Optimization in the design and control of robotic manipulators: A survey

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Robotics is a relatively new and evolving technology being applied to manufacturing automation and is fast replacing the special-purpose machines or *hard automation* as it is often called. Demands for higher productivity, better and uniform quality products, and better working environments are primary reasons for its development. An industrial robot is a multifunctional and computer-controlled mechanical manipulator exhibiting a complex and highly nonlinear behavior. Even though most current robots have anthropomorphic configurations, they have far inferior manipulating abilities compared to humans. A great deal of research effort is presently being directed toward improving their overall performance by using optimal mechanical structures and control strategies. The optimal design of robot manipulators can include kinematic performance characteristics such as workspace, accuracy, repeatability, and redundancy. The static load capacity as well as dynamic criteria such as generalized inertia ellipsoid, dynamic manipulability, and vibratory response have also been considered in the design stages. The optimal control problems typically involve trajectory planning, time-optimal control, energy-optimal control, and mixed-optimal control. The constraints in a robot manipulator design problem usually involve link stresses, actuator torques, elastic deformation of links, and collision avoidance. This paper presents a review of the literature on the issues of optimum design and control of robotic manipulators and also the various optimization techniques currently available for application to robotics.

INTRODUCTION

Robotic manipulators have seen an increasing application in industrial manufacturing and assembly operations in the last decade, especially in the automotive and the electric machinery industries. This trend is going to continue into the foreseeable future along with a broader use of robotic devices in several new applications. Primary motives for robotic development include higher productivity, lower production costs, better product quality, and relief to humans monotonous or hazardous tasks. Several books, dealing with various aspects of robotics, have been published in recent years. A general introduction to the subject can be found in [1–8]. References [9–15] discuss the aspects of trajectory planning and control of robotic manipulators. The mechanical design of robotic manipulators has been considered in [16–18]. Salisbury [19] presented a detailed study of robotic grippers. Some handbooks have also been published on the subject [20–22]. A robotic system consists of a manipulator (mechanical arm), an end-effector (robot hand), and a controller. A manipulator is a series of links connected by either revolute or prismatic joints. Based on the configuration a manipulator can be categorized as rectangular, cylindrical, spherical, or articulated (also called anthropomorphic) type. The end-effector is a gripper or tool, often interchangeable, that performs the actual task (eg, picking, welding, etc). The controller's task is to provide inputs to the joints to produce the desired motion of the end-effector. Robot controllers can be broadly classified into two categories: nonservo (or open loop) and servo controllers. Nonservo controllers are simple and generally use stepper motors but do not provide good trajectory control. Servo controllers are more common and often use sensing devices to get feedback information on certain variables such as joint positions, velocities, end-effector forces, etc.

In spite of the technical advances made in the areas of design and control methodologies and computational procedures, efficient design, and control of robotic manipulators remains a challenging task. A robotic manipulator in motion has highly complex and nonlinear dynamic characteristics. Even a complete closed-form kinematic solution of a general 6-degree-of-freedom manipulator is nearly impossible to obtain. In addition to these difficulties, the designer also has to deal with a large number of design variables. In order to overcome these obstacles, many robotic researchers have resorted to nonlinear programming techniques in the design and control of robotic manipulators. This paper provides a brief summary of such attempts and also outlines some popular optimization techniques available to researchers.

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OPTIMUM ROBOT DESIGN

In the past, the mechanical design of robotic manipulators has been based on some simple criteria, such as number of joints, link sizes and weights, and payload capacity. The design process includes deciding upon a robot configuration largely by intuition or experience. Various mechanical components are selected to meet the static strength and stiffness requirements. Actuators are selected based on static load ratings and acceleration allowances. This results in a manipulator with inefficient and unpredictable performance.

Recent years have seen an increasing demand for high precision, high speed, and task-oriented robots. Consequently, the design methodology is undergoing important changes. A complete knowledge of the various characteristics of a manipulator arm is required. For example, the dynamic performance of the manipulator arm must be considered even in the early design stages. Often the design decisions are based on complex and conflicting requirements. Therefore, optimization techniques, especially nonlinear programming methods, are proving to be important tools in the design of high performance robots. This section deals with the optimum design of robotic structures. An optimum design problem can be expressed mathematically as [23]:

$$\begin{aligned}&\text{Find } \mathbf{x} = [x_1\, x_2 \cdots x_n]^T\\&\text{which minimizes } f(\mathbf{x})\\&\text{such that } g_j(\mathbf{x}) \le 0;\ j = 1, m\\&h_k(\mathbf{x}) = 0;\ k = 1, l,\end{aligned} \tag{1}$$

where $\mathbf{x}$ is the design (or decision) variable vector, f is the objective function, and g_j and h_k are the inequality and the equality constraints, respectively. In robot design, the design variables include the kinematic characterizations (geometric parameters, joint travel ranges, and tolerances) and dynamic characterizations (link masses, inertias, material properties, and actuator specifications). The objective function is a measure of the manipulator performance which is to be optimized, and design constraints might include upper and/or lower limits on sizes, weights, actuator torques, etc. Over the years researchers have proposed and investigated different criteria for evaluating manipulator performance, as summarized below.

Kinematic criteria

Manipulator workspace is probably the most widely investigated kinematic property of robot arms [24–42]. Tsai and Soni [43] proposed a procedure for designing two- and three-link revolute joint manipulators based on workspace. The procedure gives the dimensions and the location of the arm which will enclose within its workspace a given set of end-points. Yang and Lee [44] defined a performance index, called volume index, based on workspace volume and link dimensions, and used a heuristic optimization technique (HOT) [45,46] to optimize this index with various kinematic parameters as design variables. They demonstrated the technique by optimizing some commercially available robots for better workspace. Gupta and Roth [47] determined the primary and secondary workspaces of manipulators with three orthogonal mutually intersecting axes and studied the effects of end-effector size. Tsai and Soni [48] studied the effects of various kinematic parameters on the workspace shape and size for a 3-degree-of-freedom robot, and selected the optimum parameters for maximizing the workspace. These results were applied to optimize higher-degree-of-freedom robots by considering two separate structures, namely the *regional* structure (first three links) and the *orientational* structure (last three links for a six-link robot) [49]. Vijaykumar et al [50] determined the optimum regional and orientational structures of a six-revolute joint manipulator based on workspace shape and volume criteria and subject to a constraint on the total length. Similar results were obtained in a recent work by Paden and Sastry [51], who have shown that a six-revolute joint manipulator has optimal workspace properties if it is an *elbow* manipulator. Lin and Freudenstein proposed a search technique to maximize the criteria of workspace volume and the workspace-to-void ratio [52,53]. They numerically evaluated these criteria for a number of arm configurations based on some selected values of kinematic parameters and discussed the resulting workspaces.

The ability of a manipulator to reach a point in its workspace from different directions, also known as *dexterity*, has been investigated in various reports. Roth [54] used the term *approach angle* to study dexterity. Yang and Lai [55] discussed this property by using the concept of *service angle*, a term introduced by Vinogradov et al [56], which is defined as the total range of approach angle around a point in the workspace. The terms *dextrous* (*or primary*) *workspace* and *secondary workspace* were also introduced and investigated in this context [24,25,47].

Another index for kinematic performance is the determinant of the Jacobian matrix used by Paul and Stevenson as a measure of degeneracy in the analysis of robot wrists [57]. A generalized form of this concept was proposed by Yoshikawa [58] as the *manipulability measure* to obtain a quantitative measure of the manipulating ability of a robot arm in positioning and orienting the end-effector. He discussed some properties of this measure and used it to determine the best postures of some simple 2- and 3-degree-of-freedom manipulators [59]. Uchiyama et al [60] also used the determinant of the Jacobian as a performance index and obtained its distribution in the whole workspace for two 6-degree-of-freedom robot arms. They also illustrated the use of this index to obtain optimal trajectories between two end-points using a random search technique [61]. A more general approach was suggested by Lilov and Bekjarov [62]. They proposed a mathematical technique for introducing geometrical and kinematical qualitative characteristics for robotic systems and interpreted the geometrical and kinematical criteria used at present as special cases. They suggested a package of programs, called CAMS, capable of calculating and visualizing these characteristics in a three-dimensional workspace.

The concept of *condition number* of the Jacobian matrix, based on its norm, has also been used in the kinematic investigation of robotic manipulators [63,64]. Salisbury and Craig [65] first used this concept to study the manipulating ability of articulated hands. Angeles and Lopez-Cajun [66] defined a dexterity index based on the minimum value of the condition number of the Jacobian in the workspace, and proposed a numerical procedure to compute its value. Two industrial manipulators were analyzed using this criteria. They also reported that their conclusions regarding two-link arm configurations with maximum dexterity did not agree with those obtained by Yoshikawa [59]. Another criteria, called *global conditioning index*, based on the average value of the condition number of the Jacobian in the workspace has been proposed by Gosselin and Angeles [67].

Another desirable property of robot manipulators relates to the accuracy and repeatability criteria. Colson and Perreira [68] presented definitions of these criteria, and discussed sources of poor performance and techniques to improve the performance. Bhatti and Rao [69] presented a probabilistic approach to robot kinematics and proposed the concept of *manipulator reliability* as a quantitative measure of robot accuracy and repeatability. They also presented techniques to compute this measure and discussed its relationship to the geometric parameters such as tolerances and arm configuration. Optimum assignment of tolerances based on manipulator reliability is a topic that needs investigation.

Incorporation of *redundancy* has been suggested by some researchers to improve the manipulator performance. The presence of redundant degrees of freedom offers more flexibility in trajectory planning, which can be particularly useful in avoiding obstacles and arm singularities, and in generating optimum trajectories [70–75]. Yoshikawa [58] applied the concept of manipulability to develop a control algorithm for redundant manipulators for avoiding singularities. Hollerbach [76] investigated some kinematic arrangements for 7-degree-of-freedom manipulators and arrived at an optimum arrangement by considering various aspects such as elimination of singularities, mechanical realizability, kinematic simplicity, and workspace shape. Yahsi and Özgören [77] optimized the motion of manipulators with redundant degrees of freedom by minimizing the total joint movement using Davidon–Fletcher–Powell (DFP) method [23].

Load carrying capacity

Certain manipulation tasks require generation of forces or torques in certain directions at the end-effector. The static force and torque available at the end-effector specify the static load carrying capacity of the manipulator, and are functions of actuator characteristics and arm configuration. Thomas et al [78] investigated the static payload capacity and the maximum velocity and acceleration available at the end-effector due to the actuator torque bounds for a given arm posture. Since the load carrying capacity varies within the workspace, Chong and Kok [79] suggested a method for plotting contours of constant load carrying capacities, based on actuator characteristics. They also derived expressions for computing the static and dynamic load carrying capacities. However, in the computation of dynamic load carrying capacity, they neglected the effect of the Coriolis and centrifugal forces. Wang and Ravani [80, 81] defined the dynamic load carrying capacity (DLCC) as the maximum load that the manipulator can carry while executing a given trajectory. They proposed an algorithm to compute DLCC based on the superposition of the dynamic models of the manipulator and the load. They also considered the problem of trajectory generation for optimum load carrying capacity, which was solved by linearizing the dynamic model and using an iterative linear programming method (based on the cutting plane techniques [23]). A three-link robot arm was considered for numerical illustration.

Dynamic criteria

Dynamic performance is a significant factor in the design of high performance robots. A robot arm in motion exhibits a highly complex and nonlinear behavior due to inertial, centrifugal, Coriolis, and gravity forces acting on it. The designer needs a tool for characterizing the dynamic features of robot arms. One such measure was proposed by Asada [82] as *generalized inertia ellipsoid* (GIE). He extended a concept of inertia ellipsoid, a geometrical representation of the inertia tensor of a single rigid body, to the dynamics of robot arms. GIE indicates the positioning and orienting abilities of the end-effector (bounds on the end-effector accelerations) when external forces are applied to it. The shape and the distribution of GIE in workspace indicate directional dependency and nonlinearity in dynamic capabilities of the arm. To reduce these two effects, it is desirable to have a uniform and isotropic GIE configuration over the workspace. Asada obtained a GIE configuration of a two-link robot arm and also determined the optimum link lengths and mass distributions to achieve a more uniform and isotropic GIE distribution over a wide range of the workspace. Lee and Tortorelli [83] developed an objective function based on these considerations. They optimized this criteria for a 5-degree-of-freedom robot arm by changing the gear ratios of the first three actuators. Their algorithm used a quasi-Newton method [23] based subroutine developed by the IMSL [84].

As mentioned above, GIE relates the end-effector accelerations to the external forces acting on it. However, in robot control, the actuating forces are applied at the robot joints (actuator efforts) instead of at the end-effector. This problem was overcome by Yoshikawa [85], who extended the concept of kinematic manipulability proposed earlier [58] to *dynamic manipulability* by taking arm dynamics into consideration. This measure relates the actuator torques to the maximum available end-effector accelerations. He discussed some properties of this measure, and analyzed a two-link arm to obtain its best posture from the viewpoint of dynamic manipulability. This measure can be optimized in the design process to achieve large isotropic and uniform bounds on the end-effector acceleration, which will enhance the dynamic capabilities of the manipulator. Khatib and Burdick [86] derived a similar measure for evaluation of the dynamic performance, but used the dynamic model of the arm in Cartesian space (space in which task is described) instead of in joint-space as done by Yoshikawa [85]. They formulated the optimization problem based on this measure and illustrated its use by optimizing a two-link arm. The objective was to achieve large isotropic and uniform bounds on the end-effector acceleration with link lengths, masses, locations of centers of mass, and inertias as design variables.

The criteria presented in [82, 85, 86] defined the dynamic capability of the manipulator at a given location of the end-effector. In order to apply these measures in the design optimization of a manipulator, they must be specified over the entire or a part of the workspace. This was achieved by Graettinger and Krogh [87], who proposed the concept of *acceleration radius* to describe the dynamic capability over an operating region defined by a range of joint positions and velocities. Acceleration radius provides a lower bound on the magnitude of the acceleration available at the end-effector for any state (joint positions and velocities) in the operating region, subject to constraints on the actuator torques. Algorithm for the computation of the acceleration radius was formulated as a two-level optimization problem, which was solved numerically using a quasi-Newton method [23] based subroutine from the IMSL [84]. The use of this measure was illustrated through three examples dealing with workspace optimization and actuator specification.

Stiffness criteria

Structural elasticity of links and elasticity of actuating mechanisms cause errors in the end-effector location. Manufacturers try to reduce link deflections by using heavy link structures, which, due to their large inertias, cause slow performance and large overshoots. These effects become significant at high speeds. Also large actuators are needed to move these heavy links which add to the system's weight and further detriment the performance. Optimum link design with varying cross section and use of high-strength lightweight materials have been suggested to improve the arm stiffness and the vibrational characteristics. Numerical techniques, such as finite element models, can be used to carry out a detailed stiffness analysis of robotic arms.

Mendelson and Rinderle [88] investigated the use of an aluminum beam coated with a damping material as a compliant link to improve the vibration response of a robot arm. Rivin and Anis [89] proposed a composite link made of two different materials (steel and aluminum), and optimized it to obtain an increase in the link stiffness and a reduction in the required driving torque. Sung and Thompson [90] and Liao et al [91] suggested an approach for improving the dynamic response of robot arms by fabricating arm links with optimally tailored composite laminates. They developed a procedure for determining the optimum material characteristics and the cross-sectional geometries of the links to maximize the damping capacity of the links subject to constraints on the flexural rigidity and cross-sectional area of each link. The procedure was based on Biggs' recursive quadratic programming algorithm [92] and a finite element formulation [93]. An industrial robot was analyzed using this approach and the proposed design was shown to have a superior response with smaller deflections and shorter settling-times.

Saravanos et al [94] presented an optimization method for improving the end-effector stiffness based on finite element analysis and a response surface optimization (RSO) method [95]. They minimized the static end-effector deflections in different arm postures by optimizing the link lengths and cross sections of an existing robot arm and achieved significant increases in end-effector stiffness and stiffness to mass ratio without increasing the total mass.

Potkonjak and Vukobratović [96] considered the optimum design problem of the minimization of the time or the energy consumed by a manipulator to execute a certain task subject to constraints on link stresses, actuator torques, and elastic deformations of the links. Method of feasible directions [23] was used to determine the optimum diameters of the second and the third link of a 6-degree-of-freedom industrial robot. Scheinman and Roth [97] determined the optimum ratio of link lengths for a 2-degree-of-freedom planar robot to maximize the distance travelled in a fixed time interval (or to minimize the travel time between two points) under the constraints of total manipulator length and actuator torques.

Multiobjective optimization in robot design

A multiobjective optimization problem can be expressed as [23]:

$$\begin{aligned}&\text{Find } \mathbf{x} = [x_1\, x_2 \cdots x_n]^T\\ &\text{which minimizes } f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\\ &\text{such that } g_j(\mathbf{x}) \le 0;\ j = 1, m\\ &h_j(\mathbf{x}) = 0;\ j = 1, l.\end{aligned} \tag{2}$$

Optimum robotic structures have been obtained with respect to different individual criteria as stated earlier. However, some of these criteria impose conflicting requirements on the design. A good example is the speed of operation, which is related to dynamic manipulability, and end-effector accuracy (or arm stiffness). If a robot is to perform satisfactorily in all aspects, different criteria must be optimized simultaneously in the design process, which can be handled by using a multiobjective optimization procedure. However, no work has been reported in the literature that applies multiobjective optimization techniques to robot design.

Robot configuration

Most work in optimum robot design has excluded one important characteristic as design variable, namely the *structural configuration* of the robot arm as determined by the kinematic parameters such as the twist angles of the links, distances between the links, and link lengths. This is different from the *arm posture* which is determined by the joint variables (joint angles for a revolute arm). Most researchers have tried to optimize the existing robot configurations by changing parameters such as link dimensions, weights, inertias, and actuator specifications. It should be noted that of the many different kinematic arrangement possibilities only few simple ones are being used in commercially available robots [98]. A possible reason for this might be to maintain the kinematic equations, in particular, the inverse kinematic equations simple. Until 1983 it was widely believed that for a 6-degree-of-freedom arm the inverse kinematics problem could only be solved analytically if the last three joint axes intersected at a point. With recent advances such as efficient iterative procedures and faster computers, it seems possible to obtain the inverse kinematic solution in real time for any robot configuration [99–105]. Therefore, there exists no reason why designers should not explore other kinematic arrangements.

OPTIMAL ROBOT CONTROL

During the past several years the role of robots in industry has been slowly changing from position control devices to motion control manipulators. The desire for increased productivity presented the problems of optimal control and optimal trajectory planning for industrial manipulators. An optimal control problem can be mathematically expressed as [106]:

Find the control vector $\mathbf{u} = [u_1\, u_2 \cdots u_m]^T$ which minimizes the performance index,

$$J = h\big(\mathbf{x}(t_f), t_f\big) + \int_{t_0}^{t_f} g(\mathbf{x}(t), \mathbf{u}(t), t)\, dt, \tag{3}$$

where $\mathbf{x} = [x_1\, x_2 \cdots x_n]^T$ is called the state variable vector and t is the time parameter. The state variable vector and the control variable vector are related by the system equations as

$$\dot{\mathbf{x}}(t) = \mathbf{a}(\mathbf{x}(t), \mathbf{u}(t), t). \tag{4}$$

The problem of robot control is traditionally divided into two parts, namely *motion planning* and *motion control*. When the objective is restricted to obtaining a time history of the end-effector/joint positions, velocities and accelerations *without a consideration of the manipulator dynamics*, the problem is one of motion (or trajectory) planning. The motion control deals with computing the necessary actuator efforts (torques/forces) so that the manipulator end-effector follows the desired trajectory, and usually involves manipulator dynamics. The commonly used motion control techniques include computed torque method, resolved motion control, adaptive control, and nonlinear decoupled feedback control. It is the motion planning part where optimization methods play an important role. However, a number of recently proposed optimal motion planning algorithms take manipulator dynamics into consideration, thereby representing a more balanced approach to the overall problem of robot control. To avoid any confusion, such algorithms are discussed under the general category of optimal control in this study.

Optimal trajectory planning

The minimum-time trajectory planning aims at moving the manipulator end-effector from an initial location to a final location in space at the maximum possible speed and along the best possible path subject to constraints such as bounds on the actuator efforts, joint limits, and obstacles. Some researchers have attempted to solve this problem using kinematic approaches [107–110]. The end-effector path is specified as a set of knot points (end-effector positions and orientations), and the total trajectory is approximated by a sequence of segments (straight lines or smooth curves such as cubic splines) between the knot points. The problem is formulated as the minimization of the total travelling time for the entire path subject to constraints on position, velocity, and acceleration. The problem can be formulated either in the Cartesian space [107–109] or in the joint space [110]. Optimization procedures in [107–109] were based on linear programming methods [23]. Since the dynamics is not considered here, the constraints on joint/end-effector velocities and accelerations have to be obtained experimentally for the worst arm configuration. Since, in reality, the bounds on velocities and acceleration vary with the position, the solution obtained with these techniques will be optimal only when the arm is operating near its worst configuration. Further the path described by the knot points may not be the best path between the initial and the final positions.

Time-optimal control

The problem of optimal motion control of a manipulator between two points or along a specified path has drawn considerable attention in the research community. The classical control theory provides a mathematical tool to solve the two-point boundary value problem (TPBVP) of optimal motion control between two points in the form of Pontryagin's extremum principle [111]. This principle when applied to the dynamic model of a robot arm generates a set of coupled nonlinear differential equations which cannot be solved analytically. Kahn and Roth [112] and Kahn [113] obtained an approximate or *suboptimal* minimum-time solution by applying Pontryagin's principle to the linearized and uncoupled equations of motion of a 3-degree-of-freedom manipulator. The linearization was obtained by neglecting the velocity product terms. This problem was solvable analytically, and the solution has the familiar bang-bang form with multiple switching points. One drawback of this method is the computational difficulty for higher-degree-of-freedom manipulators. Also the kinematic constraints on the motion were not considered. Further, it has been shown that the linearization of the arm dynamics results in significant errors [114].

Due to the computational difficulties in applying Pontryagin's principle to robot motion control problem, Kiriazov and Marinov [115] proposed a direct approach. They considered the problem with actuator constraints and a set of dynamic constraints, and used a control synthesis algorithm to generate all feasible motions and the corresponding travel times of the TPBVP. This feasible domain is then searched using the steepest descent method [23] to obtain the minimum travel time. The procedure was illustrated for a two-link cylindrical robot. The control synthesis algorithm requires a time-consuming iterative method (Newton's or bisection method) to generate feasible motions. Also the set of feasible solutions might become too large for higher-degree-of-freedom manipulators. Furthermore, this method does not guarantee a global optimal solution.

Bobrow et al [116, 117] proposed a minimum-time control algorithm, also known as the *phase-plane method*, to compute the actuator efforts for a manipulator moving on a specified path. In this technique, the end-effector path is described in a parametric form, the distance along the path being the parameter. The manipulator dynamics is then expressed as functions of this parameter. From this model and given the limits on actuator efforts, a velocity limit curve is plotted in the phase plane (velocity vs position plane) which gives the maximum *admissible* velocity at each point along the path. The minimum-time path is then shown to be a path, in the admissible region of the phase plane, consisting of a sequence of switching points from maximum to minimum acceleration and vice versa [118]. An algorithm is presented for selecting these switching points and the technique is demonstrated for a three-link manipulator. Shiller [119] and Dubowsky and Shiller [120] applied this technique to a six-link manipulator and discussed its use in design of robots and their work environments. They presented a computer program called OPTARM (optimal time control of articulated robotic manipulators) based on the extension of the technique which includes payload and gripper constraints [121]. The program can model end-effector paths as straight lines, circular curves, and splines, provides useful information for design and motion planning, and has interactive graphics capabilities. A similar approach was independently proposed by Shin and McKay [122–124] where the path is approximated by a curve in the joint space. They considered the effect of viscous friction at the joints, showed the existence of *islands of inadmissibility* in the admissible regions, something Bobrow's algorithm might have difficulties with, and suggested another procedure to find minimum-time paths.

The main criticism of the kinematic approaches [107–110] described in the previous section centered around the usage of constant, worst-case bounds on the velocities and accelerations, resulting in underutilization of manipulator's capabilities, which prompted some researchers [125–127] to include arm dynamics in these methods to convert the bounds on actuator efforts to those on velocity and acceleration. However, the basic idea remained the same, specification of path by several knot points and construction of the path using straight lines or smooth curves in either the joint space or the Cartesian space. The algorithms vary in detail as to how to distribute velocities and accelerations along these paths to minimize the total traveling time.

The techniques described above [116–127] suffer from a common weakness, which is the requirement that the path be described by some approximating function in space. This limitation, though helpful in avoiding contact with the obstacles, does not guarantee a true minimum-time trajectory. This problem was overcome by Sahar and Hollerbach [128]. They discretized the joint space into a grid used the dynamic scaling concept developed earlier [129] along with the actuator bounds to calculate the minimum time to travel from one point to a neighboring point on this grid. The minimum-time trajectory is then found by using a graph search technique on this grid. This technique was demonstrated successfully for a two-link planar manipulator. However, large computational costs even for simple problems and combinatorial explosion of the search space for higher degrees of freedom make the implementation of this technique impractical.

Due to the computational difficulties experienced with both optimal control theory based algorithms and the graph search based techniques, Rajan [130] attempted to develop a more efficient algorithm by combining the two approaches. He emphasized that the minimum-time path has to be smooth and therefore parametrized the path in the joint-space using cubic splines and knot points. He then used the algorithm of Bobrow et al [116] to calculate the minimum time for this path. The path is then changed by varying the spline parameters and knot points and the process is repeated. The path that gives the shortest minimum time is the time-optimal trajectory. However, this method does not guarantee a global minimum. He implemented this method for a two-link manipulator, and claimed that it can be extended to higher-degree-of-freedom manipulators with reasonable efficiency.

Shin and McKay [131] proposed two methods for determining near-minimum time paths between two points. One method is based on the concept of lower bound on the travel time and the other is based on the concept of velocity bounds developed in [122–124]. They showed that the resulting paths in both methods are *geodesics* in the inertia space which can be determined by solving a two-point boundary value problem. The first three joints of the Bendix PACS arm and the Stanford arm were considered for numerical illustration.

Tan and Potts [132] proposed an algorithm for determining minimum-time control for a specified path, based on a discrete dynamic model of the manipulator arm [133]. They considered the constraints on actuator torques, joint velocities, and jerks, and the resulting optimization problem was solved using an iterative linear programming method [23].

Energy-optimal control

This problem concerns with the minimization of energy consumption by a manipulator along the trajectory, and is significant for the fully automated factory of the future where a large number of robots will be working at high speeds, handling heavy loads. Vukobratović and Kirćanski [134] developed an algorithm for obtaining the velocity distribution of the end-effector on a specified path to minimize the energy consumed. They combined the dynamic model of the arm with quasilinear models of actuators to construct a dynamic model of the system. The path is parametrized in the Cartesian space. Since it is difficult to determine the optimal actuator inputs as functions of time, the problem formulation is discretized in time and is reduced to a series of minimizations. The optimization procedure was based on dynamic programming [135]. The method was applied to a six-link industrial manipulator; however, only the first three joints were considered for optimization.

Schmitt [136] and Schmitt et al [137] proposed a method based on calculus of variations, in particular the Rayleigh–Ritz scheme, to obtain optimum motion between two points by minimizing the joint torques. The path was approximated in the joint space using shape functions satisfying the boundary conditions. The problem was formulated as determination of the total traveling time and the constant coefficients of the shape functions that would minimize an objective function based on joint torques. The problem was solved using a penalty function optimization technique [138]. The method was implemented for a three-link manipulator and the results were compared with some standard motion planning algorithms.

Mixed optimal control

The optimization of the total cost of operating a robot implies the minimization of the travel time and energy consumption along the trajectory simultaneously. This problem is sometimes called the mixed optimal control problem. One such technique was proposed by Marinov and Kiriazov [139] for the case when the starting and the end points of the path are specified. The technique is based on a method presented by them earlier [115]. The performance index used is composed of two terms: the travel time and the energy consumed with suitable weighting factors, which is minimized over the set of feasible motions. Based on the choice of these weighting factors, the problem can be solved as time-optimal, energy-optimal, or mixed optimal control. A two-link cylindrical manipulator is considered for numerical illustration; however, results are presented only for time-optimal and energy-optimal controls.

Kim and Shin [140] proposed a feedback method to solve the mixed optimal control problem. The method applies Pontryagin's principle [111] to the averaged dynamics of the manipulator. The inertia matrix and the centrifugal, Coriolis, and gravitational force vectors in the dynamic equations are assumed to be constant over each sampling interval, and are updated at regular intervals using the input and feedback information on positions and velocities. The resulting problem is solved analytically. Due to the approximations involved in the dynamics, the solution is referred to as near minimum time-fuel control. The method is illustrated by simulating the first three joints of a six-link PUMA 600 manipulator, and the results indicate that the method can be implemented in real time using a mainframe computer.

Shin and McKay [141] extended their earlier work [122–124] to include a performance index that can take a general form to minimize the total operating cost. They also considered the interdependency of the actuator constraints and the constraints on the jerk. To handle the increased complexity, dynamic programming technique [135] was used for locating the optimal path in the phase-plane grid. The method was demonstrated for the first three joints of the Bendix PACS arm, and the results were compared with those of their earlier work. It was found that the algorithm converges reasonably fast; however, converging time depends strongly on the grid size used, and also offers flexibility in the choice of the performance index and the constraints.

Singh and Leu [142] also considered the optimal control problem with a general performance index. The problem was formulated by discretizing the kinematics and the dynamics equations along the Cartesian path, which was specified as a sequence of points. The algorithm derives the bounds on the joint velocities from the actuator torque constraints and the dynamics equations in a recursive manner. The problem was solved using dynamic programming [135], and the technique was illustrated for a two-link manipulator.

Feedback control

Spong et al [143,144] presented an optimal control law for feedback control that minimizes the deviation between system velocity (or acceleration) vector and a desired velocity (or acceleration) vector, subject to constraints on actuator inputs. The formulation was based on state space description of Lagrangian equations of motion and the resulting quadratic programming problem was solved using the primal–dual method [145]. The simulation results were presented for a two-link manipulator. They also showed that some of the feedback control techniques developed earlier [146–151] were special cases of their control law with different choices of desirable velocity vector. Kim et al [152] obtained the optimal feedback control law using a parameter sensitivity method [153]. Kao et al [154] proposed a feedback control algorithm to obtain near-minimum-time trajectories.

Collision avoidance

Another important issue in robot motion planning relates to avoiding contact with the obstacles present in the workspace.

Several approaches have been proposed for planning collision-free motions. Some examples can be seen in [155–165]. This problem has also been considered in the framework of an optimal control problem. Johnson and Gilbert [166] used the concept of distance functions [159] between the robot and the obstacles, expressed them as constraints in the time-optimal motion planning problem, and solved the resulting optimization problem using a penalty function method. The technique was illustrated for two-link Cartesian robot and a three-link cylindrical robot. Dubowsky et al [167] investigated the problem of time-optimal motion planning [116–121] by including path constraints due to obstacles, and proposed a computer software package called OPTARM II, based on a penalty function method. Ozaki and Mohri [168] considered constraints due to obstacles and formulated an optimization problem with a quadratic objective function that minimizes the distance between actual and desired end-effector positions. The objective function and the constraints were linearized, and a linear programming method was used to solve the problem. Bobrow [169] considered obstacle avoidance in the framework of a time-optimal control algorithm developed earlier [116–118] by using distance functions [159].

Mayne [170] formulated the positioning and trajectory planning of a robot with redundant degrees of freedom in the presence of obstacles in a confined workspace as a nonlinear optimization problem. He defined the measure of *distance-to-contact*, similar to distance functions of [159], and developed a method to calculate this measure between various objects. The distance-to-contact measure appears as an inequality constraint in the final formulation. He also investigated a method to reduce the number of constraints to be considered in a given arm posture. The problem of optimum positioning and trajectory planning for a 3-degree-of-freedom planar robot were formulated as two separate problems. The optimum positioning problem was formulated so that the distance between the robot and the obstacle boundaries is maximized. In the optimum trajectory planning problem the movements of various joints were minimized from the initial to the final location. The expressions for computing the gradients of the objective and constraint functions were also derived and a gradient-based constrained minimization method was used to solve these problems.

OPTIMIZATION TECHNIQUES

The *classical* optimization methods, based on principles of differential calculus and calculus of variations, can provide closed-form solutions for certain optimization problems where the objective and the constraint functions are simple explicit expressions. Kuhn–Tucker conditions provide necessary conditions of optimality in the presence of inequality constraints. Objective functions of an integral form, as found in optimal control problems, can be optimized by applying techniques of calculus of variations and Pontryagin's minimum principle, resulting in a two-point boundary value problem (TPBVP). Another popular technique used in optimal control theory is based on the principle of optimality, and is known as dynamic programming algorithm. This method leads to either a recurrence equation, providing an exact optimal solution to the discretized system, or a partial differential equation, also known as Hamilton–Jacobi–Bellman equation, requiring a numerical solution.

The *numerical* optimization techniques are iterative in nature, starting from an initial estimate and progressing toward the optimum solution. Depending on the type of problem they can be classified as *linear*, *quadratic*, *geometric*, *integer*, or *nonlinear* programming methods. Most optimization problems in robotic design and control fall into the category of nonlinear programming, although some researchers have also used linear and quadratic programming techniques. A nonlinear programming algorithm consists of a sequence of improved approximations to the optimum point, and involves the iterative process

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + \alpha^{(i)*}\mathbf{S}^{(i)},$$

where $\mathbf{x}^{(i)}$ is the starting point, $\mathbf{S}^{(i)}$ is the search direction, and $\alpha^{(i)*}$ is the optimal step length. The problem of finding $\alpha^{(i)*}$ can be posed as: find $\alpha^{(i)}$ that minimizes $f(\mathbf{x}^{(i+1)}) = f(\mathbf{x}^{(i)} + \alpha^{(i)}\mathbf{S}^{(i)}) = f(\alpha^{(i)})$ for fixed $\mathbf{x}^{(i)}$ and $\mathbf{S}^{(i)}$. This problem can be solved by using an *elimination* method such as Fibonacci or golden section method or an *interpolation* method such as quadratic, cubic, or direct root method.

If the minimization problem is an *unconstrained* one, the search directions $\mathbf{S}^{(i)}$ can be generated by using a *direct search* or a *descent* method. The direct search methods do not require the derivatives of the function and include the *random search*, *pattern search* (Hooke and Jeeves' [171] and Powell's [172]) and *heuristic* (simplex [173,174]) methods. The descent methods make use of the gradient of the objective function and are, in general, more efficient than the direct search methods. The *steepest descent* [175–177], *Fletcher–Reeves* [178], *Newton's*, and the *variable metric methods* fall in this category. The Newton's method uses the iterative process

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} - \alpha^{(i)*}[\mathbf{J}^{(i)}]^{-1}\mathbf{g}^{(i)} \tag{5}$$

and $\mathbf{g}^{(i)} = \nabla f(\mathbf{x}^{(i)})$. The *quasi-Newton* methods approximate $[\mathbf{J}^{(i)}]^{-1}$ by $[\mathbf{H}^{(i)}]$, where, for example, the *Davidon-Fletcher-Powell (DFP) variable metric method* [179,180] generates $[\mathbf{H}^{(i+1)}]$ as

$$[\mathbf{H}^{(i+1)}] = [\mathbf{H}^{(i)}] + \frac{\sigma\sigma^T}{\sigma^T\mathbf{y}} - \frac{[\mathbf{H}^{(i)}]\mathbf{y}\mathbf{y}^T[\mathbf{H}^{(i)}]}{\mathbf{y}^T[\mathbf{H}^{(i)}]\mathbf{y}}, \tag{6}$$

where

$$\sigma = -\alpha^{(i)*}[\mathbf{H}^{(i)}]\nabla f(\mathbf{x}^{(i)}), \tag{7}$$

$$\mathbf{y} = \nabla f(\mathbf{x}^{(i+1)}) - \nabla f(\mathbf{x}^{(i)}), \tag{8}$$

and the *Broyden–Fletcher–Goldfarb–Shanno (BFGS) variable metric method* [181–185] uses

$$[\mathbf{H}^{(i+1)}] = \left([I] - \frac{\sigma\mathbf{y}^T}{\sigma^T\mathbf{y}}\right)[\mathbf{H}^{(i)}]\left([\mathbf{I}] - \frac{\sigma\mathbf{y}^T}{\sigma^T\mathbf{y}}\right) + \frac{\sigma\sigma^T}{\sigma^T\mathbf{y}}, \tag{9}$$

where [I] is the identity matrix. The initial value $[\mathbf{H}^{(i)}]$ is usually taken as the identity matrix [I].

If the optimization problem is a *constrained* one, any of the *direct* methods (such as the complex [186], cutting plane, generalized reduced gradient, recursive quadratic programming, and the feasible direction methods) or the *indirect* methods (such as the penalty function and augmented Lagrangian method) can be used for solution. *The cutting plane method* [187,188] is applicable to problems with linear objective functions; however, any problem with nonlinear objective function can be expressed as one with linear objective function by adding a new variable and a new constraint. All the constraints are then linearized about the current design vector, and the resulting problem is solved using a linear programming technique to get the improved design vector. The process is repeated until the convergence criteria are met. In the *generalized reduced gradient method* [189,190], a "reduced gradient" resulting from perturbations of a set of independent variables is defined while maintaining the feasibility by adjusting the dependent variables. The problem is then solved as an unconstrained problem in terms of the independent variables. The *sequential quadratic programming method* uses a quadratic approximation for $f(\mathbf{x})$ and linear approximations for $g_j(\mathbf{x})$ and $h_k(\mathbf{x})$. The search directions in each iteration are then found by solving a quadratic programming problem [191]. In the *methods of feasible directions*, different schemes are used to find usable feasible directions and acceptable step lengths so that no constraints are violated. *Zoutendijk's method* [192] and *Rosen's gradient projection method* [193,194] fall under this category. The *penalty function methods* [138,195,196], also known as the *sequential unconstrained minimization techniques* (SUMT), involve transforming the original problem into an equivalent problem which is solved through a sequence of unconstrained minimizations. In these methods a new objective function is constructed by adding a penalty term to the original objective function of the constrained problem as shown below:

$$\phi^{(k)} = \phi(\mathbf{x}, r^{(k)}) = f(\mathbf{x}) + r^{(k)} \sum_{j=1}^{m} G_j\left[g_j(\mathbf{x})\right], \qquad (10)$$

where $f(\mathbf{x})$ is the original objective function, G_j is a function of the constraint g_j, and $r^{(k)}$ is the penalty parameter. This new function is then minimized for a sequence of values of $r^{(k)}$. In the interior penalty function method, the minima of function $\phi^{(k)}$ lie in the feasible domain while in the exterior penalty function method they lie in the infeasible domain. Eventually they both converge to the optimum of the original problem as $r^{(k)}$ is varied. Improvements in convergence of these methods have been achieved through linear, quadratic, and cubic extended penalty methods [197–199].

In the *augmented Lagrange multiplier method* [200], also known as the *primal–dual method*, an augmented Lagrangian is constructed as

$$A^{(i)} = f(\mathbf{x}^{(i)}) + \sum_{j=1}^{m} \left\{ \lambda_j^{(i)} \psi_j^{(i)} + r^{(i)} \left[\psi_j^{(i)} \right]^2 \right\}$$
$$+ \sum_{k=1}^{l} \left\{ \lambda_{k+m}^{(i)} h_k(\mathbf{x}^{(i)}) + r^{(i)} \left[h_k(\mathbf{x}^{(i)}) \right]^2 \right\}, \qquad (11)$$

where

$$\psi_j^{(i)} = \max\left[g_j(\mathbf{x}^{(i)}), \frac{-\lambda_j^{(i)}}{2r^{(i)}} \right] \qquad (12)$$

and λ's are the Lagrange multipliers. The original problem is solved by a sequence of unconstrained minimizations of the augmented Lagrangian for increasing values of $r^{(i)}$.

CONCLUSIONS

An overview of the literature on the subject of optimum design and control of robotic manipulators is presented. In optimum robot design, although researchers have proposed and investigated a number of different performance characteristics, the same enthusiasm has not been shown in applying these criteria to optimize robotic structures. This can be attributed to the computationally expensive criteria and the large number of design variables that must be handled in the optimization of a practical 6-degree-of-freedom industrial robot. On the other hand, of all the techniques presented for optimal control, only a few are capable of providing truly optimal solutions. And none of the time- or energy-optimal control schemes have proved widely successful in terms of real-time application and providing satisfactory results. Some of the main issues that will attract attention in the future research are summarized below:

1. The design optimization should include all pertinent kinematic and dynamic parameters of the robot arm in order to achieve optimum configurations. Due to computational difficulties, many researchers have considered only those parameters which they believe to be dominant in the optimization work.
2. Use of composite materials with optimum properties in robot construction is a promising concept and should be fully utilized.
3. Optimum robotic configurations should be obtained with respect to various criteria discussed in this study and possibly new ones. These structures might not be practically useful, but they will provide good insight into the nature of these characteristics.
4. One difficulty the researchers are facing is *where* in the workspace should they optimize a given criteria. Techniques must be developed to integrate these criteria over the entire workspace.
5. It is not sufficient to consider only one criteria in the optimization process. Multiobjective techniques must be used to simultaneously consider more than one criterion at a time [201,202].
6. A great deal of effort has been spent in finding minimum-time and minimum-energy trajectories and controls between two specified points in the workspace. A fresh look at this problem will be to investigate and locate the optimum trajectories and controls *within the workspace*. This can prove helpful in setting up an optimum work environment for a given robot.
7. The control aspects must be considered in the design stages. This takes us into the realm of *task-oriented robots* [203–205]. The designer should be aware of the future application of the robot and its work environment and should utilize that information in order to build a robot best suited for that environment.

Finally a true global optimization of a robotic system can be achieved by integrating structural, control, and environmental aspects. This not only requires a knowledge of the various disciplines such as mechanical, structural, control, materials, and industrial engineering but also the availability of efficient algorithms and computers to handle the complexity of the task.

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