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Analysis of an inventory system under backorder correlated deterministic demand and geometric supply process

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Abstract

In this article we propose a single item, periodic review model that investigates the effects of changes in the demand process that occur after stockout realizations. We investigate a system where the demands in successive periods are deterministic but affected by the backorder realizations. In order to capture the effects of changes in the demand process we use a geometric type supply availability. We analytically derive the necessary components for obtaining profit related performance measures and provide computational analysis. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Backorder correlated demand; Supply uncertainty

1. Introduction and literature review

One of the costs that is frequently used in inventory models is the penalty cost, also known as the shortage or stockout cost, as a hedging against stockouts. The stockout cost has a different interpretation depending on whether excess demand is backordered or lost. In the backorder case, the stockout cost includes book-keeping and/or delay costs. In the lost sales case, it includes the lost profit that would have been made from the sale. In either case, it would also include the 'loss of goodwill' cost, which is a measure of customer satisfaction. Unfortunately, estimating the loss of goodwill component of the stockout cost can be very difficult in practice, since it is not easy to express customer dissatisfaction in monetary terms.

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In this article we propose a single item, periodic review model that investigates the effects of changes in the demand patterns that occur after stockout realizations. Obviously, if there is no restriction on the supply or capacity availability in an ordering period, then there will be no successive backorder periods, hence the demand pattern is not expected to change. It is either supply/capacity restrictions or the lead-time effect that would introduce unmet customer demand and therefore shifts in the demand pattern for the product. In order to capture capacity/supply or lead time effects we incorporate a supply process where the number of periods between two supply realizations is random. This would correspond to either uncertainties in the supply/capacity process (imperfect production, machine breakdowns, supplier's capacity constraints) or random lead time where once the order arrives all the outstanding orders during the lead-time period are also met.

We specifically make three contributions. First, we provide a simple model that captures the

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changes in the demand pattern after stockout realizations and the relations between supply availability and system response to post-backorder demand patterns. Then, we derive analytical expressions for the steady-state behavior of the system. These expressions enable us to explicitly characterize the cost/revenue related performance measures for the model. Finally, we perform computational analysis that help us in understanding the essential characteristics of the system.

Several attempts have been made in the literature to model the relation between stockouts and customer satisfaction to eliminate the loss of goodwill component. A common substitute for stockout costs is the use of service levels. Another approach to eliminate the loss of goodwill component of stockout cost is considering the effect of stockouts on the customer behavior, i.e., future demand. The reasoning behind this approach is that stockouts tend to modify demand patterns. Robinson [1] provides a detailed review of studies that support the idea that customers alter their joint behavior according to meeting stockouts, delivery time or service provided. In a highly competitive market, a customer who experiences stockouts will be less likely to buy again from that supplier. In a monopolistic market, on the other hand, stockouts may attract more demand. In this study, both of these cases are investigated.

The majority of all those studies in the literature to model the relation between a system parameter and future demand relate the demand pattern to either the physical on-hand inventory level or the service level provided by the retailer. The idea of inventory level dependent demand was born due to the fact that stock level had motivational effect on the customers. Baker and Urban [2], Datta and Pal [3], Urban [4], and Paul et al. [5] constitute a series of articles that study inventory level dependent demand patterns. In each of these articles, a continuous-time deterministic inventory system is investigated assuming that the demand rate of the item has a polynomial functional form dependent on the inventory level. Our work also assumes a deterministic demand process, however we consider an uncertain supply structure.

The concept of service level dependent demand within a production/inventory system is incorporated by Schwartz [6], Ernst and Cohen [7], and Ernst and Powell [8]. According to these studies, demand rate is proportional to the service level provided by the retailer. This property of service level related demand models requires that the market immediately has full information about the service level, which is not very realistic. On the contrary, information on a retailer's being out of stock is more readily available in the market. Fewer studies have been made on the case that demand changes due to stockouts dynamically throughout the planning horizon. Ref. [1] is an article that contributes to the inventory literature with stockout dependent nonstationary demand. In [1] both the expectation and the variability of demand change over time in response to the actual (not expected) number of satisfied and dissatisfied customers. In the preceding articles, the demand parameters (demand rate or mean demand) are affected from the stockout realizations. Moreover, approaches in these articles do not allow tractable analysis of the system performance. Contrary to theirs, our approach allows us to model situations where the demand distribution, rather than the mean demand, can change after stockout realizations.

Besides the models in which the demand depends on some system parameter, there are also models with external nonstationarity in demand. The first detailed study on nonstationary demand models is due to Karlin [9,10]. Like Karlin [9,10], Zipkin [11] and Morton and Pentico [12] attempt to analyze similar problems in which independent demand varies from period to period. In the literature, there is another approach to incorporate the external nonstationarity in demand. This approach assumes that demand varies with an underlying state-of-the-world variable that can represent economic fluctuations, uncertain market conditions, or stages in the product life cycle. Song and Zipkin [13] and Sethi and Cheng [14] take such an approach to describe the nonstationarity in demand. Certainly, this line of research is similar to ours, as we also consider time-varying demands. However, in our model, nonstationarity of demand is determined by the internal dynamics of the system, the backorders.

Queueing systems with state dependent arrival rates may provide another way to incorporate the nonstationary dependent demand. Arrival rate to a queue can be thought to be the demand rate of the production/inventory system, whereas the number of customers waiting in the queue coincides with the outstanding orders of the inventory systems. There are numerous studies on the arrival rate that depends on the number of customers in the queue. One of the ideas behind these studies is that new customers are discouraged from joining long queues (backorders or outstanding orders diminish demand in a production/inventory system). Some of these studies are Courtois and Georges [15], Winston [16], Giorno et al. [17], Shantikumar and Sumita [18], and Li et al. [19]. The model that we analyze here is a discrete-time model, whereas the queueing models cited above deal with continuous time systems.

Besides the demand pattern, another important element in an inventory model is the supply process. An implicit assumption underlying most inventory systems is that whatever is ordered is received exactly. Such an assumption is not realistic due to many uncertainties in the supply process. Lots containing defective items, uncertain capacity due to machine breakdowns and strikes are some possible causes that lead to partial or full supply unavailabilities. Yano and Lee [20] provide a comprehensive review of the literature on approaches for determining lot sizes when production or procurement yield are random. Some of the other studies on uncertain supply/random yield are Shih [21], Ehrhardt and Taube [22], Henig and Gerchak [23], Parlar et al. [24], Güllü et al. [25,26], Ciarallo et al. [27], Güllü [28], Wang and Gerchak [29], Parlar and Berkin [30], and Parlar and Perry [31].

The rest of the paper is organized as follows. Section 2 is devoted to the formulation and analysis of the infinite horizon periodic review inventory model under order-up-to level type (base stock) ordering policies. In Section 3, we provide our computational experience on the model. Finally, in Section 4 concluding remarks are made and future research areas are discussed.

2. Problem formulation

The problem of this study is characterized by its demand and supply processes. Supply is either fully available in a period with probability p or it is completely unavailable with probability $(1-p)$ and the supply in a period is realized independent of the previous period. Furthermore, it is assumed that

- lead time is zero,
- capacity is infinite whenever supply is available,
- partial supply availability is not possible.

We characterize the demand process with two deterministic values whose realizations depend on whether backorders are observed or not. As we have already mentioned, deterministic demand is sensitive to backorders. There are two values that the demand can take according to the backorders. For the sake of simplicity, we normalize the problem so that the demand in a period before backorders is equal to unity and d is the per period demand after the first backorder period until the period in which inventory level becomes positive again. Inventory level vs. time plot presented in Fig. 1 is a realization of this inventory model operating under a base stock (order-up-to type) policy for which the order-up-to level is y.

A supply cycle starts whenever a supply availability occurs, because then the system restarts itself. The planning cycle constitutes an infinite number of supply cycles. Starting each supply cycle at the same inventory level and demand structure, cycles are statistically independent and identical to each other.

System state, X_n, denotes the shortfall inventory level right after the demand occurrence in period n.

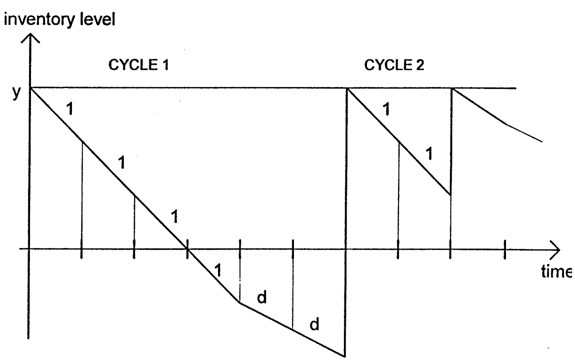

Fig. 1. A realization of the system: inventory level vs. time.

We assume the following sequence of events in a period: first, demand realization takes place, then, X_n is recorded and finally, the replenishment of inventory occurs if supply is available. By definition,

$$X_n = y - I_n > 0,$$

where I_n is the inventory level at the end of period n. As X_{n+1} depends only on X_n and the supply status of period n, it can be concluded that $\{X_n,\ n = 1, 2, 3, \ldots\}$ is a Markov Chain.

The objective of this problem is maximizing the expected per period profit, which is the difference of the expected contribution of demand per period and the expected costs per period, namely holding and backordering costs. Notation required to write the expected profit per period are as follows:

$P(y)$: expected profit per period,
$C(y)$: sum of expected holding and backordering costs per period,
c: unit contribution to profit, i.e. difference of unit selling price and the unit purchasing or production cost,
h: unit inventory holding cost per period,
b: unit backordering cost per period (b includes the cost of processing, paperwork, etc. It does not cover the cost of loss of goodwill as this component is incorporated via the backorder correlated demand structure.)
$\gamma_i(y)$: stationary probability of X_n being in state i under an order-up-to level of y.

$$P(y) = cE[\text{Demand per Period}] - C(y). \tag{1}$$

In this problem, we choose maximization of profit as the objective, since the contribution term of $P(y)$, cE[Demand per Period] in Eq. (1), is a function of the decision variable, y, as per period demand will be affected by the order-up-to level. Otherwise, minimization of cost and maximization of profit type objectives would be identical.

It is necessary to write $C(y)$ in detail for obtaining $P(y)$. For this purpose, note that whenever shortfall, X_n, is less than or equal to y, a holding cost which is linearly proportional with the positive ending inventory is charged. On the other hand, a backordering cost which is linearly proportional with the ending backorder level is charged whenever shortfall, X_n, is greater than y. Then, $C(y)$ can be written as follows:

$$\begin{aligned} C(y) &= h\sum_{i=1}^{y}(y-i)\gamma_i(y) + b\sum_{i=y+1}^{\infty}(i-y)\gamma_i(y) \\ &= (h+b)\sum_{i=1}^{y}(y-i)\gamma_i(y) + b\left[\sum_{i=1}^{\infty} i\gamma_i(y) - y\right]. \end{aligned} \tag{2}$$

From (2), it can be observed that only $\gamma_i(y)$ for $i = 1, \ldots, y$ and expected value of the shortfall are required to calculate $C(y)$. Next, the limiting probability distribution of shortfall, $\{\gamma_i(y),\ i \geqslant 1\}$, the expected value of shortfall, $E[SF] = \sum_{i=1}^{\infty} i\gamma_i(y)$, and expected per period demand will be obtained to compute the terms of $P(y)$.

X_n can take a set of values. The set is $\{1, 2, \ldots, y,\ y+1, y+1+d, y+1+2d, y+1+3d, \ldots\}$. The transition probabilities for X_n is as follows. For $i = 1, 2, 3, \ldots, y$

$$\Pr\{X_{n+1} = j | X_n = i\} = \begin{cases} p & \text{for } j = 1, \\ 1-p & \text{for } j = i+1, \\ 0 & \text{otherwise.} \end{cases}$$

For $i = y+1, y+1+d, y+1+2d, \ldots$

$$\Pr\{X_{n+1} = j | X_n = i\} = \begin{cases} p & \text{for } j = 1, \\ 1-p & \text{for } j = i+d, \\ 0 & \text{otherwise.} \end{cases}$$

According to these transition probabilities remaining equations can be obtained:

$$\begin{aligned} &\gamma_1(y) = p, \\ &\gamma_2(y) = (1-p)\gamma_1(y) = p(1-p), \\ &\gamma_3(y) = (1-p)\gamma_2(y) = p(1-p)^2, \\ &\quad\vdots \\ &\gamma_y(y) = (1-p)\gamma_{y-1}(y) = p(1-p)^{y-1}, \\ &\gamma_{y+1}(y) = (1-p)\gamma_y(y) = p(1-p)^y, \\ &\gamma_{y+1+d}(y) = (1-p)\gamma_{y+1}(y) = p(1-p)^{y+1}, \\ &\gamma_{y+1+2d}(y) = (1-p)\gamma_{y+1+d}(y) = p(1-p)^{y+2}, \\ &\qquad\vdots \end{aligned}$$

Using the above equations for $\gamma_i(y), E[SF]$ is derived as

$$E[SF] = \sum_{i=1}^{y} ip(1-p)^{i-1} + \sum_{i=y+1}^{\infty} (y+1+(i-y-1)d)p(1-p)^{i-1} = \frac{1-(1-d)(1-p)^{y+1}}{p}. \quad (3)$$

Next, $E[SF]$ and $\gamma_i(y)$ for $i = 1, 2, 3, \ldots, y$ are used for finding $C(y)$:

$$C(y) = (h+b)\sum_{i=1}^{y} (y-i)p(1-p)^{i-1} + b\left(\frac{1-(1-d)(1-p)^{y+1}}{p}\right) - by = h\left(y + \frac{(1-p)^y - 1}{p}\right) + b(1-p)^y\left[1 + d\left(\frac{1-p}{p}\right)\right]. \quad (4)$$

At this point, we only need to calculate E[Demand per Period] to obtain the objective function, $P(y)$. The basic concepts of renewal theory are used for this purpose.

As we have discussed earlier a supply cycle is defined as the time between two consecutive supply availability periods. Following the renewal theory, expected per period demand can be written as

$$E[\text{Demand per Period}] = \frac{E[\text{Demand per Cycle}]}{E[N]}, \quad (5)$$

where N is the random variable denoting the number of periods in a cycle. Note that cycle length, N, is geometrically distributed, i.e., $\Pr\{N = i\} = p(1-p)^{i-1}$ for $i = 1, 2, 3, \ldots$. Then,

$$E[N] = \frac{1}{p}. \quad (6)$$

For obtaining E[Demand per Cycle], define ρ as the random variable denoting the first period in a cycle in which a stockout occurs, such that $\rho \leqslant N$. Note that by definition $\rho = N$ if no stockout occurs in a cycle. Then,

$$E[\text{Demand per Cycle}|N = n] = E[\rho + (n-\rho)d] = E[\rho] + (n - E[\rho])d, \quad (7)$$

E[Demand per Cycle] can be obtained by taking expectation of (7) over N.

$$E[\text{Demand per Cycle}] = E[\rho] + (E[N] - E[\rho])d = \frac{d}{p} + (1-d)E[\rho]. \quad (8)$$

For deriving $E[\rho]$, the conditional expectation of ρ on N is written first:

$$E[\rho|N = n] = \begin{cases} n & \text{for } n \leqslant y, \\ y+1 & \text{for } n > y. \end{cases} \quad (9)$$

Then,

$$E[\rho] = \sum_{n=1}^{y} np(1-p)^{n-1} + \sum_{n=y+1}^{\infty} (y+1)p(1-p)^{n-1} = \frac{1-(1-p)^{y+1}}{p}. \quad (10)$$

Having completed the calculation of $E[\rho]$, we have all the necessary ingredients to investigate $P(y)$:

$$P(y) = c - h\left(y - \frac{1}{p}\right) - \frac{(1-p)^y}{p}[h + b + (1-d)(1-p)(cp-b)]. \quad (11)$$

The next step should be the analysis of $P(y)$. For this purpose, first and second differences of $P(y)$ are derived.

Let $\Delta P(y)$ be the first difference of $P(y)$, such that

$$\Delta P(y) = P(y+1) - P(y) \quad \text{for } y = 0, 1, 2, \ldots. \quad (12)$$

Then, for $y = 0, 1, 2, \ldots,$

$$\Delta P(y)$$
$$= (1-p)^y[h + b + (1-d)(1-p)(cp-b)] - h. \tag{13}$$

Second difference of $P(y)$, on the other hand, is given by the following equation. For $y = 0, 1, 2, \ldots,$

$$\Delta P(y+1) - \Delta P(y)$$
$$= -p(1-p)^y[h + b + (1-d)(1-p)(cp-b)]. \tag{14}$$

From Eq. (14), it can be noted that $P(y)$ is concave if

$$[h + b(p + d(1-p)) + (1-d)(1-p)cp]$$

is nonnegative. As this condition is not always satisfied, it may be concluded that $P(y)$ is concave only for special values of the problem parameters. For instance for $d \leqslant 1$, i.e. for a competitive market (since buyers shift to other alternative locations, demand decreases), it is obvious that $P(y)$ is concave. We construct numerical examples that give rise to different structures of $P(y)$ in the next section.

Before the numerical examples, an assumption underlying our analysis should be discussed. This assumption is the non-negativity of y. It seems reasonable to assume a nonnegative order-up-to level for most of the inventory problems. However, for this problem in concern, analysis of $P(y)$ for negative values of y should not be overlooked. Because, for some special values of the model parameters, such as a high value of c, a high value of d or a low value of b, $P(y)$ can be maximized for a negative value of y. For instance, for a monopolistic market, i.e., for d greater than 1 (when shortages are encountered, with the anticipation of shortages in the future, buyers tend to place orders in higher quantities), it may be profitable to operate with a negative order-up-to level, hence increasing the demand and consecutively increasing the revenues per period.

A simplifying observation for negative values of y is that $P(y-1) \leqslant P(y)$ for $y \leqslant -1$. Specifically,

$$P(y-1) = P(y) - b \quad \text{for } y = -1, -2, -3, \ldots. \tag{15}$$

Hence, considering $P(y)$ only for $y = -1, 0, 1, 2, \ldots$ will be adequate for a complete analysis.

$y = -1$ means that the system always operates with backorders; inventory is never held. Hence, inventory holding costs are not necessary. Such an operating style coincides with the 'make-to-order' systems in production systems. The expected profit per period when order-up-to level is -1 is

$$\begin{aligned} P(-1) &= cE[\text{Demand per Period}] \\ &\quad - \sum_{i=0}^{\infty} b(i+1)\gamma_i(-1) \\ &= cd - b - b\sum_{i=0}^{\infty} i\gamma_i(-1) \\ &= cd - b - b\sum_{i=1}^{\infty} idp(1-p)^{i-1} \\ &= \left(c - \frac{b}{p}\right)d - b. \end{aligned} \tag{16}$$

3. Computational results

In this section, we examine some numerical examples to provide insights about the operation of our model. Since the objective function, $P(y)$, is not concave for all cases, we evaluate $P(y)$ for a reasonable set of y values that may provide the optimal solution and plot $P(y)$ vs. y for special parameters of the problem. The base setting for the parameters is selected as: $c = 8$, $h = 1$, $b = 5$, $p = 0.90$, and $d = 2$, so that we assume a monopolistic market. For observing different structures of $P(y)$, $P(y)$ is plotted for different values of c, as all other parameters are set at their base levels in Fig. 2.

For $c = 20$ and $c = 50$, optimum value of y turns out to be -1, since d is greater than 1. From the plots of $P(y)$, it can also be observed that $P(y)$ is neither convex nor concave. In Fig. 3, we plot $P(y)$ for different values of d.

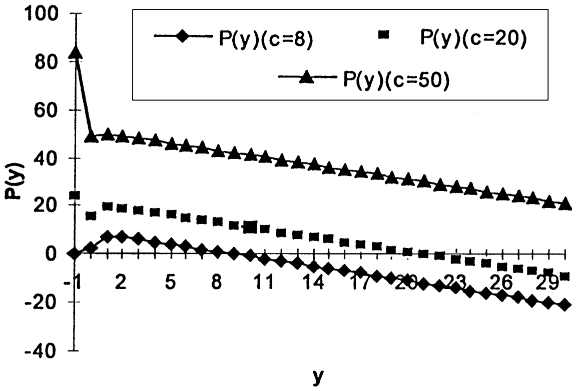

Fig. 2. $P(y)$ vs. y.

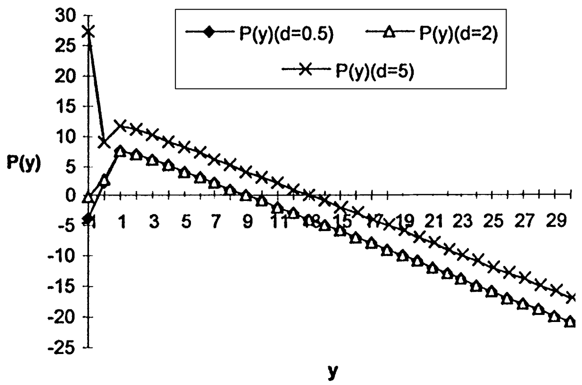

Fig. 3. $P(y)$ vs. y.

$d = 0.5$ implies that the market is competitive, so that the demand decreases as stockouts occur. For this case, $P(y)$ is concave as it is proved analytically in the previous section. However, for $d = 2$ and $d = 5$, $P(y)$ is neither concave nor convex. Some of our other observations are as follows:

- As p increases, expected per period profit always improves and order-up-to level decreases or stays the same.
- For $d > 1$ and small b, $y = -1$ provides the maximum profit. The reason is that unit backordering cost is relatively small and more revenues can be earned as demand increases.
- For $d > 1$, an increase in c causes y that maximizes $P(y)$, to decrease or stay the same, because in this case stockouts increase average demand and more demand with a higher c make more profit.
- As b increases y that maximizes $P(y)$ also increases.
- Comparing the plots for different values of b, it can be concluded that b affects the plots and hence ordering policies, therefore estimation of parameter b is critical.

4. Conclusion

In this study, we formulated and analyzed a periodic review inventory problem under backorder dependent deterministic demand and uncertain supply. The motivation behind this study is due to two factors:

1. customers alter their joint behavior after stockouts, hence, demand is affected by stockouts;
2. supply may not be available every time an order is given due to machine breakdowns, strikes, etc. Therefore, it will be more realistic to assume an uncertain supply process.

Consecutive supply unavailabilities may cause stockouts, thus, changes in demand patterns. This means that, the second factor affects the first one. In this research, we formulated a simple model that investigates the interaction between supply unavailabilities and post-backorder demand patterns.

Under geometrically distributed intersupply times and backorder dependent deterministic demand, the infinite horizon periodic review stochastic inventory problem is formulated for a single item. The objective is selected as maximizing the expected profit per period. It is observed that the objective function is neither concave nor convex. Numerical examples also show that the objective function does not have a specific structure although it is possible to obtain the order-up-to level which maximizes the expected per period profit.

We currently investigate two extensions of the model under the general supply structures and/or random demand. Capacity restrictions modeled as partial supply availability may be another model for which a modified base stock policy would be appropriate.

Throughout this study lead time is assumed to be zero. Although random supply may imitate lead time to a certain extent, the actual lead time effects cannot be observed. Assuming a positive lead time at times of supply availability may be a good practice.

Another interesting case might be analyzing the model under lost sales assumption instead of backorders for shortages. One may expect that lost sales case is more applicable to competitive markets rather than monopolistic markets.

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