

# Reduced Complexity Joint User and Receive Antenna Selection Algorithms for SLNR-based Precoding in MU-MIMO Systems

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**Abstract**—This paper investigates the user scheduling strategy of multi-user MIMO (MU-MIMO) systems using a transmit precoding scheme based on maximum signal to leakage and noise ratio (SLNR). When users are equipped with multiple antennas, joint user and receive antenna selection may be performed and is shown to potentially provide superior performance to user selection schemes where users utilise all available antennas, especially at high SNR. To overcome the impractical computational burden of exhaustive methods, two suboptimal algorithms are presented in this paper. The simulation results show that the proposed suboptimal algorithms perform very close to the exhaustive joint user and antenna selection algorithm.

## I. INTRODUCTION

Multuser multiple input multiple output (MU-MIMO) schemes have recently attracted research attention due to their capability of offering significant gain in system capacity by enabling simultaneous multiplexing of multuser data streams into the same frequency and time resources. One of the major issues in MU-MIMO systems is how to design appropriate transmit precoding schemes to avoid co-channel interference (CCI) among the users.

Dirty Paper Coding (DPC) [1] is known to be an optimal scheme achieving the theoretical sum capacity [2]; however, it suffers from high complexity due to its non-linear processing. Zero-Forcing (ZF) and Block Diagonalisation (BD) [3] are linear precoding techniques aiming to perfectly cancel CCI for each user. ZF and BD algorithms require sufficient degrees of freedom in the spatial domain in order to force the CCI to zero. Generally, the number of transmit antennas is required to be larger than the sum of receive antennas of all users; otherwise, time scheduling is necessary. Another linear approach recently proposed in [4] is aimed to maximise a new performance criterion, referred to as the signal to leakage and noise ratio (SLNR). In contrast to interference which quantifies an amount of unwanted signal power from other users perceived at the desired user, leakage is a measure of how much signal power intended to a given user leaks into the others. Aiming to maximise SLNR instead of signal to interference and noise ratio (SINR) leads to suboptimal performance; however, it decouples the collective design criterion into individual user objectives and results in a closed-form solution for each user. In addition, it poses no restrictions on the number of antennas and offers superior performance in terms of bit error rate (BER) and outage probability compared to ZF solutions [4].

Although there is no limitation on the number of antennas and data streams in the SLNR-based solutions, [5] shows that

increasing the number of receive antennas at a user can lead to additional signal leakage considered by other users, causing the degradation of their SLNR. As a result, the system may not benefit from simultaneously scheduling maximum data streams to all users, deploying all of their receive antennas. Therefore, user selection (possibly joint with receive antenna selection) remains necessary in MU-MIMO systems based on SLNR precoding design. Some user scheduling algorithms have been proposed in [7]–[9] for the SLNR-based precoding scheme with single-antenna receivers. The problem becomes more complicated when users are equipped with multiple antennas. In addition to choosing active users, it is possible to dynamically select the set of active receive antennas and allocate different numbers of data streams. In this paper, it is shown that a joint user and receive antenna selection (URAS) scheme potentially provides significant gain over a user selection (US) scheme, where users utilise all receive antennas when scheduled for data transmission, especially at high SNR. Note that the number of data streams may be chosen to be lower than the number of selected antennas in order to exploit receive beamforming. Two suboptimal joint user and antenna selection algorithms with dynamic data stream allocation are also proposed in this paper and are shown to achieve very similar performance compared to the exhaustive method.

*Notation:*  $(\cdot)^T$ ,  $(\cdot)^*$ ,  $Tr(\cdot)$  denote transpose, Hermitian transpose and trace operations, respectively.  $|\cdot|$  represents the cardinality of a set and  $\|\cdot\|$  denotes a vector norm.  $[\mathbf{A}]_{ll}$  denotes the entry  $(l, l)$  of matrix  $\mathbf{A}$ .  $vertcat\{\cdot\}$ ,  $horzcat\{\cdot\}$  represent the row and column concatenation operations, respectively.  $x+=a$  denotes an increment of  $x$  by  $a$ .

## II. SYSTEM MODEL

Consider a single-cell single-carrier downlink MU-MIMO system where the base station (BS) has  $N_t$  antennas and communicates with  $K$  users, each of which is equipped with multiple antennas as depicted in Fig. 1. Let  $N_{r,j}$  be the number of receive antennas of user  $j \in \{1, \dots, K\}$ . The full channel information between the BS and user  $j$ , denoted as  $\mathbf{H}_j^F \in \mathbb{C}^{N_{r,j} \times N_t}$ , is assumed to be available at the BS. Each entry of  $\mathbf{H}_j^F$  is assumed to be independent complex Gaussian variables with zero mean and unit variance. Based on available channel information from all users, the BS dynamically selects a subset of receive antennas and corresponding number of data streams, aiming to maximise an objective function, e.g. sum capacity. A user who has at least one receive antenna selected can be scheduled for data transmission. Let  $M_j$  denote the number of

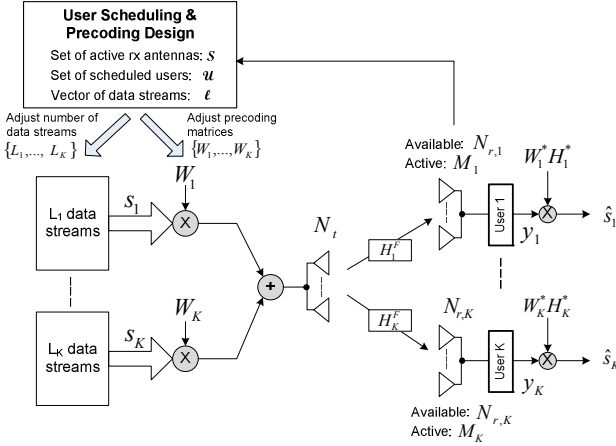


Figure 1. Block diagram of the downlink MU-MIMO system.

receive antennas selected by the BS for the user  $j$ ,  $M_j \leq N_{r,j}$ , and  $L_j$  denote the corresponding number of data streams,  $L_j \leq \min(N_t, M_j)$ . Thus, the set of scheduled users can be defined as  $\mathcal{U} = \{j: j \in \{1, \dots, K\} \text{ and } L_j > 0\}$ . The total number of data streams transmitted by the BS is given by  $L = \sum_{j \in \mathcal{U}} L_j$ .

Data streams from each scheduled user  $j$ ,  $\mathbf{s}_j \in \mathbb{C}^{L_j \times 1}$ , are pre-multiplied by a precoding matrix  $\mathbf{W}_j \in \mathbb{C}^{N_t \times L_j}$ . The BS indicates to each scheduled user the set of selected antennas so that only selected receive antennas can be involved in the decoding process. Let  $\mathbf{H}_j$  be the channel matrix obtained by choosing the rows of  $\mathbf{H}_j^T$  associated to the selected antennas for user  $j$ ,  $\mathbf{H}_j \in \mathbb{C}^{M_j \times N_t}$ . The received signal vector for each user  $j$  in the scheduled set can be given by

$$\mathbf{y}_j = \mathbf{H}_j \mathbf{W}_j \mathbf{s}_j + \mathbf{H}_j \sum_{k \in \mathcal{U}, k \neq j} \mathbf{W}_k \mathbf{s}_k + \mathbf{v}_j \quad (1)$$

where  $\text{Tr}(\mathbf{W}_j^* \mathbf{W}_j) = L_j$ ,  $E\{\mathbf{s}_j \mathbf{s}_j^*\} = (E_j/L_j) \mathbf{I}$  and  $\sum_{j \in \mathcal{U}} E_j = E_{BS}$ . The first two conditions generalise the power constraints in [4] to limit the allocated power of user  $j$  to  $E_j$ . The third condition is added to ensure that the total transmitted power for all scheduled users does not exceed the maximum available power at the base station ( $E_{BS}$ ). The additive noise vector, denoted as  $\mathbf{v}_j$ , is assumed to have identical and independent complex Gaussian elements with variance  $\sigma_j^2$ , i.e.  $E\{\mathbf{v}_j \mathbf{v}_j^*\} = \sigma_j^2 \mathbf{I}$ . According to [4], the criterion of the SLNR precoding design is to obtain a precoding matrix which maximises the SLNR. This corresponds to the following optimisation problem.

$$\mathbf{W}_j^o = \arg \max_{\mathbf{W}_j \in \mathbb{C}^{N_t \times L_j}} \frac{\text{Tr}(\mathbf{W}_j^* \mathbf{H}_j^* \mathbf{H}_j \mathbf{W}_j)}{\text{Tr}[\mathbf{W}_j^* ((M_j \sigma_j^2 / E_j) \mathbf{I} + \tilde{\mathbf{H}}_j^* \tilde{\mathbf{H}}_j) \mathbf{W}_j]} \quad (2)$$

subject to  $\text{Tr}(\mathbf{W}_j^* \mathbf{W}_j) = L_j$

where  $\tilde{\mathbf{H}}_j = \text{vertcat}\{\mathbf{H}_k: \forall k \in \mathcal{U} \text{ and } k \neq j\}$  ( $\sum_{k \neq j} M_k \times N_t$ ).

It is shown in [4] that the solution of (2) can be written as

$$\mathbf{W}_j^o = \rho \cdot \text{eigvec}_{L_j}\{\mathbf{H}_j^* \mathbf{H}_j, (M_j \sigma_j^2 / E_j) \mathbf{I} + \tilde{\mathbf{H}}_j^* \tilde{\mathbf{H}}_j\} \quad (N_t \times L_j) \quad (3)$$

where the function  $\text{eigvec}_{L_j}\{\mathbf{A}, \mathbf{B}\}$  returns a matrix containing  $L_j$  columns of eigenvectors (corresponding to the  $L_j$  largest eigenvalues), satisfying the generalised eigenvalue problem  $\mathbf{A}\mathbf{x} = \lambda \mathbf{B}\mathbf{x}$ .  $\rho$  is a normalisation factor so that  $\text{Tr}(\mathbf{W}_j^* \mathbf{W}_j) = L_j$ .

For the decoding process, the matched filter,  $\mathbf{G}_j = \mathbf{W}_j^* \mathbf{H}_j^*$ , is applied as in [4]. The decoding vector can be given by

$$\begin{aligned} \hat{\mathbf{s}}_j &= \mathbf{G}_j \mathbf{y}_j = \mathbf{W}_j^* \mathbf{H}_j^* \mathbf{H}_j \mathbf{W}_j \mathbf{s}_j + \mathbf{W}_j^* \mathbf{H}_j^* \sum_{k \in \mathcal{U}, k \neq j} \mathbf{W}_k \mathbf{s}_k + \mathbf{W}_j^* \mathbf{H}_j^* \mathbf{v}_j \\ &= \mathbf{D}_j \mathbf{s}_j + \mathbf{Q}_j \sum_{k \in \mathcal{U}, k \neq j} \mathbf{W}_k \mathbf{s}_k + \mathbf{W}_j^* \mathbf{H}_j^* \mathbf{v}_j \end{aligned} \quad (4)$$

where  $\mathbf{D}_j = \mathbf{W}_j^* \mathbf{H}_j^* \mathbf{H}_j \mathbf{W}_j$  is a diagonal matrix and  $\mathbf{Q}_j = \mathbf{W}_j^* \mathbf{H}_j^* \mathbf{H}_j$ .

In this case, the signal to interference and noise ratio (SINR) for the data stream  $l$  of user  $j$  can be calculated as

$$\begin{aligned} \text{SINR}_j^l &= \frac{E[\mathbf{D}_j \mathbf{s}_j \mathbf{s}_j^* \mathbf{D}_j^*]_{ll}}{E[\mathbf{W}_j^* \mathbf{H}_j^* \mathbf{v}_j \mathbf{v}_j^* \mathbf{H}_j \mathbf{W}_j]_{ll} + E[\sum_{k \in \mathcal{U}, k \neq j} \mathbf{Q}_j \mathbf{W}_k \mathbf{s}_k \mathbf{s}_k^* \mathbf{W}_k^* \mathbf{Q}_j^*]_{ll}} \\ &= \frac{(E_j/L_j) [\mathbf{D}_j \mathbf{D}_j^*]_{ll}}{\sigma_j^2 [\mathbf{D}_j]_{ll} + [\mathbf{Q}_j (\sum_{k \in \mathcal{U}, k \neq j} (E_k/L_k) \mathbf{W}_k \mathbf{W}_k^*) \mathbf{Q}_j^*]_{ll}} \end{aligned} \quad (5)$$

Note that the SLNR precoding design in (2) also depends on the amount of power allocated to each user. Two simple power allocation schemes, namely Equal Power per User (EPU) and Equal Power per data Stream (EPS), are considered in this paper. The former assumes equal power allocation among all users, i.e.  $E_j = \frac{E_{BS}}{|\mathcal{U}|}$ ,  $\forall j \in \mathcal{U}$ , while the latter allocates power to users relatively to the number of scheduled data streams, i.e.  $E_j = \left(\frac{L_j}{L}\right) E_{BS}$ ,  $\forall j \in \mathcal{U}$ . It will be shown in Section IV that EPS provides slightly higher sum capacity than EPU. Thus, EPS will be assumed for joint user and antenna selection schemes in Section III. In this case, SINR expression in (5) can be rewritten as

$$\begin{aligned} \text{SINR}_j^l &= \frac{(E_{BS}/L) [\mathbf{D}_j \mathbf{D}_j^*]_{ll}}{\sigma_j^2 [\mathbf{D}_j]_{ll} + (E_{BS}/L) [\mathbf{Q}_j (\sum_{k \in \mathcal{U}, k \neq j} \mathbf{W}_k \mathbf{W}_k^*) \mathbf{Q}_j^*]_{ll}} \\ &= \frac{[\mathbf{D}_j \mathbf{D}_j^*]_{ll}}{[(L \sigma_j^2 / E_{BS}) \mathbf{D}_j + \mathbf{Q}_j \bar{\mathbf{W}}_j \bar{\mathbf{W}}_j^* \mathbf{Q}_j^*]_{ll}} \end{aligned} \quad (6)$$

where  $\bar{\mathbf{W}}_j = \text{horzcat}\{\mathbf{W}_k: \forall k \in \mathcal{U} \text{ and } k \neq j\}$  ( $N_t \times \sum_{k \neq j} L_k$ ). Thus, the sum capacity can be calculated as

$$C_{\text{sum}} = \sum_{j \in \mathcal{U}} \sum_{l=1}^{L_j} \log_2 \left( 1 + \frac{[\mathbf{D}_j \mathbf{D}_j^*]_{ll}}{[(L \sigma_j^2 / E_{BS}) \mathbf{D}_j + \mathbf{Q}_j \bar{\mathbf{W}}_j \bar{\mathbf{W}}_j^* \mathbf{Q}_j^*]_{ll}} \right) \quad (7)$$

### III. JOINT USER AND ANTENNA SELECTION ALGORITHMS

Let  $N_r = \sum_j N_{r,j}$  denote the total number of receive antennas and  $\mathcal{R} = \{1, \dots, N_r\}$  denote the set of receive antenna indices of all users. The aim of the joint user and receive antenna selection algorithms is to find the subset of receive antennas  $\mathcal{S} \subseteq \mathcal{R}$  and the vector of corresponding number of scheduled data streams for all users,  $\mathbf{l} = [L_1, \dots, L_K]^T$ , which maximise the sum capacity in (7). The set of scheduled users is then derived by  $\mathcal{U} = \{j: j \in \{1, \dots, K\} \text{ and } L_j > 0\}$ . An exhaustive algorithm requires a search over  $2^{N_r} - 1$  antenna combinations, each of which involves at least one possible data stream combination. The overall complexity is clearly computationally prohibitive for any significant number of receive antennas. This motivates the search for reduced-complexity suboptimal algorithms.

#### A. Suboptimal Algorithm 1 (SA1)

This suboptimal algorithm can be divided into two phases. The first phase extends the ideas of the capacity-based iterative user selection algorithm as proposed in [9]. The algorithm first

selects a receive antenna with the highest capacity. Then, from the remaining unselected antennas, it finds the next receive antenna providing the largest sum capacity. In this phase, the algorithm increases the number of data streams as it increases the number of antennas. This phase terminates when the sum capacity would reduce as a result of adding one more receive antenna (equivalent to one more data stream). It is clear, at the end of the first phase, that no further benefit can be obtained from multiplexing more data streams into the system. Nevertheless, the system may still achieve an extra gain from receive beamforming by adding more receive antennas to the selected users. Hence, in the second phase, the algorithm researches the remaining unselected antennas of the selected users without increasing the number of allocated data streams. The algorithm terminates when no extra sum capacity is achieved by the receive beamforming. The pseudo code of the algorithm can be summarised in Table I.

TABLE I. SUBOPTIMAL ALGORITHM 1

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*Initialisation:*  
 $UsrId = \text{mapping between rx antenna id and user id}$   
 $\mathcal{R} = \{1, \dots, N_r\}, \mathcal{S} = \emptyset, \mathcal{U} = \emptyset, \mathbf{l} = \mathbf{0}, \tilde{\mathbf{H}} = \emptyset$   
 $C_{max} = 0, flag = 1, phase = 1$

Do while  $flag = 1$   
a) for every  $r \in \mathcal{R}$   
i) Let  $\mathcal{S}_{tmp} = \mathcal{S} + \{r\}, \mathbf{W} = \emptyset, \mathbf{H} = \tilde{\mathbf{H}}, \mathbf{l}_{tmp} = \mathbf{l}$   
ii) Find the candidate user:  
 $u = UsrId(r), \mathcal{U}_{tmp} = \mathcal{U} \cup \{u\}, \mathbf{H}_u = \mathbf{H}\{u\} = [\mathbf{H}\{u\}; \mathbf{h}_r]$   
iii) If  $phase = 1, \mathbf{l}_{tmp}(u) += 1; \text{end}$ ,  $L_{tmp} = \text{sum}(\mathbf{l}_{tmp})$   
iv) Find precoding  $\mathbf{W}_j$  for every user  $j \in \mathcal{U}_{tmp}$   
 $M_j = \text{size}(\mathbf{H}_j, 1), E_j = L_j \cdot E_{BS}/L_{tmp}$   
 $\mathbf{W}_j \propto \text{eigvec}_{L_j} \left( \mathbf{H}_j^* \mathbf{H}_j, \frac{M_j \sigma_j^2}{E_j} \mathbf{I} + \tilde{\mathbf{H}}_j^* \tilde{\mathbf{H}}_j \right), \text{Tr}(\mathbf{W}_j^* \mathbf{W}_j) = L_j$   
 $\mathbf{W}\{j\} = \mathbf{W}_j$   
v) Calculate sum capacity, denoted as  $C_r$   

$$C_r = \sum_{j \in \mathcal{U}_{tmp}} \sum_{i=1}^{M_j} \log_2 \left( 1 + \frac{[\mathbf{D}_j \mathbf{D}_j^*]_{ii}}{[(L_{tmp} \sigma_j^2 / E_{BS}) \mathbf{D}_j + \mathbf{Q}_j \mathbf{W}_j \mathbf{W}_j^* \mathbf{Q}_j]_{ii}} \right)$$
  
end  
b)  $\bar{r} = \arg \max_{r \in \mathcal{R}} C_r$   
c) if  $C_{\bar{r}} > C_{max}$   
 $C_{max} = C_{\bar{r}}, \mathcal{S} = \mathcal{S} + \{\bar{r}\}, \mathcal{R} = \mathcal{R} - \{\bar{r}\}$   
 $\bar{u} = UsrId(\bar{r}), \mathcal{U} = \mathcal{U} \cup \{\bar{u}\}, \tilde{\mathbf{H}}\{\bar{u}\} = [\tilde{\mathbf{H}}\{\bar{u}\}; \mathbf{h}_{\bar{r}}]$   
if  $phase = 1, \mathbf{l}(\bar{u}) += 1; \text{end}$   
elseif  $phase = 1$   
 $\mathcal{R} = \text{remaining antennas of users in } \mathcal{U}, \text{ not been selected in } \mathcal{S}$   
 $phase = 2$   
else  
 $flag = 0$   
end  
end  
end

*Output:*  $\mathcal{S}, \mathcal{U}, \mathbf{l}$

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### B. Suboptimal Algorithm 2 (SA2)

It is seen that the main computational burden of SA1 focuses on updating users' precoding matrices and evaluating the sum capacity [step iv) and v) in TABLE I, respectively]. The complexity can be further reduced by ignoring the effect of power leakage from previously selected antennas to the candidate antenna. Thus, the algorithm is only required to compute the beamforming vector of the candidate antenna without updating the precoding matrices of the selected ones. Subsequent to this observation, it was found that treating each receive antenna as an individual user provides more robustness to the errors from outdated precoding matrices than considering multiple antennas at each user. In this case, the problem is transformed into a user selection problem with a single receive antenna per user. The solution of SLNR precoding design for a

candidate antenna  $r$  in (3) is reduced to the maximum eigenvector, expressed as

$$\mathbf{w}_r^o = \rho \cdot \max. \text{eigenvector} \left\{ \left( (L \sigma_r^2 / E_{BS}) \mathbf{I} + \tilde{\mathbf{H}}^* \tilde{\mathbf{H}} \right)^{-1} \mathbf{h}_r^* \mathbf{h}_r \right\} (N_t \times 1) \quad (8)$$

where  $\mathbf{h}_r$  and  $\tilde{\mathbf{H}}$  contain the channel vector of the candidate antenna  $r$  and the previously selected antennas, respectively. The sum capacity expression in (7) can also be rewritten as

$$C_{sum} = \sum_{r \in \mathcal{S}} \log_2 \left( 1 + \frac{\|\mathbf{h}_r \mathbf{w}_r\|^2}{(L \sigma_r^2 / E_{BS}) + \sum_{q \in \mathcal{S}, q \neq r} \|\mathbf{h}_r \mathbf{w}_q\|^2} \right) \quad (9)$$

The details of SA2 are given in TABLE II. By treating each antenna as a separate user, no receive beamforming can be exploited. Thus, procedures in the second phase are excluded from SA2 and the number of data streams is always equal to the number of selected antennas. Remark that the resulting algorithm of SA2 becomes similar to the user selection algorithm proposed in [6], except that the sum power constraint ( $\sum_{j \in \mathcal{U}} E_j = E_{BS}$ ) is considered in the algorithm proposed here. It is also noted that once the selection process is done, the final precoding matrices used for data transmission are computed using (3) as in the conventional multi-antenna approach although it has been ignored during the selection process.

TABLE II. SUBOPTIMAL ALGORITHM 2

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*Initialisation:*  
 $\mathcal{R} = \{1, \dots, N_r\}, \mathcal{S} = \emptyset, L = 0, \tilde{\mathbf{H}} = \emptyset, \mathbf{W} = \emptyset$   
 $C_{max} = 0, flag = 1$

Do while  $flag = 1$   
a) for every  $r \in \mathcal{R}$   
i) Let  $\mathcal{S}_{tmp} = \mathcal{S} + \{r\}$   
ii) - skip -  
iii)  $L_{tmp} = L + 1$   
iv) Find precoding only for the candidate antenna  
 $\mathbf{w}_r \propto \max. \text{eigenvector} \left( (L_{tmp} \sigma_r^2 / E_{BS}) \mathbf{I} + \tilde{\mathbf{H}}^* \tilde{\mathbf{H}} \right)^{-1} \mathbf{h}_r^* \mathbf{h}_r$   
 $\text{Tr}(\mathbf{w}_r^* \mathbf{w}_r) = 1$   
 $\mathbf{W}_{tmp} = [\mathbf{W}, \mathbf{w}_r]$   
v) Calculate sum capacity, denoted as  $C_r$   

$$C_r = \sum_{i \in \mathcal{S}_{tmp}} \log_2 \left( 1 + \frac{\|\mathbf{h}_i \mathbf{W}_{tmp}(:, i)\|^2}{(L_{tmp} \sigma_i^2 / E_{BS}) + \sum_{i \in \mathcal{S}_{tmp}, i \neq i} \|\mathbf{h}_i \mathbf{W}_{tmp}(:, i)\|^2} \right)$$
  
end  
b)  $\bar{r} = \arg \max_{r \in \mathcal{R}} C_r$   
c) if  $C_{\bar{r}} > C_{max}$   
 $C_{max} = C_{\bar{r}}, \mathcal{S} = \mathcal{S} + \{\bar{r}\}, \mathcal{R} = \mathcal{R} - \{\bar{r}\}$   
 $L = L + 1$   
 $\mathbf{W} = [\mathbf{W}, \mathbf{w}_{\bar{r}}], \tilde{\mathbf{H}} = [\tilde{\mathbf{H}}; \mathbf{h}_{\bar{r}}]$   
else  
 $flag = 0$   
end  
end

*Output:*  $\mathcal{S}$  ( $\mathcal{U}, \mathbf{l}$  are derived from  $\mathcal{S}$ )

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### C. Complexity Analysis

As pointed out, the computational load of the proposed algorithms is mainly contributed to computing the precoding matrices in (3) and calculating the sum capacity in (7). From the experimental observation, the algorithms tend to terminate before the complete  $N_r$  iterations. The first phase of the algorithms is expected to end when the total number of data streams are in the order of the spatial-domain degree of freedom of the MIMO channel,  $L = \sum L_j \approx \min(N_t, N_r) \approx N_t$  (assuming  $N_t \leq N_r$ ). The second phase tests the possibility to exploit receive beamforming for the selected users. Assuming  $L$  different users have been selected at the end of the first phase and each user is equipped with  $m$  receive antenna ( $N_r =$

$\sum_j N_{r,j} = m \cdot K$ ), thus  $(m-1)L$  iterations are expected in the second phase. The total number of iterations can be estimated as  $mL$ . Based on this assumption, the overall complexity is approximated as given in Table III. It can be seen that the complexity of these two suboptimal algorithms seems to be linearly increase with the total number of receive antennas  $N_r$ , which is significantly reduced compared to the exponential function in the exhaustive algorithms.

TABLE III. AN APPROXIMATION OF COMPUTATIONAL COMPLEXITY

Function	Suboptimal Algorithm 1	Suboptimal Algorithm 2
Computation of precoding matrices	$O(2m^2 N_t^5 N_r)$	$O(2N_t^4 N_r)$
Calculation of the sum capacity	$O(4m^2 N_t^4 N_r)$	$O(4N_t^4 N_r)$
Overall complexity	$O((2N_t + 4)m^2 N_t^4 N_r)$	$O(6N_t^4 N_r)$

#### IV. SIMULATION RESULTS

This section presents simulation results comparing the performance of the following algorithms:

- Iterative water-filling for DPC [10] as an upper bound.
- User selection, i.e. scheduled users deploy all of their receive antennas, using the exhaustive method (SLNR US EXH).
- Joint user and antenna selection algorithms using the exhaustive and suboptimal methods (SLNR URAS EXH, SLNR URAS SA1, and SLNR URAS SA2, respectively).
- No-selection scheme, i.e. the maximum number of data streams are transmitted to all users simultaneously (SLNR NS).

Simulation results are conducted over 1,000 channel realisations. The noise variance is assumed to be equal for all users,  $\sigma_j^2 = \sigma^2$ ,  $\forall j \in \{1, \dots, K\}$ . The SNR is defined as  $E_{BS}/\sigma^2$ .

##### A. User Selection vs. Joint User and Antenna Selection

This subsection examines the potential gain of URAS over US in terms of sum capacity. Fig. 2 shows the ergodic sum capacity versus SNR for a MIMO system where  $N_t = 4$ ;  $N_{r,j} = 2, \forall j$ ;  $K = 5$ . URAS yields significant gain compared to US, especially at high SNR. As the system is interference-limited at high SNR, allowing the system to disable low-benefit receive antennas at some users improves SLNR to the others, resulting in an improvement in system capacity. At low SNR, on the other hand, the system is noise-limited. Thus, it tends to exploit receive beamforming by allocating a single data stream with all available receive antennas to each scheduled user to obtain a maximal receive signal and thereby overcome noise. This leads to a similar solution as user selection, resulting in the same sum capacity as shown in Fig. 2. Moreover, two power allocation schemes assumed in this paper contribute to very close performance. EPS results in slightly higher sum capacity compared to EPU, therefore, EPS is assumed for the remaining simulation results.

Note that, although possible, multiplexing the maximum number of data streams to all users simultaneously (SLNR NS) does not provide an advantage in terms of sum capacity. The SLNR precoding design cannot find a solution to efficiently cancel inter-user interference leading to the saturation of capacity at high SNR, and no receive beamforming can be exploited causing low capacity at low SNR. With user and/or antenna selection, the system tries to allocate a reasonable number of data streams at every time instant. Fig. 3 shows the average achievable sum capacity as a function of the number of

data streams multiplexed. At high SNR, achievable capacity seems to reduce when attempting to multiplex a number of data streams larger than the available spatial-domain degree of freedom,  $\min(N_t, \sum_j N_{r,j})$  (equal to 4 in this example). At low SNR, the peak capacity may be obtained with less data streams due to the exploitation of receive beamforming. It can be seen again that URAS is superior to US at high SNR, but similar performance at low SNR as previously discussed.

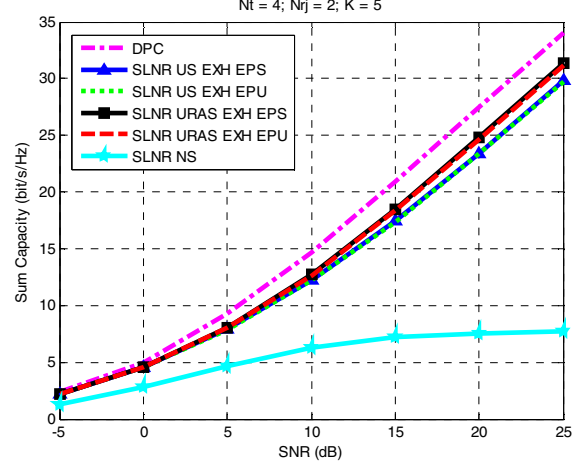


Figure 2. Comparison of ergodic sum capacity between user selection and joint user/receive antenna selection schemes.

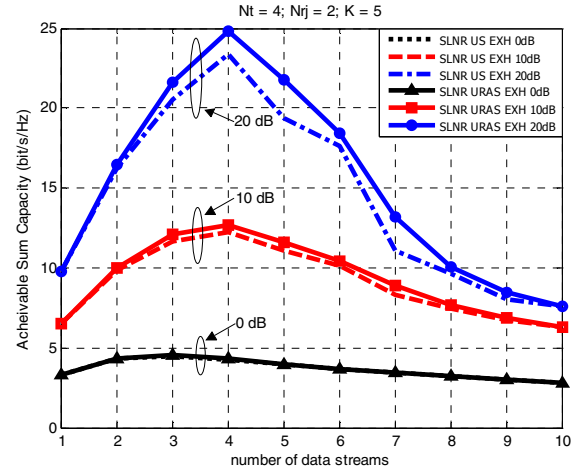


Figure 3. Achievable ergodic sum capacity as a function of the number of multiplexed data streams at SNR = 0, 10 and 20 dB.

##### B. Suboptimal Joint User and Antenna Selection Algorithms

The comparison of sum capacity between the exhaustive search and the proposed suboptimal algorithms is given in Fig. 4. SA1 appears to have close performance to URAS EXH throughout the whole range of SNR, while SA2 fails to achieve similar capacity at high SNR due to the lack of precoding matrices update leading to degraded CCI cancellation capability at this range. Comparing to US EXH, the proposed algorithms offer very close performance at low SNR and outperform US EXH at high SNR although little gain is seen for SA2. This trend remains valid for different number of users as shown in Fig. 5. Note that the simulation results for the exhaustive schemes are provided up to 10 users due to the high complexity of the exhaustive algorithms.

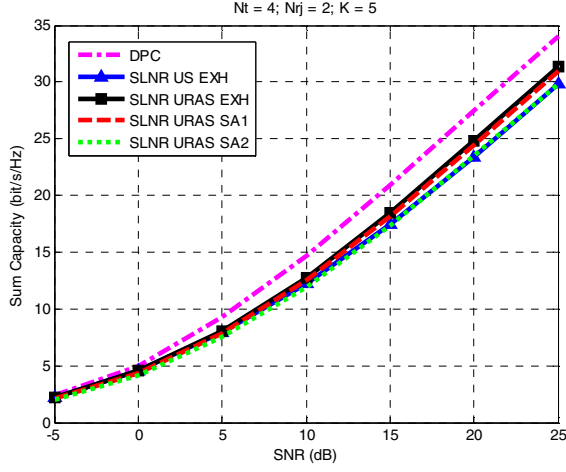


Figure 4. Comparison of ergodic sum capacity between the exhaustive search and suboptimal algorithms.

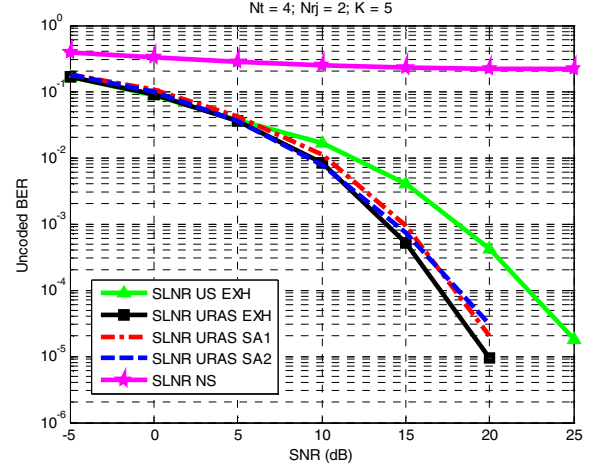


Figure 6. Average uncoded BER over all scheduled users for different scheduling schemes with QPSK modulation.

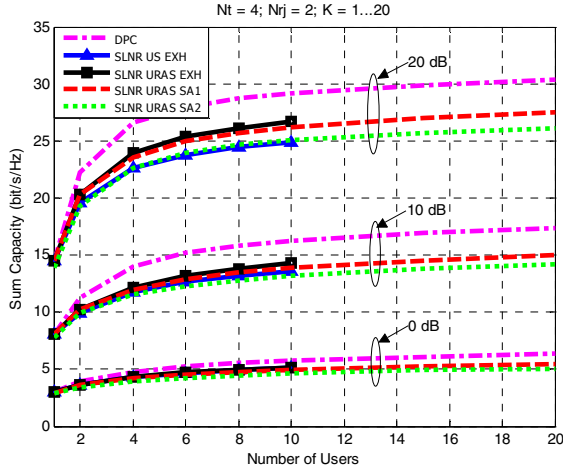


Figure 5. Ergodic sum capacity versus the number of users at SNR = 0, 10 and 20 dB

### C. BER Performance

BER results from software simulation are presented in Fig. 6. User selection and the joint user and antenna selection schemes offer similar performance at low SNR. At high SNR, the proposed algorithms outperform the US EXH and still perform very close to the URAS EXH. No-selection scheme (SLNR NS) delivers poor BER performance throughout the whole SNR range due to the inefficiency of the SLNR precoding design to eliminate CCI when the total number of data streams exceeds the available spatial-domain degree of freedom.

### V. CONCLUSIONS

This paper evaluated the potential gain of joint user and receive antenna selection over user selection in a MU-MIMO system using an SLNR-based precoding scheme with multiple antenna receivers. The performance gain is shown to be significant at high SNR. Two suboptimal joint user and receive antenna selection algorithms were then proposed with dynamic data stream allocation. They are shown to perform very close to

the exhaustive algorithm and even outperform the exhaustive user-only selection at high SNR, yet offer significantly lower complexity. Although the proposed algorithms initially aim to maximise the sum capacity, they can be easily extended to incorporate fairness by modifying the objective function from the sum capacity to other weighted sum capacity criteria such as the rate proportional and max-min fairness.

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