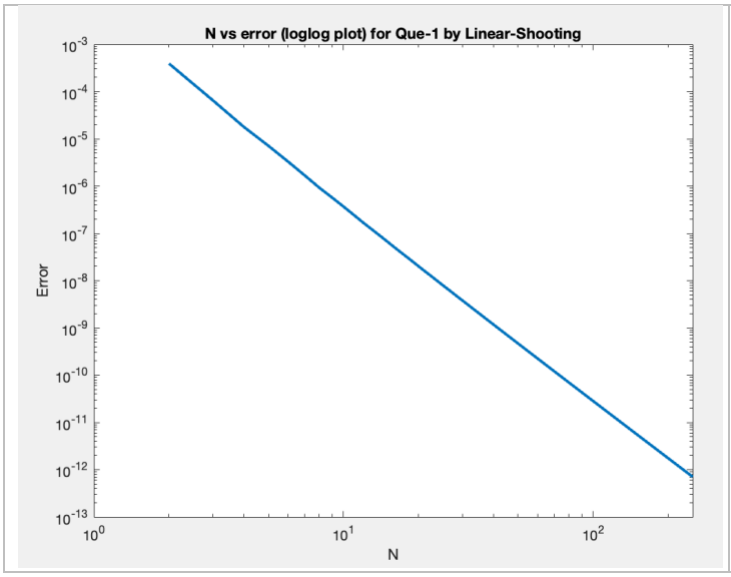
Scientific Computing (MA322)

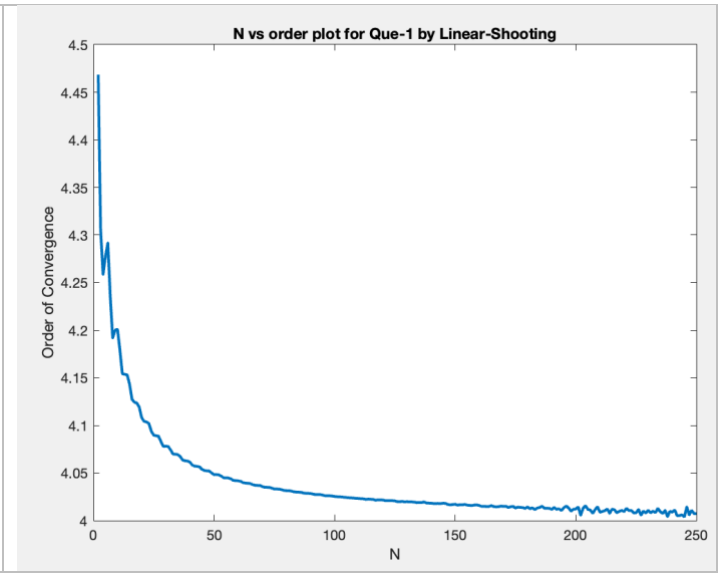
Lab 13

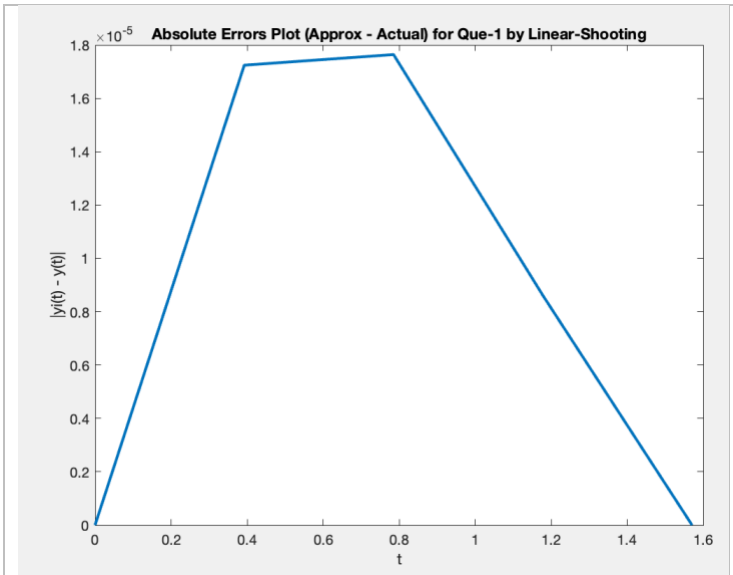
Naveen Kumar A G 210123075

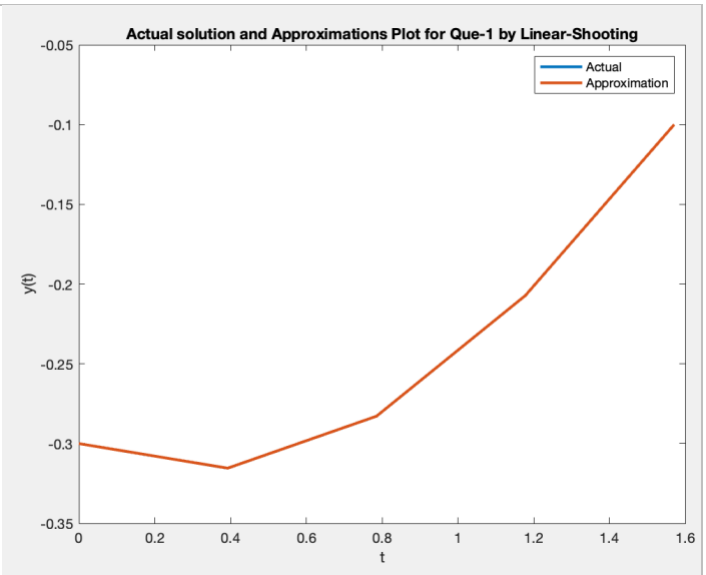
Q1

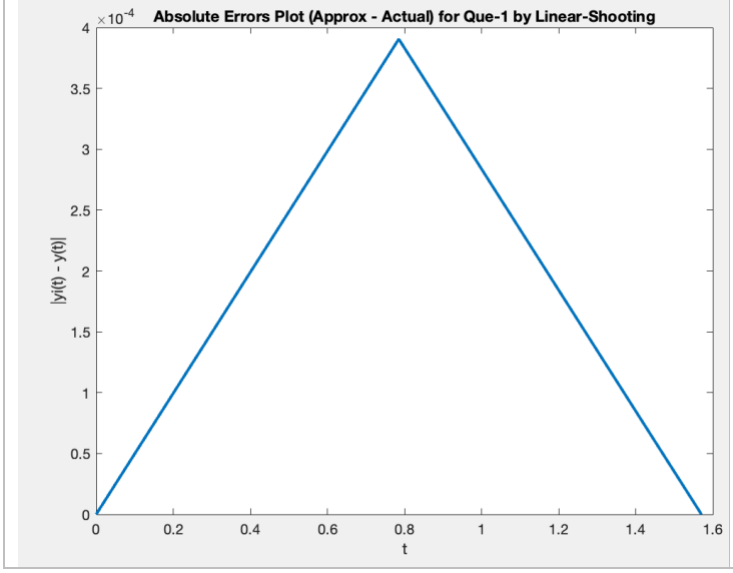
```
 Que-1 by Linear-Shooting
For h = 7.853982e-01
t                       Approximation           Exact                   Error(|Exact - Approx.|)
0.000000                -0.3000000              -0.3000000              0.0000000
0.785398                -0.2824522              -0.2828427              0.0003905
1.570796                -0.1000000              -0.1000000              0.0000000

For h = 3.926991e-01
t                       Approximation           Exact                   Error(|Exact - Approx.|)
0.000000                -0.3000000              -0.3000000              0.0000000
0.392699                -0.3154150              -0.3154322              0.0000172
0.785398                -0.2828251              -0.2828427              0.0000176
1.178097                -0.2071844              -0.2071930              0.0000086
1.570796                -0.1000000              -0.1000000              0.0000000
```

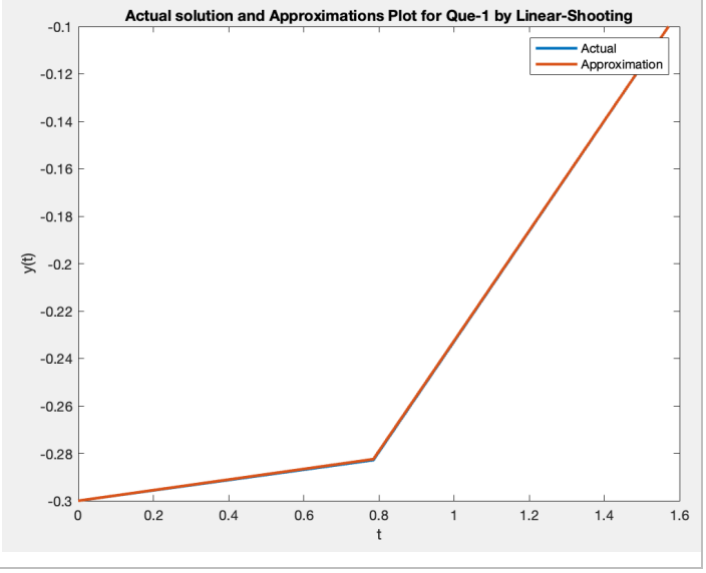





Absolute Errors Plot (Approx - Actual) for Que-1 by Linear-Shooting
×10^-5
|y(t) - y(t)|
t


Actual solution and Approximations Plot for Que-1 by Linear-Shooting
Actual
Approximation
y(t)
t

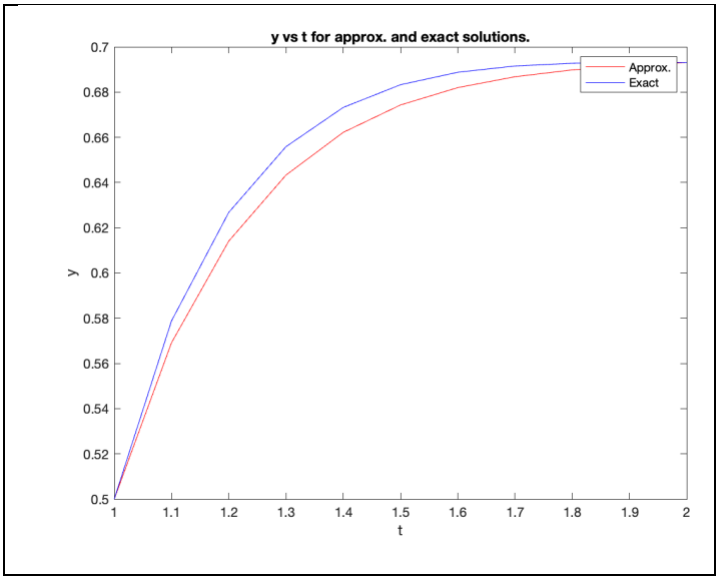

Absolute Errors Plot (Approx - Actual) for Que-1 by Linear-Shooting
×10^-4
|y(t) - y(t)|
t

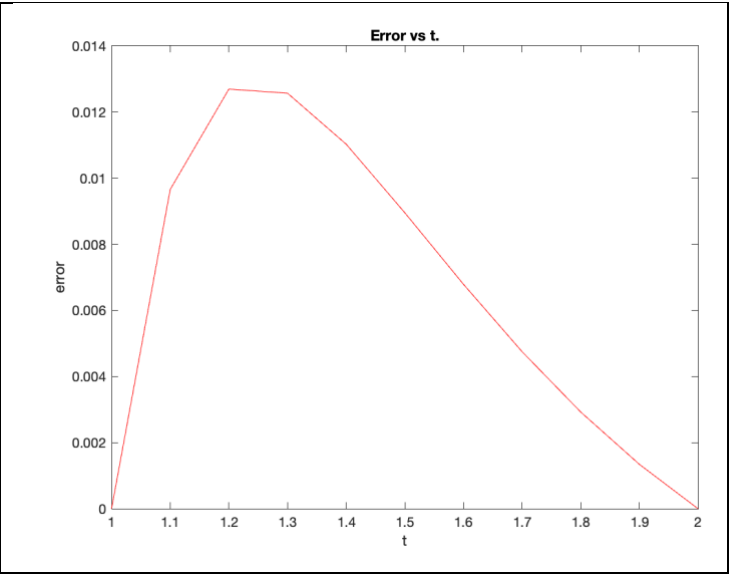

Actual solution and Approximations Plot for Que-1 by Linear-Shooting
Actual
Approximation
y(t)
t

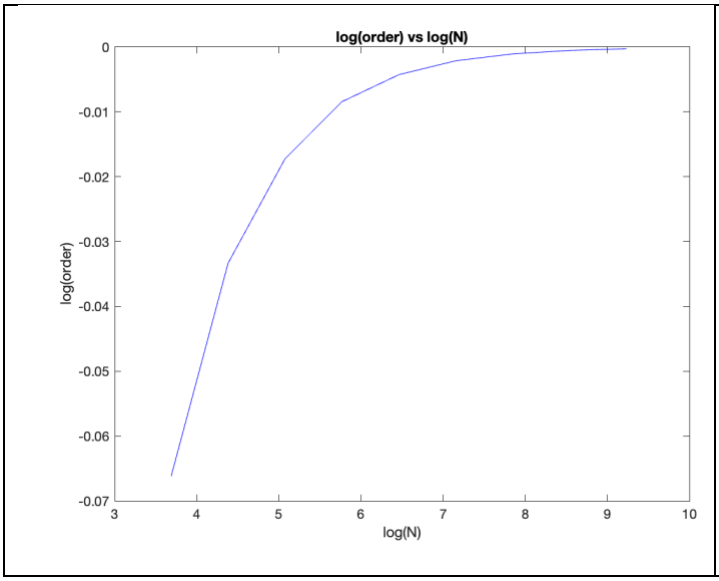
Q2 (a)
Forward difference for 1st order and central for 2nd order.

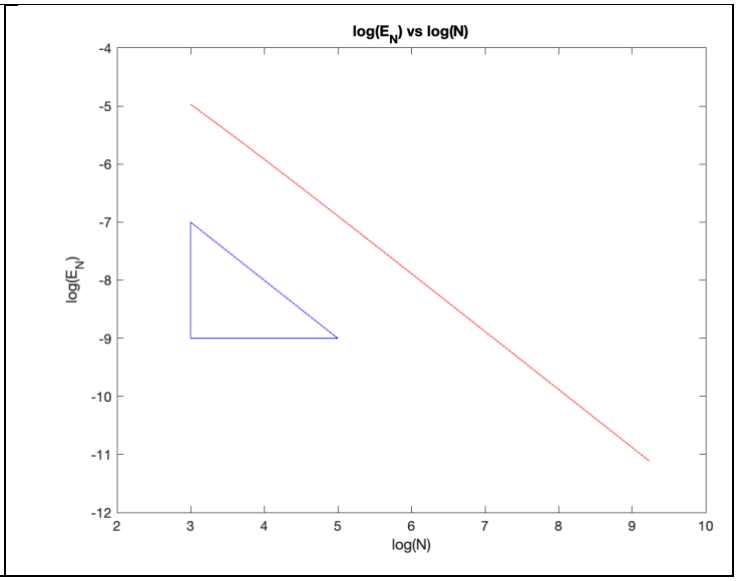
t_i	Approx.	Exact	Error
1	0.500000000000000	0.500000000000000	0
1.10000000000000	0.569125339656444	0.578781254184490	0.00965591452804637
1.20000000000000	0.614074041345689	0.626766001238399	0.0126919598927101
1.30000000000000	0.643288145898542	0.655855388727846	0.0125672428293039
1.40000000000000	0.662181068552706	0.673206930498764	0.0110258619460581
1.50000000000000	0.674290573220773	0.683242885885942	0.00895231266516916
1.60000000000000	0.681964213993019	0.688753629245736	0.00678941525271715
1.70000000000000	0.686778372956144	0.691527905041409	0.00474953208526430
1.80000000000000	0.689800758660782	0.692724936507057	0.00292417784627586
1.90000000000000	0.691758396996097	0.693100423568517	0.00134202657241922
2	0.693147180559945	0.693147180559945	1.11022302462516e-16

N	E_N	Order
20	0.00696950490740000	0
40	0.00364290933759404	0.935965084378142
80	0.00186332593698713	0.967211034974386
160	0.000942764825191289	0.982910218312758
320	0.000474133787627506	0.991603726478728
640	0.000237767885116980	0.995740339771056
1280	0.000119058815708506	0.997879417103730
2560	5.95732864496767e-05	0.998936994074685
5120	2.97976173557757e-05	0.999468573602959
10240	1.49015850755552e-05	0.999731178373621





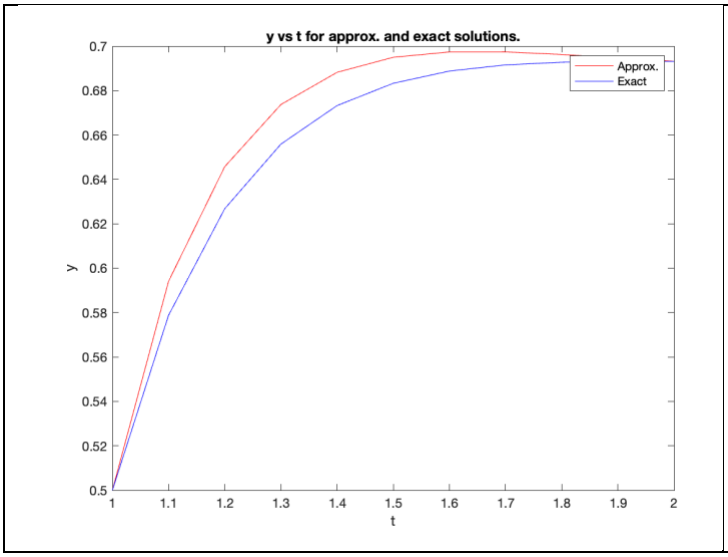

log(order) vs log(N)
log(order)
log(N)
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
3
4
5
6
7
8
9
10

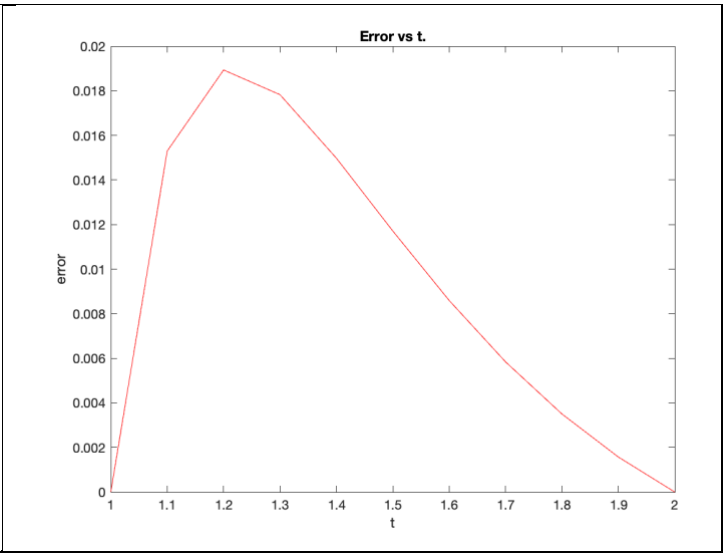

log(E_N) vs log(N)
log(E_N)
log(N)
-4
-5
-6
-7
-8
-9
-10
-11
-12
2
3
4
5
6
7
8
9
10

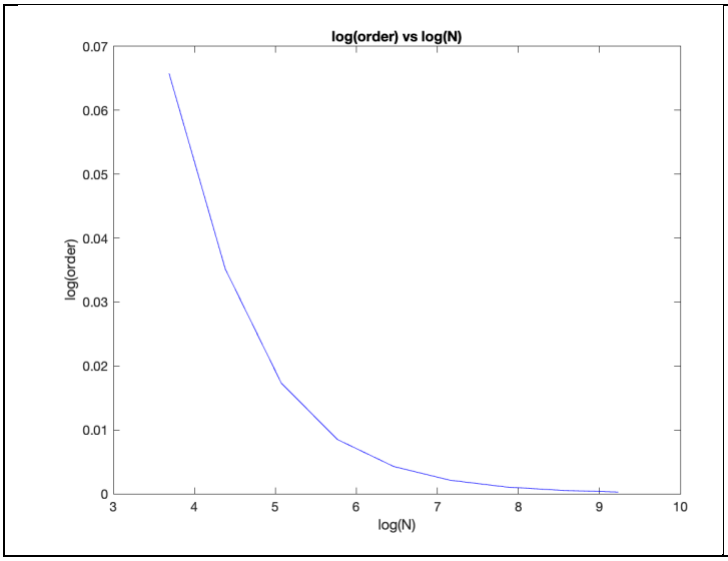
Backward difference for 1st order and central for 2nd order.

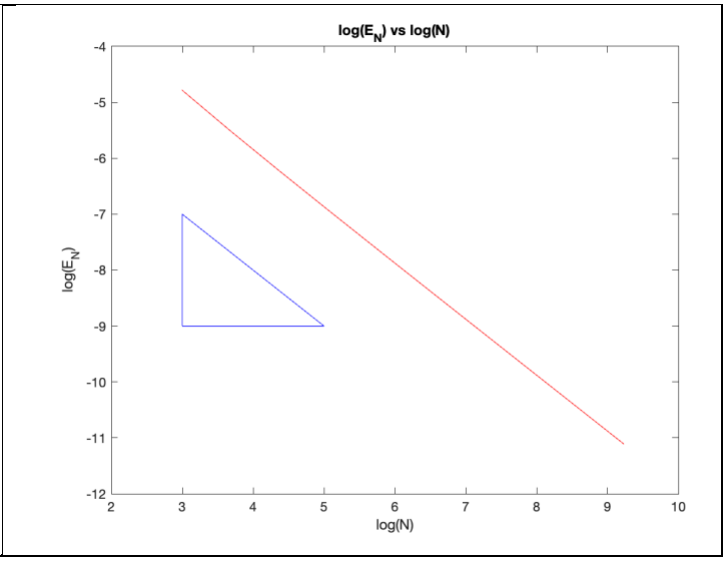
t_i	Approx.	Exact	Error
1	0.500000000000000	0.500000000000000	0
1.10000000000000	0.594078168494148	0.578781254184490	0.0152969143096580
1.20000000000000	0.645701994912906	0.626766001238399	0.0189359936745072
1.30000000000000	0.673682039773760	0.655855388727846	0.0178266510459133
1.40000000000000	0.688185174082087	0.673206930498764	0.0149782435833227
1.50000000000000	0.694955648205781	0.683242885885942	0.0117127623198382
1.60000000000000	0.697347413317844	0.688753629245736	0.00859378407210820
1.70000000000000	0.697365113838828	0.691527905041409	0.00583720879741945
1.80000000000000	0.696224761207632	0.692724936507057	0.00349982470057419
1.90000000000000	0.694668449307408	0.693100423568517	0.00156802573889092
2	0.693147180559945	0.693147180559945	1.11022302462516e-16

N	E_N	Order
20	0.00839852144945330	0
40	0.00400604239595048	1.06795767608186
80	0.00195391259597533	1.03581175634541
160	0.000965216765105637	1.01744105349279
320	0.000479767668177589	1.00851703747491
640	0.000239173832248252	1.00427638239128
1280	0.000119410432469658	1.00213066915492
2560	5.96612002180530e-05	1.00106397919762
5120	2.98195915130473e-05	1.00053250615211
10240	1.49070367578830e-05	1.00026698970564





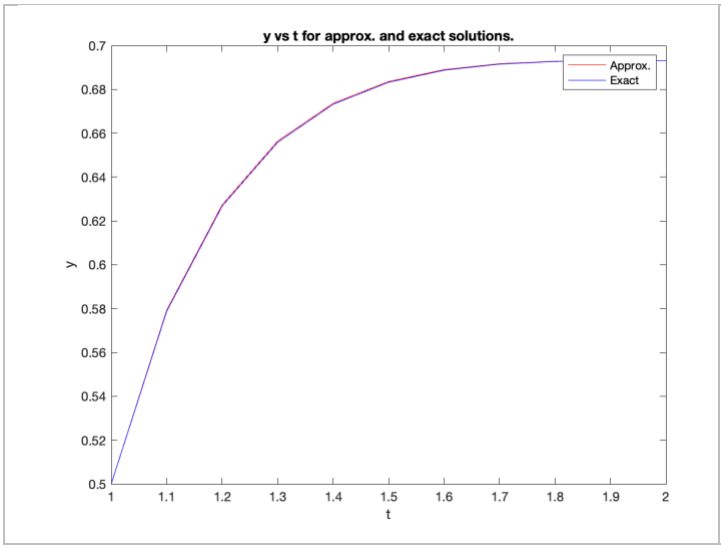

log(order) vs log(N)
log(order)
log(N)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
3
4
5
6
7
8
9
10

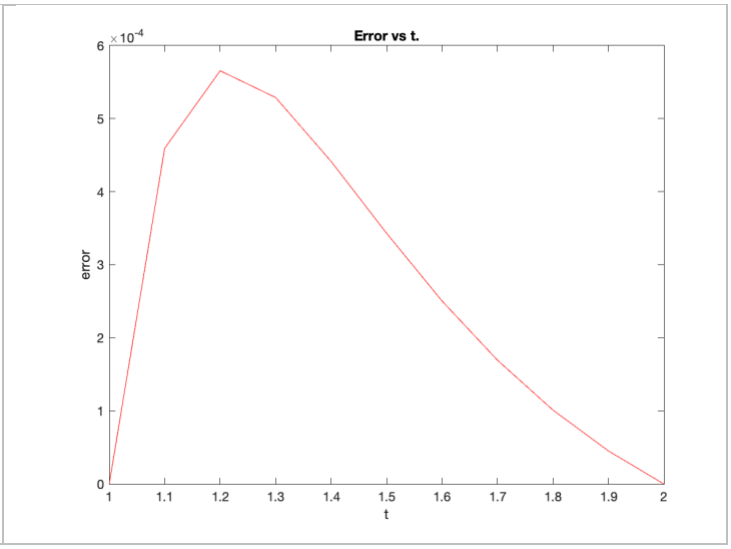

log(E_N) vs log(N)
log(E_N)
log(N)
-4
-5
-6
-7
-8
-9
-10
-11
-12
2
3
4
5
6
7
8
9
10

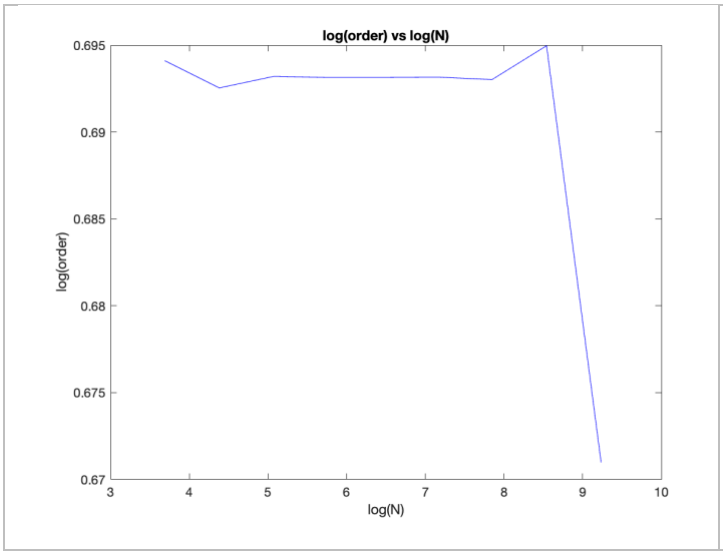
Central difference for 1st order and central for 2nd order.

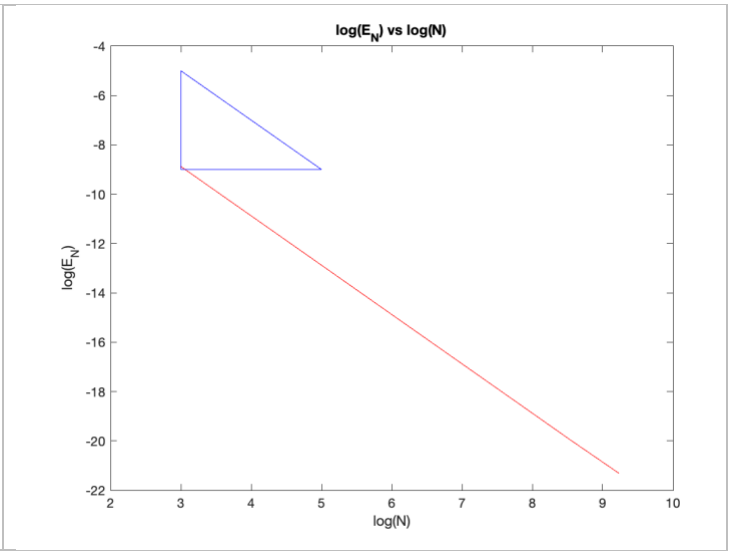
t_i	Approx.	Exact	Error
1	0.500000000000000	0.500000000000000	0
1.10000000000000	0.579240771919053	0.578781254184490	0.000459517734562875
1.20000000000000	0.627331507833436	0.626766001238399	0.000565506595037069
1.30000000000000	0.656384295974192	0.655855388727846	0.000528907246346022
1.40000000000000	0.673648442851704	0.673206930498764	0.000441512352939988
1.50000000000000	0.683586051168494	0.683242885885942	0.000343165282551516
1.60000000000000	0.689004057974978	0.688753629245736	0.000250428729241858
1.70000000000000	0.691697226958290	0.691527905041409	0.000169321916880905
1.80000000000000	0.692826081877647	0.692724936507057	0.000101145370589473
1.90000000000000	0.693145613496602	0.693100423568517	4.51899280852208e-05
2	0.693147180559945	0.693147180559945	1.11022302462516e-16

N	E_N	Order
20	0.000140622228987808	0
40	3.51087081846835e-05	2.00192384904064
80	8.78455961361713e-06	1.99878704546396
160	2.19596070005768e-06	2.00011772762190
320	5.48994835147809e-07	1.99998775365552
640	1.37249556053654e-07	1.99999109395122
1280	3.43116732803850e-08	2.00003009394828
2560	8.57943460719213e-09	1.99974500263662
5120	2.13944384430675e-09	2.00364676400961
10240	5.51366063916703e-10	1.95615343327164






log(order) vs log(N)
log(order)
log(N)
0.695
0.69
0.685
0.68
0.675
0.67
3
4
5
6
7
8
9
10

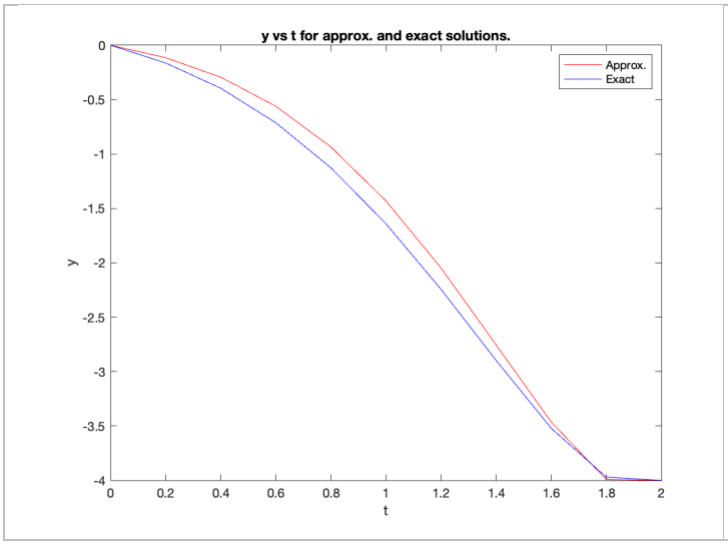

log(E_N) vs log(N)
log(E_N)
log(N)
-4
-6
-8
-10
-12
-14
-16
-18
-20
-22
2
3
4
5
6
7
8
9
10

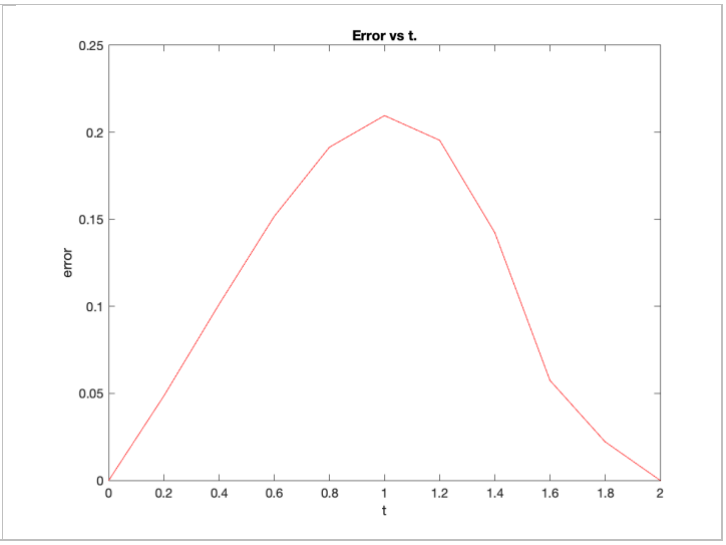
Q2 (b)

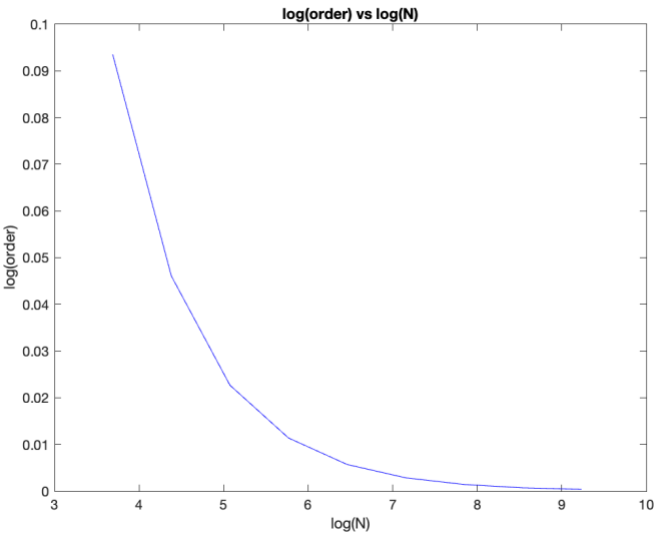
Forward difference for 1st order and central for 2nd order.

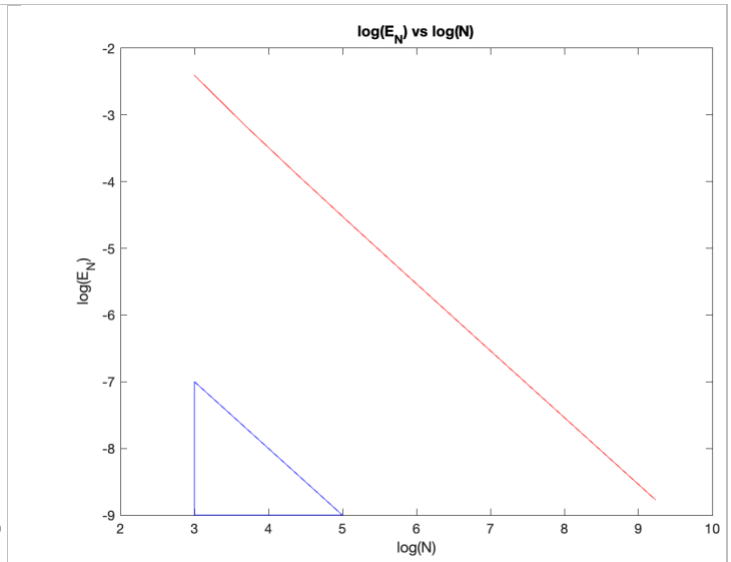
t_i	Approx.	Exact	Error
0	0	0	0
0.200000000000000	-0.114151716314232	-0.162700199388837	0.0485484830746048
0.400000000000000	-0.293842425641534	-0.394987606370133	0.101145180728598
0.600000000000000	-0.560622120873835	-0.712284922795433	0.151662801921598
0.800000000000000	-0.934995386187128	-1.12639322177366	0.191397835586536
1	-1.43125561977721	-1.64085908577048	0.209603465993268
1.20000000000000	-2.04838684587826	-2.24380632625187	0.195419480373614
1.40000000000000	-2.75477041250254	-2.89715520411126	0.142384791608722
1.60000000000000	-3.46327299913733	-3.52075148120968	0.0574784820723484
1.80000000000000	-3.99156898498401	-3.96939012900356	0.0221788559804521
2	-4	-4	0

N	E_N	Order
20	0.0905725641244310	0
40	0.0423115986106593	1.09802090667061
80	0.0204754448477529	1.04715839834291
160	0.0100761724403691	1.02294707886087
320	0.00499828328577423	1.01144314128927
640	0.00248924289131569	1.00572566066218
1280	0.00124215842400610	1.00285782455284
2560	0.000620465162297412	1.00142707188846
5120	0.000310079557779064	1.00071179023317
10240	0.000154998391351358	1.00038517577618





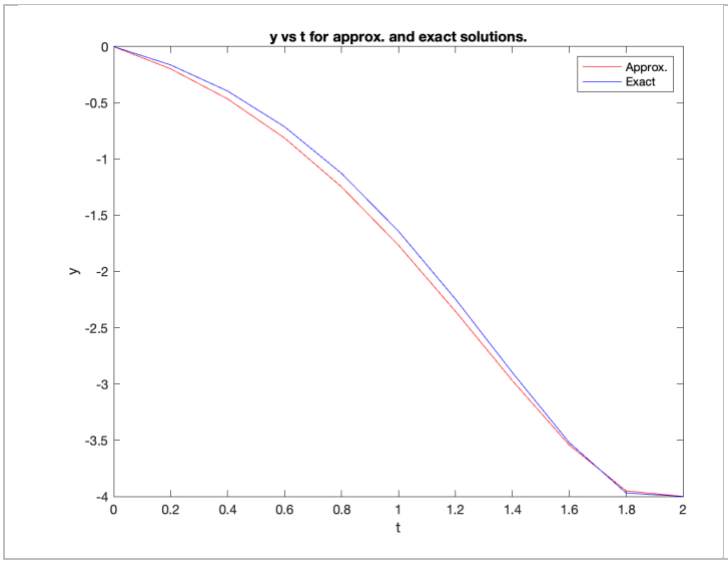

log(order) vs log(N)
log(order)
log(N)
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
3
4
5
6
7
8
9
10

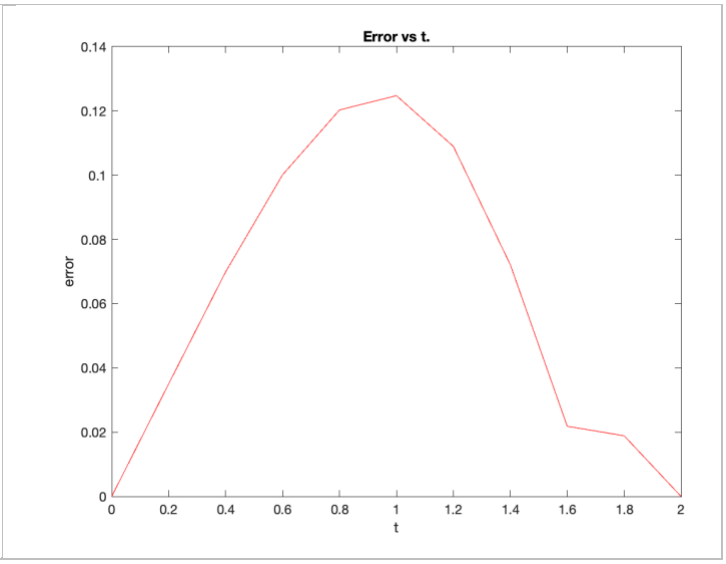

log(E_N) vs log(N)
log(E_N)
log(N)
-2
-3
-4
-5
-6
-7
-8
-9
2
3
4
5
6
7
8
9
10

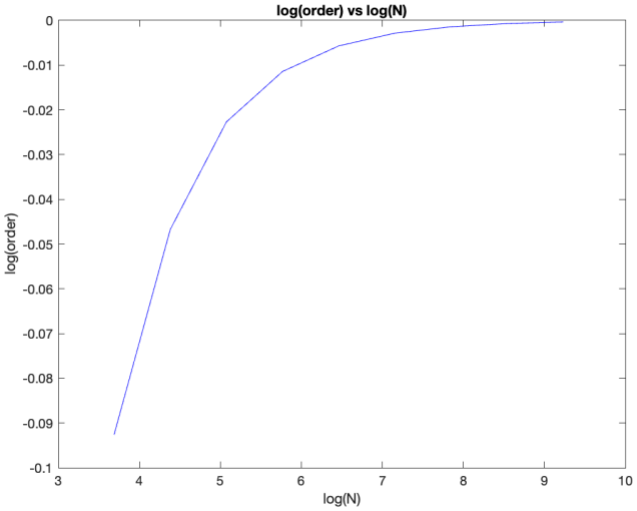
Backward difference for 1st order and central for 2nd order.

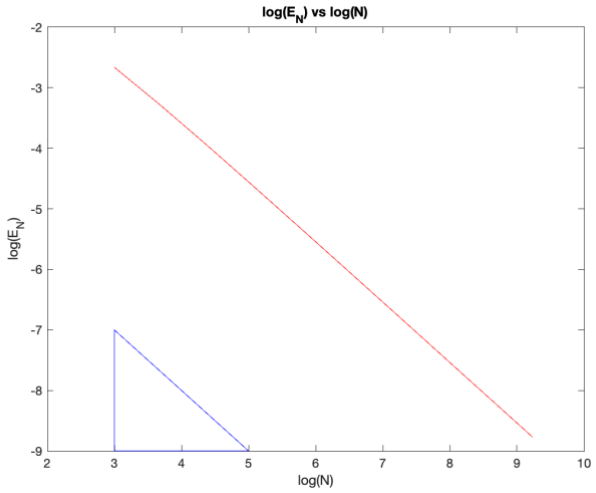
t_i	Approx.	Exact	Error
0	0	0	0
0.200000000000000	-0.197721437478672	-0.162700199388837	0.0350212380898349
0.400000000000000	-0.464851370384384	-0.394987606370133	0.0698637640142513
0.600000000000000	-0.812370026474745	-0.712284922795433	0.100085103679313
0.800000000000000	-1.24667049273289	-1.12639322177366	0.120277270959225
1	-1.76560701607322	-1.64085908577048	0.124747930302739
1.20000000000000	-2.35276259496838	-2.24380632625187	0.108956268716508
1.40000000000000	-2.96930428933152	-2.89715520411126	0.0721490852202638
1.60000000000000	-3.54259929172336	-3.52075148120968	0.0218478105136835
1.80000000000000	-3.95051424824171	-3.96939012900356	0.0188758807618505
2	-4	-4	0

N	E_N	Order
20	0.0700712299413442	0
40	0.0372498832615551	0.911586315409768
80	0.0192238051109419	0.954341976526603
160	0.00976256219677008	0.977562188022450
320	0.00491983427971388	0.988650114147394
640	0.00246963128556765	0.994314056065772
1280	0.00123725567329691	0.997152006868788
2560	0.000619239677033479	0.998573839039455
5120	0.000309772174553080	0.999290354270086
10240	0.000154924305483917	0.999644058302614





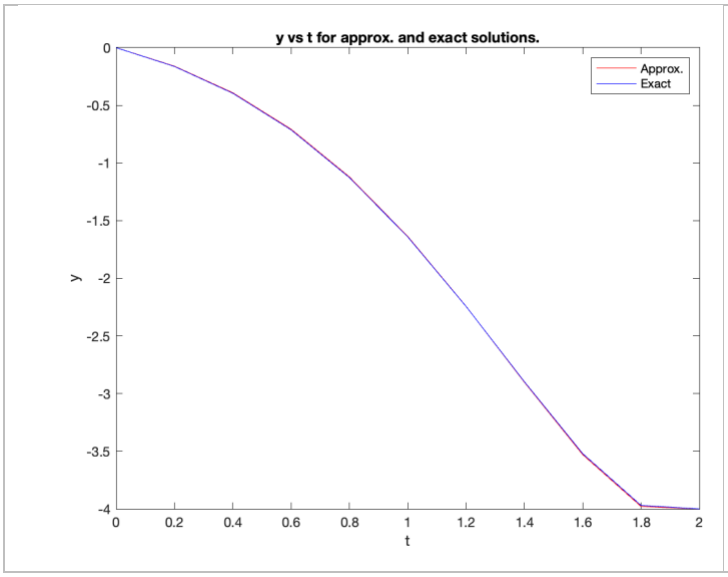

log(order) vs log(N)
log(order)
log(N)
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
3
4
5
6
7
8
9
10

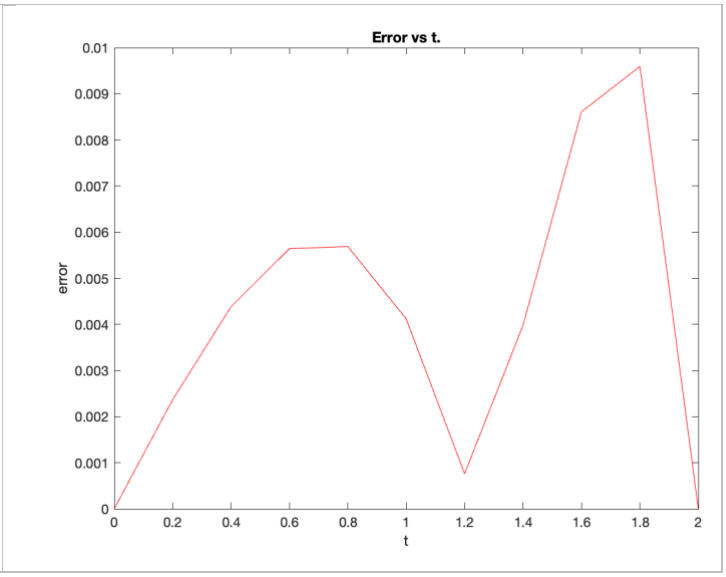

log(E_N) vs log(N)
log(E_N)
log(N)
-2
-3
-4
-5
-6
-7
-8
-9
2
3
4
5
6
7
8
9
10

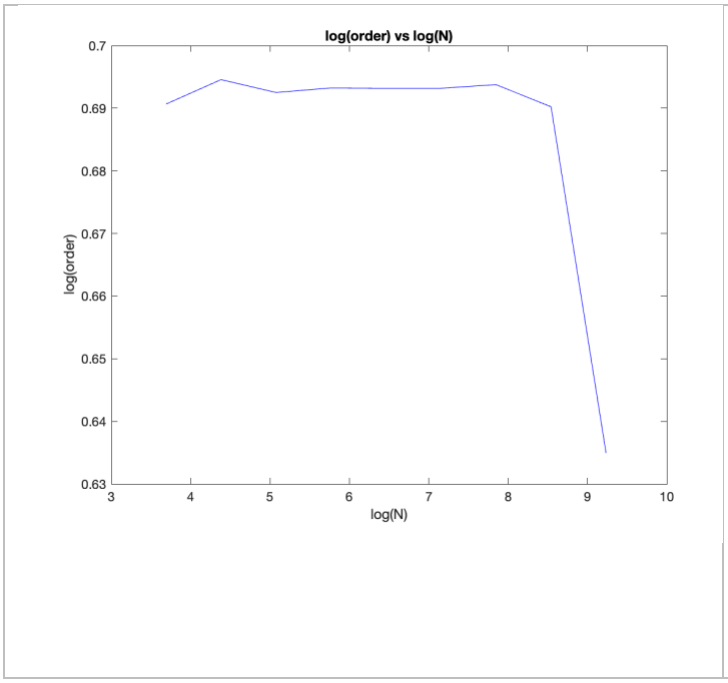
Central difference for 1st order and central for 2nd order.

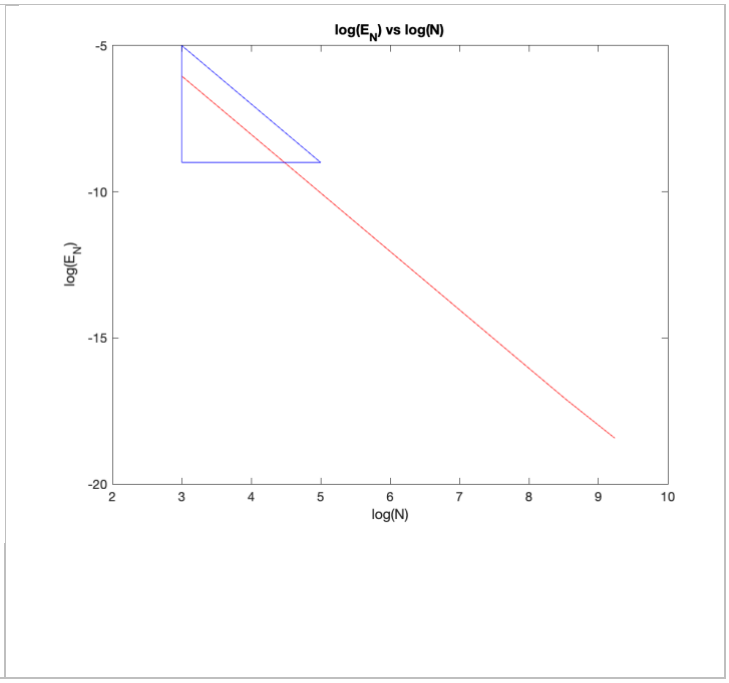
t_i	Approx.	Exact	Error
0	0	0	0
0.200000000000000	-0.160333873947962	-0.162700199388837	0.00236632544087489
0.400000000000000	-0.390603963590905	-0.394987606370133	0.00438364277922776
0.600000000000000	-0.706642405922949	-0.712284922795433	0.00564251687248407
0.800000000000000	-1.12070438511315	-1.12639322177366	0.00568883666051256
1	-1.63674049750310	-1.64085908577048	0.00411858826737710
1.20000000000000	-2.24304354978992	-2.24380632625187	0.000762776461959547
1.40000000000000	-2.90113893536645	-2.89715520411126	0.00398373125518825
1.60000000000000	-3.52936106928380	-3.52075148120968	0.00860958807412304
1.80000000000000	-3.97898362274403	-3.96939012900356	0.00959349374046603
2	-4	-4	0

N	E_N	Order
20	0.00239244126282134	0
40	0.000600177014040249	1.99502353346951
80	0.000149749993434423	2.00283212698387
160	3.74707768111549e-05	1.99871814798803
320	9.36653970740053e-06	2.00017781189040
640	2.34156165657140e-06	2.00004514295563
1280	5.85363621130597e-07	2.00006603290657
2560	1.46216833307733e-07	2.00122367662889
5120	3.67021555369718e-08	1.99417271114293
10240	9.92352244821859e-09	1.88694058090498





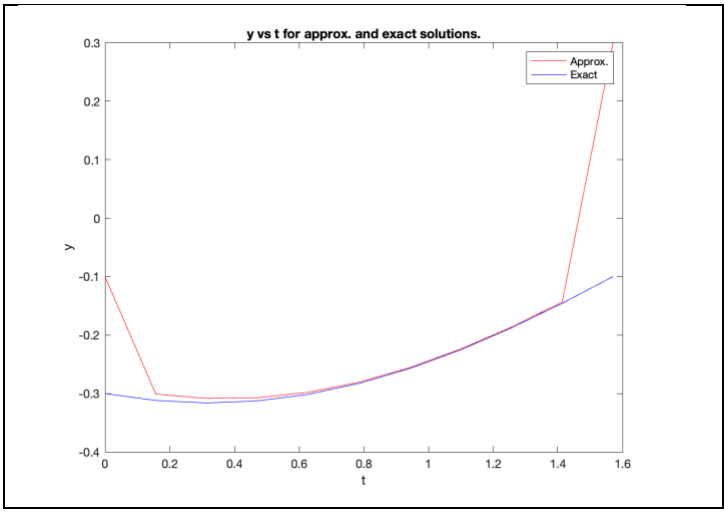

log(order) vs log(N)
log(order)
log(N)
0.7
0.69
0.68
0.67
0.66
0.65
0.64
0.63
3
4
5
6
7
8
9
10

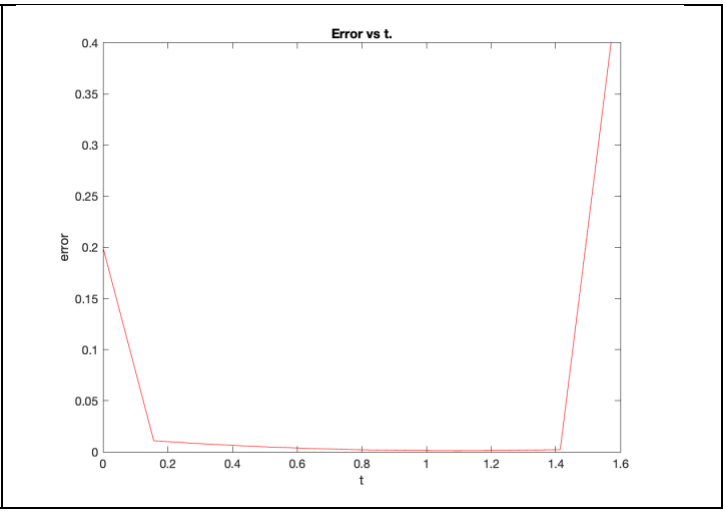

log(E_N) vs log(N)
log(E_N)
log(N)
-5
-10
-15
-20
2
3
4
5
6
7
8
9
10

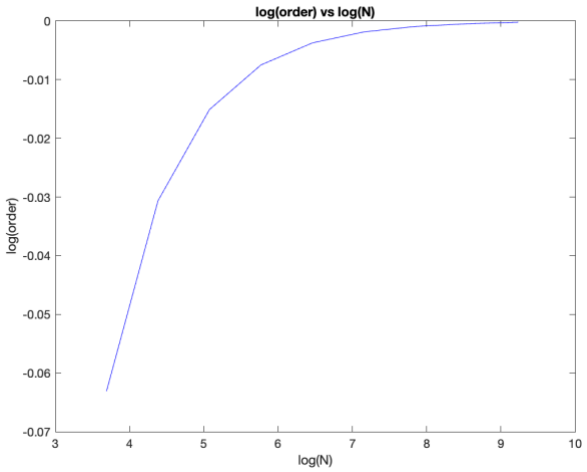
Q2 (c)
Forward difference for 1st order and central for 2nd order.

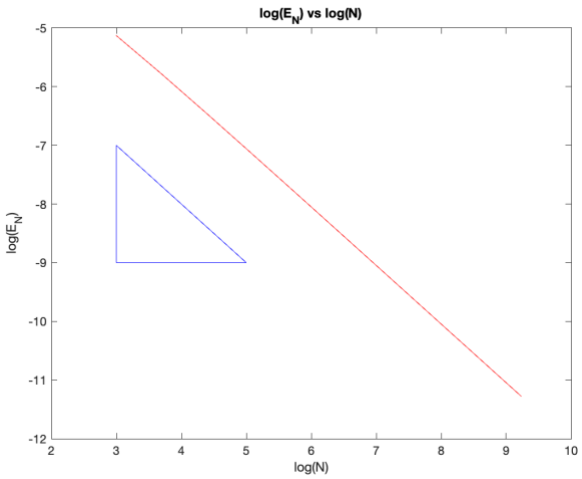
t_i	Approx.	Exact	Error
0	-0.100000000000000	-0.300000000000000	0.200000000000000
0.157079632679490	-0.301050373319115	-0.311949948682564	0.0108995753634492
0.314159265358979	-0.308398600948057	-0.316218654326041	0.00782005337798369
0.471238898038469	-0.307331726297535	-0.312701007230465	0.00536928093292977
0.628318530717959	-0.297976908280019	-0.301483623541732	0.00350671526171242
0.785398163397448	-0.280642007169912	-0.282842712474619	0.00220070530470717
0.942477796076938	-0.255808185998556	-0.257237275125237	0.00142908912668044
1.09955742875643	-0.224116920729045	-0.225297802340701	0.00118088161165625
1.25663706143592	-0.186351425489584	-0.187810749942000	0.00145932445241564
1.41371669411541	-0.143412482362673	-0.145699173571583	0.00228669120891009
1.57079632679490	0.300000000000000	-0.100000000000000	0.400000000000000

N	E_N	Order
20	0.00594195053394936	0
40	0.00309959420825839	0.938857241407236
80	0.00158255385338357	0.969824758988001
160	0.000799535560268072	0.985020488784322
320	0.000401840761328431	0.992538287203530
640	0.000201439648209845	0.996276244780188
1280	0.000100849770469424	0.998139868247331
2560	5.04573673426600e-05	0.999070960917781
5120	2.52369715564860e-05	0.999526137494205
10240	1.26202581693380e-05	0.999797373446730





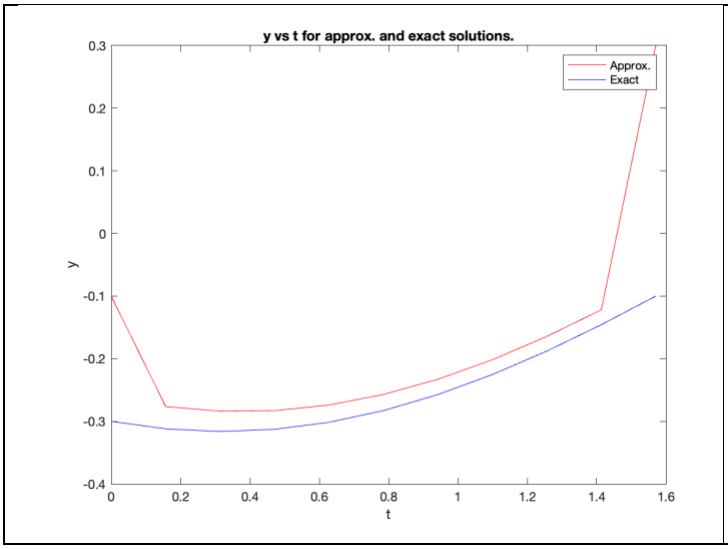

log(order) vs log(N)
log(order)
log(N)
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
3
4
5
6
7
8
9
10

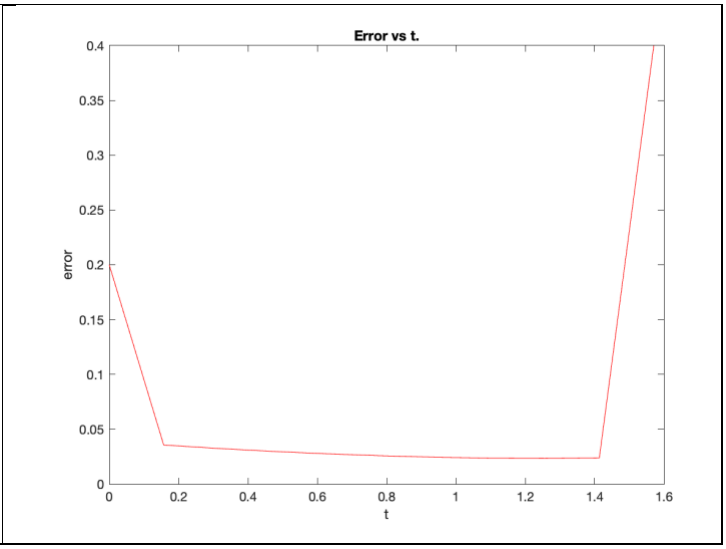

log(E_N) vs log(N)
log(E_N)
log(N)
-5
-6
-7
-8
-9
-10
-11
-12
2
3
4
5
6
7
8
9
10

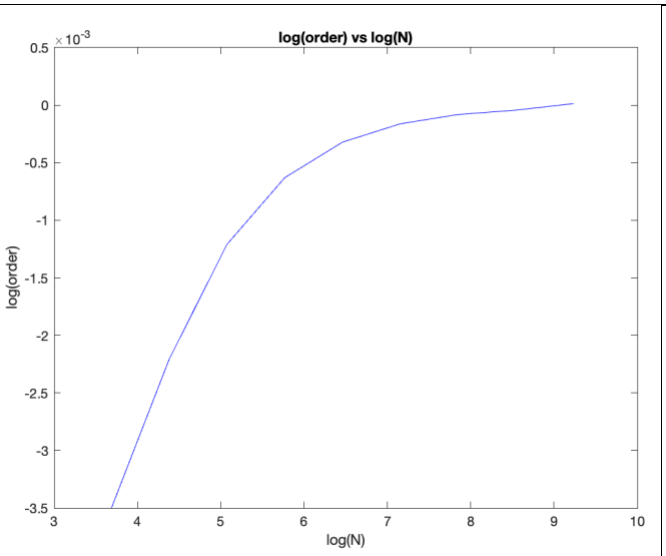
Backward difference for 1st order and central for 2nd order.

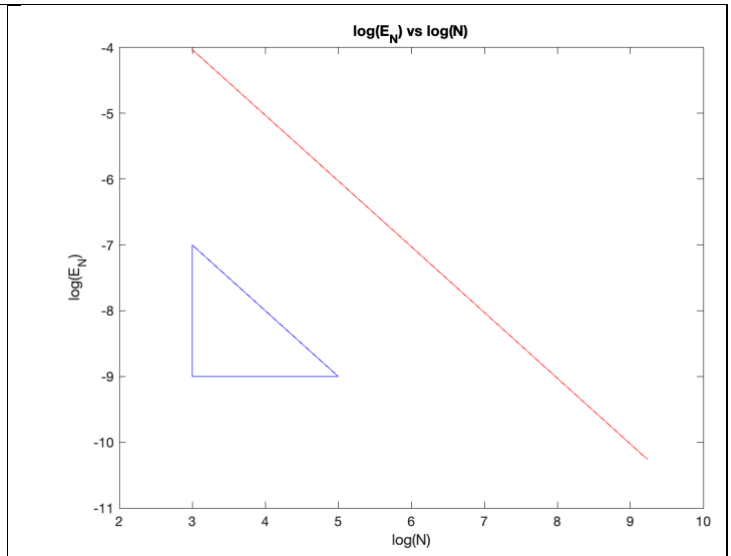
t_i	Approx.	Exact	Error
0	-0.100000000000000	-0.300000000000000	0.200000000000000
0.157079632679490	-0.276354579936568	-0.311949948682564	0.0355953687459966
0.314159265358979	-0.283797263213699	-0.316218654326041	0.0324213911123422
0.471238898038469	-0.282947495089028	-0.312701007230465	0.0297535121414372
0.628318530717959	-0.273942440132188	-0.301483623541732	0.0275411834095433
0.785398163397448	-0.257079697793741	-0.282842712474619	0.0257630146808778
0.942477796076938	-0.232807376167584	-0.257237275125237	0.0244298989576526
1.09955742875643	-0.201707930915957	-0.225297802340701	0.0235898714247439
1.25663706143592	-0.164475517055477	-0.187810749942000	0.0233352328865229
1.41371669411541	-0.121886502017797	-0.145699173571583	0.0238126715537865
1.57079632679490	0.300000000000000	-0.100000000000000	0.400000000000000

N	E_N	Order
20	0.0178318912945321	0
40	0.00893756257298517	0.996506384471935
80	0.00447561500086763	0.997795498445886
160	0.00223968314368134	0.998791295578242
320	0.00112033070035927	0.999369991934889
640	0.000560290116014661	0.999678703781387
1280	0.000280176544004473	0.999837862071281
2560	0.000140096183929073	0.999918521549847
5120	7.00502097505940e-05	0.999956383205138
10240	3.50247808942328e-05	1.00001334494324





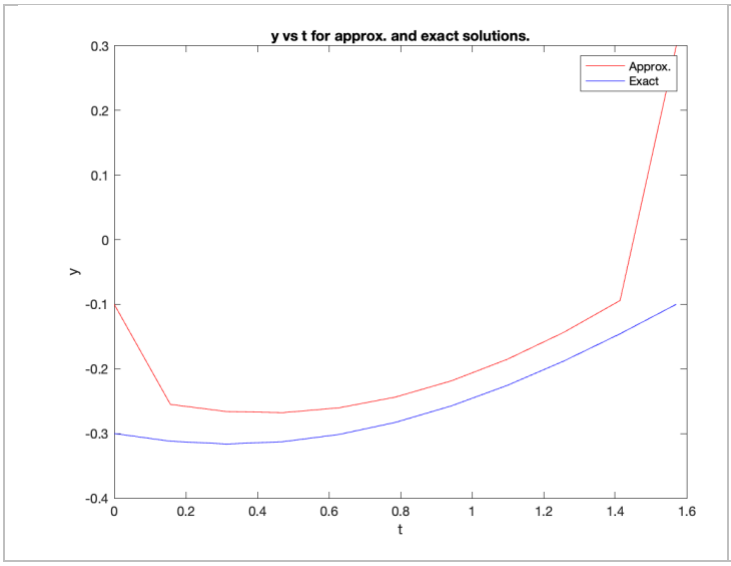

log(order) vs log(N)
×10^-3
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
log(order)
3
4
5
6
7
8
9
10
log(N)

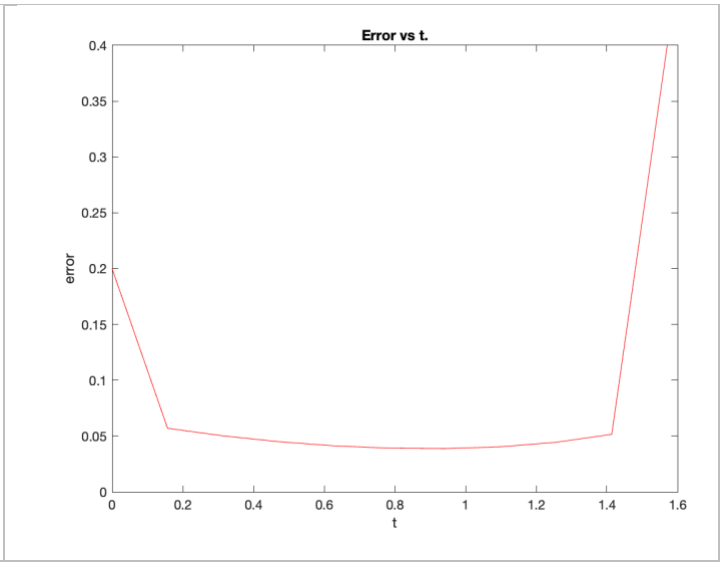

log(E_N) vs log(N)
-4
-5
-6
-7
-8
-9
-10
-11
log(E_N)
2
3
4
5
6
7
8
9
10
log(N)

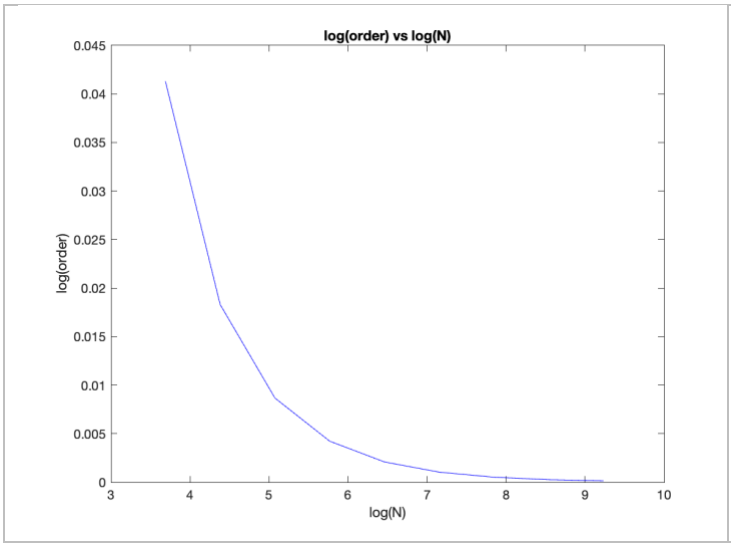
Central difference for 1st order and central for 2nd order.

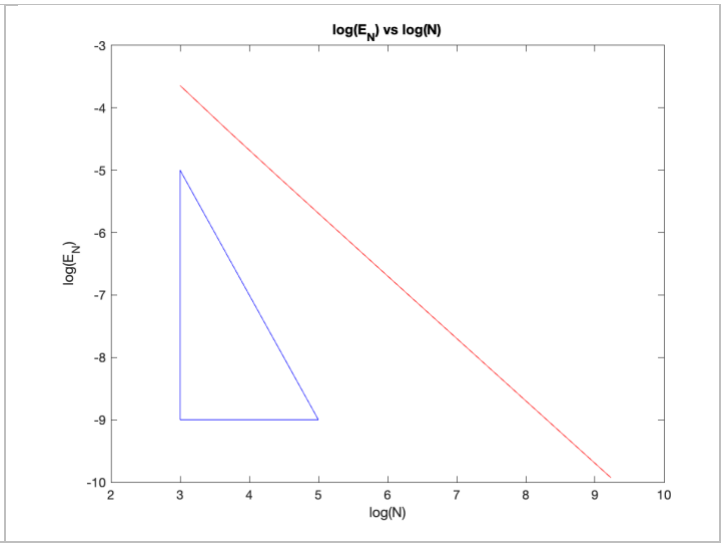
t_i	Approx.	Exact	Error
0	-0.100000000000000	-0.300000000000000	0.200000000000000
0.157079632679490	-0.254980289453444	-0.311949948682564	0.0569696592291203
0.314159265358979	-0.266028223247428	-0.316218654326041	0.0501904310786127
0.471238898038469	-0.267739880658861	-0.312701007230465	0.0449611265716037
0.628318530717959	-0.260223353399749	-0.301483623541732	0.0412602701419826
0.785398163397448	-0.243698428448824	-0.282842712474619	0.0391442840257952
0.942477796076938	-0.218473340026033	-0.257237275125237	0.0387639350992033
1.09955742875643	-0.184909160722147	-0.225297802340701	0.0403886416185541
1.25663706143592	-0.143369472177665	-0.187810749942000	0.0444412777643351
1.41371669411541	-0.0941520478015715	-0.145699173571583	0.0515471257700115
1.57079632679490	0.300000000000000	-0.100000000000000	0.400000000000000

N	E_N	Order
20	0.0262890968886179	0
40	0.0127657994353013	1.04218069502841
80	0.00630169080376242	1.01847301225685
160	0.00313188805737763	1.00870632188475
320	0.00156135485301684	1.00423418802400
640	0.000779547865144670	1.00208894557894
1280	0.000389493687957820	1.00103766029121
2560	0.000194677035828983	1.00051723521856
5120	9.73207705078916e-05	1.00026306574702
10240	4.86555807853173e-05	1.00014245109723






log(order) vs log(N)
log(order)
log(N)
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
3
4
5
6
7
8
9
10

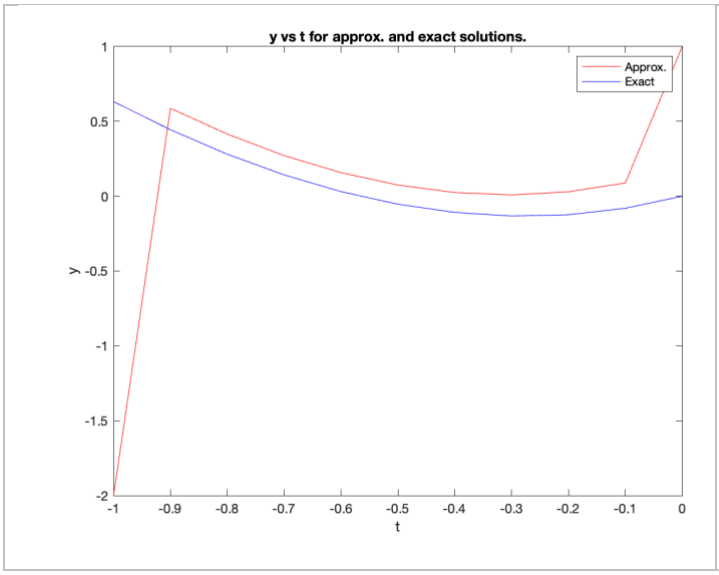

log(E_N) vs log(N)
log(E_N)
log(N)
-3
-4
-5
-6
-7
-8
-9
-10
2
3
4
5
6
7
8
9
10

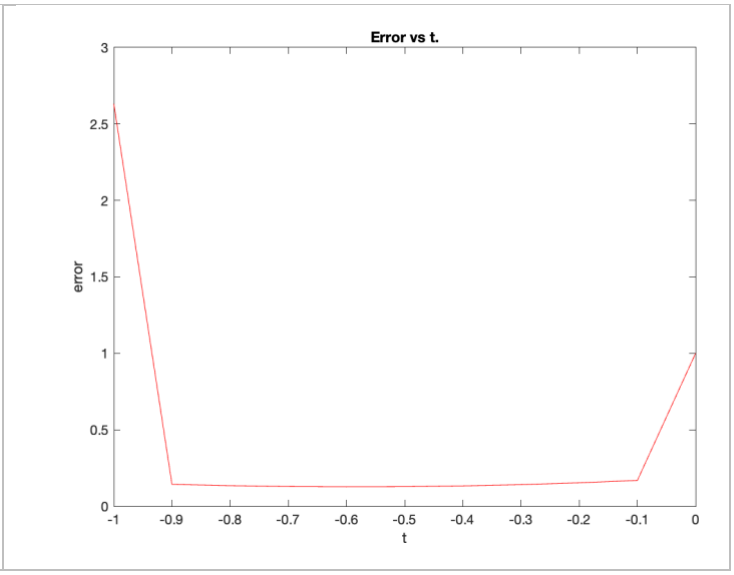
Q2 (d)

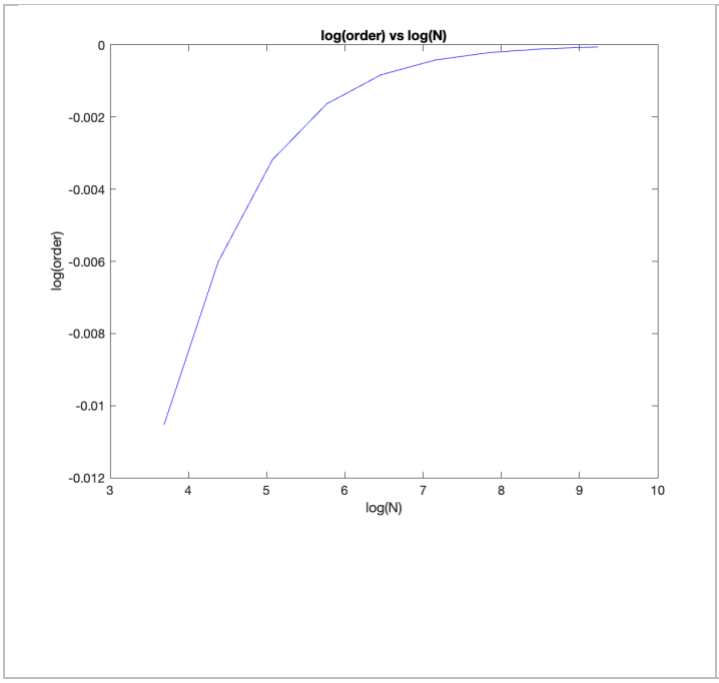
Forward difference for 1st order and central for 2nd order.

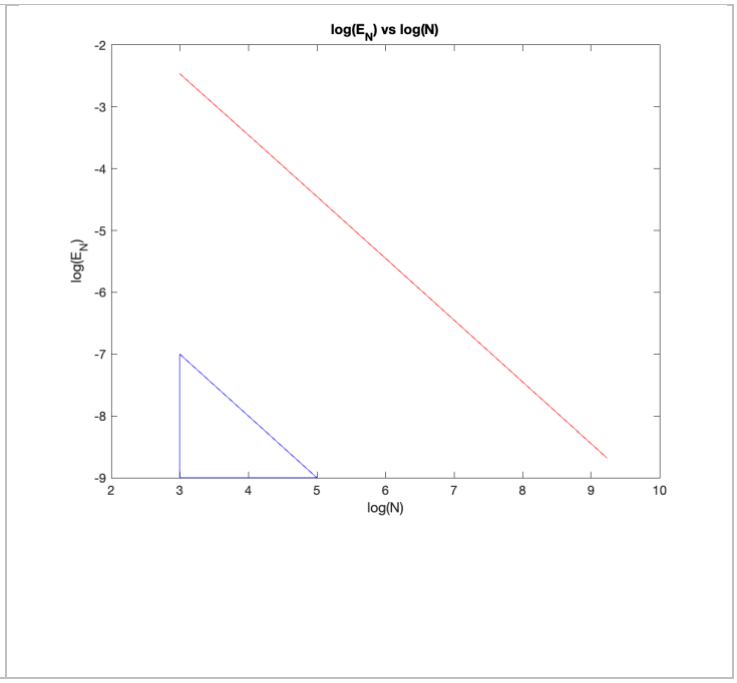
t_i	Approx.	Exact	Error
-1	-2	0.632120558828558	2.63212055882856
-0.900000000000000	0.587395133400223	0.444087306233461	0.143307827166763
-0.800000000000000	0.415057232418592	0.280536828706223	0.134520403712369
-0.700000000000000	0.271389328417978	0.142390287346013	0.128999041071965
-0.600000000000000	0.157545088237496	0.0307130183435841	0.126832069893912
-0.500000000000000	0.0748414884460984	-0.0532653298563167	0.128106818302415
-0.400000000000000	0.0247785093103710	-0.108128018414256	0.132906527724627
-0.300000000000000	0.00906132602375635	-0.132245466204515	0.141306792228272
-0.200000000000000	0.0296253920506722	-0.123746150615596	0.153371542666269
-0.100000000000000	0.0886648626942641	-0.0804837418035960	0.169148604497860
0	1	0	1

N	E_N	Order
20	0.0854170018666032	0
40	0.0430198398626333	0.989521105857311
80	0.0215994742247236	0.994005959797347
160	0.0108235423601260	0.996823448837621
320	0.00541789672063891	0.998367948484641
640	0.00271050134911792	0.999173167233002
1280	0.00135564161118638	0.999583899116499
2560	0.000677918863852393	0.999791304823919
5120	0.000338983946480971	0.999895663774454
10240	0.000169498092930018	0.999947910888126





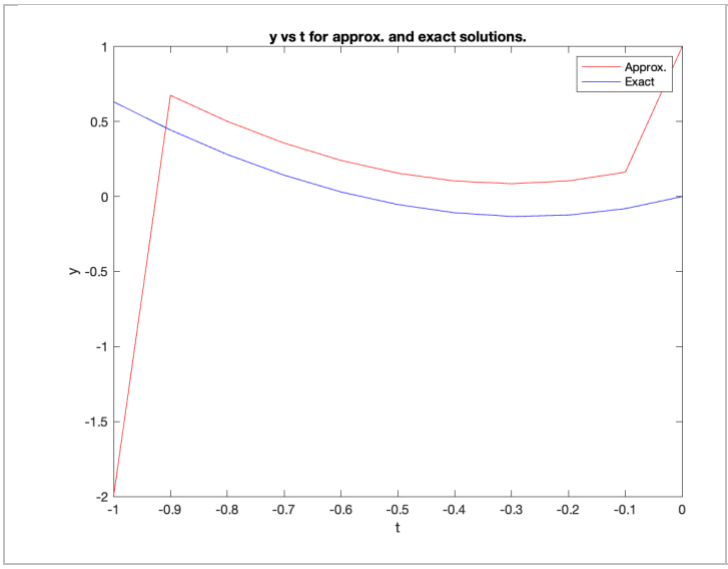

log(order) vs log(N)
log(order)
log(N)
0
-0.002
-0.004
-0.006
-0.008
-0.01
-0.012
3
4
5
6
7
8
9
10

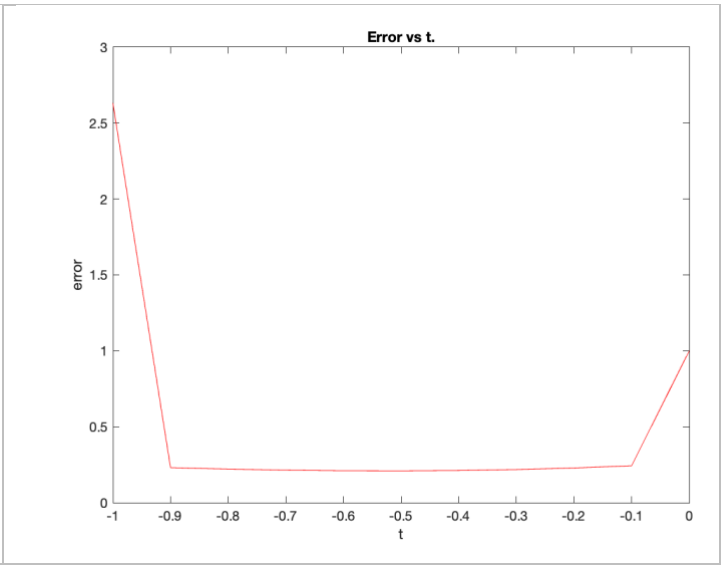

log(E_N) vs log(N)
log(E_N)
log(N)
-2
-3
-4
-5
-6
-7
-8
-9
2
3
4
5
6
7
8
9
10

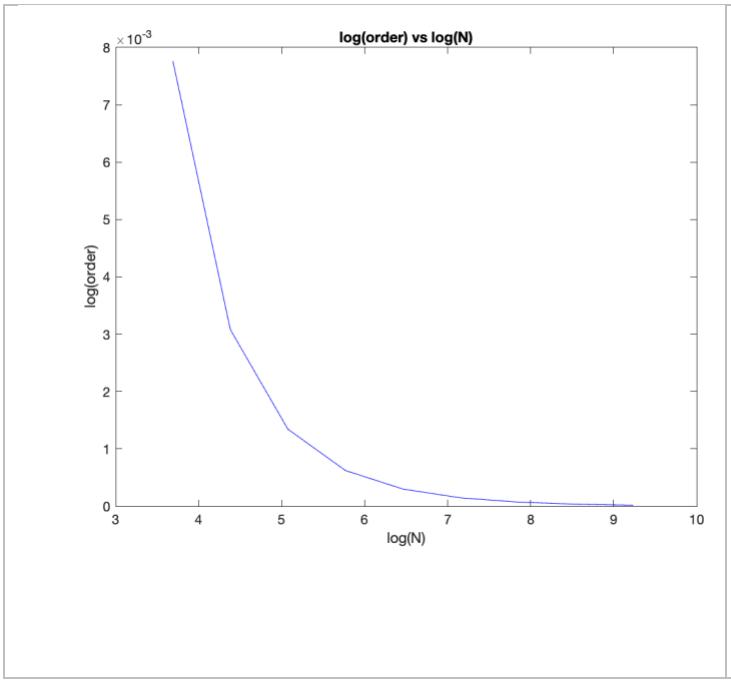
Backward difference for 1st order and central for 2nd order.

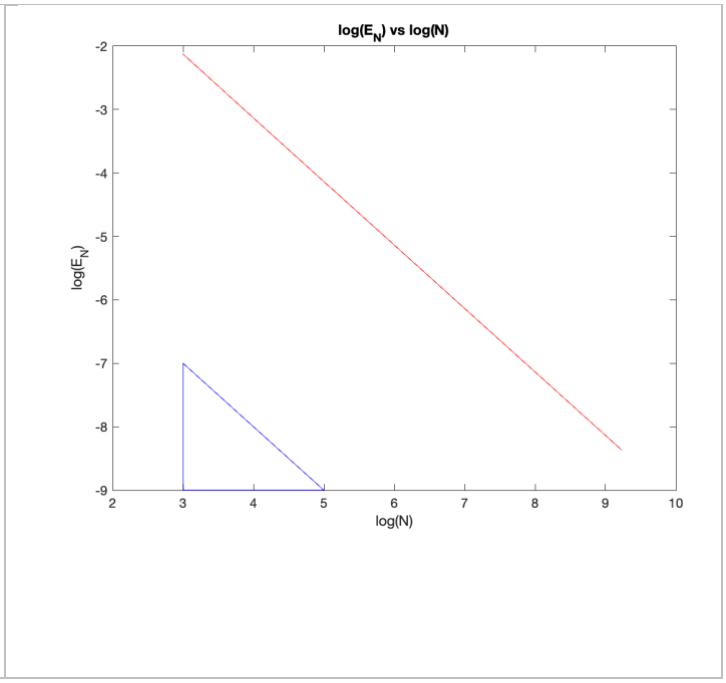
t_i	Approx.	Exact	Error
-1	-2	0.632120558828558	2.63212055882856
-0.900000000000000	0.674792721545637	0.444087306233461	0.230705415312177
-0.800000000000000	0.501713183415261	0.280536828706223	0.221176354709038
-0.700000000000000	0.356683830555454	0.142390287346013	0.214293543209441
-0.600000000000000	0.241001073670976	0.0307130183435841	0.210288055327392
-0.500000000000000	0.156149327458668	-0.0532653298563167	0.209414657314984
-0.400000000000000	0.103824920328451	-0.108128018414256	0.211952938742707
-0.300000000000000	0.0859629483139655	-0.132245466204515	0.218208414518481
-0.200000000000000	0.104767476917572	-0.123746150615596	0.228513627533169
-0.100000000000000	0.162745552994393	-0.0804837418035960	0.243229294797989
0	1	0	1

N	E_N	Order
20	0.119736461234125	0
40	0.0595456240809986	1.00779514208338
80	0.0297092255290456	1.00308449351886
160	0.0148407761964264	1.00134444776235
320	0.00741718213116046	1.00062344882197
640	0.00370782095566864	1.00029961489496
1280	0.00185372186632964	1.00014678309899
2560	0.000926814289673677	1.00007260423244
5120	0.000463395470774446	1.00003634454983
10240	0.000231694827100433	1.00001810892651





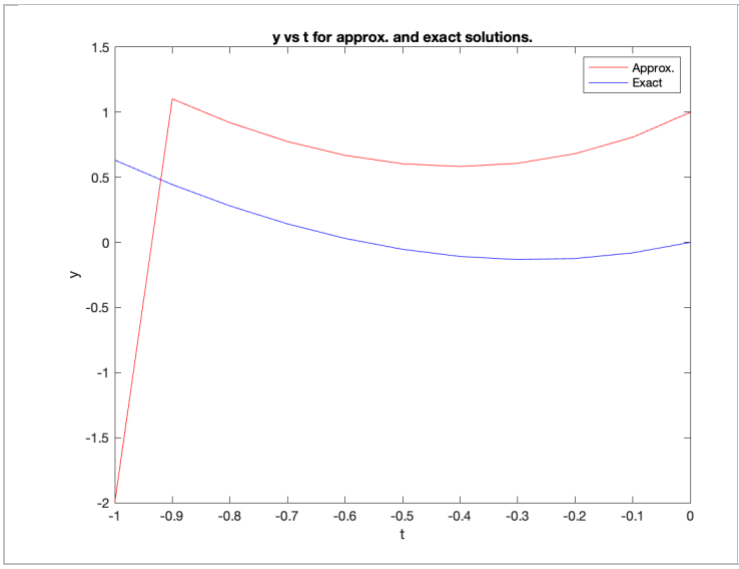

log(order) vs log(N)
×10^-3
log(order)
log(N)

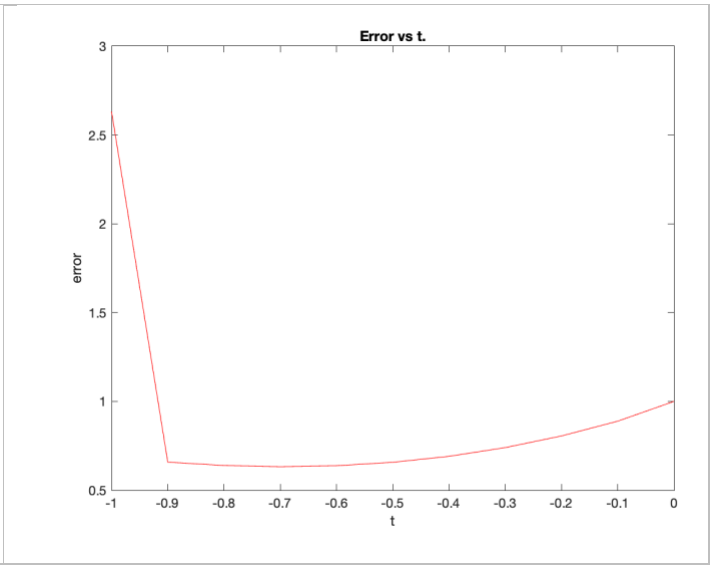

log(E_N) vs log(N)
log(E_N)
log(N)

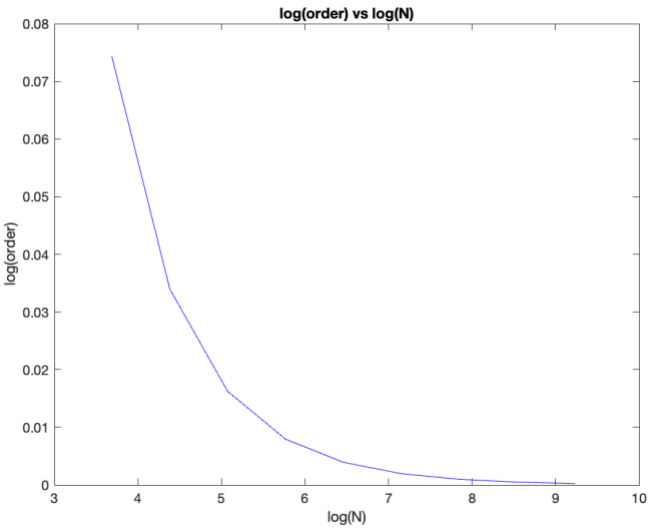
Central difference for 1st order and central for 2nd order.

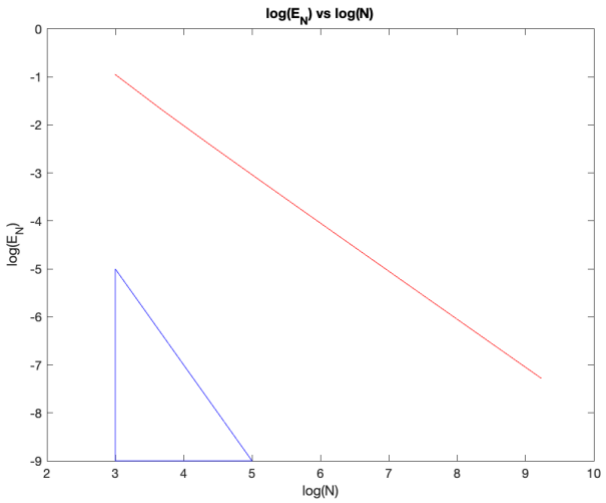
t_i	Approx.	Exact	Error
-1	-2	0.632120558828558	2.63212055882856
-0.900000000000000	1.10213954790551	0.444087306233461	0.658052241672047
-0.800000000000000	0.919873247103918	0.280536828706223	0.639336418397695
-0.700000000000000	0.774771993730085	0.142390287346013	0.632381706384072
-0.600000000000000	0.668741544711526	0.0307130183435841	0.638028526367941
-0.500000000000000	0.603912083904240	-0.0532653298563167	0.657177413760557
-0.400000000000000	0.582655657436704	-0.108128018414256	0.690783675850960
-0.300000000000000	0.607605117789987	-0.132245466204515	0.739850583994502
-0.200000000000000	0.681675072537834	-0.123746150615596	0.805421223153430
-0.100000000000000	0.808085439314473	-0.0804837418035960	0.888569181118069
0	1	0	1

N	E_N	Order
20	0.388496082754032	0
40	0.184132560126802	1.07715529111846
80	0.0898872202815118	1.03455684052794
160	0.0444368244508510	1.01636029272299
320	0.0220961553543603	1.00796035429079
640	0.0110180505310196	1.00392638356744
1280	0.00550158454631676	1.00194988115971
2560	0.00274894030668407	1.00097161922351
5120	0.00137400810974888	1.00048505968975
10240	0.000686888660373756	1.00024234654195





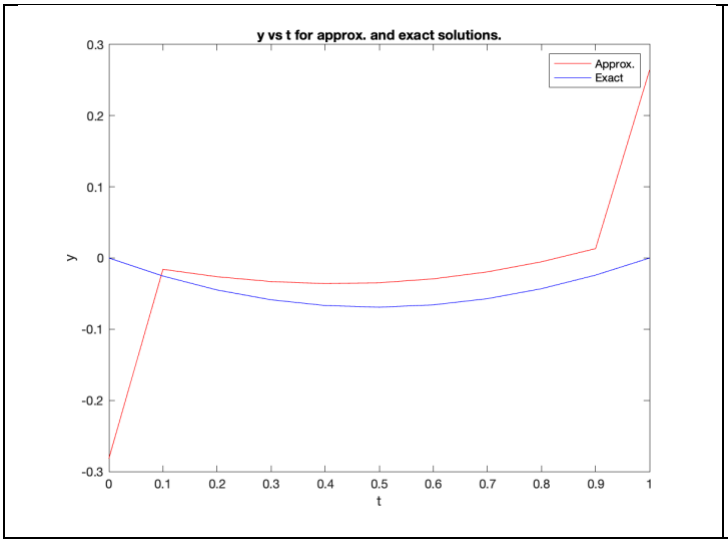

log(order) vs log(N)
log(order)
log(N)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
3
4
5
6
7
8
9
10

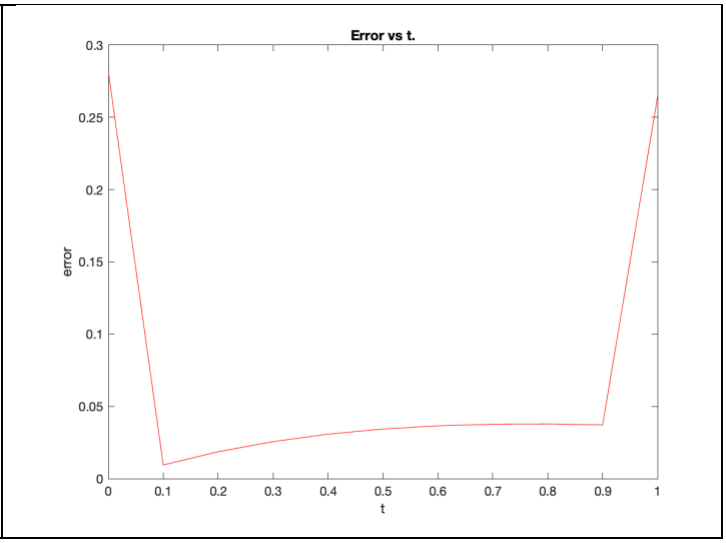

log(E_N) vs log(N)
log(E_N)
log(N)
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
2
3
4
5
6
7
8
9
10

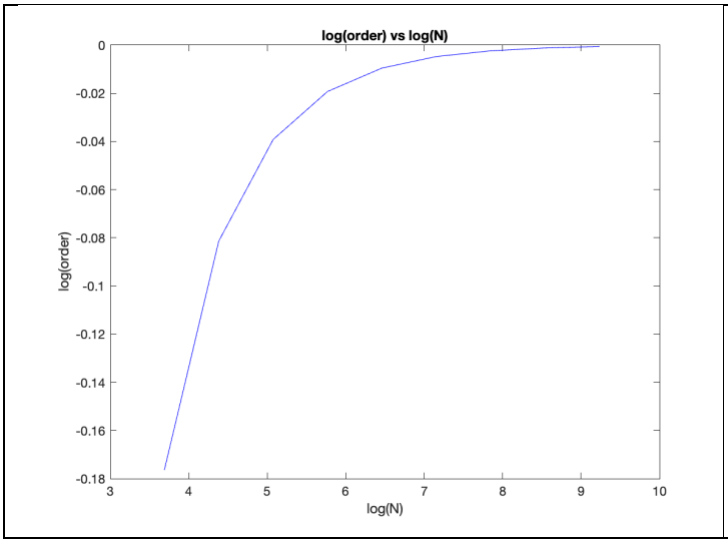
Q2 (e)
Forward difference for 1st order and central for 2nd order.

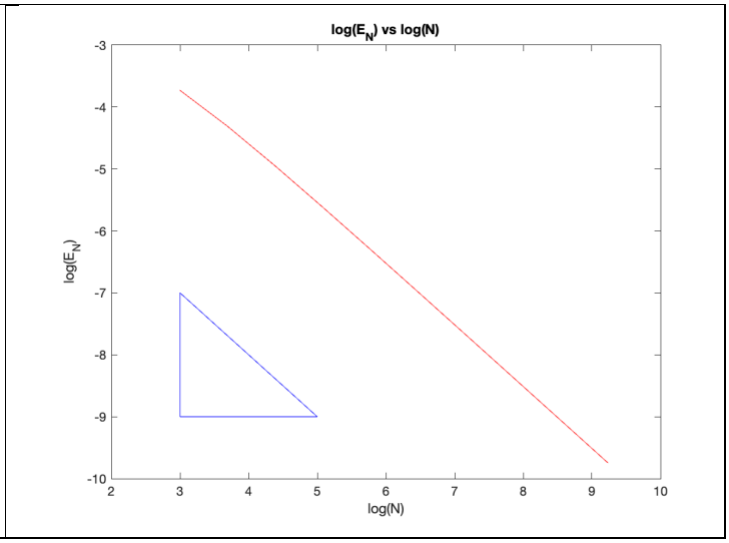
t_i	Approx.	Exact	Error
0	-0.281718171540955	0	0.281718171540955
0.100000000000000	-0.0158704253199524	-0.0253323364195779	0.00946191109962551
0.200000000000000	-0.0262950160664927	-0.0449226093767356	0.0186275933102430
0.300000000000000	-0.0330963832213889	-0.0587286788044523	0.0256322955830635
0.400000000000000	-0.0359883859902908	-0.0667684246010292	0.0307800386107384
0.500000000000000	-0.0347651517477900	-0.0691087049373025	0.0343435531895125
0.600000000000000	-0.0292894136144744	-0.0658558725400167	0.0365664589255423
0.700000000000000	-0.0194822200565908	-0.0571476524219521	0.0376654323653613
0.800000000000000	-0.00531387359121620	-0.0431462078249756	0.0378323342337594
0.900000000000000	0.0132040307431894	-0.0240322417837975	0.0372362725269869
1	0.264241117657115	0	0.264241117657115

N	E_N	Order
20	0.0240765003543437	0
40	0.0134659936119603	0.838305018491373
80	0.00710780898176579	0.921843870141766
160	0.00364987452971208	0.961558040406729
320	0.00184921479193617	0.980934061987743
640	0.000930713505934988	0.990503759898846
1280	0.000466887891163753	0.995260970426957
2560	0.000233827302828241	0.997632776575973
5120	0.000117009533375734	0.998817317416387
10240	5.85287437151055e-05	0.999408860955168





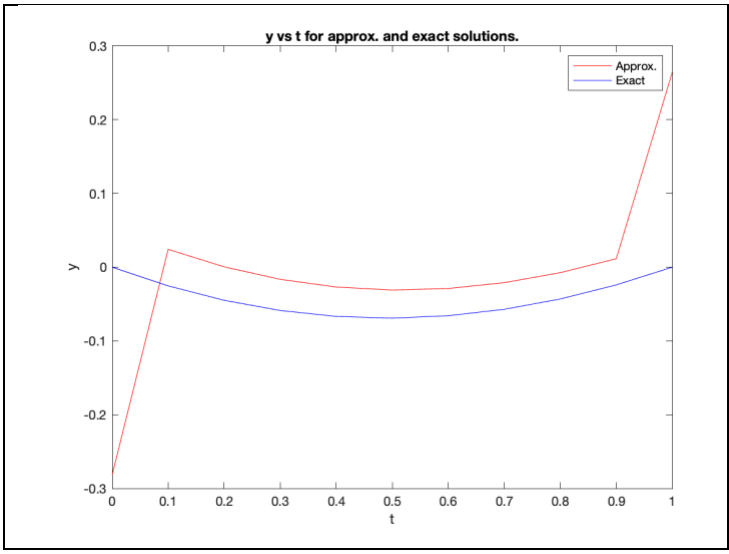

log(order) vs log(N)
log(order)
log(N)
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
-0.16
-0.18
3
4
5
6
7
8
9
10

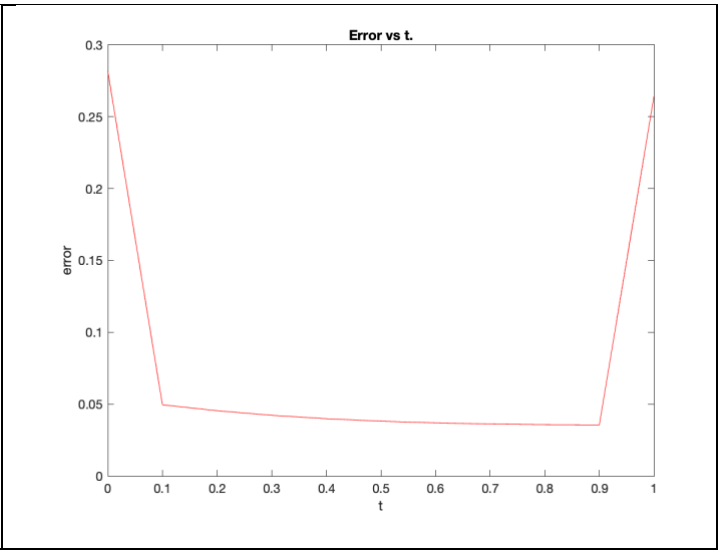

log(E_N) vs log(N)
log(E_N)
log(N)
-3
-4
-5
-6
-7
-8
-9
-10
2
3
4
5
6
7
8
9
10

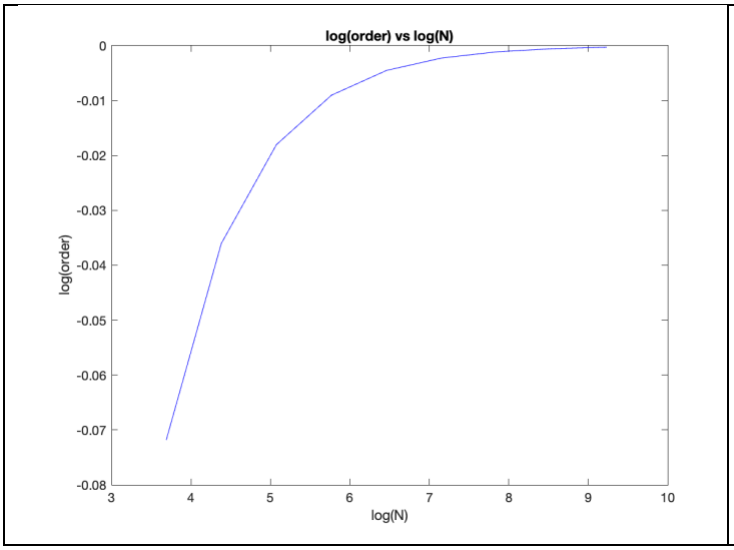
Backward difference for 1st order and central for 2nd order.

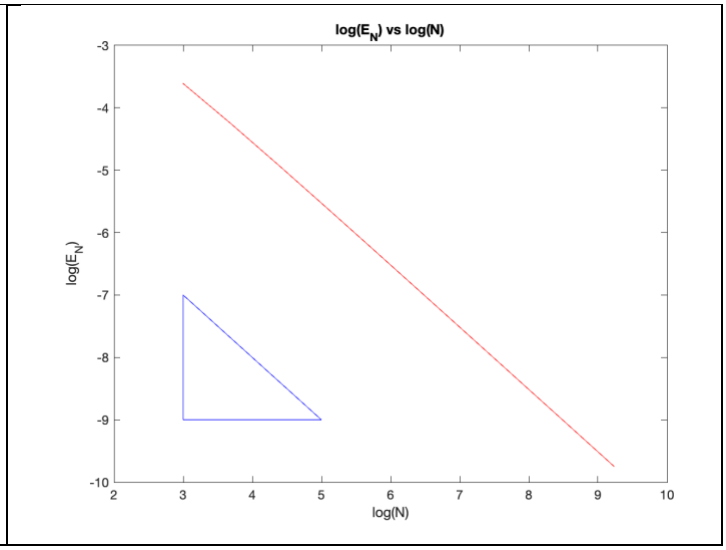
t_i	Approx.	Exact	Error
0	-0.281718171540955	0	0.281718171540955
0.100000000000000	0.0241958258891633	-0.0253323364195779	0.0495281623087412
0.200000000000000	0.000480747835862219	-0.0449226093767356	0.0454033572125979
0.300000000000000	-0.0164961220851373	-0.0587286788044523	0.0422325567193150
0.400000000000000	-0.0269126568010855	-0.0667684246010292	0.0398557677999437
0.500000000000000	-0.0309767580058332	-0.0691087049373025	0.0381319469314693
0.600000000000000	-0.0289182713895731	-0.0658558725400167	0.0369376011504436
0.700000000000000	-0.0209822993826692	-0.0571476524219521	0.0361653530392829
0.800000000000000	-0.00742369878331944	-0.0431462078249756	0.0357225090416561
0.900000000000000	0.0114974186839936	-0.0240322417837975	0.0355296604677911
1	0.264241117657115	0	0.264241117657115

N	E_N	Order
20	0.0271493947417634	0
40	0.0142429067239380	0.930676430745681
80	0.00729831909567941	0.964607470105988
160	0.00369466356845528	0.982121229224899
320	0.00185887285275151	0.991014907283452
640	0.000932342594125603	0.995496009827307
1280	0.000466900453490511	0.997745188243458
2560	0.000233632834567814	0.998871946189979
5120	0.000116862287346698	0.999433611093763
10240	5.84425284605537e-05	0.999718931119488





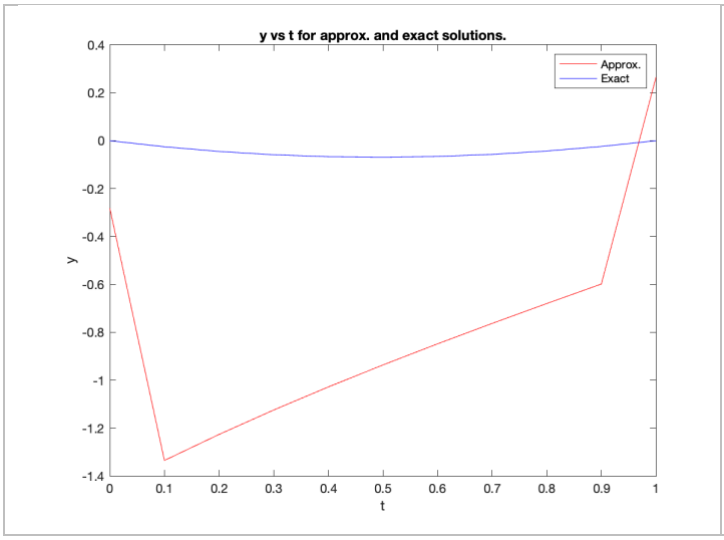

log(order) vs log(N)
log(order)
log(N)
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
3
4
5
6
7
8
9
10

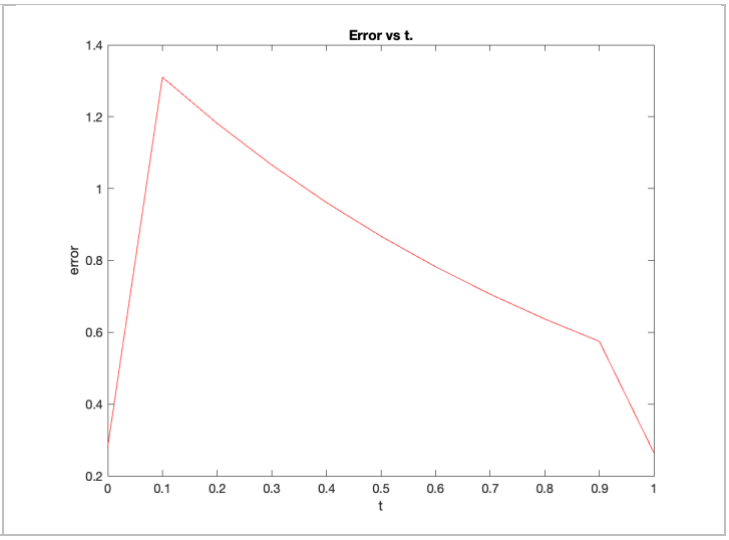

log(E_N) vs log(N)
log(E_N)
log(N)
-3
-4
-5
-6
-7
-8
-9
-10
2
3
4
5
6
7
8
9
10

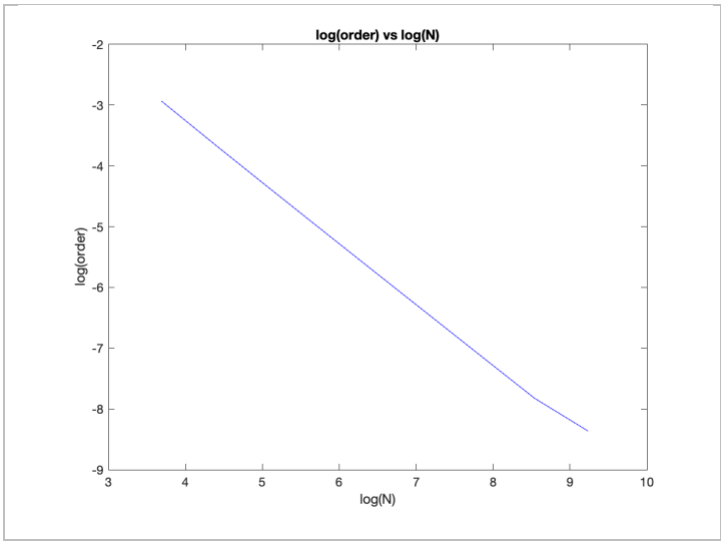
Central difference for 1st order and central for 2nd order.

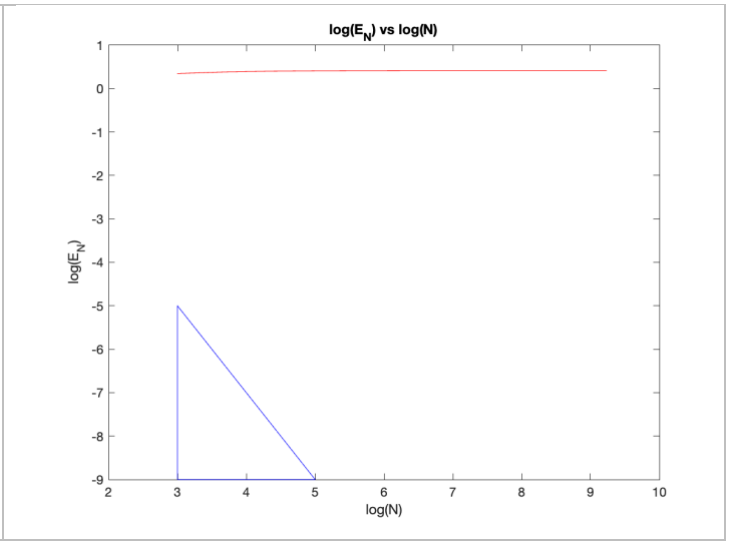
t_i	Approx.	Exact	Error
0	-0.281718171540955	0	0.281718171540955
0.100000000000000	-1.33522771882574	-0.0253323364195779	1.30989538240617
0.200000000000000	-1.22655958238186	-0.0449226093767356	1.18163697300512
0.300000000000000	-1.12468056545157	-0.0587286788044523	1.06595188664711
0.400000000000000	-1.02837336464086	-0.0667684246010292	0.961604940039832
0.500000000000000	-0.936591351571730	-0.0691087049373025	0.867482646634428
0.600000000000000	-0.848437055864517	-0.0658558725400167	0.782581183324500
0.700000000000000	-0.763143204323483	-0.0571476524219521	0.705995551901531
0.800000000000000	-0.680056023932423	-0.0431462078249756	0.636909816107448
0.900000000000000	-0.598620548849443	-0.0240322417837975	0.574588307065646
1	0.264241117657115	0	0.264241117657115

N	E_N	Order
20	1.41441680796833	0
40	1.46753179912424	-0.0531844406729775
80	1.49427980951974	-0.0260585588767745
160	1.50769837655149	-0.0128975148018768
320	1.51441840622878	-0.00641601246129264
640	1.51778105764700	-0.00319984392311142
1280	1.51946303669285	-0.00159788479359121
2560	1.52030376080177	-0.000798027358300127
5120	1.52072480293870	-0.000399493391199323
10240	1.52097096830327	-0.000233515498058471






log(order) vs log(N)
log(order)
log(N)
-2
-3
-4
-5
-6
-7
-8
-9
3
4
5
6
7
8
9
10

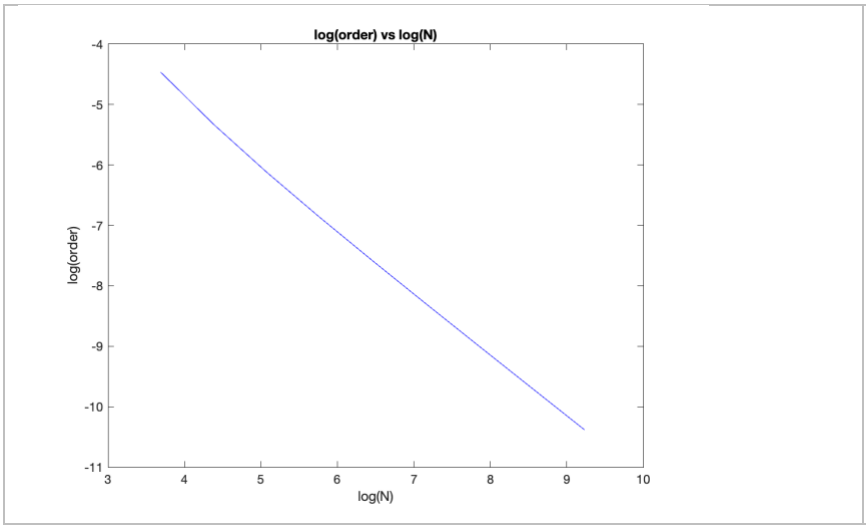

log(E_N) vs log(N)
log(E_N)
log(N)
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
2
3
4
5
6
7
8
9
10

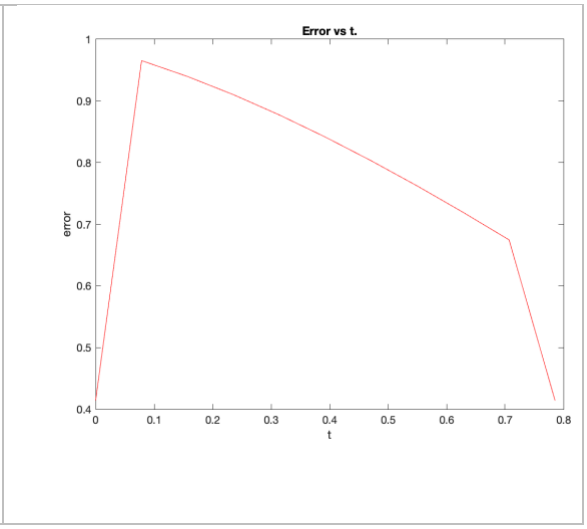
Q2 (d)

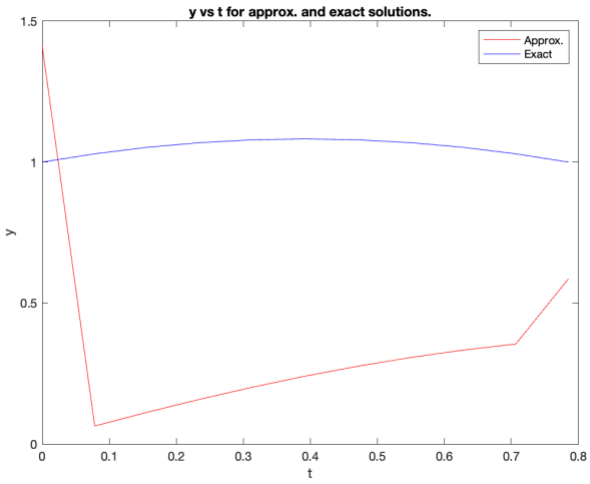
Forward difference for 1st order and central for 2nd order.

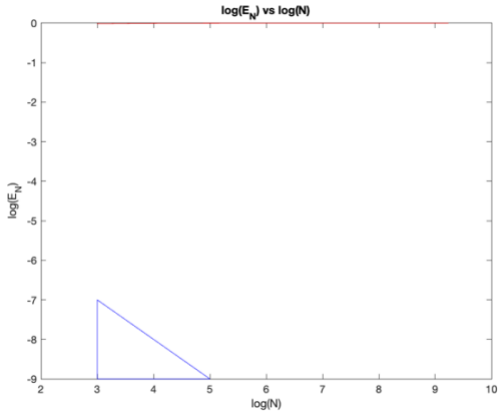
t_i	Approx.	Exact	Error
0	1.41421356237310	1	0.414213562373095
0.0785398163397448	0.0640975733211083	1.02941615527513	0.965318581954022
0.157079632679490	0.112915082718207	1.05248561763738	0.939570534919175
0.235619449019234	0.159000557226727	1.06906615617991	0.910065598953188
0.314159265358979	0.201848558304565	1.07905554636903	0.877206988064463
0.392699081698724	0.241023466557935	1.08239220029239	0.841368733734459
0.471238898038469	0.276161927478368	1.07905554636903	0.802893618890660
0.549778714378214	0.306973920666331	1.06906615617991	0.762092235513584
0.628318530717959	0.333242667296519	1.05248561763738	0.719242950340862
0.706858347057704	0.354823576232574	1.02941615527513	0.674592579042556
0.785398163397448	0.585786437626905	1	0.414213562373095

N	E_N	Order
20	0.985808585484767	0
40	0.993711033965570	-0.0115188389013856
80	0.997060016343009	-0.00485396331733028
160	0.998581512366534	-0.00219984918293053
320	0.999303681495111	-0.00104297270840129
640	0.999655078352538	-0.000507222579073545
1280	0.999828349369802	-0.000250041820298938
2560	0.999914377332852	-0.000124128083167008
5120	0.999957239340698	-6.18407758326643e-05
10240	0.999978632340832	-3.08645648517514e-05





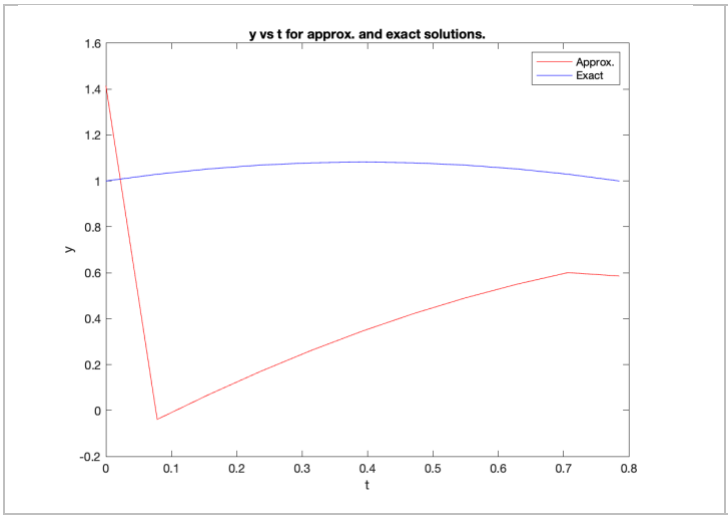

y vs t for approx. and exact solutions.
Approx.
Exact
y
t
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

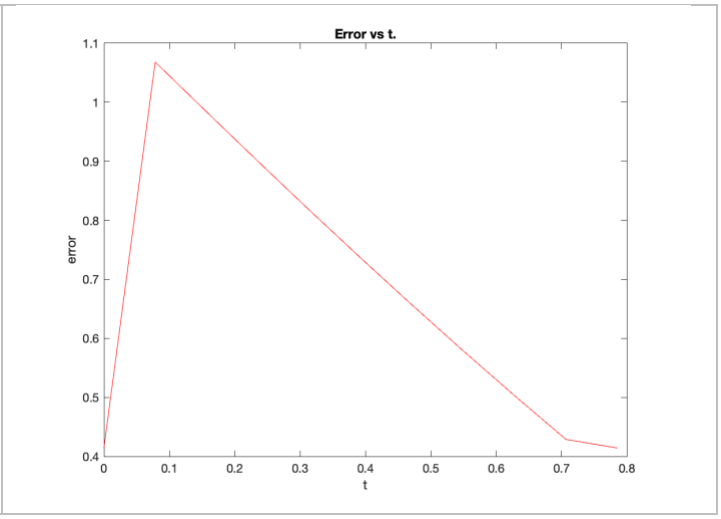

log(E_N) vs log(N)
log(E_N)
log(N)
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
2
3
4
5
6
7
8
9
10

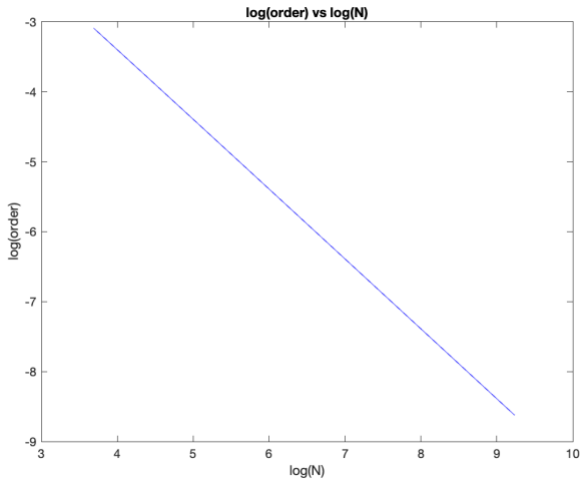
Backward difference for 1st order and central for 2nd order.

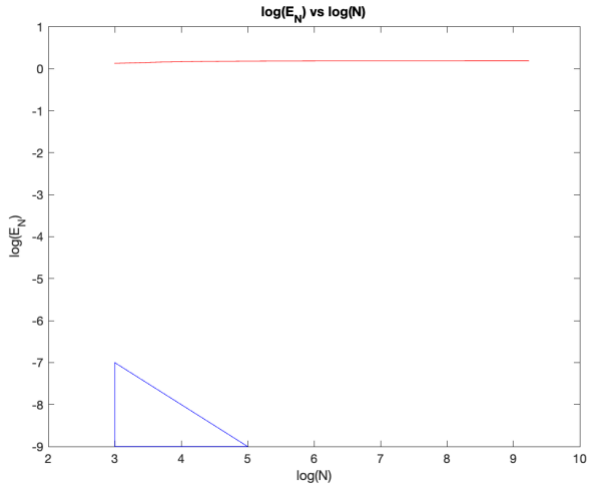
t_i	Approx.	Exact	Error
0	1.41421356237310	1	0.414213562373095
0.0785398163397448	-0.0387178078217774	1.02941615527513	1.06813396309691
0.157079632679490	0.0687560686835236	1.05248561763738	0.983729548953858
0.235619449019234	0.169008231332583	1.06906615617991	0.900057924847332
0.314159265358979	0.261577246432697	1.07905554636903	0.817478299936330
0.392699081698724	0.346116416385222	1.08239220029239	0.736275783907172
0.471238898038469	0.422386291763520	1.07905554636903	0.656669254605508
0.549778714378214	0.490246594659917	1.06906615617991	0.578819561519998
0.628318530717959	0.549647904450115	1.05248561763738	0.502837713187266
0.706858347057704	0.600623370229705	1.02941615527513	0.428792785045425
0.785398163397448	0.585786437626905	1	0.414213562373095

N	E_N	Order
20	1.13714475352969	0
40	1.17358024693676	-0.0455005790130259
80	1.19233574007004	-0.0228740360318782
160	1.20185533479189	-0.0114727221371576
320	1.20665155521247	-0.00574587732426364
640	1.20905889356389	-0.00287539117344589
1280	1.21026488522557	-0.00143831788786264
2560	1.21086846346725	-0.000719315484678376
5120	1.21117039795530	-0.000359696450603400
10240	1.21132140658386	-0.000179863890091211





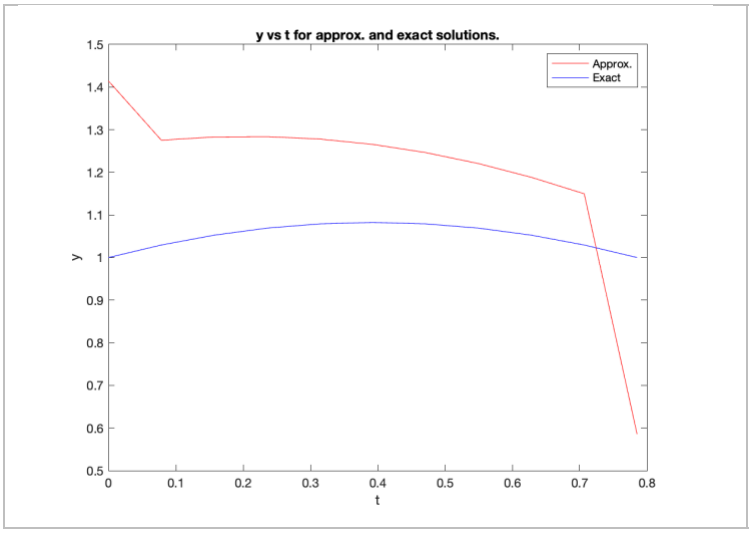

log(order) vs log(N)
log(order)
log(N)
-3
-4
-5
-6
-7
-8
-9
3
4
5
6
7
8
9
10

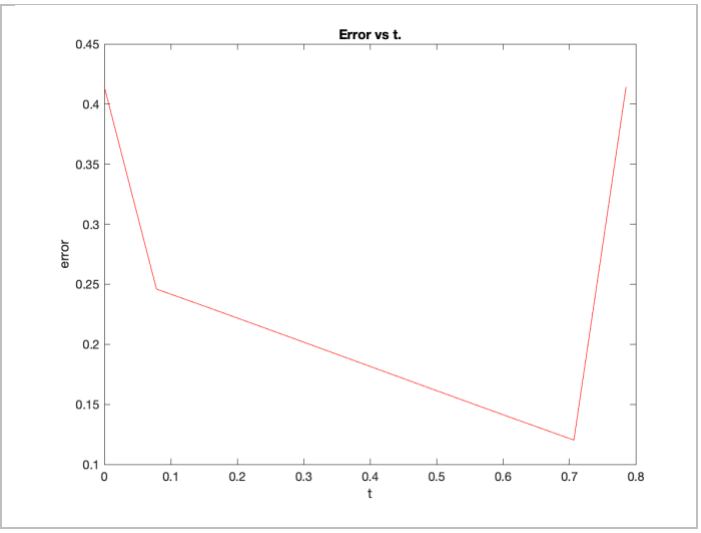

log(E_N) vs log(N)
log(E_N)
log(N)
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
2
3
4
5
6
7
8
9
10

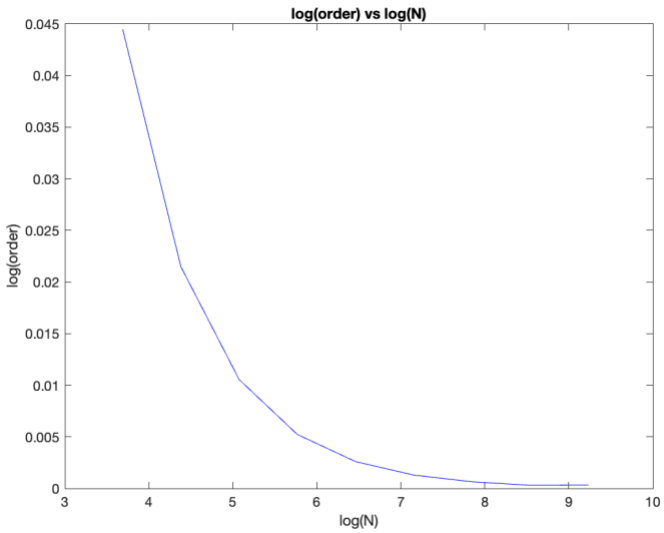
Central difference for 1st order and central for 2nd order.

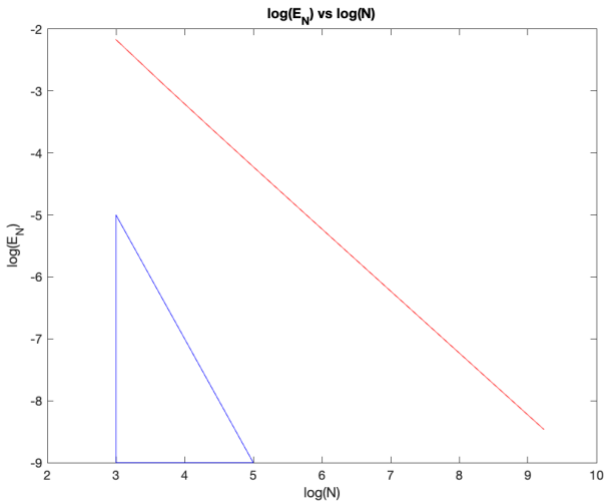
t_i	Approx.	Exact	Error
0	1.41421356237310	1	0.414213562373095
0.0785398163397448	1.27532586317697	1.02941615527513	0.245909707901838
0.157079632679490	1.28291416621367	1.05248561763738	0.230428548576286
0.235619449019234	1.28379960760057	1.06906615617991	0.214733451420657
0.314159265358979	1.27797032098778	1.07905554636903	0.198914774618748
0.392699081698724	1.26544171166982	1.08239220029239	0.183049511377430
0.471238898038469	1.24625777394905	1.07905554636903	0.167202227580021
0.549778714378214	1.22049230536637	1.06906615617991	0.151426149186458
0.628318530717959	1.18824994958727	1.05248561763738	0.135764331949891
0.706858347057704	1.14966701441905	1.02941615527513	0.120250859143916
0.785398163397448	0.585786437626905	1	0.414213562373095

N	E_N	Order
20	0.114713612433170	0
40	0.0555759255744230	1.04550462264944
80	0.0273728296107905	1.02171549234399
160	0.0135861408674902	1.01060886017164
320	0.00676842624979845	1.00524338602630
640	0.00337810431626151	1.00260654735641
1280	0.00168753143895040	1.00129949939360
2560	0.000843387241395543	1.00064727795336
5120	0.000421602640924501	1.00031129289717
10240	0.000210752737264563	1.00033253499711






log(order) vs log(N)
log(order)
log(N)
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
3
4
5
6
7
8
9
10


log(E_N) vs log(N)
log(E_N)
log(N)
-2
-3
-4
-5
-6
-7
-8
-9
2
3
4
5
6
7
8
9
10