1) Boundary Condition are the pre assumptio
that are made in order to derive
any formula and Equations. These
are the criteria which are considered
to be followed by the System which
is used to derive the formula or equation

2) Stoke's theorem :-

$$\int_L \vec{A}\cdot\vec{dl} = \int_S (\vec{\nabla}\times\vec{A})\cdot\vec{ds}$$

Consider any surface S which is divided into many differential surfaces that are $ds_1, ds_2, ds_3, \ldots ds_n$.

And the surface is enclosed by the line L,

So, $$\oint_L A\cdot dl = \int_{dS_1} \vec{A}\cdot\vec{dl} + \int_{ds_2} \vec{A}\cdot\vec{dl} + \int_{dS_3} \vec{A}\cdot\vec{dl} + \int_{dS_n} \vec{A}\cdot\vec{dl}$$

Now, from the definition of curl,

$$\vec{\nabla}\times\vec{A} = \int \frac{\vec{A}\,dl}{ds}$$

$$(\vec{\nabla}\times\vec{A})\,ds = \int \vec{A}\cdot\vec{dl}$$

$$\Rightarrow \int_L \vec{A}\cdot\vec{dl} = (\vec{\nabla}\times\vec{A})\,ds + (\nabla\times A)\,ds_2 + (\nabla\times A)\,dS_3 + \ldots (\vec{\nabla}\times\vec{A})\,dS_2$$

$$\Rightarrow \int_L \vec{A}\cdot\vec{dl} = (\vec{\nabla}\times\vec{A})(ds_1 + ds_2 + ds_3 + \ldots ds_n).$$

$$\therefore \boxed{\int_L \vec{A}\cdot\vec{dl} = \int_S (\vec{\nabla}\times\vec{A})\,ds}$$

Proved

3) (i) Electric flux & Electric flux density :-

Elective flux is the no. of electric field lines that is produced due to flow of electrical current inside a curve.

Electric flux density is the no. of amount flux passing through a unit Surface area i.e., The no. of electric field lines passing through the unit Surface area,

(ii) Displacement current is the current which flows in an open ckt. ie, where there is no close path for the charges to flow.

(iii) The laplacian operator is a differential operator given by the divergence of the gradient of a function an Euclidean space.

(iv) Poison's and laplace Eq^n :-

Poison's Equation is the partial differential Equation which is used to decide the potential field caused by a given charge as man density distribution.

Laplace eq^n is a second order diff. eq which is often written as $\nabla^2 f = 0$, ∇ is the gradient operator $f(x,y,z)$.
Laplace eq^n maps the scalar function to scalar function.

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(4) Using Gauss's Law of Electrostatic

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

where, $E \rightarrow$ Electric field intensity
ρ = Total charge density
$\epsilon_0 \rightarrow$ Electric permiability of the material.

then, $U = \frac{1}{2} \int \rho \phi \, dv$ (Energy & Potential)

$$= \frac{1}{2} \int \epsilon_0 (\nabla \cdot E) \phi \, dv$$

we have,

$$U = \frac{\epsilon_0}{2} \int \nabla \cdot (E \phi) \, dv - \frac{\epsilon_0}{2} \int (\nabla \phi) \cdot E \, dv.$$

Using divergence theorem and taking the area to be at infinity where $\phi(\infty) = 0$.

$$\therefore U = \frac{\epsilon_0}{2} \int \phi E \cdot dA - \frac{\epsilon_0}{2} \int (-E) E \, dv.$$

$$= \frac{1}{2} \int \epsilon_0 E_c^2 \, dv.$$

So The energy density $\Rightarrow$ $\boxed{U_c = \frac{1}{2} \epsilon_0 |E|^2}$

5) Maxwell's Equation:-

① Maxwell's 1^{st} Eqⁿ,

According to Gauss theorem,

$$\int_S E \cdot ds = \frac{q}{\epsilon_0} \quad \text{--- (i)}$$

where, q is the total energy which can be written.

$$q = \iiint_V \rho \, dv \quad \text{--- (2)}$$

ρ = is the volume charge density.

From eqⁿ (i),

$$\int_V (\vec{\nabla} \cdot \vec{E}) dv = \iiint_V \frac{\rho}{\epsilon_0} dv$$

$$\therefore \boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

In case of charge free surface,

$$\boxed{\therefore \nabla \cdot \vec{E} = 0}$$

But in vaccumn, $\vec{D} = \epsilon_0 \vec{E}$ eq (3) becomes

$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho}$$

② Maxwell's 2^{nd} Eq^n:-

Since there is no isolated magnetic poles exists.
Thus the no. of magnetic lines of force entering a surface in a magnetic field is equal to the no. of magnetic line leaving the surface.
So, Mathematically,

$$\boxed{\iint_S B \cdot \vec{ds} = 0}$$

$$\boxed{\iiint_V (\vec{\nabla} \cdot \vec{B}) \vec{dv} = 0}$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

③ Maxwell's 3rd Equation :-

According to faraday's Law of EMI wherever there is change in magnetic flux link with a closed ckt., an emf is induced in its which is a directly proportional to the negative ~~of~~ rate of change of magnetic flux.

$$\therefore \boxed{e = \frac{-d\phi}{dt}}$$

$$\Rightarrow \frac{-d}{dt} \iint_S \vec{B} \cdot \vec{ds}$$

But the electric flux produced due to change in magnetic flux, the emf induced in the closed ckt will be equal to line integration of the electric field along with the closed ckt

$$\mathcal{E} = \int_C \vec{E} \cdot \vec{dl}$$

$$\oint_C \vec{F} \, \vec{dL} = -\int_S \frac{\vec{dB}}{dt} \, \vec{ds}$$

$$\iint (\nabla \times \vec{E}) \, \vec{ds} = -\iint_S \frac{\vec{dB}}{dt} \, \vec{dS}$$

$$(\nabla \times \vec{E}) = -\frac{\vec{dB}}{dt}.$$

(4) Maxwell's ~~B~~ 4th equation

According to Ampere's law curl of v is equal to

$$\nabla \times B = \mu_0 (J + J_d)$$

$$div\ (\vec{\nabla} \times \vec{B}) = \mu_0\ div\ (J + J_d)$$

$$\Rightarrow div\ \vec{J} = -div\ \vec{J_d}$$

$$\text{div} \cdot \vec{J} = -\frac{d\rho}{dt}$$

from maxwell first equation

$$\nabla \times E = \frac{\rho}{\epsilon_0}$$

$$\rho = \epsilon_0 (\vec{\nabla} \cdot \vec{E})$$

$$\text{di} \ \vec{Jd} = \frac{d(\epsilon_0 (\vec{\nabla} \times \vec{E}))}{dt}$$

$$\boxed{\vec{Jd} = \epsilon_0 \frac{dE}{dt}}$$

Putting in early eq

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{dE}{dt} \right)$$

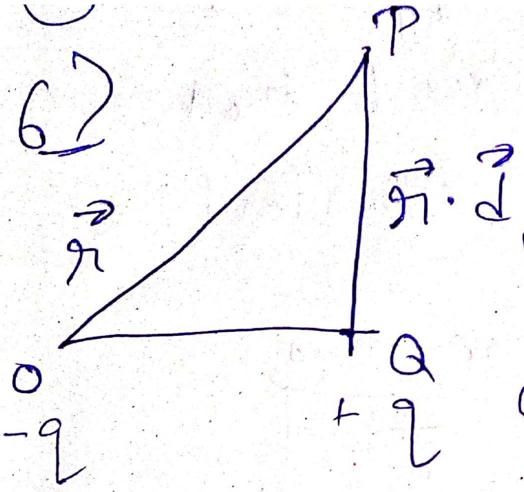
in case of vacuum.

$$\vec{B} = \mu_0 \vec{H} \text{ and } \vec{D} = \epsilon_0 \vec{E}$$

$$\nabla \times (\mu_0 \vec{H}) = \mu_0 (J + \epsilon_0 \vec{E})$$

$$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

6)



Two equal and opposite point charges are repeated by a distance, small compare of distance at which the potential due to the change conf^n is demand are said to be contribute an electric dipole.

The potential is given by:

$$\phi_{os} = \frac{q}{4\pi\epsilon_0 (\vec{r}\cdot\vec{d})} - \frac{q}{4\pi\epsilon_0 (\vec{r})}$$

$$\frac{1}{(\vec{r}-\vec{d})} = \left[l^2 - 2\vec{l}\cdot\vec{d} + d^2\right]^{-1/2} \left[a^2 - 2ab + c\right]$$

$$= \frac{1}{r}\left[1 - \frac{2\vec{l}\vec{d}}{l^2} + \frac{d^2}{r^2}\right]^{-1/2}$$

$$= \frac{1}{r}\left[\frac{1 - 2\vec{r}\vec{d}}{r^2}\right]^{-1/2}$$

$$\frac{1}{\vec{r}-\vec{d}} \approx \frac{1}{r}\left[1 + \frac{\vec{r}\cdot\vec{d}}{l^2}\right]$$

$$\phi_s = \frac{q}{4\pi\epsilon_0} = \frac{\vec{r}\,(\vec{d})}{r_0^3}$$

For dipole, $\vec{d}$ is approached towards 0 once q is allowed to approach towards infinity (∞) such that the term $q.\vec{d}$ remain constant.

so, $\frac{q.\vec{d}}{\phi}$ So, eq^n (3) becoms

$$\boxed{\phi_r = \frac{\vec{P}\cdot\vec{r}}{4\pi\epsilon r^2}}$$

7) The Point charges are $-4nc$, $-5\,nc$, $-1\,nc$ at $(0,0,0)$, $(0,0,2)$, $(2,0,0)$ coordinate.

$$WE = W_1 + W_2 + W_3$$

$$= 0 + (-)1 \times \phi_{21} + (-1)3(\phi_{31} + \phi_{32})$$

$$= \frac{1}{4\pi\epsilon_0}(24) = \frac{6}{\pi\epsilon_0}$$

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8 (a) $F = 2x^2 \hat{a}_x - 3xz\hat{a}_y - 6y^2\hat{a}_z$

from $(1,0,0)$ to $(0,0,0)$

$$\int F \cdot dL = \int 2x^2 \, dx - \int 3x \cancel{z}^{0} dy - \int 6y^2 \cancel{dz}^{0}$$

$$\int_0^1 2x^2 \hat{a}_x \left[\frac{2x^3}{3}\right]_0^1 = \frac{2}{3}$$

(b) $(0,0,0)$ to $(0,1,0)$

$$\int F \cdot dl = \int 2x^2 \cancel{dx}^{0} - \int 3x \cancel{2dy}^{0} - \int 6\cancel{y}^{0} dz$$

$\therefore x = 0$

$= \underline{0}$

(c) $(0,1,0)$ to $(1,1,1)$.

$$\int F \cdot dL = \int_0^1 2x^2 \, d\hat{x} - \int 3x \cancel{z}^{0} dy - \int_0^1 6y^2 d\hat{z}$$

$$= \left[\frac{2x^3}{3}\right]_0^1 - \left[\frac{6y^3}{3}\right]_0^1$$

$$= \frac{2}{3} - \frac{\cancel{6}^{2}}{\cancel{3}} = \frac{-4}{3}$$

(d) $(1,1,1)$ to $(1,0,0)$.

$$\int F \cdot dL = \int \cancel{2x^2 d\hat{x}}^{0} - \int 3x\hat{z}\,dy - 6y^2 dz.$$

$$- \int_1^0 3xz \, dy - \int_1 6y^2 \, d\hat{z}$$

Now using $\frac{x - x_1}{x - x_2} = \frac{y - y_1}{y - y_2} = \frac{z - z_1}{z - z_2}$

$x = 0, y = z$

$\rightarrow -\int_1^0 3y\,y - \int_1^0 6z\,z^2$

$= -3\int_1^0 y^2 - \int_1^0 6z^3$

$= 1 + \frac{6}{4} = \frac{10}{4}$

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10) $E \cdot ds = 4xz\,\hat{a}_x - y^2\hat{a}_y + yz\,\hat{a}_z$ and the cube bounded by $x=0, x=2, y=0, y=2, z=0, z=2$.

$\oint E\,ds_1 = \iint y^2\,dx\,dz$

$= -\int_0^2\int_0^2 y^2\,dx\,dz$ at $y=2$

$= -\int_0^2 [y^2 x]_0^2\,dz = -\int_0^2 [4\times 2]_0^2\,dz$

$= -\int_0^2 16\,dz$

$= -32$

$$\oint E\,ds_4 = \int_0^2\int_0^2 yz\,dx\,dy$$

$$= 2\int_0^2\int_0^2 y\,dy\,dx$$

$$= 2\int_0^2 \left[\frac{y^2}{2}\right]_0^2 dx$$

$$= 4\int_0^2 dx = 4[x]_0^2 = \underline{8}$$

$$\oint E\,ds_3 = -4xz\,dz = -8\left[\frac{z^2}{2}\right]_0^2 dy$$

$$= 2\times -8 \times [y]_0^2\,dy$$

$$= 16 \times 2 = \underline{32}$$