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Integrated Production-Inventory-Distribution System Design with Risk Pooling: Model Formulation and Heuristic Solution

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In this paper, we consider a multiproduct two-echelon production-inventory-distribution system design model that captures risk-pooling effects by consolidating the safety-stock inventory of the retailers at distribution centers (DCs). We propose a model that determines plant and DC locations, shipment levels from plants to the DCs, safety-stock levels at DCs, and the assignment of retailers to DCs by minimizing the sum of fixed facility location costs, transportation costs, and safety-stock costs. The model is formulated as a nonlinear mixed-integer programming problem and linearized using piecewise-linear functions. The formulation is strengthened using redundant constraints. Lagrangean relaxation is applied to decompose the problem by echelon. A lower bound is provided by the Lagrangean relaxation, while a heuristic is proposed that uses the solution of the subproblems to construct an overall feasible solution. Computational results reveal that the Lagrangean relaxation provides a sharp lower bound and a heuristic solution that is within 5% of the optimal solution.

Key words: production-inventory-distribution system design; risk pooling; piecewise linearization; Lagrangean relaxation
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1. Introduction

Supply chain planning decisions can be broadly classified into strategic, tactical, and operational levels. While production-distribution system design decisions typically lie at the strategic level, inventory models are part of tactical-level planning. Although some recent work on the development of large-scale integrated models for strategic design and tactical planning (for example, Dogan and Goeotschalckx 1999) is indicative of a trend to model more realistic problems that integrate various decisions, most of the strategic production-distribution system designs fail to consider inventory decisions (Nozick and Turnquist 1998; Erengüç, Simpson, and Vakharia 1999). A recent review (see Erengüç, Simpson, and Vakharia 1999) identifies the integration of inventory decisions in production-distribution system design as an area of future research. Because of the complexity of the two problems, the literature tends to handle them separately. In this paper, we integrate the two decisions to consider inventory decisions explicitly and analyze the trade-off among facility location cost, transportation cost, and safety-stock inventory-holding cost. In particular, we look at the risk-pooling benefits arising from centralizing inventory at DCs and its effect on the design of production-distribution system.

Research on integrated production-inventory-distribution system design is relatively new. Barahona and Jensen (1998) present a model that combined plant location decisions with inventory decisions, where the inventory decisions include whether or not to stock certain parts at a particular location. Nozick and Turnquist (1998) propose a joint location-inventory model by embedding an approximate safety-stock cost function (expressed as a linear function of number of DCs used) in the fixed cost of the uncapacitated facility location problem. Nozick and Turnquist (2001) consider a model of a production-distribution system to determine the number and location of DCs and the allocation of safety stock to DCs and plants. They proposed two heuristics; the first heuristic determines the inventory location and quantity while trying to minimize the penalty cost for stockouts and inventory-holding cost, whereas the second heuristic determines the number and location of DCs. Erlebacher and Meller (2000) formulate a joint location-inventory model with nonlinear objective function for locating DCs and allocating customers to DCs. The model was solved using continuous approximation and two heuristics. The first heuristic determines the optimal number of DCs based on simplifying assumptions, whereas the second heuristic determines the location of these DCs and allocates customers to DCs. While the heuristic approaches proposed above often found good solutions to the problem, it is difficult to evaluate the quality because there is no way of knowing how far from the optimality these solutions are.

Daskin, Coullard, and Shen (2002) and Shen, Coullard, and Daskin (2003) develop a single-commodity joint location-inventory model with risk pooling that simultaneously determines DC locations, shipment sizes and frequencies from the DCs to retailers, the working inventory and safety-stock levels at the DCs, and the assignment of retailers to the DCs to minimize the fixed facility location costs, transportation costs, and inventory costs. They propose a Lagrangean relaxation-based heuristic. They also present a set-covering model and a branch-and-price based solution procedure. Snyder, Daskin, and Teo (2003) present a stochastic version of the location model with risk pooling proposed by Daskin, Coullard, and Shen (2002) and Shen, Coullard, and Daskin (2003). However, it is worthwhile noting that in order to alleviate the nonlinearity due to the risk-pooling considerations, they discuss solutions to two special cases, i.e., either (1) the demand is deterministic or (2) the demand has mean identically proportional to its variance for all the retailers. Shu, Song, and Sun (2004) and Shu, Teo, and Shen (2005) extend the model of Snyder, Daskin, and Teo (2003) to the more general case of demand patterns. While Shu, Song, and Sun (2004) present primal-dual algorithms to illustrate the structure of the pricing problem, Shu, Teo, and Shen (2005) use concepts from computational geometry to solve the pricing problem efficiently. Teo and Shu (2004) consider a distribution network design problem integrating transportation and infinite-horizon multiechelon inventory cost. The two-echelon inventory cost function considers the cost arising from coordination of replenishment activities between the warehouses and the retailers and does not include safety-stock costs. In this paper, we study the effect of considering risk pooling and safety-stock inventory in the design of multiproduct production-inventory-distribution systems. The focus is to capture the benefits of risk pooling by consolidating the safety-stock inventory of the retailers at the distribution centers (DCs). The model can be extended to consider other inventory-holding cost components such as the fixed cost of placing and shipping an order. Also, unlike the models above, where the DCs are assumed to be uncapacitated, we consider capacity constraints on the DCs and plants.

The basic premise of this paper is to consider safety-stock inventory together with facility location costs and transportation costs in determining the optimal location of plants and DCs, and the assignment of retailers to DCs. We consider a two-echelon production-inventory-distribution system design problem that accounts for safety stock at DCs. Each DC serves as a direct intermediary between the plants and the retailers for the shipment of products. Retailers face uncertain demand, and safety stocks are maintained at DCs to provide appropriate service levels to the retailers and to protect against short-term variations in demand and transportation lead time between DCs and retailers. To achieve risk-pooling benefits and inventory-cost reductions, safety stocks for all the retailers served by a DC are maintained at that DC. Therefore, less total safety stock is required in the system than in the case in which every retailer maintains its own safety stock. As demand uncertainty increases, there is potential for considerable savings in total cost. Therefore, the objective is to determine the plant and DC locations, shipment sizes from plants to DCs, assignment of retailers to DCs, and the safety-stock levels at DCs by minimizing the fixed facility location costs, transportation costs, and safety-stock costs. The model is formulated as a nonlinear mixed-integer programming (NLMIP) problem and is linearized using piecewise-linear functions. Given the fact that both the location/allocation and inventory problems are difficult to solve separately, it is not surprising that an integrated model that handles these problems simultaneously would be hard to solve to optimality. Therefore, we propose an efficient solution methodology based on Lagrangean relaxation and redundant constraints that computes a lower bound and a heuristic solution and improves them iteratively.

The paper is organized as follows. Section 2 details the problem formulation and the linearization procedure. Section 3 presents the Lagrangean relaxation and the heuristic solution procedure. Section 4 presents computational results and discussion. Finally, §5 concludes with some directions for future research.

2. Problem Formulation

Consider the problem of configuring a production-inventory-distribution system, where a set of plants and DCs are to be established to distribute various products to a set of retailers. The DCs act as intermediate facilities between the plants and the retailers and facilitate the shipment of products between the two echelons, as depicted in Figure 1. We assume that the demand for each product at each retailer is independent and normally distributed, and the safety-stock inventory for all the retailers served by a DC is held at that DC.

The strategic decision involves two components: The first component is the location/allocation problem, which determines the number and location of the plants and DCs and assigns retailers to DCs, whereas the second component deals with determining the location and level of safety-stock inventory at the DCs

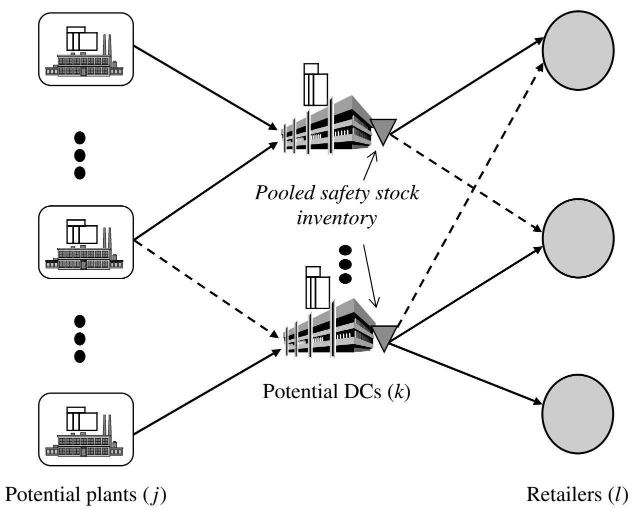

Figure 1 Production-Inventory-Distribution System

in order to provide a certain level of service to the retailers. To model this problem, we define the following notation:

Indices:

i Index for products, $i = 1, 2, \ldots, I$.
j Index for potential plant locations, $j = 1, 2, \ldots, J$.
k Index for potential DC locations, $k = 1, 2, \ldots, K$.
l Index for retailers, $l = 1, 2, \ldots, L$.

Parameters:

c_{ijk} Unit cost of producing and shipping product i from plant j to DC k.
t_{ikl} Unit cost of shipping product i from DC k to retailer l.
f_j Fixed (annual) cost of opening a plant at site j.
g_k Fixed (annual) cost of opening a DC at site k.
h_i Inventory-holding cost per unit of product i per year (at plants and DCs).
α Service level for the retailers.
P_j Production capacity of plant j.
V_k Maximum storage capacity of DC k.
μ_{il} Mean of daily demand for product i at retailer l.
σ_{il} Standard deviation of daily demand for product i at retailer l.
z_α Standard normal value so that $P(z \le z_\alpha) \le \alpha$.
$\mathscr{K}$ Number of production days in a year.
$\mathscr{L}$ Average transportation lead time from a DC to retailer (in days).

Decision Variables:

x_{ijk} Number of units of product i produced at plant j and shipped to DC k.
y_{kl} 1 if DC k serves retailer l; 0 otherwise.
w_j 1 if plant j is opened; 0 otherwise.
z_k 1 if DC k is opened; 0 otherwise.
SS_{ik} Safety stock for product i at DC k.

Risk pooling refers to the consolidation of multiple inventory locations into a single one. The risk-pooling effect achieved by grouping multiple inventory locations has been documented to benefit inventory systems by reducing the need for safety stock and, consequently, lowering the inventory-holding costs (Eppen 1979; Chen and Lin 1989; Cherikh 2000). Eppen (1979) showed that a pooled system yields a lower cost than a distributed system, and the difference increases with the variance of demands and decreases with the correlation between these demands, with the difference reducing to zero when the demands have perfect positive correlation. Apart from saving in total inventory costs, systems with risk pooling provide higher service levels. However, the increase in service level depends on the coefficient of variation and the correlation between the demand from various retailers.

To model this, let us assume for a moment that the assignment of retailers to a DC is known a priori. Assume that the demand for a product i at retailer l is independent and normally distributed, with mean μ_{il} and standard deviation σ_{il}. Also, let $\mathscr{L}$ be the average lead time in days for shipping from DCs to retailers. From basic inventory theory, we know that if the demands at each retailer are uncorrelated, then the demand during lead time at the DC is normally distributed with a mean of $\mathscr{L}\sum_{l=1}^{L}\mu_{il}$ and a variance of $\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2$. Let us consider the centralized mode, which refers to holding the safety stock pooled at the DCs, versus decentralized mode, which refers to maintaining safety stock at the retailer's end or maintaining a separate safety stock for each retailer at the DC. The total amount of safety stock under the decentralized mode (no risk pooling) is $z_\alpha \mathscr{L}\sum_{l=1}^{L}\sigma_{il}$, whereas under the centralized mode (with risk pooling) it is $z_\alpha\sqrt{\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2}$, where z_α is the standard normal deviate. The pooled safety-stock inventory is less than the decentralized safety-stock inventory. Hence, considerable savings in inventory cost can be achieved by centralized inventory policy/risk pooling. Clearly, this depends on the assignment of retailers to DCs, which is not known in advance and must be determined endogenously. In order to simultaneously determine the assignment of retailers to DCs, and the safety-stock levels (SS_{ik}), we use the expression $z_\alpha\sqrt{\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}}$, where y_{kl} is a binary decision variable that equals one if the retailer l is served by DC k and zero otherwise.

The production-inventory-distribution system design model formulated below simultaneously determines the plant and DC locations, production and shipment levels from plants to DCs, safety-stock levels at DCs, and the assignment of retailers to DCs. The objective is to minimize the fixed facility location costs, transportation costs between echelons, and safety-stock costs. The resulting nonlinear MIP formulation is

$$[\mathrm{P}_{NL}] \quad \min \sum_{j=1}^{J} f_j w_j + \sum_{k=1}^{K} g_k z_k + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} c_{ijk} x_{ijk} + \mathscr{X}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{l=1}^{L} t_{ikl}\mu_{il} y_{kl} + \sum_{i=1}^{I}\sum_{k=1}^{K} h_i\left(z_\alpha\sqrt{\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}}\right) \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijk} \le P_j w_j \quad \forall j \tag{2}$$

$$\sum_{j=1}^{J} x_{ijk} = z_\alpha\sqrt{\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}} + \mathscr{X}\sum_{l=1}^{L}\mu_{il} y_{kl} \quad \forall i,k \tag{3}$$

$$\mathscr{X}\sum_{i=1}^{I}\sum_{l=1}^{L}\mu_{il} y_{kl} + \sum_{i=1}^{I}\left(z_\alpha\sqrt{\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}}\right) \le V_k z_k \quad \forall k \tag{4}$$

$$\sum_{k=1}^{K} y_{kl} = 1 \quad \forall l \tag{5}$$

$$w_j, z_k, y_{kl} \in \{0,1\}, \quad x_{ijk} \ge 0 \ \forall i,j,k,l. \tag{6}$$

The objective function (1) minimizes the sum of the fixed costs of opening plants and DCs (amortized annually), the cost of production and transportation of products from plants to DCs, the cost of shipping and handling between DCs and retailers, and the cost of holding pooled safety stock at the DCs. Constraints (2) and (4) are capacity restrictions on the opened plants and DCs, respectively, and permit the use of opened facilities only. Constraints (3) are product flow conservation equations at the DCs, which ensure that for every product that flows through the DC, a part of it is held in safety stock and the rest is used to satisfy demand at the retailers. Constraints (5) are single-sourcing requirements that restrict a retailer's demand to be served by a single DC. Constraints (6) are integrality and nonnegativity requirements.

In the model $[\mathrm{P}_{NL}]$, given that a retailer's demand is to be entirely served by a DC, $(\sum_k y_{kl} = 1)$, the inventory holding cost $(\sum_i \sum_k \sum_l h_i \mu_{il} y_{kl})$ is equivalent to $\sum_i \sum_l h_i \mu_{il}$, which is a constant term, and hence it does not appear in the objective function. However, if an EOQ ordering policy cost is used with an order-setup cost (as in Daskin, Coullard, and Shen 2002; Shen, Coullard, and Daskin 2003; Shen and Daskin 2005), we will have an additional term in the objective function $\sqrt{2F_k \sum_i \sum_l h_i \mu_{il} y_{kl}}$, where F_k is the fixed cost of placing and shipping an order from DC k. Note that this term is similar in structure to the safety-stock term $z_\alpha\sqrt{\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}}$ in the objective function and can be easily incorporated in the model. We choose not to restrict the model to a particular ordering policy; hence, this is ignored in the sequel.

Problem $[\mathrm{P}_{NL}]$ has a nonlinear (safety-stock) term $z_\alpha\sqrt{\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}}$ in the objective function that makes the model intractable. It can, however, be linearized using piecewise-linear and concave functions. Let L_{ik}^r, $r = 1, 2, \ldots, R$ be a set of points where in range $[L_{ik}^{r-1}, L_{ik}^r]$ the function $\sqrt{\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}}$ is approximated by the linear function $F_{ik}^r + C_{ik}^r \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}$, with $C_{ik}^1 > C_{ik}^2 > C_{ik}^3 > \cdots > C_{ik}^R$ and $F_{ik}^1 < F_{ik}^2 < F_{ik}^3 < \cdots < F_{ik}^R$. For the continuity of the function, F_{ik}^{r+1} is set to $F_{ik}^r + (C_{ik}^r - C_{ik}^{r+1})L_{ik}^r$ for all i, k, and r (see Figure 2). To use this linearization, we must introduce the binary variables u_{ik}^r, which are one if the range r is active; zero otherwise. With these definitions, the safety stock can be approximated using the following set of constraints:

$$SS_{ik} = z_\alpha \sum_{r=1}^{R}\left(F_{ik}^r + C_{ik}^r \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}\right)u_{ik}^r \quad \forall i,k \tag{7}$$

$$L_{ik}^{r-1}u_{ik}^r \le \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl} \le L_{ik}^r u_{ik}^r + M(1-u_{ik}^r) \quad \forall i,k,r \tag{8}$$

$$\sum_{r=1}^{R} u_{ik}^r = 1 \quad \forall i,k, \tag{9}$$

where M represents the usual big-M.

Note that Equation (7) is nonlinear in y_{kl} and u_{ik}^r. To deal with that, we define binary auxiliary variables δ_{ikl}^r that equal one if both $u_{ik}^r = 1$ and $y_{kl} = 1$, and are zero otherwise. This replaces (7) with the following equivalent set of linear constraints:

$$SS_{ik} = z_\alpha \sum_{r=1}^{R}\left(F_{ik}^r u_{ik}^r + C_{ik}^r \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 \delta_{ikl}^r\right) \quad \forall i,k \tag{10}$$

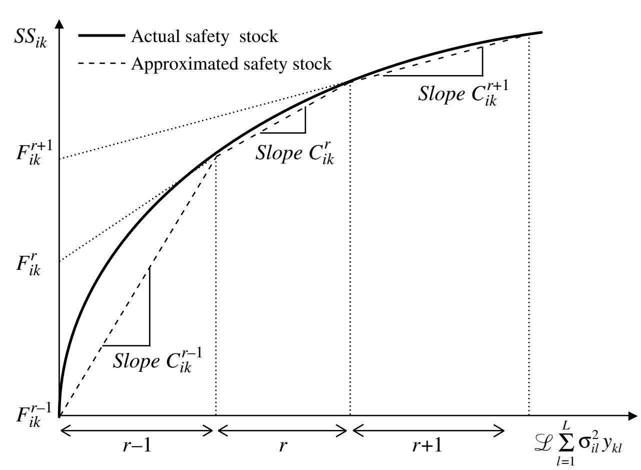

Figure 2 Piecewise-Linear Safety Stock

$$u_{ik}^r + y_{kl} - 2\delta_{ikl}^r \geq 0 \quad \forall\, i, k, l, r \tag{11}$$

$$u_{ik}^r + y_{kl} - \delta_{ikl}^r \leq 1 \quad \forall\, i, k, l, r. \tag{12}$$

The number of linear segments required to approximate the nonlinear term is set by the user, based on the degree of approximation desired. The higher the number of linear segments, the better is the approximation. The resulting linear MIP formulation is

$$[\mathrm{P}_L] \quad \min \sum_{j=1}^{J} f_j w_j + \sum_{k=1}^{K} g_k z_k + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} c_{ijk} x_{ijk} + \mathscr{X}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{l=1}^{L} t_{ikl}\mu_{il} y_{kl} + z_\alpha \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} h_i \cdot \left(F_{ik}^r u_{ik}^r + C_{ik}^r \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 \delta_{ikl}^r\right) \tag{13}$$

$$\text{s.t.} \quad \sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijk} \leq P_j w_j \quad \forall\, j \tag{14}$$

$$\sum_{j=1}^{J} x_{ijk} = z_\alpha \sum_{r=1}^{R}\left(F_{ik}^r u_{ik}^r + C_{ik}^r \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 \delta_{ikl}^r\right) + \mathscr{X}\sum_{l=1}^{L}\mu_{il} y_{kl} \quad \forall\, i, k \tag{15}$$

$$\mathscr{X}\sum_{i=1}^{I}\sum_{l=1}^{L}\mu_{il} y_{kl} + z_\alpha \sum_{i=1}^{I}\sum_{r=1}^{R}\left(F_{ik}^r u_{ik}^r + C_{ik}^r \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 \delta_{ikl}^r\right) \leq V_k z_k \quad \forall\, k \tag{16}$$

$$\sum_{k=1}^{K} y_{kl} = 1 \quad \forall\, l \tag{17}$$

$$u_{ik}^r + y_{kl} - 2\delta_{ikl}^r \geq 0 \quad \forall\, i, k, l, r \tag{18}$$

$$u_{ik}^r + y_{kl} - \delta_{ikl}^r \leq 1 \quad \forall\, i, k, l, r \tag{19}$$

$$L_{ik}^{r-1} u_{ik}^r \leq \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl} \leq L_{ik}^r u_{ik}^r + M(1 - u_{ik}^r) \quad \forall\, i, k, r \tag{20}$$

$$\sum_{r=1}^{R} u_{ik}^r = 1 \quad \forall\, i, k \tag{21}$$

$$w_j, z_k, y_{kl}, u_{ik}^r, \delta_{ikl}^r \in \{0, 1\}, \quad x_{ijk} \geq 0 \quad \forall\, i, j, k, l. \tag{22}$$

The formulation can be strengthened by adding the following set of redundant constraints:

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk} \leq V_k \quad \forall\, k \tag{23}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \geq \mathscr{X}\sum_{l=1}^{L}\mu_{il} \quad \forall\, i \tag{24}$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \leq \mathscr{X}\sum_{l=1}^{L}\mu_{il} + z_\alpha \left\{\sum_{k=1}^{K}\left(\max_r F_{ik}^r\right) + \left(\max_{k,r} C_{ik}^r\right) \cdot \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\right\} \quad \forall\, i. \tag{25}$$

Constraints (23) are derived from (15) and (16), whereas constraints (24) and (25) are derived from (16). Constraints (23) imply that the total flow of products through a DC should not exceed the DC capacity, whereas constraints (24) ensure that the flow of every product from plants to DC is at least equal to the demand of that product from all the retailers. Constraints (25) serve as an upper bound on the flow of product from plants to DCs. These constraints are redundant in the original formulation, but nonredundant when the problem is decomposed.

Model $[\mathrm{P}_L]$ could be solved using an off-the-shelf MIP solver. In general, such an approach will yield excessive runtimes, even for moderately sized problems. For example, even for a problem with three products, five potential plant locations, 10 potential DC locations, 30 retailers, and five linear segments, the linear MIP model has 5,865 variables (900 continuous and 4,965 binary variables) and 10,000 constraints. Our experiments indicate that MIP solver (CPLEX) fails to provide the optimal solution (to the linearized MIP model) within a time limit of 1,000 minutes even for a moderate-size problem. This motivates the development of the Lagrangean-relaxation based approach. It is worth noting that constraints (15) and (23)–(25) relate to the plant-DC echelon, whereas constraints (16)–(21) relate to the DC-retailer echelon, and constraints (15) are the flow balance constraints that link the two echelons. We exploit this structure using Lagrangean relaxation and decompose the model into two subproblems that are relatively easy to solve.

3. Lagrangean Relaxation

Lagrangean relaxation is one of the most widely used techniques for solving MIP problems in general (Fisher 1981) and production-distribution system design problems in particular (Elhedhli and Goffin 2005). The relaxation usually targets constraints that, if removed, result in subproblems that are easy to solve, either because of their special structure or because they have efficient solution procedures (see Pirkul and Jayaraman 1996, 1998; Elhedhli and Goffin 2005; Elhedhli and Gzara 2005 and some of the references therein). However, there is a trade-off between the quality of the relaxation bound and the difficulty of the subproblems. The decomposition here exploits the echelon structure of the problem, and is aimed at subproblems that preserve most of the characteristics of the original problem, rather than getting easy subproblems.

3.1. Lower Bound

Upon relaxing the flow balance constraints, the problem decomposes into two subproblems—one for the plant-DC echelon, and the other for the DC-retailer echelon. Associating dual multipliers λ_{ik} with flow conservation constraint (10) in $[\mathrm{P}_L]$ leads to the following two subproblems:

$$[\mathrm{SP}_1]\quad \min\ \sum_{j=1}^{J} f_j w_j + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(c_{ijk}-\lambda_{ik})x_{ijk}$$

$$\text{s.t.}\ \sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijk} \le P_j w_j \quad \forall j$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk} \le V_k \quad \forall k$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \ge \mathscr{X}\sum_{l=1}^{L}\mu_{il} \quad \forall i$$

$$\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \le \mathscr{X}\sum_{l=1}^{L}\mu_{il} + z_\alpha\left\{\sum_{k=1}^{K}\left(\max_r F_{ik}^r\right) + \left(\max_{k,r} C_{ik}^r\right)\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\right\} \quad \forall i$$

$$w_j \in \{0,1\};\quad x_{ijk} \ge 0 \quad \forall i,j,k$$

$$[\mathrm{SP}_2]\quad \min\ \sum_{k=1}^{K} g_k z_k + \mathscr{X}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{l=1}^{L}(t_{ikl}+\lambda_{ik})(\mu_{il}y_{kl}) + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R}(z_\alpha h_i+\lambda_{ik}) \cdot\left(F_{ik}^r u_{ik}^r + C_{ik}^r\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\delta_{ikl}^r\right)$$

$$\text{s.t.}\ \mathscr{X}\sum_{i=1}^{I}\sum_{l=1}^{L}\mu_{il}y_{kl} + z_\alpha\sum_{i=1}^{I}\sum_{r=1}^{R}\left(F_{ik}^r u_{ik}^r + C_{ik}^r\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\delta_{ikl}^r\right) \le V_k z_k \quad \forall k$$

$$u_{ik}^r + y_{kl} - 2\delta_{ikl}^r \ge 0 \quad \forall i,k,l,r$$

$$u_{ik}^r + y_{kl} - \delta_{ikl}^r \le 1 \quad \forall i,k,l,r$$

$$L_{ik}^{r-1}u_{ik}^r \le \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl} \le L_{ik}^r u_{ik}^r + M(1-u_{ik}^r) \quad \forall i,k,r$$

$$\sum_{r=1}^{R} u_{ik}^r = 1 \quad \forall i,k$$

$$z_k, y_{kl}, u_{ik}^r, \delta_{ikl}^r \in \{0,1\} \quad \forall i,k,l,r.$$

Subproblem $[\mathrm{SP}_1]$ determines the location of plants and the flow of products into the DCs, whereas subproblem $[\mathrm{SP}_2]$ provides the location of DCs, the assignment of retailers to DCs, and the amount of safety stock to be held at DCs. For a given Lagrange multiplier λ, the sum of the objectives of the subproblems $v(\mathrm{SP}_1)+v(\mathrm{SP}_2)$ gives a lower bound to $v(\mathrm{P}_L)$, where $v(\bullet)$ denotes the optimal objective value of the problem $(\bullet)$. The best lower bound is the solution of the Lagrangean dual problem: $\max_\lambda\{v(\mathrm{SP}_1)+v(\mathrm{SP}_2)\}$, which is a concave and nondifferentiable optimization problem that can be solved using cutting-plane methods or subgradient optimization. In this paper, we use a cutting-plane method because it is guaranteed to provide the best Lagrangean bound. The Lagrangean dual problem is equivalent to

$$\max_\lambda\left\{\min_{h\in I_x}\sum_{j=1}^{J} f_j w_j^h + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}(c_{ijk}-\lambda_{ik})x_{ijk}^h + \min_{h\in I_y}\sum_{k=1}^{K} g_k z_k^h + \mathscr{X}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{l=1}^{L}(t_{ikl}+\lambda_{ik})(\mu_{il}y_{kl}^h) + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R}(z_\alpha h_i+\lambda_{ik})\left(F_{ik}^r u_{ik}^{rh} + C_{ik}^r\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\delta_{ikl}^{rh}\right)\right\}, \tag{26}$$

where I_x is the index set of feasible points in the set:

$$\{(x_{ijk}, w_j)\colon (9), (18), (19), (20);\ w_j\in\{0,1\};\ x_{ijk}\ge 0\}$$

and I_y is the index set of feasible integer points in the set:

$$\{(y_{kl}, z_k, u_{ik}^r, \delta_{ikl}^r)\colon (11), (12), (13), (14), (15), (16);\ y_{kl}, z_k, u_{ik}^r, \delta_{ikl}^r \in\{0,1\}\}.$$

We can write problem (26) as a linear program with an exponential number of constraints:

[MP]

$$\max_\lambda\ \theta_1+\theta_2$$

$$\text{s.t.}\ \theta_1 + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk}^h\lambda_{ik} \le \sum_{j=1}^{J} f_j w_j^h + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} c_{ijk}x_{ijk}^h \quad h\in I_x \tag{27}$$

$$\theta_2 - \sum_{i=1}^{I}\sum_{k=1}^{K}\left[\mathscr{X}\sum_{l=1}^{L}\mu_{il}y_{kl}^h + \sum_{r=1}^{R}\left(F_{ik}^r u_{ik}^{hr} + C_{ik}^r\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\delta_{ikl}^{rh}\right)\right]\lambda_{ik} \le \sum_{k=1}^{K} g_k z_k^h + \mathscr{X}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{l=1}^{L} t_{ikl}\mu_{il}y_{kl}^h$$

$$+\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} z_\alpha h_i\left(F_{ik}^r u_{ik}^{hr}+C_{ik}^r \mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\delta_{ikl}^{rh}\right)$$
$$h\in I_y. \tag{28}$$

The relaxation of [MP] defined on subsets $\bar{I}_x\subset I_x$ and $\bar{I}_y\subset I_y$ results in a relaxed master problem [RMP]. We use Kelley's classical cutting plane (Kelley 1960), where the solution of the relaxed master problem [*RMP*] is used to solve the subproblems [SP_1] and [SP_2], and generate two cuts of the form:

$$\theta_1+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk}^{\bar{i}}\lambda_{ik}\le\sum_{j=1}^{J} f_j w_j^{\bar{i}}+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} c_{ijk}x_{ijk}^{\bar{i}} \tag{29}$$

and

$$\theta_2-\sum_{i=1}^{I}\sum_{k=1}^{K}\left[\mathscr{X}\sum_{l=1}^{L}\mu_{il}y_{kl}^{\bar{j}}+\sum_{r=1}^{R}\left(F_{ik}^r u_{ik}^{\bar{j}r}+C_{ik}^r\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\delta_{ikl}^{\bar{j}r}\right)\right]\lambda_{ik}$$
$$\le\sum_{k=1}^{K} g_k z_k^{\bar{j}}+\mathscr{X}\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{l=1}^{L} t_{ikl}\mu_{il}y_{kl}^{\bar{j}}$$
$$+\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{r=1}^{R} h_i z_\alpha\left(F_{ik}^r u_{ik}^{\bar{j}r}+C_{ik}^r\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\delta_{ikl}^{\bar{j}r}\right). \tag{30}$$

The index sets $\bar{I}_x$ and $\bar{I}_y$ are updated as $\bar{I}_x\cup\{\bar{i}\}$ and $\bar{I}_y\cup\{\bar{j}\}$, respectively, as we proceed through the iterations of the algorithm. The algorithm terminates when $(v(\text{MP})-v(\text{SP}_1)-v(\text{SP}_2))\cdot(v(\text{SP}_1)+v(\text{SP}_2))^{-1}$ is less than some optimality tolerance ϵ specified by the user.

3.2. Heuristic

At each iteration, the solution of the Lagrangean dual problem provides a lower bound to [P_L]. To generate a feasible solution, we propose a heuristic. The first subproblem [SP_1] provides the location of plants (w_j) and the flow of products into the DCs (x_{ijk}), whereas the second subproblem [SP_2] provides the location of DCs (z_k), the assignment of retailers to DCs (y_{kl}), and the amount of safety stock to be held at the DCs (u_{ik}^r). The link between the two subproblems is the flow balance of products in and out of DCs. Hence, a feasible solution to problem [P_L] can be constructed by solving one of the two subproblems with the additional flow balance constraints. We build a feasible solution to the original problem by solving [SP_1] with an additional set of constraints

$$\sum_{j=1}^{J} x_{ijk}=z_\alpha\sum_{r\in R}\left(F_{ik}^r\bar{u}_{ik}^r+C_{ik}^r\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2\bar{y}_{kl}\right)+\mathscr{X}\sum_{l=1}^{L}(\mu_{il}\bar{y}_{kl}),$$

where $\bar{z}_k$, $\bar{y}_{kl}$, and $\bar{u}_{ik}^r$ are obtained from the solution of [SP_2]. The heuristic is activated at the final iteration of the Lagrangean procedure. The performance of the heuristic is reported in the next section.

Note that one can also use $\bar{w}_j$ and $\bar{x}_{ijk}$, an optimal solution of [SP_1], to solve [SP_2] with the additional set of constraints

$$z_\alpha\sum_{r\in R}\left(F_{ik}^r u_{ik}^r+C_{ik}^r\mathscr{L}\sum_{l=1}^{L}\sigma_{il}^2 y_{kl}\right)+\mathscr{X}\sum_{l=1}^{L}(\mu_{il}y_{kl})=\sum_{j=1}^{J}\bar{x}_{ijk}$$

in order to generate a feasible solution. However, this might fail to find a feasible solution to the problem when the production level $\bar{x}_{ijk}$ obtained by solving [SP_1] is not enough to simultaneously satisfy demand and hold safety stocks. This could be fixed using some greedy procedure.

4. Computational Results and Discussion

The proposed solution procedure is coded in Gams, where the master problems and subproblems are solved using Cplex 9.1. The tests were carried on a Sun Blade 2500 workstation with 1.6-GHz UltraSPARC IIIi processors.

4.1. Test Problems

The test problems are generated according to a procedure similar to the one used by Elhedhli and Goffin (2005). We use random sets of coordinates from a unit square to generate the coordinates for plants, DCs, and retailers. The transportation costs c_{ijk} and t_{ijk} are set to: $c_{ijk}=10\times d_{jk}$ and $t_{ikl}=10\times d'_{kl}$, where d_{jk} and d'_{kl} are the Euclidean distances between plant j and DC k, and DC k and retailer l, respectively. The average transportation lead time $\mathscr{L}$ between DCs and retailers is set to one. The expected demand μ_{il} is generated uniformly on U[50, 300] and its variance σ_{il}^2 is generated uniformly on U[0, 100]. The capacities of plants and DCs are set to

$$P_j=\beta_j^1\times\sum_i\sum_l\mu_{il},\quad j=1,\ldots,J$$
$$V_k=\beta_j^2\times\sum_i\sum_l\mu_{il},\quad k=1,\ldots,K,$$

where β_j^1 and β_j^2 are parameters that can be used to vary the ratio of total demand to plant and DC capacities, respectively. The fixed cost of plants and DCs and the inventory-holding costs are set to

$$f_j=\gamma_1\times\{\text{U}[500,1{,}000]+\text{U}[1{,}000,2{,}000]\times P_j^{0.5}\},\quad j=1,\ldots,J$$
$$g_k=\gamma_2\times\{\text{U}[100,500]+\text{U}[500,1{,}000]\times V_k^{0.25}\},\quad k=1,\ldots,K$$
$$h_i=\gamma_3\times\text{U}[5,10],\quad i=1,\ldots,I,$$

where γ_1, γ_2, and γ_3 are parameters that can be used to vary the fixed facility location costs of plants and

DCs, and inventory-holding costs, respectively. The concavity of the fixed facility location costs accounts for economies of scale. The scaling parameters β_j^1, β_j^2, γ_1, and γ_2 are generated uniformly on U[0.7, 1]. The service level is set to 95%, which corresponds to $z_\alpha = 1.645$. All the data have been rounded to the nearest integer. The test problems are denoted by $(I.J.K.L)$, where I is the number of products, J is the number of potential plants, K is the number of potential DCs, and L is the number of retailers. The values of the parameters are set to $I = 1, 3$; $J = 5, 10, 15$; $K = 10, 20, 30$; $L = 30, 40, 50$; and $R = 5$.

The optimality tolerance ϵ for the Lagrangean problem was set to 10^{-6}. The starting Lagrangean multipliers (λ_{ik}) were set to dual variables of the flow conservation constraints of the LP relaxation of $[P_L]$. The number of linear segments R required to approximate safety stock is set to five. The plant capacity utilization (PCU) is defined as the ratio of total demand to the total capacity of open plants, and the DC capacity utilization (DCU) is defined as the ratio of total demand to the total capacity of open DCs. The plant fixed cost (PFC), the DC fixed cost (DCFC), the variable cost (VC), and safety-stock cost (SSC) are expressed as percentage of total costs. We perform the testing on 54 problem instances of the model by varying the parameter settings. All computation times reported here are the time used by the solver (CPLEX), and hence exclude input times.

4.2. Performance

In this section, we report on the performance of the Lagrangean-relaxation based solution method in terms of the lower bound, heuristic solution, and the computational time over a wide range of problems. In Tables 1–6, the quality of the Lagrangean bound with all the redundant constraints (LAG3) is compared with the Lagrangean bound without the redundant constraints (LAG0) and the linear programming bound (LP) for different scenarios. The lower bounds (LP and LAG0) are expressed as the percentage of Lagrangean bound with all the redundant constraints (LAG3). The quality of the heuristic solution (GAP) is also expressed as the percentage of the Lagrangean bound with redundant constraints (LAG3): $100 \times (\text{Heuristic Solution} - \text{LAG3})/\text{LAG3}$. The percentage error due to the linear approximation of safety stock (SSE) is expressed as: $100 \times (\text{Nonlinear Safety Stock} - \text{Linear Safety Stock})/\text{Nonlinear Safety Stock}$. The columns marked PCU, DCU, #DC give the plant capacity utilization, the DC capacity utilization, and the number of DCs opened, respectively. The columns marked PFC, DFC, VC, and SSC indicate the plant fixed costs, DC fixed cost, production and transportation cost, and safety-stock cost, respectively (expressed as percentage of total cost). In all these test problems, the heuristic is activated at the termination of the Lagrangean problem. The tables also show the computational time of subproblems (SP_1 and SP_2), the master problem (MP), and the heuristic (HR) as the percentage of the total computational time (CPU) for various scenarios.

For the base-case scenario, the parameters β_1, β_2, γ_1, γ_2, and γ_3 are set to $\beta_1 \sim$ U[0.35, 0.75], $\beta_2 \sim$ U[0.35, 0.75], $\gamma_1 \sim$ U[0.8, 1], $\gamma_2 \sim$ U[0.8, 1], and $\gamma_3 \sim$ U[0.8, 1]. To test the solution procedure under a wide variety of settings, the plant capacity, the DC capacity, fixed cost of opening plants and DCs, and the safety-stock cost were varied by changing β_1, β_2, γ_1, γ_2, and γ_3 as follows:

- Tight capacities scenario: $\beta_1 \sim$ U[0.25, 0.5] and $\beta_2 \sim$ U[0.25, 0.5],
- Excess capacities scenario: $\beta_1 \sim$ U[1, 3] and $\beta_2 \sim$ U[1, 3],
- Dominant fixed-cost scenario: The variable cost are scaled down to $c_{ijk} = 0.01 \times d_{jk}$ and $t_{ikl} = 0.01 \times d'_{kl}$.
- Dominant variable-cost scenario: $\gamma_1 \sim$ U[0.01, 0.1], $\gamma_2 \sim$ U[0.01, 0.1], and $\gamma_3 \sim$ U[0.05, 0.1],
- Dominant safety-stock cost scenario: $\gamma_1 \sim$ U[0.01, 0.1], $\gamma_2 \sim$ U[0.01, 0.1], and $\gamma_3 \sim$ U[5, 10]. The variable cost are scaled down to $c_{ijk} = 0.01 \times d_{jk}$ and $t_{ikl} = 0.01 \times d'_{kl}$.

The lower capacity utilization level for plants and DCs approximates an unconstrained environment, whereas higher capacity utilization represents a constrained environment, and is important for comparative analysis of the results indicating the computational difficulty of solving tightly capacitated problems. Table 7 summarizes the average bounds and the heuristic performance for various scenarios in terms of solution quality and computational time. From these results, we make the observations:

- The LP bound can be as low as 40.22% (Table 6) and is, on average, 61.06% for all instances tested (Table 7).
- The addition of the redundant constraints helps to improve the quality of the Lagrangean lower bound by 14.67% on average (Table 7).
- As far as the quality of the heuristic solution is concerned, it is within a maximum of 5% of the optimal solution. For the base, tight capacity, excess capacity, dominant fixed cost, dominant variable cost, and dominant safety-stock costs, the heuristic solution is, on average, 2.52%, 2.50%, 1.48%, 1.74%, 1.57%, and 2.40% from the optimal solution, respectively (Table 7). The heuristic performs better in the case of excess capacity and dominant fixed-cost scenarios, where the heuristic solution is within a maximum of 2.96% (Table 3) and 2.78% (Table 4) from the optimal solution, respectively.

Table 1 Comparison of the Bounds and Heuristic Performance: Base-Case Scenario

					Cost components (%)				LB		UB	Computational time					
No.	*I.J.K.L*	PCU	DCU	#DC	PFC	DFC	VC	SSC	LP	LAG0	Gap (%)	SP_1	SP_2	MP	HR	CPU (M)	SSE (%)
1.	1.5.10.30	0.45	0.42	4	18.84	18.53	55.50	7.13	61.07	86.50	0.32	6.33	93.44	0.12	0.10	22.30	6.42
2.	1.5.10.40	0.29	0.37	4	11.00	27.56	57.80	3.64	65.89	82.70	0.73	5.66	94.22	0.08	0.04	51.38	5.62
3.	1.5.10.50	0.45	0.45	6	20.18	18.73	53.12	7.97	53.44	91.03	1.16	4.51	90.87	0.41	4.22	36.81	3.01
4.	1.5.20.30	0.34	0.43	5	14.92	27.83	54.96	2.29	51.98	91.62	3.64	2.01	97.96	0.02	0.01	58.21	2.14
5.	1.5.20.40	0.62	0.46	5	28.64	35.46	30.73	5.17	47.41	81.55	1.94	10.22	86.51	2.70	0.57	60.17	3.27
6.	1.5.20.50	0.48	0.48	6	23.07	36.14	37.51	3.28	65.34	83.64	0.34	5.32	92.57	0.20	1.91	61.11	1.71
7.	1.5.30.30	0.38	0.46	5	17.69	22.34	52.17	7.80	45.88	82.15	2.36	13.39	83.70	2.16	0.75	92.39	2.75
8.	1.5.30.40	0.61	0.52	5	31.49	18.67	43.14	6.71	54.44	84.16	3.28	4.21	94.53	0.54	0.72	132.98	0.82
9.	1.5.30.50	0.48	0.49	8	23.71	17.93	55.44	2.92	59.64	82.14	4.83	9.24	87.83	1.99	0.94	88.20	4.28
10.	1.10.10.30	0.36	0.49	4	17.43	22.68	51.52	8.36	57.11	87.23	4.07	13.78	85.40	0.64	0.17	145.72	5.16
11.	1.10.10.40	0.53	0.51	4	27.18	9.73	53.63	9.47	45.56	92.37	3.75	1.70	94.13	3.49	0.69	129.70	7.68
12.	1.10.10.50	0.47	0.43	5	20.20	9.34	61.77	8.70	48.79	84.09	3.68	13.47	82.62	3.19	0.73	124.83	0.45
13.	1.10.20.30	0.27	0.52	5	13.99	17.32	64.65	4.04	62.36	83.14	0.60	11.87	83.79	2.51	1.82	164.12	4.65
14.	1.10.20.40	0.30	0.53	5	15.99	17.24	56.83	9.93	68.96	88.38	2.56	7.03	89.11	3.05	0.82	225.67	3.44
15.	1.10.20.50	0.57	0.46	6	26.14	18.52	49.71	5.64	60.02	83.78	0.09	9.61	87.83	2.30	0.26	217.91	7.46
16.	1.10.30.30	0.39	0.39	5	14.92	16.78	65.85	2.45	71.94	79.53	4.20	9.88	87.44	1.45	1.23	198.44	2.73
17.	1.10.30.40	0.28	0.45	5	12.60	13.72	70.98	2.70	55.30	83.78	4.08	9.56	88.37	0.76	1.31	223.26	5.30
18.	1.10.30.50	0.55	0.42	7	23.32	21.30	49.01	6.38	64.45	80.64	2.81	11.19	84.56	2.33	1.91	192.00	0.33
19.	1.15.10.30	0.47	0.48	4	22.72	20.87	52.64	3.77	54.38	82.65	4.47	5.18	91.82	2.62	0.37	259.02	5.00
20.	1.15.10.40	0.44	0.54	4	23.82	14.18	53.68	8.32	61.08	84.78	3.40	3.78	92.68	1.79	1.76	253.82	3.65
21.	1.15.10.50	0.32	0.45	5	14.64	15.86	60.21	9.30	50.76	84.79	4.33	3.80	93.11	2.36	0.73	329.33	1.12
22.	1.15.20.30	0.56	0.50	5	27.94	12.62	53.77	5.67	70.17	86.71	4.21	5.22	92.70	0.78	1.29	245.24	1.22
23.	1.15.20.40	0.54	0.48	5	25.89	10.29	55.17	8.64	62.03	87.85	1.73	10.09	85.13	3.30	1.48	250.11	1.56
24.	1.15.20.50	0.46	0.51	6	23.42	10.31	58.27	8.00	53.78	93.00	4.27	8.73	88.73	2.43	0.10	238.41	4.37
25.	1.15.30.30	0.49	0.47	5	23.15	15.61	54.78	6.46	49.23	86.78	2.94	0.25	95.42	3.82	0.52	265.54	4.67
26.	1.15.30.40	0.44	0.57	5	24.79	18.37	48.65	8.19	40.88	86.34	4.58	9.90	86.95	1.29	1.86	211.75	7.41
27.	1.15.30.50	0.31	0.52	6	16.16	20.71	54.05	9.08	64.94	89.64	3.03	12.63	83.20	3.81	0.36	274.27	4.91
28.	3.5.10.30	0.63	0.53	7	33.05	17.56	46.06	3.33	68.81	88.27	3.45	7.48	89.33	1.71	1.47	313.65	3.64
29.	3.5.10.40	0.43	0.40	6	17.11	11.01	65.47	6.41	46.64	88.13	4.27	16.32	82.19	0.65	0.84	455.34	6.58
30.	3.5.10.50	0.48	0.38	8	18.33	20.10	53.43	8.14	48.31	85.72	4.22	16.49	82.26	0.02	1.23	493.93	4.51
31.	3.5.20.30	0.44	0.49	6	21.40	22.66	48.54	7.41	42.46	85.45	1.36	5.99	90.29	2.31	1.41	524.97	4.98
32.	3.5.20.40	0.35	0.42	7	14.74	10.98	70.74	3.53	43.19	82.99	2.49	16.48	81.49	1.08	0.95	419.28	1.77
33.	3.5.20.50	0.59	0.40	7	23.66	11.94	55.68	8.72	43.39	84.89	3.02	6.71	91.42	1.67	0.21	446.69	5.21
34.	3.5.30.30	0.57	0.53	8	30.39	13.00	47.88	8.73	56.20	83.40	0.72	3.77	92.99	1.54	1.70	559.05	0.79
35.	3.5.30.40	0.60	0.47	7	28.15	12.98	54.45	4.43	65.62	91.94	0.85	3.99	91.81	4.11	0.10	452.94	0.84
36.	3.5.30.50	0.46	0.50	8	23.19	10.54	61.10	5.17	59.58	83.23	3.74	4.58	94.08	0.53	0.81	408.19	1.80
37.	3.10.10.30	0.49	0.58	5	28.55	14.00	48.45	9.00	60.55	89.53	3.75	14.36	82.33	2.83	0.48	460.99	3.05
38.	3.10.10.40	0.44	0.42	6	18.16	22.46	52.37	7.01	52.05	77.13	1.91	12.23	86.03	1.03	0.71	484.29	5.59
39.	3.10.10.50	0.32	0.42	6	13.41	20.73	56.47	9.39	61.77	89.92	4.49	9.94	86.27	3.53	0.26	503.12	0.28
40.	3.10.20.30	0.50	0.46	7	22.92	10.79	56.36	9.93	50.74	90.83	0.09	9.72	86.85	1.51	1.92	559.11	0.21
41.	3.10.20.40	0.53	0.43	8	23.13	11.10	61.82	3.95	65.31	81.78	0.18	2.73	92.64	2.66	1.96	557.62	5.38
42.	3.10.20.50	0.30	0.43	8	12.85	11.55	71.83	3.78	45.69	82.28	0.04	15.63	81.84	0.83	1.70	591.42	5.20
43.	3.10.30.30	0.53	0.54	6	28.11	17.37	45.89	8.63	54.46	84.62	0.07	16.08	82.06	0.32	1.54	544.15	2.88
44.	3.10.30.40	0.52	0.48	8	25.29	15.18	56.79	2.74	59.91	79.65	3.15	2.80	94.31	2.39	0.50	681.38	1.14
45.	3.10.30.50	0.61	0.47	9	28.32	20.45	48.70	2.53	48.15	90.20	0.58	11.60	86.35	0.15	1.90	659.15	3.09
46.	3.15.10.30	0.47	0.41	6	19.43	16.90	55.16	8.52	74.30	85.89	1.69	6.85	91.16	0.54	1.44	634.84	4.53
47.	3.15.10.40	0.37	0.53	6	19.74	16.90	54.16	9.20	55.80	81.81	1.10	3.40	93.26	2.75	0.59	582.57	4.21
48.	3.15.10.50	0.37	0.52	7	19.35	14.49	59.42	6.74	47.02	87.62	2.70	12.11	84.16	2.29	1.44	546.92	6.94
49.	3.15.20.30	0.52	0.45	8	23.19	11.60	56.29	8.92	49.96	86.08	0.65	4.47	94.31	0.04	1.18	617.21	4.37
50.	3.15.20.40	0.34	0.48	6	16.19	21.24	53.93	8.65	42.97	88.54	1.14	2.69	93.61	3.50	0.20	592.28	5.93
51.	3.15.20.50	0.64	0.50	9	32.13	21.35	43.76	2.76	61.79	82.32	2.74	4.40	91.39	2.66	1.56	812.80	3.94
52.	3.15.30.30	0.50	0.42	7	20.98	14.28	58.28	6.46	63.59	92.50	2.93	8.06	90.10	1.38	0.46	681.41	2.77
53.	3.15.30.40	0.50	0.55	8	27.33	10.69	57.95	4.03	50.11	82.49	3.43	7.97	90.42	0.74	0.87	792.90	3.54
54.	3.15.30.50	0.63	0.63	9	39.77	21.90	29.59	8.74	71.73	91.14	3.93	15.26	81.25	2.26	1.24	624.80	2.67
Min		0.27	0.37	4	11.00	9.34	29.59	0.47	40.88	77.13	0.04	0.25	81.25	0.02	0.01	22.30	0.21
Max		0.64	0.63	9	39.77	36.14	71.83	8.19	74.30	93.00	4.83	16.49	97.96	4.11	4.22	812.80	7.68
Average		0.46	0.47	6.15	22.09	17.27	54.19	2.35	56.24	85.73	2.52	8.35	88.87	1.76	1.02	362.11	3.65

Table 2 Comparison of the Bounds and Heuristic Performance: Tight Capacities Scenario

				Cost components (%)				LB			Computational time					
No.	*I.J.K.L*	PCU	DCU	PFC	DFC	VC	SSC	LP	LAG0	UB Gap (%)	SP_1	SP_2	MP	HR	CPU (M)	SSE (%)
1.	1.5.10.30	0.90	0.79	38.41	21.64	33.66	6.29	59.08	90.27	0.32	7.77	86.07	2.56	3.60	12.19	3.65
2.	1.5.10.40	0.85	0.75	30.34	23.23	42.86	3.58	76.35	76.09	0.69	6.17	87.72	5.16	0.95	34.42	0.31
3.	1.5.10.50	0.93	0.76	34.04	34.46	23.73	7.77	60.61	90.02	1.18	9.37	78.66	8.41	3.56	47.89	5.02
4.	1.5.20.30	0.94	0.82	39.07	39.86	19.08	1.98	61.22	76.74	3.36	8.45	88.52	1.97	1.06	54.89	2.06
5.	1.5.20.40	0.86	0.90	33.60	17.25	44.36	4.79	56.19	93.97	2.07	9.10	86.51	3.02	1.37	55.12	6.13
6.	1.5.20.50	0.79	0.83	26.51	25.80	44.70	2.99	73.53	81.26	0.36	1.89	90.79	5.23	2.09	57.28	4.61
7.	1.5.30.30	0.80	0.87	22.80	34.05	34.96	8.19	48.04	76.91	2.40	8.67	76.91	9.77	4.65	78.81	2.66
8.	1.5.30.40	0.78	0.87	28.67	20.67	44.72	5.94	62.17	89.13	3.02	4.33	85.00	5.70	4.97	89.46	1.93
9.	1.5.30.50	0.94	0.81	27.34	15.03	56.50	1.14	64.22	75.82	3.47	9.42	79.53	9.80	1.25	116.51	5.53
10.	1.10.10.30	0.87	0.78	30.51	34.41	33.28	1.80	57.45	85.05	4.30	3.88	87.86	4.12	4.14	123.18	2.62
11.	1.10.10.40	0.79	0.78	33.83	17.04	45.36	3.77	44.60	89.80	3.64	6.94	87.72	2.96	2.38	124.00	2.57
12.	1.10.10.50	0.78	0.86	35.20	25.47	37.07	2.27	57.61	82.33	3.52	10.45	81.39	7.17	0.99	141.78	0.96
13.	1.10.20.30	0.82	0.87	15.20	20.68	62.61	1.51	68.25	83.83	0.63	3.42	92.32	0.75	3.51	163.42	3.53
14.	1.10.20.40	0.88	0.84	19.60	38.20	39.54	2.66	78.05	91.80	2.45	1.23	87.97	7.01	3.79	185.07	3.45
15.	1.10.20.50	0.81	0.85	32.31	19.72	47.07	0.91	69.07	91.20	0.10	2.60	83.51	8.54	5.36	190.05	5.67
16.	1.10.30.30	0.81	0.87	22.23	20.59	56.54	0.63	83.39	75.39	4.45	2.36	86.01	7.63	4.00	191.02	3.27
17.	1.10.30.40	0.94	0.79	23.77	22.88	52.65	0.71	53.98	83.21	4.01	5.19	80.58	9.36	4.88	216.96	4.14
18.	1.10.30.50	0.82	0.80	19.54	22.98	54.99	2.50	73.97	93.35	2.59	8.07	84.70	2.68	4.54	217.46	1.31
19.	1.15.10.30	0.93	0.77	37.50	38.87	22.11	1.51	57.57	78.25	4.14	9.49	79.79	5.87	4.85	218.04	2.42
20.	1.15.10.40	0.89	0.80	31.32	28.01	38.44	2.23	65.16	92.58	3.57	6.51	90.67	0.76	2.05	219.41	5.55
21.	1.15.10.50	0.77	0.74	37.80	19.99	39.70	2.50	48.38	89.09	4.65	1.96	86.60	8.01	3.42	221.89	7.23
22.	1.15.20.30	0.77	0.80	37.12	19.31	42.29	1.29	72.88	86.04	4.34	6.45	89.91	2.99	0.65	224.62	0.81
23.	1.15.20.40	0.79	0.81	35.65	39.23	23.39	1.73	73.23	79.19	1.87	5.91	88.04	1.42	4.64	236.10	6.85
24.	1.15.20.50	0.79	0.85	21.52	17.12	59.66	1.69	58.75	80.11	4.30	7.31	87.68	3.80	1.21	237.52	1.34
25.	1.15.30.30	0.90	0.71	33.64	34.33	29.70	2.33	48.20	83.76	2.92	6.04	89.44	3.20	1.32	252.79	0.96
26.	1.15.30.40	0.92	0.72	22.30	38.23	36.43	3.04	47.78	80.60	4.35	2.64	85.49	9.03	2.85	253.18	1.20
27.	1.15.30.50	0.85	0.83	30.10	17.75	49.41	2.73	61.81	87.14	2.98	9.94	83.79	3.15	3.12	307.65	8.13
28.	3.5.10.30	0.78	0.75	37.67	37.16	24.31	0.85	82.35	88.21	3.25	7.60	85.73	4.40	2.26	308.16	4.96
29.	3.5.10.40	0.92	0.75	29.27	19.56	48.66	2.50	49.56	80.49	4.06	6.09	86.77	5.08	2.06	371.92	1.65
30.	3.5.10.50	0.77	0.72	34.36	38.06	24.51	3.07	51.08	76.44	3.87	6.91	89.00	0.00	4.09	389.17	2.73
31.	3.5.20.30	0.93	0.76	23.17	17.37	57.61	1.85	45.17	84.76	1.25	2.44	89.16	5.38	3.02	420.63	1.34
32.	3.5.20.40	0.81	0.84	19.81	24.11	55.25	0.82	47.86	86.92	2.49	2.63	89.65	5.55	2.17	434.14	1.82
33.	3.5.20.50	0.88	0.73	37.80	34.96	24.28	2.96	50.20	90.15	2.96	5.09	87.19	5.85	1.86	440.48	3.20
34.	3.5.30.30	0.92	0.78	21.41	20.39	55.43	2.77	58.72	75.05	0.78	6.47	87.92	4.06	1.55	441.49	5.81
35.	3.5.30.40	0.81	0.76	36.02	23.12	39.84	1.03	70.51	82.49	0.92	2.17	86.12	8.19	3.51	447.21	5.27
36.	3.5.30.50	0.88	0.78	39.89	23.25	35.62	1.24	56.82	92.75	3.42	4.25	85.98	4.39	5.38	453.81	7.85
37.	3.10.10.30	0.76	0.71	31.53	19.16	45.87	3.44	66.20	85.77	4.00	9.46	85.83	2.11	2.59	456.21	1.93
38.	3.10.10.40	0.87	0.75	32.28	37.44	28.42	1.85	57.03	88.31	1.78	7.58	78.65	8.51	5.26	465.15	4.61
39.	3.10.10.50	0.85	0.77	29.55	16.53	52.37	1.55	65.08	76.70	4.70	10.14	81.47	4.27	4.12	478.10	4.61
40.	3.10.20.30	0.94	0.76	36.41	29.60	31.25	2.73	59.65	88.05	0.09	3.72	84.27	6.57	5.43	482.99	5.31
41.	3.10.20.40	0.81	0.86	25.30	26.76	46.51	1.43	77.95	90.10	0.19	7.91	86.59	0.25	5.24	495.35	6.34
42.	3.10.20.50	0.87	0.78	20.00	16.45	62.56	0.99	49.20	93.07	0.04	9.73	82.51	4.41	3.36	498.61	5.89
43.	3.10.30.30	0.87	0.83	35.77	20.11	42.37	1.74	63.59	83.22	0.06	2.98	92.09	4.22	0.71	498.84	2.44
44.	3.10.30.40	0.92	0.78	38.46	23.68	36.92	0.94	60.42	89.66	3.06	6.15	87.11	1.58	5.16	505.27	1.58
45.	3.10.30.50	0.86	0.85	39.10	19.35	40.74	0.82	53.21	84.92	0.55	10.18	82.96	4.83	2.03	523.24	4.15
46.	3.15.10.30	0.94	0.81	22.10	24.64	50.97	2.29	84.34	79.69	1.75	9.98	86.76	1.95	1.31	535.69	1.10
47.	3.15.10.40	0.94	0.85	22.63	16.90	58.53	1.94	55.07	93.07	1.04	4.51	89.77	1.66	4.06	580.58	4.07
48.	3.15.10.50	0.78	0.72	30.96	24.94	41.86	2.25	49.99	79.20	2.81	1.21	88.83	5.68	4.28	596.11	7.76
49.	3.15.20.30	0.87	0.78	32.72	20.49	43.83	2.96	48.47	87.75	0.60	7.74	84.58	4.33	3.35	602.71	1.17
50.	3.15.20.40	0.89	0.84	19.35	39.64	39.10	1.91	44.47	87.77	1.16	10.43	78.14	7.96	3.47	614.54	2.10
51.	3.15.20.50	0.88	0.72	18.92	34.19	46.42	0.47	61.99	90.50	2.58	10.92	83.69	2.81	2.58	636.12	5.59
52.	3.15.30.30	0.86	0.71	38.04	27.54	33.39	1.03	75.21	87.91	2.67	4.62	86.63	5.28	3.46	653.36	6.84
53.	3.15.30.40	0.91	0.73	24.56	18.62	56.19	0.63	58.50	94.00	3.33	8.61	84.64	3.18	3.57	696.61	7.33
54.	3.15.30.50	0.94	0.70	38.47	20.70	38.69	2.14	76.93	88.30	4.11	2.09	80.21	2.26	4.27	761.52	1.16
Min		0.76	0.70	15.20	15.03	19.08	0.47	44.47	75.05	0.04	1.21	76.91	0.00	0.65	12.19	0.31
Max		0.94	0.90	39.89	39.86	62.61	8.19	84.34	94.00	4.70	10.92	92.32	9.80	5.43	761.52	8.13
Average		0.86	0.79	29.92	25.58	42.15	2.35	61.13	85.34	2.50	6.24	85.66	4.72	3.17	334.42	3.75

Table 3 Comparison of the Bounds and Heuristic Performance: Excess Capacities Scenario

				Cost components (%)				LB			Computational time					
No.	*I.J.K.L*	PCU	DCU	PFC	DFC	VC	SSC	LP	LAG0	UB Gap (%)	SP_1	SP_2	MP	HR	CPU (M)	SSE (%)
1.	1.5.10.30	0.31	0.16	35.35	12.65	46.45	5.55	66.77	81.65	2.70	21.19	71.79	6.62	0.39	41.15	3.65
2.	1.5.10.40	0.43	0.21	23.35	23.20	51.30	2.15	51.75	80.36	1.86	13.62	75.35	7.05	3.98	53.73	2.47
3.	1.5.10.50	0.12	0.12	25.00	23.76	43.92	7.32	52.43	90.62	2.11	24.18	66.72	8.68	0.43	65.74	2.44
4.	1.5.20.30	0.28	0.21	36.41	29.91	31.87	1.81	77.95	88.14	2.33	27.51	62.38	7.76	2.36	68.71	1.85
5.	1.5.20.40	0.22	0.10	39.59	18.43	39.56	2.42	64.31	86.46	2.29	12.28	81.52	4.69	1.52	75.07	0.92
6.	1.5.20.50	0.37	0.26	35.28	28.67	33.69	2.36	60.87	75.50	0.63	16.45	77.13	2.17	4.25	80.48	1.50
7.	1.5.30.30	0.43	0.12	36.30	16.48	42.78	4.44	50.91	75.28	0.34	27.28	60.50	10.08	2.14	110.75	3.82
8.	1.5.30.40	0.34	0.28	35.48	23.03	36.32	5.17	68.85	92.94	2.47	22.41	63.26	9.47	4.86	117.15	2.39
9.	1.5.30.50	0.43	0.18	30.10	13.00	56.05	0.85	67.73	89.58	2.33	21.78	69.17	5.33	3.72	139.51	4.50
10.	1.10.10.30	0.44	0.29	35.39	13.61	49.65	1.36	69.69	82.11	0.47	29.94	60.93	7.63	1.50	150.72	6.77
11.	1.10.10.40	0.39	0.30	26.39	26.02	44.70	2.90	55.42	93.02	0.89	26.80	61.21	9.39	2.60	166.21	3.39
12.	1.10.10.50	0.11	0.11	25.94	27.56	45.26	1.24	66.23	86.52	1.69	17.70	70.67	9.39	2.24	191.27	1.21
13.	1.10.20.30	0.43	0.09	29.34	29.32	39.96	1.38	64.05	91.56	1.29	13.15	75.88	7.52	3.45	179.98	7.81
14.	1.10.20.40	0.41	0.31	38.50	24.05	35.44	2.01	62.65	93.71	2.16	16.36	76.44	2.48	4.72	191.18	2.08
15.	1.10.20.50	0.17	0.32	25.46	23.71	49.93	0.90	51.66	92.99	0.33	21.92	66.33	10.39	1.35	202.51	4.22
16.	1.10.30.30	0.40	0.11	29.81	12.06	57.70	0.42	63.65	90.79	1.43	22.33	65.50	10.76	1.40	211.74	6.03
17.	1.10.30.40	0.24	0.29	36.41	25.43	37.69	0.47	56.21	81.09	1.82	27.25	64.15	7.43	1.16	245.71	3.17
18.	1.10.30.50	0.12	0.19	25.28	17.40	55.23	2.10	67.13	83.64	1.57	23.77	69.29	6.49	0.45	240.45	8.31
19.	1.15.10.30	0.36	0.30	30.22	12.82	55.62	1.34	62.00	82.81	2.27	15.81	78.94	4.18	1.08	269.90	4.09
20.	1.15.10.40	0.22	0.16	28.80	12.00	57.14	2.06	74.22	84.95	2.15	22.23	63.26	10.61	3.91	268.75	6.75
21.	1.15.10.50	0.19	0.24	32.28	23.14	42.63	1.96	68.52	86.10	0.12	29.29	58.62	9.66	2.44	237.16	0.91
22.	1.15.20.30	0.29	0.15	23.54	21.56	53.66	1.24	56.26	87.61	2.96	17.64	71.10	7.87	3.39	277.80	3.16
23.	1.15.20.40	0.20	0.12	21.15	21.94	55.36	1.55	62.27	80.77	1.52	23.48	63.90	9.04	3.58	286.51	8.56
24.	1.15.20.50	0.40	0.18	21.38	22.19	55.54	0.90	76.74	90.61	0.16	23.16	66.98	5.27	4.60	263.58	3.22
25.	1.15.30.30	0.25	0.22	21.68	14.25	62.82	1.24	78.38	81.01	2.88	16.83	79.64	1.24	2.28	277.19	5.13
26.	1.15.30.40	0.18	0.11	24.77	26.75	46.87	1.61	54.06	75.19	1.63	16.88	76.80	3.15	3.17	272.37	2.01
27.	1.15.30.50	0.40	0.22	27.45	12.94	58.09	1.52	55.39	75.78	1.25	24.16	69.69	2.08	4.07	363.13	3.45
28.	3.5.10.30	0.13	0.23	39.78	27.59	31.87	0.75	55.45	77.05	1.56	17.11	79.09	1.77	2.02	344.74	1.43
29.	3.5.10.40	0.23	0.26	35.19	10.51	52.51	1.79	50.95	75.99	0.16	19.75	77.91	1.57	0.78	425.35	7.34
30.	3.5.10.50	0.19	0.28	33.05	23.60	40.87	2.48	57.93	85.38	2.84	22.47	70.55	2.94	4.04	467.52	5.49
31.	3.5.20.30	0.13	0.13	29.80	24.65	44.35	1.19	75.92	79.68	0.01	20.02	74.04	5.55	0.39	515.87	7.78
32.	3.5.20.40	0.36	0.24	29.94	14.88	54.53	0.65	68.41	84.52	2.10	25.61	63.95	8.08	2.36	537.56	7.53
33.	3.5.20.50	0.15	0.16	33.55	25.60	38.37	2.49	71.48	75.58	0.46	16.12	78.33	5.06	0.50	441.95	4.35
34.	3.5.30.30	0.44	0.23	28.31	24.80	44.32	2.57	76.05	84.05	1.08	28.48	68.35	2.94	0.23	472.45	6.40
35.	3.5.30.40	0.20	0.11	23.05	29.23	46.72	1.00	51.53	86.34	0.41	12.87	80.29	3.26	3.58	512.85	6.16
36.	3.5.30.50	0.30	0.20	24.29	17.90	56.88	0.92	69.90	84.61	2.38	15.96	70.87	9.74	3.44	505.91	4.18
37.	3.10.10.30	0.37	0.23	21.99	18.94	57.26	1.81	72.20	86.54	2.81	10.81	80.30	8.37	0.53	553.10	5.96
38.	3.10.10.40	0.11	0.08	30.46	21.47	47.13	0.95	59.59	85.20	2.83	20.57	77.23	1.54	0.66	505.84	2.79
39.	3.10.10.50	0.20	0.24	30.70	10.71	57.12	1.47	55.37	89.82	0.95	20.45	72.17	7.15	0.23	573.68	8.24
40.	3.10.20.30	0.20	0.15	25.26	25.46	46.78	2.50	66.81	77.45	1.84	20.64	70.49	8.31	0.55	584.79	7.16
41.	3.10.20.40	0.20	0.32	23.67	10.98	64.35	1.00	53.93	89.58	0.74	27.65	62.74	9.18	0.43	531.23	5.47
42.	3.10.20.50	0.14	0.19	25.83	26.62	46.95	0.59	68.21	92.74	1.68	28.47	68.14	3.05	0.34	609.83	8.57
43.	3.10.30.30	0.32	0.21	37.72	21.60	39.12	1.57	52.95	91.00	1.20	26.67	61.99	6.35	5.00	514.59	5.95
44.	3.10.30.40	0.42	0.12	39.36	11.42	48.40	0.83	77.77	81.53	0.23	27.49	63.51	8.80	0.20	514.39	8.34
45.	3.10.30.50	0.41	0.21	37.93	16.70	44.75	0.62	78.99	89.65	1.18	15.27	77.61	5.64	1.48	556.28	1.17
46.	3.15.10.30	0.24	0.19	36.81	27.42	34.14	1.63	68.73	80.25	0.53	24.27	65.90	8.42	1.42	563.86	6.17
47.	3.15.10.40	0.15	0.22	27.31	14.10	56.90	1.68	79.75	77.95	1.82	20.86	73.32	1.40	4.41	650.03	6.91
48.	3.15.10.50	0.41	0.10	23.98	12.44	61.43	2.15	50.27	92.81	2.32	17.82	73.80	4.07	4.32	651.47	5.26
49.	3.15.20.30	0.25	0.16	38.80	29.47	29.46	2.28	60.97	77.61	1.19	12.99	78.59	4.21	4.21	691.71	1.18
50.	3.15.20.40	0.33	0.31	29.97	29.19	39.36	1.49	61.80	88.31	0.13	21.69	64.91	9.72	3.68	655.87	1.74
51.	3.15.20.50	0.30	0.18	38.81	23.63	37.26	0.30	54.06	76.21	1.12	11.96	78.41	9.01	0.62	649.01	1.50
52.	3.15.30.30	0.40	0.26	31.69	21.46	46.09	0.76	62.50	77.95	1.92	10.08	82.27	4.79	2.86	794.69	3.32
53.	3.15.30.40	0.29	0.28	29.14	23.50	46.89	0.47	58.76	92.42	2.33	24.86	59.65	10.44	5.04	724.58	8.86
54.	3.15.30.50	0.31	0.21	25.85	15.69	56.67	1.79	71.41	81.51	0.58	14.26	76.30	9.30	0.14	894.49	3.52
Min		0.11	0.08	21.15	10.51	29.46	0.30	50.27	75.19	0.01	10.08	58.62	1.24	0.14	41.15	0.91
Max		0.44	0.32	39.78	29.91	64.35	7.32	79.75	93.71	2.96	29.94	82.27	10.76	5.04	894.49	8.86
Average		0.28	0.20	30.24	20.66	47.33	1.78	63.66	84.49	1.48	20.57	70.70	6.43	2.30	379.48	4.57

Table 4 Comparison of the Bounds and Heuristic Performance: Dominant Fixed-Cost Scenario

				Cost components (%)				LB		UB	Computational time					
No.	*I.J.K.L*	PCU	DCU	PFC	DFC	VC	SSC	LP	LAG0	Gap (%)	SP_1	SP_2	MP	HR	CPU (M)	SSE (%)
1.	1.5.10.30	0.68	0.53	76.57	22.30	1.13	0.04	73.41	93.79	2.69	11.52	78.80	7.45	2.23	35.85	8.45
2.	1.5.10.40	0.45	0.69	71.72	22.11	6.17	0.26	82.32	88.00	0.98	11.42	78.24	2.42	7.92	58.61	4.30
3.	1.5.10.50	0.48	0.44	73.44	18.30	8.27	0.67	72.58	87.25	1.20	13.15	82.39	1.15	3.32	69.29	5.95
4.	1.5.20.30	0.70	0.76	78.63	14.14	7.23	0.69	71.17	90.77	1.73	12.73	83.30	3.29	0.68	59.03	1.23
5.	1.5.20.40	0.62	0.50	71.67	23.14	5.19	0.27	66.63	89.39	1.71	12.31	79.86	4.41	3.42	68.25	4.57
6.	1.5.20.50	0.68	0.35	75.55	12.62	11.83	0.14	68.08	88.92	0.94	9.72	71.55	4.51	14.22	90.90	2.09
7.	1.5.30.30	0.71	0.46	77.54	17.59	4.87	0.36	66.69	88.33	1.25	16.35	71.59	5.61	6.46	90.92	6.71
8.	1.5.30.40	0.61	0.66	81.72	15.98	2.30	0.11	73.79	85.32	2.29	8.53	76.31	4.78	10.37	123.60	8.18
9.	1.5.30.50	0.43	0.72	83.74	13.66	2.60	0.04	69.81	94.89	1.15	12.09	80.42	4.85	2.64	156.95	7.04
10.	1.10.10.30	0.56	0.58	77.82	19.02	3.17	0.17	69.43	91.61	2.41	13.11	77.92	2.97	6.00	161.99	5.66
11.	1.10.10.40	0.64	0.66	71.24	16.52	12.24	1.13	73.97	94.65	1.77	12.90	80.23	3.92	2.95	161.20	6.40
12.	1.10.10.50	0.64	0.53	79.94	18.66	1.40	0.09	76.07	85.21	1.40	9.53	72.20	8.80	9.46	165.69	8.21
13.	1.10.20.30	0.61	0.56	75.44	20.66	3.89	0.29	66.44	89.60	1.62	12.29	78.65	5.30	3.76	187.27	7.47
14.	1.10.20.40	0.44	0.62	77.37	10.35	12.28	0.07	67.34	86.95	2.45	14.32	71.83	8.75	5.09	202.84	3.98
15.	1.10.20.50	0.57	0.53	75.76	21.21	3.04	0.21	66.42	93.00	1.09	10.98	80.21	3.12	5.69	224.65	2.07
16.	1.10.30.30	0.79	0.74	71.17	12.04	16.80	0.08	77.37	90.96	1.42	15.36	77.78	2.45	4.41	251.81	6.81
17.	1.10.30.40	0.69	0.43	80.57	17.88	1.55	0.02	70.24	91.91	2.78	12.32	80.80	5.83	1.04	265.36	5.45
18.	1.10.30.50	0.41	0.54	71.85	12.70	15.46	1.05	83.30	86.03	2.30	10.74	70.79	4.40	14.06	210.89	7.83
19.	1.15.10.30	0.41	0.50	80.50	18.27	1.24	0.11	67.24	87.51	2.41	8.23	83.52	7.04	1.20	284.13	4.79
20.	1.15.10.40	0.71	0.59	78.86	19.20	1.94	0.01	80.37	94.60	2.76	12.61	72.13	5.66	9.61	258.53	6.68
21.	1.15.10.50	0.48	0.71	79.43	18.02	2.55	0.08	73.02	93.10	1.95	16.27	70.40	3.47	9.85	253.20	4.19
22.	1.15.20.30	0.64	0.50	83.72	14.58	1.70	0.08	69.06	94.78	1.15	19.89	74.70	5.13	0.28	315.69	7.57
23.	1.15.20.40	0.62	0.40	77.92	16.29	5.79	0.32	67.42	90.17	2.48	14.71	80.41	4.15	0.72	327.36	1.75
24.	1.15.20.50	0.67	0.46	81.40	15.89	2.71	0.06	72.02	93.81	0.99	10.67	71.85	2.46	15.02	240.31	7.73
25.	1.15.30.30	0.49	0.48	78.09	14.07	7.84	0.30	79.70	94.78	2.51	8.94	72.85	2.13	16.08	233.67	3.82
26.	1.15.30.40	0.71	0.45	70.88	13.87	15.24	0.66	83.97	92.60	2.21	9.67	72.25	3.63	14.45	232.36	1.89
27.	1.15.30.50	0.42	0.60	70.83	20.80	8.37	0.72	73.61	92.79	1.40	9.54	77.02	3.48	9.96	325.31	2.18
28.	3.5.10.30	0.76	0.44	78.76	19.22	2.02	0.13	70.38	94.46	2.54	16.35	74.14	8.77	0.75	384.46	2.84
29.	3.5.10.40	0.46	0.76	71.08	16.00	12.92	0.14	74.69	91.98	1.01	19.03	74.47	5.27	1.23	398.34	4.02
30.	3.5.10.50	0.53	0.56	82.82	16.90	0.28	0.03	66.90	94.32	2.40	17.05	71.36	7.19	4.40	513.68	7.59
31.	3.5.20.30	0.69	0.38	75.96	20.40	3.64	0.34	71.99	93.90	1.74	15.32	79.03	5.06	0.60	588.73	2.92
32.	3.5.20.40	0.55	0.53	79.02	20.89	0.09	0.00	68.22	88.81	1.58	14.37	81.43	1.77	2.43	482.70	4.55
33.	3.5.20.50	0.73	0.54	80.02	18.18	1.81	0.08	83.69	94.57	1.58	10.56	84.46	4.36	0.62	519.43	6.30
34.	3.5.30.30	0.61	0.68	71.45	23.60	4.95	0.29	69.73	88.48	1.57	8.32	79.83	1.48	10.37	566.63	5.08
35.	3.5.30.40	0.49	0.73	81.87	16.25	1.88	0.06	75.54	89.27	2.59	15.75	74.30	6.41	3.55	434.93	2.38
36.	3.5.30.50	0.63	0.48	79.94	19.00	0.08	0.98	82.39	90.05	1.56	12.73	81.08	1.39	4.80	515.40	1.10
37.	3.10.10.30	0.56	0.49	74.38	14.87	10.75	0.60	82.27	90.54	2.58	10.77	83.92	4.11	1.21	491.82	1.77
38.	3.10.10.40	0.53	0.57	78.79	20.38	0.83	0.02	74.36	89.61	2.30	12.28	78.91	4.63	4.18	551.88	8.19
39.	3.10.10.50	0.71	0.51	83.06	11.83	5.10	0.11	83.12	90.86	1.20	13.46	73.21	7.10	6.22	559.72	1.42
40.	3.10.20.30	0.59	0.40	81.86	13.49	4.65	0.17	83.90	89.97	1.09	15.76	73.96	4.59	5.69	515.10	7.82
41.	3.10.20.40	0.65	0.44	81.90	16.69	1.42	0.00	83.59	94.58	1.99	8.11	82.43	5.58	3.89	558.10	6.33
42.	3.10.20.50	0.41	0.78	70.21	16.79	13.00	0.95	69.79	91.66	1.08	10.32	71.12	8.59	9.97	580.05	4.42
43.	3.10.30.30	0.41	0.77	72.84	22.66	4.50	0.43	68.77	91.50	1.12	18.79	73.97	5.77	1.47	613.00	5.22
44.	3.10.30.40	0.72	0.56	83.89	12.93	3.18	0.02	83.34	87.26	0.97	17.00	79.46	1.91	1.62	596.22	6.67
45.	3.10.30.50	0.60	0.57	81.18	14.71	4.11	0.04	66.09	94.56	0.87	9.32	78.45	3.70	8.53	643.84	7.20
46.	3.15.10.30	0.45	0.55	82.49	15.83	1.67	0.11	73.88	89.03	2.68	14.19	78.29	1.76	5.76	471.53	8.89
47.	3.15.10.40	0.49	0.36	73.64	13.18	13.18	1.02	80.93	86.70	1.26	10.01	78.54	7.04	4.42	581.70	7.27
48.	3.15.10.50	0.75	0.53	71.56	15.37	13.07	0.46	84.64	86.47	1.56	13.13	76.11	4.33	6.43	705.90	5.10
49.	3.15.20.30	0.72	0.78	72.60	15.14	12.26	0.36	84.93	87.87	1.22	14.52	80.38	4.52	0.58	812.46	2.61
50.	3.15.20.40	0.78	0.67	84.85	12.42	2.73	0.07	81.36	94.56	1.96	9.32	77.07	3.48	10.12	761.94	7.11
51.	3.15.20.50	0.47	0.61	81.24	13.91	4.85	0.46	65.93	87.80	1.24	16.12	74.65	7.82	1.41	624.06	5.29
52.	3.15.30.30	0.51	0.40	74.90	10.44	14.66	1.25	70.20	88.34	2.08	8.70	72.60	1.73	16.97	841.27	8.02
53.	3.15.30.40	0.61	0.42	77.39	20.67	1.94	0.00	77.79	87.31	2.20	18.59	78.96	1.53	0.93	802.34	3.85
54.	3.15.30.50	0.60	0.57	76.69	17.24	6.07	0.56	65.56	92.62	1.19	9.61	75.23	4.85	10.31	903.51	4.30
Min		0.41	0.35	70.21	10.35	0.08	0.00	65.56	85.21	0.87	8.11	70.40	1.15	0.28	35.85	1.10
Max		0.79	0.78	84.85	23.60	16.80	1.25	84.93	94.89	2.78	19.89	84.46	8.80	16.97	903.51	8.89
Average		0.59	0.56	77.37	16.83	5.79	0.31	74.18	90.70	1.74	12.77	76.95	4.55	5.73	390.27	5.21

Table 5 Comparison of the Bounds and Heuristic Performance: Dominant Variable-Cost Scenario

				Cost components (%)				LB		UB	Computational time					
No.	*I.J.K.L*	PCU	DCU	PFC	DFC	VC	SSC	LP	LAG0	Gap (%)	SP_1	SP_2	MP	HR	CPU (M)	SSE (%)
1.	1.5.10.30	0.45	0.24	7.28	9.95	80.13	2.64	54.30	81.75	2.06	4.12	76.56	12.81	6.50	26.34	1.43
2.	1.5.10.40	0.28	0.44	8.01	11.51	78.60	1.89	55.20	82.60	1.26	9.31	76.65	6.82	7.22	42.76	1.97
3.	1.5.10.50	0.20	0.42	10.11	6.47	73.04	10.38	58.78	87.06	2.96	10.84	84.70	0.98	3.49	49.07	3.25
4.	1.5.20.30	0.42	0.24	10.38	5.80	80.90	2.91	54.53	79.95	0.62	5.73	70.38	15.82	8.07	39.89	0.79
5.	1.5.20.40	0.21	0.32	12.73	6.12	73.68	7.47	58.05	84.29	2.35	7.44	81.83	3.67	7.07	46.26	1.01
6.	1.5.20.50	0.38	0.40	13.15	12.19	69.40	5.25	69.13	76.79	2.97	8.51	73.50	12.78	5.22	63.30	2.80
7.	1.5.30.30	0.26	0.47	7.12	12.69	74.45	5.75	71.30	90.86	2.32	7.49	75.99	15.45	1.07	60.79	2.70
8.	1.5.30.40	0.41	0.50	9.24	14.34	67.54	8.88	69.62	77.46	3.01	3.30	70.38	23.39	2.94	79.71	1.41
9.	1.5.30.50	0.39	0.40	6.63	10.41	74.21	8.75	54.16	74.04	0.60	9.26	81.12	6.70	2.92	109.12	2.38
10.	1.10.10.30	0.34	0.36	14.61	7.96	66.72	10.72	54.55	82.85	1.19	9.93	84.96	2.43	2.68	112.39	2.72
11.	1.10.10.40	0.25	0.35	14.13	10.50	72.64	2.73	54.43	77.23	2.36	5.59	81.32	6.18	6.91	105.23	1.15
12.	1.10.10.50	0.33	0.34	11.90	9.25	71.63	7.21	64.94	75.68	2.83	6.20	79.37	14.16	0.28	120.98	2.75
13.	1.10.20.30	0.32	0.47	11.07	11.28	74.57	3.07	53.12	78.05	2.80	8.29	71.36	18.62	1.73	117.81	1.96
14.	1.10.20.40	0.41	0.23	13.50	11.99	65.30	9.21	67.81	76.80	1.98	8.91	82.88	2.49	5.71	136.93	0.92
15.	1.10.20.50	0.24	0.39	14.24	11.36	65.76	8.65	61.26	82.31	0.34	4.07	72.73	16.59	6.61	150.44	3.12
16.	1.10.30.30	0.29	0.28	5.22	6.25	83.28	5.24	50.53	76.81	1.28	2.02	77.75	14.32	5.91	169.31	0.98
17.	1.10.30.40	0.21	0.21	10.39	11.80	74.83	2.98	66.14	92.47	0.65	5.54	72.79	16.53	5.14	202.07	2.93
18.	1.10.30.50	0.35	0.28	11.06	10.45	75.09	3.40	51.58	76.00	0.35	7.55	84.43	1.05	6.98	150.14	1.37
19.	1.15.10.30	0.39	0.25	6.08	13.65	74.14	6.12	51.43	72.66	2.72	4.43	71.98	18.44	5.16	187.90	3.26
20.	1.15.10.40	0.36	0.40	8.83	12.68	70.13	8.37	56.56	85.30	0.48	8.34	77.48	13.44	0.74	154.85	2.22
21.	1.15.10.50	0.30	0.23	11.88	6.81	78.89	2.42	50.28	86.16	1.80	10.14	74.56	9.45	5.85	164.56	1.19
22.	1.15.20.30	0.40	0.27	14.63	8.99	70.89	5.50	51.67	89.74	2.32	6.44	80.96	11.37	1.23	201.07	3.15
23.	1.15.20.40	0.21	0.36	5.74	12.27	71.55	10.44	68.29	72.73	2.29	2.20	76.68	17.09	4.03	212.73	2.62
24.	1.15.20.50	0.31	0.28	8.02	11.54	78.78	1.65	55.22	83.00	2.37	6.19	80.36	8.70	4.76	165.22	1.59
25.	1.15.30.30	0.33	0.47	6.65	5.18	78.00	10.17	58.61	91.21	2.45	8.76	84.34	0.92	5.98	174.57	1.00
26.	1.15.30.40	0.26	0.36	7.84	5.93	83.92	2.31	69.11	82.56	3.03	8.70	77.02	10.57	3.71	152.94	3.05
27.	1.15.30.50	0.23	0.25	12.90	11.18	73.42	2.50	54.58	80.51	0.50	8.80	79.71	9.19	2.30	255.41	3.71
28.	3.5.10.30	0.38	0.43	8.54	7.72	76.97	6.77	73.55	90.84	0.26	10.87	72.57	13.41	3.16	272.92	1.74
29.	3.5.10.40	0.30	0.30	11.09	9.38	69.83	9.70	70.49	82.11	2.21	10.69	81.23	3.55	4.52	294.97	3.39
30.	3.5.10.50	0.42	0.48	6.30	6.50	83.19	4.01	66.09	79.97	0.50	9.84	79.36	4.53	6.27	353.64	0.89
31.	3.5.20.30	0.30	0.48	8.90	13.03	72.98	5.09	51.24	86.24	2.74	3.45	74.67	16.09	5.79	391.33	2.61
32.	3.5.20.40	0.21	0.37	6.78	6.98	83.83	2.42	52.26	91.55	2.88	2.24	84.37	10.92	2.47	297.11	2.81
33.	3.5.20.50	0.34	0.42	6.12	10.45	77.46	5.97	63.07	82.28	2.70	7.42	80.04	7.86	4.68	348.48	3.20
34.	3.5.30.30	0.40	0.22	9.16	14.46	65.98	10.39	66.28	78.85	2.02	2.21	80.53	16.38	0.89	373.41	3.29
35.	3.5.30.40	0.29	0.30	10.66	10.71	70.34	8.29	68.25	72.64	2.83	8.35	82.56	2.20	6.88	263.22	2.02
36.	3.5.30.50	0.21	0.25	10.84	9.57	69.60	9.99	72.85	76.13	0.36	5.28	73.02	19.70	2.00	329.09	2.36
37.	3.10.10.30	0.25	0.33	13.45	13.15	66.35	7.05	56.08	79.82	0.15	3.45	71.43	19.86	5.27	320.49	1.57
38.	3.10.10.40	0.42	0.34	13.21	6.71	78.04	2.04	57.34	85.67	1.09	5.35	70.13	19.98	4.54	390.26	2.02
39.	3.10.10.50	0.41	0.39	11.78	5.02	79.95	3.24	51.15	91.80	0.49	9.50	77.79	10.33	2.38	436.63	0.89
40.	3.10.20.30	0.43	0.41	11.29	7.94	74.59	6.18	65.49	91.09	0.50	5.71	83.39	7.16	3.75	360.94	2.97
41.	3.10.20.40	0.21	0.24	14.11	9.43	68.06	8.39	56.66	81.82	0.23	7.38	77.03	8.03	7.56	416.16	1.47
42.	3.10.20.50	0.32	0.45	10.00	10.61	77.67	1.73	72.31	92.22	0.12	9.55	72.06	15.83	2.56	387.06	2.96
43.	3.10.30.30	0.44	0.28	5.90	11.00	73.78	9.32	68.22	79.30	2.16	2.81	72.84	17.14	7.21	417.08	3.11
44.	3.10.30.40	0.39	0.36	11.38	6.37	72.91	9.34	51.18	85.57	1.50	5.13	82.45	8.21	4.22	430.55	1.11
45.	3.10.30.50	0.26	0.47	7.02	10.11	75.36	7.51	74.15	72.18	0.14	7.18	75.93	14.51	2.37	469.81	0.83
46.	3.15.10.30	0.26	0.33	13.72	12.87	62.63	10.77	63.29	77.22	1.15	10.04	77.91	10.51	1.53	320.71	2.15
47.	3.15.10.40	0.42	0.44	5.77	10.35	73.51	10.38	54.09	78.11	2.97	10.65	75.01	7.63	6.72	426.88	1.77
48.	3.15.10.50	0.20	0.33	7.23	5.35	83.05	4.37	70.23	70.91	2.92	3.26	84.40	4.21	8.12	451.26	0.85
49.	3.15.20.30	0.35	0.30	11.60	9.12	68.89	10.39	73.98	86.40	0.61	6.15	72.03	18.22	3.60	575.06	2.47
50.	3.15.20.40	0.43	0.47	6.33	5.04	80.70	7.93	59.15	85.79	2.03	3.37	75.39	16.92	4.32	501.29	2.79
51.	3.15.20.50	0.21	0.49	6.18	5.12	78.18	10.52	62.23	72.06	0.78	5.05	72.01	15.75	7.19	480.79	3.27
52.	3.15.30.30	0.42	0.24	9.58	7.15	72.44	10.83	65.92	70.89	0.25	6.94	84.16	1.69	7.20	622.18	1.47
53.	3.15.30.40	0.28	0.21	9.95	14.30	70.38	5.36	74.15	75.72	0.25	6.47	78.47	13.18	1.88	476.57	2.75
54.	3.15.30.50	0.27	0.34	9.62	9.41	76.23	4.74	61.56	80.01	1.15	4.51	82.24	11.71	1.54	667.66	1.92
Min		0.20	0.21	5.22	5.02	62.63	1.65	50.28	70.89	0.12	2.02	70.13	0.92	0.28	26.34	0.79
Max		0.45	0.50	14.63	14.46	83.92	10.83	74.15	92.47	3.03	10.87	84.96	23.39	8.12	667.66	3.71
Average		0.32	0.35	9.81	9.56	74.16	6.47	61.23	81.37	1.57	6.68	77.68	11.21	4.42	269.02	2.15

Table 6 Comparison of the Bounds and Heuristic Performance: Dominant Safety-Stock Cost Scenario

					Cost components (%)				LB		UB	Computational time					
No.	*I.J.K.L*	PCU	DCU	#DC	PFC	DFC	VC	SSC	LP	LAG0	Gap (%)	SP_1	SP_2	MP	HR	CPU (M)	SSE (%)
1.	1.5.10.30	0.53	0.75	3	14.91	22.21	19.80	43.08	53.47	84.92	1.79	1.65	89.75	0.95	7.64	23.47	7.14
2.	1.5.10.40	0.53	0.75	3	18.83	2.06	23.09	56.03	57.16	79.81	2.86	9.53	83.26	3.45	3.75	44.21	7.31
3.	1.5.10.50	0.71	0.76	5	16.86	0.95	32.19	50.00	47.05	79.60	2.37	5.95	81.69	3.20	9.17	34.95	9.26
4.	1.5.20.30	0.70	0.89	5	17.62	6.39	21.22	54.76	43.97	92.99	3.40	8.69	76.61	1.85	12.85	52.64	10.05
5.	1.5.20.40	0.52	0.76	5	9.91	25.97	22.60	41.52	45.90	79.64	3.00	3.58	93.65	2.05	0.73	60.53	2.96
6.	1.5.20.50	0.50	0.77	5	11.16	14.91	32.34	41.58	47.07	78.43	3.70	7.57	75.29	0.99	16.15	62.54	3.82
7.	1.5.30.30	0.66	0.78	4	9.01	13.05	20.70	57.25	54.83	78.97	3.11	3.39	89.10	2.24	5.27	87.52	1.28
8.	1.5.30.40	0.61	0.87	4	14.43	6.34	30.19	49.05	56.15	85.09	0.81	3.65	90.25	1.03	5.07	119.35	8.09
9.	1.5.30.50	0.52	0.70	5	12.26	13.21	30.01	44.52	43.76	89.18	2.19	7.24	84.30	1.29	7.17	80.71	3.40
10.	1.10.10.30	0.65	0.69	3	16.72	4.22	29.86	49.19	50.17	78.35	2.63	3.39	80.22	1.82	14.58	124.61	4.37
11.	1.10.10.40	0.68	0.87	3	15.62	7.33	25.26	51.78	50.40	82.40	2.09	9.74	77.42	2.37	10.47	109.24	7.29
12.	1.10.10.50	0.70	0.76	5	15.95	7.72	22.02	54.32	58.54	78.04	3.53	6.69	84.18	2.22	6.91	110.80	4.23
13.	1.10.20.30	0.56	0.86	5	13.02	18.81	26.59	41.58	46.15	80.63	2.60	10.63	81.93	1.23	6.21	153.90	2.92
14.	1.10.20.40	0.55	0.68	5	13.10	7.64	17.78	61.48	58.93	88.73	2.98	7.97	89.68	2.30	0.05	231.64	8.38
15.	1.10.20.50	0.68	0.84	5	13.75	7.55	27.16	51.55	54.05	83.30	3.18	1.71	84.17	1.63	12.49	191.03	2.97
16.	1.10.30.30	0.51	0.66	4	11.73	20.90	22.75	44.63	53.25	89.63	2.58	6.41	89.90	3.23	0.46	169.69	2.54
17.	1.10.30.40	0.74	0.86	4	23.50	16.70	19.54	40.25	43.49	75.28	1.18	5.26	82.61	1.65	10.48	223.35	9.24
18.	1.10.30.50	0.73	0.83	6	17.52	10.75	30.93	40.80	42.82	93.58	2.30	9.58	86.63	3.37	0.42	178.81	10.12
19.	1.15.10.30	0.64	0.68	3	14.17	24.33	21.40	40.10	40.54	89.79	2.11	1.90	87.21	3.41	7.47	243.11	1.63
20.	1.15.10.40	0.66	0.68	3	18.86	0.02	32.92	48.20	59.04	88.62	2.17	5.36	90.65	3.50	0.50	230.90	6.92
21.	1.15.10.50	0.70	0.74	5	12.94	12.81	26.20	48.05	54.44	75.40	3.69	8.18	89.31	2.38	0.13	282.37	7.27
22.	1.15.20.30	0.68	0.88	5	20.07	0.86	23.53	55.53	57.83	93.32	3.12	9.52	84.42	1.13	4.92	233.14	7.43
23.	1.15.20.40	0.66	0.69	5	17.69	11.44	25.33	45.55	52.62	75.55	2.51	8.13	88.99	2.28	0.61	234.55	1.14
24.	1.15.20.50	0.63	0.82	5	10.67	13.95	21.44	53.95	51.88	81.61	2.09	9.43	84.08	3.67	2.82	218.04	9.35
25.	1.15.30.30	0.71	0.86	4	23.45	0.26	32.00	44.29	52.40	83.30	3.18	9.72	78.66	2.92	8.70	233.82	4.58
26.	1.15.30.40	0.54	0.73	4	16.90	21.35	18.57	43.18	51.20	91.83	3.50	4.86	90.91	2.76	1.47	222.18	10.21
27.	1.15.30.50	0.63	0.73	5	15.96	4.65	33.08	46.31	40.22	92.82	1.06	8.69	84.60	1.47	5.24	264.71	4.53
28.	3.5.10.30	0.68	0.69	3	16.46	7.45	27.68	48.41	40.43	84.43	2.74	1.79	86.09	3.63	8.49	323.31	2.85
29.	3.5.10.40	0.73	0.68	3	18.30	7.41	16.29	58.00	59.22	88.58	2.93	7.30	75.32	2.34	15.04	438.77	7.66
30.	3.5.10.50	0.73	0.73	5	14.75	1.53	29.75	53.97	59.38	78.05	0.89	1.44	75.05	3.74	19.77	420.38	7.58
31.	3.5.20.30	0.63	0.67	5	23.04	1.30	27.31	48.35	43.35	76.58	0.95	1.25	78.18	1.64	18.93	460.73	4.75
32.	3.5.20.40	0.71	0.75	5	9.78	3.24	23.10	63.88	41.76	89.94	2.87	10.67	86.01	2.90	0.42	382.68	3.08
33.	3.5.20.50	0.73	0.88	5	11.99	16.86	29.16	41.99	51.27	80.27	2.91	5.32	91.18	2.32	1.18	449.38	10.35
34.	3.5.30.30	0.59	0.77	4	19.12	14.05	23.22	43.60	59.00	77.01	3.59	4.77	82.76	1.38	11.09	579.59	3.12
35.	3.5.30.40	0.56	0.72	4	20.53	1.92	12.58	64.97	57.58	90.65	2.30	1.26	82.77	1.79	14.18	400.71	6.38
36.	3.5.30.50	0.63	0.66	5	17.78	5.34	12.02	64.86	41.18	81.61	2.86	6.47	86.10	3.38	4.06	416.40	8.89
37.	3.10.10.30	0.59	0.76	3	8.34	9.31	22.62	59.73	42.83	81.27	1.30	5.46	76.67	3.14	14.74	385.00	6.15
38.	3.10.10.40	0.57	0.89	3	15.44	7.46	18.56	58.53	58.60	89.29	2.56	1.55	94.08	1.61	2.76	488.61	1.33
39.	3.10.10.50	0.51	0.70	5	14.19	6.06	27.02	52.74	52.94	75.99	1.37	3.51	91.00	1.12	4.37	529.39	9.77
40.	3.10.20.30	0.60	0.86	5	15.47	0.97	28.20	55.36	43.40	91.80	3.35	7.98	75.05	3.63	13.34	515.17	5.38
41.	3.10.20.40	0.63	0.80	5	10.82	0.67	24.22	64.30	53.72	83.57	2.24	3.50	79.54	3.56	13.40	564.54	2.00
42.	3.10.20.50	0.51	0.79	5	11.54	17.44	27.01	44.01	48.20	84.78	1.62	2.37	76.32	1.90	19.41	587.34	8.84
43.	3.10.30.30	0.59	0.70	4	18.11	7.05	20.00	54.84	44.56	86.06	1.84	4.65	92.40	1.67	1.28	459.25	6.19
44.	3.10.30.40	0.63	0.80	4	8.25	5.85	21.18	64.71	56.62	94.55	3.03	2.18	83.82	2.96	11.04	610.99	2.93
45.	3.10.30.50	0.56	0.90	5	22.83	9.42	22.43	45.32	54.67	80.04	2.44	9.44	86.96	1.84	1.76	641.67	1.22
46.	3.15.10.30	0.52	0.82	3	20.14	4.83	33.37	41.65	41.38	88.16	1.61	1.08	87.83	1.83	9.27	643.17	3.61
47.	3.15.10.40	0.68	0.68	3	14.73	23.33	20.56	41.38	44.35	91.31	1.22	10.57	84.73	3.23	1.47	496.92	6.27
48.	3.15.10.50	0.70	0.82	5	13.93	19.28	22.79	44.00	48.86	85.75	1.52	4.48	91.81	2.71	1.00	505.80	8.04
49.	3.15.20.30	0.63	0.82	5	12.32	1.39	22.63	63.67	42.97	78.69	2.09	4.53	78.42	0.89	16.16	619.25	3.41
50.	3.15.20.40	0.66	0.84	5	17.46	0.30	30.28	51.96	47.33	89.63	3.42	3.45	80.95	3.09	12.51	510.54	5.09
51.	3.15.20.50	0.66	0.80	5	18.16	13.84	22.15	45.84	52.06	85.31	3.24	4.91	81.25	2.85	10.99	844.59	3.78
52.	3.15.30.30	0.69	0.75	4	16.91	4.73	13.46	64.90	44.73	81.78	1.05	2.25	75.72	1.09	20.94	597.82	2.07
53.	3.15.30.40	0.63	0.78	4	12.34	10.50	26.54	50.63	54.65	88.28	1.07	8.11	86.95	1.20	3.74	815.65	9.19
54.	3.15.30.50	0.54	0.72	5	11.45	14.78	13.22	60.55	43.00	81.05	2.65	3.39	82.31	1.99	12.32	566.00	8.19
Min		0.50	0.66	3	8.25	0.02	12.02	40.10	40.22	75.28	0.81	1.08	75.05	0.89	0.05	23.47	1.14
Max		0.74	0.90	6	23.50	25.97	33.37	64.97	59.38	94.55	3.70	10.67	94.08	3.74	20.94	844.59	10.35
Average		0.63	0.77	4.35	15.38	9.49	24.18	50.94	49.91	84.36	2.40	5.59	84.24	2.29	7.88	342.14	5.68

Table 7 Comparison of the Average Bounds and Heuristic Performance for Various Scenarios

No.	Scenario	PCU	DCU	Cost components (%) PFC	DFC	VC	SSC	LB LP	LAG0	UB Gap (%)	Computational time SP_1	SP_2	MP	HR	CPU (M)	SSE (%)
1.	Base case	0.46	0.47	22.09	17.27	54.19	2.35	56.24	85.73	2.52	8.35	88.87	1.76	1.02	362.11	3.65
2.	Tight capacities	0.86	0.79	29.92	25.58	42.15	2.35	61.13	85.34	2.50	6.24	85.66	4.72	3.17	334.42	3.75
3.	Excess capacities	0.28	0.20	30.24	20.66	47.33	1.78	63.66	84.49	1.48	20.57	70.70	6.43	2.30	379.48	4.57
4.	Dominant fixed cost	0.59	0.56	77.37	16.83	5.79	0.31	74.18	90.70	1.74	12.77	76.95	4.55	5.73	390.27	5.21
5.	Dominant variable cost	0.32	0.35	9.81	9.56	74.16	6.47	61.23	81.37	1.57	6.68	77.68	11.21	4.42	269.02	2.15
6.	Dominant safety-stock cost	0.63	0.77	15.38	9.49	24.18	50.94	49.91	84.36	2.40	5.59	84.24	2.29	7.88	342.14	5.68
Average		0.52	0.52	30.80	16.57	41.30	10.70	61.06	85.33	2.04	10.03	80.68	5.16	4.09	346.24	4.17

- The error caused due to approximation of the safety stock can be as high as 10.35% (Table 6) and is, on average, 4.17% for all instances tested (Table 7). In most cases this error is fairly small compared to the objective function value. Hence, the piecewise-linear safety stock is a good approximation of the actual safety stock.
- In terms of computational time, the solution methodology proposed succeeds in finding feasible solutions that are within close range of the optimal (average of 2.04%) in reasonable computational time (average of 346.24 minutes) for problems with up to five products, 15 plants, 30 DCs, and 50 retailers. This efficiency is in part due to preserving most of the characteristics of the original model in the subproblems. The total computational time can be as high as 903.5 minutes (Table 4) in some cases.
- It is evident that the solution of the subproblems accounts for most of the computational time, SP_1: 10.03% and SP_2: 80.68% on average for the instances tested (Table 7). The master problem accounts for 5.16%, whereas the heuristic accounts for 4.09% on average (Table 7). This supports the claim that the difficulty of the original problem has been transferred to the subproblems, while keeping them still computationally tractable. The benefit is in getting a lower bound and a heuristic solution close to optimal solution.
- The results in Table 6 suggest that as holding cost increases relative to plant and DC location cost and transportation cost, the number of DCs opened decreases. For example, the maximum number of DCs opened under dominant safety-stock cost scenario is six (Table 6), compared to nine under the base-case scenario (Table 1). Therefore, the model prescribes opening few DCs so that the system can make better use of the risk-pooling effect.

4.2.1. Approximation Error. In Table 8, the number of linear segments (R) of the piecewise-linear safety-stock term is varied to see how well the approximation of the nonlinear cost function performs and whether the solution obtained is in fact in the region where the linear approximation is expected to be accurate. The results suggest that this error is within a maximum of 7.68% and 3.65%, on average, for $R=5$ for the selected instances of the base case. Upon increasing the number of linear segments R to 10, this error is within a maximum of 6.60% and 2.77%, on average. Furthermore, increasing the number of linear segments R to 15, the error reduced to a maximum of 4.19% and 1.84%, on average. Hence, the error decreases gradually as the number of linear segments increases, but at the expense of computational time (on average from a base case of 362.11 minutes for $R=5$ to 429.86 minutes for $R=10$ and to 489.67 minutes for $R=15$). Increasing R increases the number of constraints and binary variables almost exponentially.

5. Conclusions

In this paper, we presented a model that integrates safety-stock decisions in the design of a multiproduct production-inventory-distribution system. The model determines the plant and DC locations, shipment levels from plants to the DCs, safety-stock levels at the DCs, and the assignment of retailers to DCs by minimizing the fixed facility location costs, transportation costs, and safety-stock costs. It was evident that the inclusion of safety-stock decisions introduced significant computational complexity in the problem, which was alleviated by using a piecewise-linear approximation. We employed Lagrangean relaxation to the model by exploiting the echelon structure of the problem, and obtained subproblems that preserved most of the characteristics of the original problem. We presented an efficient heuristic to produce a good feasible solution to the problem. We also added redundant constraints to the subproblems, which improved the quality of the Lagrangean bound. The computational results on a wide variety of problems demonstrate that the Lagrangean bound is quite close to our heuristic solution, which is within 5% of the optimal solution. The results also show that the linearized safety-stock values provide a good approximation of actual nonlinear safety stock, as the error is within 9%. It is worthwhile to note that some of the difficulty of

Table 8 Approximation Quality

		$R=5$		$R=10$		$R=15$	
No.	*I.J.K.L*	SSE (%)	CPU (M)	SS error (%)	CPU (M)	SS error (%)	CPU (M)
1.	1.5.10.30	6.42	22.30	5.33	31.00	3.47	43.09
2.	1.5.10.40	5.62	51.38	4.85	51.37	2.98	52.87
3.	1.5.10.50	3.01	36.81	2.77	35.61	1.54	38.48
4.	1.5.20.30	2.14	58.21	1.19	52.60	0.96	62.64
5.	1.5.20.40	3.27	60.17	2.48	82.27	2.22	113.05
6.	1.5.20.50	1.71	61.11	1.50	78.98	0.76	87.78
7.	1.5.30.30	2.75	92.39	2.07	119.54	1.11	163.22
8.	1.5.30.40	0.82	132.98	0.50	144.46	0.41	131.10
9.	1.5.30.50	4.28	88.20	4.01	86.10	2.28	95.25
10.	1.10.10.30	5.16	145.72	3.39	185.69	3.09	209.10
11.	1.10.10.40	7.68	129.70	5.88	170.89	3.11	208.18
12.	1.10.10.50	0.45	124.83	0.39	133.55	0.17	179.12
13.	1.10.20.30	4.65	164.12	4.22	194.42	1.97	205.99
14.	1.10.20.40	3.44	225.67	2.30	262.88	1.57	312.15
15.	1.10.20.50	7.46	217.91	6.60	217.68	4.19	257.23
16.	1.10.30.30	2.73	198.44	2.51	260.82	1.05	328.57
17.	1.10.30.40	5.30	223.26	3.24	237.76	3.43	325.66
18.	1.10.30.50	0.33	192.00	0.23	230.37	0.13	290.72
19.	1.15.10.30	5.00	259.02	3.16	233.85	3.20	257.14
20.	1.15.10.40	3.65	253.82	2.52	248.64	1.78	312.02
21.	1.15.10.50	1.12	329.33	0.70	375.04	0.68	359.46
22.	1.15.20.30	1.22	245.24	1.10	284.88	0.51	309.04
23.	1.15.20.40	1.56	250.11	1.48	235.53	0.68	258.94
24.	1.15.20.50	4.37	238.41	3.90	272.02	1.72	366.31
25.	1.15.30.30	4.67	265.54	3.04	361.64	2.10	503.18
26.	1.15.30.40	7.41	211.75	6.47	275.41	3.82	261.95
27.	1.15.30.50	4.91	274.27	4.06	369.85	3.20	407.34
28.	3.5.10.30	3.64	313.65	2.43	324.00	2.08	344.73
29.	3.5.10.40	6.58	455.34	4.88	561.88	3.18	510.75
30.	3.5.10.50	4.51	493.93	3.31	538.42	2.88	712.48
31.	3.5.20.30	4.98	524.97	3.57	604.36	2.59	674.26
32.	3.5.20.40	1.77	419.28	1.02	527.34	1.03	561.12
33.	3.5.20.50	5.21	446.69	3.53	510.15	2.60	593.46
34.	3.5.30.30	0.79	559.05	0.51	761.19	0.44	716.90
35.	3.5.30.40	0.84	452.94	0.65	587.10	0.27	696.71
36.	3.5.30.50	1.80	408.19	1.16	430.19	0.70	527.14
37.	3.10.10.30	3.05	460.99	1.97	441.25	1.27	570.62
38.	3.10.10.40	5.59	484.29	4.14	551.23	3.30	713.68
39.	3.10.10.50	0.28	503.12	0.23	567.24	0.13	769.58
40.	3.10.20.30	0.21	559.11	0.16	760.99	0.10	1,023.04
41.	3.10.20.40	5.38	557.62	3.41	678.20	2.79	700.49
42.	3.10.20.50	5.20	591.42	3.23	736.03	2.01	947.83
43.	3.10.30.30	2.88	544.15	1.97	587.74	1.51	670.10
44.	3.10.30.40	1.14	681.38	0.95	896.42	0.55	1,156.77
45.	3.10.30.50	3.09	659.15	1.90	710.14	2.02	674.20
46.	3.15.10.30	4.53	634.84	2.69	793.07	1.79	820.25
47.	3.15.10.40	4.21	582.57	3.44	544.80	2.08	624.03
48.	3.15.10.50	6.94	546.92	4.90	697.74	2.60	953.54
49.	3.15.20.30	4.37	617.21	4.08	776.62	1.91	855.94
50.	3.15.20.40	5.93	592.28	5.31	694.48	2.98	745.03
51.	3.15.20.50	3.94	812.80	3.48	1,113.25	2.23	1,085.52
52.	3.15.30.30	2.77	681.41	2.29	937.71	1.78	906.19
53.	3.15.30.40	3.54	792.90	2.23	877.97	1.45	922.70
54.	3.15.30.50	2.67	624.80	2.19	770.06	1.05	825.58
Min		0.21	22.30	0.16	31.00	0.10	38.48
Max		7.68	812.80	6.60	1,113.25	4.19	1,156.77
Average		3.65	362.11	2.77	429.86	1.84	489.67

the original problem was transferred to the subproblems, while keeping them computationally tractable.

This research can be extended in a number of important ways. Inclusion of other inventory decisions: frequency and size of the shipments from plants to the DCs and from DCs to the retailers based on different replenishment policies, and lead time in addition to safety-stock inventory in the model, would be a direction worth pursuing. Another would be to include stockout and backorder costs in the model. From a computational point of view, further improvement of the solution methodology is another future research venue.

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