Computational MR imaging **Name:**
Laboratory 7: Parallel Imaging III: Non-Cartesian Imaging and Iterative Reconstruction
21/12/2022

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1. **Derivation of gradient descent (analytical):**

`Images/3akspace.png`

Figure 1: Derivation proof.

2. **Iterative image reconstruction with gradient descent:**

   2.1 Figure 2. shows the sampling trajectory, singlw channel radial kdata (512 read out points, 64 spokes) and its respective receive coil sensitivity map.

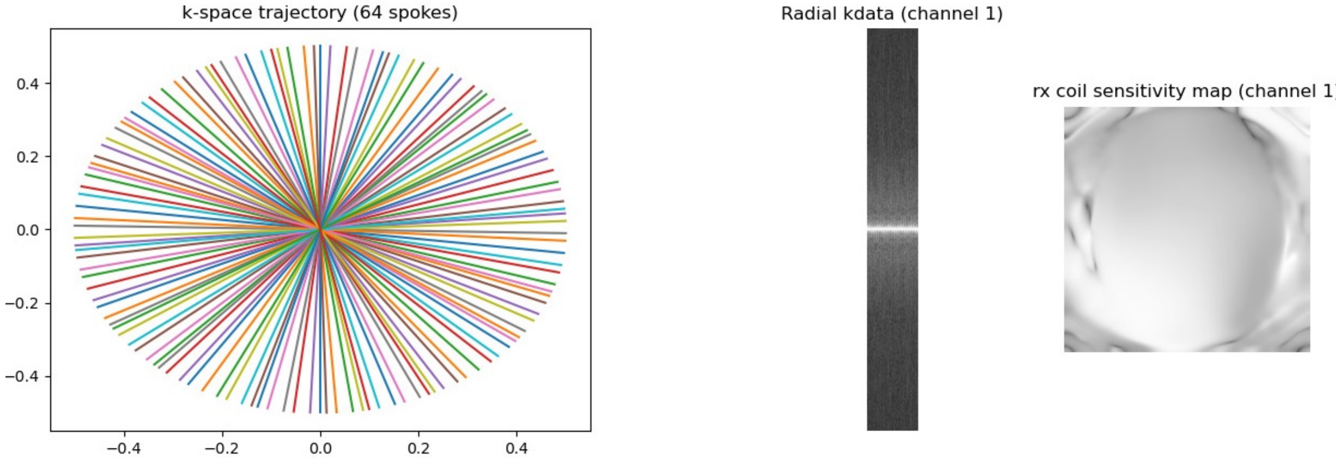

Figure 2: sampling trajectory, kspace data (1st channel) and sensitivity map (1st channel).

   2.2 Results of regridding using NUFFT operator along with density compensation is presented in the figure below. It is presented along with the ground truth for comparison. We can see that regridding reconstruction shows severe streaking artifacts in both the foreground and inside the ROI. Hence, we will attempt an iterative reconstruction using the gradient descent to improve upon the result in this step.

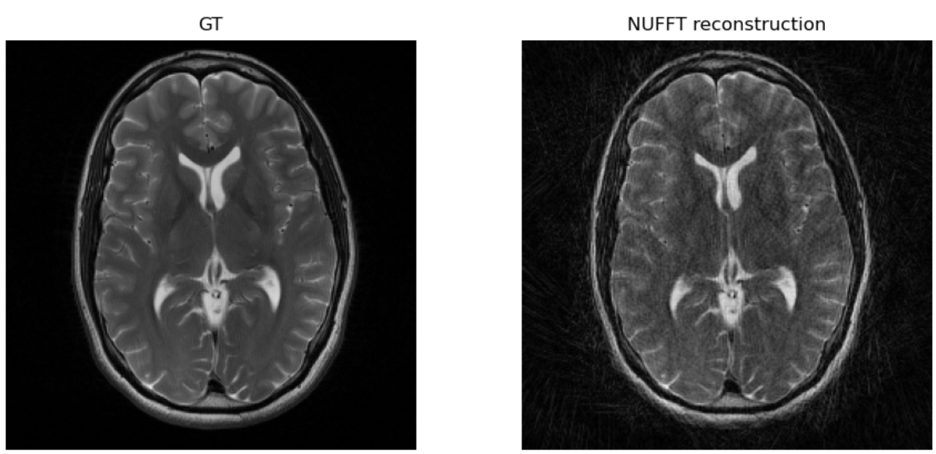

Figure 3: Gound truth vs NUFFT .

   2.3 Now we implement gradient descent reconstruction and set the initial hyperparameters (step size=$10^{-}2$ and 300 iterations). This result is then compared with results in 2.2 in summarized in Figure 4. It is also seen that algorithm despite being run over 300 iterations converges soon after 200 iterations and this is clearly seen in mean square error (mse) plot to the bottom right of this figure.

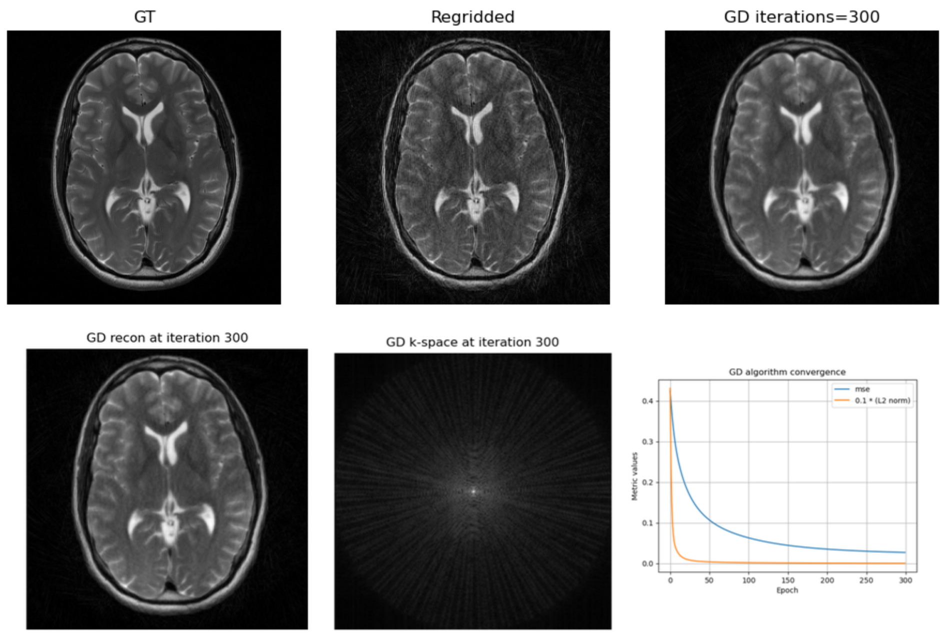

Figure 4: Evaluating GD reconstruction and it's convergence.

2.4 Figure 5. compares the reconstruction errors of the NUFFT operation and GD reconstruction after wrt to the GT. Here, we see that GD result is an improvement over NUFFT operation but there are still a few errors inside the image structures. When comparing the absolute reconstructions we see the image appears a little brighter and shows poorer resolution inside certain image structures.

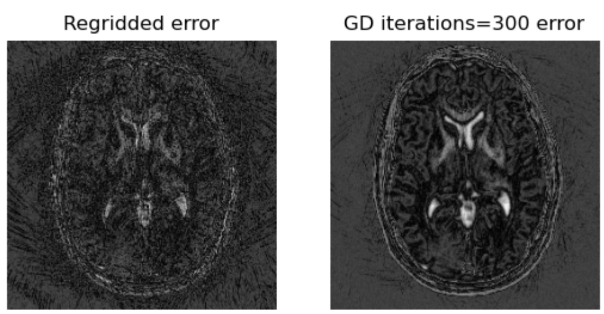

Figure 5: mse and L2 norm measured over iterations.

2.5 Now we experiment with different hyperparameters to not only study convergence behavior but also see how the GD algorithm performs accounts for density compensation without requiring any additional density information or filter operation. The hyperparameter combinations used were:
(i) step = 1×10^{-3}, iterations=100
(ii) step = 1×10^{-2}, iterations=200
(iii) step = 3×10^{-2}, iterations=300

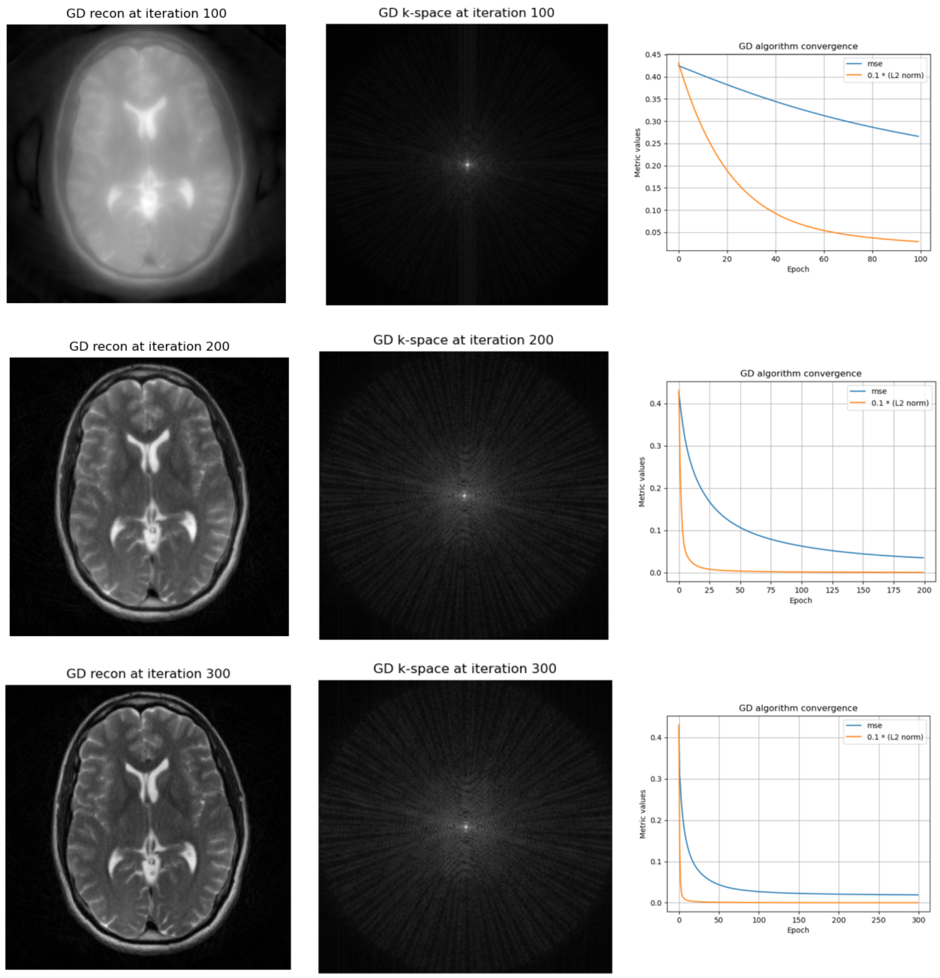

Figure 6: Comparative study of hyperparameters.

(a) We see that the efficacy of the reconstruction result is largely dependent on not only the iterations but also the step size. This can be justified by comparing the convergence of the mse values. From the above study we see that selected hyperparameters in (iii) show the best result and also completely converge.

3. **CG-SENSE:**

3.1 CG SENSE algorithm was implemented and run for 30 iterations. From the results below we see that the results are relatively similar to GD but the algorithm converges after 25 iterations. This is due to the use of non-orthogonal search directions during the gradient update.

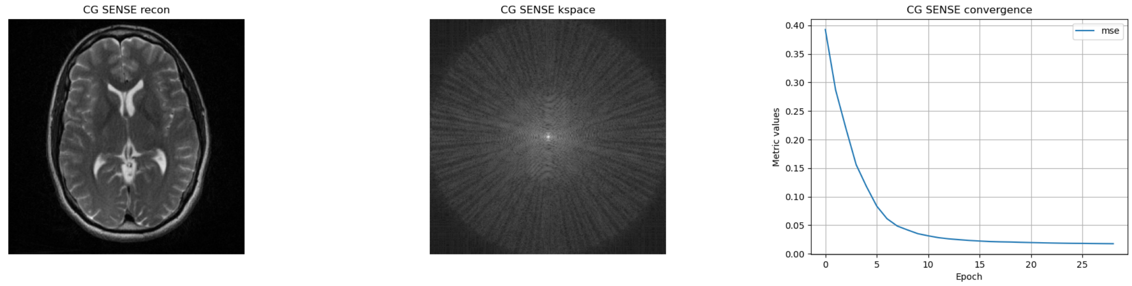

Figure 7: CG SENSE results.

3.2 Now we compare the results of CG sense with NUFFT operation, GD and GT. We see that the CG SENSE result is fairly comparable to the GD recon which was run for 300 iterations. We can also see that the error in reconstruction also shows that the CG SENSE algorithm offers a better result than the NUFFT result which corrupted with streak artifacts

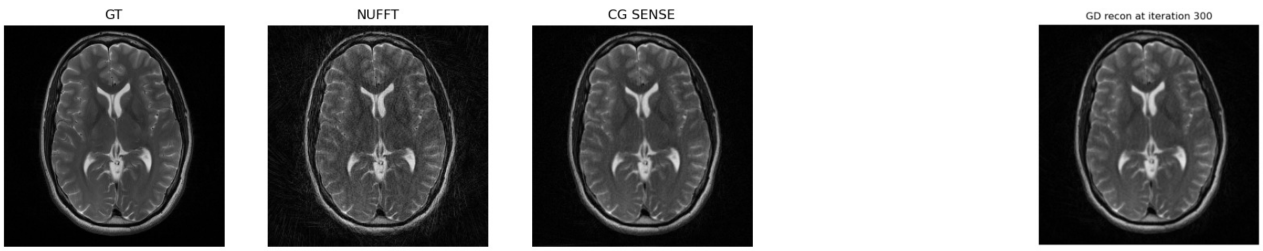

Figure 8: Comparing CG SENSE with other methods.

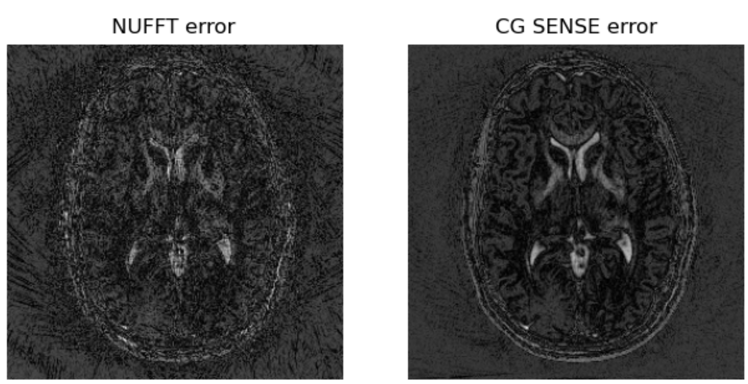

Figure 9: Comparing CG SENSE with other methods.

3.3 The figure below shows the evolution of noise and it's k-space estimated over 500 iterations. This is a true estimate of noise and can be used to calculate SNR calculations.

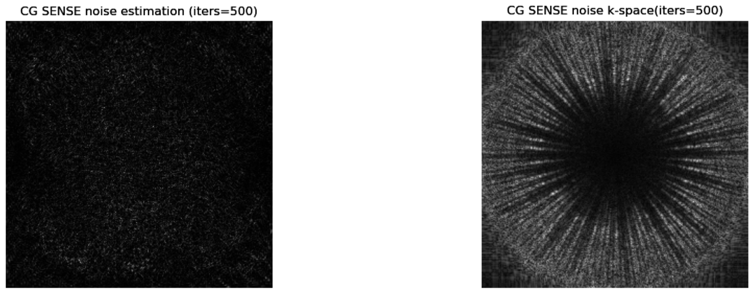

Figure 10: Comparing CG SENSE with other methods.