Assignment 2 - Navneet Kaur - 18110106

1. Show that the columns of the rotation matrix are orthogonal.

Any rotation matrix can be written as a product of $R_{x,\theta}$, $R_{y,\phi}$ and $R_{z,\psi}$. If we prove that these three have orthogonal columns (they are orthogonal matrix), then R would also have orthogonal columns (since it would be an orthogonal matrix).

Imp: If A and B are orthogonal matrices then AB is also orthogonal.

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\theta & -s_\theta \\ 0 & s_\theta & c_\theta \end{bmatrix} = [C_1 \quad C_2 \quad C_3]$$

$C_1 \cdot C_2 = 0 \qquad C_2 \cdot C_3 = -C_\theta S_\theta + S_\theta C_\theta = 0$

$C_1 \cdot C_3 = 0$

Thus, $R_{x,\theta}$ is orthogonal. Similarly, $R_{y,\phi}$ and $R_{z,\psi}$ will also be orthogonal matrix.

$\Rightarrow$ R is orthogonal matrix

$\Rightarrow$ R has orthogonal columns.

2. Show that $\det(R_0^1) = 1$.

$\det(I) = 1 = \det(R R^{-1})$ $\quad (RR^{-1} = I)$

$= \det(R R^T)$ $\quad (R^T = R^{-1})$

$$= \det(R^2)$$

$$= \det(R) \cdot \det(R)$$

$$\Rightarrow \det(R_0^1) = 1 \quad \{\text{for right handed systems}\}$$

3. Done

4. Done

5. Show that $RS(a)R^T = S(Ra)$

$$RS(a)R^T = R(a \times R^T) = (Ra) \times (RR^T)$$

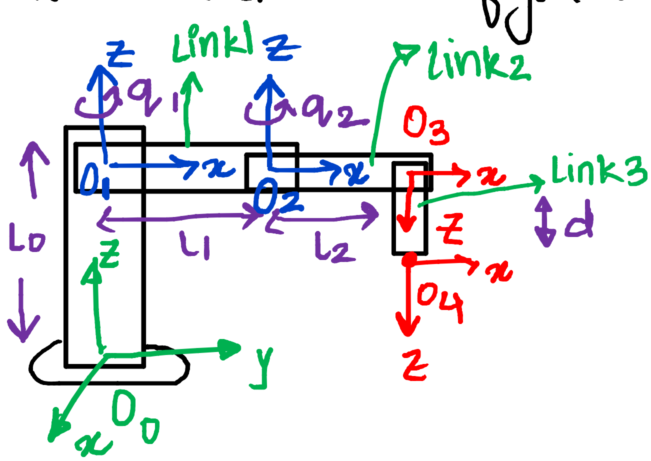
$$= (Ra) \times I$$

$\{S(a)R = a \times R\}$

$$= S(Ra) I$$

$$= \boxed{S(Ra)}$$

6. RRP SCARA configuration.

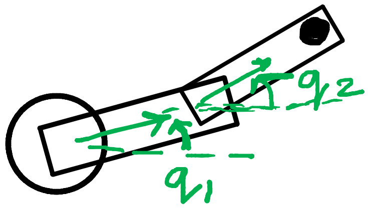


Top view

0^{th} frame – Base frame

1^{st} frame – x along link 1 rotated by q_1 translated in z direction $= l_0$

2^{nd} frame – x along link 2 (rotated by q_2) and translated in x $= l_1$



3rd frame – z along link3 (rotated about x-direction by 180°) translated in x direction $= l_2$

4th frame – end effector frame

$$\begin{bmatrix} P_4^0 \\ 1 \end{bmatrix} = H_0^1 \; H_1^2 \; H_2^3 \begin{bmatrix} P_4^3 \\ 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \quad H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \quad H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$$

$$P_4^3 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix} \rightarrow \text{Variable}$$

$$\begin{bmatrix} p_4^0 \\ 1 \end{bmatrix} = H_0^1 \; H_1^2 \; H_2^3 \begin{bmatrix} p_4^3 \\ 1 \end{bmatrix}$$

$$= H_0^1 H_1^2 \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$= H_0^1 \begin{bmatrix} c_2 & -s_2 & 0 & l_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 c_2 + l_1 \\ l_2 s_2 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_2 c_1 c_2 + l_1 c_1 - l_2 s_1 s_2 \\ l_2 c_2 s_1 + l_1 s_1 + l_2 s_2 c_1 \\ -d + l_0 \\ 1 \end{bmatrix}$$

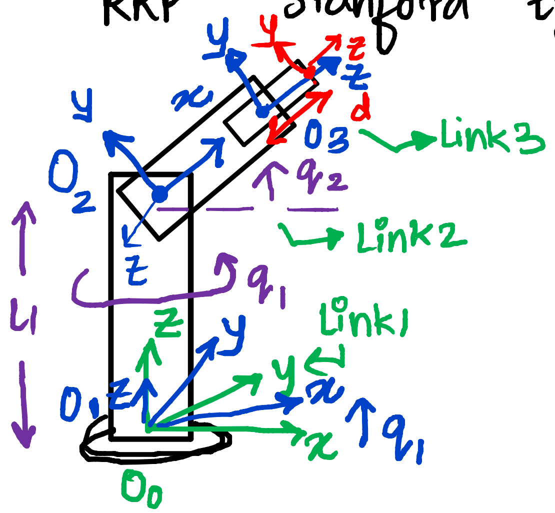
$c_{12} = \cos(q_1 + q_2)$

$s_{12} = \sin(q_1 + q_2)$

$$= \begin{bmatrix} l_2 c_{12} + l_1 c_1 \\ l_2 s_{12} + l_1 s_1 \\ l_0 - d \\ 1 \end{bmatrix}$$

$$\Rightarrow p_4^0 = \begin{bmatrix} l_2 c_{12} + l_1 c_1 \\ l_2 s_{12} + l_1 s_1 \\ l_0 - d \end{bmatrix}$$

8. RRP Stanford type configuration



0^{th} frame – Base frame

1^{st} frame – At base with rotation along z by q_1

2^{nd} frame – x along link2

2 rotations: along x by $90°$

along new z by q_2

translation by l_1 along old z

3^{rd} frame – z along link3 (prismatic joint) so rotation along y by $90°$. + translation by l_2 in old x

4^{th} frame – end effector frame

$$\begin{bmatrix} P_4^0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_4^3 \\ 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\pi/2 & -S\pi/2 \\ 0 & S\pi/2 & C\pi/2 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_2 & -S_2 & 0 \\ 0 & 0 & -1 \\ S_2 & C_2 & 0 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} C_{\pi/2} & 0 & S_{\pi/2} \\ 0 & 1 & 0 \\ -S_{\pi/2} & 0 & C_{\pi/2} \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$P_4^3 = \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix} \qquad H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \qquad H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_4^0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_4^3 \\ 1 \end{bmatrix}$$

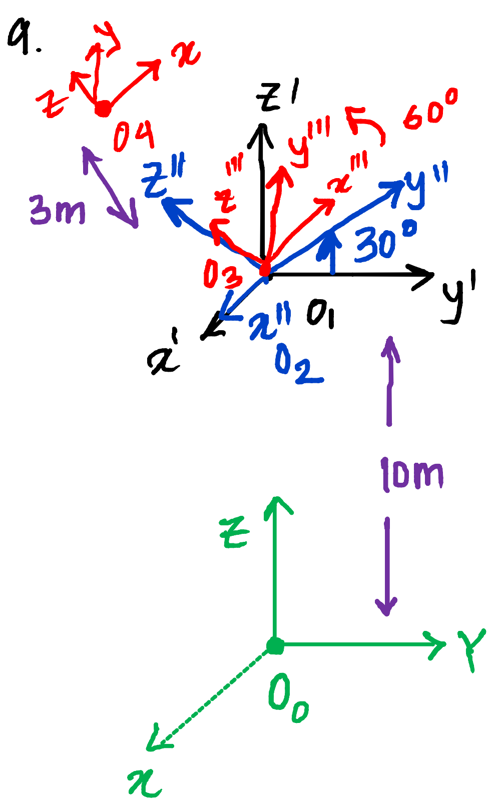
$$= H_0^1 H_1^2 \begin{bmatrix} 0 & 0 & 1 & L_2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$= H_0^1 \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d+l_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (d+l_2)c_2 \\ 0 \\ (d+l_2)s_2+l_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (d+l_2)c_1c_2 \\ (d+l_2)s_1c_2 \\ (d+l_2)s_2+l_1 \\ 1 \end{bmatrix} \qquad P_4^0 = \begin{bmatrix} (d+l_2)c_1c_2 \\ (d+l_2)s_1c_2 \\ (d+l_2)s_2+l_1 \end{bmatrix}$$

9.



0^{th} frame – on the base

1^{st} frame – 10 m above (in z - direction)
no rotation

2^{nd} frame – Rotation of 30° about x-axis

3^{rd} frame – Rotation of 60° about z-axis (new)

4^{th} frame – obstacle frame

$$\begin{bmatrix} P_4^0 \\ 1 \end{bmatrix} = H_0^1 \; H_1^2 \; H_2^3 \begin{bmatrix} P_4^3 \\ 1 \end{bmatrix}$$

$$R_0^1 = I \qquad\qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30^\circ & -\sin 30^\circ \\ 0 & \sin 30^\circ & \cos 30^\circ \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} \cos 60^\circ & -\sin 60^\circ & 0 \\ \sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_4^3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \qquad H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \qquad H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$P_4^3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \qquad H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \qquad H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_4^0 \\ 1 \end{bmatrix} = H_0^1 \; H_1^2 \; H_2^3 \begin{bmatrix} P_4^3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$= H_0^1 H_1^2 \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= H_0^1 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -3/2 \\ 10 + 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

$$p_4^0 = \begin{bmatrix} 0 \\ -3/2 \\ 10 + \frac{3\sqrt{3}}{2} \end{bmatrix}$$

Coordinates of obstacle

10.

A. Planetary Gearbox - They are compact and produce high gear ratios. They have coaxial configuration. But they have heavy losses owing to high virtual powers.

B. Harmonic Gearbox - It has zero backlash and is again compact and light weight. It is used to provide a boundary condition to the robotic joint. With continuous use and varied payload and speed, their average efficiency drops below 50%.

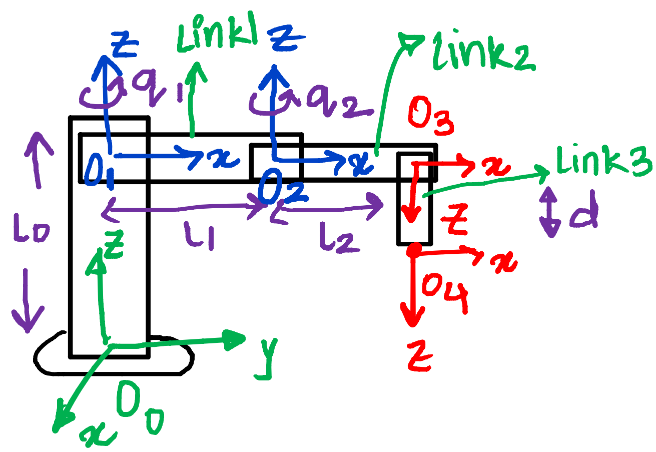
C. Cycloid Gearbox - it has high robustness and torsional stiffness (their torque-to-weight ratio are larger than that of Planetary Gearbox but lower than Harmonic Gearbox). But their efficiency is higher than that of Harmonic Gearbox. There is some backlash that is found in these but it is overlooked by their torsional rigidity. High input speeds tend to cause problems due to the presence of a large cam (and thus, larger inertia).

Reference - link

Typically, a gear box would not be seen with the motors in a drone due to the following reasons:

1. The drones need to be lightweight and gearboxes are heavy and not compact.
2. The required rpm for the motors of the drone does not require any gear to work efficiently (for example, for diesel engines, gearbox is used so that the motors rotate at their maximum efficiency). Without losing much efficiency, the electrical energy can be converted to mechanical energy in the case of drones.

11.



3 dof system

Jacobian will be 6x3 matrix

For revolute $\rightarrow$ $J_i = \begin{bmatrix} z_{i-1} \times (O_n - O_{i-1}) \\ z_{i-1} \end{bmatrix}$

For prismatic $\rightarrow$ $J_i = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$

1st and 2nd joint are revolute and 3rd joint is prismatic.

$$J = \begin{bmatrix} z_0 \times (O_3 - O_1) & z_1 \times (O_3 - O_2) & z_3 \\ z_1 & z_2 & 0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_3 = -\hat{k} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

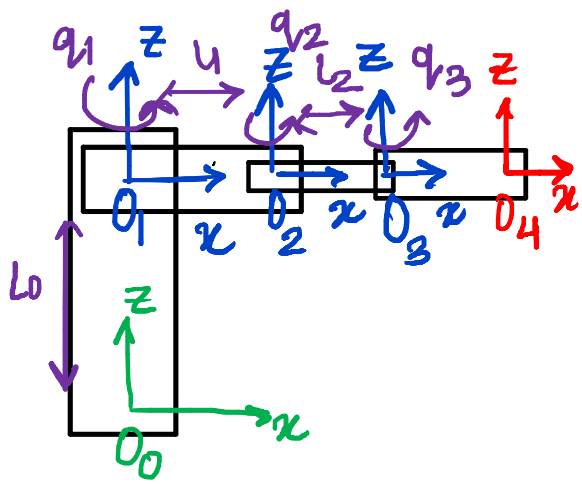
$$O_1 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix} \qquad O_2 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ l_0 \end{bmatrix} \qquad O_3 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ l_0 \end{bmatrix}$$

$$\Rightarrow J = \begin{bmatrix} z_0 \times (O_3 - O_1) & z_1 \times (O_3 - O_2) & z_3 \\ z_1 & z_2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_2 c_{12} \\ l_2 s_{12} \\ 0 \end{bmatrix} & \begin{matrix} 0 \\ 0 \\ -1 \end{matrix} \\ \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

13.



$$R_0^1 = I$$

$$R_1^2 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3^4 = \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ L_0 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \quad d_3^4 = \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

For revolute $\rightarrow$ $J_i = \begin{bmatrix} z_{i-1}^0 \times (o_n - o_{i-1}) \\ z_{i-1}^0 \end{bmatrix}$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_0 \times (O_3 - O_1) & z_0 \times (O_3 - O_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ L_0 \end{bmatrix} \quad O_1 = \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ L_0 \end{bmatrix} \quad O_2 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ L_0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 c_{12} + l_3 s_{123} \\ L_0 \end{bmatrix}$$

$$z_0 = z_1 = z_2 = \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_1 = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ 0 \end{bmatrix} \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_2 c_{12} + l_3 c_{123} \\ l_2 s_{12} + l_3 s_{123} \\ 0 \end{bmatrix} \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_3 = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_3 c_{123} \\ l_3 s_{123} \\ 0 \end{bmatrix} \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$