Int. J. Production Economics 166 (2015) 103–118

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Coordination and competition in a common retailer channel: Wholesale price versus revenue-sharing mechanisms

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ARTICLE INFO

Article history:
Received 23 December 2013
Received in revised form
26 August 2014
Accepted 9 April 2015
Available online 1 May 2015

Keywords:
Supply chain
Coordination
Competition
Wholesale price contract
Revenue sharing contract
Newsvendor problem with pricing
Game theory

ABSTRACT

In this paper, we study a supply chain comprising two competing manufacturers who sell their products through a common retailer. The retailer sells two competing brands with varying degrees of product substitutability. Under a linear stochastic demand, which is dependent on the retailer's price of its own brand as well as on the competing brand's retail price, we present a newsvendor model to determine the price and the quantity and study non-cooperative games among channel members. We establish a Stackelberg game where the common retailer acts as the Stackelberg leader. Later, we consider the case where manufacturers act as Stackelberg leaders. The basic model is developed based on wholesale price contract. We present some analytical results and establish the equilibrium of the system. We compare our equilibrium solution with that of the integrated system where a manufacturer produces two brands of product and sells them to the customer through its own retail channel. To enable supply chain coordination, we consider a revenue sharing contract. Finally, a numerical analysis is conducted to illustrate the impact of model parameters on the optimal decision variables.

1. Introduction

Most firms have started to realize that the profit across the supply chain can be improved through greater cooperation and better coordination. In general, a system of independent profit-maximizing firms earns lower profit than an integrated supply chain, where the objective is to maximize the total profit across the chain. But there are some situations where the members of a supply chain system can obtain a Pareto improvement. For example, to coordinate the supply chain, the members involved in the supply chain may work under incentives, such as quantity discounts (QD), buyback policies (BB), revenue-sharing (RS) contract, and sales rebate (SR) contracts and thus improve their own profit in addition to the supply chain performance. Several research efforts have been devoted to improving channel coordination and cooperation. There are two streams of literature related to this work: the first stream relates to channel competition in supply chains, and the second stream deals with the various supply chain contracts.

1.1. Channel competition

Channel competition has received lots of attention in the supply chain and marketing literature in the last two decades. Channel competition can be categorized into upstream competition and downstream competition. Upstream competition refers to the competition among the members of the upstream part of a supply chain e.g. suppliers/manufacturers, whereas downstream competition refers to the competition among members of the downstream part of a supply chain e.g. retailers. Table 1 lists papers on channel competition under deterministic and stochastic demand settings.

Jeuland and Shugan (1983) were one of the earliest to study channel competition. They develop a simple framework where channel competition is operationalized as a set of reaction function of competing channels. Through these reaction functions, they incorporate the nature of demand (e.g. degree of product differentiation) and channel structure (e.g. whether the channel is a vertical system or not). Since then, several researchers contributed to the literature on this topic. To the best of our knowledge, the upstream competition was first studied by Choi (1991). He developed a duopoly model of two competing manufacturers who sell their products through a common independent retailer. By considering price dependent demand, three non-cooperative games of different power structures between two manufacturers and a retailer are studied. Recently, Cachon and Kok (2010) develop a

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http://dx.doi.org/10.1016/j.ijpe.2015.04.010
0925-5273/

Table 1
Literature on upstream and downstream competition.

Competition type	Deterministic demand	Stochastic demand
Downstream competition	Ingene and Parry (1995), Padmanabhan and Png (1997), and Cachon and Lariviere (2005)	Yao et al. (2008)
Upstream competition	Choi (1991) and Pan et al. (2010)	Our model

model by considering three types of contract and two part tariffs and found that in their market structure those two sophisticated contracts i.e., quantity discounts and two-part tariffs force the manufacturers to compete more aggressively relative to when they only offer wholesale price contract. Pan et al. (2010) present a model of supply chain with one retailer and two suppliers with unreliable supply. They investigate the impacts of supply chain disruption on the retailer's sourcing strategy and supplier's pricing strategy using theoretical and computational analysis. On the other hand, downstream competition was studied by Ingene and Parry (1995). They present a model with competing retailers. To coordinate the channel, they mainly focus on quantity discount and two part tariff mechanism. Padmanabhan and Png (1997) present a similar model where two competing retailers carry one product that must be purchased from a common manufacturer. In their model, the retailers are assumed to have linear price dependent demand with an uncertain interrupt. By considering a downstream competition, Cachon and Lariviere (2005) found that revenue sharing contract performs well in coordinating a supply chain with retailers competing in terms of sales volumes.

Choi (1996) presents a model where upstream as well as downstream competition is considered. Their model deals with a channel structure in which there are duopoly manufacturers and duopoly retailers. Their major finding is that, while (horizontal) product differentiation helps manufacturers, it hurts retailers. Conversely, while (horizontal) store differentiation helps retailers, it hurts manufacturers. Most of the papers mentioned earlier assume that the customer demand faced by the retailer is price dependent and deterministic. Recently, Yao et al. (2008) present a model with one manufacturer and two competing retailers facing price dependent and stochastic demand.

1.2. Channel coordination and cooperation

Channel coordination and Pareto efficiency are the two important aspects of supply chain contracting. In a coordinated supply chain, contractual terms between the members of the supply chain ensure that the total expected profit of the supply chain is maximized. A contract is said to be Pareto efficient if all the members of the supply chain are no worse off (and at least one of them is strictly better off) with the existing contract than any other different contracts.

In general, a system of independent profit-maximizing firms earn lower profits than an integrated supply chain. However, there are some situations, where the members of a supply chain can obtain a Pareto improvement. For example, to coordinate the supply chain, the members may use some incentives, such as quantity discount, buyback or return policy, revenue sharing contract, and sales rebate contract to maximize their profits and in the process they can also improve the supply chain performance. In practice, in a supply chain, different manufacturers may adopt different types of contracts with the retailers to sell their product. Table 2 lists papers on supply chain contracts under deterministic and stochastic demand settings.

The expected profit of the wholesale price (WP) contract or price only contract is used as a benchmark to evaluate the expected outcomes of any contract. A WP contract is one in which retailer bears all the risk for all the unsold units. Supply chain model with WP contract has been studied under deterministic (Shugan and Jeuland, 1988; Choi, 1991, 1996) as well as stochastic demand settings (Petruzzi and Dada, 1999; Pan et al., 2009). It is well known that WP contract cannot coordinate a supply chain (Lariviere, 1999). Hence, researchers have studied other classes of contracts to establish better coordinations among supply chain members.

A buy-back (BB) contract or return policy is a contract between buyer and seller in which the seller allows the buyer to return unsold stock at the end of the selling season for a partial or complete refund. BB contract has been studied under deterministic demand (Padmanabhan and Png, 1997) and stochastic demand settings (Pasternack, 1985; Emmons and Gilbert, 1998; Lau and Lau, 1999; Donohue, 2000; Bose and Anand, 2007; Yao et al., 2008).

Supply chain coordination with revenue sharing (RS) contract has been well studied (see Cachon, 2003, and references therein). In RS contract, the supplier or manufacturer charges w_r per unit purchased from the retailer plus the retailers give the supplier/manufacturer a certain percentage of their revenue from that unit. Under the assumption that all revenues are shared, the salvage revenue is also shared between the firms. It is also possible to design a coordinating RS contract under the assumption that only regular revenue is shared. If f is the retailer's share of revenue generated from each unit then $(1-f)$ is the fraction of the revenue that the supplier earns. Recently, RS contracts have been applied successfully in the video cassette rental industry. Studies on revenue sharing contract have considered deterministic demand (Giannoccaro and Pontrandolfo, 2004) and stochastic demand settings (Wang et al., 2004; Cachon and Lariviere, 2005; Yao et al., 2008; Qin and Yang, 2008; Pang et al., 2014; Sammi and Panos, 2014).

Although several studies have addressed the above two issues separately, very little attention has been given to the joint effect of channel competition and channel coordination in supply chain management. The objective of this paper is to analyze the integrated effect of competitive and cooperative pricing behaviour in supply chains and study revenue sharing contract policy coordinating the supply chain. The research questions addressed in this paper are (1) Is it beneficial for the competitive manufacturers to offer a RS contract to the retailer instead of WP contract in the case of Stackelberg game? (2) What are the effects of competitive factors on the supply chain efficiency? To address these questions, we develop a model based on WP contract. Then in order to improve the channel coordination, we reformulate that model taking into account the RS contract. We also develop a corresponding model for integrated channel system and compare the optimal results with our decentralized system.

The following are the summary of contributions of this paper. Analytically, we have shown that (1) there exists an optimal stock level (z_1^*, z_2^*) that will maximize the expected profit of the common retailer, (2) there exists at least one Nash equilibrium between two competitive manufacturers, (3) there exists a Stackelberg game between each

Table 2
Literature on supply chain contracts with deterministic and stochastic demand.

Contract	Deterministic demand	Stochastic demand
Wholesale price/price only contract	Shugan and Jeuland (1988), Choi (1991, 1996)	Petruzzi and Dada (1999) and Pan et al. (2009)
Return policy/buy-back contract	Padmanabhan and Png (1997)	Pasternack (1985), Emmons and Gilbert (1998), Lau and Lau (1999), Donohue (2000), Bose and Anand (2007), Yao et al. (2008)
Revenue sharing contract	Giannoccaro and Pontrandolfo (2004), Cachon and Lariviere (2005), and Pan et al. (2010)	Cachon et al. (2001), Wang et al. (2004), Qin and Yang (2008), Pang et al. (2014), Tang and Kouvelis (2014), and **Our model**

manufacturer and the retailer where the retailer is the Stackelberg leader. Through numerical examples, we observe the following conclusions: (1) when manufacturers are Stackelberg leader and two manufacturers are competitive in nature, then it is beneficial for the manufacturers to offer wholesale price contract though RS contract improves the channel performance. (2) when retailer is the Stackelberg leader then he must offer salvage RS contract to increase his profit as well as to improve the channel coordination. (3) With the increase of the price sensitivity of demand, manufacturer's share $(1-\phi_i), i=1,2$ increases and subsequently it increases the chance of acceptance of RS contract of manufacturers offered by the retailer. (4) With the increase in the competition factor γ_i, the channel efficiency increases for both contracts. This implies that the competition between two manufacturers can improve the channel efficiency.

The remainder of the paper is organized as follows. In Section 2, we describe the model and present an equilibrium analysis. Integrated model is developed in Section 3. To improve the supply chain coordination, RS contract is considered in Section 4. In Section 5, we provide managerial insights. Finally, we conclude with future research directions. All the proofs are presented in the Appendices.

2. The model

We consider a supply chain consisting of two competing manufacturers, $M_i, i=1,2$, who sell their products through a common retailer, denoted by R (refer to Fig. 1). The two manufacturers are assumed to comprise a duopoly. Demand faced by the retailer is stochastic and dependent on the retail prices charged by the retailer. The retailer sells the two competing brands with varying degree of product substitutability.

The market under consideration consists of two level channels structure: manufacturer level and retailer level. Each manufacturer is assumed to produce single product (brand) whereas the retailer is not constrained to sell only one manufacturer's product. The competing manufacturers are essentially duopoly.

Linear duopoly price-dependent stochastic demand function that captures product substitution is considered in our model. Due to the stochastic nature of the customer's demand, the common retailer may face shortage or leftover.

We first define the price dependent stochastic demand function. Then, we formulate the manufacturers and retailers problem, and analyze the equilibrium behaviour of the system in a game theoretic setting. Non-cooperative game is considered among the channel members i.e., each member of the channel chooses strategies simultaneously and thereafter committed to their chosen strategies. Each player of the game is assumed to be rational i.e. each member of the channel is assumed to seek to maximize its own profit.

Each manufacturer's production cost is c_i per unit. Here we assume the unlimited capacity of each manufacturer. Thus the manufacturers are always able to produce the order quantity $Q_i, i=1,2$, in time for the start of the selling season. The lead times of both products are assumed to be zero.

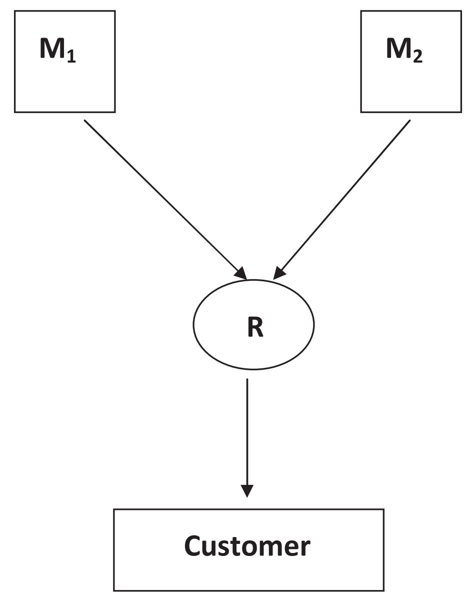

Fig. 1. Structure of the channel.

To model the problem, we define the following notation:

i	index for product, $i = 1, 2$
d_i	price dependent deterministic demand for ith product, $i = 1, 2$
D_i	demand faced by the retailer for ith product
α_i	initial market size of ith product
β_i	product i's own-price sensitivity
γ_i	product i's competitor-price sensitivity
ϵ_i	random variable denoting the random factor of the customer demand faced by the retailer for ith product
$f_i(\cdot)$	probability density function (pdf) of the random variable ϵ_i
$F_i(\cdot)$	cumulative distribution function (cdf) of the random variable ϵ_i
A_i, B_i	lower bound and upper bound on ϵ_i
v_i	salvage value or disposal cost per unit leftovers of the ith product
s_i	shortage cost per unit faced by the retailer for ith product
c_i	production cost of ith product

The decision variables are:

Q_i	order quantity of the common retailer for ith product
P_i	retail price of ith product
z_i	retailer's safety stock for ith product
w_i	wholesale price per unit for ith product

2.1. Demand function

To focus on brand substitutability, we assume that the manufacturer's cost structure and brand's demand structure are symmetric. Our basic model uses the duopoly static demand function (Choi, 1991, 1996; Jeuland and Shugan, 1983), given by

$$d_i(P_i,P_j)=\alpha_i-\beta_i P_i+\gamma_i P_j \text{ for } i=1,2 \text{ and } j=3-i,$$

where d_i is the price dependent demand function of ith product at the retail price P_i, given that the price of the other brand is P_j. α_i is the primary demand of ith product, β_i is the product i's own price sensitivity and γ_i is the product i's competitor-price sensitivity for $i=1, 2$. The parameters are required to satisfy $\beta_i, \gamma_i > 0$. We assume that the demand of ith product, $d_i(P_i,P_j)$ is a linearly decreasing function of its retail price P_i. The demand is also affected by the competitive price sensitivity factor γ_i, which captures the substitutability between the two products. Higher values of γ_i imply that the products are viewed as closer substitutes. To account for the randomness in demand, we consider the additive function (Petruzzi and Dada, 1999) as

$$D_i(P_i,P_j)=d_i(P_i,P_j)+\epsilon_i \text{ for } i=1,2 \text{ and } j=3-i,$$

where ϵ_i for $i=1, 2$ are the random variables defined on the range $[A_i,B_i]$. Let $f_i(\cdot)$ and $F_i(\cdot)$ denote the pdf and cdf of ϵ_i for $i=1, 2$. To ensure that for all values of P_j, there exists some range of P_i, such that $D_i(P_i,P_j)>0$, we require $A_i > -\alpha_i, i=1,2$.

2.2. Basic model

The sequence of events in our model (shown in Fig. 2) is as follows:

1. Before the start of the selling season, each manufacturer sets a wholesale price w_i for ith product.
2. For a given wholesale price, the retailer forecasts the demand, sets the unit retail price P_i and orders Q_i units of ith product from each manufacturer.
3. At the end of the selling period, excess inventory (if there is any) is salvaged by the retailer at a cost v_i per unit. Let s_i is the shortage cost per unit due to excess demand of the customers.

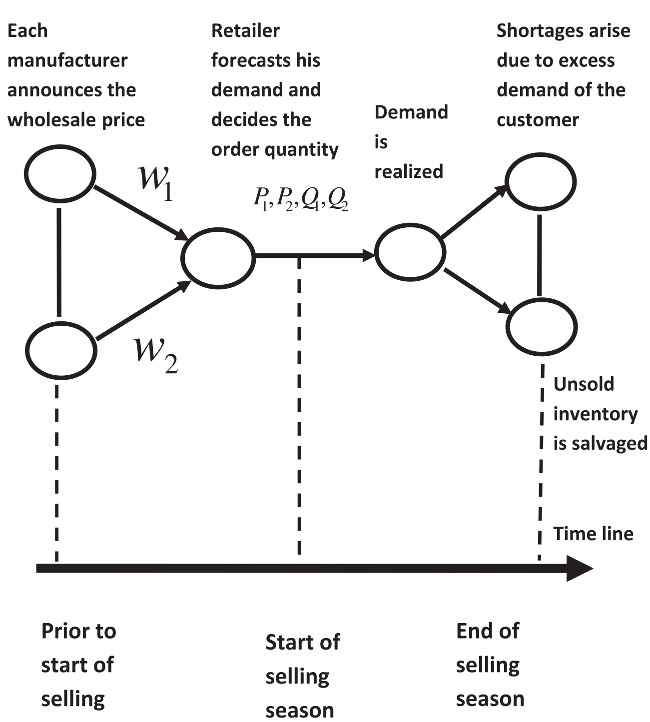

Fig. 2. Sequence of events on timeline and decisions.

2.2.1. Retailer's problem

Let us assume that the order quantity of the retailer for ith product is given by

$$Q_i=\alpha_i-\beta_i P_i+\gamma_i P_j+z_i \text{ for } i=1,2 \text{ and } j=3-i, \tag{1}$$

where z_i represents the amount of safety stocks of retailer for the ith product. Using the newsvendor model, the profit function of the common retailer for a given inventory level Q_i and retail price P_i of product i can be expressed as

$$\Pi_R(P_i,Q_i)=\begin{cases} P_iD_i(P_i,P_j)-w_iQ_i+v_i\big(Q_i-D_i(P_i,P_j)\big), D_i(P_i,P_j)\le Q_i \\ P_iQ_i-w_iQ_i-s_i\big(D_i(P_i,P_j)-Q_i\big), D_i(P_i,P_j)>Q_i. \end{cases}$$

The total expected profit is

$$\begin{aligned} E[\Pi_R(P_i,Q_i)]=\sum_{i=1}^{2}\Bigg[\int_{A_i}^{Q_i}&\big(P_iD_i(P_i,P_j)-w_iQ_i \\ &+v_i\big(Q_i-D_i(P_i,P_j)\big)\big)f_i(\epsilon_i)\,d\epsilon_i. \\ &+\int_{Q_i}^{B_i}\big(P_iQ_i-w_iQ_i-s_i\big(D_i(P_i,P_j)-Q_i\big)\big)f_i(\epsilon_i)\,d\epsilon_i\Bigg]. \end{aligned}$$

Substituting Q_i from (1) in the above expression with the safety stock z_i and retail price P_i as decision variables, the expected profit function of the retailer can be rewritten as shown above

$$\begin{aligned} E[\Pi_R(z_i,P_i)]=\sum_{i=1}^{2}&\big[(P_i-w_i)\big(d_i(P_i,P_j)+\mu_i\big)-(w_i-v_i)\Lambda(z_i) \\ &-(P_i-w_i+s_i)\Theta(z_i)\big], \end{aligned}$$

where

$$\Lambda(z_i)=\int_{A_i}^{z_i}(z_i-\epsilon_i)f_i(\epsilon_i)\,d\epsilon_i, \Theta(z_i) = \int_{z_i}^{B_i}(\epsilon_i-z_i)f_i(\epsilon_i)\,d\epsilon_i \text{ and } E(\epsilon_i)=\mu_i$$

for $j=3-i$.

Since this is a newsvendor model with two competitive suppliers and one common retailer with price and quantity as the decision variables and additive demand uncertainty, the results obtained by Petruzzi and Dada (1999) are applicable here. Our model with two competitive suppliers and a single retailer with symmetric demand function is the same as obtained by Petruzzi and Dada (1999) in Section 1.1 but with the following notation: $h=-v_i; s=s_i;\ \mu_i=E(\epsilon_i); a=\alpha_i+\gamma_iP_j, j=3-i;\ b=\beta_i; c=w_i; p=P_i$.

The optimal retail price P_i for a given z_i is obtained by setting $\frac{\partial E[\Pi_R]}{\partial P_i}=0$ and is given by

$$P_i^*=P_i(z_i)=P_i^0-\frac{\Theta(z_i)}{2\beta_i},$$

where

$$P_i^0=\frac{\alpha_i+\beta_iw_i+(\gamma_i+\gamma_j)P_j-\gamma_jw_j+\mu_i}{2\beta_i} \text{ and } \Theta(z_i) = \int_{z_i}^{B_i}(\epsilon_i-z_i)f_i(\epsilon_i)\,d\epsilon_i.$$

The first order optimality condition for z_i is

$$\begin{aligned}\frac{dE[\Pi_R(z_i,P_i(z_i))]}{dz_i}&=-(w_i-v_i)+[1-F(z_i)]\left[P_i^0+s_i-v_i-\frac{\Theta(z_i)}{2\beta_i}\right] \\ &=0 \text{ for } i=1,2.\end{aligned}$$

Due to the complexity of the above expression, there is no closed form solution z_i^* of the above problem. But for a given z_i^*, the optimal order quantity for ith product can be obtained from the expression: $Q_i^* = \alpha_i - \beta_i P_i + \gamma_i P_j + z_i^*$, $i=1,2, j=3-i$. Solving the above two equations for the retail prices as a function of z_1 and z_2, we get

$$P_1^*(z_1,z_2) = \left(\frac{\gamma_1+\gamma_2}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}\right)(\alpha_2+\beta_2 w_2-\gamma_1 w_1+\mu_2-\Theta(z_2)) + \left(\frac{2\beta_2}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}\right)(\alpha_1+\beta_1 w_1-\gamma_2 w_2+\mu_1-\Theta(z_1))$$

and

$$P_2^*(z_1,z_2) = \left(\frac{\gamma_1+\gamma_2}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}\right)(\alpha_1+\beta_1 w_1-\gamma_2 w_2+\mu_1-\Theta(z_1)) + \left(\frac{2\beta_1}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}\right)(\alpha_2+\beta_2 w_2-\gamma_1 w_1+\mu_2-\Theta(z_2)).$$

Replacing P_1 and P_2 by $P_1^*(z_1,z_2)$ and $P_2^*(z_1,z_2)$ in the retailer's objective function, we get

$$\begin{aligned} E[\Pi_R(z_1,z_2)] = &\left(P_1^*(z_1,z_2)-w_1\right)(\alpha_1-\beta_1 P_1^*(z_1,z_2) \\ &+\gamma_1 P_2^*(z_1,z_2)+\mu_1)-(w_1-v_1)\Lambda(z_1) \\ &-\left(P_1^*(z_1,z_2)-w_1+s_1\right)\Theta(z_1)+\left(P_2^*(z_1,z_2)-w_2\right) \\ &\times\left(\alpha_2-\beta_2 P_2^*(z_1,z_2)+\gamma_2 P_1^*(z_1,z_2)+\mu_2\right)-(w_2-v_2)\Lambda(z_2) \\ &-\left(P_2^*(z_1,z_2)-w_2+s_2\right)\Theta(z_2). \end{aligned} \tag{2}$$

The expression (2) is a function of z_1 and z_2 only. Upon substituting the optimal $P_1^*(z_1,z_2)$ and $P_2^*(z_1,z_2)$ into the retailer's profit function, we can simplify the retailer's four dimensional space into two dimensions for further analysis. Thus, the retailer's problem takes the form

$$\max_{z_1,z_2} E[\Pi_R(z_1,z_2)] \tag{3}$$

Depending on the parameters of the problem, multiple sets of (z_1,z_2) may satisfy the first order optimality condition. In the following proposition, we show that the objective function $E[\Pi_R(z_1,z_2)]$ is a concave function of z_1,z_2 and hence there exists an optimal solution $\{z_1^*,z_2^*\}$ that will maximize the expected profit of the common retailer. Moreover, due to concavity of the objective function, the above solution will be a global optimal solution.

Proposition 1. *If the cumulative distribution functions $F_i(\cdot)$ for $i=1$, 2 satisfy the following condition*

$A1: \ cz_i r_i(z_i) > 1$ *where* $r_i(z_i) = \dfrac{f_i(z_i)}{1-F_i(z_i)}$ *is the hazard rate,*

and the demand and cost parameters satisfy the following conditions:

$A2: \ 4\beta_1\beta_2-(\gamma_1+\gamma_2)^2 > 0,$
$A3: \ \gamma_1=\gamma_2,$

then there exists an optimal solution $\{z_1^*,z_2^*\}$ that will maximize the expected profit of the common retailer.

Note 1. The assumption A1 is satisfied for variety of distributions including exponential, uniform etc. The assumption A2 i.e., $4\beta_1\beta_2-(\gamma_1+\gamma_2)^2>0$ only be satisfied if $\beta_1>\gamma_1$, $\beta_2>\gamma_2$. In other words, the above inequality holds if the demand faced by the common retailer for the ith product is more sensitive to its own retail price than the competing brand's retail price .

Note 2. We assumed that $\beta_i-\gamma_i>0$ and as $\beta_i-\gamma_i$ decreases (i.e., the products are less differentiated) the more substitutable the two products, therefore more price competition between them. That implies the difference between P_1 and P_2 decreases.

2.2.2. Manufacturer's problem

Prior to the start of the selling season, each manufacturer will choose a wholesale price (per unit) for their product to charge to the retailer. Each manufacturer's expected profit is given by

$$\Pi_{M_i} = (w_i-c_i)Q_i = (w_i-c_i)\left(z_i+d_i(P_i,P_j)\right) \ i=1,2 \text{ and } j=3-i. \tag{4}$$

In the decentralized system, every member of the channel wants to maximize his/her own profit. The behaviour of the manufacturers and the retailer in the market can be described using the Stackelberg equilibrium, whereas the behaviour of the two competitive manufacturers can be represented using horizontal Nash game. We will discuss the following two possible power balance scenarios: Nash game and Stackelberg game.

2.3. Equilibrium analysis

In this section, we will show that the game between two competing manufacturers is a supermodular game and there exists a Stackelberg game between each manufacturer and the common retailer. Any equilibrium must satisfy the following first order conditions:

$$P_1 = P_1^0 - \frac{\Theta(z_1)}{2\beta_1}, \tag{5}$$

$$P_2 = P_2^0 - \frac{\Theta(z_2)}{2\beta_2}, \tag{6}$$

$$\frac{\partial E[\Pi_R]}{\partial z_1} = -(w_1-v_1)+(1-F_1(z_1))(P_1+s_1-v_1)=0, \tag{7}$$

$$\frac{\partial E[\Pi_R]}{\partial z_2} = -(w_2-v_2)+(1-F_2(z_2))(P_2+s_2-v_2)=0, \tag{8}$$

$$\frac{\partial E\left[\Pi_{M_1}\right]}{\partial w_1} = (z_1+d_1)=0, \tag{9}$$

and

$$\frac{\partial E\left[\Pi_{M_2}\right]}{\partial w_2} = (z_2+d_2)=0 \tag{10}$$

where P_1^0 and P_2^0 are the same as defined earlier. Solving Eqs. (5) and (6) for the retailer prices as a function of z_1 and z_2, we get

$$P_1^*(z_1,z_2) = \left(\frac{\gamma_1+\gamma_2}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}\right)(\alpha_2+\beta_2 w_2-\gamma_1 w_1+\mu_2-\Theta(z_2)) + \left(\frac{2\beta_2}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}\right)(\alpha_1+\beta_1 w_1-\gamma_2 w_2+\mu_1-\Theta(z_1)) \tag{11}$$

and

$$P_2^*(z_1,z_2) = \left(\frac{\gamma_1+\gamma_2}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}\right)(\alpha_1+\beta_1 w_1-\gamma_2 w_2+\mu_1-\Theta(z_1)) + \left(\frac{2\beta_1}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}\right)(\alpha_2+\beta_2 w_2-\gamma_1 w_1+\mu_2-\Theta(z_2)). \tag{12}$$

Thus, the game is defined by Eqs. (7)–(12) and we get the following propositions.

Proposition 2. *If Assumption* A2 *is satisfied, then the game between two competitive manufacturers is a supermodular game and hence there exists at least one Nash equilibrium.*

2.3.1. Existence of the retailer Stackelberg game

Under the retailer Stackelberg game, the retailer is the leader and manufacturers are followers. In this market, retailer's objective will be to maximize his profit taking into consideration the reaction function of the manufacturers. The manufacturers' reaction functions are obtained from the following first order optimality conditions:

$$\frac{\partial \Pi_{M_1}}{\partial w_1} = (\alpha_1 - \beta_1(m_1 + w_1) + \gamma_1 P_2 + z_1) - \beta_1(w_1 - c_1) = 0,$$
$$\frac{\partial \Pi_{M_2}}{\partial w_2} = (\alpha_2 - \beta_2(m_2 + w_2) + \gamma_2 P_1 + z_2) - \beta_2(w_2 - c_2) = 0, \tag{13}$$

where $m_i = P_i - w_i$ is the retail margin for ith product. For the retailer Stackelberg game M_i knows retail margin of his own product. Again, from Proposition 2 we have already established the existence of the Nash equilibrium between the manufacturers. Hence, resulting reaction functions are given by

$$\begin{aligned} w_1 &= \frac{1}{2\beta_1}[\alpha_1 - \beta_1 m_1 + \gamma_1 P_2 + z_1 + \beta_1 c_1] \\ &= \frac{1}{\beta_1}[\alpha_1 - \beta_1 P_1 + \gamma_1 P_2 + z_1 + \beta_1 c_1], \\ w_2 &= \frac{1}{2\beta_2}[\alpha_2 - \beta_2 m_2 + \gamma_2 P_1 + z_2 + \beta_2 c_2] \\ &= \frac{1}{\beta_2}[\alpha_2 - \beta_2 P_2 + \gamma_2 P_1 + z_2 + \beta_2 c_2]. \end{aligned}$$

Substituting these reaction functions into the retailer's profit function, the expected profit takes the form

$$\begin{aligned} E[\Pi_R^s(z_1, z_2)] = &\left(P_1^*(z_1, z_2) - w_1(z_1, z_2)\right)\left(\alpha_1 - \beta_1 P_1^*(z_1, z_2)\right. \\ &\left. + \gamma_1 P_2^*(z_1, z_2) + \mu_1\right) - (w_1(z_1, z_2) - v_1)\Lambda(z_1) \\ &- \left(P_1^*(z_1, z_2) - w_1(z_1, z_2) + s_1\right)\Theta(z_1) \\ &+ \left(P_2^*(z_1, z_2) - w_2(z_1, z_2)\right) \\ &\times \left(\alpha_2 - \beta_2 P_2^*(z_1, z_2) + \gamma_2 P_1^*(z_1, z_2) + \mu_2\right) \\ &- (w_2(z_1, z_2) - v_2)\Lambda(z_2) \\ &- \left(P_2^*(z_1, z_2) - w_2(z_1, z_2) + s_2\right)\Theta(z_2). \end{aligned} \tag{14}$$

The following proposition can be deduced immediately.

Proposition 3. *If the Assumptions* A2 and A3 *are satisfied and if the following condition holds*

$A4: \ 2\beta_1\beta_2 - \gamma_1(\gamma_1 + \gamma_2) \geq 0$ *and* $2\beta_1\beta_2 - \gamma_2(\gamma_1 + \gamma_2) \geq 0$

then there exists a Stackelberg game between each manufacturer and the common retailer where the retailer is the Stackelberg leader and the manufacturer is the follower.

Note 3. It is to be noted that under Assumption A3, Assumptions A2 and A4 are equivalent.

3. Integrated system

Let us consider an integrated system under the assumption that the manufacturer produces two types of products at unit cost c_1 and c_2 and sells them to the customer through its own retail channel. Any excess inventory will be salvaged and there will be shortage cost for excess demand. The expression of the profit of the integrated channel system will be similar to the retailer's profit of the decentralized system except that w_1, w_2 will be replaced by c_1 and c_2. Thus, the expected profit $E[\Pi_I]$ for the integrated system can be written as

$$E[\Pi_I] = \sum_{i=1}^{2} \left[(P_i - c_i)(d_i(P_i, P_j) + \mu_i) - (c_i - v_i)\Lambda(z_i) - (P_i - c_i + s_i)\Theta(z_i)\right] \tag{15}$$

where

$$\Lambda(z_i) = \int_{A_i}^{z_i} (z_i - \epsilon_i) f_i(\epsilon_i)\, d\epsilon_i, \quad \Theta(z_i) = \int_{z_i}^{B_i} (\epsilon_i - z_i) f_i(\epsilon_i)\, d\epsilon_i \text{ and } E(\epsilon_i) = \mu_i,\ j = 3 - i$$

Here the decision variables are P_1, P_2, z_1 and z_2. The integrated profit function $E[\Pi_I]$ is concave with respect to each decision variable as the second derivative of the function with respect to each decision variables is negative. Now, jointly solving the first order optimality conditions of P_1 and P_2, we can write the optimal prices $P_1^I(z_1, z_2), P_2^I(z_1, z_2)$ of the integrated system as functions of z_1 and z_2 as follows:

$$\begin{aligned} P_1^I(z_1, z_2) = &\left(\frac{\gamma_1 + \gamma_2}{4\beta_1\beta_2 - (\gamma_1 + \gamma_2)^2}\right)(\alpha_2 + \beta_2 c_2 - \gamma_1 c_1 + \mu_2 - \Theta(z_2)) \\ &+ \left(\frac{2\beta_2}{4\beta_1\beta_2 - (\gamma_1 + \gamma_2)^2}\right)(\alpha_1 + \beta_1 c_1 - \gamma_2 c_2 + \mu_1 - \Theta(z_1)) \end{aligned}$$

and

$$\begin{aligned} P_2^I(z_1, z_2) = &\left(\frac{\gamma_1 + \gamma_2}{4\beta_1\beta_2 - (\gamma_1 + \gamma_2)^2}\right)(\alpha_1 + \beta_1 c_1 - \gamma_2 c_2 + \mu_1 - \Theta(z_1)) \\ &+ \left(\frac{2\beta_1}{4\beta_1\beta_2 - (\gamma_1 + \gamma_2)^2}\right)(\alpha_2 + \beta_2 c_2 - \gamma_1 c_1 + \mu_2 - \Theta(z_2)). \end{aligned}$$

Again, it can be easily verified that under Assumption A2, $E[\Pi_I]$ is jointly concave with respect to P_1 and P_2. Thus, it is possible to substitute P_1 and P_2 with $P_1^I(z_1, z_2)$ and $P_2^I(z_1, z_2)$ in Eq. (15) and rewrite the expected integrated profit function as

$$\begin{aligned} E[\Pi_I(z_1, z_2)] = &\left(P_1^I(z_1, z_2) - c_1\right)\left(\alpha_1 - \beta_1 P_1^I(z_1, z_2) + \gamma_1 P_2^I(z_1, z_2) + \mu_1\right) \\ &- (c_1 - v_1)\Lambda(z_1) \\ &- \left(P_1^I(z_1, z_2) - c_1 + s_1\right)\Theta(z_1) + \left(P_2^I(z_1, z_2) - c_2\right) \\ &\times \left(\alpha_2 - \beta_2 P_2^I(z_1, z_2) + \gamma_2 P_1^I(z_1, z_2) + \mu_2\right) - (c_2 - v_2)\Lambda(z_2) \\ &- \left(P_2^I(z_1, z_2) - c_2 + s_2\right)\Theta(z_2) \end{aligned} \tag{16}$$

Next, we can show that integrated profit function is a concave function of z_1, z_2 which will imply that there exists an optimal solution (z_1^{I*}, z_2^{I*}) which will maximize the expected integrated profit function $E[\Pi_I(z_1, z_2)]$. The proof is similar to Proposition 1.

4. Supply chain coordination and revenue sharing contract

In the previous section, we have developed and analyzed our model under WP contract. To enable supply chain coordination, we will consider revenue sharing contract. Under this contract, the retailer pays each manufacturer a wholesale price for each unit purchased, plus a percentage of revenue generated from the sale of each unit at the end of the selling season. If ϕ_i be the fraction of the revenue generated from ith product the retailer sells, then $(1 - \phi_i)$ is the fraction of revenue the ith manufacturer earns. We assume that under this contract the manufacturer charges lower wholesale price $w_i^R = \phi_i c_i$ per unit at the start of the selling season. We assume that the salvage revenue is also shared (Cachon and Lariviere, 2005). Depending on the leadership of the Stackelberg game, two situations will arise: (1) manufacturers are the Stackelberg leaders and (2) retailer is the Stackelberg leader. Each of these cases are discussed in the following subsections.

4.1. Manufacturers as Stackelberg leaders

We consider a market scenario where two competing manufacturers sell their product through a common retailer. Let us analyze the case when manufacturers are Stackelberg leaders. When one manufacturer offers the retailer a RS contract while the other offers a WP contract, the retailer would prefer the manufacturer who allows the retailer to keep all the revenue until the end of the selling season. To avoid this situation, the manufacturer who offers the RS contract should confirm that the following condition holds A5: $\phi_i P_i^R - w_i^R = \phi_i(P_i^R - c_i) \geq m_i = P_i - w_i$, where P_i^R is the retail price for ith product for RS contract. Therefore, when manufacturers are Stackelberg leaders, depending on WP and RS contract, two cases arise.

4.1.1. When both manufacturers offer salvage RS contracts

When manufacturers are the Stackelberg leaders, the sequence of the events is as follows. Prior to the selling season, the retailer will choose her order quantity Q_i for the product i for which the manufacturers will charge a wholesale price $w_i^R = \phi_i c_i$ per unit subject to the condition A5. Then, at the end of the selling season the retailer keeps the fraction ϕ_i of the revenue generated from each product and share the fraction $(1-\phi_i)$ of that revenue with the manufacturers. Therefore, in this case the retailer's expected profit is

$$E\left[\Pi_R^{RS}\right] = \sum_{i=1}^{2} \left[(\phi_i P_i - w_i^R)\big(d_i(P_i, P_j) + \mu_i\big) - (w_i^R - \phi_i v_i)\Lambda(z_i) - (\phi_i P_i - w_i + s_i)\Theta(z_i)\right] \tag{17}$$

where

$$\Lambda(z_i) = \int_{A_i}^{z_i} (z_i - \epsilon_i) f_i(\epsilon_i)\, d\epsilon_i, \quad \Theta(z_i) = \int_{z_i}^{B_i} (\epsilon_i - z_i) f_i(\epsilon_i)\, d\epsilon_i \text{ and } E(\epsilon_i) = \mu_i,$$

and the profit of each manufacturer is

$$\Pi_{M_i}^{RS} = (w_i^R - c_i)(d_i(P_i, P_j) + z_i) + (1-\phi_i)P_i\big(d_i(P_i, P_j) + \mu_i - \Theta(z_i)\big) + (1-\phi_i)v_i\Lambda(z_i) \tag{18}$$

subject to the condition A5. Here z_i and P_i are the decision variables of the retailer and ϕ_i is the decision variable of ith manufacturer. We can find the optimal retail prices as functions of z_1 and z_2 as follows:

$$\begin{aligned} P_1^R(z_1, z_2) = {} & \left(\frac{\phi_1\gamma_1 + \phi_2\gamma_2}{4\phi_1\phi_2\beta_1\beta_2 - (\phi_1\gamma_1 + \phi_2\gamma_2)^2}\right)(\phi_2\alpha_2 + \phi_2\mu_2 + \beta_2 w_2^R \\ & - \phi_2\Theta(z_2) - \gamma_1 w_1^R) \\ & + \left(\frac{2\phi_2\beta_2}{4\phi_1\phi_2\beta_1\beta_2 - (\phi_1\gamma_1 + \phi_2\gamma_2)^2}\right)(\phi_1\alpha_1 + \phi_1\mu_1 + \beta_1 w_1^R \\ & - \phi_1\Theta(z_1) - \gamma_2 w_2^R) \\ P_2^R(z_1, z_2) = {} & \left(\frac{\phi_1\gamma_1 + \phi_2\gamma_2}{4\phi_1\phi_2\beta_1\beta_2 - (\phi_1\gamma_1 + \phi_2\gamma_2)^2}\right)(\phi_1\alpha_1 + \phi_1\mu_1 \\ & + \beta_1 w_1^R - \phi_1\Theta(z_1) - \gamma_2 w_2^R) \\ & + \left(\frac{2\phi_1\beta_1}{4\phi_1\phi_2\beta_1\beta_2 - (\phi_1\gamma_1 + \phi_2\gamma_2)^2}\right)(\phi_2\alpha_2 + \phi_2\mu_2 \\ & + \beta_2 w_2^R - \phi_2\Theta(z_2) - \gamma_1 w_1^R). \end{aligned}$$

4.1.2. When one manufacturer offers salvage RS contract and the other offers wp contract

Without loss of generality, let us assume that M_1 offers WP contract and M_2 offers salvage RS contract. Then, the manufacturers' respective profits are

$$\begin{aligned} \Pi_{M_1} &= (w_1 - c_1)(\alpha_1 - \beta_1 P_1 + \gamma_1 P_2 + z_1) \\ \Pi_{M_2}^{RS} &= (w_2^R - c_2)(d_2(P_i, P_j) + z_2) + (1-\phi_2)P_2\big(d_2(P_i, P_j) \\ &\quad + \mu_2 - \Theta(z_2)\big) + (1-\phi_2)v_2\Lambda(z_2). \end{aligned}$$

subject to the condition $\phi_2(P_2^R - c_2) \geq m_2 = P_2 - w_2$. Otherwise, the retailer will not accept the salvage RS contract or would prefer to buy the product from manufacturer M_1. In this case, M_2 expects that the retailer will prefer to share his revenue as small as possible. Hence, M_2 would like to set $\phi_2(P_2^R - c_2) = m_2 = P_2 - w_2$. In this case, the retailer's expected profit is

$$\begin{aligned} E\left[\Pi_R^{RSW}\right] = {} & \left[(P_1 - w_1)\big(d_1(P_1, P_2) + \mu_1\big) - (w_1 - v_1)\Lambda(z_1) - (P_1 - w_1 + s_1)\Theta(z_i)\right] \\ & + \left[(\phi_2 P_2 - w_2^R)\big(d_2(P_1, P_2) + \mu_2\big) - (w_2^R - \phi_2 v_2)\Lambda(z_2) - (\phi_2 P_2 - w_2^R + s_2)\Theta(z_2)\right], \end{aligned}$$

where $\Lambda(z_i)$ and $\theta_(z_i)$ are the same as defined earlier. The optimal retail prices as functions of z_1 and z_2 are as follows:

$$\begin{aligned} P_1^{RW}(z_1, z_2) = {} & \left(\frac{\gamma_1 + \phi_2\gamma_2}{4\phi_2\beta_1\beta_2 - (\gamma_1 + \phi_2\gamma_2)^2}\right)(\phi_2\alpha_2 \\ & + \phi_2\mu_2 + \beta_2 w_2^R - \phi_2\Theta(z_2) - \gamma_1 w_1) \\ & + \left(\frac{2\phi_2\beta_2}{4\phi_2\beta_1\beta_2 - (\gamma_1 + \phi_2\gamma_2)^2}\right)(\alpha_1 + \mu_1 \\ & + \beta_1 w_1 - \Theta(z_1) - \gamma_2 w_2^R), \\ P_2^{RW}(z_1, z_2) = {} & \left(\frac{\gamma_1 + \phi_2\gamma_2}{4\phi_2\beta_1\beta_2 - (\gamma_1 + \phi_2\gamma_2)^2}\right)(\alpha_1 + \mu_1 \\ & + \beta_1 w_1 - \Theta(z_1) - \gamma_2 w_2^R) \\ & + \left(\frac{2\beta_1}{4\phi_2\beta_1\beta_2 - (\gamma_1 + \phi_2\gamma_2)^2}\right)(\phi_2\alpha_2 + \phi_2\mu_2 \\ & + \beta_2 w_2^R - \phi_2\Theta(z_2) - \gamma_1 w_1). \end{aligned}$$

4.2. Retailer as the Stackelberg leader

Consider the case when the retailer is the Stackelberg leader (e.g. retail giant, Walmart). When retailer is the Stackelberg leader, he will announce the retail price of both products with the aim of optimizing her own expected profit by taking into consideration the manufacturers' reaction functions. In this case, the manufacturer decides Q_1 and Q_2 to be provided to the retailer to maximize manufacturer's own expected profit. The decision variables of the retailer are ϕ_i, P_i, where $w_i^R = \phi_i c_i$. The expected profit of the retailer is given by

$$E\left[\Pi_R^{RS}\right] = \sum_{i=1}^{2} \left[(\phi_i P_i - w_i^R)\big(d_i(P_i, P_j) + \mu_i\big) - (w_i^R - \phi_i v_i)\Lambda(z_i) - (\phi_i P_i - w_i + s_i)\Theta(z_i)\right],$$

where

$$\Lambda(z_i) = \int_{A_i}^{z_i} (z_i - \epsilon_i) f_i(\epsilon_i)\, d\epsilon_i, \quad \Theta(z_i) = \int_{z_i}^{B_i} (\epsilon_i - z_i) f_i(\epsilon_i)\, d\epsilon_i \text{ and } E(\epsilon_i) = \mu_i, \tag{19}$$

and the profit of each manufacturer is

$$\Pi_{M_i}^{RS} = (w_i^R - c_i)(d_i(P_i, P_j) + z_i) + (1-\phi_i)P_i\big(d_i(P_i, P_j) + \mu_i - \Theta(z_i)\big) + (1-\phi_i)v_i\Lambda(z_i) \tag{20}$$

In this case, reaction function of manufacturers can be obtained using $\frac{\partial \Pi_{M_1}^{RS}}{\partial z_1} = 0$ and $\frac{\partial \Pi_{M_2}^{RS}}{\partial z_2} = 0$. Because, the retailer is the Stackelberg leader, he prefers to share his revenue as small as possible. Hence, the retailer will maximize his profit subject to the conditions $\phi_i(P_i^R - c_i) = m_i = P_i - w_i$.

5. Numerical study and managerial observations

We conduct numerical analysis to investigate the effect of key parameters on the optimal equilibrium solutions as well as compare the performance of our decentralized model with that of the integrated system. Due to the lack of closed form analytical solutions, we find the equilibrium solutions numerically. To do so we have followed the approach outlined in Section 2.3. We formulate each of these problems as non-linear programming problems and solve using Mathematica 7.0 (Wolfram, 1996). With the random parametric values, we evaluate the optimal decisions of manufacturers and the common retailer subject to profit maximization in all possible scenarios.

In our study, the stochastic demand follows truncated normal distribution and hence the actual demand also follows truncated normal distribution. The probability density function of the stochastic demand is given by

$$f(z;\mu,\sigma) = \frac{\frac{1}{\sigma}\phi\left(\frac{z-\mu}{\sigma}\right)}{\Phi\left(\frac{B-\mu}{\sigma}\right) - \Phi\left(\frac{A-\mu}{\sigma}\right)}, \quad A \le z \le B$$

where $\phi(z) = \frac{1}{\sqrt{2\pi}}\exp[-\frac{1}{2}z^2]$, $-\infty < z < \infty$ and $\Phi(\frac{z-\mu}{\sigma}) = \int_{-\infty}^{(z-\mu)/\sigma} \phi(t)\,dt$. We have fixed the values of c_i, v_i and s_i to investigate the effect of other parameters on the performance of the contracts though these parameters have some impact on the performance of contract. Similar parameter selection was found in the existing literature on channel coordination and demand substitution with a linear demand model: Boyaci and Ray (2003), Wang et al. (2004), Qin and Yang (2008), and Zhao (2008). The variable parameters those have significance impact in contract performance and those we are interested to find out are α, β, γ and σ. Through experiment we have found that if $\beta - \gamma \le 15$, then the obtained results will be difficult to interpret. Again the different combinations of α and β lead to different variations in demand, the selection of σ also depends on different values of α and β. The selection of σ also depends on the leadership of the Stackelberg game. When manufacturers are the Stackelberg leaders then $\sigma \ge 34$. When retailers is the Stackelberg leader then $0.007 \le \sigma \le 1.5$ for our following parametric selection. By varying key parameters, we created a set of problems i.e., the manufacturers' own price sensitivity, $\beta_i \in (95, 105, 115)$, the manufacturers' competitor price-sensitivity, $\gamma_i \in (75, 80, 85)$. Since positive demand realization requires $A_i > -\alpha_i$, we set $A_i = -(\alpha_i - 1)$ and $B_i = (\alpha_i - 1)$. The other parametric values are set to $c_1 = c_2 = 1, s_1 = s_2 = 0.75$, $\alpha_1 = \alpha_2 = 200, v_1 = v_2 = 0.5$. We obtain the optimal solutions for $\beta_i = 95,\ 105,\ 115$ with $\gamma_i = 70$ and for $\gamma_i = 75,\ 80,\ 85$, with $\beta_i = 115$ to analyze the key characteristics of the WP contract and RS contract and compare them. The effect of different important parameters on the supply chain efficiency is depicted in Table 13.

In the numerical study, we have considered Stackelberg game (i.e. sequential movement) between each manufacturer and the common retailer and a Nash game (simultaneous movement) between two manufacturers. We evaluate the optimal results in tabulated form for both the two possible cases: (i) when manufacturers are the Stackelberg leader and (ii) when retailer is the Stackelberg leader. The optimal results of integrated channel system are given on the right hand side of Tables 3 and 4. In all Tables 3–12, results are obtained under the assumption of symmetric demand. Here we have discussed the effects of the following factors on the optimal solutions separately:

(i) Performance of RS contract.
(ii) Effect of price sensitivity factor.
(iii) Effect of competition factor.

5.1. Performance of revenue sharing contract

In our numerical analysis, emphasis is given on the RS contract. We now turn to supply chain efficiency (Lariviere and Porteus, 2001). To represent the supply chain efficiency in channel performance, we define it in the following two aspects.

First, we will define the efficiency of the decentralized system with respect to the integrated system as $E_f = \frac{\Pi_D^*}{\Pi_I^*}$, where Π_D^* represents the expected profit of the decentralized system and Π_I^* is that for the integrated channel system. Secondly, we define the efficiency of the Stackelberg leader with respect to the expected profit of the decentralized system as $E_{DM} = \frac{2\Pi_M^*}{\Pi_D^*}$ (when manufacturers are Stackelberg leaders) and $E_{DR} = \frac{\Pi_R^*}{\Pi_D^*}$ (when retailer is the Stackelberg leader). By comparing these efficiency factors we can obtain (i) the advantage of the RS contract on the WP contract, and (ii) whether it is beneficial for the Stackelberg leader to offer the RS contract to the follower. When

Table 3
Effect of changing price sensitivity factor β_i on the optimal solutions under WP contract where manufacturers are the Stackelberg leader ($\gamma_1 = \gamma_2 = 70$).

σ	WP contract							Integrated channel					
	w^*	P^*	z^*	Q^*	Π_R^*	Π_M^*	Π_D^*	P_I^*	z_I^*	Q_I^*	Π_I^*	E_f	E_{DM}
$\beta = 95$													
50	3.218	5.609	4.533	64.309	285.37	142.629	570.628	4.500	1.252	88.753	612.465	0.932	0.500
60	3.104	5.552	7.713	68.913	298.964	144.992	588.948	4.500	1.252	88.753	612.471	0.961	0.492
70	3.012	5.506	11.151	73.499	310.180	147.890	605.960	4.500	1.252	88.753	612.475	0.989	0.488
$\beta = 105$													
50	2.525	4.119	4.599	60.419	177.676	92.114	361.904	3.357	1.087	83.587	388.897	0.931	0.509
60	2.450	4.082	7.520	64.649	186.008	93.727	373.462	3.357	1.087	83.587	388.902	0.960	0.502
70	2.389	4.052	10.664	68.852	192.896	95.655	384.201	3.357	1.087	83.587	388.906	0.988	0.498
$\beta = 115$													
50	2.137	3.291	4.733	56.654	119.507	64.405	248.317	2.722	0.961	78.462	266.915	0.930	0.518
60	2.084	3.264	7.434	60.553	125.612	65.615	256.242	2.722	0.961	78.462	266.92	0.960	0.512
70	2.040	3.242	10.329	64.418	129.573	67.025	263.623	2.722	0.961	78.462	266.923	0.988	0.508

$\beta_1 = \beta_2 = \beta, w_1^* = w_2^* = w^*, P_1^* = P_2^* = P^*, z_1^* = z_2^* = z^*, Q_1^* = Q_2^* = Q^*, \Pi_R^*, \Pi_{M_1}^* = \Pi_{M_2}^* = \Pi_M^*$ and $P_{1I}^* = P_{2I}^* = P_I^*, z_{1I}^* = z_{2I}^* = z_I^*, Q_{1I}^* = Q_{2I}^* = Q_I^*, E_f = \frac{\Pi_D^*}{\Pi_I^*}, E_{DM} = \frac{2\Pi_M^*}{\Pi_D^*}$.

Table 4
Effect of changing competitor price sensitivity factor γ_i on the optimal solutions under WP contract where manufacturers are the Stackelberg leader ($\beta_1=\beta_2=115$).

σ	WP contract							Integrated channel					
	w^*	P^*	z^*	Q^*	Π_R^*	Π_M^*	Π_D^*	P_I^*	z_I^*	Q_I^*	Π_I^*	E_f	E_{DM}
$\gamma=75$													
50	2.307	3.653	4.659	58.523	144.529	76.478	297.685	3.000	1.020	81.021	319.969	0.930	0.514
60	2.244	3.622	7.465	62.582	151.458	77.863	307.183	3.000	1.020	81.021	319.974	0.960	0.507
70	2.193	3.597	10.479	66.610	157.025	79.496	316.017	3.000	1.020	81.021	319.978	0.988	0.503
$\gamma=80$													
50	2.525	4.119	4.599	60.419	177.676	92.114	361.904	3.357	1.087	83.587	388.897	0.931	0.509
60	2.450	4.082	7.520	64.649	186.008	93.727	373.462	3.357	1.087	83.587	388.902	0.960	0.502
70	2.389	4.052	10.664	68.852	192.896	95.655	384.201	3.357	1.087	83.587	388.906	0.988	0.498
$\gamma=85$													
50	2.814	4.740	4.557	62.347	222.230	113.095	448.420	3.833	1.163	86.163	481.634	0.931	0.504
60	2.723	4.695	7.601	66.758	232.734	115.018	462.770	3.833	1.163	86.163	481.639	0.961	0.497
70	2.649	4.658	10.886	71.146	241.411	117.345	476.101	3.833	1.163	86.163	481.643	0.988	0.493

$\gamma_1=\gamma_2=\gamma, w_1^*=w_2^*=w^*, P_1^*=P_2^*=P^*, z_1^*=z_2^*=z^*, Q_1^*=Q_2^*=Q^*, \Pi_R^*, \Pi_{M_1}^*=\Pi_{M_2}^*=\Pi_M^*$ and $P_{1I}^*=P_{2I}^*=P_I^*, z_{1I}^*=z_{2I}^*=z_I^*, Q_{1I}^*=Q_{2I}^*=Q_I^*, E_f=\frac{\Pi_D^*}{\Pi_I^*}, E_{DM}=\frac{2\Pi_M^*}{\Pi_D^*}$.

Table 5
Effect of changing price sensitivity factor β_i on the optimal solutions when both manufacturers offer RS contract ($\gamma_1=\gamma_2=70$).

σ	w_R^*	ϕ^*	P_R^*	z_R^*	Q_R^*	Π_R^*	$\Pi_{M_R}^*$	Π_{DR}^*	E_f	E_{DM}
$\beta=95$										
50	0.683166	0.683166	4.5	64.62	152.120	417.556	76.761	571.079	0.932427	0.268829
60	0.699429	0.699429	4.5	77.257	164.757	427.499	69.022	565.544	0.923381	0.244092
70	0.712551	0.712551	4.5	89.409	176.909	435.523	62.515	560.553	0.915226	0.223048
$\beta=105$										
50	0.676603	0.676603	3.357	57.249	139.749	262.275	44.560	351.396	0.90357	0.253619
60	0.692470	0.692470	3.357	68.373	150.873	268.532	38.952	346.437	0.890807	0.224874
70	0.705303	0.705303	3.357	79.113	161.613	273.513	34.161	341.835	0.878975	0.199869
$\beta=115$										
50	0.669960	0.669960	2.722	51.897	129.397	178.147	27.095	232.336	0.870449	0.233235
60	0.685405	0.685405	2.722	61.908	139.408	182.257	22.676	227.609	0.852725	0.199255
70	0.697930	0.697930	2.722	71.590	149.091	185.592	18.848	223.288	0.836524	0.168821

$\beta_1=\beta_2=\beta, w_{1R}^*=w_{2R}^*=w_R^*, \phi_1^*=\phi_2^*=\phi^*, P_{1R}^*=P_{2R}^*=P_R^*, z_{1R}^*=z_{2R}^*=z_R^*, Q_{1R}^*=Q_{2R}^*=Q_R^*, \Pi_{M_{1R}}^*=\Pi_{M_{2R}}^*=\Pi_{MR}^*, E_f=\frac{\Pi_{DR}^*}{\Pi_I^*}, E_{DM}=\frac{2\Pi_{MR}^*}{\Pi_{DR}^*}$.

Table 6
Effect of changing competitor price sensitivity factor γ_i on the optimal solutions when both manufacturers offer RS contract ($\beta_1=\beta_2=115$).

σ	w_R^*	ϕ^*	P_R^*	z_R^*	Q_R^*	Π_R^*	$\Pi_{M_R}^*$	Π_{DR}^*	E_f	E_{DM}
$\gamma=75$										
50	0.673295	0.673295	3	54.381	134.381	214.722	34.684	284.090	0.887866	0.244175
60	0.688955	0.688955	3	64.911	144.911	219.720	29.745	279.211	0.872605	0.213068
70	0.701640	0.701640	3	75.087	155.087	223.769	25.495	274.759	0.858682	0.185582
$\gamma=80$										
50	0.676603	0.676603	3.357	57.249	139.749	262.275	44.560	351.396	0.90357	0.253619
60	0.692470	0.692470	3.357	68.373	150.873	268.532	38.952	346.437	0.890807	0.224874
70	0.705303	0.705303	3.357	79.113	161.613	273.513	34.161	341.835	0.878975	0.199869
$\gamma=85$										
50	0.679892	0.679892	3.833	60.607	145.607	326.657	57.886	442.429	0.91860	0.261673
60	0.695958	0.695958	3.833	72.423	157.423	334.379	51.387	437.154	0.907638	0.23510
70	0.708939	0.708939	3.833	83.813	168.813	340.620	45.878	432.375	0.897709	0.212212

$\gamma_1=\gamma_2=\gamma, w_{1R}^*=w_{2R}^*=w_R^*, \phi_1^*=\phi_2^*=\phi^*, P_{1R}^*=P_{2R}^*=P_R^*, z_{1R}^*=z_{2R}^*=z_R^*, Q_{1R}^*=Q_{2R}^*=Q_R^*, \Pi_{M_{1R}}^*=\Pi_{M_{2R}}^*=\Pi_{MR}^*, E_f=\frac{\Pi_{DR}^*}{\Pi_I^*}, E_{DM}=\frac{2\Pi_{MR}^*}{\Pi_{DR}^*}$.

manufacturers are the Stackelberg leaders then the supply chain efficiency factors have the following relations depending on different contracts: $E_f^{WPRS} > E_f^{WP} > E_f^{RS}$ and $E_{DM}^{WP} > E_{DM}^{WPRS} > E_{DM}^{RS}$ (Tables 3–8). From the first relation we observe that when manufactures are the Stackelberg leaders then channel efficiency E_f is higher in WP contract (E_f^{WP}) than RS contract E_f^{RS}. Moreover, E_f is the highest when one manufacturer offers WP contract and the other offers RS contract (E_f^{WPRS}) to the retailer. Again, from the second relation we notice that in this case the Stackelberg leaders manufacturers' share (E_{DM}) is the highest in WP contract. Hence, it implies that when two competitive manufacturers are Stackelberg leaders, the RS contract is not beneficial though it increases the channel efficiency E_f. Thus, from our results, we can conclude that *in the Stackelberg game when manufacturers are competitive in nature and they are the Stackelberg leaders, then it is always profitable for them to offer WP contract.*

On the other hand, if retailer is the Stackelberg leader then supply chain efficiency factors possess the following relations (Tables 9–12): $E_f^{RS} > E_f^{WP}$ and $E_{DR}^{RS} > E_{DR}^{WP}$. From the first relation

Table 7
Effect of changing price sensitivity factor β_i on the optimal solutions when one manufacturer offers RS contract while the other offers WP contract ($\gamma_1=\gamma_2=70$).

σ	w_i^*	P_i^*	z_i^*	Q_i^*	Π_R^*	$\Pi_{M_i}^*$	Π_D^*	E_f	E_{DM}
$\beta=95$									
50	2.643	5.028	11.883	65.327	380.240	107.354	594.531	0.970718	0.360437
	0.663	4.731	66.041	168.626		106.937			
60	2.588	5.011	15.671	70.187	387.434	111.481	597.267	0.975175	0.351321
	0.674	4.723	78.965	181.063		98.351			
70	2.528	4.997	19.742	75.031	394.514	115.43	600.420	0.980317	0.342936
	0.686	4.714	91.328	193.225		90.4758			
$\beta=105$									
50	2.185	3.762	10.087	60.856	234.572	72.133	369.026	0.948904	0.3604348
	0.653	3.511	58.693	153.356		62.321			
60	2.493	3.749	13.279	65.165	238.394	74.894	369.608	0.950389	0.355009
	0.661	3.509	70.187	164.247		56.320			
70	2.114	3.739	16.818	69.497	242.894	77.417	370.802	0.953448	0.344949
	0.673	3.503	81.171	175.016		50.490			
$\beta=115$									
50	1.914	3.048	8.970	56.742	156.938	51.881	247.164	0.926002	0.365045
	0.644	2.833	53.355	140.911		38.345			
60	1.887	3.038	11.841	60.655	159.567	53.781	246.805	0.924641	0.353470
	0.652	2.832	63.768	150.806		33.753			
70	1.858	3.029	15.123	64.6223	163.219	55.437	247.281	0.0.921412	0.339944
	0.666	2.826	73.629	160.631		28.624			

$\beta_1=\beta_2=\beta, E_f=\frac{\Pi_D^*}{\Pi_I^*}, E_{DM}=\frac{2\Pi_M^*}{\Pi_D^*}$ and corresponding to each β first row and second row represent the optimal results corresponding to the manufacturer who offers wholesale price contract and one who offers RS contract, respectively.

Table 8
Effect of changing competitor price sensitivity factor γ_i on the optimal solutions when one manufacturer offers RS contract while the other offers WP contract ($\beta_1=\beta_2=115$).

σ	w_i^*	P_i^*	z_i^*	Q_i^*	Π_R^*	$\Pi_{M_i}^*$	Π_D^*	E_f	E_{DM}
$\gamma=75$									
50	2.025	3.354	9.677	58.776	191.074	60.278	300.413	0.93888	0.363961
	0.649	3.131	55.837	147.322		49.061			
60	1.995	3.344	12.725	62.890	194.229	62.559	300.448	0.938978	0.353536
	0.657	3.129	66.751	157.680		43.660			
70	1.966	3.334	16.038	67.001	197.636	64.714	300.926	0.940459	0.343241
	0.667	3.125	77.210	167.869		38.577			
$\gamma=80$									
50	2.166	3.746	10.484	60.873	235.590	70.980	369.795	0.950881	0.362917
	0.653	3.514	58.720	154.229		63.224			
60	2.129	3.734	13.825	65.245	239.999	73.635	370.542	0.952791	0.352304
	0.663	3.509	70.173	165.212		56.908			
70	2.094	3.723	17.439	69.607	244.515	76.156	371.727	0.955828	0.342219
	0.675	3.504	81.149	176.004		51.056			
$\gamma=85$									
50	2.335	4.262	11.792	63.206	298.750	84.404	464.314	0.96404	0.356578
	0.663	4.082	61.976	162.153		81.160			
60	2.296	4.250	15.321	67.799	303.437	87.878	465.638	0.966777	0.348341
	0.672	4.015	74.125	173.703		74.322			
70	2.257	4.239	19.186	72.407	308.703	91.054	467.524	0.970686	0.339707
	0.683	4.008	85.747	185.107		67.766			

$\gamma_1=\gamma_2=\gamma, E_f=\frac{\Pi_D^*}{\Pi_I^*}, E_{DM}=\frac{2\Pi_M^*}{\Pi_D^*}$ and corresponding to each β first row and second row represent the optimal results corresponding to the manufacturer who offers wholesale price contract and one who offers RS contract, respectively.

Table 9
Effect of changing price sensitivity factor β_i on the optimal solutions under WP contract where the common retailer is the Stackelberg leader ($\gamma_1=\gamma_2=70$).

σ	w^*	P^*	z^*	Q^*	Π_R^*	Π_M^*	Π_D^*	E_f	E_{DR}
$\beta=95$									
0.1	1.73395	5.239	0.708	69.725	450.913	51.175	553.263	0.929543	0.815007
1	1.73320	5.228	0.361	69.653	480.804	51.070	582.944	0.954443	0.824786
1.5	1.73168	5.226	0.189	69.509	482.034	50.649	583.751	0.954862	0.825753
$\beta=105$									
0.1	1.59322	3.952	0.596	62.288	263.696	36.950	337.597	0.905116	0.781098
1	1.51209	3.945	0.249	62.170	288.453	36.810	362.073	0.934782	0.796671
1.5	1.59907	3.944	0.072	62.023	289.471	37.156	363.783	0.939123	0.795723
$\beta=115$									
0.1	1.48786	3.209	0.522	56.104	167.827	27.371	222.569	0.883169	0.754044
1	1.48651	3.20526	0.186	55.949	189.157	27.220	243.597	0.917674	0.776517
1.5	1.48527	3.204	0.010	55.805	190.028	27.081	244.189	0.918185	0.778199

$\beta_1=\beta_2=\beta, w_1^*=w_2^*=w^*, z_1^*=z_2^*=z^*, Q_1^*=Q_2^*=Q^*, P_1^*=P_2^*=P^*, E_f=\frac{\Pi_D^*}{\Pi_I^*}, E_{DR}=\frac{\Pi_R^*}{\Pi_D^*}$.

Table 10
Effect of changing price sensitivity factor γ_i on the optimal solutions under WP contract where the common retailer is the Stackelberg leader ($\beta_1=\beta_2=115$).

σ	w^*	P^*	z^*	Q^*	Π_R^*	Π_M^*	Π_D^*	E_f	E_{DR}
$\gamma=75$									
0.1	1.51981	3.520	0.567	59.778	211.856	30.073	272.002	0.892993	0.778876
1	1.51857	3.515	0.232	59.636	234.496	30.925	296.347	0.930565	0.791289
1.5	1.51733	3.514	0.059	59.492	235.420	30.777	296.975	0.931035	0.814708
$\gamma=80$									
0.1	1.55392	3.912	0.620	63.700	270.838	35.285	341.408	0.880224	0.793297
1	1.55283	3.906	0.289	63.575	294.999	35.146	365.292	0.943093	0.807571
1.5	1.55160	3.005	0.120	63.434	295.984	34.990	365.965	0.943536	0.808778
$\gamma=85$									
0.1	1.59045	4.426	0.684	67.901	352.563	40.092	432.748	0.930436	0.814708
1	1.58953	4.419	0.359	67.796	378.522	39.968	458.458	0.955100	0.825642
1.5	1.58833	4.418	0.1967	67.658	379.576	39.805	459.187	0.95552	0.826627

$\gamma_1=\gamma_2=\gamma$, $w_1^*=w_2^*=w^*$, $z_1^*=z_2^*=z^*$, $Q_1^*=Q_2^*=Q^*$, $P_1^*=P_2^*=P^*$, $E_f=\frac{\Pi_D^*}{\Pi_I^*}$, $E_{DR}=\frac{\Pi_R^*}{\Pi_D^*}$.

Table 11
Effect of changing price sensitivity factor β_i on the optimal solutions when the common retailer offer RS contract with ($\gamma_1=\gamma_2=70$) at $\sigma=1$.

β	w_R^*	ϕ^*	P_R^*	z_R^*	Q_R^*	Π_R^*	$\Pi_{M_R}^*$	Π_{DR}^*	E_f	E_{DR}
95	0.99896	0.99896	4.499	1.150	88.681	610.125	0.318	610.760	0.999986	0.998960
105	0.99883	0.99883	3.356	0.934	83.481	386.863	0.226	387.315	0.999950	0.998834
115	0.99882	0.99882	2.721	0.755	78.320	265.104	0.156	265.104	0.997069	0.998821

$\beta_1=\beta_2=\beta, w_{1R}^*=w_{2R}^*=w_R^*, \phi_1^*=\phi_2^*=\phi^*, P_{1R}^*=P_{2R}^*=P_R^*, z_{1R}^*=z_{2R}^*=z_R^*, Q_{1R}^*=Q_{2R}^*=Q_R^*, \Pi_{M_{1R}}^*=\Pi_{M_{2R}}^*=\Pi_{MR}^*, E_f=\frac{\Pi_{DR}^*}{\Pi_I^*}, E_{DR}=\frac{\Pi_R^*}{\Pi_{DR}^*}$.

Table 12
Effect of changing competitor price sensitivity factor γ_i on the optimal solutions when the common retailer offer RS contract with $\beta_1=\beta_2=115$ at $\sigma=1$.

γ	w_R^*	ϕ^*	P_R^*	z_R^*	Q_R^*	Π_R^*	$\Pi_{M_R}^*$	Π_{DR}^*	E_f	E_{DR}
75	0.99897	0.99897	2.998	0.841	80.896	318.104	0.164	318.432	0.999917	0.998968
80	0.99893	0.99893	3.356	0.934	83.481	386.899	0.108	387.314	0.999949	0.998984
85	0.99998	0.99998	3.832	1.036	86.067	480.009	0.0005	480.010	0.999995	0.999998

$\gamma_1=\gamma_2=\gamma, w_{1R}^*=w_{2R}^*=w_R^*, \phi_1^*=\phi_2^*=\phi^*, P_{1R}^*=P_{2R}^*=P_R^*, z_{1R}^*=z_{2R}^*=z_R^*, Q_{1R}^*=Q_{2R}^*=Q_R^*, \Pi_{M_{1R}}^*=\Pi_{M_{2R}}^*=\Pi_{MR}^*, E_f=\frac{\Pi_{DR}^*}{\Pi_I^*}, E_{DR}=\frac{\Pi_R^*}{\Pi_{DR}^*}$.

Table 13
Effect of important parameters on the supply chain efficiency.

Effect of	Manufacturer–Stackelberg leader	Retailer–Stackelberg leader
WP and RS contract	Supply chain efficiency factors E_f and E_{DM} both are higher in WP contract than that of the RS contract	E_f and E_{DR} both are higher in RS contract than that of the WP contract
Price sensitivity parameter	With an increase of β_i E_f decreases in both contract. Again, for the same case E_{DM} increases in WP contract while it decreases in RS contract	With an increase of β_i efficiency factors E_f and E_{DR} both decrease
Competitive factor	With an increase of γ_i E_f increases but E_{DM} decreases in both contracts	With an increase of γ_i E_f and E_{DR} both moves in the same direction of γ_i for both contracts
σ	With an increase of σ E_f increases but E_{DM} decreases in WP contract. But those both decrease in RS contract	With an increase of σ E_f and E_{DR} both increase in WP contract. On the other hand, the changes of E_f and E_{DR} are almost negligible in RS contract

we can observe that the channel efficiency is higher in case of RS contract than WP contract when retailer is the Stackelberg leader. Consequently, in that case the expected profit of the decentralized system is always higher in RS contract than WP contract. Hence, *it is always profitable to offer RS contract on the aspect of decentralized channel performance with respect to integrated channel performance highly supports the results of existing literature* (Wang et al., 2004; Cachon and Lariviere, 2005; Yao et al., 2008, etc.). Again, from the column E_{DR} of Tables 9–12, we observe that the proportion of the retailer's profit is higher in RS contract than the WP contract. But in case of RS contract manufacturer's share of revenue becomes very small compared to WP contract though expected profit of decentralized system increases significantly in RS contract. Hence, we can conclude that *when retailer is the Stackelberg leader then retailer will always be encouraged to offer RS contract to the manufacturers but from manufacturers' point of view it is not beneficial to accept the RS contract offered by the retailer*.

5.2. Effect of price sensitivity factor

We consider two cases: (1) when manufactures are Stackelberg leaders and (2) when retailer is the Stackelberg leader, separately.

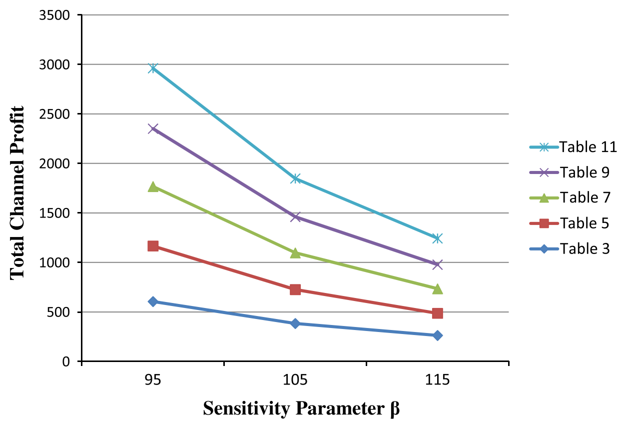

Fig. 3. Variation of decentralized expected channel profit with respect to the sensitivity parameters (total expected costs are corresponding to the tables as indicated in the figure).

When manufacturers are Stackelberg leaders, with the increase of price sensitivity factor of demand (β_i), the channel efficiency E_f decreases for both contracts. Also with the increase of β_i, E_{DM} increases in WP contract while in RS contract E_{DM} decreases with the increase of β_i. This implies that *higher the price sensitivity of the product, it is beneficial for the manufacturers to offer WP contract*. Further, as β_i increases, retail price P_i decreases in both the contracts and consequently Q_i decreases.

On the other hand, for case 2 (similar to the case 1), the higher the price sensitivity of demand, the lower the channel efficiency would be in both contracts. It is to be noted that for RS contract, the channel trends to obtain perfect coordination ($E_f = 100\%$) (Tables 9–12) and is called perfect coordination (Cachon and Lariviere, 2005). Further, E_{DR} moves opposite direction of β_i in both contracts. If we compare Tables 9 and 11, we can observe a significant increase in retailer's expected profit (Π_R^*) in RS contract compared to WP contract. Hence, *it is always profitable for the retailer to offer RS contract*. Moreover, with the increase of the price sensitivity of demand, manufacturers' share $(1-\phi_i)$ increases implying it increases the chances of acceptance of RS contract of manufacturers offered by the retailer. But the rate of increase is very small and manufacturers' profit decreases significantly in RS contract as compared to WP contract. *This is the reason why manufacturers will not be encouraged to accept the RS contract offered by retailer*.

Fig. 3 shows the total channel profit variation with respect to the sensitivity parameters. We can observe that for both the contracts, total channel profit decreases with the increase of β_i. This can be interpreted as follows. With the increase of sensitivity factor β_i, demand decreases and hence retail price increases in order to increase the demand. All these factors decrease the total channel profit.

5.3. Effect of competitive factor

For cases 1 and 2, with the increase in competition factor γ_i, the channel efficiency increases for both contracts. This implies that *the competition between two manufacturers can improve the channel efficiency*. Again while E_{DM} decreases with the increase in competition factor, E_{DR} moves in the same direction of γ_i. This implies that for case 2 i.e., *when retailer is the Stackelberg leader then competition between the manufacturers can improve the efficiency factor E_{DR}*.

6. Conclusions

In this paper, we have studied a single channel duopoly market consisting of two competitive manufacturers who sell their products through a common retailer. Demand faced by the retailer is stochastic in nature and dependent on the retail prices charged by the retailer. Basic model is developed based on WP contract. In order to investigate the channel inefficiency, we have considered an integrated system which is controlled by single decision maker. In this market scenario, a single manufacturer produces two different types (brand) of products and sells them through its own retail channel. In order to enable the supply chain coordination, we have considered a salvage RS contract. We have analyzed the characteristics of the system both analytically and numerically in a Nash game (between manufacturers) and a Stackelberg game (between retailer and each manufacturer).

The following is the summary of contributions of this paper. Analytically, we have proved that (1) there exists an optimal stock level that will maximize the expected profit of the common retailer, (2) there exists at least one Nash equilibrium between two competitive manufacturers, (3) there exists a Stackelberg game between each manufacturer and the retailer where retailer is the Stackelberg leader. Through numerical experimentation, we observe the following: (1) when manufacturers are the Stackelberg leader and two manufacturers are competitive in nature, then it is beneficial for the manufacturers to offer WP contract though RS contract improves the channel performance. (2) When retailer is the Stackelberg leader then it is beneficial for retailer to offer salvage RS contract to improve its profit as well as to improve the channel coordinations. (3) With the increase of the price sensitivity of demand β, supply chain efficiency factors always decrease. This implies that less price sensitivity of demand can improve the supply chain efficiency. (4) With the increase in the competition factor γ_i, the channel efficiency increases for both the contracts. This implies that the competition between two manufacturers can improve the channel efficiency.

Our work can be extended in a number of ways. We considered a single channel scenario, this model can be extended by considering dual channel. In our model, we have taken only upstream competition scenario, the consideration of both stream competitions can be another important extension of this model.

Acknowledgements

We would like to thank the editor and the reviewer of this paper for their valuable and constructive comments. This work was done when the first author visited Concordia University as Post doctoral fellow. The authors are grateful for the generous support provided for this research by Natural Sciences and Engineering Research Council of Canada (NSERC) under grant numbers 1092 and 386501.

Appendix A. Proof of Proposition 1

To prove proposition 1, it is sufficient to prove that the retailer's expected profit function is concave with respect to decision variables z_1 and z_2. Again $E[\Pi_R(z_1, z_1)]$ will be concave if the principal minors of the hessian matrix is alternatively $(-)ve$, $(+)ve$ and $(-)ve$ in order *i.e.*, $\frac{\partial^2 E[\Pi_R(z_1,z_2)]}{\partial z_1^2} < 0$ and $\frac{\partial^2 E[\Pi_R(z_1,z_2)]}{\partial z_1^2}\frac{\partial^2 E[\Pi_R(z_1,z_2)]}{\partial z_2^2} - \left[\frac{\partial^2 E[\Pi_R(z_1,z_2)]}{\partial z_1 \partial z_2}\right]^2 > 0$. Differentiating Eq. (4) w.r.t. z_1 gives

$$\frac{\partial E[\Pi_R(z_1,z_2)]}{\partial z_1} = \frac{\partial P_1^*(z_1,z_2)}{\partial z_1}(\alpha_1 - \beta_1 P_1^*(z_1,z_2) + \gamma_1 P_2^*(z_1,z_2) + \mu_1) + (P_1^*(z_1,z_2) - w_1) \times \left(-\beta_1 \frac{\partial P_1^*(z_1,z_2)}{\partial z_1} + \gamma_1 \frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\right)$$

$$-(w_1-v_1)\frac{\partial\Lambda(z_1)}{\partial z_1}-\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\Theta(z_1)$$
$$-\left(P_1^*(z_1,z_2)-w_1+s_1\right)\frac{\partial\Theta(z_1)}{\partial z_1}+\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}$$
$$\times\left(\alpha_2-\beta_2P_2^*(z_1,z_2)+\gamma_2P_1^*(z_1,z_2)+\mu_2\right)$$
$$+\left(P_2^*(z_1,z_2)-w_2\right)$$
$$\times\left(-\beta_2\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}+\gamma_2\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\right)-\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\Theta(z_2).$$

Differentiating Eq. (4) w.r.t. z_1, we get

$$\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_1^2}=\frac{\partial^2P_1^*(z_1,z_2)}{\partial z_1^2}(\alpha_1-\beta_1P_1^*(z_1,z_2)$$
$$+\gamma_1P_2^*(z_1,z_2)+\mu_1)+2\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}$$
$$\times\left(-\beta_1\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}+\gamma_1\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\right)+\left(P_1^*(z_1,z_2)-w_1\right)$$
$$\times\left(-\beta_1\frac{\partial^2P_1^*(z_1,z_2)}{\partial z_1^2}+\gamma_1\frac{\partial^2P_2^*(z_1,z_2)}{\partial z_1^2}\right)$$
$$-(w_1-v_1)\frac{\partial^2\Lambda(z_1)}{\partial z_1^2}-\frac{\partial^2P_1^*(z_1,z_2)}{\partial z_1^2}\Theta(z_1)$$
$$-2\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\frac{\partial\Theta(z_1)}{\partial z_1}-(P_1^*(z_1,z_2)-w_1$$
$$+s_1)\frac{\partial^2\Theta(z_1)}{\partial z_1^2}+\frac{\partial^2P_2^*(z_1,z_2)}{\partial z_1^2}$$
$$\times\left(\alpha_2-\beta_2P_2^*(z_1,z_2)+\gamma_2P_1^*(z_1,z_2)+\mu_2\right)+2\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}$$
$$\times\left(-\beta_2\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}+\gamma_2\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\right)+\left(P_2^*(z_1,z_2)-w_2\right)$$
$$\left(-\beta_2\frac{\partial^2P_2^*(z_1,z_2)}{\partial z_1^2}+\gamma_2\frac{\partial^2P_1^*(z_1,z_2)}{\partial z_1^2}\right)$$
$$-\frac{\partial^2P_2^*(z_1,z_2)}{\partial z_1^2}\Theta(z_2)-\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\frac{\partial\Theta(z_2)}{\partial z_1}.$$

Now we have the following results:

$$\frac{\partial\Lambda(z_1)}{\partial z_1}=F_1(z_1),\quad\frac{\partial^2\Lambda(z_1)}{\partial z_1^2}=f_1(z_1),\quad\frac{\partial\Theta(z_1)}{\partial z_1}$$
$$=-[1-F_1(z_1)],\quad\frac{\partial^2\Theta(z_1)}{\partial z_1^2}=f_1(z_1),$$
$$\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}=\frac{2\beta_2}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}(1-F_1(z_1)),$$
$$\frac{\partial P_1^*(z_1,z_2)}{\partial z_2}=\frac{\gamma_1+\gamma_2}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}(1-F_2(z_2)),$$
$$\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}=\frac{\gamma_1+\gamma_2}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}(1-F_1(z_1)),$$
$$\frac{\partial P_2^*(z_1,z_2)}{\partial z_2}=\frac{2\beta_1}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}(1-F_2(z_2)),$$
$$\frac{\partial^2P_1^*(z_1,z_2)}{\partial z_1\partial z_2}=0=\frac{\partial^2P_2^*(z_1,z_2)}{\partial z_1\partial z_2},\quad\frac{\partial^2P_1^*(z_1,z_2)}{\partial z_1^2}$$
$$=-\left(\frac{2\beta_2}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}\right)f_1(z_1),\quad\frac{\partial^2P_1^*(z_1,z_2)}{\partial z_2^2}$$
$$=-\left(\frac{\gamma_1+\gamma_2}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}\right)f_2(z_2),$$
$$\frac{\partial^2P_2^*(z_1,z_2)}{\partial z_1^2}=-\left(\frac{\gamma_1+\gamma_2}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}\right)f_1(z_1),\quad\frac{\partial^2P_2^*(z_1,z_2)}{\partial z_2^2}$$
$$=-\left(\frac{2\beta_1}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}\right)f_2(z_2).$$

Using the above results and after simplification, we get

$$\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_1^2}=-\left(\frac{2\beta_2f_1(z_1)}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}\right)(\alpha_1-\beta_1P_1^*(z_1,z_2)$$
$$+\gamma_1P_2^*(z_1,z_2)+\int_{A_1}^{z_1}\epsilon_1f_1(\epsilon_1)\,d\epsilon_1)$$
$$-\left(\frac{2\beta_2}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}\right)(1-F_1(z_1))^2\left[z_1f_1(z_1)-1\right]$$
$$-\frac{1}{2}(P_1^*(z_1,z_2)-w_1)f_1(z_1)$$
$$-s_1f_1(z_1)+\frac{(P_1^*(z_1,z_2)-w_1)(\gamma_2-\gamma_1)(\gamma_1+\gamma_2)f_1(z_1)}{2\left(4\beta_1\beta2-(\gamma_1+\gamma_2)^2\right)}-(w_1-v_1)f_1(z_1)$$
$$-\frac{(\gamma_1+\gamma_2)f_1(z_1)}{4\beta_1\beta2-(\gamma_1+\gamma_2)^2}\Bigg(\alpha_2-\beta_2P_2^*(z_1,z_2)+\gamma_2P_1^*(z_1,z_2)$$
$$+\int_{A_2}^{z_2}\epsilon_2f_2(\epsilon_2)\,d\epsilon_2$$
$$+\int_{z_2}^{B_2}z_2f_2(\epsilon_2)\,d\epsilon_2\Bigg)+\frac{(P_2^*(z_1,z_2)-w_2)\beta_2(\gamma_1-\gamma_2)}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}f_1(z_1)$$
<0 under assumptions $A1, A2$ and $A3$.

Similarly, it can be shown that $\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_2^2}<0$ under the same assumptions. Again

$$\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_1\partial z_2}=\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\left(-\beta_1\frac{\partial P_1^*(z_1,z_2)}{\partial z_2}+\gamma_1\frac{\partial P_2^*(z_1,z_2)}{\partial z_2}\right)$$
$$+\frac{\partial P_1^*(z_1,z_2)}{\partial z_2}\left(-\beta_1\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}+\gamma_1\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\right)$$
$$-\frac{\partial P_1^*(z_1,z_2)}{\partial z_2}\frac{\partial\Theta(z_1)}{\partial z_1}$$
$$+\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\left(-\beta_2\frac{\partial P_2^*(z_1,z_2)}{\partial z_2}+\gamma_2\frac{\partial P_1^*(z_1,z_2)}{\partial z_2}\right)$$
$$+\frac{\partial P_2^*(z_1,z_2)}{\partial z_2}\left(-\beta_2\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}+\gamma_2\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\right)$$
$$-\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\frac{\partial\Theta(z_2)}{\partial z_2},$$
$$\text{since}\frac{\partial^2P_1^*(z_1,z_2)}{\partial z_1\partial z_2}=0=\frac{\partial^2P_2^*(z_1,z_2)}{\partial z_1\partial z_2}.$$

Using partial derivatives and after some simplification, we get

$$\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_1\partial z_2}=\frac{(\gamma_1+\gamma_2)(1-F_1(z_1,))(1-F_2(z_2))}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}.$$

Now

$$\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_1^2}\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_2^2}$$
$$>\frac{2\beta_2(1-F_1(z_1))^2}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}\frac{2\beta_1(1-F_2(z_2))^2}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}$$
$$\times[z_1r_1(z_1)-1)][z_2r_2(z_2)-1)]$$
$$>\frac{4\beta_1\beta_2(1-F_1(z_1))^2(1-F_2(z_2))^2}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}^2},\ \text{under assumption }A1.$$

Using these results, we get

$$\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_1^2}\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_2^2}-\left[\frac{\partial^2E[\Pi_R(z_1,z_2)]}{\partial z_1\partial z_2}\right]^2$$
$$>\frac{(1-F_1(z_1))^2(1-F_2(z_2))^2}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}$$
>0, under assumptions $A1$ and **A2**.

Thus we get $\frac{\partial^2 E[\Pi_R(z_1,z_2)]}{\partial z_1^2} < 0$ and

$$\frac{\partial^2 E[\Pi_R(z_1,z_2)]}{\partial z_1^2}\frac{\partial^2 E[\Pi_R(z_1,z_2)]}{\partial z_2^2} - \left[\frac{\partial^2 E[\Pi_R(z_1,z_2)]}{\partial z_1 \partial z_2}\right]^2 > 0,$$

which implies the concavity of the retailer expected profit function $E[\Pi_R(z_1,z_2)]$. This completes the proof of the proposition.

Appendix B

B.1. Proof of Proposition 2

Since the profit functions are continuous and twice differentiable, to show the supermodular game between competitive manufacturers, we have to just show that $\frac{\partial^2 E\left[\Pi_{M_1}\right]}{\partial w_1 \partial w_2} \geq 0$ and $\frac{\partial^2 E\left[\Pi_{M_2}\right]}{\partial w_1 \partial w_2} \geq 0$ for $i=1,2$. We will prove the result for $i=1$, *i.e.*, for the first manufacturer. The proof is similar for $i=2$. Using the values $P_1^*(z_1,z_2)$ and $P_2^*(z_1,z_2)$ from (13) and (14), we get the expected profit of the manufacturer M_1 as

$$\Pi_{M_1} = (w_1 - c_1)\left(\alpha_1 - \beta_1 P_2^*(z_1,z_2) + \gamma_1 P_2^*(z_1,z_2)\right).$$

Now, differentiating it partially w.r.t. w_1, we get

$$\frac{\partial \Pi_{M_1}}{\partial w_1} = \left[\alpha_1 - \beta_1 P_1^*(z_1,z_2) + \gamma_1 P_2^*(z_1,z_2) + z_1\right] + (w_1 - c_1)\left[-\beta_1 \frac{\partial P_1^*(z_1,z_2)}{\partial w_1} + \gamma_1 \frac{\partial P_2^*(z_1,z)}{\partial w_1}\right].$$

Differentiating it partially w.r.t. w_2, we get

$$\frac{\partial^2 \Pi_{M_1}}{\partial w_2 \partial w_1} = \left(-\beta_1 \frac{\partial P_1^*(z_1,z_2)}{\partial w_2} + \gamma_1 \frac{\partial P_2^*(z_1,z_2)}{\partial w_2}\right) + (w_1 - c_1)\left(-\beta_1 \frac{\partial^2 P_1^*(z_1,z_2)}{\partial w_2 \partial w_1} + \gamma_1 \frac{\partial^2 P_2^*(z_1,z_2)}{\partial w_2 \partial w_1}\right).$$

Using

$$\frac{\partial P_1^*(z_1,z_2)}{\partial w_1} = \frac{2\beta_1\beta_2 - \gamma_1(\gamma_1+\gamma_2)}{4\beta_1\beta_2 - (\gamma_1+\gamma_2)^2}, \quad \frac{\partial P_2^*(z_1,z_2)}{\partial w_1} = -\frac{(\gamma_1-\gamma_2)\beta_1}{4\beta_1\beta_2 - (\gamma_1+\gamma_2)^2}$$

and the values of other partial derivatives (which were mentioned in the Appendix 1, we get

$$\frac{\partial^2 \Pi_{M_1}}{\partial w_2 \partial w_1} = \frac{(\gamma_1+\gamma_2)(\beta_1\beta_2 - \gamma_1\gamma_2)}{4\beta_1\beta_2 - (\gamma_1+\gamma_2)^2} > 0 \text{ by the assumption } A2.$$

The proof is similar for the second manufacturer. This implies that the game between the two manufacturers is a supermodular game and hence there exists at least one Nash equilibrium between two manufacturers. This completes the proof of the proposition.

Appendix C

C.1. Proof of Proposition 3

To establish the existence of the Stackelberg game, where retailer is the Stackelberg leader, it is sufficient to show that the expected profit of the retailer $E_R^s(z_1,z_2)$ is a quasiconcave function with respect to z_1 and z_2. Let $D_2(z_1,z_2)$ denotes the determinant of the Hessian matrix of the above profit function. The necessary and sufficient conditions for quasi concavity are $D_1(z_1,z_2) \leq 0, D_2(z_1,z_2) \geq 0$ and $D_1(z_1,z_2) < 0, D_2(z_1,z_2) > 0$, respectively. Now, $D_1(z_1,z_2) = -\left(\frac{\partial E\left[\Pi_R^{RS}\right]}{\partial z_1}\right) < 0$. This is automatically satisfied. Again

$$D_2(z_1,z_2) = \frac{\partial E\left[\Pi_R^s\right]}{\partial z_1}\left[\frac{\partial^2 E\left[\Pi_R^s\right]}{\partial z_1 \partial z_2}\frac{\partial E\left[\Pi_R^s\right]}{\partial z_2} - \frac{\partial E\left[\Pi_R^s\right]}{\partial z_1}\frac{\partial^2 E\left[\Pi_R^s\right]}{\partial z_2^2}\right] + \frac{\partial E\left[\Pi_R^s\right]}{\partial z_2}\left[\frac{\partial^2 E\left[\Pi_R^{RS}\right]}{\partial z_1 \partial z_2}\frac{\partial E\left[\Pi_R^s\right]}{\partial z_1} - \frac{\partial E\left[\Pi_R^s\right]}{\partial z_2}\frac{\partial^2 E\left[\Pi_R^s\right]}{\partial z_1^2}\right].$$

It is to be noted that due to symmetric nature of the customer demand function, faced by the retailer, $\frac{\partial E[\Pi_R^s]}{\partial z_1}$ and $\frac{\partial E[\Pi_R^s]}{\partial z_2}$ must be of the same sign. So, $\frac{\partial E[\Pi_R^s]}{\partial z_1}\frac{\partial E[\Pi_R^s]}{\partial z_2}$ must be greater than zero and hence to show $D_2(z_1,z_2) > 0$, it is sufficient to show that $\frac{\partial^2 E[\Pi_R^s]}{\partial z_1 \partial z_2} > 0$ and $\frac{\partial^2 E\left[\Pi_R^{RS}\right]}{\partial z_1^2} < 0$, $\frac{\partial^2 E[\Pi_R^s]}{\partial z_2^2} < 0$.

Now differentiating the profit function $E\left[\Pi_R^s\right]$ as given in Eq. (17) with respect to z_1 we get

$$\begin{aligned}
\frac{\partial E\left[\Pi_R^s\right]}{\partial z_1} = {} & \left(\frac{\partial P_1^*(z_1,z_2)}{\partial z_1} - \frac{\partial w_1(z_1,z_2)}{\partial z_1}\right)(\alpha_1 - \beta_1 P_1^*(z_1,z_2) + \gamma_1 P_2^*(z_1,z_2) + \mu_1) \\
& + \left(P_1^*(z_1,z_2) - w_1(z_1,z_2)\right)\left(\beta_1 \frac{\partial P_1^*(z_1,z_2)}{\partial z_1} + \gamma_1 \frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\right) - \frac{\partial w_1(z_1,z_2)}{\partial z_1}\Lambda(z_1) \\
& - (w_1(z_1,z_2) - v_1)\frac{\partial \Lambda(z_1)}{\partial z_1} - \left(\frac{\partial P_1^*(z_1,z_2)}{\partial z_1} - \frac{\partial w_1(z_1,z_2)}{\partial z_1}\right)\Theta(z_1) \\
& - \left(P_1^*(z_1,z_2) - w_1(z_1,z_2) + s_1\right)\frac{\partial \Theta(z_1)}{\partial z_1} \\
& + \left(\frac{\partial P_2^*(z_1,z_2)}{\partial z_1} - \frac{\partial w_2(z_1,z_2)}{\partial z_1}\right) \times (\alpha_2 - \beta_2 P_2^*(z_1,z_2) + \gamma_2 P_1^*(z_1,z_2) + \mu_2) \\
& + \left(P_2^*(z_1,z_2) - w_2(z_1,z_2)\right) \times \left(-\beta_2 \frac{\partial P_2^*(z_1,z_2)}{\partial z_1} + \gamma_2 \frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\right) - \frac{\partial w_2(z_1,z_2)}{\partial z_1}\Lambda(z_2) \\
& - \left(\frac{\partial P_2^*(z_1,z_2)}{\partial z_1} - \frac{\partial w_2(z_1,z_2)}{\partial z_1}\right)\Theta(z_2).
\end{aligned}$$

Differentiating it w.r.t. Z_2 and using the values $\frac{\partial^2 P_1^*(z_1,z_2)}{\partial z_1 \partial z_2} = \frac{\partial^2 P_2^*(z_1,z_2)}{\partial z_1 \partial z_2} = \frac{\partial^2 w_1(z_1,z_2)}{\partial z_1 \partial z_2} = \frac{\partial^2 w_2(z_1,z_2)}{\partial z_1 \partial z_2} = 0$ we get

$$\begin{aligned}
\frac{\partial^2 E\left[\Pi_R^s\right]}{\partial z_1 \partial z_2} = {} & \left(\frac{\partial P_1^*(z_1,z_2)}{\partial z_1} - \frac{\partial w_1(z_1,z_2)}{\partial z_1}\right)\left(-\beta_1 \frac{\partial P_1^*(z_1,z_2)}{\partial z_2} + \gamma_1 \frac{\partial P_2^*(z_1,z_2)}{\partial z_2}\right) \\
& + \left(\frac{\partial P_1^*(z_1,z_2)}{\partial z_2} - \frac{\partial w_1(z_1,z_2)}{\partial z_2}\right)\left(-\beta_1 \frac{\partial P_1^*(z_1,z_2)}{\partial z_1} + \gamma_1 \frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\right) \\
& - \frac{\partial w_1(z_1,z_2)}{\partial z_2}\frac{\partial \Lambda(z_1)}{\partial z_1} - \left(\frac{\partial P_1^*(z_1,z_2)}{\partial z_2} - \frac{\partial w_1(z_1,z_2)}{\partial z_2}\right)\frac{\Theta(z_1)}{\partial z_1} \\
& + \left(\frac{\partial P_2^*(z_1,z_2)}{\partial z_1} - \frac{\partial w_2(z_1,z_2)}{\partial z_1}\right)\left(-\beta_2 \frac{\partial P_2^*(z_1,z_2)}{\partial z_2} + \gamma_2 \frac{\partial P_1^*(z_1,z_2)}{\partial z_2}\right) \\
& + \left(\frac{\partial P_2^*(z_1,z_2)}{\partial z_2} - \frac{\partial w_2(z_1,z_2)}{\partial z_2}\right)\left(-\beta_2 \frac{\partial P_2^*(z_1,z_2)}{\partial z_1} + \gamma_2 \frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\right) \\
& - \frac{\partial w_2(z_1,z_2)}{\partial z_1}\frac{\partial \Lambda(z_2)}{\partial z_2} - \left(\frac{\partial P_2^*(z_1,z_2)}{\partial z_1} - \frac{\partial w_2(z_1,z_2)}{\partial z_1}\right)\frac{\Theta(z_2)}{\partial z_2}.
\end{aligned}$$

Now using the values of the partial derivatives as given in Appendix A and also using

$$\frac{\partial w_1(z_1,z_2)}{\partial z_1} = \frac{1}{\beta_1}\left[-\beta_1 \frac{\partial P_1^*(z_1,z_2)}{\partial z_1} + \gamma_1 \frac{\partial P_2^*(z_1,z_2)}{\partial z_1} + 1\right],$$

$$\frac{\partial w_1(z_1,z_2)}{\partial z_2} = \frac{1}{\beta_1}\left[-\beta_1 \frac{\partial P_1^*(z_1,z_2)}{\partial z_2} + \gamma_1 \frac{\partial P_2^*(z_1,z_2)}{\partial z_2}\right],$$

$$\frac{\partial w_2(z_1,z_2)}{\partial z_1}=\frac{1}{\beta_2}\left[-\beta_2\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}+\gamma_2\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\right],$$
$$\frac{\partial w_2(z_1,z_2)}{\partial z_2}=\frac{1}{\beta_2}\left[-\beta_2\frac{\partial P_2^*(z_1,z_2)}{\partial z_2}+\gamma_2\frac{\partial P_1^*(z_1,z_2)}{\partial z_2}+1\right]$$

and after some simplification we get

$$\frac{\partial^2[\Pi_R^s]}{\partial z_1\partial z_2}=\frac{7(\gamma_1+\gamma-2)(\beta_1\beta_2-\gamma_1\gamma-2)(1-F_1(z_1)(1-F_2(z_2)))}{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2}>0$$

under Assumptions A2 and A3. Further, differentiating again $\frac{\partial[\Pi_R^s]}{\partial z_1}$ with respect to z_1, we get

$$\frac{\partial^2[\Pi_R^s]}{\partial z_1^2}=\left(\frac{\partial^2 P_1^*(z_1,z_2)}{\partial z_1^2}-\frac{\partial^2 w_1(z_1,z_2)}{\partial z_1^2}\right)(\alpha_1$$
$$-\beta_1P_1^*(z_1,z_2)+\gamma_1P_2^*(z_1,z_2)+\mu_1)$$
$$\left(\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}-\frac{\partial w_1(z_1,z_2)}{\partial z_1}\right)\left(-\beta_1\frac{P_1^*(z_1,z_2)}{\partial z_1}+\gamma_1\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}\right)$$
$$+(P_1^*(z_1,z_2)-w_1(z_1,z_2))\left(-\beta_1\frac{\partial^2 P_1^*(z_1,z_2)}{\partial z_1^2}+\gamma_1\frac{\partial^2 P_2^*(z_1,z_2)}{\partial z_1^2}\right)$$
$$-\frac{\partial^2 w_1(z_1,z_2)}{\partial z_1^2}\Lambda(z_1)-2\frac{\partial w_1(z_1,z_2)}{\partial z_1}\frac{\partial\Lambda(z_1)}{\partial z_1}-(w_1(z_1,z_2)-v_1)\frac{\partial^2\Lambda(z_1)}{\partial z_1^2}$$
$$-\left(\frac{\partial^2 P_1^*(z_1,z_2)}{\partial z_1^2}-\frac{\partial^2 w_1(z_1,z_2)}{\partial z_1^2}\right)\Theta(z_1)$$
$$-2\left(\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}-\frac{\partial w_1(z_1,z_2)}{\partial z_1}\right)\frac{\partial\Theta(z_1)}{\partial z_1}$$
$$-(P_1^*(z_1,z_2)-w_1(z_1,z_2)+s_1)\frac{\partial^2\Theta(z_1)}{\partial z_1^2}$$
$$+\left(\frac{\partial^2 P_2^*(z_1,z_2)}{\partial z_1^2}-\frac{\partial^2 w_2(z_1,z_2)}{\partial z_1^2}\right)$$
$$\times(\alpha_2-\beta_2P_2^*(z_1,z_2)+\gamma_2P_1^*(z_1,z_2)+\mu_2)$$
$$+2\left(\frac{\partial P_2^*(z_1,z_2)}{\partial z_1}-\frac{\partial w_2(z_1,z_2)}{\partial z_1}\right)$$
$$\times\left(-\beta_2\frac{P_2^*(z_1,z_2)}{\partial z_1}+\gamma_2\frac{\partial P_1^*(z_1,z_2)}{\partial z_1}\right)-(P_2^*(z_1,z_2)-w_2(z_1,z_2))$$
$$\times\left(-\beta_2\frac{\partial^2 P_2^*(z_1,z_2)}{\partial z_1^2}+\gamma_2\frac{\partial^2 P_1^*(z_1,z_2)}{\partial z_1^2}\right)$$
$$-\frac{\partial^2 w_2(z_1,z_2)}{\partial z_1^2}\Lambda(z_2)-\left(\frac{\partial P_2^*(z_1,z_2)}{\partial z_1^2}-\frac{\partial w_2(z_1,z_2)}{\partial z_1^2}\right)\Theta(z_2).$$

Using

$$\frac{\partial^2 w_1(z_1,z_2)}{\partial z_1^2}=\frac{1}{\beta_1}\left[-\beta_1\frac{\partial^2 P_1^*(z_1,z_2)}{\partial z_1^2}+\gamma_1\frac{\partial^2 P_2^*(z_1,z_2)}{\partial z_1^2}\right],$$
$$\frac{\partial^2 w_2(z_1,z_2)}{\partial z_1^2}=\frac{1}{\beta_2}\left[-\beta_2\frac{\partial^2 P_2^*(z_1,z_2)}{\partial z_1^2}+\gamma_2\frac{\partial^2 P_1^*(z_1,z_2)}{\partial z_1^2}\right]$$

and values of all other partial derivatives as mentioned earlier and after some simplification, we get

$$\frac{\partial^2\Pi_R^s}{\partial z_1^2}<-\frac{\{4\beta_1\beta_2-\gamma_1(\gamma_1+\gamma_2)\}f_1(z_1)}{\beta_1\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}\Bigg(\alpha_1-\beta_1P_1^*(z_1,z_2)+\gamma_1P_2^*(z_1,z_2).$$
$$+\int_{A_1}^{z_1}\epsilon_1f_1(z_1)\,d\epsilon_1+\int_{z_1}^{B_1}z_1f_1(\epsilon_1)\,d\epsilon\Bigg)$$
$$-\frac{2\{2\beta_1\beta_2-\gamma_2(\gamma_1+\gamma_2)\}(1-F_1(z_1))\gamma_2(\gamma_1+\gamma2)}{\beta_1\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}^2}$$
$$-s_1f_1(z_1)-\frac{\{2\beta_1\beta_2-\gamma_2(\gamma_1+\gamma_2)\}f_1(z_1)}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}(P_1^*(z_1,z_2)-w_1(z_1,z_2))$$
$$+\frac{(\gamma_1-\gamma_2)(\gamma_1+\gamma_2)f_1(z_1)}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}\Lambda(z_1)-\frac{\{2\beta_1\beta_2-\gamma_2(\gamma_1+\gamma_2)\}}{\beta_1\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}F_1(z_1)$$
$$-(w_1(z_1,z_2)-v_1)f_1(z_1)$$
$$-\frac{2\gamma_1f_1(z_1)}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}\Bigg(\alpha_2-\beta_2P_2^*(z_1,z_2)+\gamma_2P_1^*(z_1,z_2)$$
$$+\int_{A_2}^{z_2}\epsilon_2f_2(z_2)d\epsilon_2+\int_{z_2}^{B_2}z_2f_2(\epsilon_2)d\epsilon_2\Bigg)$$
$$-\frac{2\gamma_1\beta_2(\gamma_1-\gamma_2)(1-F_1(z_1))^2}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}$$
$$+\frac{\beta_2(\gamma_1-\gamma_2)f_1(z_1)(P_2^*(z_1,z_2)-w_2(z_1,z_2))}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}$$
$$-\frac{(\gamma_1-\gamma_2)f_1(z_1)}{\left\{4\beta_1\beta_2-(\gamma_1+\gamma_2)^2\right\}}\Lambda(z_2)$$

<0, under assumptions $A2$, $A3$ and $A4$.

Hence, $E\{\Pi_R^s\}$ is a quasi concave function of z_1, z_2. Thus, there exists a Stackelberg game where retailer is the Stackelberg leader.

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