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Optimal Design of a Parallel Machine Based on Multiple Criteria*

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***Abstract*— This paper proposes to optimally design a parallel machine based on multiple criteria. Many criteria, workspace, condition number, accuracy, stiffness, maximum velocity, and maximum force, are considered. The optimal design problem is proposed as to find a set of design parameters such that (a) the Cartesian workspace generated by the resulting manipulator contains a prescribed workspace; (b) the resulting manipulator possesses a good condition number at each points in the prescribed workspace; (c) the resulting manipulator possesses good performance on accuracy, stiffness, velocity/force transmission factor. By some manipulations, the requirements on the latter four criteria are reduced to constraints on singular values of the kinematic Jacobian. A trade-off must be made since there're opposite requirements among those four criteria. The singular values of kinematic Jacobian are limited in a given interval to guarantee good properties. All the requirement are finally reduced to polynomial inequalities with respect to design parameters. The optimal design problem is transformed into a Max-Det optimization problem that can be efficiently solved. The Orthoglide is used as an example to demonstrate the procedure.**

***Index Terms*— optimal design, workspace, condition number, multi-criteria, LMI.**

I. INTRODUCTION

Generally parallel manipulators possess several superior properties over their serial counterparts, such as low inertia, high speed and high acceleration, high stiffness, and high precision. They're suitable for high-speed machining applications [1][2]. Generally there are several performance requirements when designing a manipulator. Most design related literatures, however, concerned only the two basic design criteria: workspace and condition number. Also many literatures conducted designs with respect to one or two other criteria. There're few design literature considering several criteria. But it is a must to consider multiple criteria in design for some applications, e.g., the machine tool industry. A machine tool is generally required not only a large workspace and good condition number, but also good accuracy, high stiffness, high speed, and large force, etc.

Angeles and Gosselin proposed a design approach for a planar and a spherical 3-dof parallel manipulators by considering the symmetry, global workspace, and the condition number of the jacobian of the manipulator at a home position [3][4]. In order to remove the locality of condition number, Park and Brockett [5], and also Gosselin and Angeles [6] proposed a global performance index (GCI), to measure the global performance of a parallel mechanism. Using this global condition index, Tsai *et al.* [7][8] proposed an approach based on Monte Carlo techniques to kinematically design a DELTA-like and a 3-UPU parallel manipulator. Stock and Miller [9] maximized a combined index of weighted GCI and normalized workspace volume to optimally design a linear Delta robot. Stocco, Salcudean and Sassani [10] and also Hay and Snyman [11] proposed a formulation to maximize the minimal inverse condition number in a prescribed workspace without giving a lower bound on inverse condition number. In [12], Kurtz and Hayward designed a redundant spherical parallel manipulator by considering kinematic dexterity, the forces at the actuators, and the uniformity of dexterity over the workspace. Later, Hayward *et al.* [13] proposed a hierarchical method based on multiple criteria to design a two dof haptic device.

In this paper, we consider to optimal design a machine with the topology of the Orthoglide. Several criteria including condition number, workspace, accuracy, force transmission factor, velocity transmission factor, and stiffness are applied. The optimal design problem is formulated as to find a set of design parameters of a parallel manipulator such that (a) its Cartesian workspace contains a prescribed workspace; (b) at each point in the prescribed workspace it possesses a good condition number; (c) all the singular values are in given ranges. It is found that the requirements on accuracy, stiffness, and velocity/force transmission factors are finally reduced to constraints on singular values of the kinematic Jacobian. By simple manipulations, the requirements on those criteria can be transformed into polynomial inequalities and then combined into a compact LMI (Linear Matrix Inequality). The optimal design problem is thus transformed into a Max-Det optimization problem. An algorithm proposed in [14] is used to solve the optimization problem.

The paper is organized as follows. In section II, the kinematics of the Orthoglide is given algebraicly. Section III considers to simplify condition number and workspace constraints. Section IV shows that the criteria of accuracy, force transmission factor, velocity transmission factor, stiffness are finally reduced to a constraint on singular values of the Jacobian matrix. In section V, the optimal design problem is formulated and transformed to a Max-

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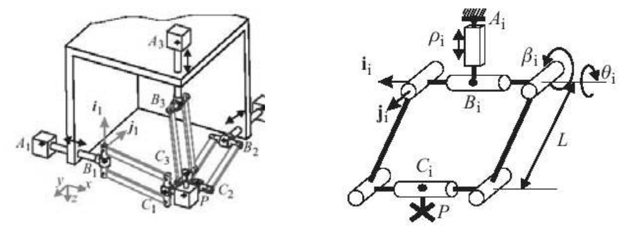


Fig. 1. (a) A schematic of the Orthoglide and its coordinate frame; (b) A subchain of the Orthoglide.

Det optimization problem. Section VI provides simulation results while a conclusion is drawn in section VII.

II. Kinematics of the Orthoglide

Wenger et al. [1] proposed the Orthoglide parallel machine, see Fig. 1. It is a Parallel manipulator with three fixed linear joints mounted orthogonally. The mobile platform is connected to the linear joints by three articulated parallelograms and moves in the Cartesian $x-y-z$ space with fixed orientation. Its workspace shape is close to a cube whose sides are parallel to the planes xy, yz and xz respectively.

We denote by O the intersection point of the three linear joint axes. At the home position the point O coincides with the tool center point P. An inertia coordinate frame is set up with the origin being at O as shown in Fig. 1-(a). Fig. 1-(b) shows a subchain of the Orthglide. Let θ_i and β_i, $i = 1, \cdots, 3$ denote the joint angles of the parallelogram about the axis $\mathbf{i}_i$ and $\mathbf{j}_i$, respectively, as shown in Fig. 1-(b). Let ρ_1, ρ_2, ρ_3 denote the linear joint variables, $\rho_i = |A_i B_i|$. Given coordinate of point P, say $X = [x, y, z]^T$, algebraicly the loop closure constraints can be derived by $\|B_i C_i\| = L$, $i = 1, \cdots, 3$, as follows.

$$(x - e + a - \rho_1)^2 + y^2 + z^2 = L^2 \quad (1)$$
$$x^2 + (y - e + a - \rho_2)^2 + z^2 = L^2 \quad (2)$$
$$x^2 + y^2 + (z - e + a - \rho_3)^2 = L^2 \quad (3)$$

Differentiating (1)-(3) with respect to time t, we obtain the instantaneous kinematic relationship of the manipulator.

$$J_x \dot{X} = J_\rho \dot{\rho}, \quad (4)$$

where J_x and J_ρ are respectively forward Jacobian and inverse Jacobian, $X = [x\ y\ z]^T$ and $\rho = [\rho_1\ \rho_2\ \rho_3]^T$.

$$J_x = \begin{bmatrix} x + b - \rho_1 & y & z \\ x & y + b - \rho_2 & z \\ x & y & z + b - \rho_3 \end{bmatrix}$$

$$J_\rho = -\begin{bmatrix} x + b - \rho_1 & 0 & 0 \\ 0 & y + b - \rho_2 & 0 \\ 0 & 0 & z + b - \rho_3 \end{bmatrix}$$

where $a = |OA_i|$, $e = |C_i P|$, and $b = a - e$.

Given the Cartesian coordinate $X = [x, y, z]$ of the reference point P, the corresponding actuation $\rho = [\rho_1, \rho_2, \rho_3]^T$ can be found by the following inverse kinematic equation.

$$\rho_1 = x + b \pm \sqrt{L^2 - y^2 - z^2};$$
$$\rho_2 = y + b \pm \sqrt{L^2 - x^2 - z^2};$$
$$\rho_3 = z + b \pm \sqrt{L^2 - x^2 - y^2}.$$

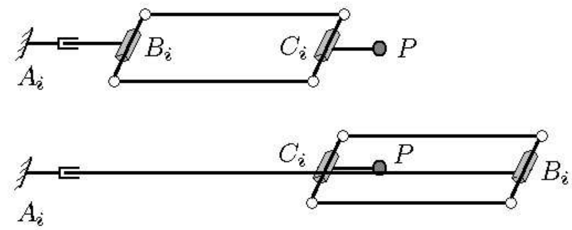


Fig. 2. A sketch of the two inverse kinematic solutions

Fig. 5 shows the configurations corresponding to the two inverse kinematic solutions for each subchain. The upper configuration of the Orthoglide in Fig. 2 corresponds to the "-" branch, while the lower one denotes the "+" branch. For practical consideration, the "-" branch is desirable. Therefore the "+" branch is abandoned. Thus

$$\rho_1 = x + b - \sqrt{L^2 - y^2 - z^2}; \quad (5)$$
$$\rho_2 = y + b - \sqrt{L^2 - x^2 - z^2}; \quad (6)$$
$$\rho_3 = z + b - \sqrt{L^2 - x^2 - y^2}. \quad (7)$$

From the inverse kinematic solutions above, we have constaints $L^2 - x^2 - y^2 \geq 0$, $L^2 - x^2 - z^2 \geq 0$, and $L^2 - y^2 - z^2 \geq 0$.

III. Design Criteria: workspace and condition number

Most of literatures designed parallel manipulators based on two basic design criteria: workspace and condition number (including its derivatives).

A. Workspace

A basic requirement in design is to contain a prescribed workspace (usually cubic) in the workspace generated by the resulting manipulator. The simplest method to check the containment of the prescribed workspace is the inverse kinematic method. First the prescribed workspace W_p is discretized as $W^* = \{X_1, X_2, \cdots\}$, for any point $X_i = [x_i, y_i, z_i]$, check its inverse kinematic solution ρ^i. If it exists and is real and is in the given actuation range $\rho^i_j \in [\rho_{jmin}, \rho_{jmax}]$, $j = 1, \cdots, 3$, then this point will indeed be in the workspace generated by the resulting manipulator.

From the inverse kinematics (5)-(7), we have constraints for real solutions as follows.

$$L^2 \geq \max_{(x,y,z) \in W_p} \{(x^2 + y^2), (x^2 + z^2), (y^2 + z^2)\}. \quad (8)$$

Here we have assumed the three subchains to be totally identical.

To restrict the inverse kinematic solutions to be in the given actuation range, we require that $\rho_i \in [\rho_{imin}, \rho_{imax}]$,

$i = 1, \cdots, 3$, with ρ_{imin} and ρ_{imax} being the minimal and maximal actuation lengths, respectively. They're all regarded as design parameters. For simplicity, we assume $\rho_{1min} = \rho_{2min} = \rho_{3min} := \rho_n$ and $\rho_{1max} = \rho_{2max} = \rho_{3max} := \rho_m$. By the investigation of the parabola generated by the quadratic inverse kinematic function, $f_i(\rho_{min}) > 0,\ i = 1, \cdots, 3$ and $f_i(\rho_{max}) < 0,\ i = 1, \cdots, 3$ guarantee that the inverse kinematic solution will be in the given actuation range. For each subchain there're two quadratic inequalities for any point in the discretized workspace W^*, e.g. the subchain # 1,

$$f_1(\rho_n) = \rho_n^2 - 2(x+b)\rho_n + (x+b)^2 + y^2 + z^2 - L^2 > 0; \quad (9)$$
$$f_1(\rho_m) = \rho_m^2 - 2(x+b)\rho_m + (x+b)^2 + y^2 + z^2 - L^2 < 0. \quad (10)$$

B. Condition number

Condition number and its derivative performance indices play an important role in most designs. A usual requirement is to restrict the manipulator to be far away from singularity manifolds or even to be in the neighborhood of isotropic configuration. In other words, it is required that $\kappa(J) = \frac{\sigma_{max}(J)}{\sigma_{min}(J)} < \gamma$, where $J = J_\rho^{-1} J_x$, γ a given number, and $\kappa(\cdot)$ the condition number function for matrices.

For a given number γ, there exist numbers γ_1 and γ_2, such that $\kappa(J_\rho) < \gamma_1,\ \kappa(J_x) < \gamma_2 \Longrightarrow \kappa(J) < \gamma$. Hence we can consider $\kappa(J_\rho) < \gamma_1,\ \kappa(J_x) < \gamma_2$ instead of of $\kappa(J) < \gamma$. This can practically simplify the analysis. Also $\kappa(J_\rho) < \gamma_1,\ \kappa(J_x) < \gamma_2$ have their physical meaning. The former implies a restriction that the resulting manipulator should be far away from the inverse singularity manifolds and be in the neighborhood of inverse isotropic configuration. While the latter constrains the resulting manipulator to be far away from the forward singularity manifolds and to be in the neighborhood of forward isotropic configuration. These properties are both desirable.

The requirement $\kappa(J_\rho) < \gamma_1$ can be expressed as

$$-\delta < d_i^2 - d_j^2 < \delta; \quad (11)$$

where δ is an appropriate positive number, $i \neq j$, and we denote $J_\rho = \mathrm{diag}\{d_1, d_2, d_3\}$.

The requirement $\kappa(J_x) < \gamma_2$ can be rewritten as

$$-\epsilon_1 < \|v_i\|^2 - \|v_j\|^2 < \epsilon_1; \quad (12)$$
$$-\epsilon_2 < v_i v_j^T < \epsilon_2. \quad (13)$$

where $J_x = [v_1^T, v_2^T, v_3^T]^T$ and $i \neq j$. It is easy to see that (11)-(13) are all polynomial (quadratic) inequalities with respect to $b = a - e$ and L. Note that the Jacobian matrices J_ρ and J_x are dependent on ρ_i, the inverse kinematic solution of $X = (x, y, z)$.

IV. Design Criteria: accuracy, velocity/force transmission factor, stiffness

In addition to workspace and condition number, there're also other criteria to help design parallel manipulators. The most frequently used are accuracy, maximum velocity, maximum force, stiffness, etc.

A. Accuracy

Consider the velocity relation as in (4), we rewrite it in incremental form.

$$J_x \delta X = J_\rho \delta\rho \quad \text{or} \quad \delta X = J^{-1}\delta\rho, \quad (14)$$

Using (14) we can compute the Cartesian error of the end-effector produced by the actuation errors (e.g., the resolution of encoder). This is meaningful when designing a high-precision machine. The amplification factor of $\delta\rho$ is the singular values of J^{-1}, $\sigma(J^{-1})$. For a fully parallel manipulator, its subchains usually have the same structure and actuators. Thus the resolutions (the errors $\delta\rho$) are the same. Assume that the error vector at actuators are bounded in a hyperball, the maximum error δX is obtained as $\delta_{max}(J^{-1})$. Usually the manipulator is required to possess the best accuracy. This can be mathematically described as

$$\begin{aligned} &\min_{\alpha} \max_{X \in W_p} \ \sigma_{max}(J^{-1}(X, \alpha)) \\ \Longleftrightarrow &\max_{\alpha} \min_{X \in W_p} \ \sigma_{min}(J(X, \alpha)) \end{aligned} \quad (15)$$

Here, σ_{max} denotes the maximum singular value of J^{-1} at the point X in the prescribed workspace, and α is the vector of design parameters. In real implementation a loose requirement is usually given to force all the maximum singular values of J^{-1} to be smaller than a given bound so that the manipulator possesses a given precision.

$$\max_{X \in W_p} \ \sigma_{max}(J^{-1}(X, \alpha)) < r_1, \quad (16)$$

This is equivalent to

$$\min_{X \in W_p} \ \sigma_{min}(J(X, \alpha)) > s_1 := \frac{1}{r_1}, \quad (17)$$

since $\sigma_{max}(J^{-1}) = 1/\sigma_{min}(J)$.

B. Velocity transmission factor

We still consider the velocity relation (4), and rewrite it as follows.

$$\dot{X} = J^{-1}\dot{\rho}. \quad (18)$$

This expression gives velocity transmission relation between the input actuation rates $\dot{\rho}$ and the Cartesian velocities $\dot{X}$ at the end-effector. For a manipulator having the same actuators, the maximum actuation rates are all the same. Hence the vector of actuation rates $\dot{\rho}$ are bounded in a n-dimensional hyper-ball. The Cartesian velocity reaches its maximum at $\sigma_{max}(J^{-1})$ and its minimum at $\sigma_{min}(J^{-1})$. The *velocity transmission factor* C_v is defined as the minimal singular value of J^-1, $C_v := \sigma_{min}(J^{-1})$. In industry, high-speed machines are always desirable. This requirement can be mathematically expressed as

$$\max_{\alpha} \ \sigma_{min}(J^{-1}(X, \alpha)) = \max_{\alpha} \ \min_{X \in W_p} \sigma_{min}(J^{-1}(X, \alpha)). \quad (19)$$

This is equivalent to

$$\min_{\alpha} \ \max_{X \in W_p} \sigma_{max}(J(X, \alpha)). \quad (20)$$

In practice, a maximum speed, which should be reached at any point in the prescribed workspace, is given as a design criterion. This requires that a desirable design α should satisfy

$$\min_{X \in W_p} \sigma_{min}(J^{-1}(X,\alpha)) > r_2. \tag{21}$$

This is equivalent to

$$\max_{X \in W_p} \sigma_{max}(J(X,\alpha)) < s_2 = \frac{1}{r_2}, \tag{22}$$

since $\sigma_{min}(J^{-1}) = \sigma_{max}(J)$.

C. Force transmission factor

The fundamental relation between the joint torque and the end-effector generalized forces for parallel manipulators, is the following:

$$\tau = J^{-T} F, \tag{23}$$

where τ is the vector of the joint torques, F is the vector of the end-effector generalized forces. Therefore

$$F = J^T \tau. \tag{24}$$

Easy to see J^T is the linear transmission from the joint torque τ and the generalized force F. The amplification factors of τ are $\sigma(J^T)$. We define the *force transmission factor* C_F as the smallest amplification factor, $C_F := \sigma_{min}(J^T)$. A general requirement on machine tools is to produce a large generalized force at end-effector for a given input joint torque, and the larger, the better. Therefore we expect to find a set of design parameters in the space of design parameters such that the smallest minimum singular value in the prescribed workspace is the largest. Mathematically,

$$\max_{\alpha} \min_{X \in W_p} \sigma_{min}(J(X,\alpha)), \tag{25}$$

Note that in above expression we apply $\sigma(J^T) = \sigma(J)$ and $\sigma_{min}(J^T) = \sigma_{min}(J)$. In real implementation, usually a loose requirement is given such that the manipulator can produce a given maximum force in all points in the prescribed workspace. This requires a desirable design parameters α should satisfy

$$\min_{X \in W_p} \sigma_{min}(J(X,\alpha)) > s_3. \tag{26}$$

D. Stiffness

The stiffness of a parallel manpulator may be evaluated by using an elastic model for the variations of the joint variables as functions of the forces that are applied on the link. In this model the change $\delta\rho$ in the joint variable ρ when a joint torque τ is applied on the link is

$$\delta\tau_i = k_i \delta\rho_i, \tag{27}$$

where k_i is the elastic stiffness of the i-th link. Also we have

$$\delta\rho = J\delta X, \quad \delta F = J^T \delta\tau, \tag{28}$$

which leads to

$$\delta F = J^T K J \delta X, \tag{29}$$

where $K = \mathrm{diag}\{k_1, k_2, \cdots\}$. The stiffness matrix of the parallel manipulator S is therefore

$$S = J^T K J. \tag{30}$$

Generally fully parallel manipulators possess identical subchains, and all subchains have the same elastic coefficient k. Hence $K = kI$ and

$$S = kJ^T J. \tag{31}$$

For a given joint, k is constant. Therefore we can remove the coefficient k and denote $S = J^T J$ if there is no confusion. In machine tool industry, high stiffness is desirable. We can obtain high stiffness by maximizing the minimum eigenvalue of $J^T J$ in the prescribed workspace, i.e.,

$$\max_{\alpha} \min_{X \in W_p} \lambda_{min}(J^T(X,\alpha)J(X,\alpha)) \tag{32}$$

Note that $\lambda(J^T J) = \sigma(J)^2$ and $\lambda_{min}(J^T J) = \sigma^2_{min}(J)$. (32) is equivalent to

$$\max_{\alpha} \min_{X \in W_p} \sigma_{min}(J(X,\alpha)) \tag{33}$$

We loose the requirement to

$$\min_{X \in W_p} \sigma_{min}(J(X,\alpha)) > s_4 \tag{34}$$

E. A trade-off consideration

The constraints on accuracy, force transmission, and stiffness are finally reduced to the same requirement, to maximize the smallest minimum singular value in the prescribed workspace, see (15), (25), and (33). The requirement on velocity transimission is finally reduced to minimize the largest maximum singular value (20). These two requirements are opposite. This is the case in physical world since a high speed always produces larger errors. Reaching the best for either of them will surely lead to a very bad performance for the other one. A trade-off has to be made between these requirements. According to the above analysis, A desirable set of design parameters α should satisfies the following conditions.

$$\min_{X \in W_p} \sigma_{min}(J(X,\alpha)) > s_1 \quad \text{and} \quad \max_{X \in W_p} \sigma_{max}(J(X,\alpha)) < s_2 \tag{35}$$

This can be expressed as that given design parameters α, for any point in the prescribed workspace, all the singular values are in the range of s_1 and s_2.

$$s_1 < \sigma(J(X,\alpha)) < s_2, \quad \forall\ X \in W_p \tag{36}$$

This is guaranteed if the following is satisfied

$$s_1 < \sigma(J_\rho^{-1})\sigma(J_x) < s_2 \tag{37}$$

since $\sigma(J) = \sigma(J_\rho^{-1} J_x) \leq \sigma_{max}(J_\rho^{-1})\sigma_{max}(J_x)$. Given proper number r_1, r_2, t_1, t_2, the satisfaction of (38) and (39) will ensure (37).

$$r_1 < \sigma(J_\rho^{-1}) < r_2 \tag{38}$$

$$t_1 < \sigma(J_x) < t_2 \tag{39}$$

The expression (38) can be further simplified as

$$\frac{1}{r_2} < \sigma(J_\rho) < \frac{1}{r_1} \tag{40}$$

Since the inverse Jacobian J_ρ is diagonal, the constraint (40) is reduced to

$$\frac{1}{r_2^2} < d_i^2(X, \rho, \alpha) < \frac{1}{r_1^2}. \tag{41}$$

The constraint (39) is equivalent to

$$t_1^2 < \lambda(J_x J_x^T) < t_2^2, \tag{42}$$

where $\lambda(\bullet)$ denotes the eigenvalue function of square matrics. Assume the characteristic polynomial of $J_x J_x^T$ is $g(\lambda) = \lambda^3 + A\lambda^2 + B\lambda + C$, where A, B, and C are corresponding coefficients dependent on X, ρ, and α. By change of variables and some simple manipulation, we obtain two equivalent characteristic polynomials, $g_1(u) = u^3 + A_1 u^2 + B_1 u + C_1$ and $g_2(v) = v^3 + A_2 v^2 + B_2 v + C_2$. By Routh stbility criterion, the restriction of the eigenvalues to be in the range of (t_1^2, t_2^2) is equivalent to the requirement that the solutions to $g_1(u) = 0$ and $g_2(v) = 0$ are all in the open left half plane, or equivalently

$$A_i(X, \rho, \alpha) > 0 \tag{43}$$

$$C_i(X, \rho, \alpha) > 0 \tag{44}$$

$$A_i B_i - C_i > 0 \quad i = 1, 2. \tag{45}$$

V. Formulation of the optimal design problem

By considering all the design specifications, the optimal design problem is formulated as to find a set of design parameters α such that (a) the workspace generated by the resulting parallel manipulator contains a prescribed workspace; (b) the resulting manipulator possesses good condition number at each point of the prescribed workspace; (c) the resulting manipulator possesses good properties of accuracy, Cartesian force, Cartesian velocity, and stiffness at each point of the prescribed workspace. Mathematically this is expressed as follows.

$$\begin{aligned}
\min \quad & \Phi(\alpha) & (46)\\
\text{subject to} \quad & \rho_{min} < \rho(X) < \rho_{max}, & (47)\\
& \kappa(J(X)) < \gamma, & (48)\\
& s_1 < \sigma(J(X)) < s_2, \quad \forall X \in W_p & (49)
\end{aligned}$$

where $\Phi(\alpha)$ is some appropriate objective function. Note that expressions (8)-(10), (11)-(13), and (43)-(45), which are sufficient conditions of constraints (48), (47), and (49), are all polynomial inequalities of design parameters $\alpha = [b, L, \rho_m, \rho_n]^T$. For simplicity those polynomial inequalities are combined into a compact LMI with respect to the vector of *design variable* ξ by the diagonal matrix technique.

$$G(X, \rho, \xi) = G_0(X, \rho) + G_1(X, \rho)\xi_1 + \cdots G_n(X, \rho)\xi_n \succ 0,$$

where G_i, $i = 1, \cdots, n$ are square (in fact diagonal) matrices, and ξ_i, $i = 1, \cdots, n$ are independent of the joint variables ρ and the Cartesian coordinates X, and are in general only polynomial functions of design parameters α.

There are many possibilities for the choice of the cost function $\Phi(\alpha)$, e.g., any criteria introduced in the previous sections. In order to give an overall consideration of all the criteria, a good candidate for Φ is the log-det function as proposed in [14]

$$\Phi(\alpha) = \mathrm{logdet} G^{-1}.$$

Such chosen Φ is convex, which guarantees a global optimum given a convex domain. This choice of objective function is advantageous in several aspects [14]. Minimization of the log-det function of G is in fact an anlytic centering problem [15]. The optimal solution, or the analytic center of the constraint polyhedron, is the farthest point away from the combined boundary of workspace, condition number, and singular value. Not only it satisfies the condition number constraints, but also it reaches best condition numbers with respect to the workspace and singular value constraints. To some extent it optimizes condition number in the prescribed workspace given the singular value constraint. Therefore, the optimal design problem is reduced to the following Max-Det optimization problem.

$$\begin{aligned}
\min \quad & \Phi(\xi) = \mathrm{logdet} G^{-1} & (50)\\
\text{subject to} \quad & G = G_0 + G_1\xi_1 + \cdots G_n\xi_n \succ 0, & (51)
\end{aligned}$$

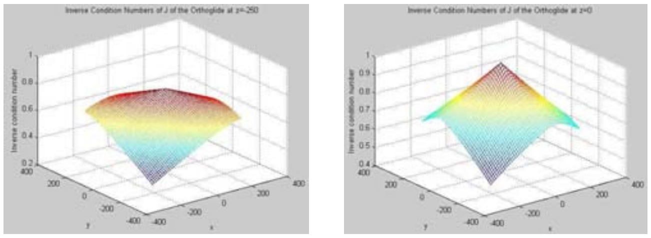
where $G_i = G_i(X, \rho)$. By the algorithm provided in [14], the Max-Det problem can be solved readily.

VI. Simulation Results

In the simulation, a prescribed workspace is given as $W_p = 500 \times 500 \times 500$. The home position, or the isotropic point, is required to be in the center of W_p. Therefore $W_p = [-250, 250] \times [-250, 250] \times [-250, 250]$. In simulation we add a linear penalty term $c^T\xi$ to the objective function with $c = [1, 1, 1, 1]$ to constrain the size of the manipulator. And the optimal design parameters are $(b^*, L^*, \rho_n^*, \rho_m^*) = (1160.8270, 889.7011, 9.2166, 620.1582)$. We find that the transmission factor of velocity (the singular value of J) is in $[0.3876, 1.6124]$. The inverse condition numbers are in the range of $[0.2967, 1]$. Fig. 3 and Fig. 4-(a) show the inverse condition number of J in the prescribed workspace at three cross sections $z = -250$, $z = 0$, and $z = 250$, respectively. Fig.4-(b) and Fig. 5 show workspaces generated by the resulting manipulator at three cross sections $z = -250$, $z = 0$, and $z = 250$, respectively. The shaded rectangle denotes the prescribed workspace at the corresponding cross section. It shows that the workspace generated by the resulting manipulator contains the prescribed workspace perfectly.

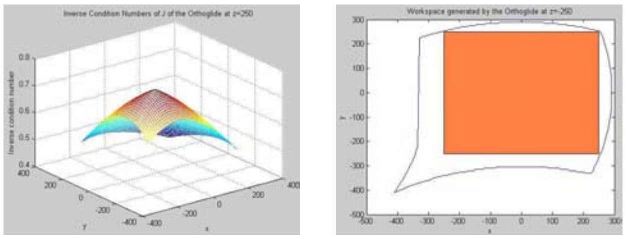
VII. Conclusion

This paper considers a multi-criteria based optimal design problem for a parallel machine. Several criteria, workspace, condition number, accuracy, stiffness, maximum velocity, and maximum force, are considered. The



(a) (b)

Fig. 3. (a) The inverse condition number of J at the cross-section $z = -250$; (b) The inverse condition number of J at the cross-section $z = 0$.



(a) (b)

Fig. 4. (a) The inverse condition number of J at the cross-section $z = 250$; (b) The workspace generated by the resulting Orthoglide at $z = -250$.

optimal design problem is proposed as to find a set of design parameters such that (a) the Cartesian workspace generated by the resulting workspace contains a prescribed workspace; (b) the resulting manipulator possesses a good condition number at each points in the prescribe workspace; (c) the resulting manipulator possesses good performance on accuracy, stiffness, velocity/force transmission factor. A trade-off must be made in order to design a parallel manipulator possessing good properties since there're opposite requirements. This gives a feasible way to formulate and solve multi-criteria optimal design problem.

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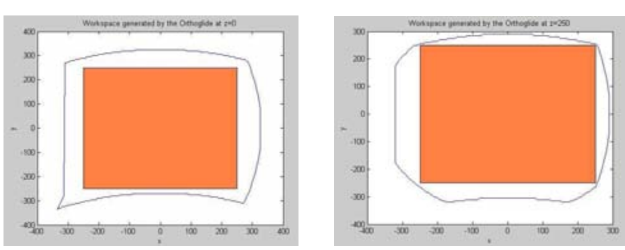

(a) (b)

Fig. 5. (a) The workspace generated by the resulting Orthoglide at $z = 0$; (b) The workspace generated by the resulting Orthoglide at $z = 250$.