Scientific Computing (MA322)

Lab 08

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Rectangle method uses f(a) as height of the rectangle and the interval length as the width.

Q1a Rectangle method: 3.125000e-02
Q1b Rectangle method: -2.500000e-01
Q1c Rectangle method: -4.000000e-01
Q1d Rectangle method: 0
Q1e Rectangle method: 2.431953e-01

Midpoint method uses f((a + b) * 0.5) as height of the rectangle and the interval length as the width.

Q21a Midpoint method: 1.582031e-01
Q21b Midpoint method: -2.666667e-01
Q21c Midpoint method: -6.753247e-01
Q21d Midpoint method: 1.803915e+00
Q21e Midpoint method: -1.189526e-02

Q22a Trapezoidal method: 2.656250e-01
Q22b Trapezoidal method: -2.678571e-01
Q22c Trapezoidal method: -8.666667e-01
Q22d Trapezoidal method: 4.143260e+00
Q22e Trapezoidal method: -3.702425e-02

Q23a Simpson method: 1.940104e-01
Q23b Simpson method: -2.670635e-01
Q23c Simpson method: -7.391053e-01
Q23d Simpson method: 2.583696e+00
Q23e Simpson method: -2.027159e-02

Q3a Rectangle method: 4
Q3b Trapezoidal method: 3
Q3c Simpson 1/3 method: 3.133333e+00
Q3d Simpson 3/8 method: 3.138462e+00

Q4 Composite Trapezoidal method: 7.125000e+00

N values are found so as have absolute errors less than 10^{-5}.

Q5 Composite Trapezoidal method with n = 92: 6.363041e-01
Corresponding h value: 1.298701e-02

Theorem 4.5 Let $f \in C^2[a,b]$, $h = (b-a)/n$, and $x_j = a + jh$, for each $j = 0, 1, \ldots, n$. There exists a $\mu \in (a,b)$ for which the **Composite Trapezoidal rule** for n subintervals can be written with its error term as

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right] - \frac{b-a}{12}h^2 f''(\mu). \quad \blacksquare$$

Q5 Composite Simpson method with n = 6: 6.362944e-01
Corresponding h value: 9.433962e-03

Theorem 4.4 Let $f \in C^4[a,b]$, n be even, $h = (b-a)/n$, and $x_j = a + jh$, for each $j = 0, 1, \ldots, n$. There exists a $\mu \in (a,b)$ for which the **Composite Simpson's rule** for n subintervals can be written with its error term as

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{(n/2)-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right] - \frac{b-a}{180}h^4 f^{(4)}(\mu). \quad \blacksquare$$

Q5 Composite Midpoint method with n = 128: 6.362845e-01
Corresponding h value: 9.259259e-03

Theorem 4.6 Let $f \in C^2[a,b]$, n be even, $h = (b-a)/(n+2)$, and $x_j = a + (j+1)h$ for each $j = -1, 0, \ldots, n+1$. There exists a $\mu \in (a,b)$ for which the **Composite Midpoint rule** for $n+2$ subintervals can be written with its error term as

$$\int_a^b f(x)\,dx = 2h\sum_{j=0}^{n/2} f(x_{2j}) + \frac{b-a}{6}h^2 f''(\mu). \quad \blacksquare$$

The values for n were calculated by substituting values of a and b into the error terms.

Q6 Composite Trapezoidal method: 9855 feet.