

Lab - 6

The exact value of I where,

$$I = E[\exp(\sqrt{U})]$$

$$U \sim U(0, 1)$$

would be calculated as,

$$\int_0^1 e^{\sqrt{x}} dx$$

Solving this integral we get the value 2.

The exact value of I is 2.

Monte Carlo Estimator I_M is used to approximate the expected value I, it is given by,

$$I_M = \frac{1}{M} \sum_{i=1}^M Y_i, \text{ where } Y_i = \exp(\sqrt{U_i}) \text{ with } U_i \sim U(0, 1).$$

The estimator was run for different values of sample size M. The observations are tabulated below.

Sample Size	95% Confidence Interval	Interval Length	I_M
100	(1.86353, 2.04259)	0.17906	1.95306
1,000	(1.95704, 2.01242)	0.05538	1.98472
10,000	(1.98279, 2.00003)	0.01724	1.99140
100,000	(1.99655, 2.00201)	0.00546	1.99928

Observations: As the sample size increases,

- the length of the 95% confidence interval for the estimation of I_M converges to 0.
- the estimated value I_M converges to the exact value I: 2.