DEPARTMENT OF MATHEMATICS, I.I.T. GUWAHATI

MA 322: Scientific Computing Lab - 8

1. Approximate the following integrals using the Rectangle rule:

   a. $\int_{0.5}^{1} x^4 dx$ *b.* $\int_{0}^{0.5} \frac{2}{x-4} dx$ *c.* $\int_{1}^{1.6} \frac{2x}{x^2-4} dx$

   d. $\int_{0}^{\pi/4} e^{3x} \sin 2x dx$ *e.* $\int_{0.75}^{1.3} \left((\sin x)^2 - 2x \sin x + 1\right) dx.$

2. Use the Midpoint rule, Trapezoidal rule and Simpson's rule to approximate the integrals given in Exercise 1.

3. Compute π from an integral of the form $\int_0^1 \frac{4}{1+x^2} dx$ by using Rectangle, Trapezoidal, Simpson's one-third and three-eighth rules. Compare and explain these numerical results to the true solution. Simpson's three-eighth rule is given by

$$\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right] - \frac{3h^5}{80} f^4(\xi), \quad \text{where } x_0 < \xi < x_3.$$

4. We want to approximate $\int_1^2 f(x)dx$ given the table of the values

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
$f(x)$	10	8	7	6	5

   Compute an estimate by the composite trapezoid rule.

5. Determine the value of n and h required to approximate

$$\int_1^2 x \ln x \, dx$$

   to within 10^{-5} and compute the approximation. Use (a) Composite Trapezoidal rule (b) Composite Simpson's rule (c) Composite Midpoint rule.

6. A car laps a race track in 84 seconds. The speed of the car at each 6-second interval is determined by using a radar gun and is given from the beginning of the lap, in feet/second, by the entries in the following table.

Time	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

   How long is the track?