Computational MR imaging **Name:** *Navaneeth N*
Laboratory 6: k-space parallel imaging
14/12/2022

1. **Prepare data and implement GRAPPA reconstruction:**

1.1 In this task, GRAPPA reconstruction for 8-coil k-space data was initialized with parameters: kernel size = 2x3, acceleration R = 4, and ACS lines of 24. Figure 1. shows the fully sampled k-space, undersampled kdata lines in PE and acquired ACS region used in the algorithm.

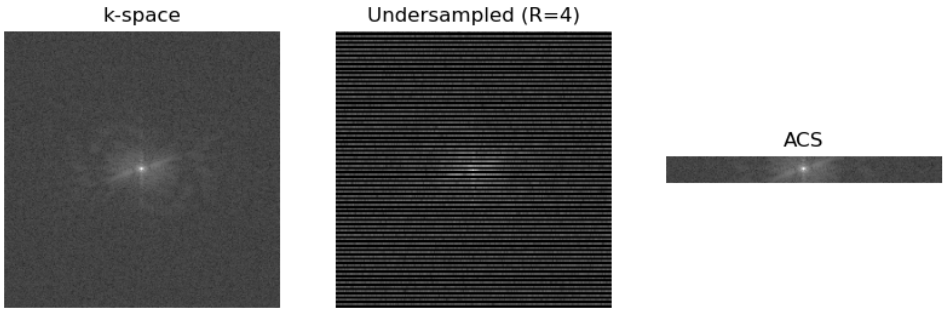

Figure 1: k-space(left), Undersampling(R=4) & ACS lines(right).

1.2 Figure 2. shows the reconstruction results for GRAPPA reconstruction with the same parameters previously mentioned but simulated for R = 2,3,4 and for a kernel size of 2x3. These reconstructions are compared to the least squares combination result (Ground truth).

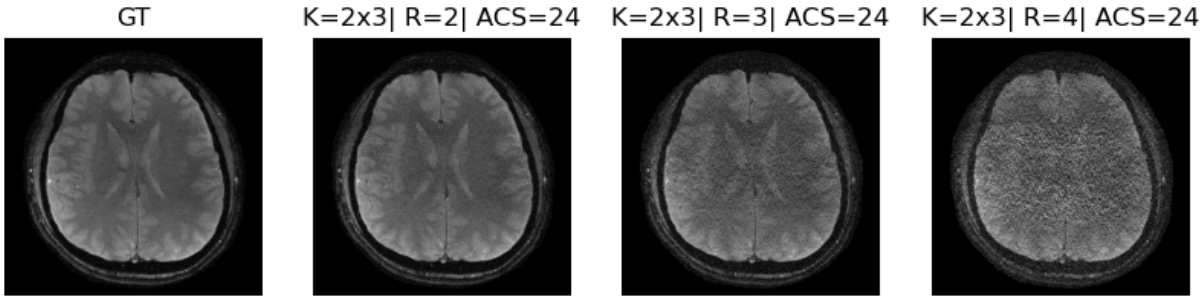

Figure 2: GRAPPA (ACS= 24, Kernel= 2x3).

1.3 When comparing these reconstruction results to the ground truth it is clear that by increasing the acceleration factor we increase the amount of spatially varying noise in the reconstruction image. This is apparent in the center region of the image which shows more noise for higher acceleration (R=3,4).

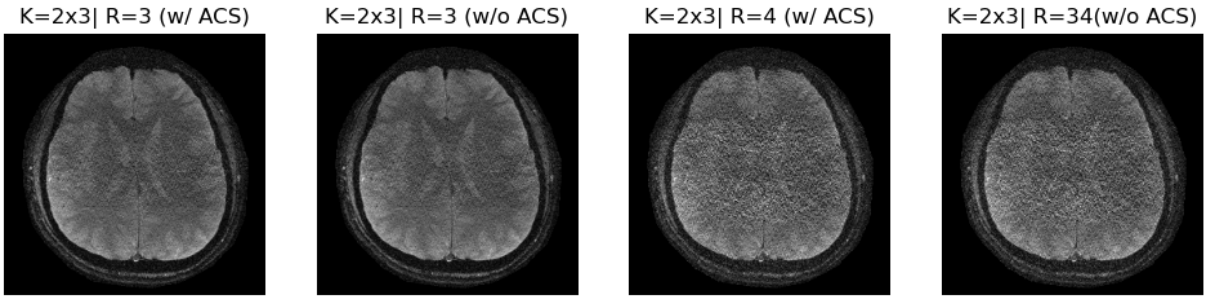

Figure 3: Comparing GRAPPA results with & without ACS lines.

1.4 An important point worth mentioning is that the results in Fig 2. do not consider the location of the ACS lines in the downsampled kspace. As a result, the interpolated kspace contains estimated values in positions where ACS lines were originally located. Replacing the central lines of interpolated kspace with the ACS lines from the original k-space will replace these estimated data with true data in this region improving reconstruction. The results of this comparison were done for R=3,4 and are summarized in Figure 3. However, the figure, it is difficult to visually discern the reconstructions in terms of image quality.

2. **Evaluating GRAPPA with varying parameters (Downsampling, kernel size, ACS lines):**

2.1 First, the GRAPPA algorithm is tested using 24 ACS lines, R $\in$ [2,3,4] and $\text{kernel}_{\textbf{size}}$ $\in$ [(2x3), (4x5), (6x7)]. These results are then discussed in terms of image quality, reconstruction error, and RMSE.

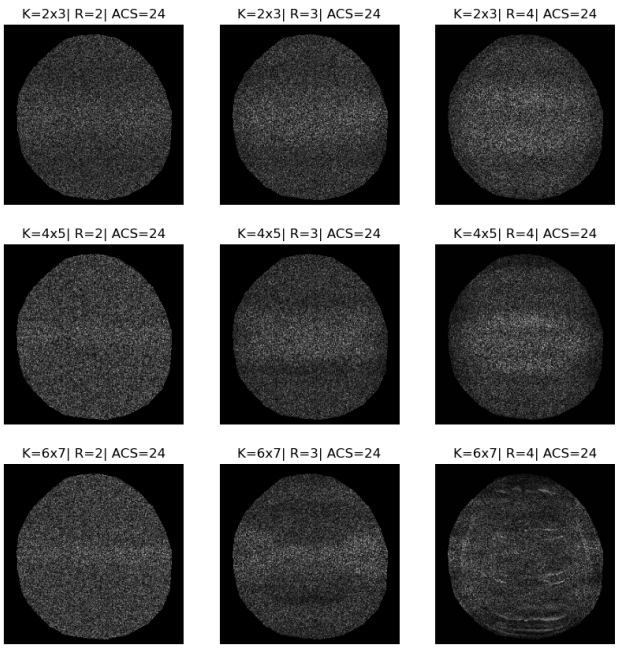

Figure 4: GRAPPA Reconstruction error .

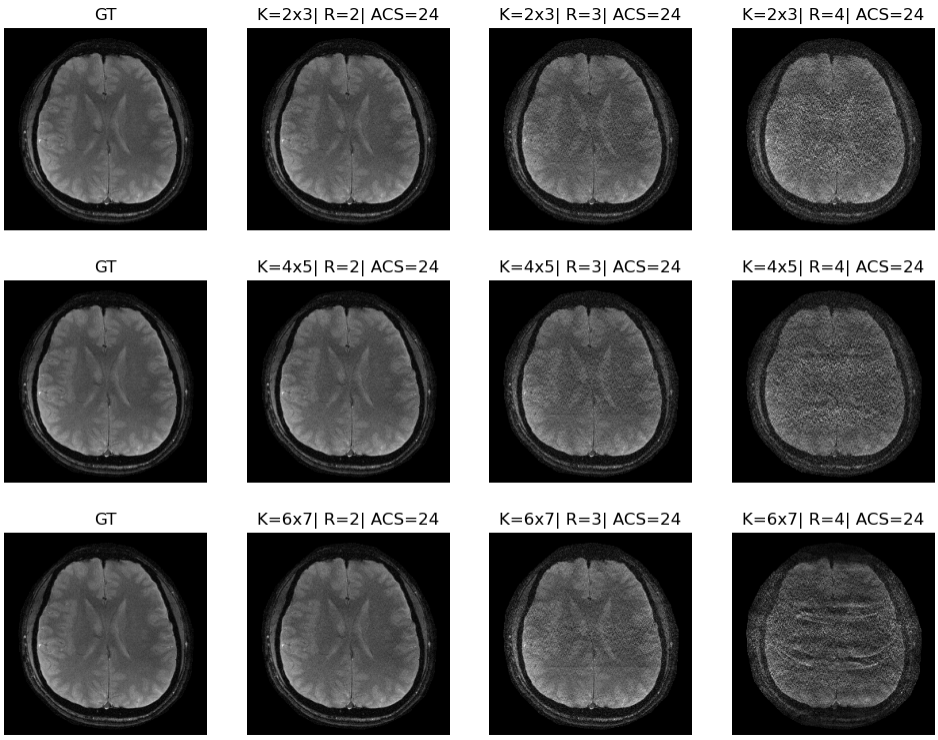

Figure 5: GRAPPA reconstruction results.

Parameters	K=2x3 and R=2	K=2x3 and R=3	K=2x3 and R=4
RMSE	0.074	0.188	0.339
Parameters	K=4x5 and R=2	K=4x5 and R=3	K=4x5 and R=4
RMSE	0.068	0.191	0.381
Parameters	K=6x7 and R=2	K=6x7 and R=3	K=6x7 and R=4
RMSE	0.067	0.191	0.416

Table 1: RMSE values for 24 ACS lines.

2.2 **Discussion:**

(a) **Image quality**: Figure 5. presents the GRAPPA results plotted for 24 ACS lines with increasing kernel size and R factor with each set of reconstructions compared to the ground truth. It is clear that increasing Rs results in noise enhancement, making structures in the image appear grainy. This noise artifact is more severe in the central region of the image. Increasing the kernel sizes for R=2 does not affect the image quality. But for higher R values we notice that noise amplifies along with residual aliasing artifacts being introduced into the image structures. Both these artifacts are clearly visible for the image with parameters: K=6x7, R=4 and ACS=24.

(b) **Reconstruction error**: Figure 4. summarizes the GRAPPA reconstruction errors of results in Figure 5. plotted against the ground truth. Similar to 2.2-(a), we see simi-

lar error profiles across kernel sizes for R=2. but for R=(3,4), error due to noise is more non-uniform (spatial SNR of some regions lower than others). In theory, larger kernel size may result in a more accurate interpolation of target points, but a very large kernel may not accurately estimate GRAPPA weights due to insufficient repetitions of the kernel through the acquired ACS. We see something similar in the error profile plot where larger kernels (4x5, 6x7) don't necessarily compensate for noise artifacts caused at higher accelerations. Instead, for K=6x7 and R=4 the image also presents aliasing artifacts.

(c) **RMSE**: Table 1. summarizes the RMSE values measured across R factors and kernel sizes. As discussed previously, increasing the R factor results in poorer image quality and the RMSE values reflect this behavior across all kernel sizes. For R=2 a larger kernel size has a lower RMSE meaning a more accurate reconstruction. But, for R=3 the RMSE shows a very small increase for larger kernels and for R=4, RMSE values show a larger increase for bigger kernels. This indicates that a larger kernel size helps in improving reconstruction at lower acceleration but is not effective at larger acceleration and in turn may introduce additional artifacts due to inaccurate GRAPPA weight estimation.

2.3 Experimenting with different ACS lines:

The same experiment discussed in (2.1)-(2.2) is repeated for ACS lines $\in$ [36,48,60] and their results are discussed with the same metrics used previously.Since R=4 gives the worst reconstruction quality, we will evaluate GRAPPA's performance for fixed R factor (R=4) but increasing ACS lines and kernel sizes.

Parameters	K=2x3 and ACS=24	K=4x5 and ACS=24	K=6x7 and ACS=24
RMSE	0.334	0.381	0.416
Parameters	K=2x3 and ACS=36	K=4x5 and ACS=36	K=6x7 and ACS=36
RMSE	0.299	0.320	0.347
Parameters	K=2x3 and ACS=48	K=4x5 and ACS=48	K=6x7 and ACS=48
RMSE	0.269	0.286	0.297
Parameters	K=2x3 and ACS=60	K=4x5 and ACS=60	K=6x7 and ACS=60
RMSE	0.244	0.256	0.263

Table 2: RMSE values for R=4.

(a) **Image quality:** Figure 6. presents the GRAPPA reconstructions a fixed R-factor of 4, $\text{kernel}_{\textbf{size}} \in$ [(2x3), (4x5), (6x7)] and ACS lines $\in$ [36,48,60]. For 24 ACS lines, increasing kernel size results in poor reconstruction with noise and folding artifacts in the image structures due to high acceleration and poor weights estimation. But, for 48 and 60 ACS lines we see that the reconstruction across larger kernel sizes is fairly decent. The level of spatially varying noise (especially at the center) is greatly

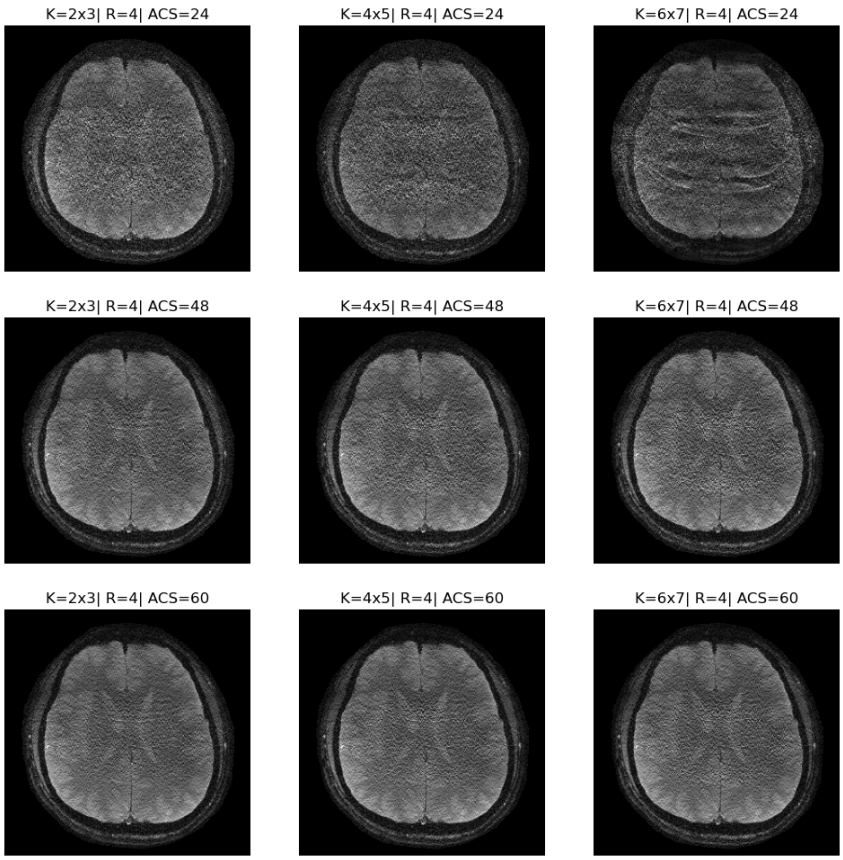

Figure 6: GRAPPA results for 24, 48 & 60 ACS lines.

reduced and aliasing artifacts are removed in comparison to results for 24 acquired ACS lines. This is due to more ACS lines being acquired, yielding more accurate GRAPPA weights.

(b) **RMSE:** RMSE values measured for this simulation are summarized in Table 2. Across all kernel sizes, the RMSE drops as more ACS lines are acquired. This is due to accurate interpolation of weights which yields an improved reconstruction. It is also interesting to note that one would expect increasing kernel sizes for ASCs of 48 and 60 would show an improvement in RMSE. On the contrary, the RMSE shows an increasing trend. However, the difference between RMSE values reduces. While this does not directly indicate that larger kernels result in better reconstruction it can be reasoned that these values were measured at high acceleration (R=4).

(c) **ACS vs kernel$_{\text{size}}$:** Increasing the ACS size results in more accurate GRAPPA weights (although more time is needed to acquire them). Using larger kernels results in better interpolation of target points but can lead to errors in GRAPPA weights for smaller ACS. This could lead to residual aliasing in the image structures. Hence, to achieve a fast but satisfactory reconstruction one will have to find a compromise

between kernel and ACS lines as well the acceleration factor.

2.4 **GRAPPA vs SENSE:**
The image reconstruction results and RMSE values for R values of 2, 3 and 4 are compared between the 2 algorithms:

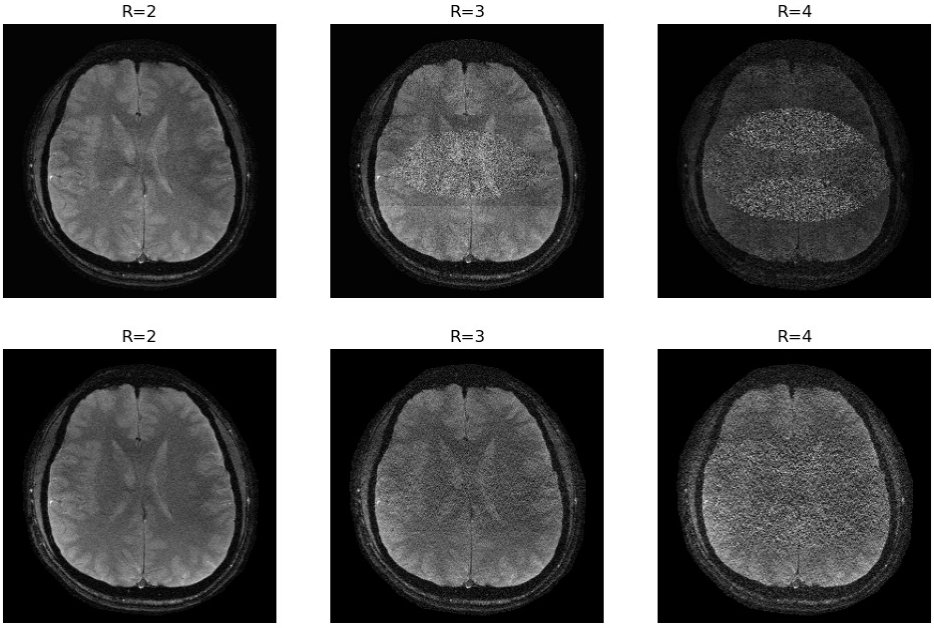

Figure 7: Cartesian SENSE (top row) vs GRAPPA (bottom row).

Parameters	R=2	R=3	R=4
$\mathbf{rmse_{SENSE}}$	0.682	0.722	0.857
$\mathbf{rmse_{GRAPPA}}$	0.074	0.188	0.339

Table 3: RMSE for SENSE vs GRAPPA.

(a) **Image quality:** Figure 7. shows the reconstruction results of Cartesian sense and GRAPPA for R=2,3,4. An ACS=24 and kernel size = 2x3 were initialized for the GRAPPA algorithm in this comparative study as these parameters yield the best RMSE values. From the images it is clear that GRAPPA is more more robust to noise and offers better results at higher acceleration.

(a) **RMSE:** It is also clear from table 3. that GRAPPA offers better reconstruction when compared to the ground truth (matched filter) across all accelerations this study was conducted for.