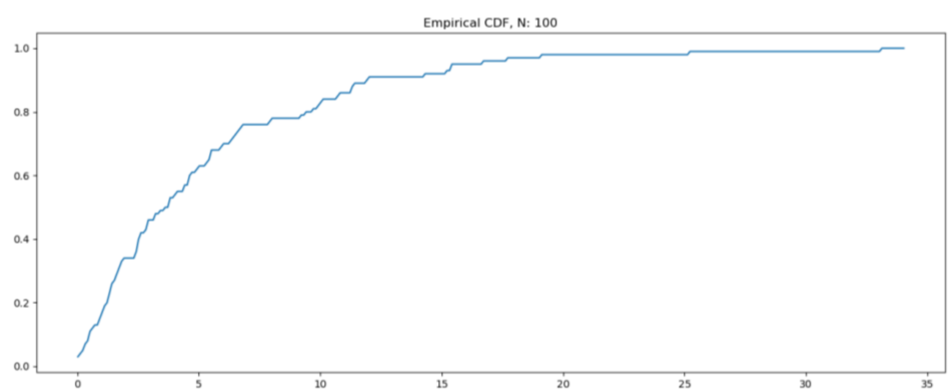
$\Theta = 5$

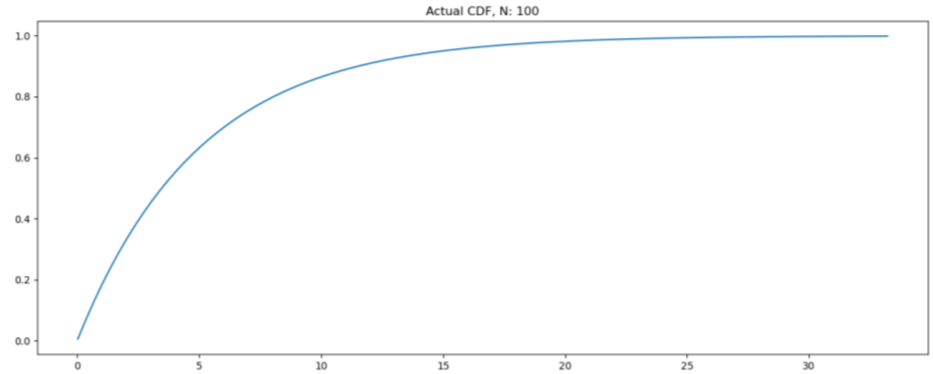
N = 10



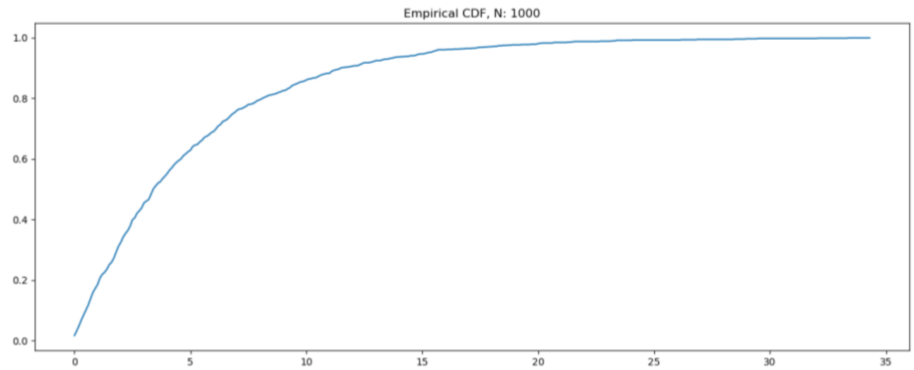


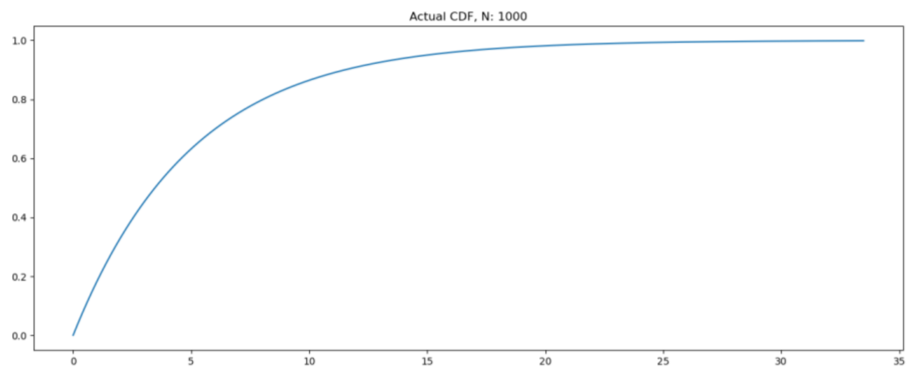
N = 100



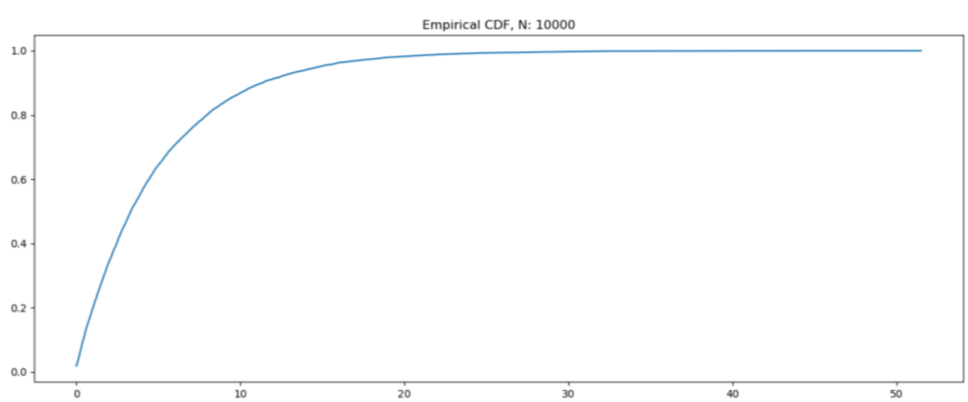


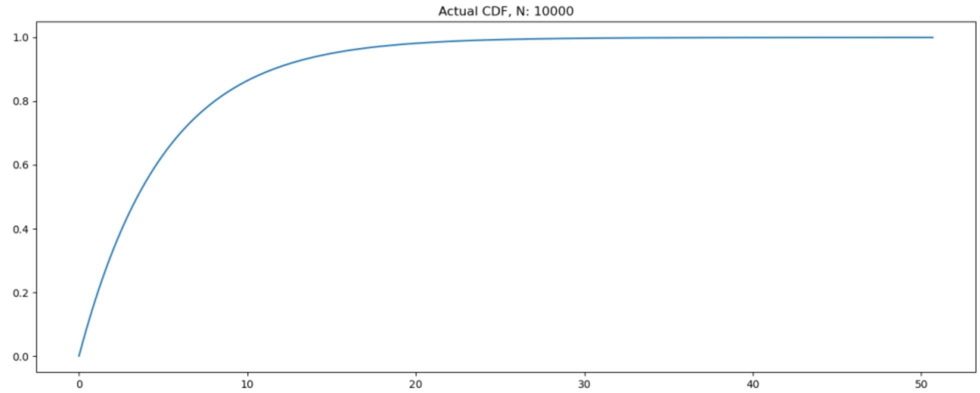
N = 1,000



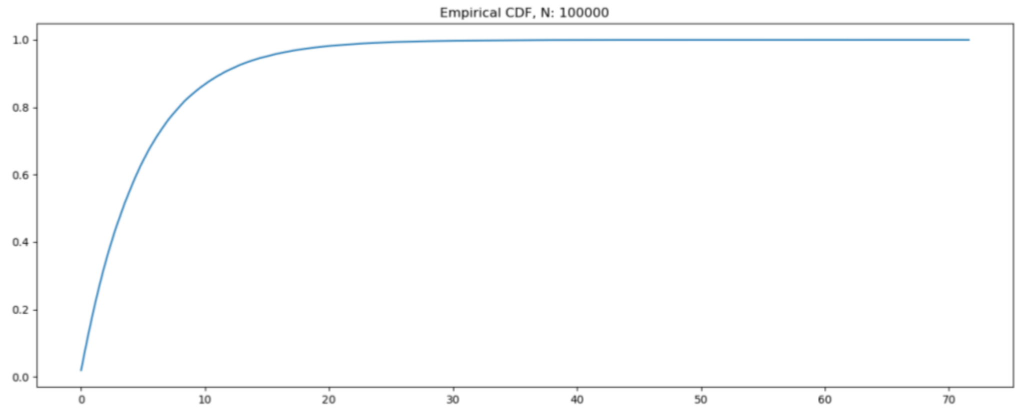


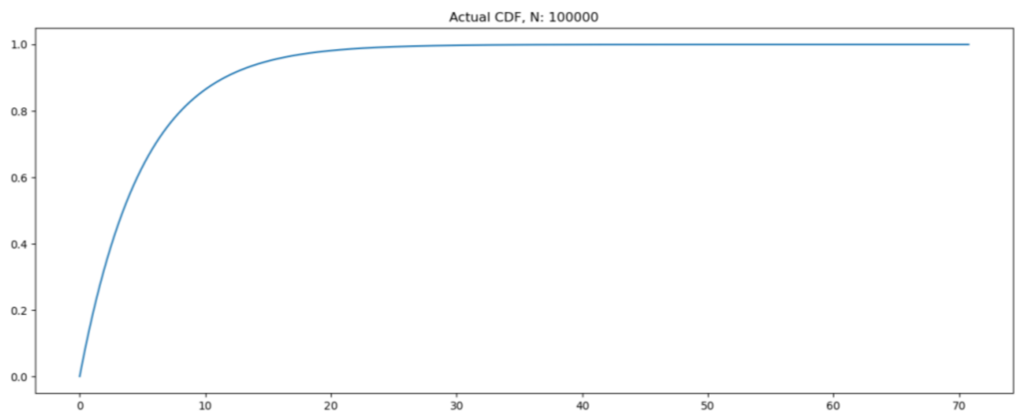
N = 10,000





N = 100,000





(c) Provide the corresponding values of the sample mean and variance. Compare the values of mean and variance to see whether they converge to actual values.

For Θ = 5, actual mean and variance are 5 and 25 respectively.

N	Mean	Variance
10.0	5.134850	19.421265
100.0	5.463334	31.870798
1000.0	5.119377	25.123556
10000.0	4.970293	24.806548
100000.0	4.985525	24.989454

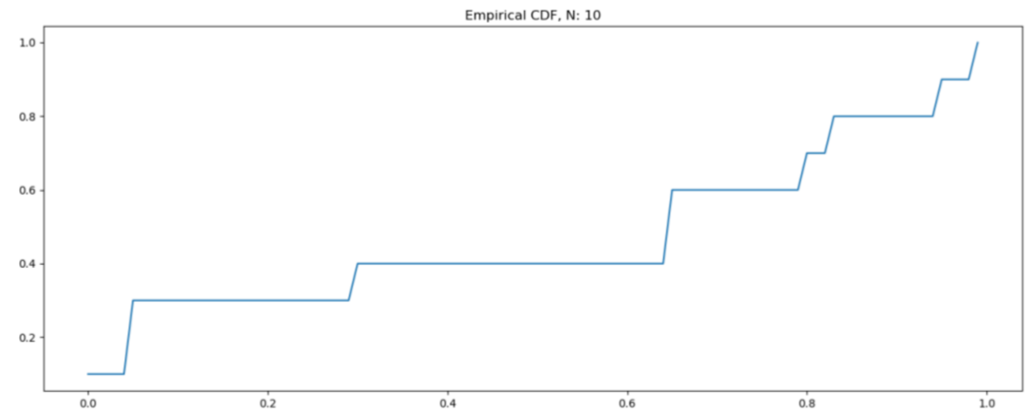
3. **Consider the Arcsin law with the distribution:**

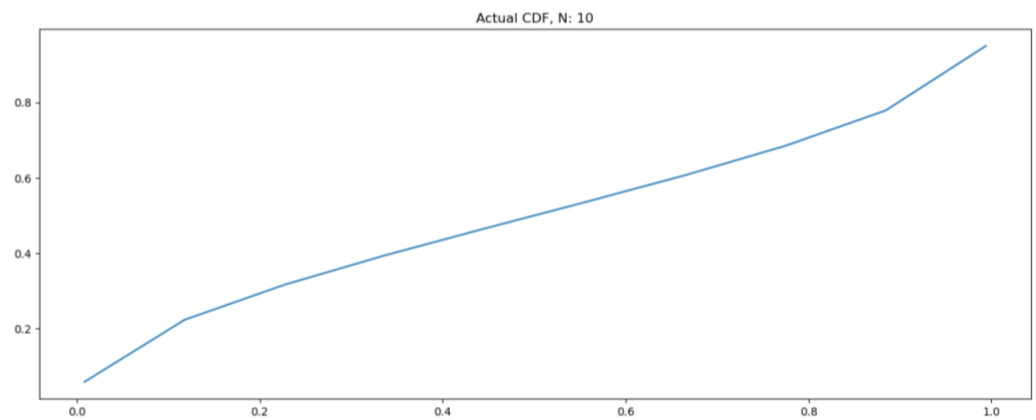
$$F(x) = \frac{2}{\pi} \text{ arc sin } \sqrt{x}\ ,\ 0 \leq x \leq 1.$$

(a) Generate X_1, X_2, . . . , X_N from the above distribution for N = 10, 100, 1000, 10000, 100000.

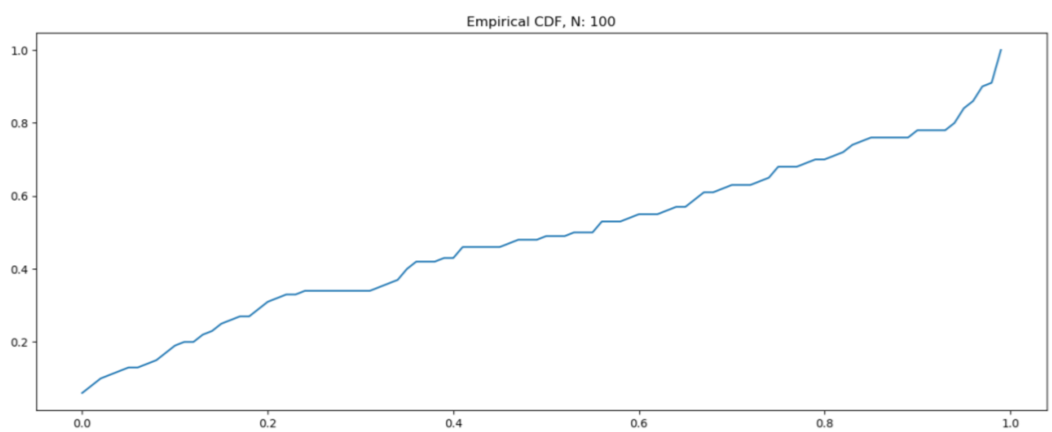
(b) For each value of N, plot the empirical CDF of these generated values, and the actual distribution function (using the above formula).

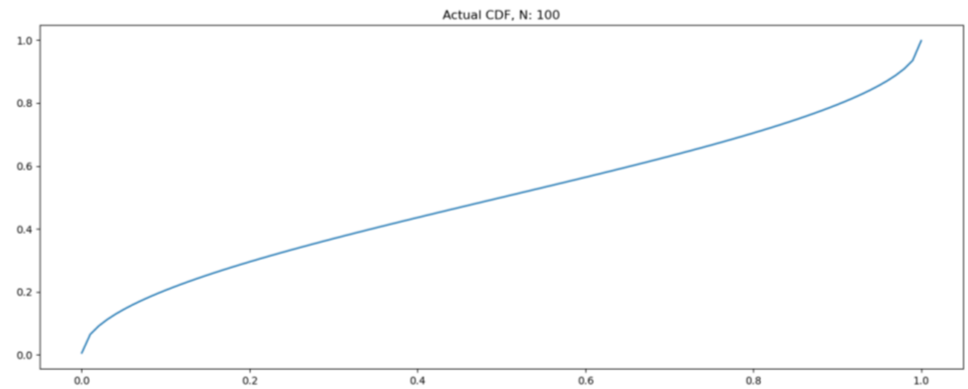
N = 10



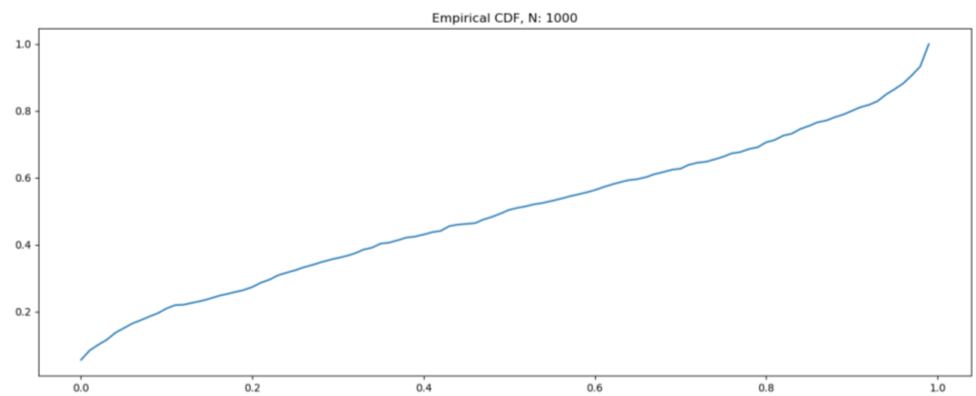


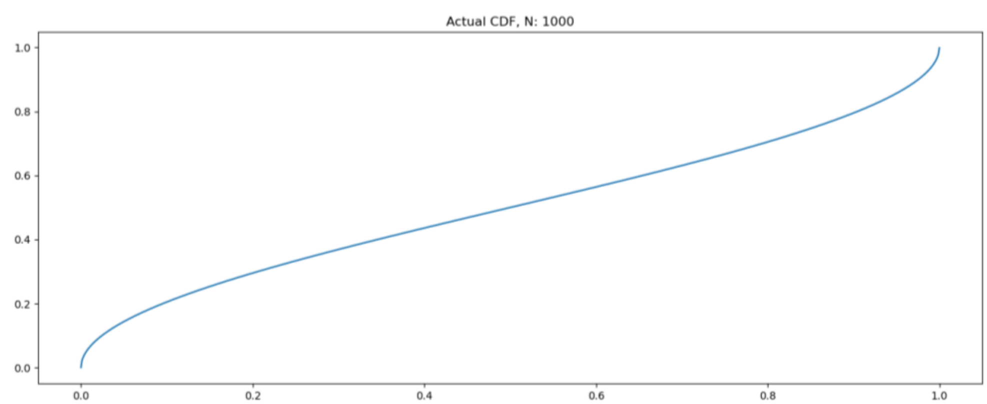
N = 100



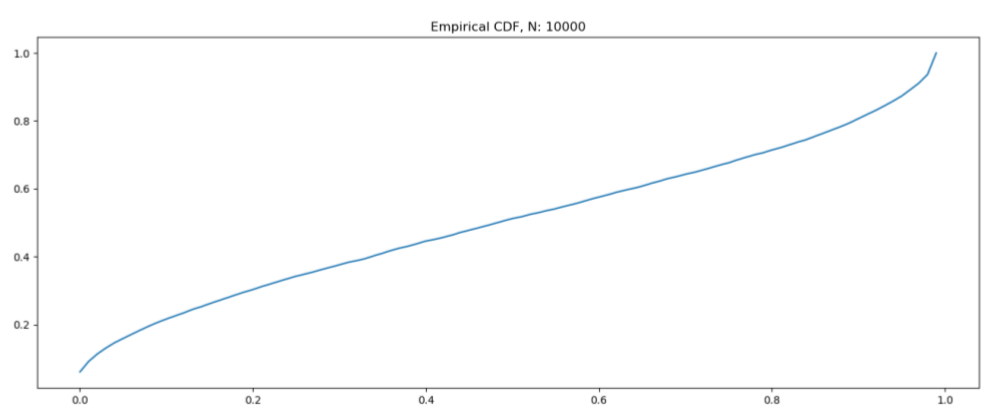


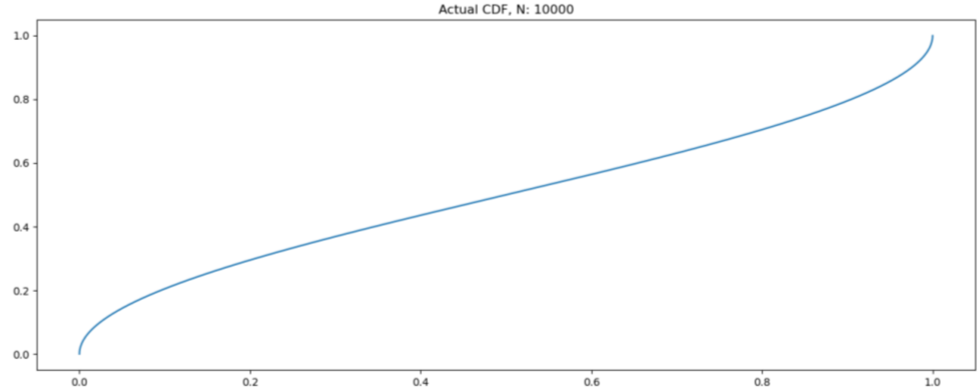
N = 1,000



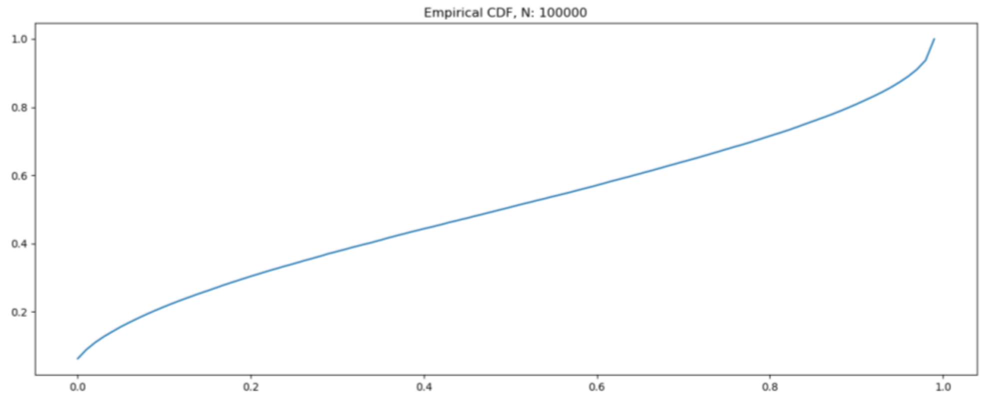


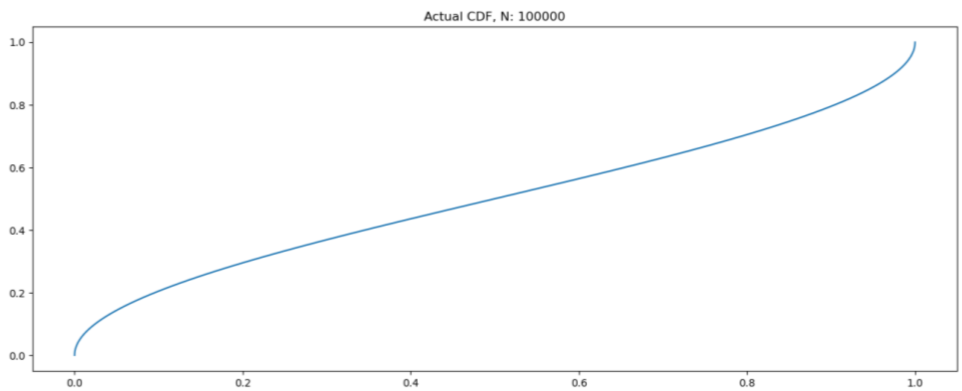
N = 10,000





N = 100,000





(c) Provide the corresponding values of the sample mean and variance.

N	Mean	Variance
10.0	0.533022	0.136034
100.0	0.517740	0.125749
1000.0	0.510495	0.122916
10000.0	0.497866	0.124823
100000.0	0.499259	0.124405

4. Using the algorithm to generate random variables from a discrete distribution, generate 100000 random numbers from a discrete uniform distribution on {1, 3, 5, . . . , 9999}. Tabulate the frequency of each observed values.