Computational MR imaging **Name:** *Navaneeth N*
Laboratory 8: Compressed Sensing
17/01/2023

1. **Sparsity/compressibility of brain images using the wavelet transform:**

   1.1 Figure 1. summarizes the magnitude plot and it's sparser representation obtained by using a Daubechies 4 tap wavelet transform. The measured L1 norm values are 130.09 and 313.52 respectively. From this metric, it is clear that the wavelet transform is much sparser as the individual non-zero entries are further spaced from each other.

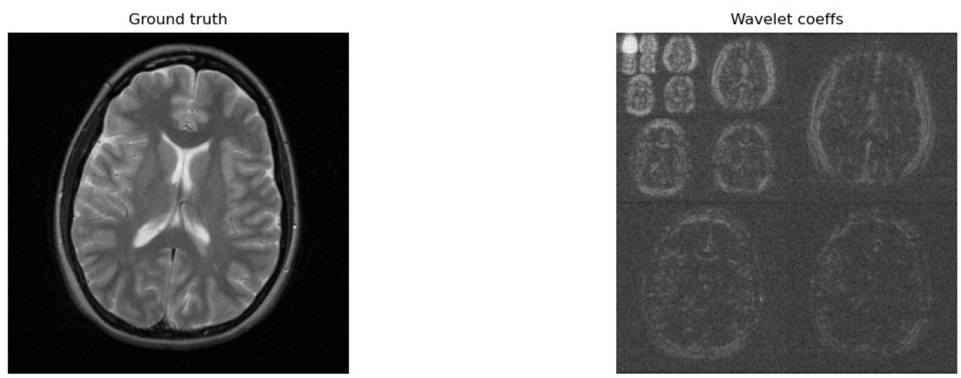

Figure 1: Magnitude image and Wavelet transform of GT.

   1.2 The results from Figure 2. are full k-space samples transformed to a sparser representation (wavelet) and compressed by factors 5, 10 and 20. These are then recovered by inverse wavelet operation and their reconstructions are performed.

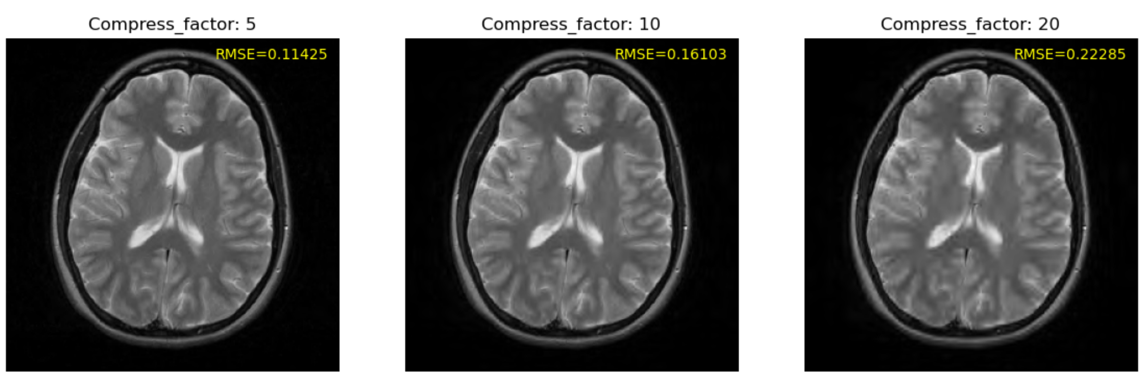

Figure 2: Compressed Images reconstructed.

   1.3 While the images individually do not show a lot of visible differences between them, the RMSE values indicate that a higher compression factor leads to a poorer reconstruction. This can be validated by taking the error plots of the reconstructed images against the ground truth. Higher compression lead to an overall reduction in image quality.

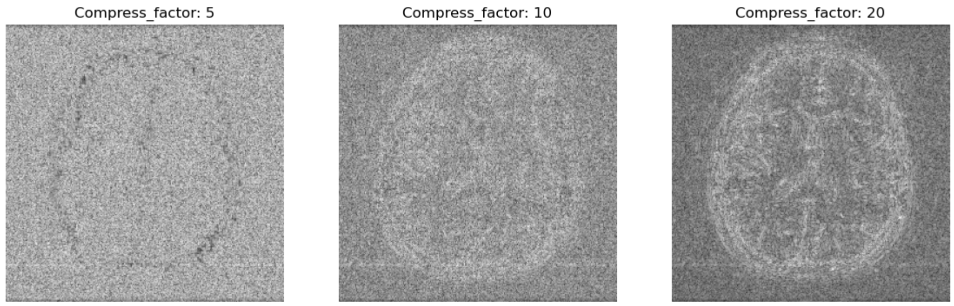

Figure 3: error plot.

2. **Compressed sensing reconstruction using iterative soft thresholding:**

   2.1 An iterative soft-thresholding approach discussed using the 4-tap Daubechies-type wavelet transform is performed. Figure 4. summarizes our undersampling scheme (Variable density for Cartesian)where fewer samples are selected to promote more sparsity and incoherence. We try to preserve as many central k-space samples as these contain a higher percentage of high-frequency information.

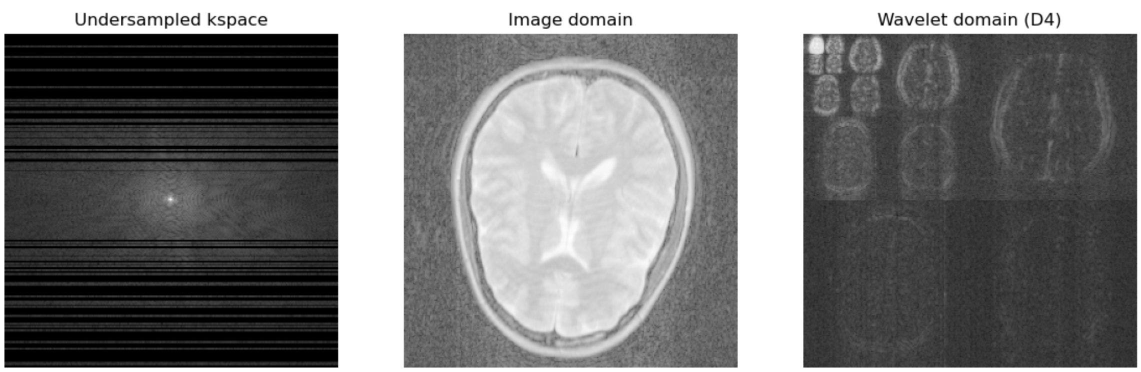

Figure 4: Under sampled k-space and respective transforms.

   2.2 The acceleration factor(R) is 2.24. Figure 5. The results of the iterative approach are presented below in Figure. 5 where the we simulate the CS based reconstruction with soft-thresholding for threshold values of 5%, 1% and 0.5% of the maximum absolute value of the starting solution.

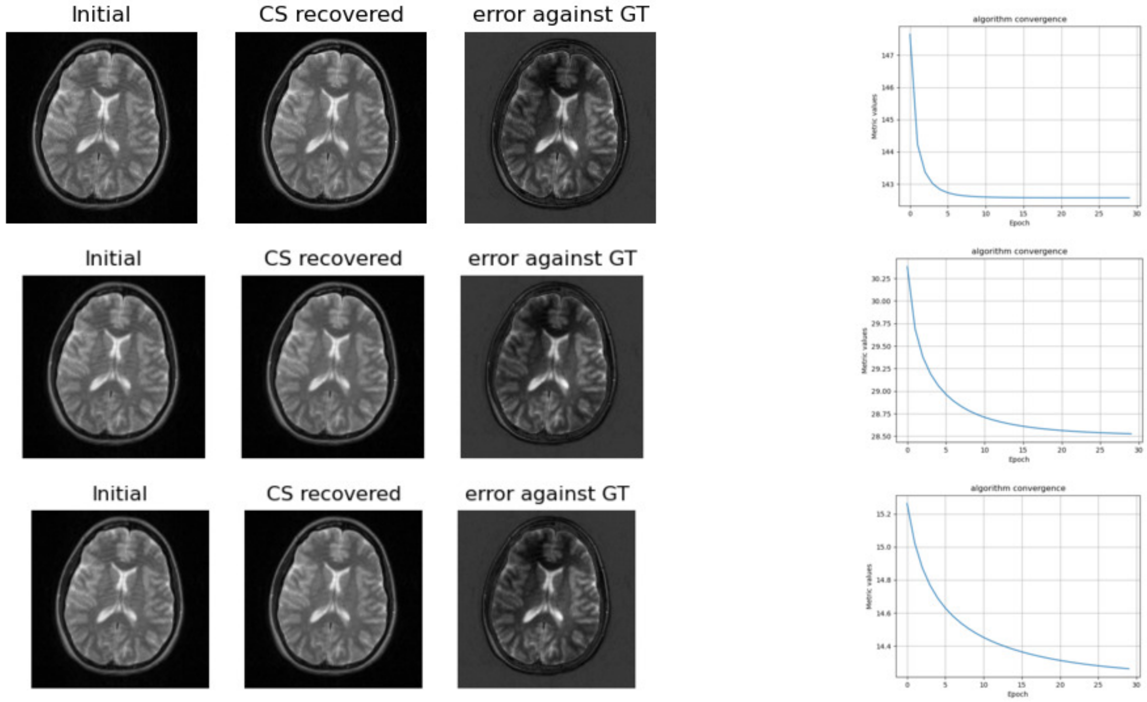

Figure 5: threshold values of 5%, 1% and 0.5% (top to bottom).

2.3 From the results above we compare the initial and final results after 30 iterations for individual lambda values. We also plot the cost function which measured the l1 and l2 norms of the sparsity and data consistency constraints. We see that with a Λ we converge faster and perform better at removing the blurring and aliasing artifacts but offer a poorer image quality. But smaller Λ does not converge completely. We see no image corruption but residual aliasing is more prevalent. Here, the trade-off is data fidelity versus artifact removal.

3. **Evaluating other under sampling schemes:**

3.1 I explored the efficacy of the iterative CS reconstruction performed earlier with alternative sampling masks/schemes that take fewer samples (higher acceleration) and also either preserve or omit central k-space samples. The two approaches are :

1. Gaussian density distribution (omits few central samples)
2. Varden's Triangular density distribution (preserves center but ignores edges)

3.2 Figure. 6 shows the k-space and it's domain representations. The image domain shows a lot of blurring artifacts. We also see that the k-space is very much like a gaussian distribution keeping most central samples but not all. The acceleration factor (R) is 2.20. Figure 7. shows the reconstruction of threshold of 1% and we see that the algorithm does well to reduce most artifacts.

3.3 Figure 7. shows the reconstruction of threshold of 1% and we see that the algorithm does well to reduce most artifacts.

3.4 Figure 8. shows the reconstruction of threshold of 1% for the 2nd scheme and we see that the algorithm offers a very good reconstruction with respect to image quality.

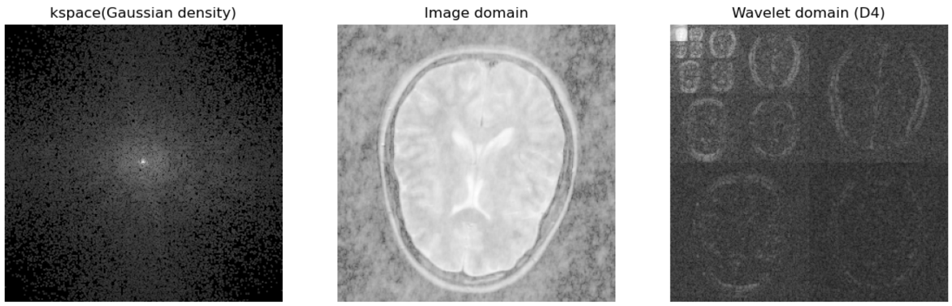

Figure 6: Gaussian distribution based sampling scheme.

Figure 7: CS recovery for Gaussian distribution based scheme .

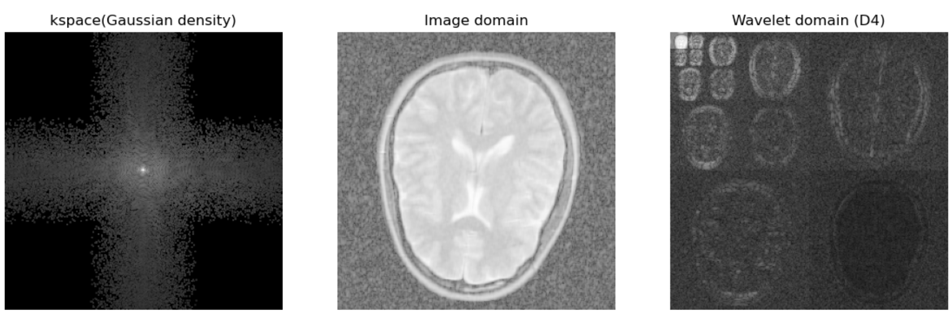

Figure 8: Varden's Triangular density sampling scheme.

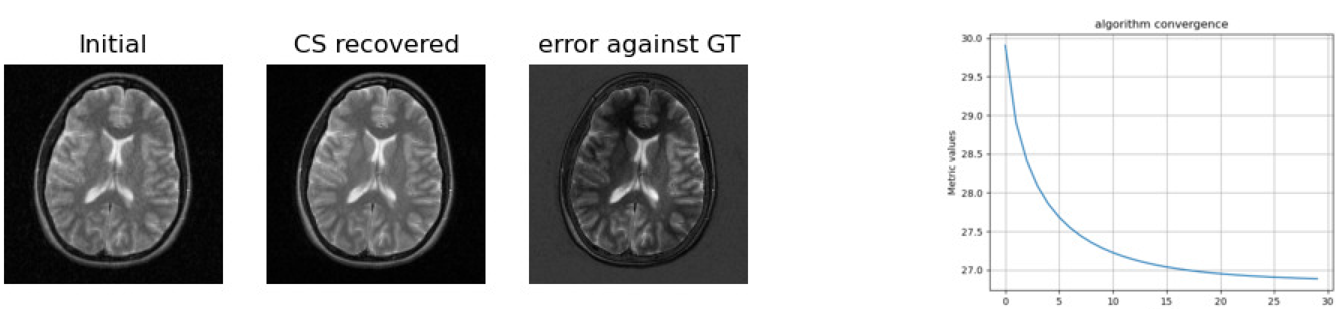

Figure 9: CS recovery for Varden's Triangular based scheme.