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Mixture inventory model involving variable lead time and controllable backorder rate

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Abstract

This paper allows the backorder rate as a control variable to widen applications of Ouyang et al.'s model [J. Oper. Res. Soc. 47 (1996) 829]. In this study, we assume that the backorder rate is dependent on the length of lead time through the amount of shortages. We discuss two models that are perfect and partial information about the lead time demand distribution, that is, we first assume that the lead time demand follows a normal distribution, and then remove this assumption by only assuming that the first and second moments of the probability distribution of lead time demand are known. For each case, we develop an algorithm to find the optimal ordering strategy. Three numerical examples are given to illustrate solution procedure. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Inventory; Lead time; Backorder rate; Minimax distribution free procedure

1. Introduction

In most of the early literature dealing with inventory problems, either in deterministic or probabilistic model, lead time is viewed as a prescribed constant or a stochastic variable, which therefore, is not subject to control (see e.g. Naddor, 1966; Montogomery, Bazaraa, & Keswani, 1973; Johnson & Montogomery, 1974; Silver, & Peterson, 1985). However, as pointed out by Tersine (1982), lead time usually consists of the following components: order preparation, order transit, supplier lead time, delivery time, and setup time. In many practical situations, lead time can be reduced at an added crashing cost; in other words, it is controllable. By shortening the lead time, we can lower the safety stock, reduce the stockout loss, improve the service level to the customer so as to increase the competitive edge in business. Through the Japanese experience of using just-in-time (JIT) production, the advantages and benefits associated with efforts to control the lead time can be clearly perceived.

Recently, some inventory model literature considering lead time as a decision variable have been developed. Liao and Shyu (1991) first presented an inventory model in which lead time is a unique

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decision variable and the order quantity is predetermined. Later, Ben-Daya and Raouf (1994) extended Liao and Shyu's (1991) model by allowing both the lead time and the order quantity as decision variables. In a recent paper, Ouyang et al. (1996) considered an inventory model with a mixture of backorders and lost sales to generalize Ben-Daya and Raouf's (1994) model, where the backorder rate is fixed.

In this study, we seek to extend Ouyang et al.'s (1996) model, and propose more general model that allow the backorder rate as a control variable. Under most market behaviors, we can often observe that many products of famous brands or fashionable commodities such as certain brand gum shoes, hi-fi equipment, and clothes may lead to a situation in which customers prefer their demands to be back-ordered while shortages occur. Certainly, if the quantity of shortages is accumulated to a degree that exceeds the waiting patience of customers, some may refuse the backorder case. This phenomenon reveals that, as shortages occur, the longer the length of lead time is, the larger the amount of shortages is, the smaller the proportion of customers can wait, and hence the smaller the backorder rate would be. Under the situation, for a vendor, how to control an appropriate length of lead time to determine a target value of backorder rate so as to minimize the inventory relevant cost and increase the competitive edge in business is worth discussing. Consequently, we here assume that the backorder rate is dependent on the length of lead time through the amount of shortages.

In this paper, we start with a lead time demand that follows a normal distribution, and try to determine the optimal ordering policy. We next relax this assumption by only assuming that the first and second moments of the probability distribution of the lead time demand to be known and finite, and then solve this inventory model by using the minimax distribution free approach.

This paper is organized as follows. In the Section 2, the model that the lead time demand has perfect information is formulated. And the model of partial information for the lead time demand is examined in Section 3. Three numerical examples are provided to illustrate solution procedure in Section 4, and Section 5 contains some concluding remarks and future research.

2. Review and extend the Ouyang et al. model

Ouyang et al. (1996) considered a mixed inventory model with variable lead time, and asserted the following total expected annual cost function:

$$\begin{aligned}\mathrm{EAC}(Q,L) &= \text{ordering cost} + \text{holding cost} + \text{stockout cost} + \text{lead time crashing cost}\\ &= A\frac{D}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)E(X-r)^+\right] + \frac{D}{Q}[\pi + \pi_0(1-\beta)]E(X-r)^+ + R(L)\frac{D}{Q}\\ &= A\frac{D}{Q} + h\left[\frac{Q}{2} + k\sigma\sqrt{L} + (1-\beta)E(X-r)^+\right] + \frac{D}{Q}[\pi + \pi_0(1-\beta)]E(X-r)^+ + R(L)\frac{D}{Q},\end{aligned} \tag{1}$$

where notations used are:

D: average demand per year
A: fixed ordering cost per order
h: holding cost per unit per year

π: stockout cost per unit short
π_0: marginal profit per unit
Q: order quantity
β: the fraction of the demand during the stockout period that will be backordered, $\beta \in [0,1]$
$E(\cdot)$: mathematical expectation
x^+: maximum value of x and 0, i.e. $x^+ = \max\{x, 0\}$
$E(X-r)^+$: expected shortage quantity at the end of cycle,

and assumptions are:

1. Consider deterministic lead time L and assume that the lead time demand X follows a normal distribution with mean μL and standard deviation $\sigma\sqrt{L}$.
2. The reorder point r = expected demand during lead time + safety stock (SS), and SS = $k \times$ (standard deviation of lead time demand), i.e. $r = \mu L + k\sigma\sqrt{L}$ where k is the safety factor and satisfies $P(X > r) = P(Z > k) = q$, Z represents the standard normal random variable and q represents the allowable stockout probability during L.
3. Inventory is continuously reviewed. Replenishments are made whenever the inventory level falls to the reorder point r.
4. The lead time L has n mutually independent components. The ith component has a minimum duration a_i and normal duration b_i, and a crashing cost per unit time c_i. Further, we assume that $c_1 \le c_2 \le \cdots \le c_n$.
5. The components of lead time are crashed one at a time starting with the component of least c_i and so on.
6. If let $L_0 \equiv \sum_{j=1}^{n} b_j$ and L_i be the length of lead time with components 1,2,…,i crashed to their minimum duration, then L_i can be expressed as $L_i = \sum_{j=1}^{n} b_j - \sum_{j=1}^{i} (b_j - a_j)$, $i = 1, 2, \ldots, n$; and the lead time crashing cost $R(L)$ per cycle for a given $L \in [L_i, L_{i-1}]$ is given by $R(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.

In this study, we use the same notation as in Ouyang et al. (1996) to avoid any possible confusion. When the lead time demand X follows a normal probability density function (p.d.f.) $f_X(x)$ with mean μL and standard deviation $\sigma\sqrt{L}$, and given that the reorder point $r = \mu L + k\sigma\sqrt{L}$, the expected shortage quantity $E(X-r)^+$ at the end of the cycle can be expressed as

$$E(X-r)^+ = \int_r^\infty (x-r) f_X(x)\,\mathrm{d}x = \sigma\sqrt{L}G(k) > 0,$$

where $G(k) = \int_k^\infty (z-k) f_z(z)\,\mathrm{d}z$ and $f_z(z)$ is the p.d.f. of the standard normal random variable Z. Hence, the total expected annual cost (1) reduces to

$$\mathrm{EAC}(Q,L) = A\frac{D}{Q} + h\left[\frac{Q}{2} + k\sigma\sqrt{L} + (1-\beta)\sigma\sqrt{L}G(k)\right] + \frac{D}{Q}[\pi + \pi_0(1-\beta)]\sigma\sqrt{L}G(k) + R(L)\frac{D}{Q}. \tag{2}$$

In model (2), the parameter β is treated as a constant used in Ouyang et al. (1996); however, in the real market as unsatisfied demands occur, the longer the length of lead time is, the larger the amount of shortages is, the smaller the proportion of customers can wait, and hence the smaller the backorder rate would be. Therefore, in this study, we consider the backorder rate, β, as a decision variable instead of the constant case to accommodate the practical inventory situation.

The assumptions in this study are exactly the same as those in Ouyang et al. (1996) except the following assumption:

During the stockout period, the backorder rate, β, is variable and is a function of L through $E(X-r)^+$. The larger the expected shortage quantity is, the smaller the backorder rate would be. Thus, we define $\beta = 1/[1+\alpha E(X-r)^+] = [1+\alpha\,\sigma\sqrt{L}G(k)]^{-1}$, where the backorder parameter α is a positive constant.

Hence, the total expected annual cost of our new model can be represented as follows

$$\mathrm{EAC}(Q,L) = A\frac{D}{Q} + h\left(\frac{Q}{2} + k\sigma\sqrt{L}\right) + \left\{\frac{h\alpha\sigma\sqrt{L}G(k)}{1+\alpha\sigma\sqrt{L}G(k)} + \frac{D}{Q}\left[\pi + \frac{\pi_0\alpha\sigma\sqrt{L}G(k)}{1+\alpha\sigma\sqrt{L}G(k)}\right]\right\}\sigma\sqrt{L}G(k) + R(L)\frac{D}{Q}. \tag{3}$$

In order to find the minimum total expected annual cost, taking the first partial derivatives of EAC(Q,L) with respect to Q and $L \in [L_i, L_{i-1}]$, we obtain

$$\frac{\partial \mathrm{EAC}(Q,L)}{\partial Q} = -\frac{AD}{Q^2} + \frac{h}{2} - \left[\pi + \frac{\pi_0\Delta(L)}{1+\Delta(L)}\right]\frac{D\Delta(L)}{\alpha Q^2} - R(L)\frac{D}{Q^2} \tag{4}$$

and

$$\frac{\partial \mathrm{EAC}(Q,L)}{\partial L} = \frac{1}{2}hk\sigma L^{-1/2} + \frac{D\pi\Delta(L)}{2\alpha QL} + \frac{[2+\Delta(L)][\Delta(L)]^2}{2\alpha L[1+\Delta(L)]^2}\left(h + \frac{D}{Q}\pi_0\right) - c_i\frac{D}{Q}, \tag{5}$$

where $\Delta(L) = \alpha\sigma\sqrt{L}G(k)$.

Notice that for fixed $L \in [L_i, L_{i-1}]$, EAC(Q,L) is convex in Q, since

$$\frac{\partial^2 \mathrm{EAC}(Q,L)}{\partial Q^2} = \frac{2AD}{Q^3} + 2\left[\pi + \frac{\pi_0\Delta(L)}{1+\Delta(L)}\right]\frac{D\Delta(L)}{\alpha Q^3} + 2R(L)\frac{D}{Q^3} > 0.$$

Setting Eq. (4) equal to zero and solving for Q, we have

$$Q = \sqrt{\frac{2D}{h}\left\{A + \left[\pi + \frac{\pi_0\Delta(L)}{1+\Delta(L)}\right]\frac{\Delta(L)}{\alpha} + R(L)\right\}}. \tag{6}$$

On the other hand, it is clear that for any given safety factor k, we have $G(k) > 0$ so as to obtain $\Delta(L) > 0$. Thus, for fixed Q, EAC(Q,L) is concave in $L \in (L_i, L_{i-1})$, because

$$\frac{\partial^2 \mathrm{EAC}(Q,L)}{\partial L^2} = -\frac{1}{4}hk\sigma L^{-3/2} - \frac{D\pi\Delta(L)}{4\alpha QL^2} - \frac{[3+\Delta(L)][\Delta(L)]^3}{4\alpha L^2[1+\Delta(L)]^3}\left(h + \frac{D}{Q}\pi_0\right) < 0.$$

Therefore, for fixed Q, the minimum total expected annual cost will occur at the end points of the interval $[L_i, L_{i-1}]$.

Consequently, we can establish the following algorithm to find the optimal lead time, optimal order quantity, and optimal backorder rate.

Algorithm 1

Step 1. For each L_i, $i = 0, 1, 2, \ldots, n$ and a given k, compute Q_i using Eq. (6).
Step 2. For each pair (Q_i, L_i), compute the corresponding total expected annual cost $\mathrm{EAC}(Q_i, L_i)$, $i = 0, 1, 2, \ldots, n$.
Step 3. Find $\min_{i=0,1,2,\ldots,n} \mathrm{EAC}(Q_i, L_i)$. If $\mathrm{EAC}(Q^*, L^*) = \min_{i=0,1,2,\ldots,n} \mathrm{EAC}(Q_i, L_i)$, then (Q^*, L^*) is the optimal solution. And the optimal backorder rate

$$\beta^* = \frac{1}{1 + \alpha\sigma\sqrt{L^*}G(k)}.$$

3. Partial demand information

In many practical situations, the probability distributional information of lead time demand is often quite limited. Hence, in this section, we relax the assumption about the normal distribution demand by only assuming that the lead time demand X has given finite first two moments (and hence, mean and variance are also given); i.e. the p.d.f. f_X of X belongs to the class $\mathscr{F}$ of p.d.f.'s with mean μL and variance $\sigma^2 L$. Now, we try to use the minimax distribution free procedure to solve this problem. The minimax distribution free approach for this problem is to find the 'most unfavorable' p.d.f. f_X in $\mathscr{F}$ for each (Q,L) and then to minimize over (Q,L); that is, our problem is to solve

$$\min_{Q,L} \max_{f_X \in \hat{\Phi}} \mathrm{EAC}(Q, L). \tag{7}$$

For this purpose, we need the following proposition which was asserted by Gallego and Moon (1993).

Proposition 1. *For any* $f_X \in \mathscr{F}$,

$$E(X - r)^+ \leq \frac{1}{2}\left[\sqrt{\sigma^2 L + (r - \mu L)^2} - (r - \mu L)\right]. \tag{8}$$

Moreover, the upper bound (8) *is tight.*

Because $r = \mu L + k\sigma\sqrt{L}$, and for any probability distribution of the lead time demand X, the above inequality always holds. Then, from the definition of β and inequality (8), we have

$$\beta \geq \left[1 + \tfrac{1}{2}\alpha\sigma\sqrt{L}\left(\sqrt{1 + k^2} - k\right)\right]^{-1}. \tag{9}$$

Therefore, problem (7) is reduced to minimize

$$\mathrm{EAC_U}(Q,L) = A\frac{D}{Q} + h\left(\frac{Q}{2} + k\sigma\sqrt{L}\right) + \frac{\sigma\sqrt{L}(\sqrt{1+k^2}-k)}{2}\left\{\frac{\frac{1}{2}h\alpha\sigma\sqrt{L}(\sqrt{1+k^2}-k)}{1+\frac{1}{2}\alpha\sigma\sqrt{L}(\sqrt{1+k^2}-k)} + \frac{D}{Q}\left[\pi + \frac{\frac{1}{2}\pi_0\alpha\sigma\sqrt{L}(\sqrt{1+k^2}-k)}{1+\frac{1}{2}\alpha\sigma\sqrt{L}(\sqrt{1+k^2}-k)}\right]\right\} + R(L)\frac{D}{Q}, \quad (10)$$

where $\mathrm{EAC_U}(Q,L)$ is the least upper bound of $\mathrm{EAC}(Q,L)$.

As discussed in the previous normal demand model, we take the partial derivatives of $\mathrm{EAC_U}(Q,L)$ with respect to Q and L, respectively, and obtain

$$\frac{\partial \mathrm{EAC_U}(Q,L)}{\partial Q} = -\frac{AD}{Q^2} + \frac{h}{2} - \frac{D\Lambda(L)}{\alpha Q^2}\left[\pi + \frac{\pi_0\Lambda(L)}{1+\Lambda(L)}\right] - R(L)\frac{D}{Q^2} \quad (11)$$

and

$$\frac{\partial \mathrm{EAC_U}(Q,L)}{\partial L} = \frac{1}{2}hk\sigma L^{-1/2} + \frac{D\pi\Lambda(L)}{2\alpha QL} + \frac{[2+\Lambda(L)][\Lambda(L)]^2}{2\alpha L[1+\Lambda(L)]^2}\left(h + \frac{D}{Q}\pi_0\right) - c_i\frac{D}{Q}, \quad (12)$$

where $\Lambda(L) = (1/2)\alpha\sigma\sqrt{\mathrm{L}}(\sqrt{1+k^2}-k)$.

It can be verified that for fixed $L \in [L_i, L_{i-1}]$, $\mathrm{EAC_U}(Q,L)$ is convex in Q. Hence, setting Eq. (11) equal to zero and solving for Q, we get

$$Q = \sqrt{\frac{2D}{h}\left\{A + \left[\pi + \frac{\pi_0\Lambda(L)}{1+\Lambda(L)}\right]\frac{\Lambda(L)}{\alpha} + R(L)\right\}}. \quad (13)$$

On the other hand, for fixed Q, $\mathrm{EAC_U}(Q,L)$ is concave in $L \in [L_i, L_{i-1}]$. Therefore, for fixed Q, the minimum total expected annual cost will occur at the end points of the interval $[L_i, L_{i-1}]$.

Theoretically, for given $D, h, A, \pi, \pi_0, \alpha, \sigma, k$ (which depends on the allowable stockout probability q and the p.d.f. $f_X(x)$), and each $L_i (i = 0, 1, 2, \ldots, n)$, from Eq. (13), we can get the optimal value of (Q_i, L_i); and then using model (10), we can obtain the corresponding total expected annual cost $\mathrm{EAC_U}(Q_i, L_i)$ for $i = 0, 1, 2, \ldots, n$. Thus, the minimum total expected annual cost can be obtained. However, in practice, since the p.d.f. f_X of X is unknown, even if the value of q is given, we can not get the exact value of k. Thus, in order to find the value of k, we need the following proposition.

Proposition 2 (Ouyang & Wu, 1997). *Let X represent the lead time demand which has a p.d.f. $f_X(x)$ with finite mean μL and standard deviation $\sigma\sqrt{L}(>0)$, then for any real number $d > 0$,*

$$P(X > d) \le \frac{\sigma^2 L}{\sigma^2 L + (d-\mu L)^2}. \quad (14)$$

Since the reorder point $r = \mu L + k\sigma\sqrt{L}$, if we take r instead of d in inequality (14), we get

$$P(X > r) \le \frac{1}{1+k^2}. \quad (15)$$

Further, it is assumed that the allowable stockout probability q during lead time is given, that is, $q = P(X > r)$, then from Eq. (15) we get $0 \le k \le \sqrt{(1/q) - 1}$.

It is easy to verify that $\mathrm{EAC_U}(Q,L)$ has a smooth curve for $k \in [0, \sqrt{(1/q) - 1}]$. Hence, we can establish the following algorithm to obtain the suitable k and hence the optimal Q, L, and β.

Algorithm 2

Step 1. For a given q, we divide the interval $[0, \sqrt{(1/q) - 1}]$ into m equal subintervals, where m is large enough. And we let $k_0 = 0$, $k_m = \sqrt{(1/q) - 1}$ and $k_j = k_{j-1} + (k_m - k_0)/m$, $j = 1, 2, \ldots, m - 1$.

Step 2. For each L_i, $i = 0, 1, 2, \ldots, n$, perform (i), (ii), and (iii).

(i) For given $k_j \in \{k_0, k_1, \ldots, k_m\}$, $j = 0, 1, 2, \ldots, m$, we can use a numerical search technique to compute Q_{i,k_j} from Eq. (13).

(ii) Compute the corresponding total expected annual cost

$$\mathrm{EAC_U}(Q_{i,k_j}, L_i) = A\frac{D}{Q_{i,k_j}} + h\left(\frac{Q_{i,k_j}}{2} + k_j\sigma\sqrt{L_i}\right) + \frac{\sigma\sqrt{L_i}(\sqrt{1 + k_j^2} - k_j)}{2}\left\{\frac{\frac{1}{2}h\alpha\sigma\sqrt{L_i}(\sqrt{1 + k_j^2} - k_j)}{1 + \frac{1}{2}\alpha\sigma\sqrt{L_i}(\sqrt{1 + k_j^2} - k_j)} + \frac{D}{Q}\left[\pi + \frac{\frac{1}{2}\pi_0\alpha\sigma\sqrt{L_i}(\sqrt{1 + k_j^2} - k_j)}{1 + \frac{1}{2}\alpha\sigma\sqrt{L_i}(\sqrt{1 + k_j^2} - k_j)}\right]\right\} + R(L_i)\frac{D}{Q_{i,k_j}}.$$

(iii) Find $\min_{k_j \in \{k_0, k_1, \ldots, k_m\}} \mathrm{EAC_U}(Q_{i,k_j}, L_i)$, and let

$$\mathrm{EAC_U}(Q_{i,k_{s(i)}}, L_i) = \min_{k_j \in \{k_0, k_1, \ldots, k_m\}} \mathrm{EAC_U}(Q_{i,k_j}, L_i).$$

Step 3. Find $\min_{i=0,1,2,\ldots,n} \mathrm{EAC_U}(Q_{i,k_{s(i)}}, L_i)$. If $\mathrm{EAC_U}(Q^{**}, L^{**}) = \min_{i=0,1,2,\ldots,n} \mathrm{EAC_U}(Q_{i,k_{s(i)}}, L_i)$, then (Q^{**}, L^{**}) is the optimal solution; the value of $k_{s(i)}$ such that $\mathrm{EAC_U}(Q^{**}, L^{**})$ exists is the optimal safety factor and we denote it by k^{**}. Thus, the optimal backorder rate

$$\beta^{**} = \left[1 + (1/2)\alpha\sigma\sqrt{\mathrm{L}^{**}}(\sqrt{1 + (k^{**})^2} - k^{**})\right]^{-1}.$$

4. Numerical examples

4.1. Perfect demand information pattern

Example 1. To illustrate the preceding theory, let us consider an inventory item with the following

Table 1
Lead time data

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

data: $D = 600$ units/year, $A = \$\ 200$ per order, $h = \$\ 20$, $\mu = 11$ units/week, $\sigma = 3$ units/week, $\pi = \$\ 50$, $\pi_0 = \$\ 100$, $\alpha = 2$, and the lead time has three components with data shown in Table 1.

We assume that the lead time demand follows a normal distribution and want to solve the case when $q = 0.1$ (in this situation, the value of safety factor k can be found directly from the standard normal table, and is 1.28). Applying the Algorithm 1 yields the results shown in Table 2.

From Table 2, the optimal inventory policy can easily be found by comparing EAC(Q_i,L_i), $i = 0, 1, 2, 3$. Therefore, we obtain the optimal order quantity $Q^* = 119$ units, optimal lead time $L^* = 6$ weeks, and optimal backorder rate $\beta^* = 0.59$. The minimum total expected annual cost EAC$(Q^*, L^*) = \$\ 2577.77$.

Example 2. Consider the same assumption and data as in Example 1. We use the Algorithm 1 to solve the cases when $\alpha = 0$, 0.5, 1, 10, 20, 40, 80, 100, and ∞. Computed results are shown in Fig. 1 and the optimal value is tabulated in Table 3.

The following inference can be made from the results in Fig. 1 and Table 3:

1. Increasing the value of backorder parameter α will result in an increase in the total expected annual cost EAC(Q,L) and the order quantity Q, but it results in a decrease in the backorder rate β.
2. The total expected annual cost (or order quantity) is more sensitive to α when its value is small.
3. As the value of α increases, the total expected annual cost becomes close to the complete lost case. (i.e. $\beta \to 0$). Conversely, decreasing the value of α, the total expected annual cost will approach the complete backordered case ($\beta \to 1$).
4. For difference parameter values α, the optimal total expected annual cost may occur at lead time $L = 6$ or 4 weeks.

Table 2
Solution procedure of Example 1 (L_i in week)

i	L_i	$R(L_i)$	Q_i	β_i	EAC(Q_i,L_i)
0	8	0	120	0.55	\$ 2612.29
1	6	5.6	119	0.59	\$ 2577.77
2	4	22.4	122	0.64	\$ 2589.96
3	3	57.4	129	0.67	\$ 2717.14

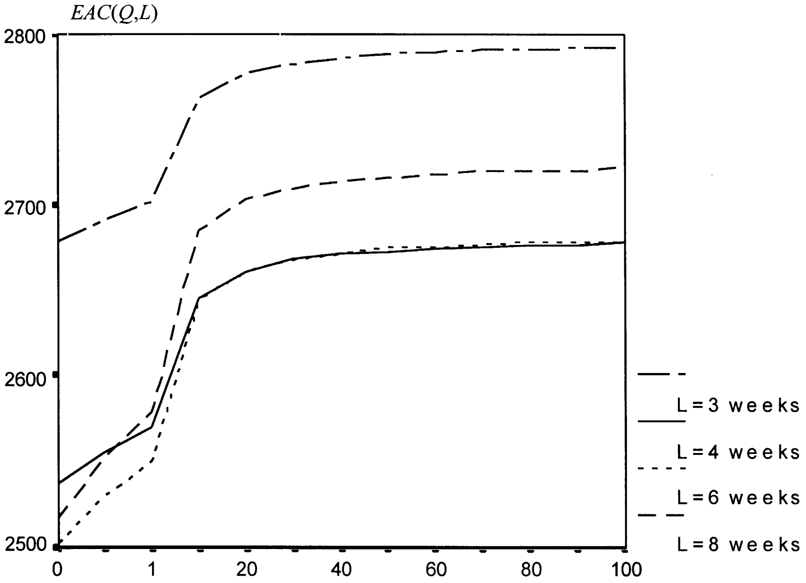

Fig. 1. The effect of backorder parameter on total expected annual cost.

4.2. Partial demand information pattern

Example 3. The data is as in Example 1. We assume here that the probability distribution of the lead time demand is free. Now, we solve the case when $q = 0.1$ (in this situation, we have $k_0 = 0$, $k_m = 3$), let $k_j = k_{j-1} + (k_m - k_0)/m$, $j = 1, 2, \ldots, m-1$ and take $m = 300$.

Applying the Algorithm 2, we tabulate the final computed results in Table 4.

From Table 4, the optimal inventory policy can be found by comparing $\mathrm{EAC_U}(Q_i, L_i)$, $i = 0, 1, 2, 3$. The optimal order quantity $Q^{**} = 129$ units, optimal lead time $L^{**} = 4$ weeks, optimal backorder rate $\beta^{**} = 0.47$, the suitable safety factor $k^{**} = 2.62$, and optimal total expected annual cost $\mathrm{EAC_U}(129, 4) = \$\ 2908.35$.

Table 3
Optimal ordering plans as α varies (L_i in week)

α	Q^*	L^*	β^*	$\mathrm{EAC}(Q^*, L^*)$
0	116	6	1.00	$ 2501.84
0.5	117	6	0.85	$ 2529.87
1	118	6	0.74	$ 2550.24
10	123	6	0.22	$ 2644.43
20	123	6	0.13	$ 2660.38
40	126	4	0.08	$ 2670.60
80	126	4	0.04	$ 2676.26
100	126	4	0.03	$ 2677.67
∞	126	4	0.00	$ 2681.91

Table 4
Solution procedure of the Example 3 (L_i in week)

i	L_i	$R(L_i)$	k_{ij}	Q_i	β_i	$\mathrm{EAC_U}(Q_i,L_i)$
0	8	0	2.68	131	0.40	$ 3077.65
1	6	5.6	2.67	129	0.43	$ 2977.80
2	4	22.4	2.62	129	0.47	$ 2908.35
3	3	57.4	2.52	136	0.50	$ 2981.17

For the optimal backorder rate β^{**} and the suitable k^{**}, if we use (Q^{**},L^{**}) instead of the optimal (Q^*,L^*) for a normal distribution demand then we can get EAC(Q^{**},L^{**}). Hence, as the p.d.f. f_X of X is a normal distribution, the added cost by using the minimax distribution free procedure instead of the normal distribution procedure is $\mathrm{EAC}(Q^{**}, L^{**}) - \mathrm{EAC}(Q^*, L^*) = \mathrm{EAC}(129, 4) - \mathrm{EAC}(119, 6) =$ $\$\,2640.80 - 2577.77 = \$\,63.03$. This is the largest amount that we would be willing to pay for the knowledge of f_X. This quantity can be regarded as the expected value of additional information (EVAI).

5. Concluding remarks

The purpose of this study is to present a mixture (Q,L) inventory model involving variable lead time and controllable backorder rate to extend that of Ouyang et al. (1996). In the real market as unsatisfied demands occur, the longer the length of lead time is, the smaller the proportion of backorder would be. Considering the reason, we assume that the backorder rate is dependent on the length of lead time through the amount of shortages. We first assume that the lead time demand follows a normal distribution, and then relax the assumption about probability distributional form of the lead time demand and apply the minimax distribution free procedure to solve the problem.

In future research on this problem, it would be of interest to treat the reorder point as a decision variable.

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