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ELSEVIER Applied Mathematics and Computation 160 (2005) 701–717

APPLIED MATHEMATICS AND COMPUTATION

www.elsevier.com/locate/amc

Inventory model involving controllable backorder rate and variable lead time demand with the mixtures of distribution

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Abstract

In the model of Ouyang and Chuang [Comput. Ind. Eng. 40 (2001) 339], they assume that the backorder rate is dependent on the length of lead time through the amount of shortages and let the backorder rate is a control variable. But, since they only assumed a single distribution for the lead time demand, when the demand of the different customers are not identical in the lead time, then we cannot use a single distribution (such as [Comput. Ind. Eng. 40 (2001) 339]) to describe the demand of the lead time. Hence, in our studies, we first assume that the lead time demand follows a mixtures of normal distribution, and then we relax the assumption about the form of the mixtures of distribution functions of the lead time demand and apply the minimax distribution free procedure to solve the problem. We develop an algorithm procedure, respectively, to find the optimal order quantity and the optimal lead time. Furthermore, two numerical examples are also given to illustrate the results.

Keywords: Order quantity; Backorder rate; Lost sales; Crashing cost; Minimax distribution free procedure; Mixtures of distribution

1. Introduction

In most of the literatures dealing with inventory problems, either in deterministic or probabilistic model, lead time is viewed as a prescribed

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0096-3003/$ - see front matter
doi:10.1016/j.amc.2003.11.039

constant or a stochastic variable, which therefore, is not subjected to control [5,8]. In fact, lead time usually consists of the following components [9]: order preparation, order transit, supplier lead time, delivery time, and setup time. In many practical situations, lead time can be reduced by an added crashing cost; in other words, it is controllable. By shortening the lead time, we can lower the safety stock, reduce the loss caused by stock-out, improve the service level to the customer, and increase the competitive ability in business. Ben-Daya and Raouf [1] presented a continuous review model in which they considered both the lead time and the order quantity as decision variables where the shortages are neglected. They gave the optimal lead time and the optimal order quantity which minimizes the sum of the ordering cost, inventory holding cost and lead time crashing cost. In a recent paper, Ouyang et al. [7], Ouyang and Chuang [6] and Wu and Tsai [10] have extended the Ben-Daya and Raouf [1] model by adding the stock-out cost. In their studies [6,7,10], they assumed a fixed stock-out cost attributable to each stock-out occasion.

In this article, the lead time and the order quantity are considered as the decision variables of a mixture of backorders and lost sales inventory model. Moreover, Ouyang and Chuang [6] also observe that many products of famous brands or fashionable goods such as certain brand gum shoes (or leather shoes), hi-fi equipment, cosmetics and clothes may lead to a situation in which customers prefer their demands to be backordered when shortages occur. Certainly, if the quantity of shortages is accumulated to a degree that exceeds the waiting patience of customers, some customers may refuse the backorder case. This phenomenon reveals that, as shortages occur, the longer the length of lead time is, the larger the amount of shortages is, the smaller the proportion of customers can wait and hence the smaller the backorder rate would be. Under the situation, for a vendor, how to control an appropriate length of lead time to determine a target value of backorder rate so as to minimize the inventory relevant cost and increase the competitive edge in business is worth discussing. Consequently, we here assume that the backorder rate is dependent on the length of lead time through the amount of shortages and let the backorder rate be a control variable (see Section 2). Since, the demand of the different customers are not identical in the lead time. So, we cannot only use a single distribution (such as [6]) to describe the demand of the lead time. Hence, we first assume that the lead time demand follows a mixtures of normal distribution, and find the optimal solutions. And then we also consider any mixtures of distribution function (d.f.), say $F_* = pF_1 + (1-p)F_2$, of the lead time demand has only known finite first and second moments (and hence, mean and variance are also known and finite) but make no assumption on the distribution form of F_*. That is, F_1 and F_2 of F_* belongs to the class Ω of all single d.f.s' with finite mean and variance. Our goal is to solve a mixture inventory model by using the minimax distribution free approach. That is, the minimax distribution free approach for our inventory model is to find the most unfavorable d.f.s F_1 and F_2 in F_* for each decision variable and then to minimize over the decision variables. Furthermore, two numerical examples are also given to illustrate the results.

2. Notations and assumptions

To establish the mathematical model, the notations are exactly the same as those in Ouyang and Chuang [6] expect the following notations:

- X the lead time demand which has a mixtures of d.f. $F_* = pF_1 + (1-p)F_2$ with finite mean $\mu_* L$ and standard deviation $\sigma_*\sqrt{L}$ (>0), a random variable.
- x^+ maximum value of x and 0, i.e., $x^+ = \max\{x, 0\}$.
- x^- maximum value of $-x$ and 0, i.e., $x^- = \max\{-x, 0\}$.

$$I_{(0<x<r)} = \begin{cases} 1, & 0 < x < r, \\ 0, & \text{o.w.} \end{cases}$$

The assumptions of the model are exactly the same as those in Ouyang and Chuang [6] expect the following assumptions: the reorder point $r =$ expected demand during lead time + safety stock (SS), and SS $= k\cdot$(standard deviation of lead time demand), i.e. $r = \mu_* L + k\sigma_*\sqrt{L}$, where $\mu_* = p\mu_1 + (1-p)\mu_2$, $\sigma_* = \sqrt{1 + p(1-p)\eta^2}\sigma$, $\mu_1 = \mu_* + (1-p)\eta\sigma/\sqrt{L}$, $\mu_2 = \mu_* - p\eta\sigma/\sqrt{L}$ (that is, $\mu_1 - \mu_2 = \eta\sigma/\sqrt{L}$, $\eta \in R$), and k is the safety factor which satisfies $p(X > r) = 1 - p\Phi(r_1) - (1-p)\Phi(r_2) = q$, where Φ represents the cumulative distribution function of the standard normal random variable, q represents the allowable stock-out probability during L, $r_1 = (r - \mu_1 L)/\left(\sigma\sqrt{L}\right) = k[1 + \eta^2 p(1-p)]^{1/2} - \eta(1-p)$, and $r_2 = (r - \mu_2 L)/\left(\sigma\sqrt{L}\right) = k[1 + \eta^2 p(1-p)]^{1/2} + \eta p$.

3. Model formulation

3.1. The mixtures of normal distribution model

First we consider the lead time demand X has a mixtures of normal distribution $F_* = pF_1 + (1-p)F_2$, where F_1 has a normal distribution with finite mean μ_1 and standard deviation $\sigma\sqrt{L}$ and F_2 has a normal distribution with finite mean μ_2 and standard deviation $\sigma\sqrt{L}$, $\mu_1 - \mu_2 = \eta\sigma/\sqrt{L}$, $\eta \in R$.

Therefore, the demand of the lead time X has the mixtures of probability density function (p.d.f.) which is given by

$$f(x) = p\frac{1}{\sqrt{2\pi}\sigma\sqrt{L}}\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu_1 L}{\sigma\sqrt{L}}\right)^2} + (1-p)\frac{1}{\sqrt{2\pi}\sigma\sqrt{L}}\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu_2 L}{\sigma\sqrt{L}}\right)^2},$$

where $\mu_1 - \mu_2 = \eta\sigma/\sqrt{L}$, $\eta \in R$, $x \in R$, $0 \leqslant p \leqslant 1$, $\sigma > 0$ [2]. Moreover, if $(\mu_1 - \mu_2)^2 < 27\sigma^2/(8L)$ or $-\sqrt{27/8} < \eta < \sqrt{27/8}$, for any $0 \leqslant p \leqslant 1$, then the mixtures of normal distribution is a unimodal distribution. When $(\mu_1 - \mu_2)^2 > 4\sigma^2/L$ or $|\eta| > 2$, at least we can find a p $(0 \leqslant p \leqslant 1)$ which makes the mixtures of normal distribution to be a bimodal distribution. Moreover, the reorder point $r = \mu_* L + k\sigma_*\sqrt{L}$, where k, μ_* and σ_* are defined as above. Then, the expected demand shortage at the end of the cycle is given by

$$B(r) = E[X-r]^+ = \int_r^\infty (x-r)\,\mathrm{d}F_*(x) = \sigma\sqrt{L}\Psi(r_1, r_2, p), \tag{1}$$

where

$$\Psi(r_1, r_2, p) \equiv p[\phi(r_1) - r_1(1 - \Phi(r_1))] + (1-p)[\phi(r_2) - r_2(1 - \Phi(r_2))],$$
$$r_1 = \frac{r - \mu_1 L}{\sigma\sqrt{L}} \quad \text{and} \quad r_2 = \frac{r - \mu_2 L}{\sigma\sqrt{L}}.$$

Here, let ϕ and Φ be the standard normal p.d.f. and cumulative distribution function (c.d.f.), respectively. Thus, the expected number of backorders per cycle is $\beta B(r)$ and the expected lost sales per cycle is $(1-\beta)B(r)$, where β $(0 \leqslant \beta \leqslant 1)$ is the fraction of the demand during the stock-out period will be ordered. Hence, the annual stock-out cost is $\frac{D}{Q}[\pi + \pi_0(1-\beta)]B(r)$.

Then, the expected net inventory level just before the order arrives is

$$\begin{aligned} E[(X-r)^- I_{(0<X<r)}] - \beta B(r) = \sigma\sqrt{L}\Bigg\{ & p\left[r_1 \cdot \Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} + (1-p)\eta\right)\right. \\ & \left. - \phi\left(\frac{\mu_*\sqrt{L}}{\sigma} + (1-p)\eta\right)\right] \\ & + (1-p)\left[r_2 \cdot \Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right. \\ & \left. - \phi\left(\frac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right]\Bigg\} + (1-\beta)B(r) \end{aligned}$$

and the expected net inventory level at beginning of the cycle is

$$\begin{aligned} Q+E[(X-r)^{-}I_{(0<X<r)}]-\beta B(r) = Q+\sigma\sqrt{L}\Big\{ & p\Big[r_1\cdot\Phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\Big) \\ & -\phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\Big)\Big] \\ & +(1-p)\Big[r_2\cdot\Phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\Big) \\ & -\phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\Big)\Big]\Big\}+(1-\beta)B(r). \end{aligned}$$

Therefore, the expected annual holding cost is

$$\begin{aligned} h\Big\{\frac{Q}{2}+\sigma\sqrt{L}\Big\{ & p\Big[r_1\cdot\Phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\Big)-\phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\Big)\Big] \\ & +(1-p)\Big[r_2\cdot\Phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\Big)-\phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\Big)\Big]\Big\}+(1-\beta)B(r)\Big\}. \end{aligned}$$

The objective of the problem is to minimize the total expected annual cost which is the sum of ordering cost, holding cost, stock-out cost and lead time crashing cost. Symbolically, the problem is

$$\begin{aligned} \underset{Q>0,L>0}{\mathrm{Min}}\ \mathrm{EAC}^N(Q,L) = A\frac{D}{Q}+h\Big[\frac{Q}{2}+\sigma\sqrt{L}\Big\{ & p\Big[r_1\cdot\Phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\Big) \\ & -\phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\Big)\Big] \\ & +(1-p)\Big[r_2\cdot\Phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\Big) \\ & -\phi\Big(\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\Big)\Big]\Big\}+(1-\beta)B(r)\Big] \\ & +\frac{D}{Q}[\pi+\pi_0(1-\beta)]B(r)+\frac{D}{Q}R(L), \end{aligned} \tag{2}$$

where the lead time crashing cost $R(L)$ per cycle for a given $L\in(L_i,L_{i-1})$ is given by

$$R(L)=c_i(L_{i-1}-L)+\sum_{j=1}^{i-1}c_j(b_j-a_j),\quad L_0=\sum_{j=1}^{n}b_j,\quad \text{and}$$

$$L_i=\sum_{j=1}^{n}b_j-\sum_{j=1}^{i}(b_j-a_j),\quad i=1,2,\ldots,n\quad \text{(also see [6, 10])}.$$

The parameter β $(0\leqslant\beta\leqslant1)$ is treated as a constant; however, in the real market as unsatisfied demands occur, the longer the length of lead time is, the larger the amount of shortages is, the smaller the proportion of customers can wait, and hence the smaller the backorder rate would be. Therefore, in model (2), we consider the backorder rate β, as a decision variable instead of the constant case to accommodate the practical inventory situation. During the stock-out period, the backorder rate β, is variable and is a function of L through $B(r) = E[X - r]^+$. The larger the expected shortage quantity is, the smaller the backorder rate would be. Thus we define

$$\beta = \frac{\theta}{1 + \varepsilon B(r)}, \quad 0 \leqslant \theta \leqslant 1, \ \ 0 \leqslant \varepsilon < \infty. \tag{3}$$

As the value of ε increases, the total expected annual cost becomes close to the complete lost case (i.e. $\beta \to 0$) for $\theta = 1$. Conversely, decreasing the value of ε, the total expected annual cost will approach the complete backordered case (i.e. $\beta \to 1$) for $\theta = 1$.

By using Eqs. (1) and (3), the model (2) is the total expected annual cost of our new model reduced to

$$\begin{aligned}\operatorname*{Min}_{Q>0,L>0} \mathrm{EAC}^N(Q,L) = {} & A\frac{D}{Q} + h\Bigg\{\frac{Q}{2} + \sigma\sqrt{L}\Bigg\{p\Bigg[r_1\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} + (1-p)\eta\right) \\ & - \phi\left(\frac{\mu_*\sqrt{L}}{\sigma} + (1-p)\eta\right)\Bigg] \\ & + (1-p)\Bigg[r_2\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} - p\eta\right) \\ & - \phi\left(\frac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\Bigg]\Bigg\}\Bigg\} + \frac{D}{Q}\pi\sigma\sqrt{L}\Psi(r_1,r_2,p) \\ & + \left(h + \frac{D}{Q}\pi_0\right)\left[1 - \frac{\theta}{1 + \varepsilon\sigma\sqrt{L}\Psi(r_1,r_2,p)}\right] \\ & \times \sigma\sqrt{L}\Psi(r_1,r_2,p) + R(L)\frac{D}{Q}. \end{aligned} \tag{4}$$

In order to find the minimum total expected annual cost, taking the first partial derivatives of $\mathrm{EAC}^N(Q,L)$ with respect to Q and L in each time interval (L_i, L_{i-1}), we obtain

$$\begin{aligned}\frac{\partial \mathrm{EAC}(Q,L)}{\partial Q} = {} & -\frac{AD}{Q^2} + \frac{h}{2} - \frac{D}{Q^2}\left[\pi + \pi_0\left(1 - \frac{\theta}{1 + \Delta(L)}\right)\right]\frac{\Delta(L)}{\varepsilon} \\ & - R(L)\frac{D}{Q^2}, \end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial \mathrm{EAC}(Q,L)}{\partial L} &= \frac{h\sigma}{2\sqrt{L}}\left\{p\left[r_1\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\right)-\phi\left(\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\right)\right]\right.\\ &\quad\left.+(1-p)\left[r_2\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\right)-\phi\left(\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\right)\right]\right\}\\ &\quad+\frac{h\mu_*}{2}\left\{p\left[r_1+\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\right]\phi\left(\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\right)\right.\\ &\quad\left.+(1-p)\left[r_2+\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\right]\phi\left(\frac{\mu_*\sqrt{L}}{\sigma}-p\eta\right)\right\}\\ &\quad+\left(h+\pi_0\frac{D}{Q}\right)\left[\frac{(1-\theta)\Delta(L)+(\Delta(L))^2[2+\Delta(L)]}{2\varepsilon L(1+\Delta(L))^2}\right]\\ &\quad+\frac{\pi D}{2LQ}\cdot\frac{\Delta(L)}{\varepsilon}-c_i\frac{D}{Q},\end{aligned}\tag{6}$$

where $\Delta(L)=\varepsilon\sigma\sqrt{L}\Psi(r_1,r_2,p)=\varepsilon B(r)$.

It is clear that for any given r_1, r_2 and p, we have $\Psi(r_1,r_2,p)>0$. Hence, we can show that for fixed $L\in(L_i,L_{i-1})$, $\mathrm{EAC}^N(Q,L)$ is convex in Q, since $\frac{\partial^2 EAC^N(Q,L)}{\partial Q^2}>0$.

However, we can also show that for fixed Q, $\mathrm{EAC}^N(Q,L)$ is concave function of $L\in[L_i,L_{i-1}]$, if $\min\left\{\frac{\mu_*\sqrt{L}}{\sigma}-p\eta,\frac{\mu_*\sqrt{L}}{\sigma}+(1-p)\eta\right\}>\sqrt{2}$, where $\eta\in R$. Hence, for fixed Q, the minimum total expected annual cost will occur at the end points of the interval (L_i,L_{i-1}). Setting Eq. (5) equal to zero and solving for Q, we have

$$\begin{aligned}\frac{\partial \mathrm{EAC}^N(Q,L)}{\partial Q} &= 0\\ &\Rightarrow -\frac{AD}{Q^2}+\frac{h}{2}-\frac{D}{Q^2}\left[\pi+\pi_0\left(1-\frac{\theta}{1+\Delta(L)}\right)\right]\frac{\Delta(L)}{\varepsilon}\\ &\quad -R(L)\frac{D}{Q^2}=0,\\ &\Rightarrow Q=\left\{\frac{2D}{h}\left\{A+\left[\pi+\pi_0\left(1-\frac{\theta}{1+\Delta(L)}\right)\right]\frac{\Delta(L)}{\varepsilon}\right.\right.\\ &\quad\left.\left.+R(L)\right\}\right\}^{\frac{1}{2}},\end{aligned}\tag{7}$$

where $\Delta(L)=\varepsilon\sigma\sqrt{L}\Psi(r_1,r_2,p)=\varepsilon B(r)$. Thus, we can establish the following iterative algorithm to find the optimal lead time L and the optimal order quantity Q.

Algorithm 1

Step 1. Input the values of A, D, h, η, σ, π, π_0, p, q, θ, ε, a_i, b_i, c_i, and $i=1,2,\dots,n$.

Step 2. Given η, p and q, by using the Microsoft Developer Studio Fortran Powerstation 4.0 and the subroutine ZREAL from IMSL [4] to solve

k of the equation $1 - p\Phi(r_1) - (1-p)\Phi(r_2) = q$ where $r_1 = k[1 + \eta^2 p(1-p)]^{1/2} - \eta(1-p)$ and $r_2 = k[1 + \eta^2 p(1-p)]^{1/2} + \eta p$. Further, we obtain r_1 and r_2.

Step 3. Use the a_i, b_i and c_i, $i = 1, 2, \dots, n$, to compute L_i and compute $B(r)$ using Eq. (1).

Step 4. For each L_i, $i = 1, 2, \dots, n$, compute Q_i by using Eq. (7).

Step 5. For each pair (Q_i, L_i), compute the corresponding total expected annual cost $\text{EAC}^N(Q_i, L_i)$, $i = 1, 2, \dots, n$.

Step 6. Find $\min_{i=0,1,\dots,n} \text{EAC}(Q_i, L_i)$. If $\text{EAC}^N(Q_*, L_*) = \min_{i=0,1,\dots,n} \text{EAC}^N(Q_i, L_i)$, then (Q_*, L_*) is the optimal solution.

3.2. The mixtures of distribution free model

The information about the form of the mixtures of d.f. of lead time demand is often limited in practices. In this subsection, we relax the restriction about the form of the mixtures of d.f. of lead time demand, i.e., we assume here that the lead time demand X has the mixtures of d.f. $F_* = pF_1 + (1-p)F_2$, where F_1 has finite mean $\mu_1 L$ and standard deviation $\sigma\sqrt{L}$ and F_2 has finite mean $\mu_2 L$ and standard deviation $\sigma\sqrt{L}$, $\mu_1 - \mu_2 = \frac{\eta\sigma}{\sqrt{L}}$, $\eta \in R$, but make no assumption on the mixtures of d.f.'s form of F_*. Now, we try to use a minimax distribution free procedure to solve this problem. If we let Ω denote the class of all single c.d.f. (included F_1 and F_2) with finite mean and standard deviation then the minimax distribution free approach for our problem is to find the most unfavorable c.d.f.s F_1 and F_2 in Ω for each decision variable and then to minimize over the decision variables; that is, our problem is to solve

$$\min_{Q>0, L>0} \max_{F_1, F_2 \in \Omega} \text{EAC}^F(Q, L), \tag{8}$$

where

$$\text{EAC}^F(Q, L) = A\frac{D}{Q} + h\left\{\frac{Q}{2} + k\sigma_*\sqrt{L} + (1-\beta)B(r)\right\} + \frac{D}{Q}[\pi + \pi_0(1-\beta)]B(r) + \frac{D}{Q}R(L)$$

and $B(r) = E(X - r)^+$.

In addition, we need the following Proposition 1 to solve the above problem of the model (8) [3].

Proposition 1. *For F_1 and $F_2 \in \Omega$,*

$$E(X_i - r)^+ \leqslant \frac{1}{2}\left\{\sqrt{\sigma^2 L + (r - \mu_i L)^2} - (r - \mu_i L)\right\}, \quad i = 1, 2, \tag{9}$$

where r.v.s X_i has a single distribution function F_i, $i = 1, 2$. Moreover, the upper bound (9) *is tight.*

So, by using $F_* = pF_1 + (1-p)F_2$ and (9), we obtain

$$\begin{aligned}
B(r) &= \int_r^\infty (x-r)\,\mathrm{d}F_* \\
&= p\int_r^\infty (x-r)\,\mathrm{d}F_1 + (1-p)\int_r^\infty (x-r)\,\mathrm{d}F_2 \\
&\leqslant p\cdot\frac{1}{2}\cdot\left\{\sqrt{\sigma^2 L + (r-\mu_1 L)^2} - (r-\mu_1 L)\right\} \\
&\quad + (1-p)\cdot\frac{1}{2}\cdot\left\{\sqrt{\sigma^2 L + (r-\mu_2 L)^2} + (r-\mu_2 L)\right\} \\
&= \frac{p}{2}(\mu_1 L - r) + \frac{(1-p)}{2}(\mu_2 L - r) + \frac{p}{2}\left[\sqrt{\sigma^2 L + (r-\mu_1 L)^2}\right] \\
&\quad + \frac{(1-p)}{2}\left[\sqrt{\sigma^2 L + (r-\mu_2 L)^2}\right] \\
&= \frac{1}{2}[p\mu_1 L - pr + (1-p)\mu_2 L - (1-p)r] + \frac{p}{2}\left[\sqrt{\sigma^2 L + (r-\mu_1 L)^2}\right] \\
&\quad + \frac{(1-p)}{2}\left[\sqrt{\sigma^2 L + (r-\mu_2 L)^2}\right] \\
&= \frac{1}{2}(\mu_* L - r) + \frac{p}{2}\left[\sqrt{\sigma^2 L + (r-\mu_1 L)^2}\right] \\
&\quad + \frac{(1-p)}{2}\left[\sqrt{\sigma^2 L + (r-\mu_2 L)^2}\right],
\end{aligned}$$

where

$$\begin{aligned}
r - \mu_1 L &= r - \mu_* L - (1-p)\eta\sigma\sqrt{L}; \\
r - \mu_2 L &= r - \mu_* L + p\eta\sigma\sqrt{L} \quad \left(\because \mu_1 - \mu_2 = \frac{\eta\sigma}{\sqrt{L}}\right).
\end{aligned}$$

Then

$$\begin{aligned}
B(r) &\leqslant \frac{1}{2}(\mu_* L - r) + \frac{p}{2}\left[\sqrt{\sigma^2 L + \left[r - \mu_* L - (1-p)\eta\sigma\sqrt{L}\right]^2}\right] \\
&\quad + \frac{(1-p)}{2}\left[\sqrt{\sigma^2 L + \left[r - \mu_* L + p\eta\sigma\sqrt{L}\right]^2}\right]. \qquad (10)
\end{aligned}$$

From the definition of the backorder rate β and inequality (10), we have

$$\beta = \left\{ \frac{\theta}{1+\varepsilon B(r)} \right\} \geqslant \theta \cdot \left\{ 1 + \varepsilon \left[\frac{1}{2}\Big(-k\sqrt{1+p(1-p)\eta^2}\sigma\sqrt{L} \Big) + \frac{p}{2}\sigma\sqrt{L}\left(\sqrt{1+\left[k\sqrt{1+p(1-p)\eta^2}-(1-p)\eta\right]^2} \right) + \frac{(1-p)}{2}\sigma\sqrt{L}\left(\sqrt{1+\left[k\sqrt{1+p(1-p)\eta^2}+p\eta\right]^2} \right) \right] \right\}^{-1}, \tag{11}$$

where $0 \leqslant \theta \leqslant 1$, $0 \leqslant \varepsilon < \infty$, $0 \leqslant p \leqslant 1$, $\sigma > 0$.

By using the inequality (10) and (11), the model (8) can be reduced to

$$\min_{Q>0,L>0} \mathrm{EAC}^{\mathrm{U}}(Q,L), \tag{12}$$

where

$$\mathrm{EAC}^{\mathrm{U}}(Q,L) = A\frac{D}{Q} + h\left(\frac{Q}{2} + k\sigma_*\sqrt{L}\right) + \left[\frac{D}{Q}\pi + \left(h + \frac{D}{Q}\pi_0\right)\cdot\left(1 - \frac{\theta}{1+\Delta^{\mathrm{U}}(L)}\right)\right]\cdot\frac{\Delta^{\mathrm{U}}(L)}{\varepsilon} + \frac{D}{Q}R(L),$$

$$\Delta^{\mathrm{U}}(L) = \varepsilon \times \left\{ \frac{1}{2}\left[-k\sqrt{1+p(1-p)\eta^2}\sigma\sqrt{L} \right] + \frac{p}{2}\sigma\sqrt{L}\left[\sqrt{1+\left[k\sqrt{1+p(1-p)\eta^2}-(1-p)\eta\right]^2} \right] + \frac{(1-p)}{2}\sigma\sqrt{L}\left[\sqrt{1+\left[k\sqrt{1+p(1-p)\eta^2}+p\eta\right]^2} \right] \right\}$$

and $\mathrm{EAC}^{\mathrm{U}}(Q,L)$ and $\Delta^{\mathrm{U}}(L)$ are the least upper bounds of $\mathrm{EAC}^F(Q,L)$ and $\Delta(L)$, respectively.

As discussed in the previous subsection, we take the partial derivatives of $\mathrm{EAC}^{\mathrm{U}}(Q,L)$ with respect to Q and L, we obtain

$$\frac{\partial \mathrm{EAC}^{\mathrm{U}}(Q,L)}{\partial Q} = -\frac{AD}{Q^2} + \frac{h}{2} - \frac{D}{Q^2}\left[\pi + \pi_0\left(1 - \frac{\theta}{1+\Delta^{\mathrm{U}}(L)}\right)\right]\frac{\Delta^{\mathrm{U}}(L)}{\varepsilon} - R(L)\frac{D}{Q^2} \tag{13}$$

and

$$\frac{\partial \mathrm{EAC}^{\mathrm{U}}(Q,L)}{\partial L} = \frac{hk}{2\sqrt{L}}\left[\sqrt{1+p(1-p)\eta^2}\sigma\right] + \left\{\left(h+\frac{D}{Q}\pi_0\right) \cdot \left[\frac{\theta[\Delta^{\mathrm{U}}(L)]^2}{2\varepsilon L[1+\Delta^{\mathrm{U}}(L)]^2}\right]\right\} + \left\{h\left(1-\frac{\theta}{1+\Delta^{\mathrm{U}}(L)}\right) + \frac{D}{Q}\left[\pi+\pi_0\left(1-\frac{\theta}{1+\Delta^{\mathrm{U}}(L)}\right)\right]\right\}\frac{\Delta^{\mathrm{U}}(L)}{2\varepsilon L} - \frac{D}{Q}c_i, \tag{14}$$

where $\Delta^{\mathrm{U}}(L)$ and $R(L)$ as the above definitions.

In addition, it can be verified that for fixed $L \in (L_i, L_{i-1})$, $\mathrm{EAC}^{\mathrm{U}}(Q,L)$ is convex in Q. For fixed Q, $\mathrm{EAC}^{\mathrm{U}}(Q,L)$ is concave in $L \in (L_i, L_{i-1})$. Therefore, for fixed Q, the minimum total expected annual cost will occur at the end points of the interval (L_i, L_{i-1}). Hence, setting Eq. (13) equal to zero and solving for Q, we get

$$Q = \left\{\frac{2D}{h}\left(A + \left[\pi+\pi_0\left(1-\frac{\theta}{1+\Delta^{\mathrm{U}}(L)}\right)\right]\frac{\Delta^{\mathrm{U}}(L)}{\varepsilon} + R(L)\right)\right\}^{\frac{1}{2}}, \tag{15}$$

where $\Delta^{\mathrm{U}}(L)$ and $R(L)$ as the above definitions.

Theoretically, for fixed A, D, h, π, π_0, σ, η, θ, p, q, ε, and each L_i ($i = 1, 2, \ldots, n$), the optimal (Q_i, L_i) pair given L_i can be obtained by solving Eq. (15) iteratively until convergence. The convergence of the procedure can be shown. Furthermore, using Eq. (12), we can obtain the corresponding total expected annual cost $\mathrm{EAC}^{\mathrm{U}}(Q_i, L_i)$. Hence, the minimum total expected annual cost is $\min_{i=0,1,\ldots,n} \mathrm{EAC}^{\mathrm{U}}(Q_i, L_i)$. However, in practice, since the p.d.f. f_X of X is unknown, even if the value of q is given, we can not get the exact value of k. Thus, in order to find the value of k, we need the following proposition.

Proposition 2. *Let Y be a random variable which has a p.d.f. $f_Y(y)$ with finite mean μL and standard deviation $\sigma\sqrt{L}$ (>0), then for any real number $d > \mu L$,*

$$P(Y > d) \leqslant \frac{\sigma^2 L}{\sigma^2 L + (d-\mu L)^2}. \tag{16}$$

So, by using $F_* = pF_1 + (1-p)F_2$, the recorder point $r = \mu_* L + k\sigma_*\sqrt{L}$ and the Proposition 2, we get

$$\begin{aligned} P(X > r) &\leqslant p\frac{\sigma^2 L}{\sigma^2 L + (r-\mu_1 L)^2} + (1-p)\frac{\sigma^2 L}{\sigma^2 L + (r-\mu_2 L)^2} \\ &= p\frac{1}{1+((r-\mu_1 L)/\sigma\sqrt{L})^2} + (1-p)\frac{1}{1+((r-\mu_2 L)/\sigma\sqrt{L})^2} \\ &= \frac{p}{1+[k\sqrt{1+p(1-p)\eta^2}-(1-p)\eta]^2} \\ &\quad + \frac{1-p}{1+[k\sqrt{1+p(1-p)\eta^2}+p\eta]^2}. \end{aligned} \tag{17}$$

Further, it is assumed that the allowable stock-out probability q during lead time is given, that is, $q = P(X > r)$, then from Eq. (17), we get $0 \leqslant k \leqslant \sqrt{\frac{1}{q}-1}+|\eta|$.

It is easy to verify that $\mathrm{EAC^U}(Q, L)$ has a smooth curve for $k \in \left[0, \sqrt{\frac{1}{q}-1}+|\eta|\right]$. Hence, we can establish the following algorithm to obtain the suitable k and hence the optimal Q and L.

Algorithm 2

Step 1. For given A, D, η, h, σ, π, π_0, p, q, θ, ε, a_i, b_i, c_i, and $i = 1, 2, \ldots, n$.

Step 2. For a given q, we divide the interval $\left[0, \sqrt{\frac{1}{q}-1}+|\eta|\right]$ into m equal subintervals, where m is large enough. And we let $k_0 = 0$, $k_m = \sqrt{(1/q)-1}+|\eta|$ and $k_j = k_{j-1} + (k_m - k_0)/m$, $j = 1, 2, \ldots, m-1$.

Step 3. Use the a_i, b_i and c_i, to compute L_i, $i = 1, 2, \ldots, n$.

Step 4. For each L_i, $i = 1, 2, \ldots, n$, compute Q_{i,k_j} by using Eq. (15) for given $k_j \in \{k_0, k_1, \ldots, k_m\}$, $j = 0, 1, 2, \ldots, m$.

Step 5. For each L_i, $i = 1, 2, \ldots, n$, compute the corresponding total expected annual cost $\mathrm{EAC^U}(Q_{i,k_j}, L_i)$, $i = 1, 2, \ldots, n$.

Step 6. Find $\min_{k_j\in\{k_0,k_1,\ldots,k_m\}} \mathrm{EAC^U}(Q_{i,k_j}, L_i)$ and let $\mathrm{EAC^U}(Q_{i,k_{s(i)}}, L_i) = \min_{k_j\in\{k_0,k_1,\ldots,k_m\}} \mathrm{EAC^U}(Q_{i,k_j}, L_i)$, then find $\min_{i=0,1,2,\ldots,n} \mathrm{EAC^U}(Q_{i,k_{s(i)}}, L_i)$. If $\mathrm{EAC^U}(Q^*, L^*) = \min_{i=0,1,2,\ldots,n} \mathrm{EAC^U}(Q_{i,k_{s(i)}}, L_i)$, then (Q^*, L^*) is the optimal solution.

4. Numerical examples

In order to illustrate the above solution procedure, let us consider an inventory system with the data: $D = 600$ units/year, $A = \$200$ per order, $h = \$20$, $\pi = \$50$, $\pi_0 = \$100$, $\mu_* = 11$ units/week, $\sigma = 3$ units/week, $\theta = 1.0$, 0.6, $\varepsilon = 0.0, 2.0, 20, 40, 100, \infty$ (backorder case), $q = 0.1$ and the lead time has three components with data shown in Table 1.

Table 1
Lead time data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Example 1. We assume here that the lead time demand follows a mixtures of normal distribution and want to solve the case when $\eta = 0.7$, $p = 0(0.2)1$, $\theta = 1.0,\ 0.6$, $\varepsilon = 0.0, 2.0, 20, 40, 100, \infty$. A summary of these optimal results is presented in (Q_*, L_*) and $\mathrm{EAC}^N(Q_*, L_*)$ of Tables 2 and 3. From Tables 2 and 3, when $\eta = 0.7$, we can get the order quantity Q_* and the minimum total expected annual cost $\mathrm{EAC}^N(Q_*, L_*)$ increase as ε increases (i.e. the backorder rate β decreases) for $\theta = 0.6,\ 1.0$ and the fixed p. Further, we also found that the minimum total expected annual cost $\mathrm{EAC}^N(Q_*, L_*)$ is more sensitive to ε when its value is small for $\theta = 0.6,\ 1.0$ and the fixed p. Moreover, for $\theta = 0.6$, 1.0 and the fixed ε, when $p = 0$ or 1, the model considers only one kind of customers' demand; when $0 < p < 1$, the model considers two kinds of customers' demand. It implies that $\mathrm{EAC}^N(Q_*, L_*)$ of two kinds of customers' demand are larger than $\mathrm{EAC}^N(Q_*, L_*)$ of one kind of customers' demand. Thus, the minimum total expected annual cost $\mathrm{EAC}^N(Q_*, L_*)$ increases and then decrease as p increases for the fixed ε. In addition, no matter what values of p, the optimal lead time L_* is approached to a certain value (4 weeks) for $\theta = 0.6$, 1.0 and $\varepsilon = 20, 40, 100, \infty$. However, no matter what values of p, the optimal lead time L_* is approached to a certain value (6 weeks) for $\theta = 0.6,\ 1.0$ and $\varepsilon = 0.0,\ 2.0$.

Example 2. The data is as in Example 1. We assume here that the probability distribution of the lead time demand is mixtures of free. By using the proposed Algorithm 2 and $m = 500$ procedure yields these optimal results shown in (Q^*, L^*) and $\mathrm{EAC}^{\mathrm{U}}(Q^*, L^*)$ of Tables 2 and 3. From Tables 2 and 3, when $\eta = 0.7$, we can get the order quantity Q^* and the minimum total expected annual cost $\mathrm{EAC}^{\mathrm{U}}(Q^*, L^*)$ increase as ε increases (i.e. the backorder rate β decreases) for $\theta = 0.6,\ 1.0$ and the fixed p. Further, we also found that the minimum total expected annual cost $\mathrm{EAC}^{\mathrm{U}}(Q^*, L^*)$ is more sensitive to ε when its value is small for $\theta = 0.6,\ 1.0$ and the fixed p. Moreover, the minimum total expected annual cost $\mathrm{EAC}^{\mathrm{U}}(Q^*, L^*)$ increase and then decrease as p increases for $\theta = 0.6,\ 1.0$ and the fixed ε.In addition, no matter what values of p, the optimal lead time L^* is approached to a certain value (4 weeks) for $\theta = 0.6,\ 1.0$ and $\varepsilon = 0.0, 2.0, 20, 40, 100, \infty$. Finally, the total expected annual cost $\mathrm{EAC}^N(Q^*, L^*)$ is obtained by substituting Q^* and L^* into Eq. (4) when the d.f.

Table 2
Summary of the optimal solution procedure (L_i in weeks and $\eta = 0.7$, $\theta = 1.0$)

p	(Q^*, L^*)	$\mathrm{EAC}^{\mathrm{U}}(Q^*, L^*)$	$\mathrm{EAC}^{N}(Q^*, L^*)$	(Q_*, L_*)	$\mathrm{EAC}^{N}(Q_*, L_*)$	EVAI	Percentage penalty
$\varepsilon = \infty$							
0.0	(134, 4)	3031.220	2686.272	(126, 4)	2681.414	4.86	1.0018
0.2	(134, 4)	3034.702	2703.502	(127, 4)	2699.361	4.14	1.0015
0.4	(134, 4)	3035.950	2707.003	(127, 4)	2702.961	4.04	1.0015
0.6	(134, 4)	3035.561	2703.404	(127, 4)	2699.155	4.25	1.0016
0.8	(134, 4)	3033.902	2695.946	(126, 4)	2691.408	4.54	1.0017
1.0	(134, 4)	3031.220	2686.272	(126, 4)	2681.414	4.86	1.0018
$\varepsilon = 100$							
0.0	(134, 4)	3026.633	2681.499	(126, 4)	2676.620	4.88	1.0018
0.2	(134, 4)	3030.114	2698.689	(126, 4)	2694.579	4.11	1.0015
0.4	(134, 4)	3031.360	2702.239	(126, 4)	2698.180	4.06	1.0015
0.6	(134, 4)	3030.971	2698.637	(126, 4)	2694.370	4.27	1.0016
0.8	(134, 4)	3029.314	2691.177	(126, 4)	2686.619	4.56	1.0017
1.0	(134, 4)	3026.633	2681.499	(126, 4)	2676.620	4.88	1.0018
$\varepsilon = 40$							
0.0	(134, 4)	3020.057	2674.891	(126, 4)	2669.997	4.89	1.0018
0.2	(134, 4)	3023.536	2692.068	(126, 4)	2687.944	4.12	1.0015
0.4	(134, 4)	3024.781	2695.617	(126, 4)	2691.545	4.07	1.0015
0.6	(134, 4)	3024.393	2692.019	(126, 4)	2687.738	4.28	1.0016
0.8	(134, 4)	3022.737	2684.563	(126, 4)	2679.991	4.57	1.0017
1.0	(134, 4)	3020.057	2674.891	(126, 4)	2669.997	4.89	1.0018
$\varepsilon = 20$							
0.0	(133, 4)	3009.840	2665.138	(125, 4)	2660.251	4.89	1.0018
0.2	(133, 4)	3013.320	2682.240	(126, 4)	2678.121	4.12	1.0015
0.4	(133, 4)	3014.565	2685.785	(126, 4)	2681.718	4.07	1.0015
0.6	(133, 4)	3014.177	2682.207	(126, 4)	2677.931	4.28	1.0016
0.8	(133, 4)	3012.521	2674.779	(125, 4)	2670.214	4.57	1.0017
1.0	(133, 4)	3009.840	2665.138	(125, 4)	2660.251	4.89	1.0018
$\varepsilon = 2.0$							
0.0	(129, 4)	2908.333	2628.754	(119, 6)	2577.513	51.24	1.0199
0.2	(129, 4)	2912.052	2645.716	(120, 6)	2596.837	48.88	1.0188
0.4	(129, 4)	2913.354	2649.240	(120, 6)	2601.086	48.15	1.0185
0.6	(129, 4)	2912.920	2645.688	(120, 6)	2597.165	48.52	1.0187
0.8	(129, 4)	2911.158	2638.292	(120, 6)	2588.697	49.60	1.0192
1.0	(129, 4)	2908.333	2628.754	(119, 6)	2577.513	51.24	1.0199
$\varepsilon = 0.0$							
0.0	(127, 4)	2715.267	2556.457	(116, 6)	2501.762	54.70	1.0219
0.2	(127, 4)	2720.988	2567.295	(116, 6)	2514.569	52.73	1.0210
0.4	(127, 4)	2722.671	2570.419	(116, 6)	2518.404	52.02	1.0207
0.6	(127, 4)	2721.823	2568.663	(116, 6)	2516.363	52.30	1.0208
0.8	(127, 4)	2719.206	2563.651	(116, 6)	2510.412	53.24	1.0212
1.0	(127, 4)	2715.267	2556.457	(116, 6)	2501.762	54.70	1.0219

Table 3
Summary of the optimal solution procedure (L_i in weeks and $\eta = 0.7$, $\theta = 0.6$)

p	(Q^*, L^*)	$\mathrm{EAC}^{\mathrm{U}}(Q^*, L^*)$	$\mathrm{EAC}^{N}(Q^*, L^*)$	(Q_*, L_*)	$\mathrm{EAC}^{N}(Q_*, L_*)$	EVAI	Percentage penalty
$\varepsilon = \infty$							
0.0	(134, 4)	3031.220	2686.272	(126, 4)	2681.414	4.86	1.0018
0.2	(134, 4)	3034.702	2703.502	(127, 4)	2699.361	4.14	1.0015
0.4	(134, 4)	3035.950	2707.003	(127, 4)	2702.961	4.04	1.0015
0.6	(134, 4)	3035.561	2703.404	(127, 4)	2699.155	4.25	1.0016
0.8	(134, 4)	3033.902	2695.946	(126, 4)	2691.408	4.54	1.0017
1.0	(134, 4)	3031.220	2686.272	(126, 4)	2681.414	4.86	1.0018
$\varepsilon = 100$							
0.0	(134, 4)	3028.469	2683.409	(126, 4)	2678.538	4.87	1.0018
0.2	(134, 4)	3031.950	2700.644	(127, 4)	2696.493	4.15	1.0015
0.4	(134, 4)	3033.197	2704.146	(127, 4)	2700.094	4.05	1.0015
0.6	(134, 4)	3032.808	2700.545	(126, 4)	2696.285	4.26	1.0016
0.8	(134, 4)	3031.150	2693.085	(126, 4)	2688.536	4.55	1.0017
1.0	(134, 4)	3028.469	2683.409	(126, 4)	2678.538	4.87	1.0018
$\varepsilon = 40$							
0.0	(134, 4)	3024.527	2679.449	(126, 4)	2674.569	4.88	1.0018
0.2	(134, 4)	3028.008	2696.628	(126, 4)	2692.517	4.11	1.0015
0.4	(134, 4)	3029.254	2700.177	(126, 4)	2696.117	4.06	1.0015
0.6	(134, 4)	3028.865	2696.579	(126, 4)	2692.310	4.27	1.0016
0.8	(134, 4)	3027.208	2689.122	(126, 4)	2684.564	4.56	1.0017
1.0	(134, 4)	3024.527	2679.449	(126, 4)	2674.569	4.88	1.0018
$\varepsilon = 20$							
0.0	(134, 4)	3018.411	2673.611	(125, 4)	2668.736	4.88	1.0018
0.2	(133, 4)	3021.892	2690.745	(126, 4)	2686.637	4.11	1.0015
0.4	(133, 4)	3023.138	2694.292	(126, 4)	2690.235	4.06	1.0015
0.6	(133, 4)	3022.750	2690.705	(126, 4)	2686.440	4.27	1.0016
0.8	(134, 4)	3021.092	2683.266	(126, 4)	2678.711	4.55	1.0017
1.0	(134, 4)	3018.411	2673.611	(125, 4)	2668.736	4.88	1.0018
$\varepsilon = 2$							
0.0	(131, 4)	2958.479	2669.870	(121, 6)	2620.069	49.80	1.0190
0.2	(131, 4)	2962.102	2687.956	(122, 6)	2640.555	47.40	1.0180
0.4	(131, 4)	2963.382	2691.541	(122, 6)	2644.874	46.67	1.0176
0.6	(131, 4)	2962.967	2687.680	(122, 6)	2640.629	47.05	1.0178
0.8	(131, 4)	2961.246	2679.906	(122, 6)	2631.716	48.19	1.0183
1.0	(131, 4)	2958.479	2669.870	(121, 6)	2620.069	49.80	1.0190
$\varepsilon = 0.0$							
0.0	(130, 4)	2865.380	2627.759	(119, 6)	2575.637	52.12	1.0202
0.2	(130, 4)	2869.816	2642.188	(120, 6)	2592.342	49.85	1.0192
0.4	(130, 4)	2871.266	2645.556	(120, 6)	2596.421	49.13	1.0189
0.6	(130, 4)	2870.691	2642.730	(120, 6)	2593.267	49.46	1.0191
0.8	(130, 4)	2868.618	2636.369	(119, 6)	2585.813	50.56	1.0196
1.0	(130, 4)	2865.380	2627.759	(119, 6)	2575.637	52.12	1.0202

of the lead time demand is the mixtures of normal distribution. The expected value of additional information (EVAI) is the largest amount that one is willing to pay for the knowledge of F_1 and F_2 and is equal to $\mathrm{EAC}^N(Q^*, L^*) - \mathrm{EAC}^N(Q_*, L_*)$. From Tables 2 and 3, we observe that for the fixed p, EVAI decreases as ε increases (i.e. the backorder rate β decreases) except $p = 0.2$, when $\theta = 1.0$, while for the fixed p, EVAI decreases as ε increases (i.e. the backorder rate β decreases) except $p = 0.2$, 0.8, when $\theta = 0.6$. Moreover, we also observe that for the given $\varepsilon = 0.0, 2.0, 20, 40, 100, \infty$, EVAI decreases and then increases as p increases when $\theta = 0.6$, 1.0. And the cost penalty$\mathrm{EAC}^N(Q^*, L^*)/\mathrm{EAC}^N(Q_*, L_*)$ of using the distribution free operating policy instead of the optimal one is decreasing as ε increases (i.e. the backorder rate β decreases) for the fixed p.

5. Concluding remarks

In this article, we propose a continuous review inventory model by considering the mixtures of distribution of the lead time demand and controllable backorder rate to extend that of Ouyang and Chuang [6]. In addition, we also develop an algorithmic procedure to find the optimal order quantity and the optimal lead time. Further, we get the significant results in the total expected annual cost.

In future research on this problem, it would be interesting to deal with the inventory model with a service level constraint or treat the reorder point as a decision variable.

Acknowledgements

This research was partially supported by the National Science Council, ROC (Plan No. NSC 91-2213-E-130-011).

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