A Local Measure of Fault Tolerance for Kinematically Redundant Manipulators

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***Abstract*— When a manipulator suffers a joint failure, its performance can be significantly affected. If the failed joint is locked, the resulting manipulator Jacobian is given by the original Jacobian, except that the column associated with the failed joint is removed. The rank of the resulting Jacobian then determines if the manipulator still has the ability to perform arbitrary end-effector motions. Unfortunately, even at an operating configuration that has a relatively high manipulability index, a joint failure may still result in a singular Jacobian. This work examines the problem of determining the reduced manipulability of a manipulator after one or more joint failures. Configurations that result in a minimal reduction of the manipulability index for any set of joint failures are determined.**

I. Introduction

KINEMATICALLY redundant manipulators offer several advantages over conventional nonredundant manipulators including the potential for obstacle avoidance, torque minimization, singularity avoidance, and greater dexterity [1], [3], [4], [9], [13], [14], [18], [19], [26]–[28]. Another advantage that has only recently been investigated is fault tolerance [8], [16], [17], [20], [24]. Obviously, a complete joint failure in a nonredundant manipulator automatically results in the loss of full end-effector control; however, with a kinematically redundant manipulator, one can design the manipulator such that the extra degrees of freedom will be able to compensate for the failure. This article examines the problem of determining the reduced manipulability after one or more joint failures have occurred.

Recall that the end-effector velocities, $\dot{\boldsymbol{x}}$, and the joint velocities, $\dot{\theta}$, are related by the equation

$$\dot{\boldsymbol{x}} = J\dot{\theta} \tag{1}$$

where J is the manipulator Jacobian. If a failed joint i is locked, the Jacobian equation becomes

$$\dot{\boldsymbol{x}} = {}^iJ\,{}^i\dot{\theta} \tag{2}$$

where

$${}^iJ = [\boldsymbol{j}_1 \quad \cdots \quad \boldsymbol{j}_{i-1} \quad \boldsymbol{j}_{i+1} \quad \cdots \quad \boldsymbol{j}_n], \tag{3}$$

$\boldsymbol{j}_j$ denotes the j-th column of J, and

$${}^i\dot{\theta} = [\dot{\theta}_1 \quad \cdots \quad \dot{\theta}_{i-1} \quad \dot{\theta}_{i+1} \quad \cdots \quad \dot{\theta}_n]^T.$$

The reduced manipulator Jacobian ${}^iJ$ then determines the kinematic properties of the degraded system. In this article, a local measure of fault tolerance is defined that measures the performance of the degraded system relative to the original system.

In the next section, necessary and sufficient conditions are derived for determining whether a manipulator with a single degree of redundancy is in a configuration for which the reduced system is singular. Using this condition, one can then develop strategies that will avoid such configurations. Section III discusses how the manipulability of a manipulator is affected by a joint failure. Once this has been determined, configurations can be identified for which the manipulability is reduced by a minimum amount due to any joint failure. The case of multiple joint failures is considered in Section IV. Section V discusses the application of these results to motion planning and Section VI presents a fully general spatial example. Finally, conclusions appear in Section VII.

II. Fault Intolerant Configurations

As mentioned earlier, a joint failure can essentially result in a manipulator being in a singular configuration, even if the original Jacobian is of full rank. It is easy to show, using column space arguments, that the rank of the reduced Jacobian satisfies

$$\mathrm{rank}(J) - 1 \le \mathrm{rank}({}^iJ) \le \mathrm{rank}(J) \tag{4}$$

so that a single joint failure at a nonsingular configuration will not result in a multiple singularity. For applications that require a manipulator to work in a hazardous environment where joint failures are not unlikely, it would be beneficial to have a simple method for determining whether a joint failure will render a manipulator to be in a singularity. Necessary and sufficient conditions for the reduced Jacobian to be singular in the case of a single degree of redundancy will be derived in this section.

A configuration θ^* will be said to be fault intolerant with respect to joint i if the reduced Jacobian ${}^iJ$ is singular. Much of this article is dedicated to identifying fault intolerant configurations and quantifying, at least locally, the fault tolerance of

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a configuration. It can be shown that a singular configuration for a planar revolute manipulator is characterized by the links being collinear. This geometric approach for identifying singularities can be applied to the problem of determining fault intolerant configurations. If there is a failure in a joint other than the first joint, then one can view the manipulator as an $(n-1)$-jointed robot with links $i-1$ and i replaced by a link connecting joints $i-1$ and $i+1$ where it is assumed that the failure has occurred in joint i. One can then check to see if the links of the new manipulator are collinear. This is illustrated for a planar 3R manipulator in Fig. 1. For this simple manipulator it is easy to geometrically identify the failure intolerant configurations and to develop a physical intuition into the meaning of failure intolerance. For example, from Fig. 1(a) it is clear that a failure in joint one will result in a singular configuration whenever $\theta_3 = k\pi$ since this failure is physically equivalent to having a two-link manipulator that is at a reach singularity. A failure in joint two will result in a singular configuration whenever the origins for link one, link three, and the end effector are collinear, which is illustrated in Fig. 1(b). A similar geometric argument identifies the family of configurations represented in Fig. 1(c) as being intolerant to failures in joint three. It is important to note that the image of the fault intolerant configurations is the entire workspace. In other words, for each end-effector position in this example there is a fault intolerant configuration. This is easy to see from Fig. 1(c) since this configuration can put the end effector at any distance from the base. This is in contrast to end-effector positions that correspond to the kinematic singularities that, for this example, partition the workspace into regions for which one would not need to worry about singularities. Thus, as this example illustrates, one cannot guarantee fault tolerance by simply restricting the workspace of the manipulator.

The ability to identify failure intolerant configurations from purely geometric arguments becomes much more difficult for more general manipulators [6], [12]. Fortunately, a more analytical method for determining fault intolerant configurations can be derived. The following theorem illustrates this for arbitrary manipulators that have a single degree of redundancy.

Theorem 1: Consider a manipulator with a single degree of redundancy. Suppose that for the configuration θ^*, $J(\theta^*)$ is of full rank and that its null vector $\boldsymbol{n}_J(\theta^*)$ is known. Then the configuration θ^* is fault intolerant with respect to joint i if and only if $n_i(\theta^*) = 0$ where n_i is the i-th component of $\boldsymbol{n}_J$.

Proof: ($\Rightarrow$) If ${}^iJ(\theta^*)$ is not of full rank, it has a nonzero null vector

$$\boldsymbol{w} = [w_1 \quad w_2 \quad \cdots \quad w_{n-1}]^T.$$

Let

$$\tilde{\boldsymbol{w}} = [w_1 \quad \cdots \quad w_{i-1} \quad 0 \quad w_i \quad \cdots \quad w_{n-1}]^T.$$

Then clearly

$$J(\theta^*)\tilde{\boldsymbol{w}} = {}^iJ(\theta^*)\boldsymbol{w} = \boldsymbol{0} \tag{5}$$

so that $\tilde{\boldsymbol{w}}$ is a nonzero null vector of $J(\theta^*)$. As the nullity of $J(\theta^*)$ is one, it follows that $\boldsymbol{n}_J(\theta^*)$ is a nonzero multiple of $\tilde{\boldsymbol{w}}$, proving that $n_i(\theta^*) = 0$.

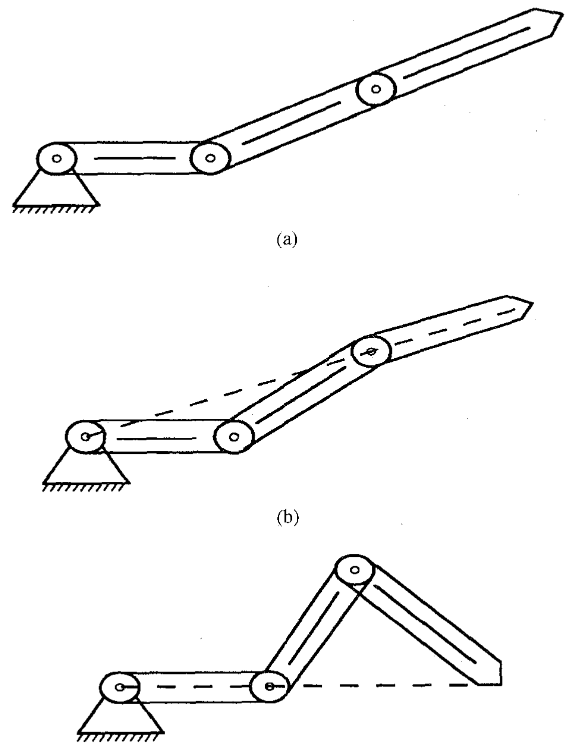

(a)
(b)
(c)

Fig. 1. The planar 3R manipulator configurations shown represent examples of the three types of fault intolerant configurations for this manipulator. These configurations correspond to situations in which a single locked joint failure will effectively result in a 2R manipulator in a singular configuration. Configurations of the type shown in (a) are intolerant with respect to failures in joint one. These configurations are characterized by $\theta_3 = k\pi$. Configurations of the type shown in (b) are intolerant to failures with respect to joint two. These configurations are characterized by the end effector, joint one, and joint three being collinear. Configurations of the type shown in (c) are intolerant to failures with respect to joint three. Analogous to the other two cases, these configurations are characterized by the end effector, joint one, and joint two being collinear.

($\Leftarrow$) Suppose $n_i(\theta^*) = 0$. Let $\boldsymbol{w}$ denote the $(n-1)$-vector that is obtained by deleting the i-th component of $\boldsymbol{n}_J(\theta^*)$. It is easy to see that $\boldsymbol{w}$ is a nonzero vector satisfying ${}^iJ(\theta^*)\boldsymbol{w} = J(\theta^*)\boldsymbol{n}_J(\theta^*) = \boldsymbol{0}$. Hence the square matrix ${}^iJ(\theta^*)$ has a nontrivial null space, proving that it is not of full rank. ∎

Thus, the question of whether a particular joint failure results in a singular Jacobian has been reduced to merely checking the corresponding component of the null vector. As an application of this result, one can see that a seven degree-of-freedom anthropomorphic arm is fault intolerant with respect to an elbow joint failure since the corresponding element of the null vector is always zero (e.g., see [25]). This is physically explained by noting that the only joint in the human arm that can change the distance from the shoulder to the wrist is the elbow joint. In most cases, the elements of the null vector will not be identically zero but will only be zero for certain special configurations. For example, consider the same simple planar manipulator shown in Fig. 1. For the configuration shown in Fig. 2, the null vector is given by $[1 \ 0 \ 0]^T$ which shows that locally all of the redundancy is located in joint one and that the manipulator is intolerant to failures in either joint two or three.

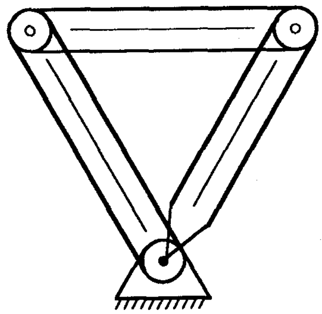


Fig. 2. A planar 3R manipulator with unit length links in a configuration for which the null vector is given by $\boldsymbol{n}_J = [1\ 0\ 0]^T$. This also corresponds to having $\boldsymbol{j}_1 = \mathbf{0}$ which physically means that the motion of joint one has no effect on the velocity of the end effector. Since all of the redundancy is effectively located in joint one for this configuration, a failure in joint one has no effect on the motion of the end effector. Conversely, the manipulator is intolerant to a failure in either joint two or joint three.

This characterization of fault intolerant configurations can also be used in conjunction with an augmented or extended Jacobian technique to devise a control strategy that will keep the manipulator away from these configurations [2], [10], [22]. For example, one could require that the product of the elements of the null vector remain at a constant nonzero value throughout the desired end-effector motion. However, as with any extended Jacobian technique, algorithmic singularities can limit the usefulness of such an approach [2], [5].

III. Relative Manipulability Indices

One shortcoming of the characterization given in Theorem 1 is that it does not say anything about the reduced performance of the resulting manipulator other than determining whether it would be in a singularity. There are a variety of kinematic measures proposed to quantify the performance of a kinematically redundant manipulator [1],[14]. These measures are often used to define optimal operating configurations. One particular measure is the manipulability index [27] defined as

$$w(J) = \sqrt{\det(JJ^T)}. \tag{6}$$

The manipulability index is a nonnegative quantity that takes on the value zero precisely at the singular configurations of the robot. Configurations that result in a relatively large manipulability index are usually considered to be good operating configurations. However, the emphasis in this section will be in determining configurations for which a joint failure will not result in a small manipulability index. This work investigates the fault tolerance of kinematically redundant manipulators by examining the manipulability indices of the reduced Jacobians ${}^iJ$ relative to the manipulability index of the original Jacobian. Configurations that result in a minimal reduction of the manipulability index for any joint failure are determined. Such configurations are locally fault tolerant in the sense that the robot would not have a substantial reduction in its manipulability index after a joint failure.

To pursue this approach, we define the i-th relative manipulability index r_i to be the ratio of the reduced manipulability index to the original manipulability; i.e.,

$$r_i(\theta) = \frac{w_i(J)}{w(J)} \qquad (w(J) \neq 0) \tag{7}$$

where $w_i(J) = w({}^iJ)$.[1] This quantity provides a measure of the amount of manipulability retained after a failure in the i-th joint. The relative manipulability indices clearly range from zero to one and are independent of the scaling applied to the linear or rotational components of J due to the normalization. In particular, if r_i is zero, then the manipulability index of ${}^iJ$ is also zero so that the manipulator is in a configuration that is fault intolerant with respect to joint i; in this case, a failure in the i-th joint is critical since it essentially renders the robot singular. At the other extreme, if r_i is one, then a failure of the i-th joint has no effect on the manipulability of the robot at that configuration. This is clearly true when the i-th column of J is the zero vector; after developing the necessary machinery, it should also be clear that the converse to this statement holds. This is precisely the situation illustrated in Fig. 2 where $\boldsymbol{j}_1 = \mathbf{0}$ and thus the local behavior of the end effector is completely unaffected by a failure in joint one.

If all joint failures are equally likely, then one possible measure of fault tolerance is to maximize the minimum relative manipulability index, i.e., to maximize

$$\min_i r_i(\theta). \tag{8}$$

If certain joints are more likely to fail than other joints, one may instead want to maximize

$$\min_i a_i r_i(\theta) \tag{9}$$

or

$$\sum_{i=1}^{n} a_i r_i(\theta) \tag{10}$$

where the nonnegative scalar quantities a_i represent some weighting [21].

To determine the relative manipulability indices, one can clearly calculate the reduced manipulability for each joint failure and divide by the manipulability of the original manipulator. However, it is possible to determine this information directly from a knowledge of the null space. This will first be done for manipulators with a single degree of redundancy and then generalized to the case of multiple degrees of redundancy. The case of multiple failures will be considered in the next section.

It is first noted that by the Binet-Cauchy Theorem,

$$\det(JJ^T) = \sum_{i_1 < i_2 < \cdots < i_m} (\det[\boldsymbol{j}_{i_1} \quad \boldsymbol{j}_{i_2} \quad \cdots \quad \boldsymbol{j}_{i_m}])^2 \tag{11}$$

where the summation is taken over the $\binom{n}{m}$ subdeterminants of J. For the case of a single degree of redundancy, this becomes

$$\det(JJ^T) = \sum_{i=1}^{n} [\det({}^iJ)]^2 \tag{12}$$

[1] The case where $w(J) = 0$ is considered in Section V.

so that

$$w(J) = \sqrt{[w_1(J)]^2 + [w_2(J)]^2 + \cdots + [w_n(J)]^2} \quad (13)$$

where once again $w_i(J) = w({}^iJ)$. This gives a simple relationship between the overall manipulability index and the resulting manipulability indices due to a single joint failure. This can be rewritten as

$$r_1^2 + \cdots + r_n^2 = 1. \quad (14)$$

Equation (14) shows how the relative manipulability indices are distributed and clearly illustrates that the overall fault tolerance to all joint failures must be considered. In particular, if $r_j = 1$ for some joint j, then the manipulator's configuration is fault intolerant with respect to any of the other joints. Once again, this is the case illustrated in Fig. 2, where $r_1 = 1$ and $r_2 = r_3 = 0$.

It is important to note that the relative manipulability indices are intimately related to the null space of the Jacobian [7]. To see this, first note that for manipulators with a single degree of redundancy, a null vector $\boldsymbol{n}_J$ can be determined by using

$$n_i = (-1)^{i+1} \det({}^iJ) \qquad i = 1, 2, \cdots, n \quad (15)$$

where n_i is the i-th component of $\boldsymbol{n}_J$. Equation (15) follows from the Laplace expansion of the determinant, which, for the special case of a 2×3 Jacobian, results in $\boldsymbol{n}_J$ being simply the cross product of the two rows of J. By taking absolute values of both sides of (15), one has

$$|n_i| = |\det({}^iJ)| = w_i(J) \quad (16)$$

which gives the result that

$$w(J) = \|\boldsymbol{n}_J\|. \quad (17)$$

By letting $\hat{\boldsymbol{n}}_J$ be the unit length null vector

$$\hat{\boldsymbol{n}}_J = \frac{\boldsymbol{n}_J}{\|\boldsymbol{n}_J\|} \quad (18)$$

one has

$$w_i(J) = |n_i| = |\hat{n}_i| w(J) \quad (19)$$

where $\hat{n}_i$ is the i-th component of $\hat{\boldsymbol{n}}_J$. Hence the relative manipulability indices are given by

$$r_i = |\hat{n}_i| \qquad i = 1, 2, \cdots, n. \quad (20)$$

The values of $|\hat{n}_i|$ and $w(J)$ are given directly by the singular value decomposition $J = U\Sigma V^T$ as $|\hat{n}_i| = |v_{in}|$ and $w(J) = \sigma_1\sigma_2\cdots\sigma_m$ where v_{in} is the (i, n) element of V. Thus

$$w_i(J) = |v_{in}|\sigma_1\sigma_2\cdots\sigma_m. \quad (21)$$

Note that (19) implies Theorem 1. One can also conclude from (20) that Jacobians for which the components of the null vector $\boldsymbol{n}_J$ are of equal magnitude are optimal in terms of maximizing the minimum relative manipulability index given in (8). Consider the optimally fault tolerant configurations for a planar 3R manipulator. If each link is of unit length, the null vector is

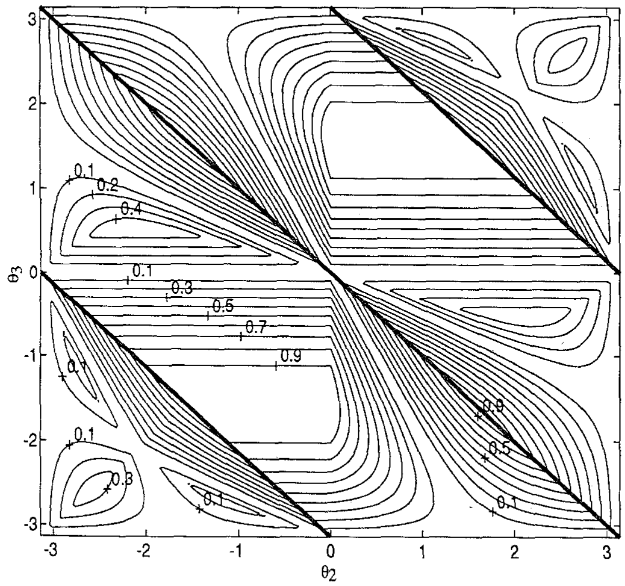


Fig. 3. A contour plot showing the minimum reduced manipulability index, $\min(w_i)$, for a planar 3R manipulator with equal link lengths. The boldface line indicates the configurations that possess the optimal value of $1/\sqrt{3}$ for the minimum relative manipulability index.

$$\boldsymbol{n}_J = \begin{bmatrix} \sin\theta_3 \\ -\sin\theta_3 - \sin(\theta_2 + \theta_3) \\ \sin\theta_2 + \sin(\theta_2 + \theta_3) \end{bmatrix}. \quad (22)$$

Optimally fault tolerant configurations have the property that each component of (22) has the same magnitude. One can show that this is equivalent to $\theta_2 + \theta_3 = k\pi$. These are illustrated by the boldface lines in the contour plots of Fig. 3 and Fig. 4. It is important to note that the image of the surface $\{\theta|\theta_2 + \theta_3 = (2k+1)\pi\}$ is the unit circle $\{\boldsymbol{x}|\ \|\boldsymbol{x}\| = 1\}$ centered at the base in the workspace while the image of the surface $\{\theta|\theta_2 + \theta_3 = 2k\pi\}$ is all of the workspace except the open unit disk centered at the base, i.e., $\{\boldsymbol{x}|1 \le \|\boldsymbol{x}\| \le 3\}$. Thus a significant portion of the workspace can be covered with the manipulator in an optimally fault tolerant configuration. Note that (20) can also be used to calculate optimal solutions to (9) and (10). In particular, the Jacobians that maximize (9) are characterized by having $a_i|n_i| = a_j|n_j|$ while the Jacobians that maximize (10) are characterized by the null vector being a nonzero multiple of $[a_1 \pm a_2\ \cdots\ \pm a_n]^T$.

One can also consider the dynamics of a manipulator experiencing a locked joint failure [15]. In this case, one is interested in the dynamic manipulability index, JH^{-1}, where H is the moment of inertia matrix [28]. This will, of course, modify the measure of failure tolerance for the various manipulator configurations due to the effect of the inertia matrix. Thus the relative dynamic manipulability will, in general, differ from the relative kinematic manipulability, as will the optimally failure tolerant configurations. However, it is important to note that the failure intolerant configurations will not be changed since they are strictly due to the singularities

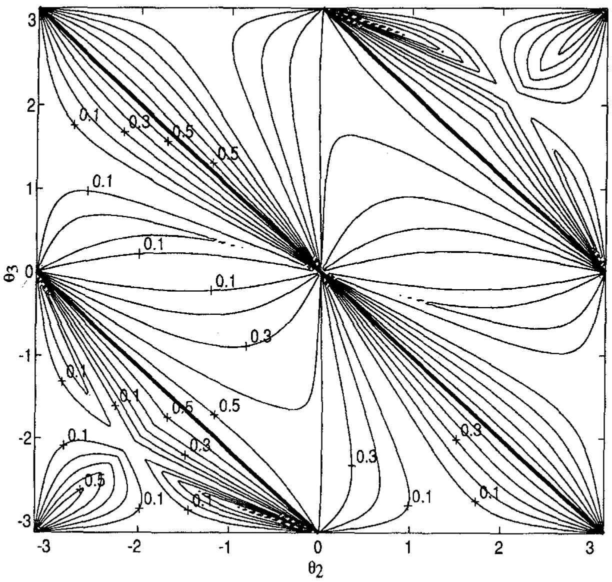


Fig. 4. A contour plot showing the minimum relative manipulability index, $\min(r_i)$, for a planar 3R manipulator with equal link lengths. The boldface line indicates the configurations that possess the optimal value of $1/\sqrt{3}$ for the minimum relative manipulability index.

in ${}^iJ$. Thus if one is concerned with identifying a region in the joint space that is guaranteed to not be failure intolerant, one need only consider the kinematics [15].

The above results have been illustrated for manipulators with a single degree of redundancy. These ideas can be easily generalized to include multiple degrees of redundancy as the following theorem shows.

Theorem 2: Let J be an m by n manipulator Jacobian of full rank and let $J = U\Sigma[V_1\ V_2]^T$ be its SVD where V_2 is a matrix of $n-m$ orthonormal n-dimensional vectors in the null space of J. Then the manipulability index after a failure of the i-th joint is given by

$$w({}^iJ) = \|\hat{\boldsymbol{N}}_i\| w(J) \tag{23}$$

where $\hat{\boldsymbol{N}}_i$ is the i-th column of V_2^T. Hence the relative manipulability index r_i is given by $\|\hat{\boldsymbol{N}}_i\|$. Furthermore,

$$\sum_{i=1}^{n} r_i^2 = n - m. \tag{24}$$

Proof: See Appendix A.

Like the one-dimensional case, this theorem has a very elegant physically intuitive interpretation. The magnitude of a joint's contribution to the null space, i.e. $\|\hat{\boldsymbol{N}}_i\|$, is effectively a measure of how much of the manipulator's total redundancy resides in that particular joint. Thus the more redundancy associated with a joint, the more tolerant the manipulator is to a failure in that joint. It is important to note that the matrix V_2 from the SVD of J is not unique; however, the space spanned by this matrix is unique and all possible V_2 matrices are related by orthogonal transformations. Since orthogonal transformations are norm-preserving, the results of the theorem are independent of the particular choice of V_2.

Once again, one should not make the manipulator completely fault tolerant to a particular failure unless it is known which particular joint failure is imminent. By Theorem 2, this would mean that $\|\hat{\boldsymbol{N}}_i\| = 1$. Due to the fact that V is orthogonal, the norm of the i-th column of V_1^T would then be 0 so that the i-th column of J is the zero vector. Hence, having the manipulator in a configuration where there would be no loss of manipulability after a failure in joint i means that joint will not be able to contribute to the end-effector motion prior to a failure. Optimal Jacobians would have similar null space properties as before. For example, a Jacobian with the property that each column of V_2^T is of equal norm is optimal in terms of maximizing the minimum relative manipulability index (see (8)). Such a Jacobian will be said to have the optimal reduced manipulability property.

IV. Multiple Joint Failures

It is possible that a configuration that is optimally fault tolerant in the sense of (8) may not be fault tolerant for two or more joint failures. For example, consider the Jacobian

$$J = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \tag{25}$$

which is clearly fault tolerant to a single joint failure since joint three duplicates the motion of joint one and joint four duplicates the motion of joint two. A planar 4R manipulator configuration that corresponds to this Jacobian is given in Fig. 5(a). One can see that J maximizes its minimum relative manipulability index since the columns of the matrix

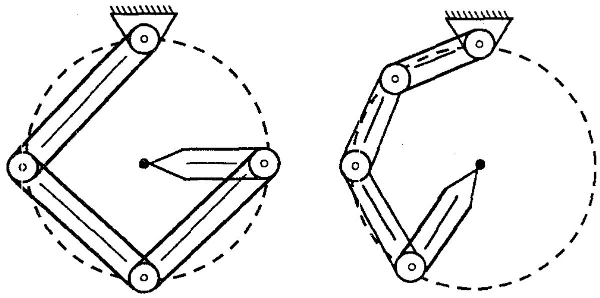
$$V_2^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{26}$$

are all of the same norm, thus satisfying the conditions of Theorem 2. However, if the second and fourth joints both fail, then the rank of the resulting Jacobian is one. Likewise, failures of the first and third joints also result in a zero manipulability index. Thus, while (25) is optimally fault tolerant to single joint failures, it is not fault tolerant to multiple failures.

In this section, conditions are derived for guaranteeing optimal fault tolerance with respect to multiple failures. As with the case of a single joint failure, the reduced manipulability resulting from multiple joint failures can also be determined from the SVD of J. For the case of two joint failures in say joints i and j, the manipulability index becomes

$$\begin{aligned} w_{ij}(J) &= w(J)\sqrt{\|\hat{\boldsymbol{N}}_i\|^2\|\hat{\boldsymbol{N}}_j\|^2 - (\hat{\boldsymbol{N}}_i \cdot \hat{\boldsymbol{N}}_j)^2} \\ &= w(J)\|\hat{\boldsymbol{N}}_i\|\|\hat{\boldsymbol{N}}_j\||\sin\phi_{ij}| \end{aligned} \tag{27}$$

where ϕ_{ij} denotes the angle between the vectors $\hat{\boldsymbol{N}}_i$ and $\hat{\boldsymbol{N}}_j$. Note that the effect of two failures on the reduction of the original manipulability is not simply the product of the individual joint failures. The manipulability will be reduced by a factor that is the product of the magnitudes of $\hat{\boldsymbol{N}}_i$ and $\hat{\boldsymbol{N}}_j$, i.e., the reduction due to considering the joint failures individually, along with the magnitude of the sine of the angle



(a) (b)

Fig. 5. Both of the planar 4R manipulator configurations shown are optimal in terms of failure tolerance with respect to a single locked joint failure. The configuration in (a), however, is intolerant to two locked joint failures if either joints one and three or joints two and four fail. The configuration in (b) is optimally failure tolerant with respect to any two locked joint failures. Note that failure tolerance can be geometrically related to the degree that the columns of the Jacobian overlap with each other.

between $\hat{\boldsymbol{N}}_i$ and $\hat{\boldsymbol{N}}_j$. Physically this makes sense, since the angle between $\hat{\boldsymbol{N}}_i$ and $\hat{\boldsymbol{N}}_j$ is related to how much the end-effector motion due to joints i and j are correlated. If the sine of the angle between these vectors is zero, then these two joints not only contribute to the same direction of end-effector motion, but they are also the only joints that contribute to this particular direction (assuming a nonsingular J), thus resulting in a zero value of reduced manipulability. This is exactly the case illustrated by the Jacobian in (25) and its corresponding null vectors given in (26). The general case for an arbitrary number of joint failures is given by the following theorem along with a relationship between the relative manipulability indices.

Theorem 3: Suppose that a manipulator is in a nonsingular configuration and that there are $f \leq n - m$ distinct joint failures occurring in joints $i_1, i_2, \cdots, i_f$. Then the reduced manipulability index is given by

$$w_{i_1,\cdots,i_f}(J) = w(J)\sqrt{\det(\hat{N}^T_{i_1,\cdots,i_f}\hat{N}_{i_1,\cdots,i_f})} \tag{28}$$

where $\hat{N}_{i_1,\cdots,i_f}$ denotes the $(n-m)\times f$ matrix composed of the columns of V_2^T associated with the failed joints. Furthermore, if the relative manipulability index $r_{i_1,\cdots,i_f}(J)$ is defined to be $w_{i_1,\cdots,i_f}(J)/w(J)$, then the following relationship holds for $n - m$ joint failures:

$$\sum_{i_1 < i_2 < \cdots < i_{n-m}} r^2_{i_1,\cdots,i_{n-m}}(J) = 1. \tag{29}$$

Proof: See Appendix B.

Once again, it is important to note that all V_2^T matrices for a Jacobian differ only by a premultiplication of an orthogonal matrix. Since any such premultiplication by an orthogonal matrix preserves the inner product of the columns as well as the column norms, (28) does not depend on the particular choice of V_2^T.

Theorem 3 can be used for identifying and designing configurations that are multi-fault tolerant. For example, using (27) and some purely geometric arguments, one can choose a V_2 that is optimally fault tolerant to any two joint failures. One such V_2 is

$$V_2^T = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{2} & 0 & \frac{-1}{2} \\ 0 & \frac{1}{2} & \frac{-1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}. \tag{30}$$

Each column of (30) has length $1/\sqrt{2}$ in order to guarantee maximum worst-case manipulability for single joint failures, and each column (or its reflection) is at an angle of $180/n = 45$ degrees from some other column in order to guarantee maximum worst-case manipulability for two joint failures. Obviously, since there are only two degrees of redundancy, any failure in three joints results in zero manipulability regardless of V_2. An example of a Jacobian corresponding to the V_2 in (30) is given by

$$J = \begin{bmatrix} 1 & \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{31}$$

which is optimally fault tolerant in a worst-case sense with respect to reduced manipulability for any set of joint failures. A planar 4R manipulator configuration that corresponds to this Jacobian is given in Fig. 5(b). It is important to note that maximizing the angle between any two $\hat{N}_i$ and $\hat{N}_j$ can allow one to also spread out the columns of the Jacobian so that for any $\boldsymbol{j}_i$ (or its reflection) there is a $\boldsymbol{j}_j$ that is 45 degrees away. This makes sense from a physical point of view since it makes the velocity that any two joints can impart to the end effector overlap as much as possible. The more the columns of J overlap, the more other joints can compensate for a failed joint. To quantify this qualitative description, note that like (26), any single joint failure results in a reduced manipulability index of $w(J)/\sqrt{2}$ so that 70.71% of the original manipulability index is retained. However, for two joint failures, the reduced manipulability index becomes either $w(J)/2$ or $w(J)/(2\sqrt{2})$, which corresponds to retaining 50% or 35.36% of the original manipulability, respectively.

Thus, (28) can be very useful for identifying configurations that optimize worst-case reduced manipulability in the presence of possible joint failures. If two particular joints are more likely to fail, then by (27), one may want to keep the corresponding columns of V_2^T orthogonal to each other. One may especially want to do this when the failure of one joint becomes more likely with the failure of the other. If statistical data are known concerning the likelihood of the individual joint failures, this could be used to modify the criterion for optimal fault tolerance, e.g., maximizing the expected value of the reduced manipulability.

V. Discussion

This work has presented a local failure tolerance measure that is based on the classic definition of dexterity given by the manipulability index. The value of the manipulability index following a locked joint failure is one useful metric for determining the absolute amount of dexterity available. The relative manipulability index was also introduced to provide a measure of fault tolerance relative to the manipulability prior to a failure. The relative index proved useful for gaining insight into the distribution of redundancy throughout the joints. However, it is important to emphasize that one should not use the relative index by itself since if the original manipulability index is small, then that configuration is probably not a desirable operating configuration, even though it may be optimal in terms of relative fault tolerance. This will be illustrated through a specific example.

Consider the standard planar 3R manipulator with unit length links. Contour plots for the minimum manipulability index following a single locked joint failure are shown in Fig. 3. Note that this measure is independent of θ_1. The corresponding contour plots for the minimum relative manipulability index are shown in Fig. 4. In comparing these two plots, the immediate obvious feature is that the zeros of these two functions coincide, as is expected, and that in general, larger values of reduced manipulability correspond to larger values of relative manipulability. However, it is imperative to appreciate that this is not always the case. For example, consider the optimal configurations in terms of relative manipulability. These configurations are characterized by lines of slope -1 that pass through the kinematically singular configurations of the original manipulator, i.e., where θ_2 and θ_3 are integer multiples of π (See Fig. 4). While the relative manipulability stays constant at its maximum value of $1/\sqrt{3}$, the reduced manipulability index (see Fig. 3) varies from its maximum to its minimum value along this line of configurations. Clearly, the optimal value of the relative manipulability in these cases is a misleading indicator of the dexterity of the manipulator configuration. A redundancy resolution scheme that attempted to simply track the optimal value of the relative manipulability would have no indication of the reduction in the original manipulability and thus could inadvertently blunder into a kinematically singular configuration.

Even though relative manipulability indices are not defined at singularities due to $w(J)$ being zero, one can extend the definition using the concept of a "constrained manipulability index." The constrained manipulability index can be defined as the product of the p nonzero singular values of J, where p is the rank of J. The i-th relative constrained manipulability index r_i can now be defined as the ratio of the constrained manipulability index of ${}^iJ$ over the constrained manipulability index of J. One can show that

$$r_i = \|\hat{\boldsymbol{N}}_i\| \tag{32}$$

where the $(n-p)$-vector $\hat{\boldsymbol{N}}_i$ is the i-th column of V_2^T. Note that (24) now becomes

$$\sum_{i=1}^{n} r_i^2 = n - p. \tag{33}$$

Similarly, the relative constrained manipulabilty index for the case of multiple failures has the same form as given in Theorem 3 where $\hat{N}_{i_1,\cdots,i_f}$ is now an $(n-p)\times f$ matrix. Like the nonsingular case, the relative constrained manipulability index at a singularity gives an indication of how much further dexterity is lost by locking a joint. In particular, when $r_i = 0$, deleting the i-th column of J results in reducing the rank of J by one. Physically, this means that locking the i-th joint would result in a further reduction in the space of possible local end-effector motions.

It is also important to note that a decrease in the minimum relative manipulability index does not inherently imply a poorer level of failure tolerance. In particular, since the sum of the squares of the relative manipulability indices are constrained to be constant (see (24)), the minimum r_i can be decreased by simply increasing the reduced manipulability of the other joints faster than that associated with the minimum r_i. Clearly, from a practical point of view, the resulting Jacobian would have increased its intuitive measure of failure tolerance.

VI. Spatial Manipulators

The results developed in the previous sections are completely general and can be applied to manipulators with an arbitrary number of degrees of freedom. However, when dealing with spatial manipulators one must be careful to consider the implications of the manipulability index. This section will comment on some of these issues and present a specific example of how to calculate a manipulator Jacobian that possesses an optimal relative manipulability index.

For a spatial manipulator, each column of the manipulator Jacobian will consist of a twist denoted

$$\boldsymbol{j}_i = \begin{bmatrix} \boldsymbol{v}_i \\ \omega_i \end{bmatrix} \tag{34}$$

where ω_i is the orientational velocity and $\boldsymbol{v}_i$ is the linear velocity at the end effector resulting from joint i. The units of the resulting manipulator Jacobian are not homogeneous so that an appropriate scaling of the rows associated with the linear components must be performed before the manipulability index is meaningful. There are many ways in which to select this scale factor that, of course, must be in units of inverse length. For example, in cases where the task being performed imposes a preferred precision in linear versus rotational errors then this scaling should be used so that least squares solutions produce the desired results. In the absence of task specific scalings, one can scale the linear velocity components by the maximum reach of the manipulator, the maximum singular value of the linear Jacobian, or by the characteristic length [23]. Once scaled by an appropriate factor whose units are of inverse length, column norms become meaningful, as does the manipulability index. It should be pointed out that when scaling the linear part of the manipulator Jacobian by λ, the manipulability index becomes λ^3 times the manipulability index of the unscaled manipulator Jacobian. Hence, as mentioned earlier, the relative manipulability indices (7) are completely independent of whatever scaling is chosen since they are ratios of manipulability indices and thus not affected by this scaling.

As a specific example of applying the results of the previous sections, consider a spatial manipulator consisting of seven rotational joints. By Theorem 2, a unit null vector corresponding to an optimally failure tolerant Jacobian for this manipulator

TABLE I
DENAVIT-HARTENBERG PARAMETERS FOR THE JACOBIAN IN (36)

i	a_i	d_i	α_i	θ_i
1	0.3970	1.3170	1.2151	-2.7616
2	-1.3051	-2.4429	1.0699	1.4139
3	0.0262	1.3302	1.7002	-1.4640
4	-0.3970	-1.7480	1.2151	-0.4220
5	1.6172	0.0162	2.2981	1.6939
6	-0.1724	-2.0224	1.2552	-2.8837
7	1.0000	1.2279	0.0000	0.9543

is given by

$$\boldsymbol{n}_J = \frac{1}{\sqrt{7}}[1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]^T. \tag{35}$$

There is a whole family of manipulator Jacobians that have this null vector. One particular example is (36), shown at the bottom of the page. As required each column satisfies the condition that the positional part is orthogonal to the orientational part. Note that for this particular example the norms of the linear velocity components are equal to the norms of the rotational components. One can interpret this to mean that each of the joint axes is constrained to be separated from the end effector by a distance that is equal to the characteristic length [23]. The reduced manipulability index is

$$w_i(J) = 4.452 = \frac{1}{\sqrt{7}}w(J) \tag{37}$$

for all joint failures i where

$$w(J) = 11.78. \tag{38}$$

The Denavit-Hartenberg parameters for a manipulator that is in a configuration that possesses this optimal Jacobian can be identified in a straightforward manner [11] and are given in Table I.

VII. CONCLUSIONS

This article discussed two local measures of fault tolerance based on the manipulability index as a measure of dexterity. The reduced manipulability index was defined as the value of the manipulability following a locked joint failure and the relative manipulability index was defined to quantify the relative loss of manipulability due to a joint failure. A convenient method was developed for determining these measures from the null space of the manipulator Jacobian. Using this result, one can determine configurations that are locally optimal with respect to these measures of fault tolerance. These results impact the design of failure tolerant manipulators as well as the design of their intended workspace. In addition, they provide a basis for utilizing manipulator redundancy in anticipation of possible joint failures.

APPENDIX A
PROOF OF THEOREM 2

Lemma 1: Let J be a full rank $m \times n$ matrix and N be a matrix whose rows are orthonormal and orthogonal to the rows of J. If

$$J_N = \begin{bmatrix} J \\ N \end{bmatrix} \tag{A1}$$

then $w(J) = w(J_N)$.

Proof: Let $J = U\Sigma[V_1 \ V_2]^T$ be the SVD of J. Then

$$J_N = \begin{bmatrix} U & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} S & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} V_1^T \\ N \end{bmatrix} \tag{A2}$$

where $S = \mathrm{diag}(\sigma_1, \cdots, \sigma_m)$. Clearly, the manipulability index of J_N is

$$w(J_N) = \sigma_1\sigma_2\cdots\sigma_m 1 \cdots 1 = \sigma_1\sigma_2\cdots\sigma_m = w(J). \tag{A3}$$

■

Lemma 2: Let V_2 and W_2 be two $n \times s$ matrices $(s < n)$, each with the property that its columns are orthonormal. If the column spaces of these two matrices are identical then there exists a $s \times s$ orthogonal matrix Q such that $W_2 = V_2 Q$.

Proof: Let V_1 be an $n \times (n-s)$ matrix whose columns are orthonormal and orthogonal to the columns of V_2. The existence of such a matrix is guaranteed by the Gram-Schmidt orthogonalization procedure. It then follows that $[V_1 \ V_2]$ and $[V_1 \ W_2]$ are $n \times n$ orthogonal matrices so that

$$V_2V_2^T = I_n - V_1V_1^T = W_2W_2^T. \tag{A4}$$

Let $Q = V_2^T W_2$. Then the $s \times s$ matrix Q is orthogonal since

$$Q^TQ = W_2^TV_2V_2^TW_2 = W_2^TW_2W_2^TW_2 = I_s \tag{A5}$$

where we have used the fact that $W_2^TW_2 = I_s$. We only need to show that $W_2 = V_2Q$; this follows from

$$V_2Q = V_2V_2^TW_2 = W_2W_2^TW_2 = W_2. \tag{A6}$$

■

Theorem 2: Let J be an m by n manipulator Jacobian of full rank and let $J = U\Sigma[V_1 \ V_2]^T$ be its SVD where V_2 is a matrix of $n-m$ orthonormal n-dimensional vectors in the null space of J. Then the manipulability index after a failure of the i-th joint is given by (23), where $\hat{\boldsymbol{N}}_i$ is the i-th column of V_2^T. Hence the relative manipulability index r_i is given by $\|\hat{\boldsymbol{N}}_i\|$. Furthermore, (24) holds.

$$J = \begin{bmatrix} 1 & -0.6839 & -0.1534 & 0.9590 & -0.4717 & -0.3391 & -0.3108 \\ 0 & 0.2330 & -0.7082 & -0.0417 & 0.4486 & 0.9365 & -0.8682 \\ 0 & 0.6914 & -0.6891 & 0.2804 & -0.7591 & 0.0896 & 0.3869 \\ 0 & -0.6051 & -0.9831 & 0.0417 & 0.7956 & -0.1432 & 0.8942 \\ 1 & 0.3482 & 0.0387 & -0.9576 & -0.1547 & -0.1455 & -0.1291 \\ 0 & -0.7159 & 0.1790 & -0.2850 & -0.5858 & 0.9789 & 0.4287 \end{bmatrix}. \tag{36}$$

Proof: Let N be an $(n-m-1)\times n$ matrix whose rows are orthonormal and orthogonal to the rows of J and whose i-th column is the zero vector. The existence of such a matrix is guaranteed by the fact that $\dim(\ker(J)) = n-m$. Let J_N be defined as in (A1). By Lemma 1

$$w(J) = w(J_N). \tag{A7}$$

Furthermore, because the i-th column of N is the zero vector, the rows of ${}^iN$ are orthonormal and orthogonal to the rows of ${}^iJ$ so that

$$w({}^iJ) = w({}^iJ_N). \tag{A8}$$

Let $\hat{\boldsymbol{n}}$ be the unit length null vector of J_N. We have already shown in the text (cf. (19)) that the reduced manipulability for the single degree of redundancy case is simply the product of the original manipulability with the absolute value of the i-th component of the unit length null vector. Thus,

$$w({}^iJ_N) = |\hat{n}_i| w(J_N) \tag{A9}$$

where $\hat{n}_i$ denotes the i-th component of $\hat{\boldsymbol{n}}$. It then follows that

$$w({}^iJ) = |\hat{n}_i| w(J). \tag{A10}$$

Now by Lemma 2, there exists an orthogonal matrix Q such that

$$\begin{bmatrix} N \\ \hat{\boldsymbol{n}}^T \end{bmatrix} = QV_2^T. \tag{A11}$$

Since Q preserves norms, the norm of the i-th column of (A11) is equal to the norm of the i-th column of V_2^T. Since the i-th column of N is the zero vector, it follows that the norm of the i-th column of (A11) is $|\hat{n}_i|$. Hence $|\hat{n}_i| = \|\hat{\boldsymbol{N}}_i\|$, proving the result.

To prove (24), note that $\Sigma_{i=1}^n r_i^2$ is the sum of the squares of the magnitudes of $\hat{\boldsymbol{N}}_i$, which is also equal to the sum of the squares of the magnitudes of the $n-m$ columns of V_2. Since each column of V_2 has unit norm, it follows that $\Sigma_{i=1}^n r_i^2 = n-m$. ∎

Appendix B
Proof of Theorem 3

Theorem 3: Suppose that a manipulator is in a nonsingular configuration and that there are $f \le n-m$ distinct joint failures occurring in joints $i_1, i_2, \cdots, i_f$. Then the reduced manipulability index is given by (28), where $\hat{N}_{i_1,\cdots,i_f}$ denotes the $(n-m)\times f$ matrix composed of the columns of V_2^T associated with the failed joints. Furthermore, if the relative manipulability index $r_{i_1,\cdots,i_f}(J)$ is defined to be $w_{i_1,\cdots,i_f}(J)/w(J)$, then the relationship for $n-m$ joint failures given by (29) holds.

Proof: It is sufficient to prove the result for the case where the failures occur in the first f joints; one can appropriately rearrange the columns of J if necessary. Furthermore, by using a slight variation of the QR factorization and Lemma 2 of Appendix A, one can assume that the null space component of J has the form

$$V_2^T = \begin{bmatrix} d_1 & * & \cdots & * & * & \cdots & * \\ 0 & d_2 & \ddots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \ddots & \ddots & * & * & \cdots & * \\ 0 & \cdots & 0 & d_{n-m} & * & \cdots & * \end{bmatrix}. \tag{B1}$$

By Theorem 2, a failure in joint 1 results in a reduced manipulability of $w_1(J) = |d_1| w(J)$. Since all but the first component of the first column of V_2^T are zero, the matrix

$$\begin{bmatrix} d_2 & * & \cdots & * & * & \cdots & * \\ 0 & d_3 & \ddots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \ddots & \ddots & * & * & \cdots & * \\ 0 & \cdots & 0 & d_{n-m} & * & \cdots & * \end{bmatrix} \tag{B2}$$

is the null space component of ${}^1J$. It then follows that a failure in joint 2 results in a manipulability index $w_{12}(J) = |d_2| w_1(J) = |d_1 d_2| w(J)$. Continuing this process, one obtains the result that

$$w_{12\cdots f}(J) = |d_1 d_2 \cdots d_f| w(J) \tag{B3}$$

for $f \le n-m$ failures.

Next, observe that for

$$\hat{N}_{1,\cdots,f} = \begin{bmatrix} d_1 & * & \cdots & * \\ 0 & d_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & d_f \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \tag{B4}$$

$$\hat{N}_{1,\cdots,f}^T \hat{N}_{1,\cdots,f} = \begin{bmatrix} d_1 & * & \cdots & * \\ 0 & d_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & d_f \end{bmatrix}^T \cdot \begin{bmatrix} d_1 & * & \cdots & * \\ 0 & d_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & d_f \end{bmatrix}. \tag{B5}$$

It then follows from the theory of determinants that

$$\det(\hat{N}_{1,\cdots,f}^T \hat{N}_{1,\cdots,f}) = d_1^2 \cdots d_n^2 \tag{B6}$$

so that $w_{1,\cdots,f}(J) = \sqrt{\det(\hat{N}_{1\cdots f}^T \hat{N}_{1\cdots f})}\, w(J)$. Note that orthogonal transformations preserve norms and inner products so that the result holds for any representative V_2 of the null space.

Now consider (29). Suppose $f = m-n$. It was shown that

$$r_{i_1,\cdots,i_{n-m}}^2(J) = \det(\hat{N}_{i_1,\cdots,i_{n-m}}^T \hat{N}_{i_1,\cdots,i_{n-m}}). \tag{B7}$$

Since $\hat{N}_{i_1,\cdots,i_{n-m}}$ is a square matrix, it follows that

$$r_{i_1,\cdots,i_{n-m}}^2(J) = [\det(\hat{N}_{i_1,\cdots,i_{n-m}})]^2. \tag{B8}$$

Summing over the $\binom{n}{n-m}$ possible indices, one has

$$\sum_{i_1 < i_2 < \cdots < i_{n-m}} r^2_{i_1,\cdots,i_{n-m}}(J) = \sum_{i_1 < i_2 < \cdots < i_{n-m}} [\det(\hat{N}_{i_1,\cdots,i_{n-m}})]^2. \quad \text{(B9)}$$

However, by the Binet-Cauchy Theorem, the right hand side of (B9) is just the determinant of $V_2^T V_2 = I_{n-m}$, so

$$\sum_{i_1 < i_2 < \cdots < i_{n-m}} r^2_{i_1,\cdots,i_{n-m}}(J) = \det(V_2^T V_2) = 1 \quad \text{(B10)}$$

proving the result. ∎

References

[1] J. Angeles, "The design of isotropic manipulator architectures in the presence of redundancies," *Int. J. Robot. Res.*, vol. 11, no. 3, pp. 196–201, June 1992.

[2] J. Baillieul, "Kinematic programming alternatives for redundant manipulators," *Proc. 1985 IEEE Int. Conf. Robotics Automat.*, St. Louis, MO, Mar. 1985, pp. 722–728.

[3] ——, "Avoiding obstacles and resolving redundancy," *Proc. 1986 IEEE Int. Conf. Robotics Automat.*, San Francisco, CA, Apr. 1986, pp. 1698–1703.

[4] J. Baillieul, J. Hollerbach, and R. Brockett, "Programming and control of kinematically redundant manipulators," *Proc. 24th IEEE Conf. Decision Control*, Las Vegas, Dec. 12–14, 1984, pp. 768–774.

[5] D. R. Baker and C. W. Wampler II, "On the inverse kinematics of redundant manipulators", *Int. J. Robotics Res.*, vol. 7, no. 2, pp. 3–21, Mar./Apr. 1988.

[6] J. W. Burdick, "A recursive method for finding revolute-jointed manipulator singularities," *ASME J. Mech. Design*, vol. 117, no. 1, pp. 55–63, Mar. 1995.

[7] Y.-C. Chen and I. D. Walker, "A consistent null-space based approach to inverse kinematics of redundant robots," *Proc. 1993 IEEE Int. Conf. Robotics Automat.*, pp. 374–381, May 1993.

[8] R. Colbaugh and M. Jamshidi, "Robot manipulator control for hazardous waste-handling applications," *J. Robotic Sys.*, vol. 9, no. 2, pp. 215–250, 1992.

[9] R. Dubey and J.Y.S. Luh, "Redundant robot control for higher flexibility," *Proc. 1987 IEEE Int. Conf. Robotics Automat.*, Raleigh, N.C., Mar.-Apr. 1987, pp. 1066–1072.

[10] O. Egeland, "Task-space tracking with redundant manipulators," *IEEE J. Robotics Automat.*, vol. RA-3, no. 5, pp. 471–475, Oct. 1987.

[11] M. A. Gonzalez-Palacios, J. Angeles, and R. Ranjbaran, "The kinematic synthesis of serial manipulators with a prescribed Jacobian," *Proc. 1993 Int. Conf. Robotics and Automat.*, Atlanta, GA, May 2-6 1993, pp. 450–455.

[12] J. M. Hollerbach, "Optimum kinematic design for a seven degree of freedom manipulator," *Robotics Research: The Second Int. Symp.*, H. Hanafusa and H. Inoue, editors, 1985, pp. 349–356.

[13] J. M. Hollerbach and K.C. Suh, "Redundancy resolution of manipulators through torque optimization," *IEEE J. Robot. Automat.*, vol. RA-3, no. 4, pp. 308–316, Aug. 1987.

[14] C. A. Klein and B. E. Blaho, "Dexterity measures for the design and control of kinematically redundant manipulators," *Int. J. Robotics Res.*, vol. 6, no. 2, pp. 72–83, Su 1987.

[15] C. L. Lewis and A. A. Maciejewski, "Optimization of the dynamic performance of redundant robots in the presence of faults," *Proc. Fourth Int. Symp. Robotics Manufacturing (ISRAM '92)*, Sante Fe, NM, Nov. 11–13, 1992, pp. 279–284.

[16] C. L. Lewis and A. A. Maciejewski, "Dexterity optimization of kinematically redundant manipulators in the presence of failures," *Computers and Electrical Engineering: An Int. J.*, (Special Issue on Fault Tolerant Robots), vol. 20, no. 3, pp. 273–288, May 1994.

[17] A. A. Maciejewski, "Fault tolerant properties of kinematically redundant manipulators," *Proc. 1990 IEEE Int. Conf. Robotics Automat.*, pp. 638–642, May 1990.

[18] A. A. Maciejewski and C. A. Klein, "Obstacle avoidance for kinematically redundant manipulators in dynamically varying environments, " *Int. J. Robotics Res.*, vol. 4, no. 3, pp. 109–117, Fall 1985.

[19] Y. Nakamura, H. Hanafusa, and T. Yoshikawa, "Task-priority based redundancy control of robot manipulators," *Int. J. Robotics Res.*, vol. 6, no. 2, pp. 3–15, Su 1987.

[20] C. J. J. Paredis, W. K. F. Au, and P. K. Khosla, "Kinematic design of fault tolerant manipulators," *Computers and Electrical Engineering: An Int. J.*, (Special Issue on Fault Tolerant Robots), vol. 20, no. 3, pp. 211–220, May 1994.

[21] D. L. Schneider, D. Tesar, and J. W. Barnes, "Development & testing of a reliability performance index for modular robotic systems," *Proc. 1994 Annual Reliability and Maintainability Symp.*, Anaheim, CA, Jan. 24–27, 1994, pp. 263–270.

[22] H. Seraji, "Configuration control of redundant manipulators: Theory and implementation," *IEEE Trans. Robot. Automat.*, vol. 5, no. 4, pp. 472–490, Aug. 1989.

[23] M. Tandirci, J. Angeles, and F. Ranjibaran, "The characteristic point and the characteristic length of robotic manipulators," *Robotics, Spatial Mechanisms, and Mechanical Systems*, pp. 203–207, Scottsdale, AZ, Sept. 13-16 1992.

[24] M. L. Visinsky, I. D. Walker, and J. R. Cavallaro, "Layered dynamic fault detection and tolerance for robots," *Proc. 1993 Int. Conf. Robotics Automat.*, Atlanta, GA, May 2-6, 1993, pp. 180–187.

[25] Z. Wang and K. Kazerounian, "Analytical determination of the null space of anthropomorphic manipulators," *IEEE Trans. Syst., Man, Cybern.*, vol. 22, no. 6, pp. 1488–1492, 1992.

[26] T. Yoshikawa, "Analysis and control of robot manipulators with redundancy," *Robotics Research: The First International Symposium*, M. Brady and R. Paul, Eds. Cambridge, MA: MIT Press, 1984, pp. 735–747.

[27] ——, "Manipulability of robotic mechanisms," *Int. J. Robotics Res.*, vol. 4, no. 2, pp. 3–9, Summer 1985.

[28] ——, "Dynamic manipulability of robot manipulators," *J. Robotic Sys.*, vol. 2, no. 1, pp. 113–124, 1985.

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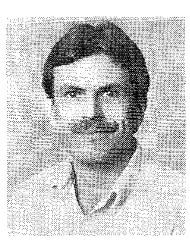

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