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An inventory model with generalized type demand, deterioration and backorder rates

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ABSTRACT

This study is motivated by the paper of Skouri et al. [Skouri, Konstantaras, Papachristos, Ganas, European Journal of Operational Research 192 (1) (2009) 79–92]. We extend their inventory model from ramp type demand rate and Weibull deterioration rate to arbitrary demand rate and arbitrary deterioration rate in the consideration of partial backorder. We demonstrate that the optimal solution is actually independent of demand. That is, for a finite time horizon, any attempt at tackling targeted inventory models under ramp type or any other types of the demand becomes redundant. Our analytical approach dramatically simplifies the solution procedure.

1. Introduction

Demand has been always one of the most influential factors in the decisions relating to inventory and production activities. Although various formations of consumption tendency have been studied, such as constant demand (e.g., Padmanabhan and Vrat, 1990; Chu et al., 2004), price dependent demand (e.g., Abad, 1996, 2001), time-dependent demand (e.g., Resh et al., 1976; Henery, 1979; Sachan, 1984; Dave, 1989; Teng, 1996; Teng et al., 2002) and time-and-price dependent demand (e.g., Wee, 1995).

The ramp type demand has attracted a great deal of interest from researchers. Since Hill (1995), one of the pioneers, developed an inventory model with ramp type demand that begins with a linear increasing demand until to the turning point, denoted as μ, proposed by previous researchers, then it becomes a constant demand. There has been a movement towards developing this type of inventory system for minimum cost and maximum profit problems. For examples, several articles of Mandal and Pal (1998) focused on decay product. Wu et al. (1999) were concerned with backlog rates relative to the waiting time. Wu and Ouyang (2000) tried to build an inventory system under two replenishment policies: starting with shortage or without shortage. Wu (2001) considered the deteriorated items satisfying Weibull distribution. Giri et al. (2003) dealt with more generalized three-parameter Weibull deterioration distribution. Deng (2005) extended the inventory model of Wu et al. (1999) for the situation where the in-stock period is shorter than μ. Manna and Chaudhuri (2006) set up a model where the deterioration is dependent on time. Panda et al. (2007) constructed an inventory model with a comprehensive ramp type demand. Deng et al. (2007) contributed to the revisions of Mandal and Pal (1998), and Wu and Ouyang (2000). Panda et al. (2008) examined the cyclic deterioration items. Wu et al. (2008) studied the maximum profit problem with the stock-dependent selling rate. They developed two inventory models all related to the conversion of the ramp type demand, and then examined the optimal solution for each case.

Recently, Skouri et al. (2009) developed an order level inventory model for deteriorating items. The model is fairly general in which the demand rate could be any function of time until reaching its stabilization while the backlog rate is any non-increasing function of the customer's waiting time up to the next replenishment.

Another factor that has an effect on the development of the inventory and production activities is deterioration. Deterioration is viewed by most managers as a trade-off among the policies of inventory level, replenishment point, and backorder. Some examples include Papachristos and Skouri (2000) and Abad (2001) assuming a time-dependent deterioration rate, and Dye et al. (2005) adopting a general-type deterioration rate and so on.

Along with all above studies that center around different types of demand and deterioration rates, this paper aims at developing a cost-minimum inventory model in the considerations of the general-type demand and deterioration with partial backorder. This paper shows that under the finite time horizon, an optimal solution is independent of demand. In other words, there is a general property that is not only shared for ramp type but also for any kind of demand behaviors. The analysis and results of this presentation thereupon points out that the lengthy discussion of Skouri et al. (2009) in considering the two-domain segments at the turning point of ramp type demand function is superfluous. This paper also

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introduces an analytical procedure that is not limited to only certain types, such as Weibull, but works for any type of deterioration rates.

The remainder of this paper is organized as follows: Section 2 defines the relative assumptions and notation. Section 3 reviews the results of Skouri et al. (2009). Section 4 provides our proposed inventory model. Section 5 presents a different approach from managerial viewpoint to discuss our findings. Two examples are employed to illustrate our finding in Section 6. Finally, we draw a conclusion in Section 7.

2. Assumptions and notation

We generalize the inventory model of Mandal and Pal (1998), Wu and Ouyang (2000), Deng et al. (2007) and Skouri et al. (2009) with the following notation and assumptions for the deterministic inventory replenishment policy with a general-type demand and backlog rate.

(1) T is the finite time horizon under consideration.
(2) t_1 is the time when the inventory level reaches zero.
(3) t_1^* is the optimal solution for t_1.
(4) c_1 is the cost of each deteriorated item.
(5) c_2 is the inventory holding cost per unit of time.
(6) c_3 is the shortage cost per unit of time.
(7) c_4 is the cost of each lost sale.
(8) $\theta(t)$ is the deterioration rate of the on-hand inventory over $[0,t_1]$.
(9) $I(t)$ is the on-hand inventory level at time t over the ordering cycle $[0,T]$.
(10) S is the maximum inventory level for each ordering cycle, that means $S = I(0)$.
(11) The demand rate $D(t)$ is assumed to be any nonnegative function.
(12) $TC(t_1)$ is the total cost that consists of holding cost, deterioration cost, shortage cost and lost sale cost.
(13) Shortage is allowed and partially backordered. We assume that the backlog function, say$k(x)$, is a decreasing function with $k(0) = 1$ and $k(T) \geqslant 0$. We also assume that $xk(x)$ is an increasing function, that is, $k(x) + xk'(x) \geqslant 0$. This condition had appeared in Skouri et al. (2009) with a slightly different expression, $k(x) + Tk'(x) \geqslant 0$.

Remark 1. The most common backlog rate functions have the following two expressions: (a) $k(x) = \frac{b}{a+x}$, with a, $b > 0$, and (b) $k(x) = e^{-(T-x)}$.

When $k(x) = \frac{b}{a+x}$, then it yields that $k(x) + xk'(x) = \frac{ab}{(a+x)^2} > 0$. When $k(x) = e^{-(T-x)}$, then we have that $k(x) + xk'(x) = (1 + x)e^{-(T-x)} > 0$. Hence, our assumption of $xk(x)$ being an increasing function is reasonable.

3. Review of previous results

Let us directly quote the results from Skouri et al. (2009). Those interested readers may consult Skouri et al. (2009) for the detailed derivation.

Eq. (14) of Skouri et al. (2009) showed that the total cost, $TC(t_1)$, takes the form

$$TC(t_1) = \begin{cases} TC_1(t_1), & \text{if } t_1 \leqslant \mu, \\ TC_2(t_1), & \text{if } \mu < t_1. \end{cases} \tag{1}$$

Eqs. (15), (16) and (21) of Skouri et al. (2009) imply that

$$\frac{d}{dt_1} TC_1(t_1) = D(t_1)g(t_1), \tag{2}$$

where $D(t_1)$ is the demand rate and

$$g(t_1) = C_1 e^{at_1^b} \int_0^{t_1} e^{-at^b} dt + C_3\left(e^{at_1^b} - 1\right) - C_2(T - t_1)\beta(T - t_1) - C_4(1 - \beta(T - t_1)), \tag{3}$$

$$\frac{d}{dt_1} TC_2(t_1) = D(\mu)g(t_1). \tag{4}$$

To find the optimal solution, they had to solve the zero for Eqs. (2) and (4). It implies to solve the same function as

$$g(t_1) = 0. \tag{5}$$

For $TC_1(t_1)$ with $0 \leqslant t_1 \leqslant \mu$, they solved

$$g(t_1) = 0 \quad \text{with} \quad 0 < t_1 < \mu. \tag{6}$$

For $TC_2(t_1)$, with $\mu \leqslant t_1 \leqslant T$, they solved

$$g(t_1) = 0 \quad \text{with} \quad \mu < t_1 < T. \tag{7}$$

They tried to show that $g(t_1)$ has the nice property such that

$$g(t_1) = 0 \quad \text{has a unique solution for} \quad 0 < t_1 < T \tag{8}$$

as proved in Eqs. (17) and (18) of Skouri et al. (2009). We find that at least one of Eqs. (6) and (7) has no solution and the minimum will occur at the boundary. The graph of $TC(t_1)$ will look like U-shape rather than W-shape.

By Skouri et al. (2009), $TC(t_1)$ is composed of two cost functions $TC_1(t_1)$ and $TC_2(t_1)$. $TC_1(t_1)$ is constructed for $0 \leqslant t_1 \leqslant \mu$ and $TC_2(t_1)$ is defined for $\mu \leqslant t_1 \leqslant T$. As a matter of fact, the rationale behind a U-shaped function obtained by Skouri et al. (2009) is that either $TC_1(t_1)$ is a monotone decreasing function and $TC_2(t_1)$ is a U-shape function or $TC_1(t_1)$ is a U-shape function and $TC_2(t_1)$ is an monotone increasing function takes place. It turns out that the minimum happens at the bottom of either U-shape function, i.e., $TC_1(t_1)$ or $TC_2(t_1)$. This phenomenon indicates that breaking down $TC(t_1)$ into two parts according to the partition of the demand function may not be a good method. The total cost function, $TC(t_1)$, can be viewed and handled as one integral whole without being segmented into two parts. In the next section, we will develop an approach to concretely show that the optimal solution is actually independent of the demand. This important demonstration indicates that there is no need to divide the entire domain into several sub-domains since the optimal solution has no connection with the type or behavior of the demand. It would be more beneficial to view the entire inventory cycle as an integral whole and consider an abstract demand rate rather than an explicit expression.

4. Our proposed inventory model

We consider the inventory model that starts with stock. This model was first proposed by Hill (1995), and then further investigated by Mandal and Pal (1998), Wu and Ouyang (2000), Deng et al. (2007) and Skouri et al. (2009). Replenishment occurs at time $t = 0$ when the inventory level attains its maximum, S. From $t = 0$ to t_1, the inventory level reduces due to both demand and deterioration, with demand rate $D(t)$, and deterioration rate $\theta(t)$. At t_1, the inventory level reaches zero, then shortage is allowed to occur during the time interval (t_1,T). The demand during the shortage period (t_1,T) is partially backlogging where the backlog rate is dependent on the waiting time until the next replenishment. The inventory levels of the model are described by the following equations:

$$\frac{d}{dt} I(t) + \theta(t)I(t) = -D(t), \quad 0 < t < t_1 \tag{9}$$

and

$$\frac{d}{dt} I(t) = -k(T - t)D(t), \quad t_1 < t < T. \tag{10}$$

We directly solve Eqs. (9) and (10) under the condition that $I(t_1) = 0$ to imply that

$$I(t) = e^{-\int_0^t \theta(x)dx}\int_t^{t_1} D(s)e^{\int_0^s \theta(x)dx}ds,\ \text{for } 0 \leqslant t \leqslant t_1, \tag{11}$$

and

$$I(t) = \int_t^{t_1} k(T-s)D(s)ds, \quad \text{for } t_1 \leqslant t \leqslant T. \tag{12}$$

The amount of deteriorated items during $[0, t_1]$ is evaluated.

$$I(0) - \int_0^{t_1} D(s)ds = \int_0^{t_1} D(s)\left(e^{\int_0^s \theta(x)dx} - 1\right)ds. \tag{13}$$

The holding cost during $[0, t_1]$ is evaluated

$$c_2\int_0^{t_1} I(t)dt = c_2\int_0^{t_1} e^{-\int_0^t \theta(x)dx}\int_t^{t_1} D(s)e^{\int_0^s \theta(x)dx}dsdt. \tag{14}$$

The shortage cost during $[t_1, T]$ is evaluated

$$c_3\int_{t_1}^T -I(t)dt = c_3\int_{t_1}^T\int_{t_1}^t k(t-s)D(s)dsdt. \tag{15}$$

The lost sales during $[t_1, T]$ is evaluated

$$c_4\int_{t_1}^T (1-k(T-t))D(t)dt. \tag{16}$$

Therefore, the total cost is expressed as:

$$\begin{aligned} TC(t_1) = \frac{1}{T}\Big\{ & A + c_1\int_0^{t_1} D(s)\left(e^{\int_0^s \theta(x)dx} - 1\right)ds \\ & + c_2\int_0^{t_1} e^{-\int_0^t \theta(x)dx}\int_t^{t_1} D(s)e^{\int_0^s \theta(x)dx}dsdt \\ & + c_3\int_{t_1}^T\int_{t_1}^t k(t-s)D(s)dsdt + c_4\int_{t_1}^T (1-k(T-t))D(t)dt\Big\}. \end{aligned} \tag{17}$$

From Eq. (17), it follows that

$$\begin{aligned} \frac{d}{dt_1}TC(t_1) = \frac{D(t_1)}{T}\Big\{ & c_1\left(e^{\int_0^{t_1} \theta(x)dx} - 1\right) + c_2\int_0^{t_1} e^{\int_t^{t_1} \theta(x)dx}dt \\ & - c_3\int_{t_1}^T k(T-t_1)dt - c_4(1-k(T-t_1))\Big\}. \end{aligned} \tag{18}$$

Motivated by Eq. (18), we assume an auxiliary function, say $f(t_1)$, where

$$\begin{aligned} f(t_1) = {} & c_1\left(e^{\int_0^{t_1} \theta(x)dx} - 1\right) \\ & + c_2\int_0^{t_1} e^{\int_t^{t_1} \theta(x)dx}dt - c_3(T-t_1)k(T-t_1) - c_4(1-k(T-t_1)). \end{aligned} \tag{19}$$

From Eq. (19), it follows that

$$f(0) = -c_3Tk(T) - c_4(1-k(T)) < 0, \tag{20}$$

$$f(T) = c_1\left(e^{\int_0^T \theta(x)dx} - 1\right) + c_2\int_0^T e^{\int_t^T \theta(x)dx}dt > 0, \tag{21}$$

and

$$\begin{aligned} \frac{d}{dt_1}f(t_1) = {} & c_1\theta(t_1)e^{\int_0^{t_1} \theta(x)dx} + c_2\left(1 + \theta(t_1)\int_0^{t_1} e^{\int_t^{t_1} \theta(x)dx}dt\right) \\ & + c_3[k(T-t_1) + (T-t_1)k'(T-t_1)] + c_4(-k'(T-t_1)). \end{aligned} \tag{22}$$

Under the assumption (13), $k(x)$ is a decreasing function, i.e., $-k'(x) \geqslant 0$. On the other hand, $k(x)$ satisfies that $k(x) + xk'(x) \geqslant 0$ and $k(T-t_1) + (T-t_1)k'(T-t_1) \geqslant 0$. Hence, $\frac{d}{dt_1}f(t_1) > 0$ to imply that $f(t_1)$ is increasing from $f(0) < 0$ to $f(T) > 0$. So there is a unique point, say t_1^*, that satisfies $f(t_1^*) = 0$. Therefore, it follows that

$$f(t_1) < 0, \quad \text{for} \quad 0 < t_1 < t_1^*, \tag{23}$$

and

$$f(t_1) > 0, \quad \text{for } t_1^* < t_1 < T. \tag{24}$$

From Eqs. (18) and (19), we know that

$$\frac{d}{dt_1}TC(t_1) = \frac{D(t_1)}{T}f(t_1). \tag{25}$$

According to Eqs. (23)–(25), we obtain $\frac{d}{dt_1}TC(t_1) \leqslant 0$ with $TC(t_1)$ non-increasing for $0 < t_1 < t_1^*$. On the other hand, as $\frac{d}{dt_1}TC(t_1) \geqslant 0$ with $TC(t_1)$ non-decreasing for $t_1^* < t_1 < T$, t_1^* is the minimum point or the optimal solution. If we carefully examine Eq. (19), the expression of $f(t_1)$, then it reveals that $f(t_1)$ does not contain the demand rate, $D(t)$. This fact specifies that the optimal solution, t_1^* is independent of the demand. We summarize our findings in the next theorem.

Theorem 1. *Suppose the deterioration rate function $k(x)$ satisfies $k(x) + xk'(x) \geqslant 0$. Then the optimal solution, t_1^*, that minimizes $TC(t_1)$ given by Eq. (17) exists and is unique. The optimal solution can be solved through some familiar numerical approaches (e.g., Newton–Raphson method or Bisection method). Moreover, it is independent of demand.*

5. Discussion

Now, let us review the solution procedure in Deng et al. (2007) and Skouri et al. (2009). They divided the inventory model according to the expression of the demand function into several problems. For example, in Skouri et al. (2009), they assumed two cases: (1) $t_1 \leqslant \mu$ and (2) $t_1 \geqslant \mu$ according to the different behaviors of the demand function $D(t)$. However, our findings point out that it is redundant to divide the entire inventory cycle into different cases. Our approach and result will significantly simplify the solution procedure.

Also, recall the numerical examples in Skouri et al. (2009) for inventory model starting with no shortages, in Example 1, as $\mu = 0.12$, they found that $t_1^* = 0.8604 > \mu$. In Example 2, as $\mu = 0.9$, they also found that $t_1^* = 0.8604 < \mu$. From our findings, the optimal solution is independent of the demand, that is, it is independent of the value of μ so that for both Example 1: $\mu = 0.12$ and Example 2: $\mu = 0.9$, they have the same optimal solution, $t_1^* = 0.8604$.

6. An alternative approach for our findings

We provide a different approach from managerial point of view to discuss our findings. Suppose there is an item with demand quantity Q that happens at time $t(Q)$. Then there are two replenishment policies: (a) fulfill demand from the stock, or (b) satisfy demand from the backorder. If we decide to fulfill demand from the stock then we need to save $Qe^{\int_0^{t(Q)} \theta(s)ds}$ at time $t = 0$. Note that the solution of $\frac{d}{dt}I(t) + \theta(t)I(t) = 0$ is $I(t) = I(0)e^{-\int_0^t \theta(s)ds}$. The beginning stock is $I(0) = Qe^{\int_0^{t(Q)} \theta(s)ds}$. After deterioration, at time $t(Q)$, the remaining stock is Q that just meets the demand. The deterioration cost is $c_1Q\left(e^{\int_0^{t(Q)} \theta(s)ds} - 1\right)$. Owing to the deterioration effect, the inventory level becomes $Qe^{\int_0^{t(Q)} \theta(s)ds}e^{-\int_0^x \theta(s)ds} = Qe^{\int_x^{t(Q)} \theta(s)ds}$ for $x \in [0, t(Q)]$. The holding cost is $c_2\int_0^{t(Q)} Qe^{\int_x^{t(Q)} \theta(s)ds}dx$. Hence, the total cost with demand Q fulfilled from the stock is $c_1Q\left(e^{\int_0^{t(Q)}\theta(s)ds}-1\right)+c_2Q\int_0^{t(Q)}e^{\int_x^{t(Q)}\theta(s)ds}dx$.

On the other hand, for the same item if the demand is replenished by backorder, then under the backlog rate, $k(T-t(Q))$, Q is divided into two parts: backlogged sales, $Qk(T-t(Q)$, and lost sales, $Q(1-k(T-t(Q)))$. Hence, the shortage cost is c_3 $(T-t(Q))Qk(T-t(Q))$ for the shortage period $[t(Q),T]$ and the lost sale cost is $c_4Q(1-k(T-t(Q)))$. It can be found that if

$$c_1Q(e^{\int_0^{t(Q)}\theta(s)ds}-1)+c_2Q\int_0^{t(Q)}e^{\int_x^{t(Q)}\theta(s)ds}dx < c_3(T-t(Q))Qk(T-t(Q))+c_4Q(1-k(T-t(Q))), \tag{26}$$

then the better policy of fulfilling the demand quantity Q is from the stock, that is $f(t(Q)) < 0$. Otherwise, if

$$c_1Q\left(e^{\int_0^{t(Q)}\theta(s)ds}-1\right)+c_2Q\int_0^{t(Q)}e^{\int_x^{t(Q)}\theta(s)ds}dx > c_3(T-t(Q))Qk(T-t(Q))+c_4Q(1-k(T-t(Q))), \tag{27}$$

then the better policy of fulfilling the demand quantity Q is from the backorder, that is $f(t(Q)) > 0$. Recall that t_1^* is the unique solution for $f(t_1^*)=0$. From Eq. (23), all demand, Q with $Q(t)<t_1^*$ that satisfies $f(t(Q)) < 0$ so they should be satisfied from stock. On the other hand, for those demand, Q with $Q(t)>t_1^*$ that satisfies $f(t(Q)) > 0$ so they should be satisfied from backorder. It implies that demands occurring during $[0,t_1^*)$ should be fulfilled from the stock; demands occurring during $(t_1^*,T]$ should be fulfilled from backorder.

The above discussion points out the demand rate, $D(t)$, has no influence on the optimal solution for the replenishment policy.

7. Conclusion

We extend several previous results with respect to ramp type demand to show the existence and uniqueness of the optimal solution. We prove that the optimal solution is independent of the demand function. It provides an explanation for a previously unexplained phenomenon that occurred in previous studies: why inventory models with different ramp type demand rates end up with the same optimal solution.

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References

Abad, P.L., 1996. Optimal pricing and lot-sizing under conditions of perishability and partial backordering. Management Science 42, 1093–1104.

Abad, P.L., 2001. Optimal price and order size for a reseller under partial backordering. Computer and Operations Research 28, 53–65.

Chu, P., Yang, K.L., Liang, S.K., Niu, T., 2004. Note on inventory model with a mixture of back orders and lost sales. European Journal of Operational Research 159, 470–475.

Dave, U., 1989. A deterministic lot-size inventory model with shortages and a linear trend in demand. Naval Research Logistics 36, 507–514.

Deng, P.S., 2005. Improved inventory models with ramp type demand and Weibull deterioration. International Journal of Information and Management Sciences 16 (4), 79–86.

Deng, P.S., Lin, R., Peter Chu, P., 2007. A note on the inventory models for deteriorating items with ramp type demand rate. European Journal of Operational Research 178, 112–120.

Dye, C.-Y., Chang, H.-J., Teng, J.-U., 2005. A deteriorating inventory model with time-varying demand and shortage-dependent partial backlogging. European Journal of Operational Research 1172, 417–429.

Giri, B.C., Jalan, A.K., Chaudhuri, K.S., 2003. Economic Order Quantity model with Weibull deterioration distribution, shortage and ramp-type demand. International Journal of Systems Science 34 (4), 237–243.

Henery, R.J., 1979. Inventory replenishment policy for increasing demand. Journal of the Operational Research Society 30, 611–617.

Hill, R.M., 1995. Inventory models for increasing demand followed by level demand. Journal of the Operational Research Society 46 (10), 250–1259.

Mandal, B., Pal, A.K., 1998. Order level inventory system with ramp type demand rate for deteriorating items. Journal of Interdisciplinary Mathematics 1, 49–66.

Manna, S.K., Chaudhuri, K.S., 2006. An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages. European Journal of Operational Research 171 (2), 557–566.

Padmanabhan, G., Vrat, P., 1990. Inventory model with a mixture of back orders and lost sales. International Journal of System Science 21, 1721–1726.

Panda, S., Saha, S., Basu, M., 2007. An EOQ model with generalized ramp-type demand and Weibull distribution deterioration. Asia–Pacific Journal of Operational Research 24 (1), 93–109.

Panda, S., Senapati, S., Basu, M., 2008. Optimal replenishment policy for perishable seasonal products in a season with ramp-type time dependent demand. Computers and Industrial Engineering 54 (2), 301–314.

Papachristos, S., Skouri, K., 2000. An optimal replenishment policy for deteriorating items with time-varying demand and partial-exponential type-backlogging. Operations Research Letters 27, 175–184.

Resh, M., Friedman, M., Barbosa, L.C., 1976. On a general solution of the deterministic lot size problem with time-proportional demand. Operations Research 24, 718–725.

Sachan, R.S., 1984. On policy inventory model for deteriorating items with time proportional demand. Journal of the Operational Research Society 35, 1013–1019.

Skouri, K., Konstantaras, I., Papachristos, S., Ganas, I., 2009. Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate. European Journal of Operational Research 192 (1), 79–92.

Teng, J.T., 1996. A deterministic replenishment model with linear trend in demand. Operations Research Letters 19, 33–41.

Teng, J.T., Chang, H.J., Dye, C.Y., Hung, C.H., 2002. An optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging. Operations Research Letters 30, 387–393.

Wee, H.M., 1995. Joint pricing and replenishment policy for deteriorating inventory with declining market. International Journal of Production Economics 40, 163–171.

Wu, J.W., Lin, C., Tan, B., Lee, W.C., 1999. An EOQ model with ramp type demand rate for items with Weibull deterioration. International Journal of Information and Management Sciences 10, 41–51.

Wu, K.S., Ouyang, L.Y., 2000. A replenishment policy for deteriorating items with ramp type demand rate. Proceeding of National Science Council ROC (A) 24, 279–286.

Wu, K.S., 2001. An EOQ inventory model for items with Weibull distribution deterioration, ramp type demand rate and partial backlogging. Production Planning and Control 12, 787–793.

Wu, K.S., Ouyang, L.Y., Yang, C.T., 2008. Retailer's optimal ordering policy for deteriorating items with ramp-type demand under stock-dependent consumption rate. International Journal of Information and Management Sciences 19, 245–262.