SRE Presentation

Circuit simulations for Hopfield Networks (TSP)

Guide: Prof. Udayan Ganguly

Mentor: Vivek Saraswat

29.11.2019

Presentation by: Navnit Kumar, Microelectronics DD student

Background

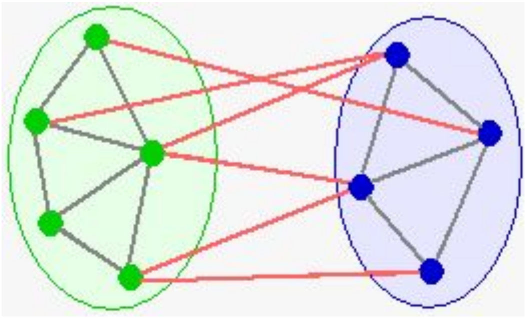
- In a paper (by Duane) , he suggested how oscillator differential equations can be used to solve NP-Hard problems.
- The role of noise was also critical there.
- The constraints of the problem became the coupling equations of the oscillator network.
- Our aim is to find out a simple circuit implementation of the differential equations as suggested by him

Motivation

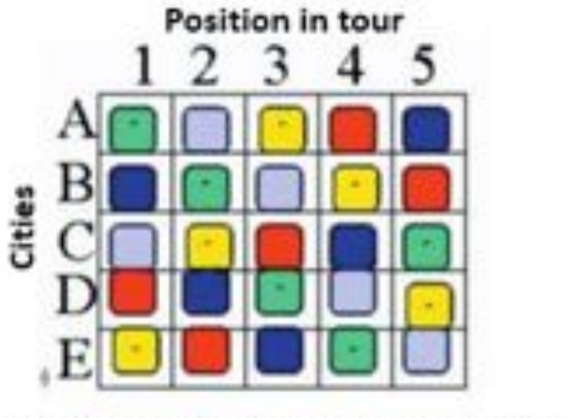
- Resource constraint based graphical optimisation problems are frequently encountered in daily life.

- Boltzmann Machines (Maximum graph Cut),
- Hopfield Networks (TSP, Oscillator Neuron)

- Although some ONN based hardware and software solutions exist in literature, a general way to design hardware for graphical problems is lacking.



Maximum graph Cut problem



Hopfield Tank representation of TSP

Approach to Lypnov’s equation :

We focused on the Hopfield - Tank representation of the TSP (mapping the graph to a network of sinusoidal oscillators)

The general approach to designing hardware involves :

1. Treat the oscillators as a complex number – magnitude, relative phase (all have same frequency)
2. Generate a function which needs to be optimised (Lyapunov function) and decide nature of coupling based on the constraints
3. Generate an oscillator evolution relation with the above function such that the Lyapunov function evolution is monotonic in the desired direction of optimum

Lyapunov Equation

$$L = A\sum_{ij}\left(|z_{ij}|^2 - 1\right)^2 + B\sum_{ij}\left|\left(\frac{z_{ij}}{|z_{ij}|}\right)^n - 1\right|^2$$
$$- C\sum_{i}\sum_{jj'}\left|\frac{z_{ij}}{|z_{ij}|} - \frac{z_{ij'}}{|z_{ij'}|}\right|^2 - D\sum_{j}\sum_{ii'}\left|\frac{z_{ij}}{|z_{ij}|} - \frac{z_{i'j}}{|z_{i'j}|}\right|^2$$
$$+ E\sum_{i}\sum_{j}\sum_{j'} d_{jj'}\,\mathrm{Re}\left(\frac{z_{ij}}{|z_{ij}|}\frac{z^*_{i+1,j'}}{|z_{i+1,j'}|}\right),$$

Lyapunov function for the TSP problem

Phase evolution ensures energy function minimization

$$\dot{z}_{ij} = -\frac{\partial L}{\partial z^*_{ij}}, \quad \dot{z}^*_{ij} = -\frac{\partial L}{\partial z_{ij}}$$

Required relation between the oscillator phase variable z and the energy function L

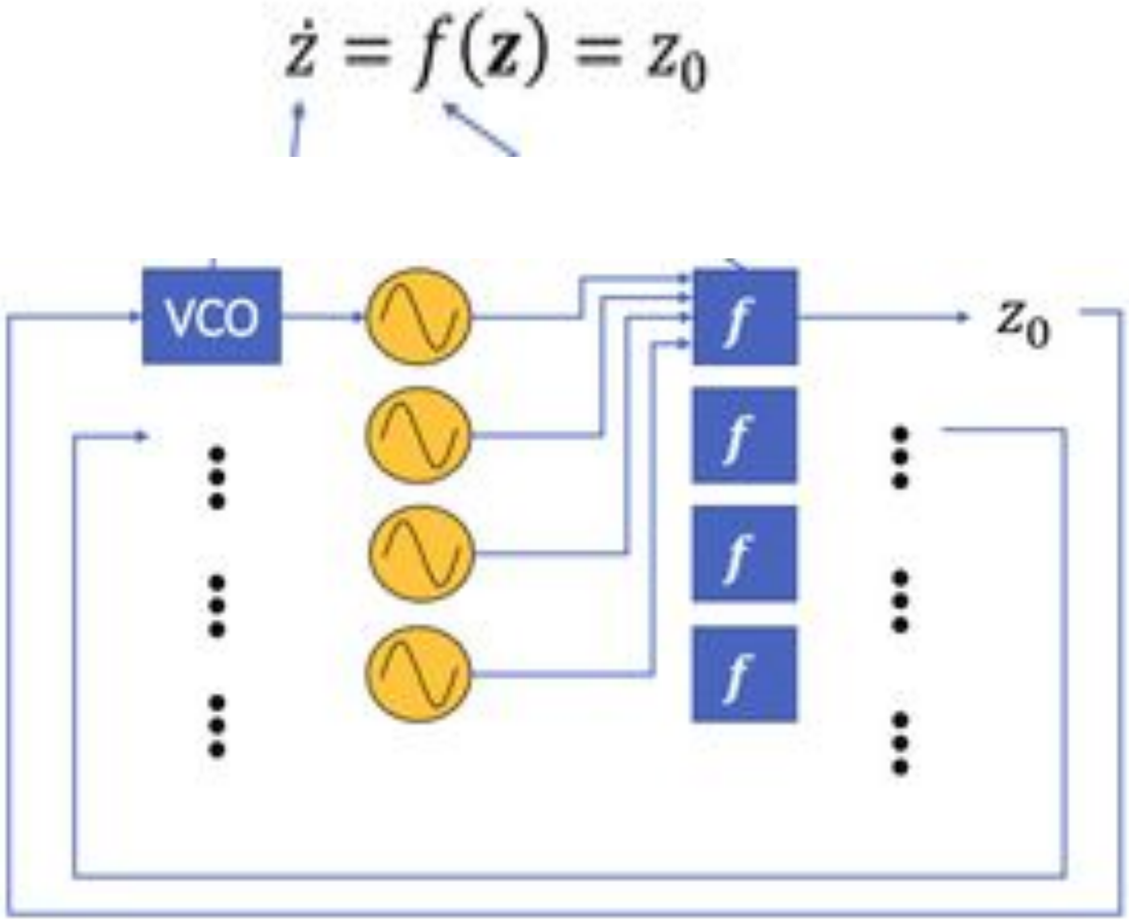
Time evolution of oscillators

Minimization condition:

$$\dot{z}_{ij} = -\frac{\partial L}{\partial z_{ij}^*}, \quad \dot{z}_{ij}^* = -\frac{\partial L}{\partial z_{ij}}$$

$$\begin{aligned}
\frac{dz_{ij}}{dt} = & -2A(|z_{ij}|^2 - 1)z_{ij} - \frac{n}{2}B\left[\left(\frac{z_{ij}}{|z_{ij}|}\right)^n - \left(\frac{z_{ij}^*}{|z_{ij}|}\right)^n\right] * \frac{1}{z_{ij}^*} \\
& + \frac{1}{2}C\sum_{j'}\left[\frac{z_{ij}}{z_{ij}^*|z_{ij}|}\frac{z_{ij'}^*}{|z_{ij'}|} - \frac{1}{|z_{ij}|}\frac{z_{ij'}}{|z_{ij'}|}\right] \\
& + \frac{1}{2}D\sum_{i'}\left[\frac{z_{ij}}{z_{ij}^*|z_{ij}|}\frac{z_{i'j}^*}{|z_{i'j}|} - \frac{1}{|z_{ij}|}\frac{z_{i'j}}{|z_{i'j}|}\right] \\
& - \frac{1}{4}E\sum_{j'} d_{jj'}\left[\frac{1}{|z_{ij}|}\left(\frac{z_{i+1,j'}}{|z_{i+1,j'}|} + \frac{z_{i-1,j'}}{|z_{i-1,j'}|}\right)\right. \\
& \left. - \frac{z_{ij}}{z_{ij}^*|z_{ij}|}\left(\frac{z_{i+1,j'}^*}{|z_{i+1,j'}|} + \frac{z_{i-1,j'}^*}{|z_{i-1,j'}|}\right)\right].
\end{aligned}$$

Network Picture



- Feedback loop reads oscillator states
- Implements ***f*** using read signals
- Outputs net frequency change expected to VCO
- VCO takes care of time evolution of oscillator
- <u>Assumption :</u> magnitude of the oscillators are all fixed (= 1) and have same frequency

Phase Domain Arithmetic method

f has Addition, Subtraction, Conjugation,Multiplication, Division of signals

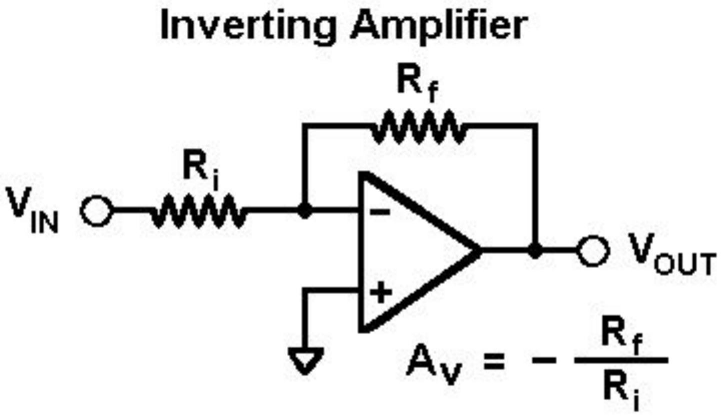
Phase domain arithmetic

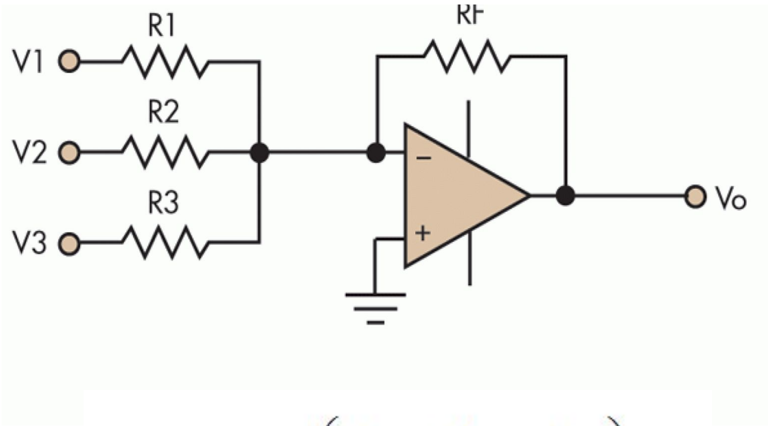
1. *Signal to phase* converter
2. *Inverting Amplifier* : For Phase inversion

3. *Summing Amplifier* : Multiplication and Division in phase domain becomes simple addition and subtraction

4. *Phase to signal* converter

OpAmp Inverting and Summing Amplifier





$$V_{out} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

- Generates inverse phase for conjugate

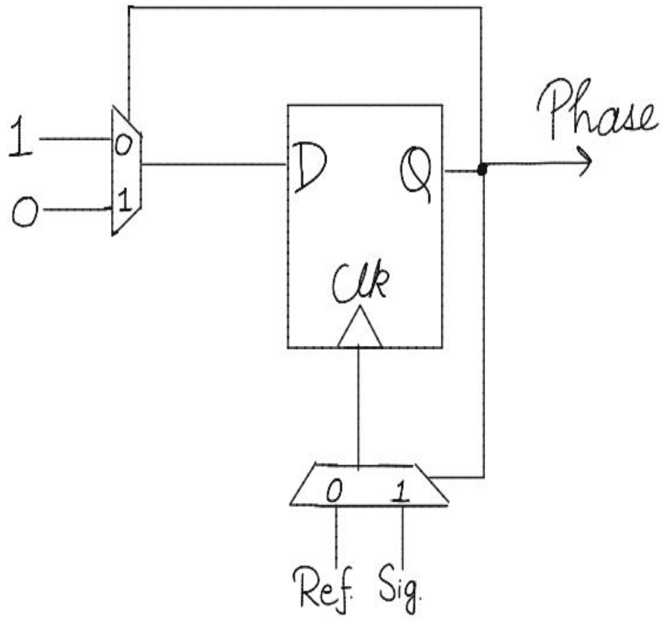
- For Multiplication of signals -> addition of phases
- For Division of signals -> subtraction of phases

Signal to Phase converter

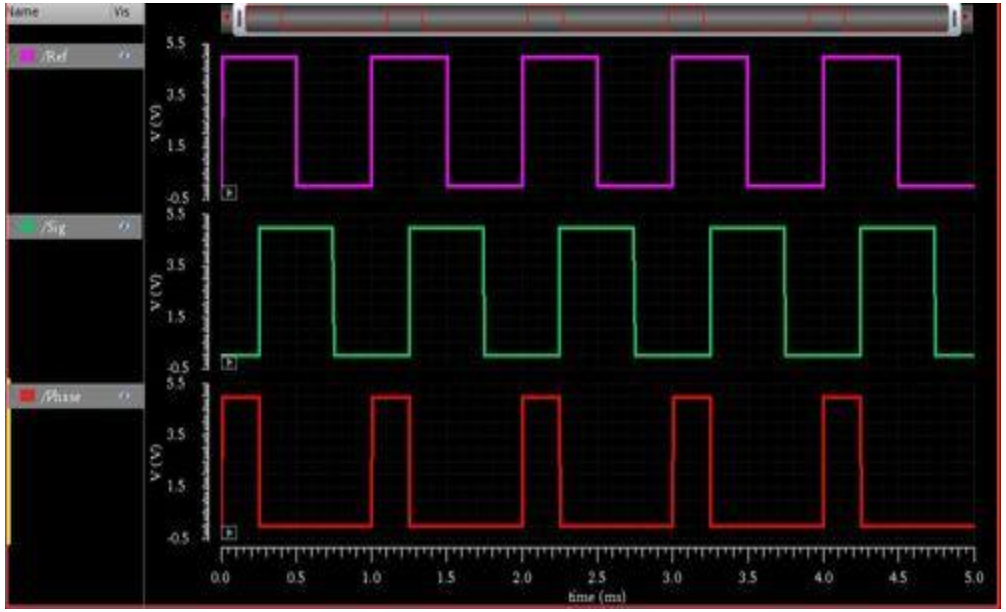
- Need Full phase detector between a reference sinusoid and a signal sinusoid.
- Inputs : digital signals Ref. and Sig

  - generated by passing Ref. and Sig. sinusoids through a DC shifter and a digital buffer (two inverters in series) with negligible delay

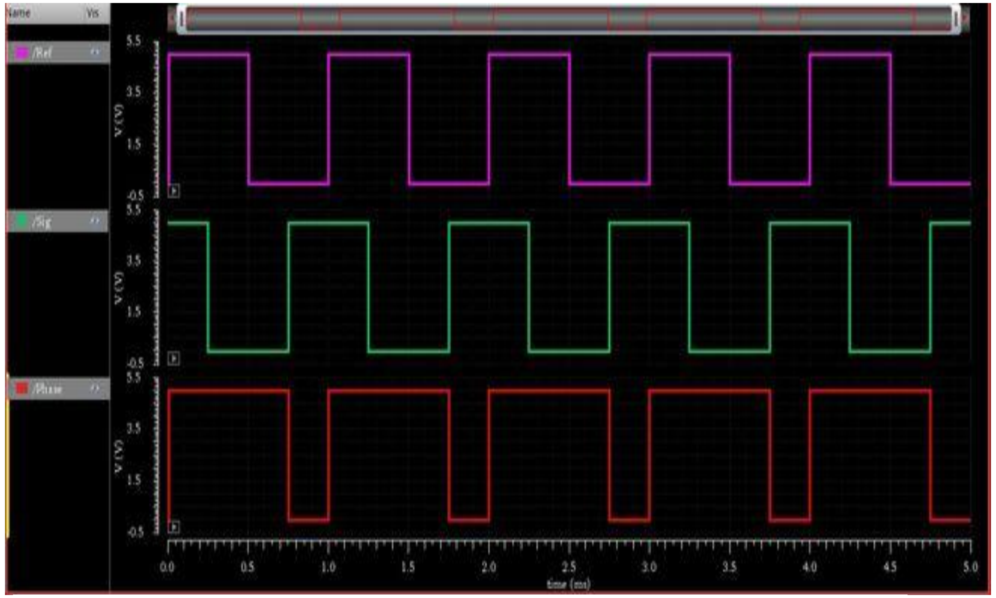
- When Output phase is 0, Ref. is used to sample D (1, for this case) phase goes from 0 to 1 whenever Ref. positive edge triggers.
- Output phase is 1, Sig. is used to sample D (0, in this case), phase going from 1 to 0 whenever Sig. positive edge triggers



Signal to Phase converter



Sig. runs 90 deg behind Ref. Phase is high for 0.25 times total period



Sig. runs 270 deg behind Ref. Phase is high for 0.75 times total period

Alternative/modifications of the circuit: Finite delay in reading phase, takes average of duty cycle which is then sampled by a clk. Alternatively, asynchronous ckt and store the duty cycle fraction as voltage on the RC circuit and read

Local Optima vs Global Optima :

- Both the TSP and associative memory problem (eg. pattern recognition) are a type of optimization problem
- *Associative memory* requires convergence to a *closest local optimum*
- *TSP requires convergence to a global optimum* and local minima need to be avoided
- there is a need for **simulated annealing**, i.e. stochasticity and noise, which help avoid local minima and create good solutions even large *n*

Conclusions

- Mapping graphical problem to a network of oscillators
- Deciding the nature of coupling based on constraints
- Using our general approach we aim to find an optimum for the differential equation of the problem
- Need to avoid local optimum and converge to global optimum value as fast as possible

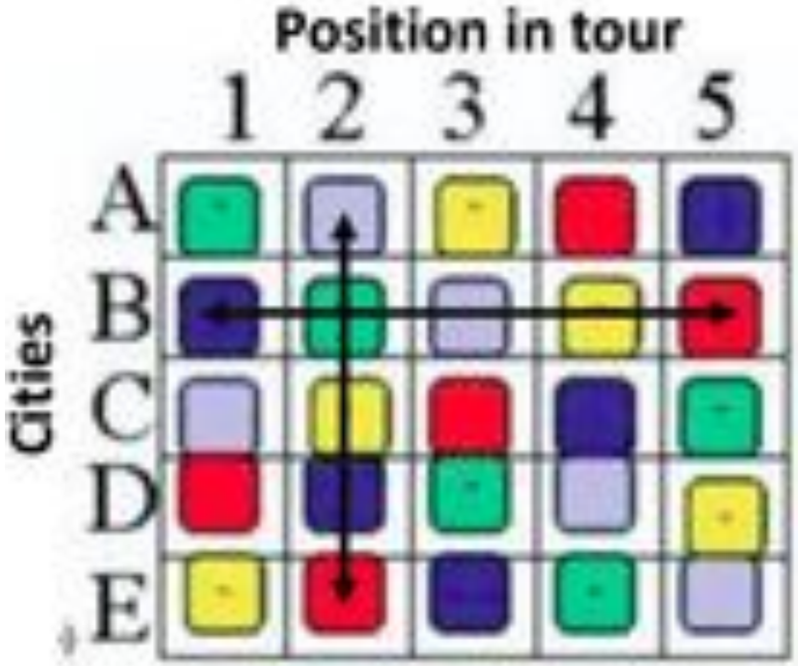
Future Work

- Complete the circuit for TSP using basic repulsion units and distance constraints – analyse analog vs. digital coupling effects

- Explore the role of stochasticity in converging to a global minimum for optimisation problems

- What simplifications in the Lyapunov equation do not affect the output correctness ?

Thank You



1. Oscillators in phase form a valid tour
2. Oscillators in the same row and column repel
3. Oscillators in phase in adjacent columns should minimise distance constraints.

$$L = Terms\ like\ -|z_1 - z_2|^2 + Distance\ constraints$$

$$\dot{z_1} = \frac{z_1^2}{z_2} - z_2 + \ \ldots\ where\ z^* = \frac{1}{z}$$

Example of the TSP