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Joint User and Receive Antenna Selection Algorithms for MU-MIMO Systems with Reduced Complexity

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Contents

Aims/Objectives

- The aim is to show that a joint user and receive antenna selection (**JURAS**) scheme potentially provides **significant gain over a user selection (US) scheme.**

- **Two sub-optimal joint user and antenna selection algorithms** with dynamic data stream allocation are also going to analyzed.

Introduction

- MU-MIMO schemes have recently attracted attention due to their capability of offering significant gain in **system capacity.**

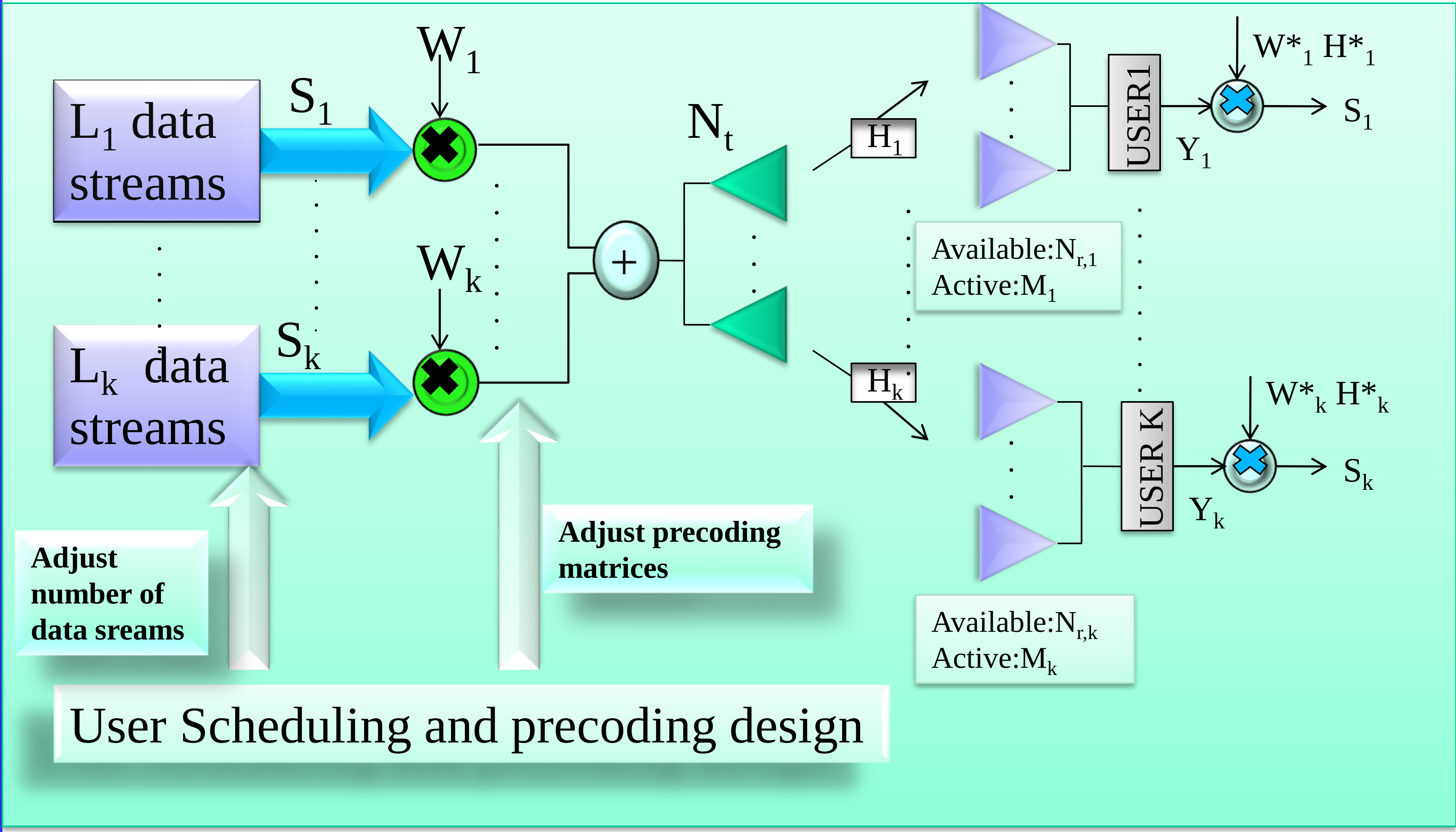
- **When users are equipped with multiple antennas**, joint user and receive antenna selection may be performed and it potentially provide **superior performance**.

- There are some techniques which provide theoretical sum capacity are DPC (**Dirty Paper Coding**) ,ZF (**Zero Forcing**), BD (**Block Diagonalization**) but they suffer due to high complexity (**exhaustive algorithms**).

Introduction

- Hence **joint user and receive antenna selection** (JURAS) is used to achieve **high sum capacity** at high SNR.
- Here two **sub optimal algorithms** are used to provide near theoretical sum capacity having **reduced complexity.**
- These suboptimal algorithms will find the best combination of user and their receive antennas which will provide **maximum possible data rates or sum capacity** thereby increasing the data transfer rates.
- Although they are suboptimal but they are very useful when number of receive antennas and users increase.

System Model



System Model

- The system model consists of a **single cell downlink MU MIMO** network.
- The BASE STATION has $\mathbf{N_t}$ number of transmitting antennas whereas there are **K** users having $\mathbf{N_r}$ receiving antennas with them where $\mathbf{M_k}$ receiving antennas will be active at a time for a particular user where $\mathbf{M_k <= Nr}$.
- Our **objective function i.e. sum capacity** is dependent on the SLNR based precoding matrix which has been given in next slides.
- We will assume **equal power per data stream power** scheme while calculating the SINR for the system.

System Model

- The **received signal** at the mobile station is given as,

$$\mathbf{y_j} = \mathbf{H_j W_j s_j} + \mathbf{H_j} \sum_{\mathbf{k \in U_j k \neq j}} \mathbf{W_k s_k} + \mathbf{v_j}$$

Received signal

Desired signal

Interfering undesired signals

White zero mean complex gaussian noise

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System Model

- For SLNR based **precoding matrix** will be,the precoding matrix is given by,

$$\boldsymbol{W_j^o} = \boldsymbol{\rho} \cdot \textbf{eigvec}_{\boldsymbol{L_j}} \{\boldsymbol{H_j^* H_j}, (\boldsymbol{M_j \sigma_j^2}/\boldsymbol{E_j})\boldsymbol{I} + \boldsymbol{\widetilde{H}_j^* \widetilde{H}_j}\}(\boldsymbol{N_t} \times \boldsymbol{L_j})$$

- After passing through **matched filter** in the receiver side, the signal becomes,

$$\boldsymbol{\hat{s}_j = G_j y_j = W_j^* H_j^* H_j W_j s_j + W_j^* H_j^* H_j \sum_{k \in U, k \neq j} W_k s_k + W_j^* H_j^* v_j}$$

$$\boldsymbol{= D_j s_j + Q_j \sum_{k \in U k * j} W_k s_k + W_j H_j v_j}$$

System Model

- **Signal to interference and noise ratio** is given by,

$$\mathrm{SINR}_j^l = \frac{\mathrm{E}\left[D_j s_j s_j^* D_j^*\right]_{ll}}{\mathrm{E}\left[W_j^* H_j^* v_j v_j^* H_j W_j\right]_{ll} + \mathrm{E}\left[\sum_{k\in U_k+j} Q_j W_k s_k s_k^* W_k^* Q_j^*\right]_{ll}}$$

$$= \frac{(E_j/L_j)\left[D_j D_j^*\right]_{ll}}{\sigma_j^2 [D_j]_{ll} + \left[Q_j\left(\sum_{k\in U, k\neq j} (E_k/L_k) W_k W_k^*\right) Q_j^*\right]_{ll}}$$

$$\mathrm{SINR}_j = \frac{(E_{BS}/L)\left[D_j D_j^*\right]_{ll}}{\sigma_j^2 [D_j]_{ll} + (E_{BS}/L)\left[Q_j\left(\sum_{k\in U_k\neq j} W_k W_k^*\right) Q_j^*\right]_{ll}}$$

$$= \frac{\left[D_j D_j^*\right]_{ll}}{\left[\left(L\sigma_j^2/E_{BS}\right) D_j + Q_j \overline{W}_j \overline{W}_j^* Q_j^*\right]_{ll}}$$

- **Here equal power per data stream** is used.

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System Model

- Now the **sum capacity** which is defined as **the data rate per unit bandwidth** is given as,

$$C_{\text{sum}} = \sum_{j \in U} \sum_{l=1}^{L_j} \log_2\left(1 + \frac{\left[D_j D_j^*\right]_{ll}}{\left[\left(L\sigma_j^2/E_{BS}\right)D_j + Q_j \widetilde{W}_j \widetilde{W}_j^* Q_j^*\right]_{ll}}\right)$$

- Thus here we have to maximize our objective function i.e. **sum capacity.**

JURAS Scheme for MU-MIMO

- **JURAS means the joint user and receive antenna selection scheme** which is being implemented in this project using two suboptimal algorithms.

- **Suboptimal algorithm-1** works by updating the user's precoding matrices and evaluate sum capacity(**complexity is somewhat more**).

- **Suboptimal algorithm-2** works by considering each receiving antenna as a single user during user selection process and then computes the precoding matrices used for data transmission (**complexity is reduced**) .

Suboptimal Algorithms

1. Suboptimal algorithm 1

- This suboptimal algorithm can be divided into two phases.The first phase extends the ideas of the **capacity-based iterative user selection algorithm.**
- It selects a receive antenna with the highest capacity. Then, from the remaining unselected antennas, it finds the next receive antenna providing the largest sum capacity.
- This phase terminates when the **sum capacity would reduce as a result of adding one more receive antenna** (equivalent to one more data stream).

Suboptimal Algorithms

- In the second phase, the algorithm researches the remaining unselected antennas of the selected users without increasing the number of allocated data streams.

- The algorithm terminates when no extra sum capacity is achieved by the receive beamforming.

Suboptimal Algorithms

- This suboptimal algorithm selects receive antenna of user having highest capacity and so on and use the below formula to calculate the sum capacity,

$$C_{\text{sum}} = \sum_{j \in U} \sum_{l=1}^{L_j} log_2\left(1 + \frac{\left[D_j D_j^*\right]_{ll}}{\left[\left(L\sigma_j^2 / E_{BS}\right) D_j + Q_j \widetilde{W}_j \widetilde{W}_j^* Q_j^*\right]_{ll}}\right)$$

- The complexity of the suboptimal algorithm 1 is more .

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Suboptimal Algorithms

❑ SA-1 PSEUDOCODE

1. **Initialization:**
2. UsrId = mapping between rx antenna id and user id
3. $\mathcal{R} = \{1, \ldots, N_r\}, \delta = \emptyset, u = \emptyset, l = 0, \widetilde{H} = \emptyset$
4. $C_{\max} = 0$, flag $= 1$, phase $= 1$
5. **Do while flag** $= 1$
    a. **for every** $r \in \mathcal{R}$
        i. Let $\delta_{\text{tmp}} = \delta + \{r\}, W = \emptyset, H = \widetilde{H}, l_{\text{tmp}} = l$
        ii. **Find the candidate user:**

$$u = U\text{srld}(r), u_{\text{tmp}} = u \cup \{u\}, H_u = H[u\} = [H[u]; h_r]$$

        iii. **If phase** $= 1$,

$$l_{\text{tmp}}(u) += 1;$$

        iv. **End**

Suboptimal Algorithms

v. $L_{\text{tmp}} = \text{sum}(l_{\text{tmp}})$

vi. Find precoding $\boldsymbol{W_j}$ for every user $\boldsymbol{j} \in \boldsymbol{U_{\text{tmp}}}$

$$M_j = \text{size}(H_j, 1), E_j = L_j \cdot E_{BS}/L_{tmp}$$

$$W_j \propto \text{eigvec}_{L_j}\left(H_j^* H_j, \left(\frac{M_j \sigma_j^2}{E_j}\right)\mathbf{I} + \tilde{H}_j^* \tilde{H}_j\right), \text{Tr}(W_j^* W_j) = L_j$$

$$W\{j\} = W_j$$

vii. Calculate sum capacity, denoted as $\boldsymbol{C_r}$

$$C_r = \sum_{j \in U_{\text{tmp}}} \sum_{i=1}^{M_j} \log_2\left(1 + \frac{\left[D_j D_j^*\right]_{ll}}{\left[\left(L_{tmp}\sigma_j^2/E_{BS}\right)D_j + Q_j \overline{W}_j \overline{W}_j^* Q_j^*\right]_{ll}}\right)$$

b. End

c. $\overline{r} = \text{argmax}_{r \in R} C_r$

d. if $\boldsymbol{C_{\vec{r}}} > \boldsymbol{C_{\max}}$

i. if phase $= 1$

$$l(\overline{u}) += 1;$$

ii. End

Suboptimal Algorithms

e. elseif phase = 1

i. $\mathcal{R}$ = remaining antennas of users in u,

not been selected in δ phase = 2

f. Else

i. flag = 0

g. End

h. End

Output: $\mathcal{S}, \mathcal{U}, \mathbf{l}$

Suboptimal Algorithms

2. Suboptimal algorithm 2

- It is seen that the **main computational burden of SA1** focuses on **updating users' precoding matrices and evaluating the sum capacity**.
- Thus, the algorithm is only required to compute the beamforming vector of **the candidate antenna without updating the precoding matrices of the selected ones.**
- Treating each receive antenna as an individual user provides **more robustness to the errors** from outdated precoding matrices than considering multiple antennas at each user.

Suboptimal Algorithms

- By treating each antenna as a separate user, no receive beamforming can be exploited. Thus, procedures in the second phase are excluded from SA2 and **the number of data streams is always equal to the number of selected antennas.**

- SA2 becomes similar to **the user selection algorithm** , except that the sum power constraint is considered in the algorithm proposed here.

Suboptimal Algorithms

- This suboptimal algorithm selects receive antenna of user having highest capacity and so on and use the below formula to calculate the sum capacity,

$$C_{sum} = \sum_{res} log_2\left(1 + \frac{\|h_r w_r\|^2}{\left(\frac{L\sigma_r^2}{E_{BS}}\right) + \Sigma_{q\in\delta, q\neq r}\|h_r w_q\|^2}\right)$$

- The complexity of the suboptimal algorithm 2 is less.

Suboptimal Algorithms

SA-2 PSEUDOCODE

1. **Initialization:**

   $\mathcal{R} = \{1, \ldots, N_r\}, \delta = \emptyset, L = 0, \mathbf{H} = \emptyset, \mathbf{W} = \emptyset$
   $C_{\max} = 0, \text{flag} = 1$

2. **Do while flag $= 1$**

   a. **for every $\boldsymbol{r} \in \boldsymbol{\mathcal{R}}$**

      i. Let $\delta_{\text{tmp}} = \delta + \{r\}$

      ii. $L_{tmp} = L + 1$

      iii. **Find precoding only for the candidate antenna**

      iv. $\mathbf{w}_r \propto$ max. eigenvector $\left(\left((L_{\text{tmp}}\, \sigma_r^2 / E_{BS})\mathbf{I} + \widetilde{\mathbf{H}}^* \widetilde{\mathbf{H}}\right)^{-1} \mathbf{h}_r^* \mathbf{h}_r\right)$

      v. $\text{Tr}(w_r^* w_r) = 1$

      vi. $W_{\text{tmp}} = [W, w_r]$

      vii. **Calculate sum capacity, denoted as $\boldsymbol{C_r}$**

$$C_r = \sum_{i \in \delta_{tmp}} \log_2\left(1 + \frac{\|\mathbf{h}_i \mathbf{W}_{\text{tmp}}(:, i)\|^2}{(L_{tmp}\, \sigma_i^2 / E_{BS}) + \sum_{i \in \delta_{tmp}, i \pm i} \|h_i W_{tmp}(:, \tilde{\imath})\|^2}\right)$$

   b. **End**

Suboptimal Algorithms

c. $\bar{r} = \text{argmax}_{r \in \mathcal{R}} C_r$

d. if $\boldsymbol{C}_{\mathbf{fr}} > \boldsymbol{C}_{\mathbf{max}}$

$$C_{\max} = C_{\bar{r}}, \delta = \delta + \{\bar{r}\}, \mathcal{R} = \mathcal{R} - \{\bar{r}\}$$
$$L = L + 1$$
$$W = [W, w_{\bar{r}}], \widetilde{H} = [\widetilde{H}\,; h_{\bar{r}}]$$

e. else

i. $\text{flag} = 0$

f. End

3. End

Output: $\boldsymbol{\delta}$($\boldsymbol{u}, \boldsymbol{l}$ are derived from $\boldsymbol{\delta}$)

Suboptimal Algorithms

- Below is the code to generate **the independent complex gaussian channel matrix i**n matlab where the mean is zero and variance is unity.

n_r=3; % number of receiving antennas

n_t=5; % number of transmitting antennas

K=5; % number of users

Hr=sqrt(1/2)*randn(n_r,n_t,k) + sqrt(1/2)*i1*randn(n_r,n_t,k);

- Here we have taken **randn** function so as to generate the random numbers having **mean=0 and variance=1.**
- This will generate **k** channel matrices having **n_r rows** and **n_t columns.**

Suboptimal Algorithms

- We can generate the precoding matrix from channel matrix using the formula given below,

$$W_j^o = \rho \cdot \text{eigvec}_{L_j}\left\{H_j^* H_j, \left(\frac{M_j \sigma_j^2}{E_j}\right) I + \widetilde{H}_j^* \widetilde{H}_j\right\}(N_t \times L_j)$$

- Here we have to find the eigen value of the expression inside the brackets and we will obtain our SLNR based precoding matrix.

Suboptimal Algorithms

- We can calculate sum capacity for every user in suboptimal algorithm 1 using channel matrix and precoding matrix by using the formula given below,

$$C_{\mathrm{sum}} = \sum_{j\in U} \sum_{l=1}^{L_j} log_2\left(1 + \frac{\left[D_j D_j^*\right]_{ll}}{\left[\left(L\sigma_j^2/E_{BS}\right)D_j + Q_j \widetilde{W}_j \widetilde{W}_j^* Q_j^*\right]_{ll}}\right)$$

- And sum capacity for suboptimal algorithm 2 can be calculated by,

$$C_{sum} = \sum_{res} log_2\left(1 + \frac{\left\|h_r w_r\right\|^2}{\left(\frac{L\sigma_r^2}{E_{BS}}\right) + \Sigma_{q\in\delta, q\neq r}\left\|h_r w_q\right\|^2}\right)$$

Work done

- In this session we have generated the **channel matrix using matlab** and the output is shown in the next page.
- we have generated the **zero mean and unity variance complex gaussian channel matrix** which will be used to find the **precoding matrices for the suboptimal algorithms and the sum capacity** using the given formulae.
- Below is the **matlab code for channel matrix.**

```
clc;
close all;
clear all;
n_r=3; % number of receiving antennas each user can have.
n_t=5; % number of transmitting antennas of the base station.
k=10;  % total number of mobile users.
H=sqrt(1/2)*randn(n_r,n_t,k) + sqrt(1/2)*i*randn(n_r,n_t,k);%channel matrix
```

Work done

Channel matrix matlab code output

```
H(:,:,1) =

   0.0565 - 0.0444i   0.4787 + 0.4331i   0.3178 - 0.4429i   0.3791 + 0.6894i  -0.1041 - 0.7636i
  -0.6707 - 1.4297i   0.6065 - 0.0388i   0.0712 + 0.1764i   0.6349 - 0.4531i   0.7126 + 0.1408i
   0.2910 - 0.6945i  -0.4887 - 0.7911i   0.5841 - 0.7022i  -0.0933 + 1.2791i  -1.5017 - 1.0755i


H(:,:,2) =

  -0.3568 - 0.5117i   0.4587 + 0.6662i  -0.3331 + 0.5766i   0.2134 + 0.4039i  -0.1250 + 0.5371i
  -0.8984 - 0.4195i   0.5839 + 0.2125i   0.0969 + 0.5649i   0.2828 + 0.2919i  -1.5076 - 0.4647i
  -0.2705 + 0.2838i  -0.7177 - 0.2638i  -0.2064 + 0.0850i  -0.6576 - 0.6979i   0.8099 - 0.4270i


H(:,:,3) =

  -0.4448 + 0.1251i  -1.0102 + 0.4210i   1.5399 + 0.2317i   0.3121 + 0.3111i   0.1163 + 0.4250i
  -0.8513 - 0.2174i  -0.0147 + 0.7402i   0.8050 - 0.1685i  -0.9886 - 0.4362i   0.5287 + 0.0653i
  -0.1796 - 0.0932i  -0.3965 - 0.1400i  -1.7656 + 0.1623i  -0.1804 + 0.1943i  -0.1931 + 1.2232i
```

Future Work

- Till now we have designed the algorithm for the two suboptimal algorithms and we have generated the **complex gaussian zero mean and unity variance channel matrix** which will be used to calculate the precoding matrix of the system and then sum capacity which is our objective function.

- In the next session we have to **implement the algorithms in matlab** and **calculate the sum capacity and carryout our simulation results** so as to analyze every corner of our suboptimal algorithms whether they provide us satisfactory results or not.

Conclusions

- The JURAS scheme enhances the **performance gain.**
- The JURAS scheme performance gain is **significant at high SNR.**
- The two JURAS suboptimal algorithms **reduced the complexity of computation.**

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Thank You