Lab 2: Active Filters using Operational Amplifiers

Objective

The purpose of the lab is to design and compare the frequency plots of Low-pass (LPF) and High-pass (HPF) active filters.

As part of performing this lab, you will

- Determine the amplitude and frequency response characteristics of Low and High Pass filters,
- Compare passive and active LPF and HPF filter configurations, and
- Compare theoretical models with actual filters.

Introduction

You will design, build, and test six filters in this lab. The configurations you will build are

- Passive RC Low-Pass Filter,
- Passive RC High-Pass Filter,
- Active RC Low-Pass Filter,
- Active RC High-Pass Filter,
- Sallen-Key second-order Active Low-Pass Filter,
- Sallen-Key second-order Active Low-Pass Filter.

For each of the configurations, you will

1. Design the filter for a specified cut-off frequency,
2. Model the filter in MatLab,
3. Simulate the design with LTSpice, and
4. Test the design in the Lab.

Steps 1, 2, and 3 are Prelab activities. Step 4 will be performed during the lab period.

In your report, you will compare the theoretical, numerical, and actually measured amplitude and phase plots for each of the six configurations.

Lab Questions

1. What are the advantages of active filters over passive filters?
2. What are the advantages of first- over second-order filters?
3. With respect to the ratio of V_o/V_i, what does +3 dB imply?
4. List two practical uses of active low- or high-pass filters.

Background

A **filter** is a circuit that is designed to pass a specified band of frequencies while attenuating all signals outside this band, see Fig.1. Filters are defined by their frequency-domain effects on signals. The most useful analytical and graphical descriptions of filters fall under the frequency domain. Thus, curves of **gain** and **phase** versus frequency are commonly used to illustrate filter characteristics, and the most widely used mathematical tools are based on the frequency domain.

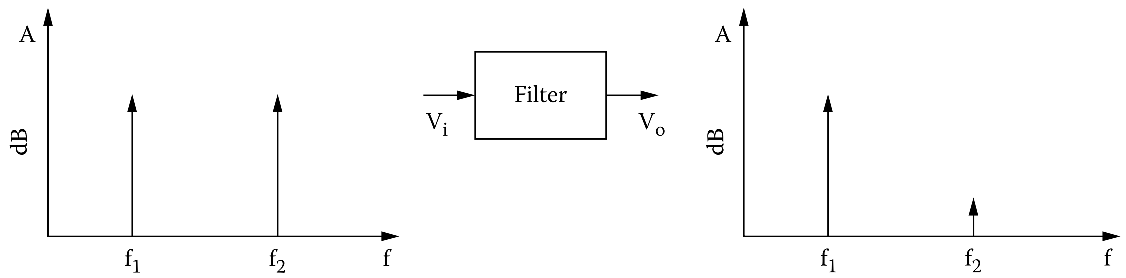

Figure 1. Using a filter to reduce the effect of an undesired signal.

The frequency-domain behaviour of a filter is described mathematically in terms of its **transfer function** or **network function**. This is the ratio of the Laplace transforms of its output and input signals.

$$H(s) = \frac{V_o}{V_i}.$$

The transfer function defines the filter's response to any arbitrary input signals, but we are most often concerned with its effect on continuous sine waves, especially the magnitude of the transfer function to signals at various frequencies. Knowing the transfer function magnitude (or gain) at each frequency allows us to determine how well the filter can distinguish between signals at different frequencies.

The transfer function magnitude versus frequency is called the **amplitude response** or sometimes, especially in audio applications, the **frequency response**.

$$A = 20\log|H(j\omega)| \ \ in\ dB$$

Similarly, the filter's phase response gives the amount of **phase shift** introduced in sinusoidal signals as a function of frequency. Because a change in the phase of a signal also represents a change in time, the phase characteristics of a filter become especially important when dealing with complex signals in which the time relationships between different frequencies are critical.

$$\arg H(j\omega) = \arg\frac{V_o(j\omega)}{V_i(j\omega)}$$

Filter networks may be either active or passive. *Passive filter networks* contain only resistors, inductors, and capacitors. *Active filters*, which are the only type covered in this text, employ operational amplifiers (op-amps) as well as resistors and capacitors.

Filter design is the process of creating a signal processing filter that satisfies a set of design specifications. The purpose of the design is to develop a type of filter that meets each of the requirements to a sufficient degree in order to make it useful.

Filter types

The first type is the **low-pass filter** (LPF). As might be expected, an LPF passes low-frequency signals, and rejects signals at frequencies above the filter's cutoff frequency, see Fig. 2. The ideal filter has a rectangular shape, indicating that the boundary between the passband and the stopband is abrupt and that the roll-off slope is infinitely steep. This type of response is ideal because it allows us to separate signals at different frequencies from one another completely.

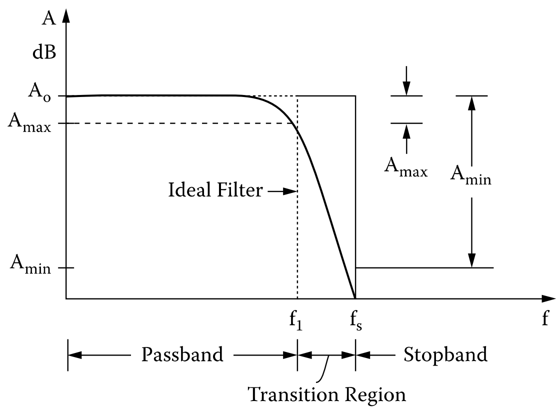

Figure 2. Low-pass frequency response.

A

$\boldsymbol{A_{max}}$ is the maximum allowable change in gain within the passband.
This quantity is also often called the maximum passband ripple.
$\boldsymbol{A_{min}}$ is the minimum allowable attenuation (referred to the maximum passband gain) within the stopband.
$\boldsymbol{f_1}$ is the cutoff frequency or passband limit.
$\boldsymbol{f_s}$ is the frequency at which the stopband begins.

The inverse of the low-pass is **the high-pass filter**, which rejects signals below its frequency, see Fig. 3.

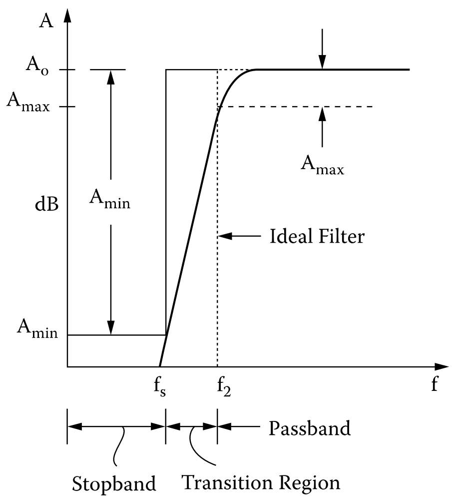

Figure 3. High-pass frequency response.

Prelab – LPF and HPF

Prelab1 – Introduction to Filtering

1. Generate a signal in MatLab composed of two sine waves using the following parameters:
   a) f_1=2000 Hz
   b) f_2 = 40000 Hz
   c) $0 \leq t \leq 2$ milliseconds
   d) T_s = 5 microseconds
   e) $y(t) = \sin(2\pi f_1 t)+0.25\cos(2\pi f_2 t)$

2. Plot the time domain and the frequency domain representations of the signal.

3. "Filter" the waveform by "zeroing out" the high-frequency components of the FFT waveform:
   There are two ways in MatLab to "filter" a waveform. One technique is to "zero out" the higher frequency elements of the FFT. The other is to use the transfer function of a low-pass filter. In the steps below, you will zero out the high-frequency elements of the FFT.

```
% create a new vector containing only the low
% frequency components of Y
L = length(Y);
NewY = zeros(L,1);
% define the number of data points to copy from the original file
NewY(1:b)=Y(1:b);
```

   As an option, instead of replacing the values of higher frequency regions with zeros, it is also feasible to replace the original values with very small values, such as 0.001, which represent out-of-band noise.

   Although you are not actually filtering the signal at this point, you perform the same function that a Low-Pass Filter performs. The performance of this filtering method depends on the number of vector elements you replace with zeros in the high-frequency components of the FFT.

4. After filtering in the frequency domain, reconstruct the signal using "*ifft*"
   In your report, compare the reconstructed signal with the original signal in the time domain.

5. Repeat this procedure to emulate a High-Pass filter to filter out the lower frequency of the FFT and observe the output.

This method introduces the concept of filtering. This filtering is considered ideal filtering in terms of its performance of the filtering. However, this filtering requires an additional procedure for reconstructing the signal. In the 'real world,' the goal is to design filters that perform as well as this simulation.

Prelab2 – LTSpice simulation

1. Build the following passive low-pass filter in LTSpice and determine its transfer function and cut-off frequency (Fig.4):

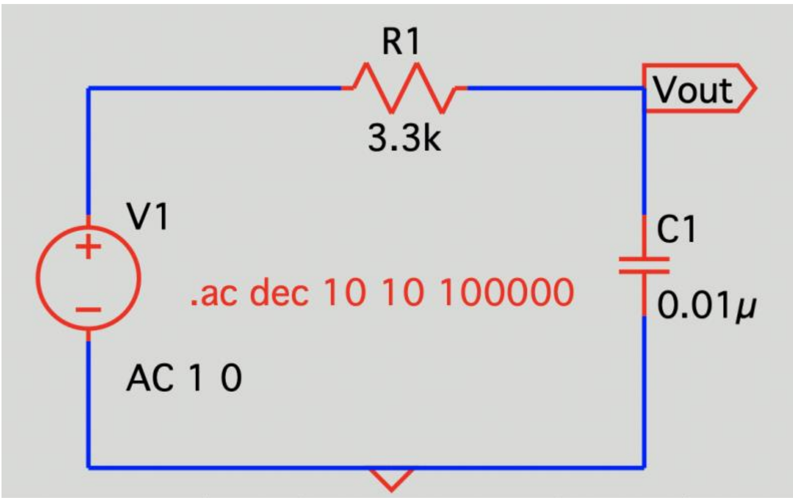

Figure 4. RC Low-pass filter.

2. Build the following passive high-pass filter in LTSpice and determine its transfer function and cut-off frequency (Fig.5):

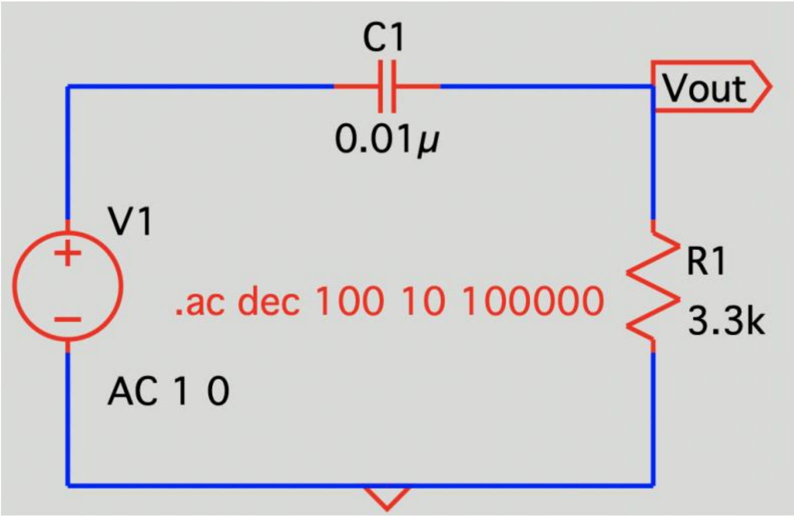

Figure 5. RC High-pass filter.

3. Build the first-order active low-pass filter in LTSpice and determine its transfer function, cut-off frequency, and maximal gain (Fig.6):

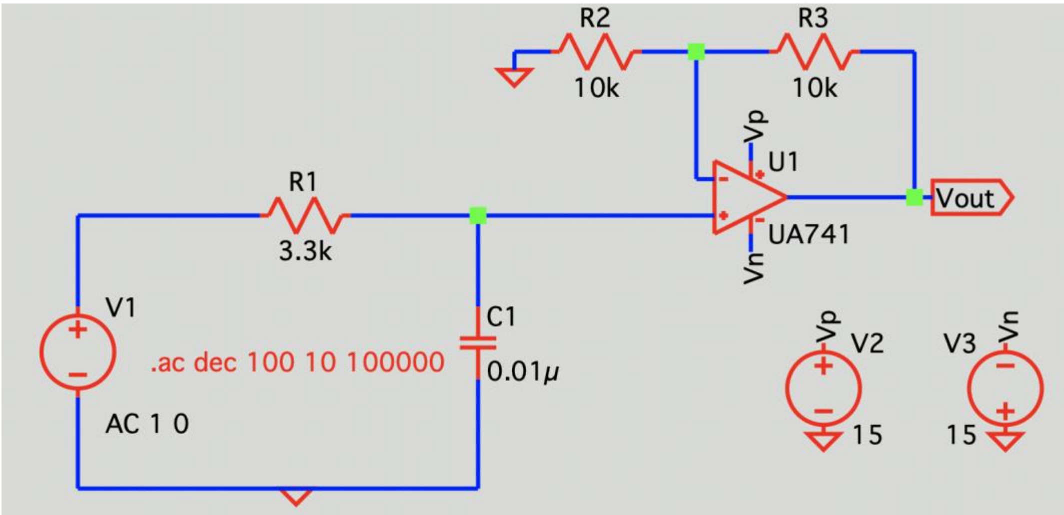

Figure 6. Active first-order Low-pass filter.

4. Build the first-order active high-pass filter in LTSpice and determine its transfer function, cut-off frequency, and maximal gain (Fig.7):

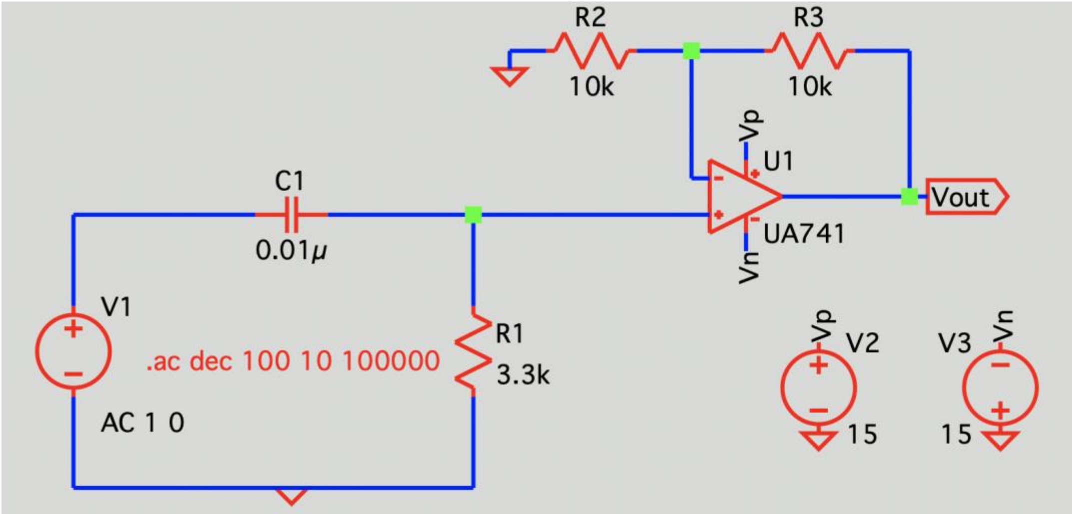

Figure 7. Active first-order High-pass filter.

5. Build the second-order active low-pass filter in LTSpice and determine its transfer function, cut-off frequency, and maximal gain (Fig.8):

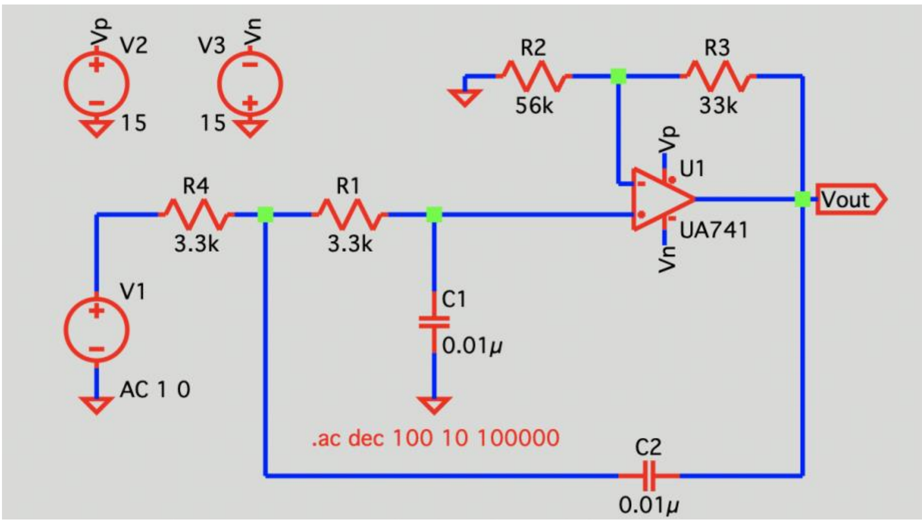

Figure 8. Sallen-Key second-order active Low-pass filter.

6. Build the second-order active high-pass filter in LTSpice and determine its transfer function, cut-off frequency, and maximal gain (Fig.9):

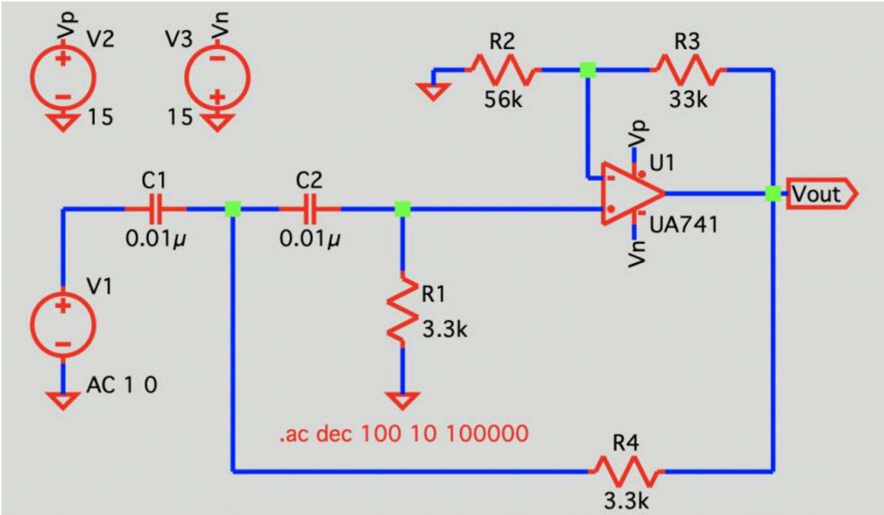

Figure 9. Sallen-Key second-order active High-pass filter.

Lab Period – Low Pass and High Pass Filters

Equipment required for this lab:

Moku:Go (or **Analog Discovery**) for signal generation and detection (oscilloscope)

Parts required for this lab:

Resistors: Determined by circuit
Capacitors: Determined by circuit
Integrated circuits: Op-amp (uA741)

Lab 3.1 Frequency response of Low-pass filters

In this section, you will determine the frequency response of the Low-Pass Filter.
Also, you will verify the filtering process by applying the sum of two sinusoids at the input and measuring the attenuation caused by the filter on the amplitudes of the two signals.

1. Build the circuit shown in Figure 4.

2. Use the *Frequency Response Analyzer* of **Moku:Go** (or *Network* of **Analog Discovery**) to visualize the amplitude and phase responses from 100Hz to 100KHz. Connect *Out*$_1$ to your $\mathbf{V_{in}}$ point and *In*$_1$ to your $\mathbf{V_{out}}$ point. (or using *Wavegen 1* and *Scope* of **Analog Discovery**)

3. Determine DC gain at low frequency, f_{DC} = 100Hz. Derive the transfer function for this circuit and calculate the theoretical DC gain

   DC gain (G_{DC}):
   Measured =
   Theoretical =

4. Manually determine its cut-off frequency. Recall that voltage at the cut-off frequency is 0.707 (- 3 dB) times the input voltage to the filter. Compare it with the theoretical value.

   Cut-off frequency (f_c):
   Measured =
   Theoretical =

5. Determine the phase shift at the cut-off frequency. Compare it with the theoretical value.

   Phase shift at the cut-off frequency (f_c):
   Measured =
   Theoretical =

6. Find the amplitudes at f_1 = 10 kHz and f_2 = 100 kHz. Record the values:
   Amplitude at f_1 (m_1) =
   Amplitude at f_2 (m_2) =

7. From the last two coordinates, estimate the slope of the magnitude response. Compare it with the theoretical value.

   Slope (dB/decade):
   Measured =
   Theoretical =

8. Use **Moku:Go** to generate a two-sine signal with frequencies f_1 = 2000Hz and f_2 = 40000 Hz of amplitude A_1=1V and A_2=0.25V, respectively, using both *Out$_1$* and *Out$_2$* and *Waveform Generator* (or *Wavegen 1* and *Wavegen 2* of **Analog Discovery**) with synchronized phases connected to your $\mathbf{V_{in}}$ point simultaneously.

   Check your input signal using the *Oscilloscope* of **Moku:Go** (or *Scope* of **Analog Discovery**)
   Using *Math* channel in the *Oscilloscope* (of *FFT* in *Scope* of **Analog Discovery**) calculate the FFT and measure the amplitudes of the main peaks in the displayed spectrum
   FFT amplitudes (dBm)
   at f_1 (pi_1) =
   at f_2 (pi_2) =

9. Connect your $\mathbf{V_{out}}$ to *In$_1$* and use the *Oscilloscope* of **Moku:Go** (or *Scope* of **Analog Discovery**) to record the filtered time signal. Using *Math* channel in the *Oscilloscope* (of *FFT* in *Scope* of **Analog Discovery**) calculate the FFT and obtain the frequency spectrum of the output signal. Measure the amplitude of the main peaks in the displayed spectrum
   FFT amplitudes (dBm)
   at f_1 (po_1) =
   at f_2 (po_2) =

10. Using the above measurements to calculate the gain/attenuation of the filtered signal at two carrying frequencies f_1 and f_2
    Gain at two carrying frequencies, f_1 and f_2 (dBm):
    G_{f1} = $po_1 - pi_1$. =
    G_{f2} = $po_2 - pi_2$ =

11. Repeat Steps 2-10for circuits in Figs. 6 and 8, properly identify the cut-off frequency condition, and then summarise all the data in the Tables below

Table 1: Measured Data

LPF type	DC Gain (dB)	Cut-off frequency, f_c (Hz)	Phase-shift at f_c (°)	Slope (dB/decade)	Filtered gain at f_1 (dBm)	Filtered gain at f_2 (dBm)
Passive RC						
Active RC						
Active Sallen-Key						

Table 2: Theoretical Data

LPF type	DC Gain (dB)	Cut-off frequency, f_c (Hz)	Phase-shift at f_c (°)	Slope (dB/decade)	Filtered gain at f_1 (dB)	Filtered gain at f_2 (dB)
Passive RC						
Active RC						
Active Sallen-Key						

Lab 3.2 Frequency response of High-pass filters

In this section, you will determine the frequency response of the High-Pass Filter.
Also, you will verify the filtering process by applying the sum of two sinusoids at the input and measuring the attenuation caused by the filter on the amplitudes of the two signals.

12. Build the circuit shown in Figure 5.

13. Use the *Frequency Response Analyzer* of **Moku:Go** (or *Network* of **Analog Discovery**)to visualize the amplitude and phase responses from 100Hz to 100KHz. Connect *Out_1* to your V_{in} point and *In_1* to your V_{out} point. (or using *Wavegen 1* and *Scope* of **Analog Discovery**)

14. Determine High-Frequency gain at 100kHz. Derive the transfer function for this circuit and calculate the theoretical gain

    HF gain (G_{HF}):
    Measured =
    Theoretical =

15. Manually determine its cut-off frequency. Recall that voltage at the cut-off frequency is 0.707 (- 3 dB) times the input voltage to the filter. Compare it with the theoretical value.

    Cut-off frequency (f_c):
    Measured =
    Theoretical =

16. Determine the phase shift at the cut-off frequency. Compare it with the theoretical value.

    Phase shift at the cut-off frequency (f_c):
    Measured =
    Theoretical =

17. Find the amplitudes at f_{s1} = 100 Hz and f_{s2} = 1000 Hz. Record the values:
    Amplitude at f_{s1} (m_1) =
    Amplitude at f_{s2} (m_2) =

18. From the last two coordinates, estimate the slope of the magnitude response. Compare it with the theoretical value.

    Slope (dB/decade):
    Measured =
    Theoretical =

19. Use **Moku:Go** to generate a two-sine signal with frequencies f_1 = 2000Hz and f_2 = 40000 Hz of amplitude A_1=1V and A_2=0.25V, respectively, using both <u>*Out_1*</u> and <u>*Out_2*</u> and <u>*Waveform Generator*</u> (or <u>*Wavegen 1*</u> and <u>*Wavegen 2*</u> of **Analog Discovery**) with synchronized phases connected to your $\mathbf{V_{in}}$ point simultaneously.

    Check your input signal using the <u>*Oscilloscope*</u> of **Moku:Go** (or <u>*Scope*</u> of **Analog Discovery**)
    Using <u>*Math*</u> channel in the <u>*Oscilloscope*</u> (of <u>*FFT*</u> in <u>*Scope*</u> of **Analog Discovery**) calculate the FFT and measure the amplitudes of the main peaks in the displayed spectrum
    FFT amplitudes (dBm)
    at f_1 (pi_1) =
    at f_2 (pi_2) =

20. Connect your $\mathbf{V_{out}}$ to <u>*In_1*</u> and use the <u>*Oscilloscope*</u> of **Moku:Go** (or <u>*Scope*</u> of **Analog Discovery**) to record the filtered time signal. Using <u>*Math*</u> channel in the <u>*Oscilloscope*</u> (of <u>*FFT*</u> in <u>*Scope*</u> of **Analog Discovery**) calculate the FFT and obtain the frequency spectrum of the output signal. Measure the amplitude of the main peaks in the displayed spectrum
    FFT amplitudes (dBm)
    at f_1 (po_1) =
    at f_2 (po_2) =

21. Using the above measurements to calculate the gain/attenuation of the filtered signal at two carrying frequencies f_1 and f_2
    Gain at two carrying frequencies, f_1 and f_2 (dBm):
    G_{f1} = $po_1 - pi_{1.}$ =
    G_{f2} = $po_2 - pi_2$ =

22. Repeat Steps 13-21 for circuits in Figs. 7 and 9, then summarise all the data in the Tables below

Table 3: Measured Data

HPF type	HF Gain (dB)	Cut-off frequency, f_c (Hz)	Phase-shift at f_c (°)	Slope (dB/decade)	Filtered gain at f_1 (dBm)	Filtered gain at f_2 (dBm)
Passive RC						
Active RC						
Active Sallen-Key						

Table 4: Theoretical Data

HPF type	HF Gain (dB)	Cut-off frequency, f_c (Hz)	Phase-shift at f_c (°)	Slope (dB/decade)	Filtered gain at f_1 (dB)	Filtered gain at f_2 (dB)
Passive RC						
Active RC						
Active Sallen-Key						

Lab Report

During this Lab, you modelled and built several different low-pass and high-pass filters. In your report,

- Compare the frequency and phase responses of *all* the Low Pass Filters.
- Compare the frequency and phase responses of *all* the High Pass Filters.
- Explain any discrepancies you noted between the models and the actual circuits.
- Note any deviations you made from the assigned procedure.