

Lab 11

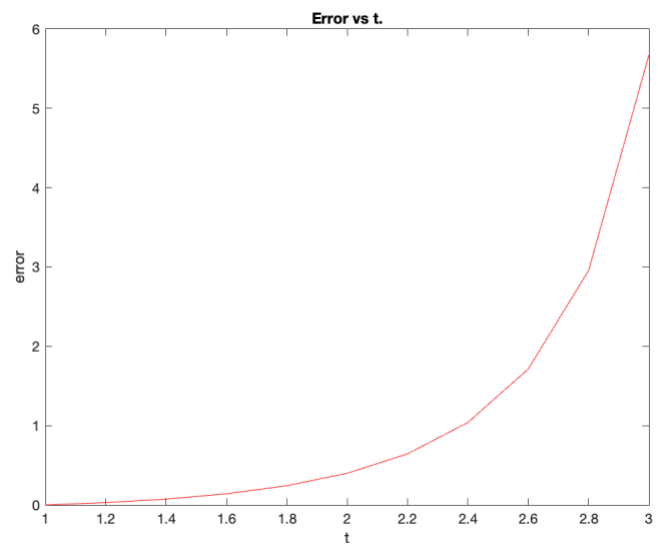
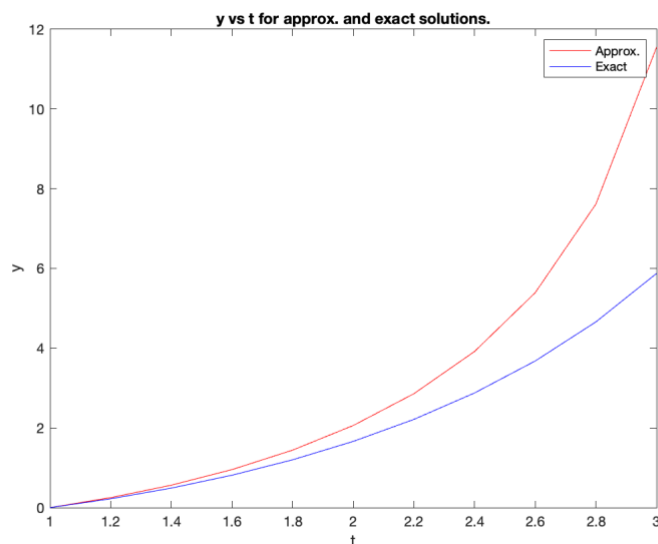
Naveen Kumar A G

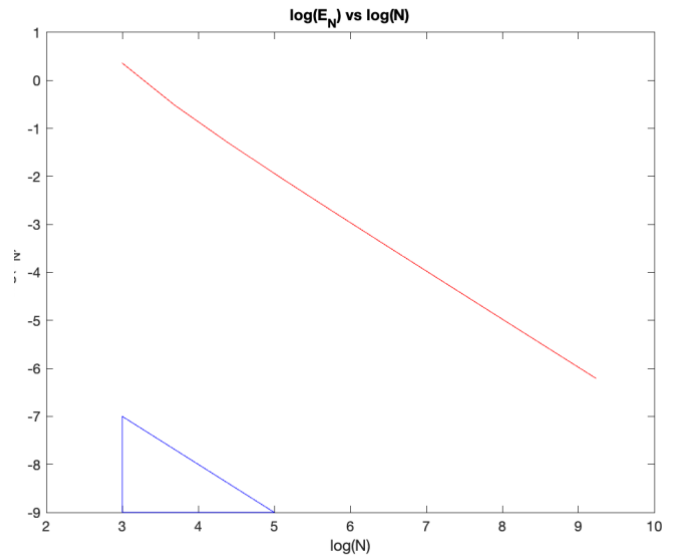
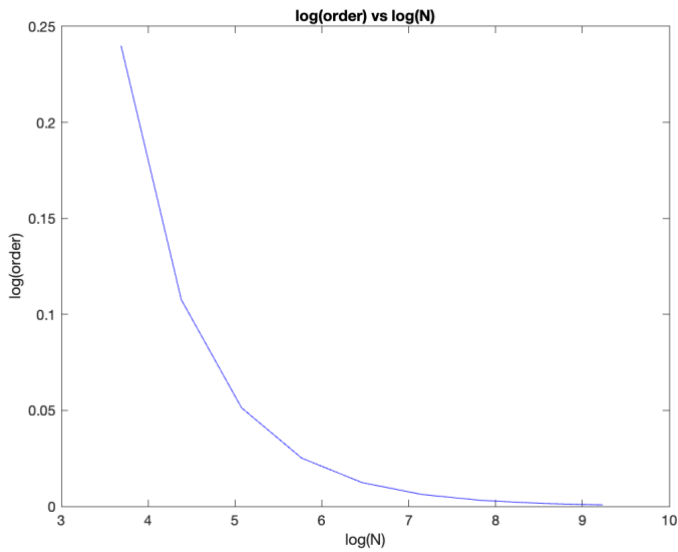
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Q1 (a) Implicit Euler Method.

t_i	Approx.	Exact	Error
1	0	0	0
1.2000000000000000	0.250454583026496	0.221242772757631	0.0292118102688648
1.4000000000000000	0.563305747476104	0.489681663750943	0.0736240837251613
1.6000000000000000	0.953529680251270	0.812752740561542	0.140776939689727
1.8000000000000000	1.44215151467760	1.19943864032594	0.242712874351662
2	2.06047770329629	1.66128175572157	0.399195947574721
2.2000000000000000	2.85773744096705	2.21350181348063	0.644235627486416
2.4000000000000000	3.91683450001300	2.87655141999484	1.04028308001815
2.6000000000000000	5.39162138733282	3.67847533085184	1.71314605648097
2.8000000000000000	7.61470635189777	4.65866505823952	2.95604129365826
3	11.5481105505513	5.87409997818417	5.67401057236710

N	E_N	Order
20	1.44268711912961	0
40	0.597742134785731	1.27116330426821
80	0.276223279564340	1.11368832951235
160	0.133156326299765	1.05271393918972
320	0.0654142920574456	1.02544319023029
640	0.0324248513954590	1.01250591518158
1280	0.0161428962350982	1.00620052339767
2560	0.00805419388198047	1.00308733253406
5120	0.00402279925177339	1.00154045601609
10240	0.00201032717815618	1.00076942822548

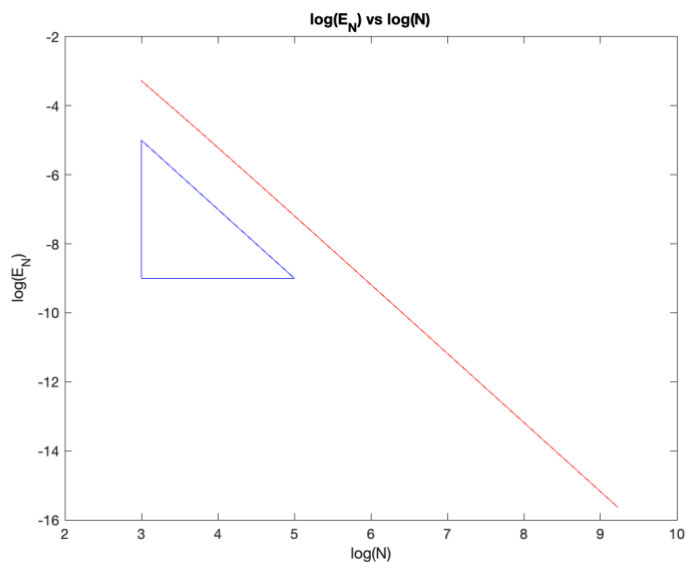
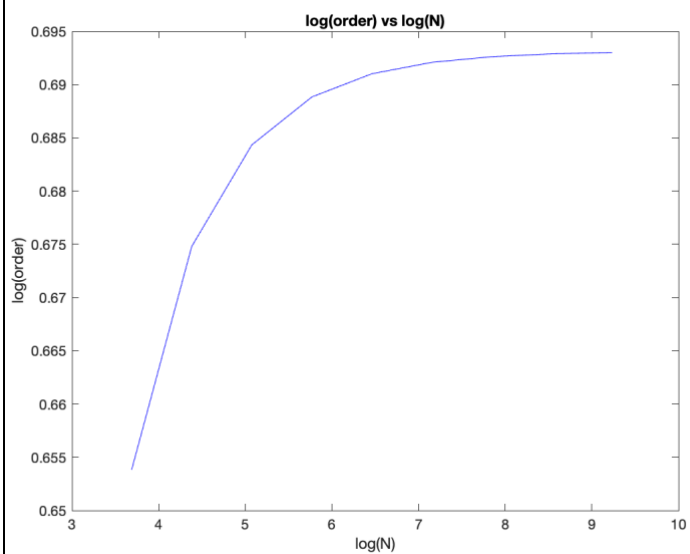
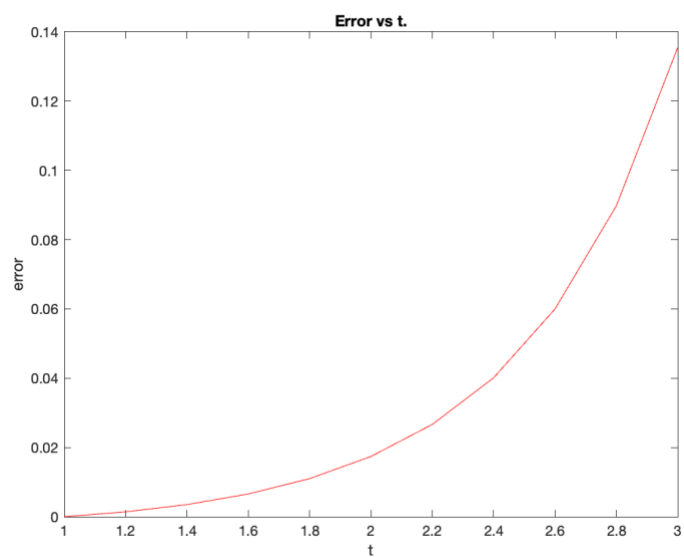
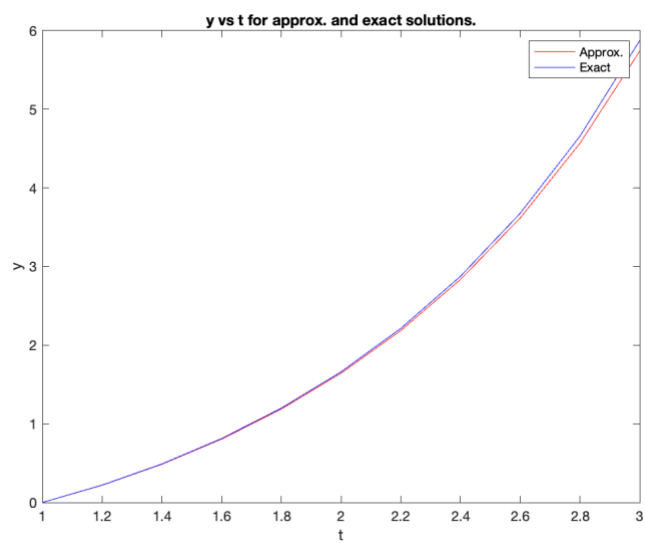




Runge-Kutta 2nd Order:

t_i	Approx.	Exact	Error
1	0	0	0
1.2000000000000000	0.219834710743802	0.221242772757631	0.00140806201382954
1.4000000000000000	0.486177046768206	0.489681663750943	0.00350461698273679
1.6000000000000000	0.806184891605961	0.812752740561542	0.00656784895558080
1.8000000000000000	1.18843925837304	1.19943864032594	0.0109993819529001
2	1.64388892124877	1.66128175572157	0.0173928344727999
2.2000000000000000	2.18686089286093	2.21350181348063	0.0266409206196993
2.4000000000000000	2.83643571306701	2.87655141999484	0.0401157069278363
2.6000000000000000	3.61849255493937	3.67847533085184	0.0599827759124790
2.8000000000000000	4.56889438371891	4.65866505823952	0.0897706745206089
3	5.73864746521413	5.87409997818417	0.135452512970039

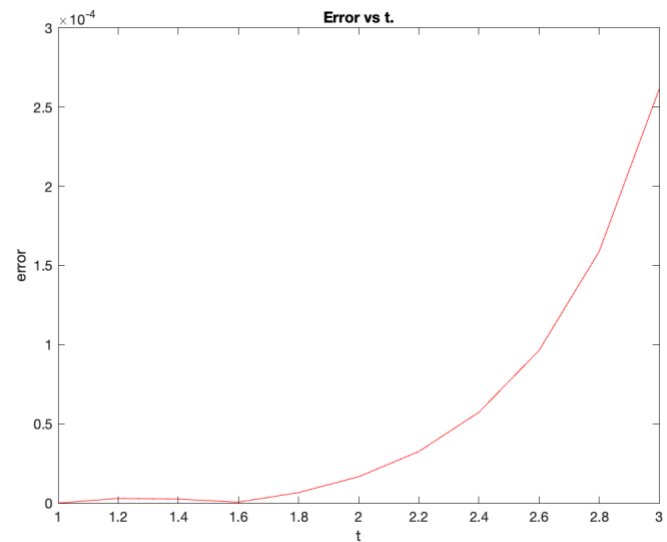
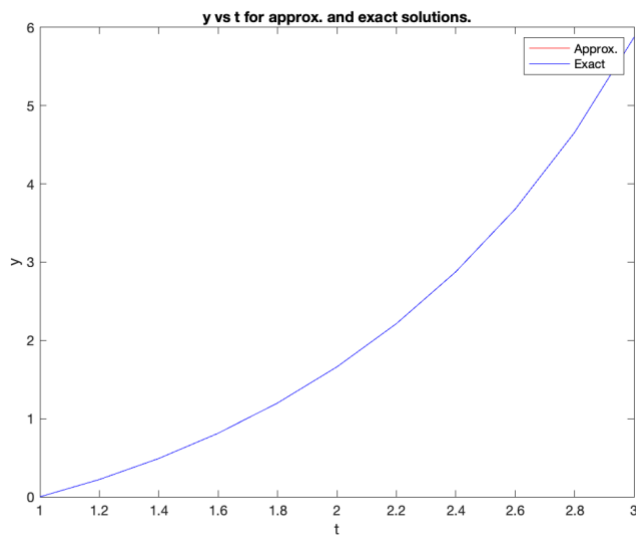
N	E_N	Order
20	0.0380119567016075	0
40	0.0100247431168368	1.92288802299014
80	0.00257015852844656	1.96363601491378
160	0.000650402203301821	1.98245329838746
320	0.000163573080298248	1.99139680543453
640	4.10141308238093e-05	1.99574237912044
1280	1.02685942413654e-05	1.99788236312875
2560	2.56902831608665e-06	1.99894399492924
5120	6.42491870157992e-07	1.99947268762195
10240	1.60652325893929e-07	1.99973633109467

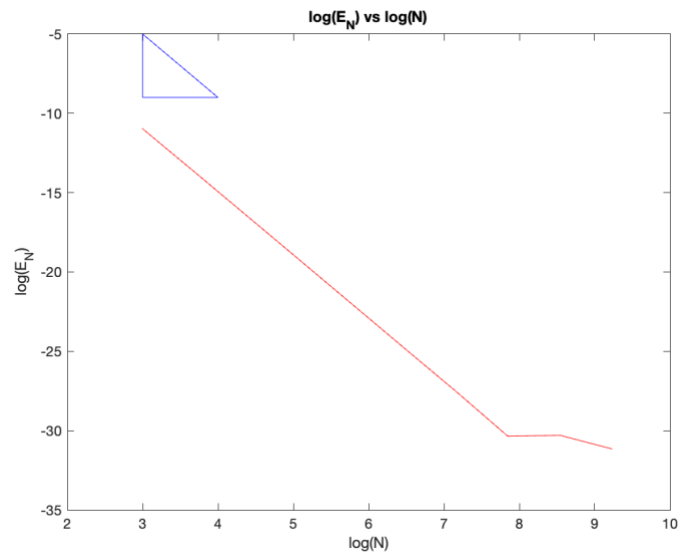
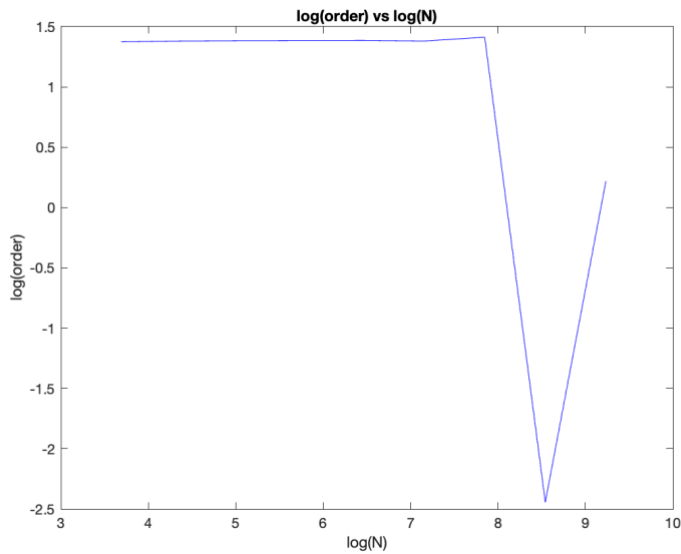


Runge-Kutta 4th Order:

t_i	Approx.	Exact	Error
1	0	0	0
1.2000000000000000	0.221245707288813	0.221242772757631	2.93453118138004e-06
1.4000000000000000	0.489684166443365	0.489681663750943	2.50269242242159e-06
1.6000000000000000	0.812752162024010	0.812752740561542	5.78537531970894e-07
1.8000000000000000	1.19943202163924	1.19943864032594	6.61868669977869e-06
2	1.66126511544869	1.66128175572157	1.66402728810697e-05
2.2000000000000000	2.21346931657636	2.21350181348063	3.24969042708339e-05
2.4000000000000000	2.87649411454154	2.87655141999484	5.73054533021028e-05
2.6000000000000000	3.67837899609925	3.67847533085184	9.63347525973452e-05
2.8000000000000000	4.65850628368228	4.65866505823952	0.000158774557231647
3	5.87383856983279	5.87409997818417	0.000261408351379229

N	E_N	Order
20	1.74579686278502e-05	0
40	1.12834635213768e-06	3.95160381577054
80	7.17129589116414e-08	3.97583232357072
160	4.51963067684776e-09	3.98795705286766
320	2.83648445199310e-10	3.99402902603964
640	1.77571362289427e-11	3.99763312017192
1280	1.12777167689722e-12	3.97685200923098
2560	6.55102869358677e-14	4.10560973847809
5120	6.95771851258612e-14	-0.0868928455743552
10240	2.93436602052167e-14	1.24556547526351

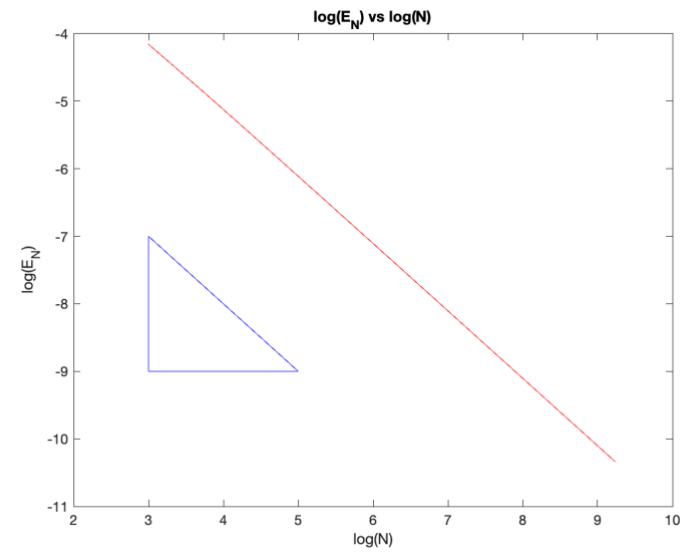
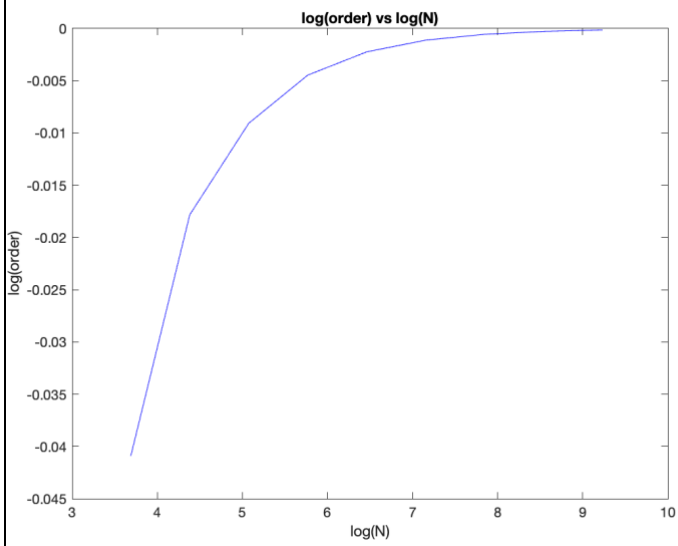
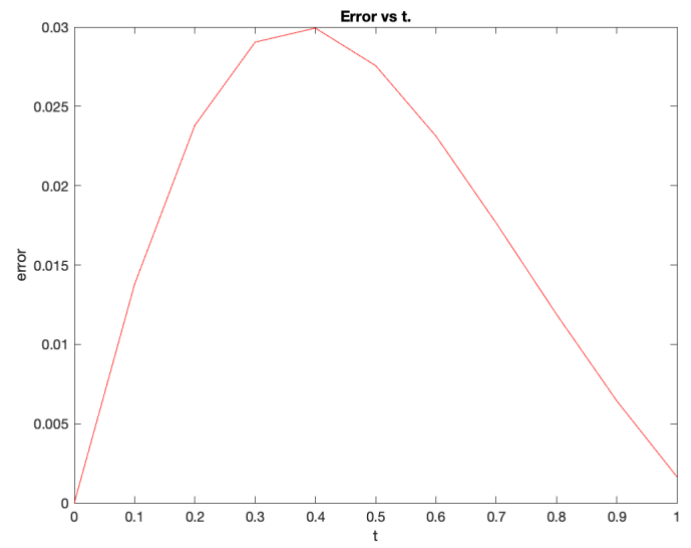
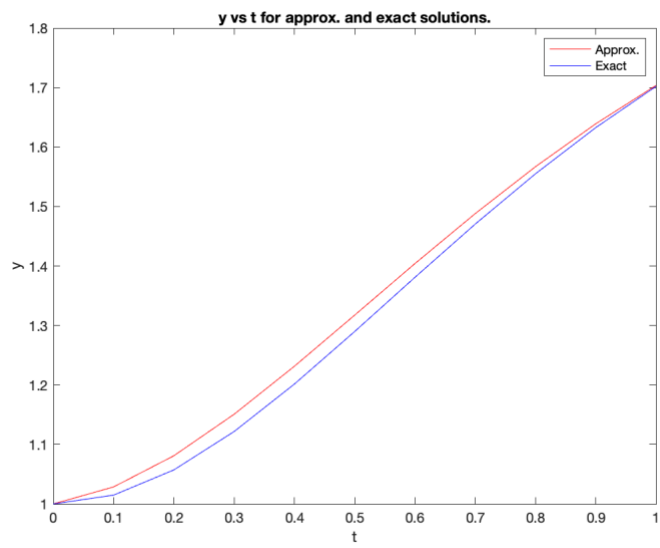




Q1 (b) Implicit Euler Method.

t_i	Approx.	Exact	Error
0	1	1	0
0.1000000000000000	1.02860172615079	1.01481549985822	0.0137862262925677
0.2000000000000000	1.08098831062131	1.05718100746420	0.0238073031571014
0.3000000000000000	1.15074609870334	1.12169801826798	0.0290480804353570
0.4000000000000000	1.23142052162560	1.20148601036440	0.0299345112611984
0.5000000000000000	1.31736970673967	1.28980527630561	0.0275644304340612
0.6000000000000000	1.40405913082250	1.38093121544373	0.0231279153787658
0.7000000000000000	1.48805948728036	1.47041518539722	0.0176443018831333
0.8000000000000000	1.56692676115531	1.55503142343518	0.0118953377201258
0.9000000000000000	1.63905137779955	1.63261318178287	0.00643819601668317
1	1.70350974414460	1.70187005276128	0.00163969138332539

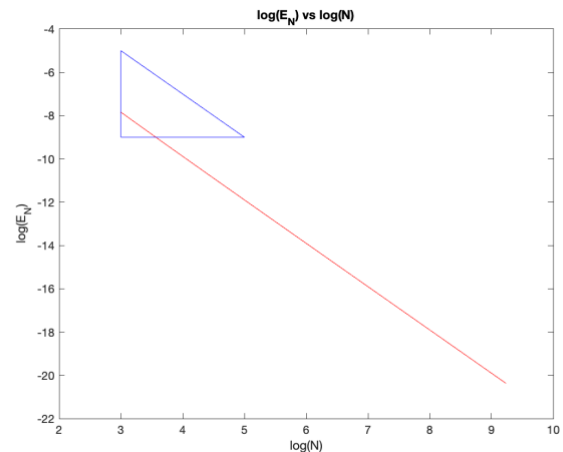
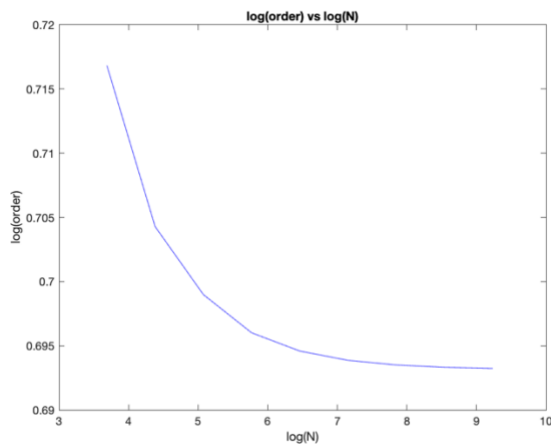
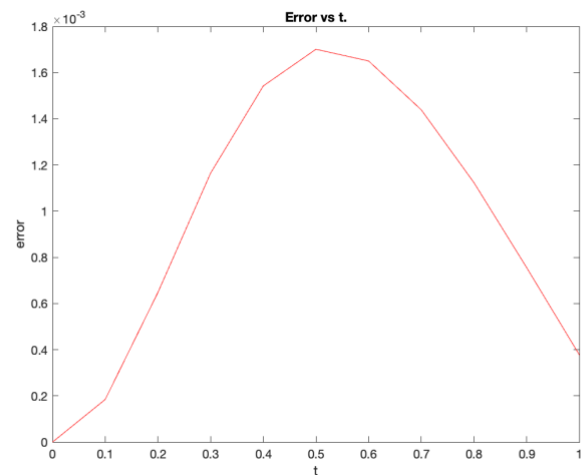
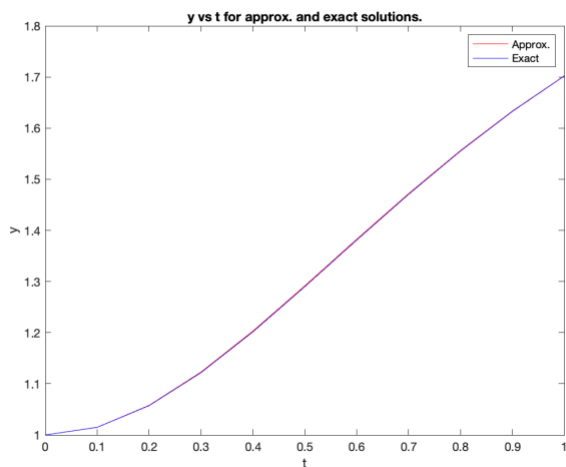
N	E_N	Order
20	0.0157343460407231	0
40	0.00808888708568283	0.959904090451583
80	0.00409427382207666	0.982333630267416
160	0.00205998490230000	0.990973828104670
320	0.00103317589888465	0.995547868980759
640	0.000517388437914982	0.997766175331208
1280	0.000258894124849358	0.998885588299537
2560	0.000129496989937911	0.999443662243025
5120	6.47609874521000e-05	0.999721675305933
10240	3.23836157371944e-05	0.999860907220482



Runge-Kutta 2nd Order:

t_i	Approx.	Exact	Error
0	1	1	0
0.1000000000000000	1.015000000000000	1.01481549985822	0.000184500141776589
0.2000000000000000	1.05782895089288	1.05718100746420	0.000647943428678609
0.3000000000000000	1.12286281792299	1.12169801826798	0.00116479965501010
0.4000000000000000	1.20302831702477	1.20148601036440	0.00154230666036412
0.5000000000000000	1.29150646848125	1.28980527630561	0.00170119217564577
0.6000000000000000	1.38258169760259	1.38093121544373	0.00165048215885258
0.7000000000000000	1.47185408694990	1.47041518539722	0.00143890155267378
0.8000000000000000	1.55615483380428	1.55503142343518	0.00112341036909905
0.9000000000000000	1.63336880578335	1.63261318178287	0.000755624000483904
1	1.70224778342493	1.70187005276128	0.000377730663653619

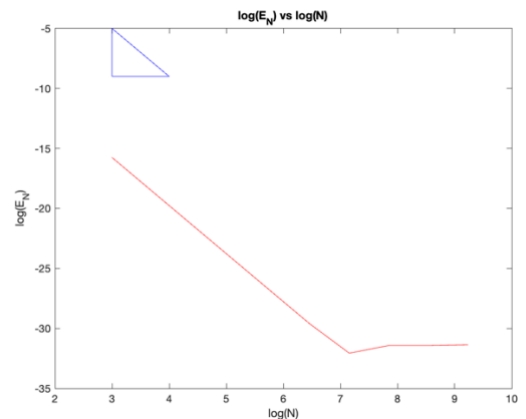
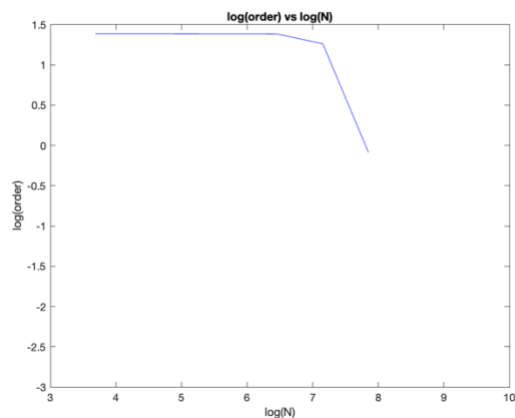
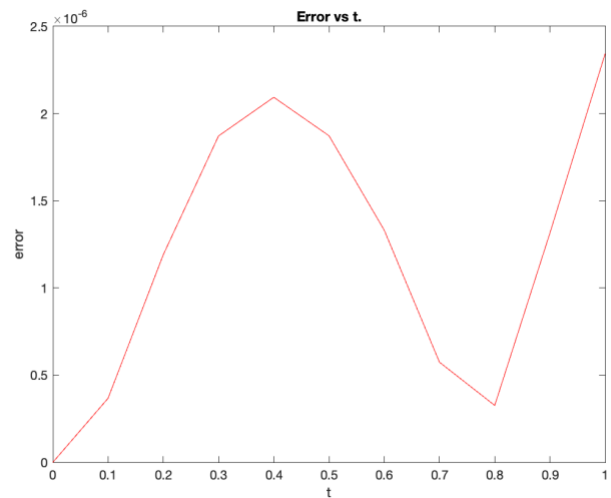
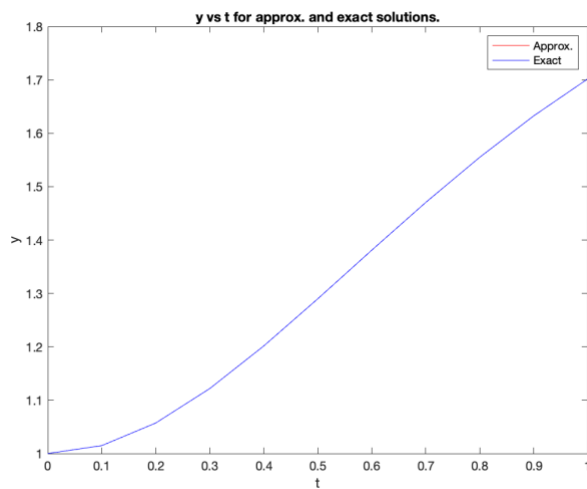
N	E_N	Order
20	0.000396717993641467	0
40	9.59394414928391e-05	2.04791788811572
80	2.36155896999070e-05	2.02238447980759
160	5.85616656315307e-06	2.01171106821280
320	1.45823299362614e-06	2.00573533876182
640	3.63829926123063e-07	2.00288513002672
1280	9.08667407348988e-08	2.00143997653823
2560	2.27053695459795e-08	2.00071881445674
5120	5.67492631642176e-09	2.00035995223725
10240	1.41855572710449e-09	2.00017883335922



Runge-Kutta 4th Order:

t_i	Approx.	Exact	Error
0	1	1	0
0.1000000000000000	1.01481586700284	1.01481549985822	3.67144615245631e-07
0.2000000000000000	1.05718219810417	1.05718100746420	1.19063996176707e-06
0.3000000000000000	1.12169988971777	1.12169801826798	1.87144978885972e-06
0.4000000000000000	1.20148810362448	1.20148601036440	2.09326007635605e-06
0.5000000000000000	1.28980714875618	1.28980527630561	1.87245057278853e-06
0.6000000000000000	1.38093254642537	1.38093121544373	1.33098163493582e-06
0.7000000000000000	1.47041575933439	1.47041518539722	5.73937165455836e-07
0.8000000000000000	1.55503109713822	1.55503142343518	3.26296961531965e-07
0.9000000000000000	1.63261186640638	1.63261318178287	1.31537648850618e-06
1	1.70186770854212	1.70187005276128	2.34421915371640e-06

N	E_N	Order
20	1.46087800112924e-07	0
40	9.07360301381880e-09	4.00901635193706
80	5.64729408112844e-10	4.00604387431433
160	3.52123112761594e-11	4.00340792971479
320	2.19760919288114e-12	4.00207317072509
640	1.37701392991628e-13	3.99631979150964
1280	1.18951804615372e-14	3.53309408975677
2560	2.25206279423167e-14	-0.920869895596479
5120	2.25206279423167e-14	0
10240	2.37739380526983e-14	-0.0781338448096464



Q2 Runge-Kutta 4th Order: domain: (0, 1) with $f(0) = 1$

t_i	Approx.	Exact	Error
0	1	1	0
0.1000000000000000	1.11422259959182	1.11422250235566	9.72361560847334e-08
0.2000000000000000	1.25293090533928	1.25293063285110	2.72488182939322e-07
0.3000000000000000	1.40845175999911	1.40845124727396	5.12725150901751e-07
0.4000000000000000	1.57122625952187	1.57122546062420	7.98897670036755e-07
0.5000000000000000	1.73049086605814	1.73048975851471	1.10754342422744e-06
0.6000000000000000	1.87508832068056	1.87508690788131	1.41279924689354e-06
0.7000000000000000	1.99434192183752	1.99434023319418	1.68864333655288e-06
0.8000000000000000	2.07891995120454	2.07891804003450	1.91117004245492e-06
0.9000000000000000	2.12161659014078	2.12161452944478	2.06069599510302e-06
1	2.11798166912361	2.11797954561299	2.12351061730271e-06

N	E_N	Order
20	1.32470363745477e-07	0
40	8.27551334165227e-09	4.00067701889637
80	5.17158938287909e-10	4.00016917028848
160	3.23196584527949e-11	4.00012387449140
320	2.01767530788570e-12	4.00164601790972
640	1.30612534275002e-13	3.94932877170998
1280	2.50470232884646e-14	2.38258229049339
2560	2.04521017995872e-14	0.292390044511038
5120	3.59321049651175e-14	-0.813024341279886
10240	2.81405276859487e-14	0.352624071419395

