Star Formation in the Multiverse

Navonil Saha
Indian Institute of Science Education and Research (IISER), Kolkata

Supervisor : Prof. Lucas Lombriser
Mentor : Dr. Daniele Sorini

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Star Formation Model

The Cosmic Star Formation rate (SFR) is one of the key components used to describe and understand the evolution of galaxies

Apart from our own universe which governs the Standard cosmology, we study the cosmological models that differ from our own in the value of one or more of the cosmological parameters , viz., the Matter density parameter, Dark energy density parameter, Baryon density parameter, the primordial power spectrum amplitude and the Hubble constant.

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Apart from our own universe which governs the Standard cosmology, we study the cosmological models that differ from our own in the value of one or more of the cosmological parameters , viz., the Matter density parameter, Dark energy density parameter, Baryon density parameter, the primordial power spectrum amplitude and the Hubble constant.

Star Formation Rate Density (SFRD) comparison of
Bousso and Leichenauer (BL) and 2021 Sorini-Peacock (SP) models

Bousso and Leichenauer (BL) Model

BL model can be applied to open, flat, and closed Friedman-Robertson- Walker (FRW) universes.

$$\dot{\rho_*}(t) = \rho_0 \int dM \int dt_{vir} \frac{\partial F}{\partial M}(M,t) \frac{A f_b M}{t_{grav}(t_{vir})} \frac{\partial P}{\partial t_{vir}}(t_{vir}, M, t)$$

The time t_{eq} at which matter and radiation have equal density is,

$$\rho_\mathrm{m}(t_\mathrm{eq}) = \rho_\mathrm{r}(t_\mathrm{eq}) \equiv \nu \frac{\pi^2}{15} T_\mathrm{eq}^4 ,$$

where

$$\nu = 1 + \frac{21}{8}\left(\frac{4}{11}\right)^{4/3} \approx 1.681$$

The temperature at equality is $T_{eq} = 0.22\xi = 0.82$ eV ; ξ is the matter mass per photon

The total density and pressure are given by $\rho = \rho_r + \rho_m + \rho_\Lambda$

$$\text{Pressure}, p = \frac{\rho_r}{3} - \rho_\Lambda$$

Cosmological perturbations are usually specified by a time-dependent power spectrum, P. The r.m.s. fluctuation amplitude, σ, within a sphere of radius R, is defined by smoothing the power spectrum with respect to an appropriate window function

$$\sigma^2 = \frac{1}{2\pi^2}\int_0^\infty \left(\frac{3sin(kR) - 3kRcos(kR)}{(kR)^3}\right)^2 \mathcal{P}(k)k^2dk$$

$$\sigma(M,t) = Qs(M)G(t)$$

Q : amplitude of primordial density perturbations
s(M) : fitting formula

The linear growth function G(t) can be found by,

$$\frac{d^2G}{dt^2} + 2H\frac{dG}{dt} = 4\pi G_N \rho_m G$$

BL model uses the extended Press-Schechter (EPS) formalism to estimate the halo mass function and the distribution of halo ages.

$$F(< M, t) = Erf\left(\frac{\delta_c}{\sqrt{2}\sigma(M,t)}\right)$$

δc = 1.68 being the critical fluctuation for collapse

Star Formation Rate

$$\dot{\rho_*}(t) = \rho_0 \int dM \int dt_{vir} \frac{\partial F}{\partial M}(M,t) \frac{A f_b M}{t_{grav}(t_{vir})} \frac{\partial P}{\partial t_{vir}}(t_{vir}, M, t)$$

ρ_0 : reference density

Normalizing the SFR

$$\int_0^{t_0} \dot{\rho}_*(dt) = \frac{f_b \rho_m}{10}$$

Parametrising the Non Standard Cosmologies

We determine the cosmological parameters of an arbitrary flat ΛCDM model, with respect to the cosmological parameters observed in our Universe, which acts as a reference cosmological model consistent with both BL and SP model.

We use the CMB temperature as an marker for for time, hence in any universe "**today**" is defined as the time when the CMB temperature is equal to the value observed in our Universe **To = 2.7255 K**

At early times the ΛCDM universes are mainly radiation dominated.

We consider a sufficiently high redshift zi such that our Universe is well into the radiation-dominated era at z = zi

$$\Omega_{rad,ref}(z_i) \approx 1$$
$$\Omega_{\Lambda,ref}(z_i) = \varepsilon_\Lambda$$
$$\Omega_{m,ref}(z_i) = \varepsilon_m$$

where ε_m, $\varepsilon_\Lambda \ll 1$.

We rescale the dark energy and matter density by some constant factors α_Λ and α_m with respect to our universe given as follows,

$$\Omega_\Lambda(z_i) = \alpha_\Lambda \varepsilon_\Lambda$$
$$\Omega_m(z_i) = \alpha_m \varepsilon_m$$
$$\Omega_{rad}(z_i) = 1 - \alpha_\Lambda \varepsilon_\Lambda - \alpha_m \varepsilon_m \approx 1$$

In our universe $\alpha_m = \alpha_\Lambda = 1$

At $z = z_i$ the Hubble constant will have the same value in all the universe, for the different universes we consider a scale factor a in terms of the Hubble constant as $H_i = H(a_i)$

$$\begin{aligned} H(a)^2 &= H_i^2 \left[(1 - \alpha_\Lambda \varepsilon_\Lambda - \alpha_\mathrm{m} \varepsilon_\mathrm{m}) \left(\frac{a_i}{a} \right)^4 + \alpha_\mathrm{m} \varepsilon_\mathrm{m} \left(\frac{a_i}{a} \right)^3 + \alpha_\Lambda \varepsilon_\Lambda \right] \\ &\approx H_i^2 \left[\left(\frac{a_i}{a} \right)^4 + \alpha_\mathrm{m} \varepsilon_\mathrm{m} \left(\frac{a_i}{a} \right)^3 + \alpha_\Lambda \varepsilon_\Lambda \right] \end{aligned}$$

In Reference Cosmology,

$$H_\mathrm{ref}(a)^2 = H_{0,\,\mathrm{ref}}^2 \left[\Omega_{\mathrm{rad,\,ref}} a^{-4} + \Omega_{\mathrm{m,\,ref}} a^{-3} + \Omega_{\Lambda,\,\mathrm{ref}} \right]$$

The Hubble constant in the new cosmology in terms of the parameters of the reference cosmology is,

$$H(a)^2 = H_{0,\,\mathrm{ref}}^2 \left[\Omega_{\mathrm{rad,\,ref}} a^{-4} + \alpha_\mathrm{m}\, \Omega_{\mathrm{m,\,ref}} a^{-3} + \alpha_\Lambda\, \Omega_{\Lambda,\,\mathrm{ref}} \right]$$

for a = 1, we have the expression for the present-day Hubble constant in the new cosmology

$$H_0^2 = H_{0,\,\mathrm{ref}}^2 \left[\Omega_{\mathrm{rad,\,ref}} + \alpha_{\mathrm{m}}\, \Omega_{\mathrm{m,\,ref}} + \alpha_\Lambda\, \Omega_{\Lambda,\,\mathrm{ref}}\right]$$

So, the evolution of the Hubble constant in the new universe in terms of the new cosmological parameters

$$H(a)^2 = H_0^2 \left[\Omega_{\mathrm{rad}} a^{-4} + \Omega_{\mathrm{m}} a^{-3} + \Omega_\Lambda\right] .$$

$$\frac{\Omega_{\mathrm{rad}}}{\Omega_{\mathrm{m}}} = \frac{\Omega_{\mathrm{rad,\,ref}}}{\alpha_{\mathrm{m}}\, \Omega_{\mathrm{m,\,ref}}}$$

$$\frac{\Omega_{\mathrm{m}}}{\Omega_\Lambda} = \frac{\alpha_{\mathrm{m}}\, \Omega_{\mathrm{m,\,ref}}}{\alpha_\Lambda\, \Omega_{\Lambda,\,\mathrm{ref}}}$$

Taking into account the flatness condition, expressions for the cosmological parameters in the new universe , will be,

$$\Omega_{\mathrm{rad}} = \frac{\Omega_{\mathrm{rad,\,ref}}}{\Omega_{\mathrm{rad,\,ref}} + \alpha_{\mathrm{m}}\Omega_{\mathrm{m,\,ref}} + \alpha_\Lambda\Omega_{\Lambda,\,\mathrm{ref}}}$$

$$\Omega_{\mathrm{m}} = \frac{\alpha_{\mathrm{m}}\Omega_{\mathrm{m,\,ref}}}{\Omega_{\mathrm{rad,\,ref}} + \alpha_{\mathrm{m}}\Omega_{\mathrm{m,\,ref}} + \alpha_\Lambda\Omega_{\Lambda,\,\mathrm{ref}}}$$

$$\Omega_\Lambda = \frac{\alpha_\Lambda\Omega_{\Lambda,\,\mathrm{ref}}}{\Omega_{\mathrm{rad,\,ref}} + \alpha_{\mathrm{m}}\Omega_{\mathrm{m,\,ref}} + \alpha_\Lambda\Omega_{\Lambda,\,\mathrm{ref}}}$$

The new σ_8 can be calculated numerically. The CAMB code in the online LAMBDA tool provides this value for certain set of cosmological parameters among the output parameters.

The baryon density Ω_b

$$\Omega_{\rm b} = f_{\rm b}\Omega_{\rm m}$$

$$f_{\rm b} = \alpha_{\rm b}\, f_{\rm b,\, ref}$$

Primordial power spectrum amplitude

$$A_{\rm s} = \alpha_{\rm s}\, A_{\rm s,\, ref}$$

$$\sigma_8 = \alpha_\sigma \sigma_{8,ref}$$

Both the BL and SP model are consistent with the Planck-2018 cosmology results

Parameters of the reference model (consistent with Planck-2018 results)

$\Omega_{\rm rad}$	9.26×10^{-5}
$\Omega_{\rm m}$	0.315
Ω_Λ	0.685
$\Omega_{\rm b}$	0.049
h	0.674
$A_{\rm s}$	2×10^{-9}
$n_{\rm s}$	0.965
$N_{\rm eff}$	2.99

Star Formation Rate Density (SFRD) Comparisons

We first develop the SFRD for our own universe or the so called Standard cosmology, and then we start to vary the cosmological parameters one by one to investigate how the SFRD changes in different universes with respect to our universe

We will take into account 4 parameters for the comparison of the different cosmologies, $\alpha_m, \alpha_b, \alpha_\lambda, \alpha_s$ these are dependent on the density parameters. We also change the σ_8 parameter which is dependent on α_σ . In the Case of Standard Cosmology (our Universe) , all the $\alpha's$ are taken to be 1. We will gradually vary this parameter one by one in case of Different Cosmologies.

Both BL and SP models are compared along with the original Hernquist & Springel (2003) along with the Madau & Dickinson (2014) Fit.

The Plots are made between Redshift in units of [Z+1] in the x-axis and the SFRD in units of $[M_\odot yr^{-1} Mpc^{-3}]$ in the y-axis.

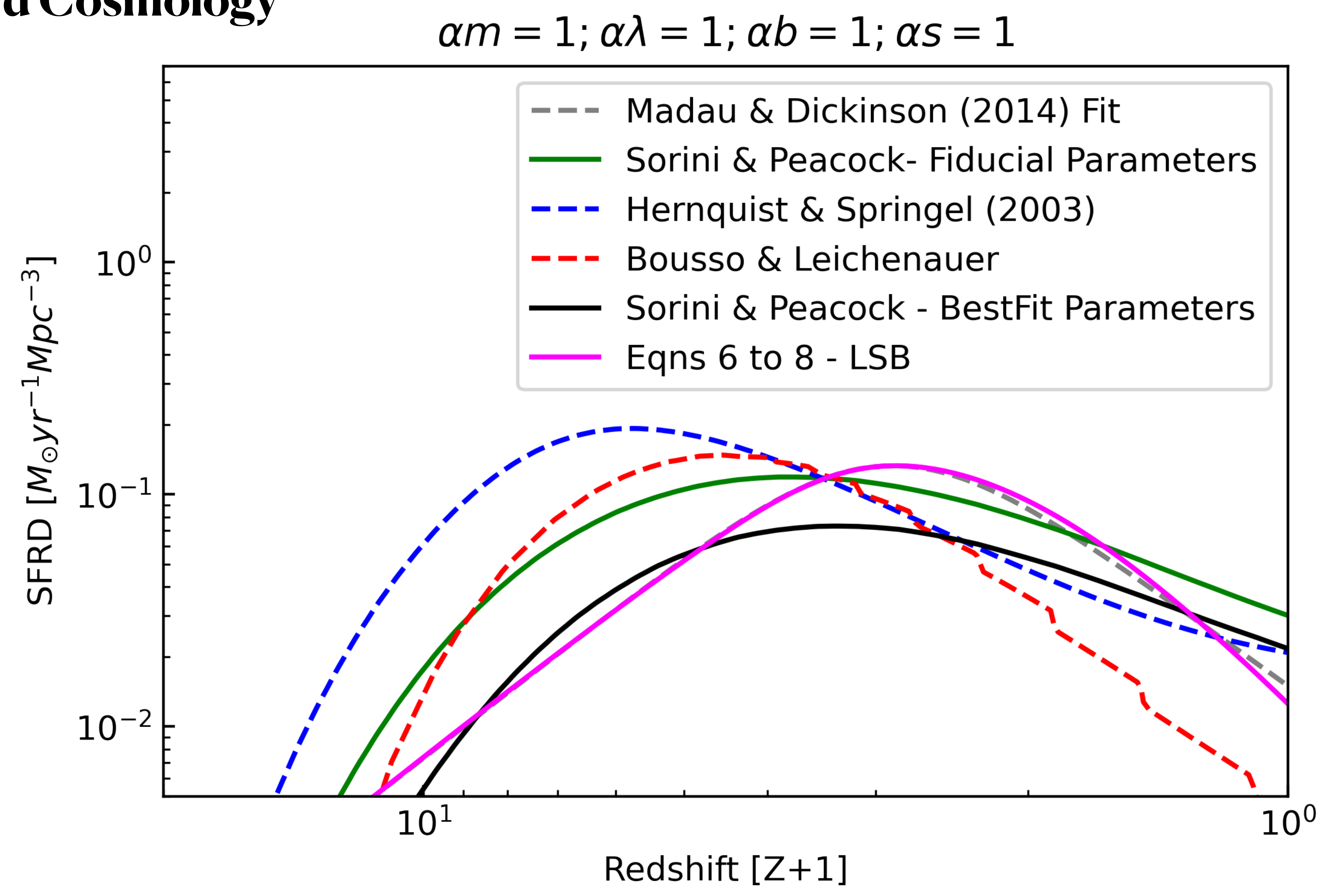
Equation 6 to 8 of Lombriser - Smer Barreto Paper

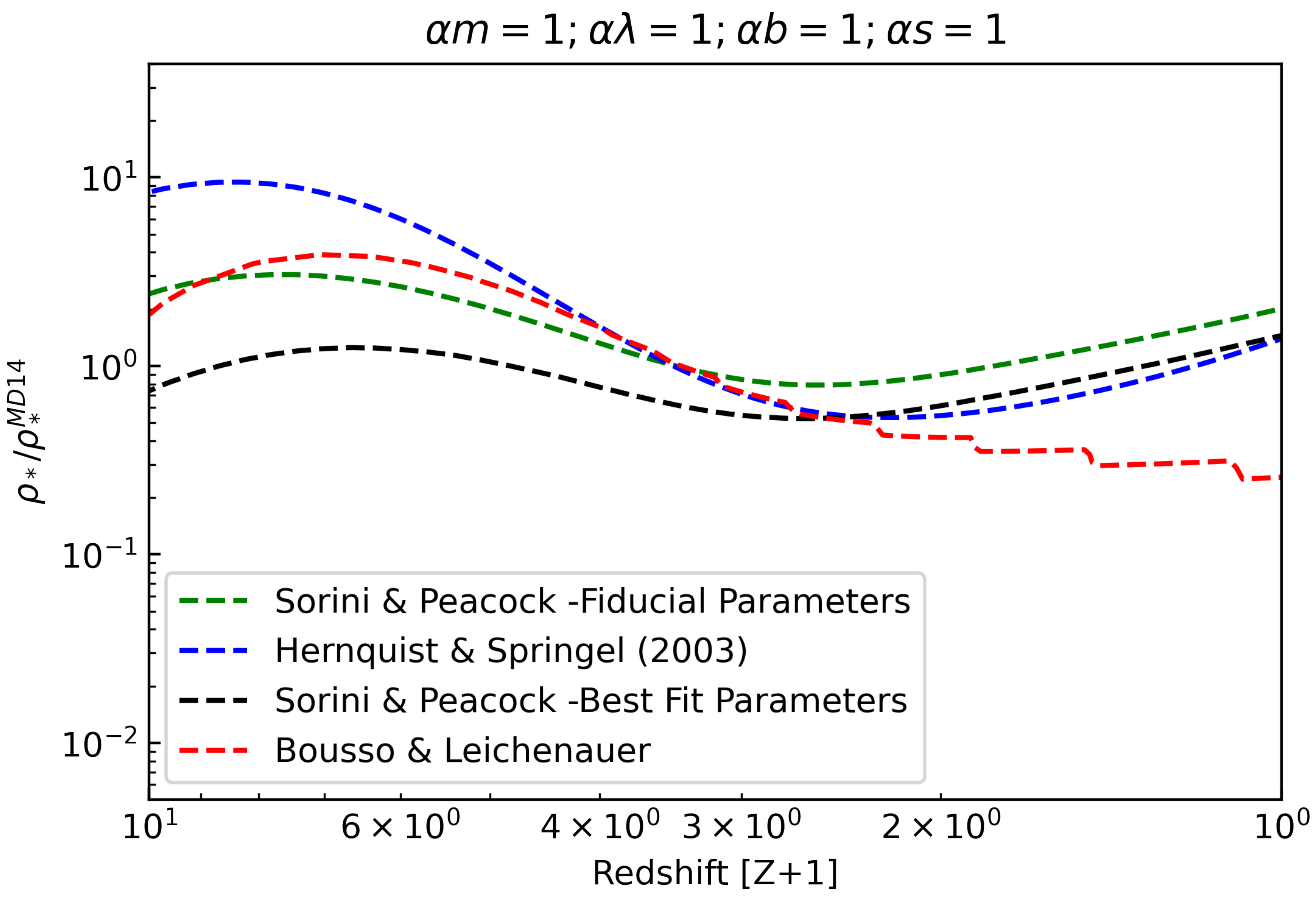
$$f_{in} = \frac{1}{2}\left(\frac{3}{2}\right)^2 \left(\frac{\Omega_b}{2\Omega_\Lambda}\right)^{1/3} (1+z) f_{out} \ ; \qquad f_{out} = \frac{4\Omega_b\Omega_\Lambda(1+z)^3}{[\Omega_b(1+z)^3+\Omega_\Lambda]^2} \ ; \qquad \frac{\dot{\rho}_*}{\rho_c} \propto \frac{f_{in} + w f_{out}}{1+w}$$

$$w(z) = [(1+z)/(1+z_{infl})]^7 \qquad (1+z_{infl}) = [(3+\sqrt{7})\Omega_\Lambda/(2\Omega_b)]^{1/3}$$

Where , normalized inner and outer func- tional behaviors and w(z) is the weighting function and z_{infl} is the inflection redshift.

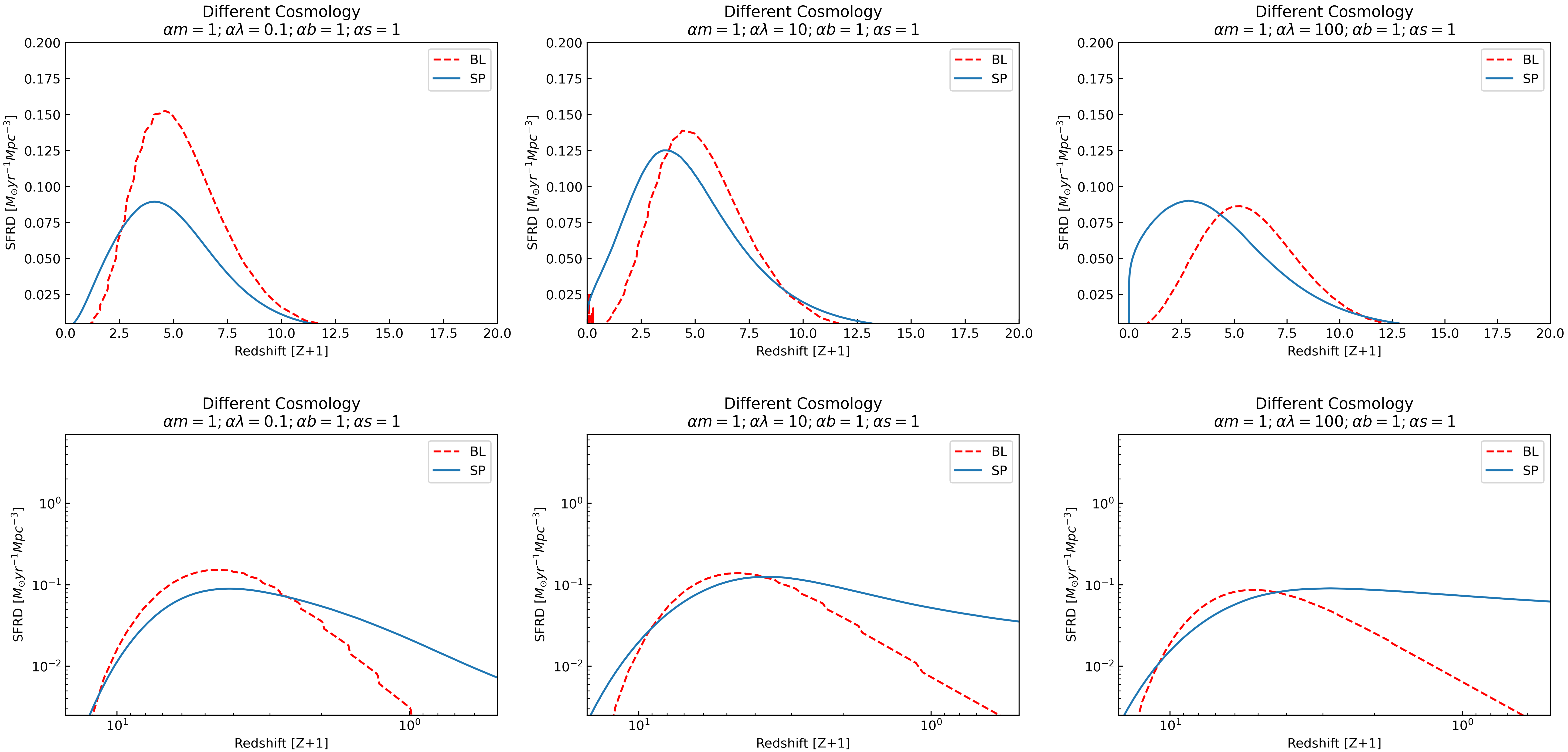
Standard Cosmology



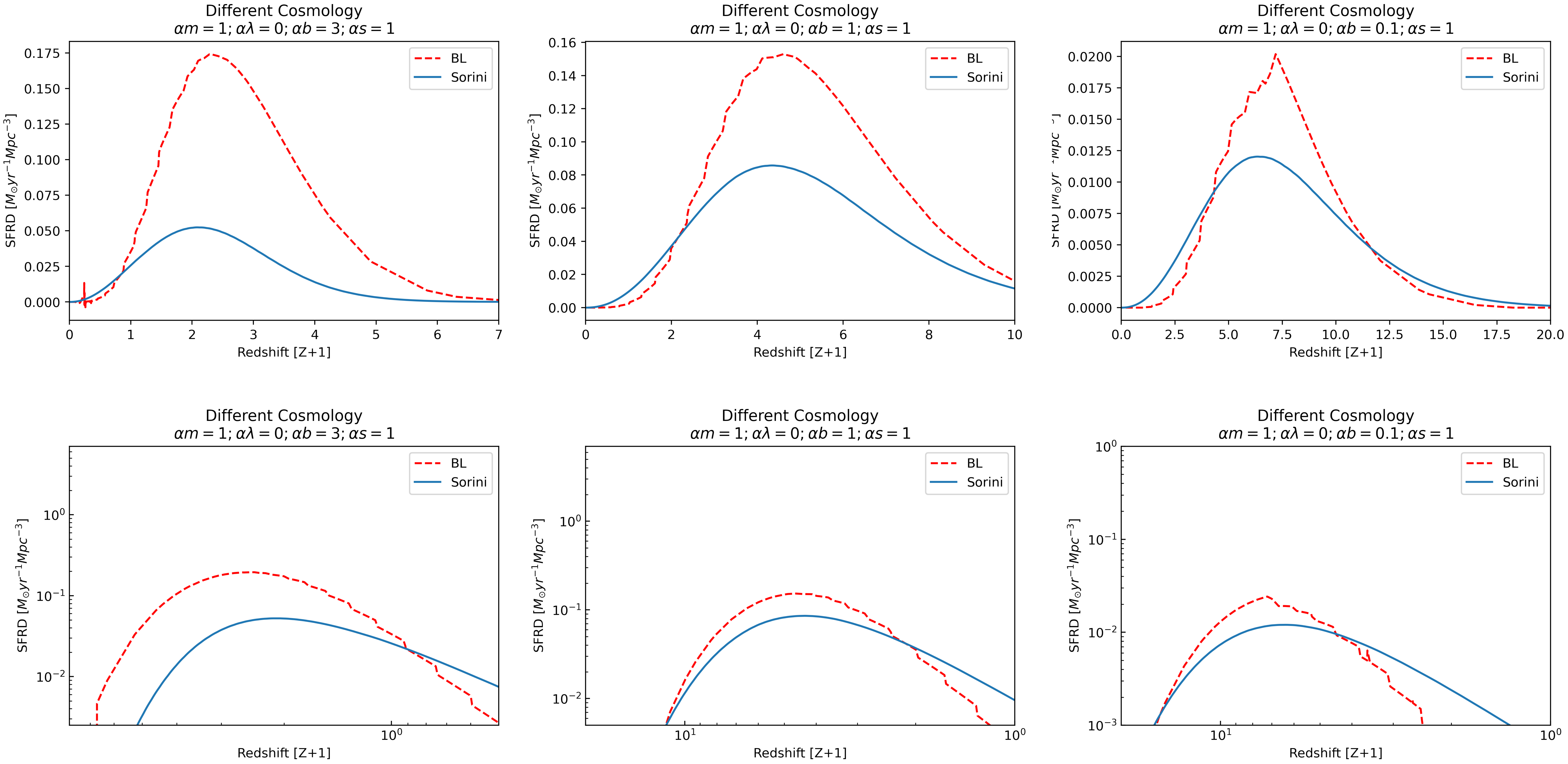


$\alpha m = 1; \alpha\lambda = 1; \alpha b = 1; \alpha s = 1$
10^1
10^0
10^{-1}
10^{-2}
ρ_*/ρ_*^{MD14}
Sorini & Peacock -Fiducial Parameters
Hernquist & Springel (2003)
Sorini & Peacock -Best Fit Parameters
Bousso & Leichenauer
10^1
6×10^0
4×10^0
3×10^0
2×10^0
10^0
Redshift [Z+1]

Different Cosmology (different α_λ)

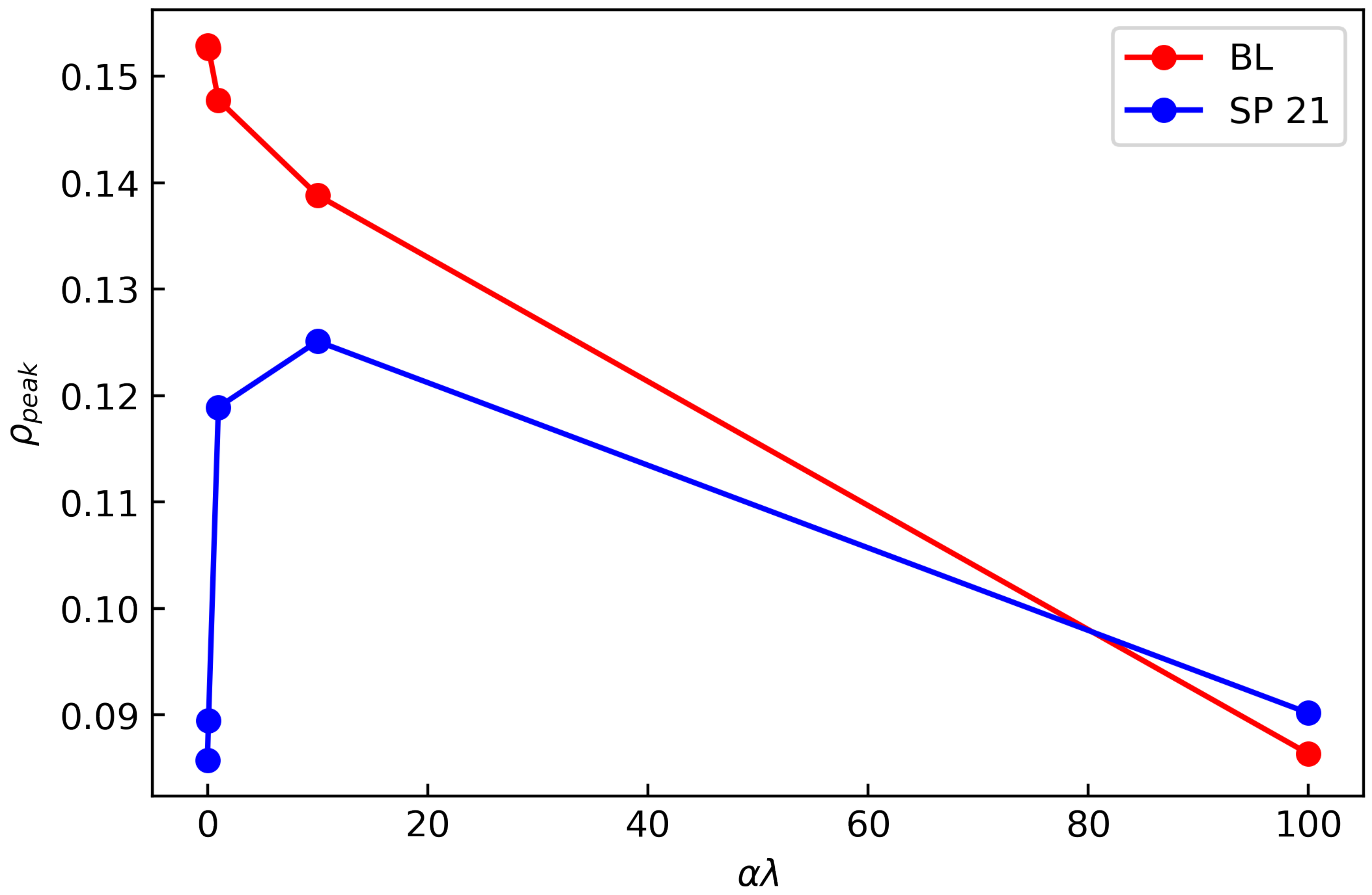


Different Cosmology (different α_b)

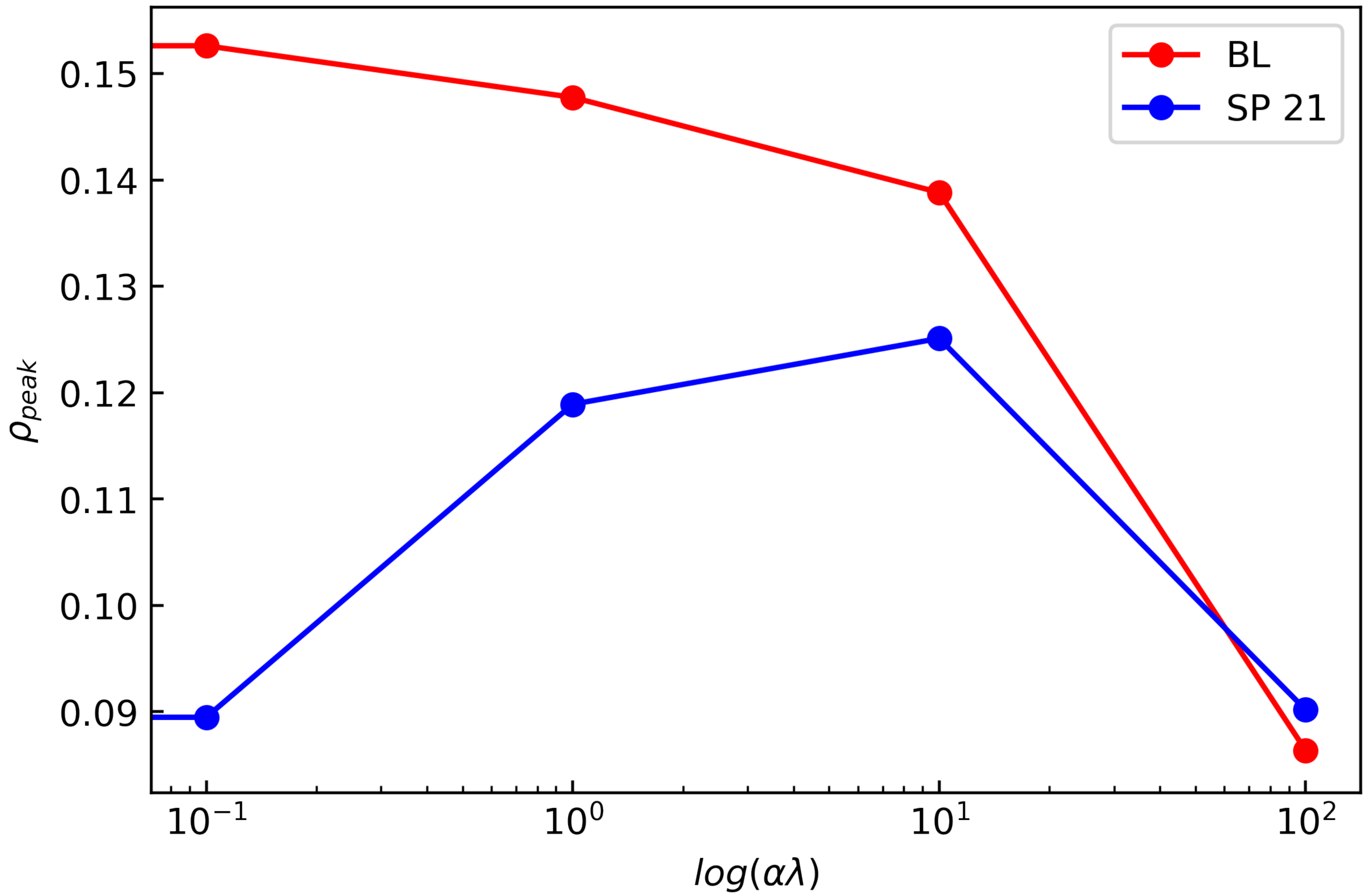


α_m	α_λ	α_b	α_s	**BL SFRD Peak**	**SP SFRD Peak**	**BL** Z_{peak}	**SP** Z_{peak}
1	0.1	1	1	0.152622711323504	0.0894832036069	3.60781747530423	3.12456668141
1	1	1	1	0.14777289335285465	0.118917741615	3.500909384007941	2.753358046
1	10	1	1	0.138827319047574	0.125107536637	3.397238132552987	2.65523114285
1	100	1	1	0.08635565908123087	0.0901803109135	4.283548202679526	1.82842712475
1	0	0.1	1	0.024208347527020863	0.0120248225632	6.2178618377821	5.344577011
1	0	1	1	0.15289244070312588	0.0857396167537	3.6103717527389456	3.3443861766
1	0	3	1	0.19465478869463002	0.0524095889847	1.4217825700881237	1.08451807725

Comparison of ρ_{peak} and different values of $\alpha\lambda$

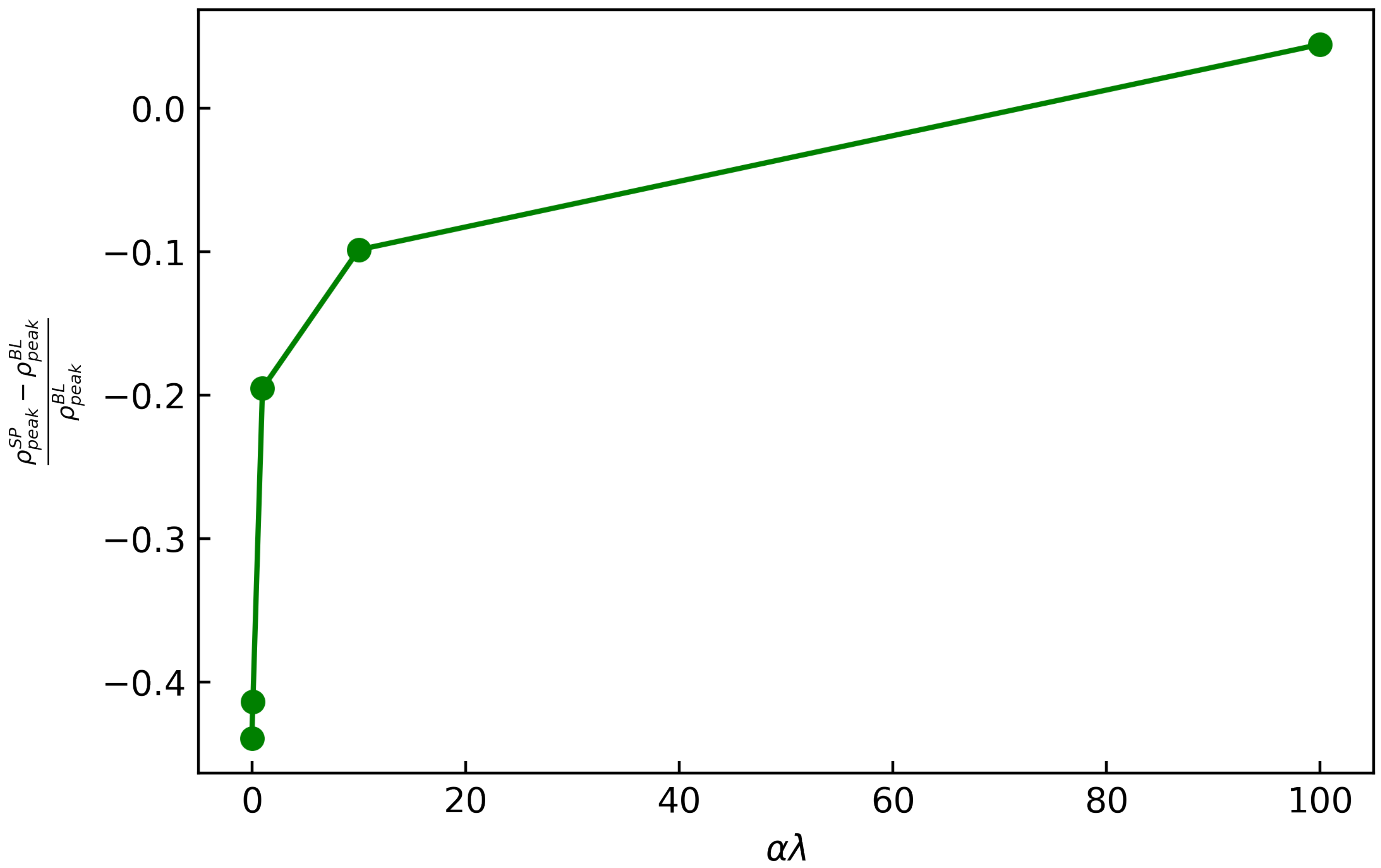


ρ_{peak} vs α_λ for 5 different values of
α_λ **= 0 , 0.1 , 1 , 10 , 100**
and $\alpha_m, \alpha_b, \alpha_s = 1$



ρ_{peak} vs $log(\alpha_\lambda)$ for 5 different values of
α_λ **= 0 , 0.1 , 1 , 10 , 100**
and $\alpha_m, \alpha_b, \alpha_s = 1$

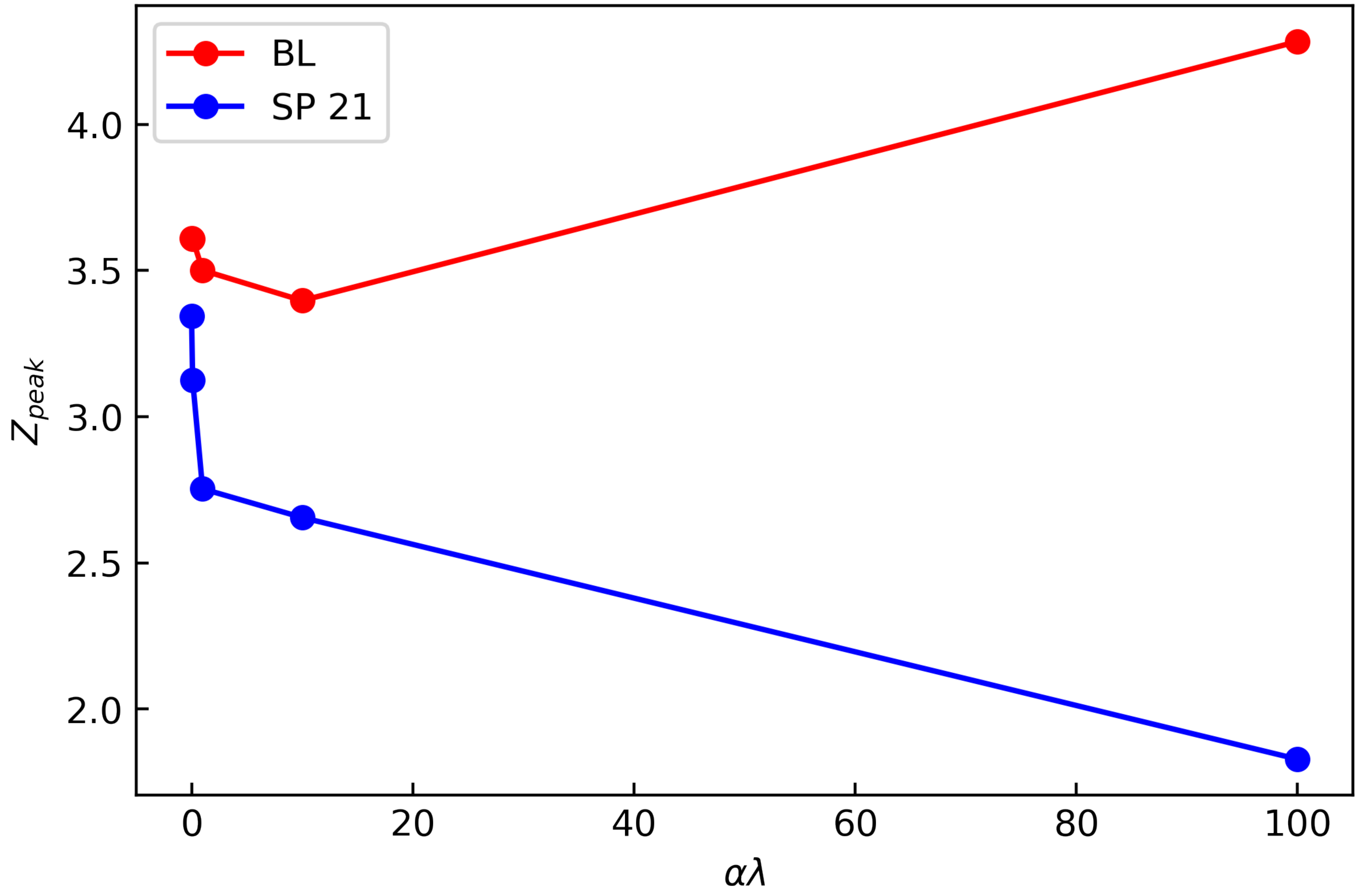
[Now, $log(0) \equiv -\infty$, so the graph extends to '-' infinity]



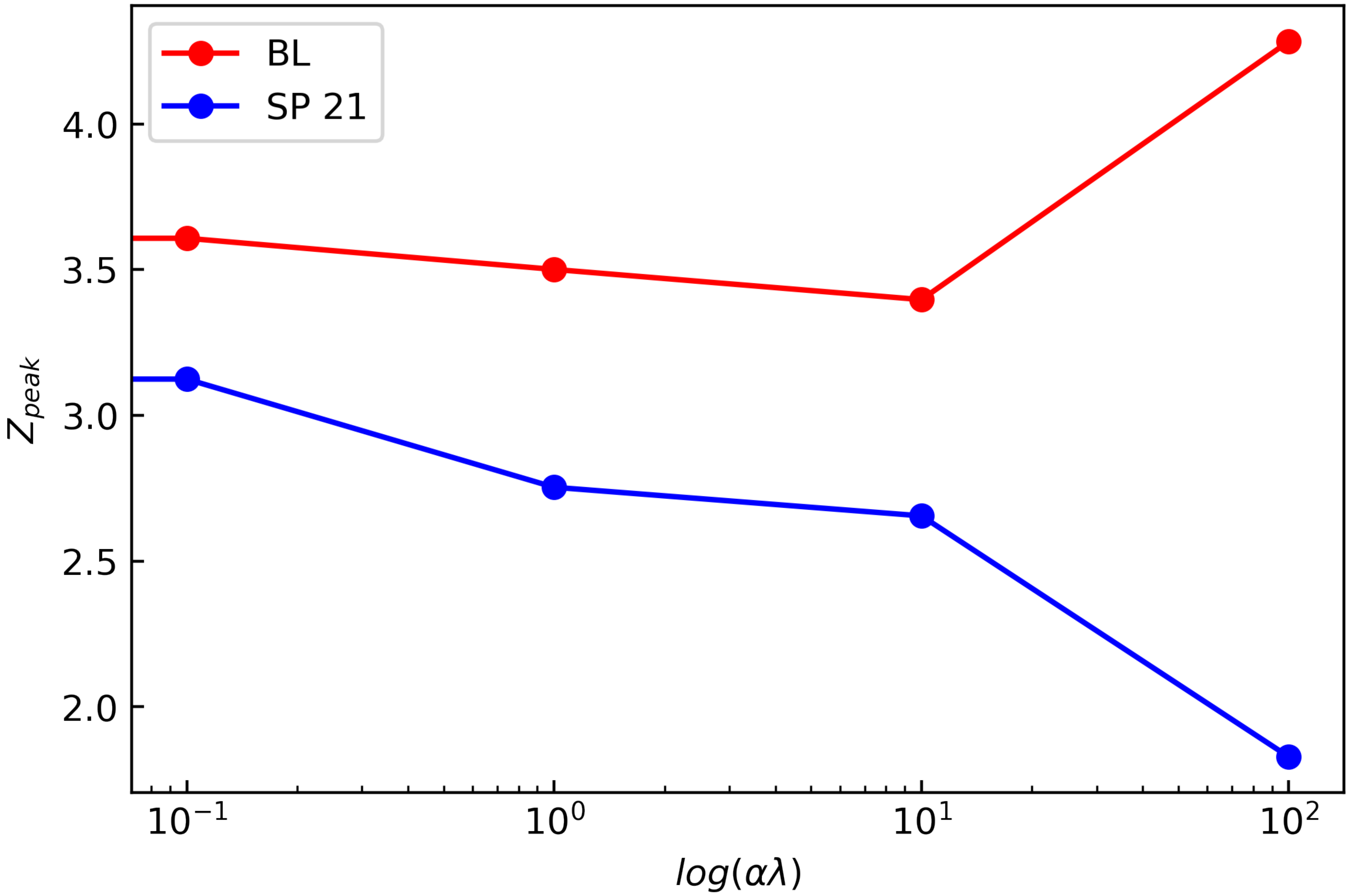
$\frac{\rho^{SP}_{peak} - \rho^{BL}_{peak}}{\rho^{BL}_{peak}}$ vs α_λ for 5 different values of

α_λ **= 0 , 0.1 , 1 , 10 , 100**

Comparison of Z_{peak} and different values of $\alpha\lambda$

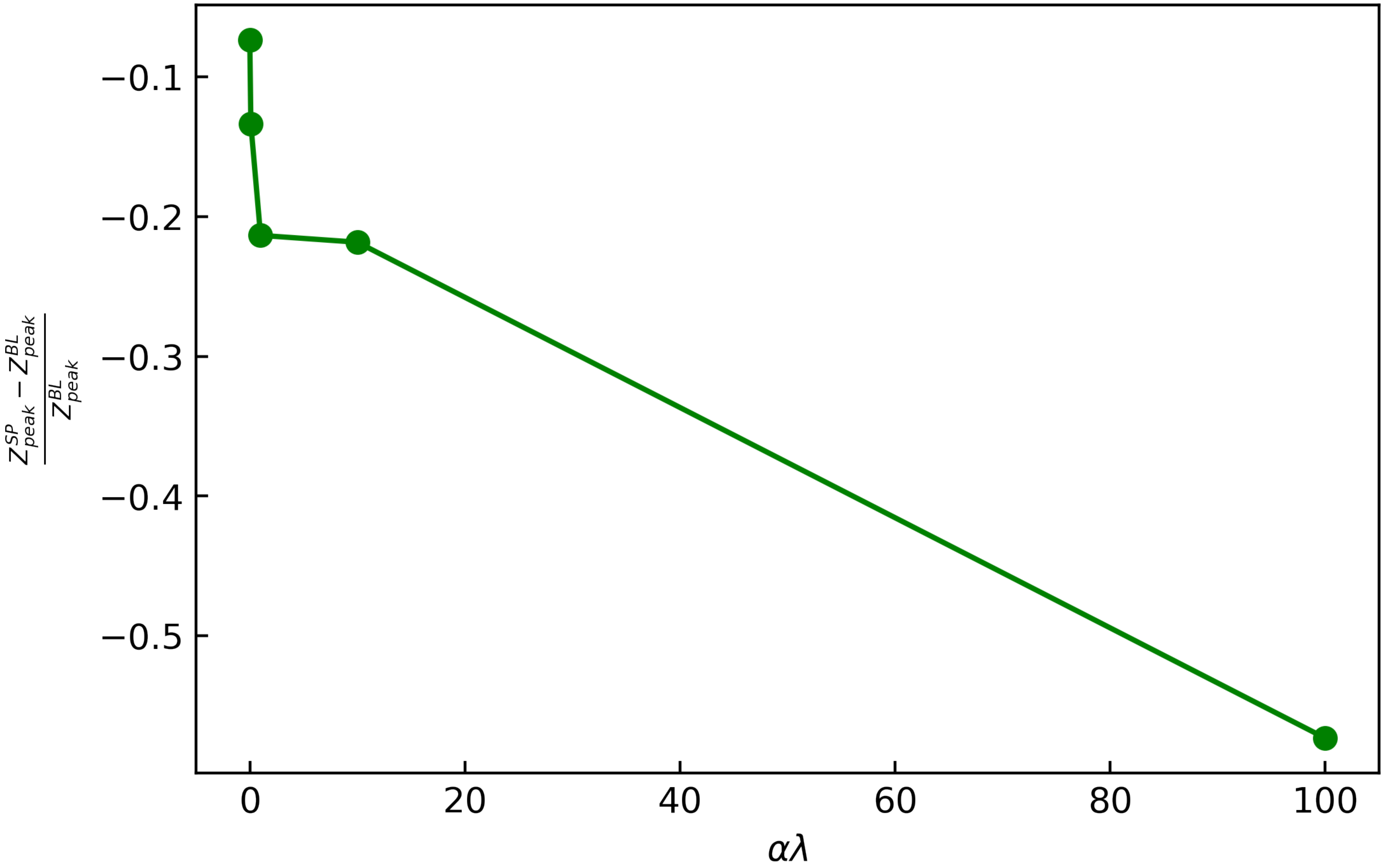


Z_{peak} vs α_λ for 5 different values of
$\alpha\lambda$ **= 0 , 0.1 , 1 , 10 , 100**
and $\alpha_m, \alpha_b, \alpha_s = 1$



Z_{peak} vs $log(\alpha_\lambda)$ for 5 different values of
$\alpha\lambda$ **= 0 , 0.1 , 1 , 10 , 100**
and $\alpha_m, \alpha_b, \alpha_s = 1$

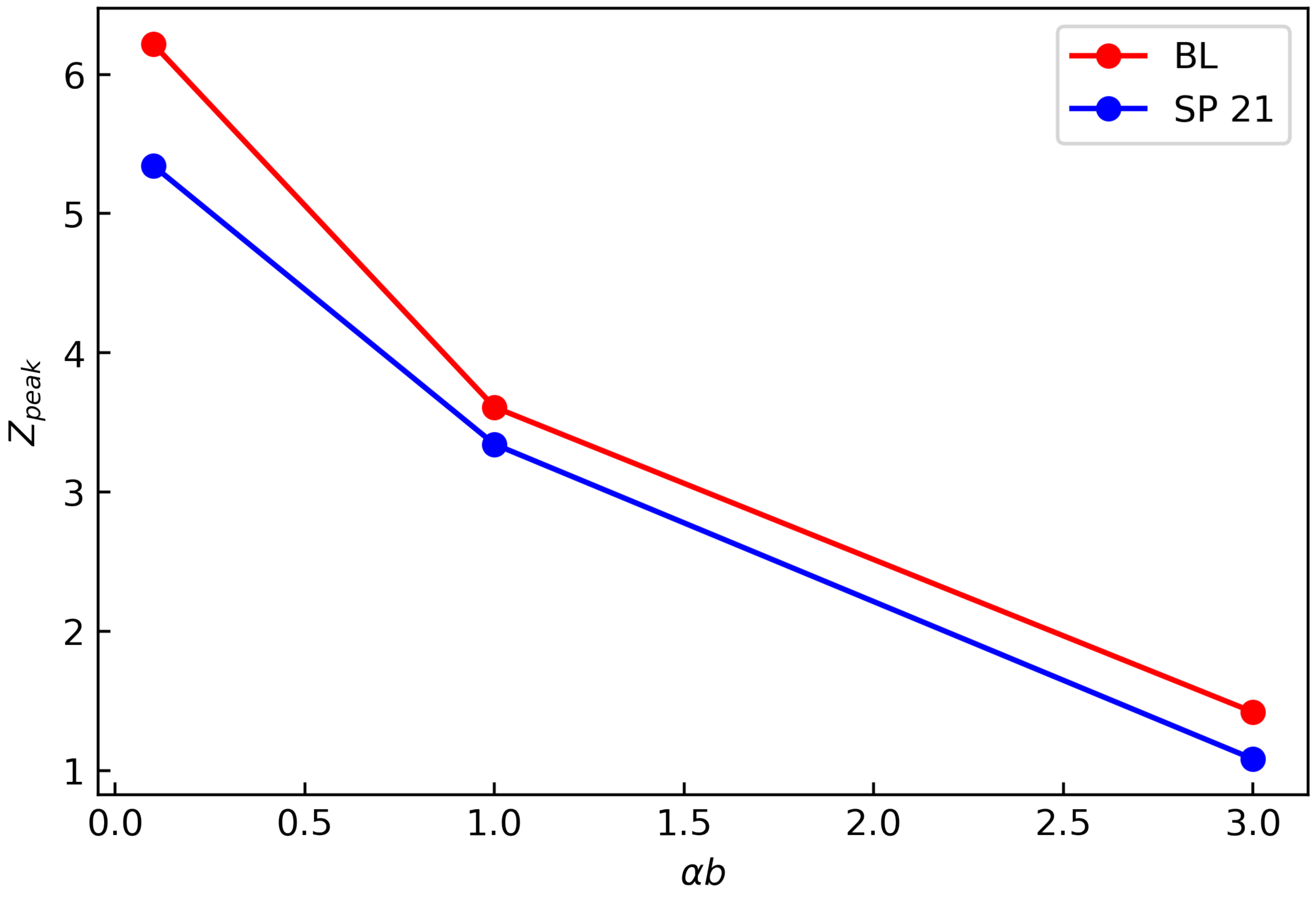
[Now, $log(0) \equiv -\infty$, so the graph extends to '-' infinity]



$\dfrac{Z^{SP}_{peak} - Z^{BL}_{peak}}{Z^{BL}_{peak}}$ vs α_λ for 5 different values of

α_λ **= 0 , 0.1 , 1 , 10 , 100**

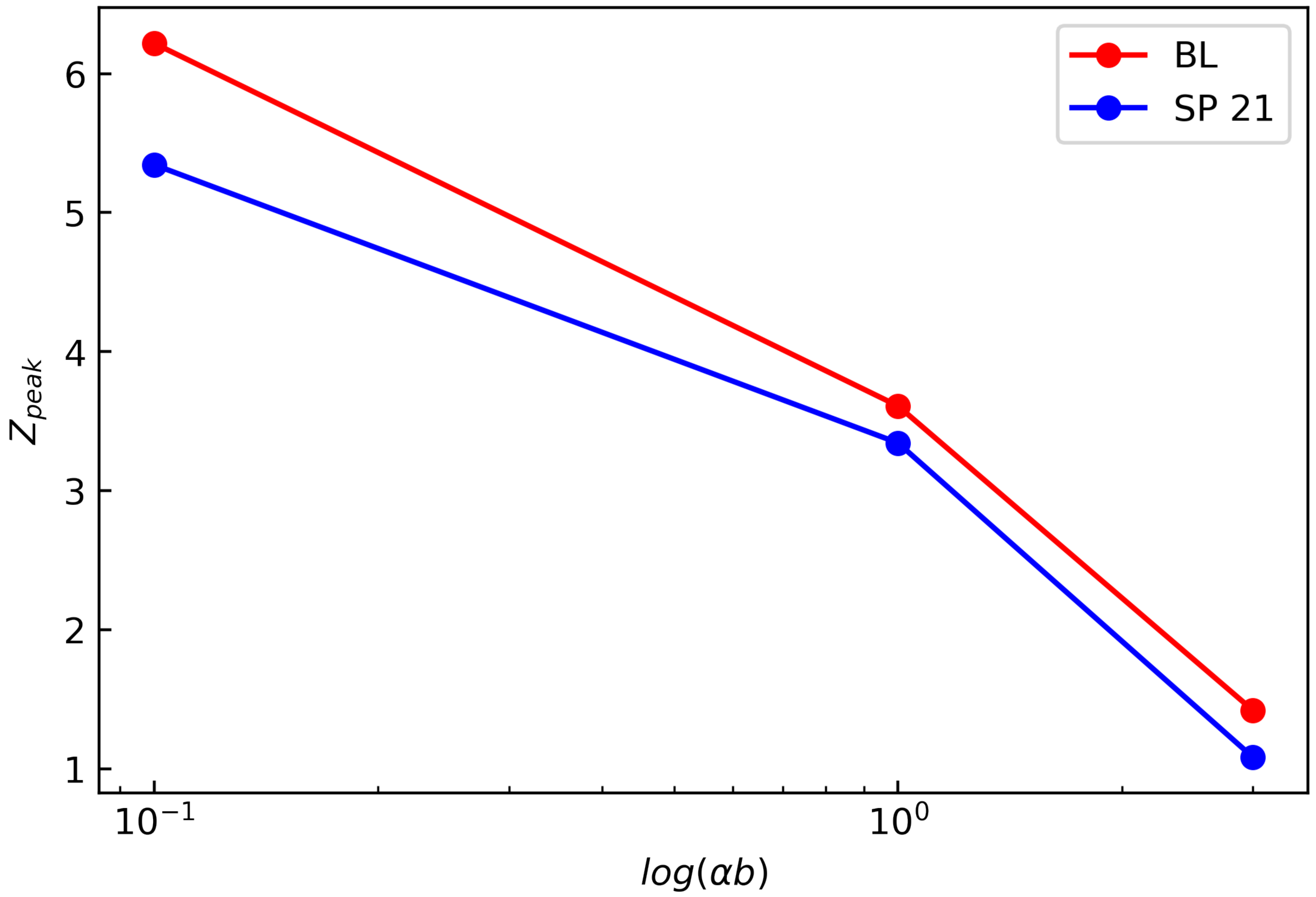
Comparison of Z_{peak} and different values of α_b



Z_{peak} vs α_b for 3 different values of

α_b **= 0.1 , 1 , 3**

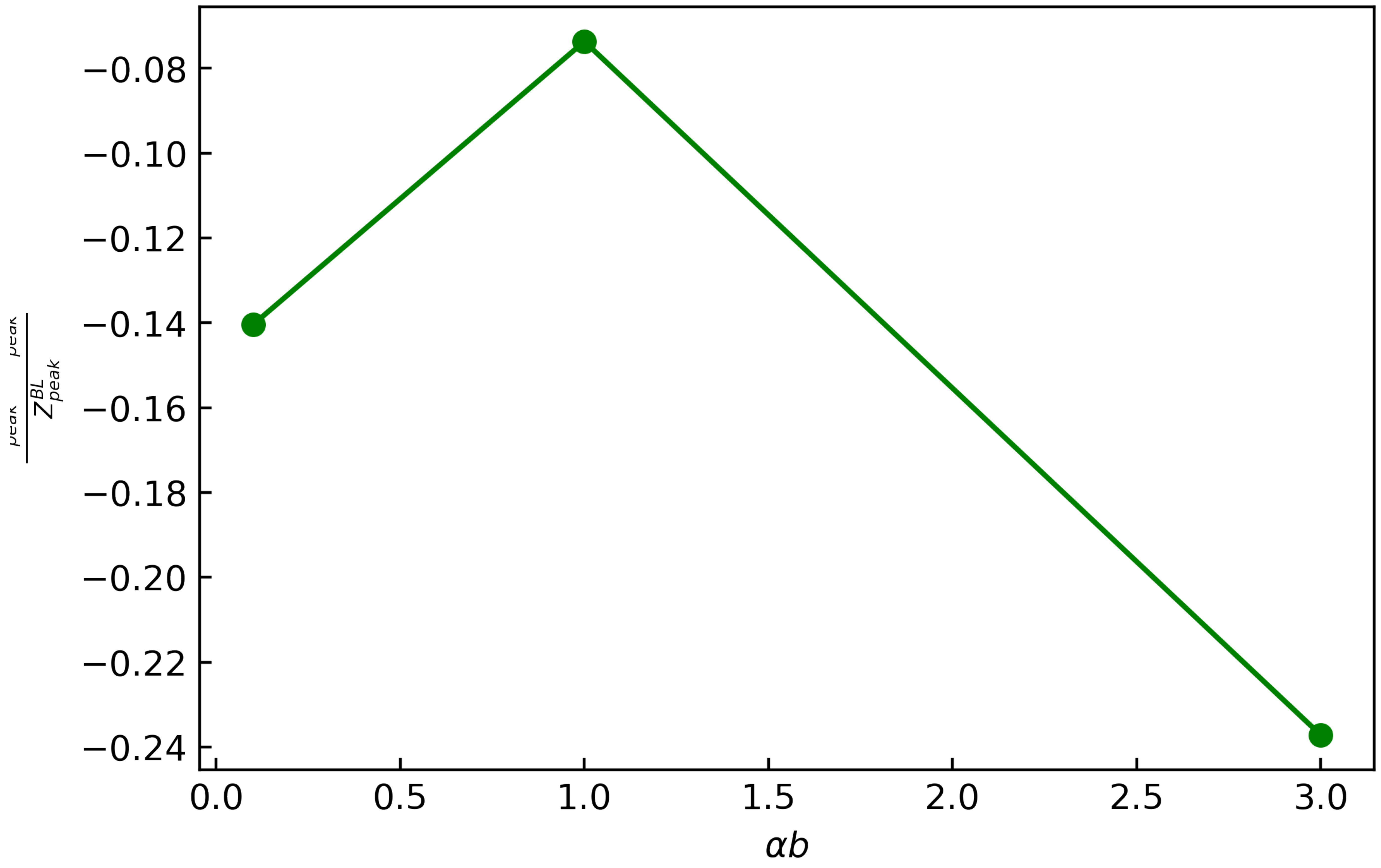
and $\alpha_\lambda = 0; \alpha_m, \alpha_s = 1$



Z_{peak} vs $log(\alpha_b)$ for 3 different values of

α_b **= 0.1 , 1 , 3**

and $\alpha_\lambda = 0; \alpha_m, \alpha_s = 1$



$\frac{Z_{peak}^{SP} - Z_{peak}^{BL}}{Z_{peak}^{BL}}$ vs α_b for 3 different values of

α_b **= 0.1, 1, 3**

Results and Discussions

- For our universe , the BL model at intermediate redshifts, is in good agreement with the data. Although, when we go to higher redshifts , the BL model as well as the SP model both seems to be slightly off with respect to the H&S model. The BL model might be off because of its crudeness as well as the different modelling.
- Once the structure formation starts , the SFRD of the BL model rises rapidly and soon hits its peak, both in the case of our universe as well as for different cosmology.
- The peak of the BL occurs earlier and the late time slope of the SFRD is also quite reasonable , although in the case of the SP model the late time slope is not steep like the BL one. The SFRD in the BL model falls faster than the SP model. The SP model on the other hand provide a peak of star formation in better agreement with data.
- In comparison with the H&S model , both the models show similar results and the peak of the CSFRD occurs in the range $2 < z < 4$. Thus in other words it improves the H&S model where the peak usually occurs at a more higher redshift of 5-6.

- In the comparison of different cosmological cases , the BL model tends to show a higher value of SFRD as compared to the SP model. Although, the peak in the BL model occurs at a higher redshift as compared to the SP model. The slope at low redshift / late time is more steep in the BL model.
- From the comparison plot of ρ_{peak} vs α_λ we can see that at higher values of α_λ the SFRD peaks of both the model show very small difference and try to converge.
- On the other hand from the comparison plot of Z_{peak} vs α_λ we can see that the Z peaks of both the model show very large difference and try to diverge on increasing the value of α_λ.

Limitations of the model

The BL model cuts-off the star formation at lower redshift , so, it can't compute the SFRD for very low redshift values (close to 0) .

The modelling of the SFRD tends to be crudely distributed when we try to compare different cosmological models with varying cosmological parameters other than the Standard Cosmological case or our Universe. The BL model does not provide a detailed analytical fit to simulations or observations of the SFR in our universe.

SFRD Comparison in the negative Redshift regions can't be computed using this model, and hence we were unable to compare the negative SFRD comparison of the BL model with that of the SP model.

The SFRD curve often starts to oscillates randomly in the lower redshift region when we change the cosmological parameters as compared to the Standard Cosmology or our Universe, and this may posses spurious result in the calculation of the SFRD and Redshift peaks from the plots.

Neglect of feedback effects from newly formed stars and from active galactic nuclei.

References

- R. Bousso and S. Leichenauer, Star Formation in the Multiverse, Phys. Rev. D 79 (2009) 063506, [arXiv:0810.3044].
- D. Sorini and J. A. Peacock, Extended Hernquist-Springel formalism for cosmic star formation, arXiv:2109.01146.
- L. Hernquist and V. Springel, An analytical model for the history of cosmic star formation, Mon. Not. Roy. Astron. Soc. 341 (2003) 1253, [astro-ph/0209183].
- Planck Collaboration, N. Aghanim et al., Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. 641 (2020) A6, [arXiv:1807.06209]. [Erratum: Astron.Astrophys. 652, C4 (2021)].
- L. Lombriser and V. Smer-Barreto, Is there another coincidence problem at the reionization epoch?, Phys. Rev. D 96 (2017), no. 12 123505, [arXiv:1707.03388].
- R. Bousso, Holographic probabilities in eternal inflation, Phys. Rev. Lett. 97 (2006) 191302, [hep-th/0605263].
- A. De Simone, A. H. Guth, M. P. Salem, and A. Vilenkin, Predicting the cosmological constant with the scale-factor cutoff measure, Phys. Rev. D 78 (2008) 063520, [arXiv:0805.2173].
- M. Tegmark, A. Aguirre, M. Rees, and F. Wilczek, Dimensionless constants, cosmology and other dark matters, Phys. Rev. D 73 (2006) 023505, [astro-ph/0511774].
- C. G. Lacey and S. Cole, Merger rates in hierarchical models of galaxy formation, Mon. Not. Roy. Astron. Soc. 262 (1993) 627–649.
- S. D. M. White, Formation and evolution of galaxies: Lectures given at Les Houches, August 1993, in Les Houches Summer School on Cosmology and Large Scale Structure (Session 60), pp. 349–430, 8, 1994. astro-ph/9410043.
- K. Nagamine, J. P. Ostriker, M. Fukugita, and R. Cen, The history of cosmological star formation: three independent approaches and a critical test using the extragalactic background light, Astrophys. J. 653 (2006) 881–893, [astro-ph/0603257].
- A. Lewis, A. Challinor, and A. Lasenby, Efficient computation of CMB anisotropies in closed FRW models, Astrophys. J. 538 (2000) 473–476, [astro-ph/9911177].
- A. Lewis and A. Challinor, CAMB: Code for Anisotropies in the Microwave Background, Feb., 2011.
- D. S. Blanco and L. Lombriser, Exploring the self-tuning of the cosmological constant from Planck mass variation, arXiv:2012.01838.

Thank you