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1) Gradient :- It is the rate of change of physical quantity with respect to the distance. The gradient of mart of the physical quantities decreases as we move away from source.

Divergence :- It is the term used to represent how much the physical quantity has diverges away away it move away from the centre. Eg - In case of top water in the water basin. In That care when water fall as the basis surface.

Curl :- It is a parameter which decides how much a physical quantity is twisting around its centre.

Eg - A tornado at the centre.

2) According to Grain "Divergence theorem".

$$\boxed{\int_S f_1\, dA = \int_V (\nabla \cdot F)\, dv}$$

According to this theorem the surface integration of my vector field f is equal to the volume integration of divergence of the vector field f.

3) $\rho = \sqrt{x^2+y^2}$; $\tan\phi = \frac{y}{x}$

Also $x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$

$$\frac{d\rho}{dx} = \frac{x}{\sqrt{x^2+y^2}} = \frac{\rho\cos\theta}{\sqrt{\rho^2}} = \frac{\cancel{\rho}\cos\phi}{\cancel{\rho}}$$

$$\frac{d\rho}{dy} = \frac{\cancel{x}\tan\phi}{\cancel{x}} \Rightarrow \sin\phi$$

$$\frac{d\phi}{dx} = \frac{dx}{d\phi} = \frac{d(\rho\cos\phi)}{dx}$$

$$\frac{d\phi}{dx} = \frac{-y}{x^2}\frac{1}{1+\left(\frac{y}{x}\right)^2}$$

$$\Rightarrow \frac{-y}{x^2+y^2} = \frac{-\sin\phi}{\rho}$$

$$\frac{d\phi}{dy} = \frac{dy}{d\phi} = \frac{d\rho\sin\phi}{d\phi} = \rho\cos\phi$$

$$\frac{d\phi}{dy} = \frac{\cos\phi}{\rho}$$

from chain rule,

$$\frac{d}{dx} = \frac{d}{d\rho}\cdot\frac{d\rho}{dx} + \frac{d}{d\phi}\cdot\frac{d\phi}{dx}$$

$$\frac{d}{dy} = \frac{d}{d\rho}\cdot\frac{d\rho}{dy} + \frac{d}{d\phi}\cdot\frac{d\phi}{dy}$$

from unit vector transformation,

$$\hat{a}_x = \cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi$$

$$\hat{a}_y = \sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi$$

$$\vec{\nabla} = \frac{d}{d\cancel{x}}\hat{a}_x + \frac{d}{dy}\hat{a}_y + \frac{d}{dz}\hat{a}_z$$

$$\rightarrow \left(\frac{d}{d\rho}\cdot\frac{d\rho}{dx} + \frac{d}{d\phi}\cdot\frac{d\phi}{dx}\right)\left(\cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi\right)$$

$$\rightarrow + \left(\frac{d}{d\rho}\cdot\frac{d\rho}{dy} + \frac{d}{d\phi}\cdot\frac{d\phi}{dy}\right)\left(\sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi\right)$$

$$\rightarrow \left(\frac{d}{d\rho}\cos\phi - \frac{d}{d\phi}\frac{\sin\phi}{\rho}\right)\left(\cos\phi\,\hat{a}_\rho - \sin\phi\,\hat{a}_\phi\right)$$

$$+ \left(\frac{d}{d\rho}\sin\phi + \frac{d}{d\phi}\frac{\cos\phi}{\rho}\right)\left(\sin\phi\,\hat{a}_\rho + \cos\phi\,\hat{a}_\phi\right)$$

$$\rightarrow \vec{\nabla} = (\sin^2\phi + \cos^2\phi)\frac{d\hat{a}_\rho}{d\rho} + \frac{1}{\rho}(\sin^2\phi + \cos^2\phi) + \frac{d}{dz}\hat{a}_z$$

$$\boxed{\vec{\nabla} = \frac{d}{d\rho}\hat{a}_\rho + \frac{1}{\rho}\frac{d}{d\phi}\hat{a}_\phi + \frac{d}{dz}\hat{a}_z}$$

(4) A scalar field is a physical quantity as region in which a scalar function has a defined value and no direction.

For eg. a hot iron rod is kept in a corner of a Room then also its heat can be felt at some distance away from it. So the region in which the heat can be feld is the scalar field.

Temperature because in all the points inside the field the temperature has a defined value.

- A vector field is a field in which the vector quantity will have a defined value and some direction.

Eg. a bullet from a gun is having a velocity field around it propagated path which is a vector field because it has a particular direction of propagation i.e, along the direction of bullet.

6) $s(x, y, z) = x^2 + y^2 - z$

$$\frac{ds}{dx} = 2x, \quad \frac{ds}{dy} = 2y, \quad \frac{ds}{dz} = 1$$

$$\hat{n} = \frac{\nabla f}{|\nabla f|} \text{ at } (1, 3, 0) \quad \boxed{\hat{n} = \text{Unit vector}}$$

$$\hat{n} = \frac{(2, 6, 1)}{\sqrt{1+4+0}} = \frac{1}{2}(2, 6, 1)$$

$$= \left(1, 3, \frac{1}{2}\right).$$

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7) $P = (-2, 6, 3)$, $A = y\hat{a}_x + (x+z)\hat{a}_y + 0\hat{a}_z$

Cartesian to cylindrical.

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$\therefore \; A_x = 6, \; A_y = 4, \; A_z = 0$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{6}{-2}\right) = -71.5^\circ$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos(-71.5^\circ) & \sin(-71.5^\circ) & 0 \\ -\sin(-71.5^\circ) & \cos(-71.5^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} 0.31 & -0.94 & 0 \\ 0.94 & 0.31 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

$A_\rho = 0.31 \times 6 - 0.94 \times 4 + 0$

$A_\phi = 0.94 \times 6 + 0.31 \times 4 + 0$

$A_z = 0$

So

$$\boxed{F = -1.96\,\hat{a}_\rho + 6.88\,a_\phi + 0\,\hat{a}_z}$$

9) $\vec{A} = (4x+9y)\hat{a}_x - 14yz\,\hat{a}_y + 8x^2z\,\hat{a}_z$

$\oint \vec{A}\cdot\vec{dL}$ from $P(0,0,0)$ to $Q(0,1,1)$.

along (a) $x = t,\ y = t^2,\ z = t^3$

$dx = dt,\ dy = 2t\,dt,\ dz = 3t^2\,dt$

$\oint \vec{A}\,dL = \int_0^1 (4t + 9t^2)\cdot dt + \int_0^1 (-14t^5)\,dt\,2t$

$+ \int_0^1 8t^2\,t^3\,dt$

$\Rightarrow \int_0^1 3t\,dt + \int_0^1 28t^6\,dt + \int_0^1 24t^7\,dt$

$\Rightarrow \left[\frac{14}{2}t^2\right]_0^1 + \left[\frac{9t^3}{3}\right]_0^1 - \left[\frac{28}{7}t^7\right]_0^1 + \left[\frac{24t^8}{8}\right]_0^1$

$= 4$

(b)

(2) from $(0,0,0)$ to $(1,0,0)$

$\oint \vec{A}\,dl = \int_0^1 (4x + 9y)\,dx + 0$

$\Rightarrow \left[\frac{4x^2}{2}\right]_0^1 = 12$

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10) According to the divergence theorem

$\vec{\phi} = 3x^2 \vec{a_x} + (3y+z)\vec{a_y} + (3z-x)\vec{a_z}$

region bounded by cylinder

$x^2 + y^2 = 9$ and the plane $x=0$;

$z=0, z=2, y=0$

$\therefore$ parameter of cylinder at,

$$\begin{cases} \rho = 3 \\ \phi = 0 \text{ to } 90^\circ \ (1^{st} \text{ quadrant}) \\ z = 2 \end{cases}$$

According to divergence theorem,

$$\boxed{\oint_S \vec{D} \cdot ds = \oint_V (\nabla \cdot \vec{D})\, dv}$$

Now, solving the RHS part,

$\int_V (\vec{\nabla} \cdot \vec{D})\, dv$

$\vec{\nabla} \cdot \vec{D} \Rightarrow \frac{\partial}{\partial x}(3x^2) + \frac{\partial}{\partial y}(3y+z) + \frac{\partial}{\partial z}(3z-x)$

$\Rightarrow (6x+6) \cdot dv$

$\Rightarrow (6x+6)\, \rho\, d\phi\, d\rho\, dz$

Also $x = \rho \cos\phi$

$\Rightarrow \oint_V (6\rho \cos\phi + 3)\, \rho\, d\phi\, d\rho\, dz,$

$\Rightarrow \left(6 \int_0^3 \rho^2 d\rho \int_0^{\pi/2} \cos\phi\, d\phi \int_0^2 dz\right)$

$+ \left[\int_0^{\pi/2} d\phi \int_0^3 \rho\, d\rho \int_0^2 dz\right]$

$$\Rightarrow 6\left(\left[\frac{\rho^3}{3}\right]_0^3 \times [\sin\phi]_0^{\pi/2} \times [z]_0^2\right)$$

$$+\left(0 + \frac{\pi}{2} + \frac{\rho^2}{2}\Big|_0^3 \times 2\right)$$

$$= 2 \times 27 \times 2 + 54 + (27 \times \pi)$$

$$= \underline{\underline{192.8}}$$

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11)

```
# include <Stdio.h>
# include <math.h>
void main()
{ float x,y,r ,thetha ;
 printf("\n Enter the value of x
                        coordinate");
 Scanf("%f" ,&x);
 printf ("\n Enter the value of y
                  coordinate");
 Scanf("%f", &y);
 r =sqrt (pow (x,2) + pow (y,2));
 thetha = pow( tan(y/x),-1);
 printf("Value of r is: %f" ,r);
 printf("\n The value of thetha is:%f",
                                thetha);
 return 0;
}
```