

1. Multicoil combination:

- 1.1 The figure below is a fully-sampled k-space (256x256) acquired with an 8-channel receive coil. It is evident from the reconstruction that each coil has a different spatial sensitivity.

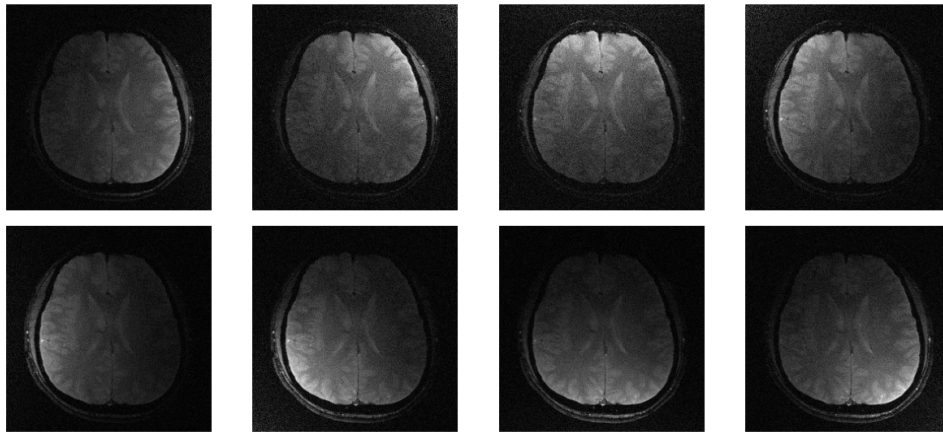


Figure 1: Multichannel coil image.

- 1.2 Now we discuss the approaches to obtain the optimal coil combination making use of coil sensitivity measures and noise maps. Figure 2. shows the noise covariance matrix Ψ . This is to show that the noise signals from the 8-coil array are correlated.

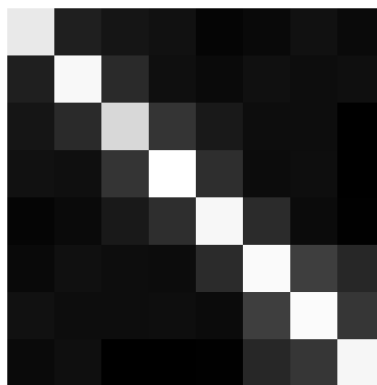


Figure 2: Ψ :noise covariance matrix .

- 1.3 Figure 3 summarizes the results of these combination approaches. It is evident that simply adding all coil images together yields the worst result while the sum of squares (SoS) and Least squares (with & without Ψ) offer better reconstruction.

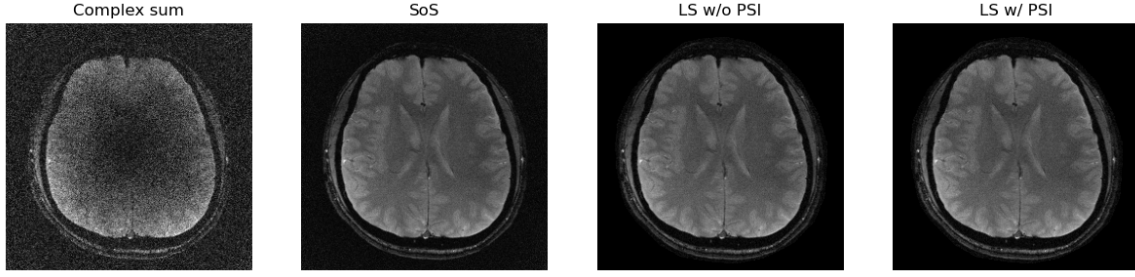


Figure 3: Reconstruction results.

- 1.4 To assess SoS and Least squares reconstructions closely, we zoom into the ROI shown in Figure 4. SoS is an approximation that considers the coil images as coil sensitivities. As a result, the SoS reconstruction suffers in terms of SNR and appears noisy.

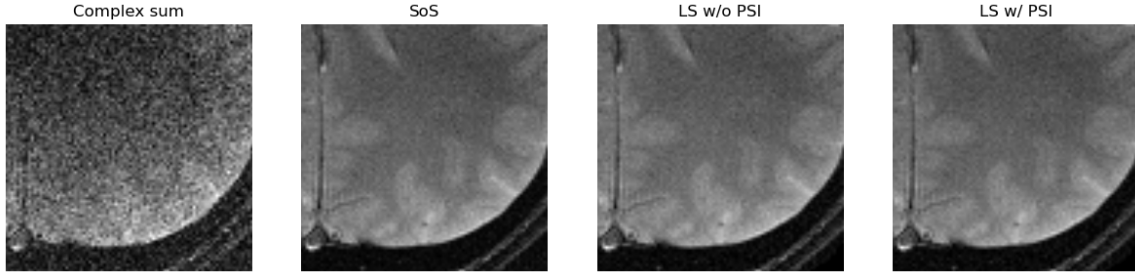


Figure 4: Reconstruction results(zoomed into ROI).

- 1.5 For the least squares (LS) approach, Ψ is combined with coil images and respective sensitivities to simulate coils with uncorrelated noise. On observing Figure 4 closely it can be seen that LS with Ψ offers marginal improvement (less grainy) in comparison to LS without Ψ .

2. Cartesian SENSE reconstruction and g-factor:

- 2.1 The Cartesian SENSE algorithm takes aliased multi-coil images and unfolds them using the sensitivity maps in the image domain. Figure 5 shows the sensitivity map of the 8-coil array.

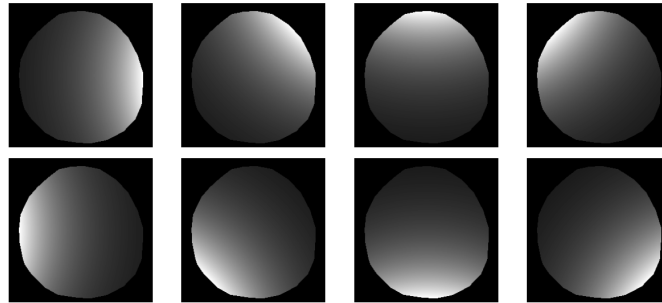


Figure 5: Sensitivity map.

2.2 The SENSE reconstruction results and respective g-factors for accelerations $R=(2,3,4)$ along PE are simulated and presented below in Figure 6. SENSE reconstruction for $R=1$ is also included for comparison.

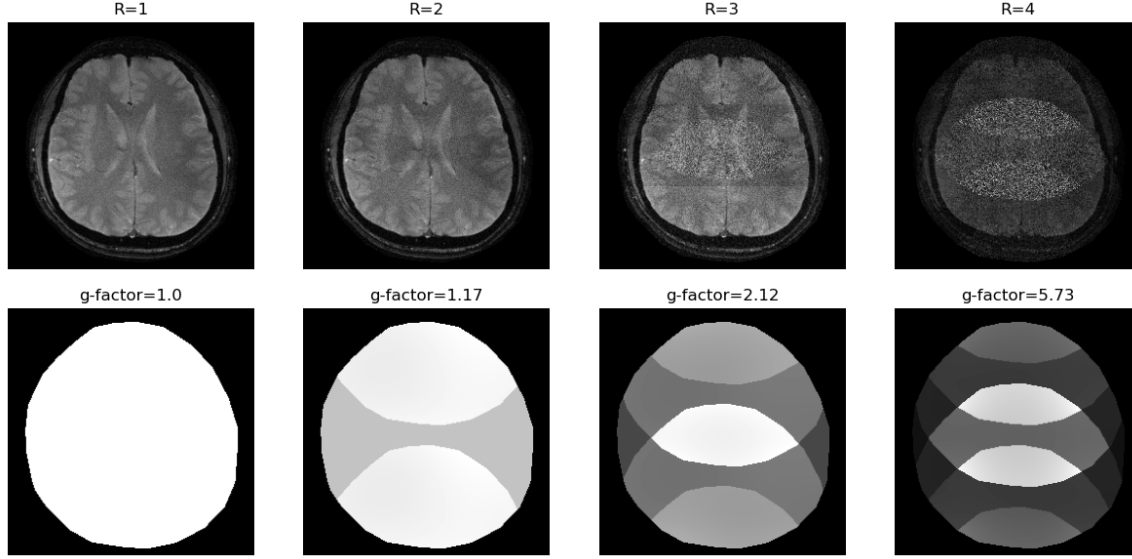


Figure 6: SENSE reconstruction & g-factors.

2.3 As the image is accelerated, we see that the average g-factor increases and the g-map shows perturbation when compared with the g-map for the unaliased image. These perturbations can be correlated to the respective noise amplification.

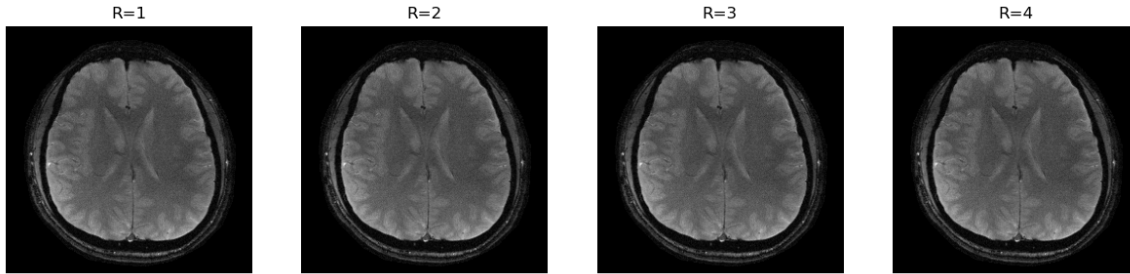


Figure 7: reconstruction error with ground truth (LS w/ Ψ).

2.4 Discussion:

(a) **Reconstruction error:** Figure 7 summarizes the reconstruction error for Cartesian SENSE w.r.t LS w/ Ψ as ground truth. However, the above error images look indiscernible, and no comment can be made on noise amplification.

(b) **RMSE:** Tables 1 presents the RMSE measures w.r.t matched filter results with Ψ . We can see that these values don't show any consistent trend (decrease/increase) with respect to noise amplification behavior. But, it can be commented that the spatial difference in coil placement could cause phase interference (constructive or destructive)

depending on the coil's sensitivity. This could explain a slight variation in individual RMSE values.

Acceleration factor (R=)	1	2	3	4
RMSE	100.99	101.69	101.95	101.79

Table 1: RMSE values.

(c) **SNR:** SNR is the best metric to explain the noise amplification behavior for increasing 1D acceleration along PE. SNR_{acc} is inversely proportional to the g-factor and inversely proportional to the square root of R. This is seen in the equation below:

$$SNR_{acc} = \frac{SNR_{no-acc}}{g\sqrt{R}} \quad (1)$$

Hence, R=4 has a very poor SNR, presents the highest noise amplification, and has the highest g-factor as well. Also, the regions perturbed by noise correlate to variations in the g-map.