Heuristic Combinatorial Optimization in the Design of Manipulator Workspace

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Abstract—In the design of manipulators, there are very few kinematic and dynamic criteria that are available for optimum design. One that has recently been established, for instance, is a workspace criterion [1], [2]. In the present study a method for the optimum design of manipulator workspace is provided based on that criterion. Analytical and computer-aided procedures are presented for the determination of the geometrical proportions of a manipulator for optimum workspace. An effective algorithm is developed by combining the heuristic optimization technique of Lee and Freudenstein [3] and the computer program of kinematic analysis of manipulators [2]. Two problems are investigated. The first is concerned with the optimum design of a manipulator that has three revolute joints and a spherical joint at the wrist, called the RRRS manipulator; the second is concerned with the problem of design improvement for five commercial robots for larger workspace volume. In the former, comparisons are also made with an analytical method that is included here, as well as the proportions of a human arm. The potential application of this procedure in the optimization of manipulator design, is demonstrated in the investigation.

I. Introduction

THE subject of optimum design of manipulators is an area of practical interest in which little has been done and reported. This is primarily due to the difficulty in establishing performance criteria for robots and manipulators. One way to evaluate the kinematic performance of manipulators is from the viewpoint of workspace generation. That is, given a number of manipulator designs, how can we evaluate or determine which one of the designs would give optimum workspace characteristics, say, in terms of its volume and consequently offer a better versatility in its use. The basic problem is, therefore, how can we design the proportions of a manipulator structure in order to optimize its workspace. This paper represents an attempt to address this question.

There have been a number of investigations on manipulator workspace [1], [2], [4]–[11]. Recently Yang and Lee [1], [2] developed an analytical representation of manipulator workspace and presented a performance index on a manipulator based on workspace. An algorithm called kinematic analysis of manipulators (KAM) was developed that implemented the theories and criteria presented in [1] and that can evaluate the volume and characteristics of a manipulator workspace. For a given manipulator structure, KAM first defines the workspace numerically, then explores the existence of hole and void, outlines the boundary, and finally calculates the total volume of workspace.

In this paper, we present an effective computational procedure for the optimum design of manipulator workspace. It is developed by combining the heuristic optimization technique (HOT) [3] with the KAM Program. HOT is used to search for the optimum proportions of a manipulator, whereas KAM is used to generate the workspace and to evaluate its volume.

To demonstrate the effectiveness of the approach, two design problems that are significant and representative of the design analysis of manipulator systems are chosen. The first problem deals with the optimization of an RRRS manipulator and the second deals with the optimization of a number of commercially available robots. A comparative study between the human arm proportions and the optimized RRRS structure is also investigated. The results show that the computational procedure is effective for reaching an optimum solution for the design parameters. The procedure presented is in a general algorithm form, and can be applied to optimum manipulator design problems of realistic complexity.

II. A Volume Index on Workspace

In this section, an evaluation index for manipulator workspace is introduced. This index is based on a definite relationship between the volume of the workspace and the total link length of the manipulator [2]. For kinematically similar manipulators, i.e., manipulators having the same geometrical proportions, this index is the same. It is a workspace volume index based on a scale of 0 to 1 to evaluate the effectiveness of a design on the geometrical proportions of a manipulator with regards to the generation of workspace volume. In the following, we begin with a brief description of manipulator geometry and a method for the analytical representation of workspace, and then, an approach is given to evaluate the workspace volume. Finally, an evaluation index based on workspace volume is introduced.

Manuscript received November 22, 1983; revised April 6, 1984. This work was supported by the National Science Foundation Grant MEA-83-07045 to the University of California, Los Angeles, by the Los Angeles Division of the University of California Academic Senate Grant no. 3847, and by the National Science Foundation Grant MEA-80-06783 to Rutgers University.

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A. Manipulator Description and Analytical Representation of Workspace

A manipulator with n unlimited revolute joints in series can be represented schematically as shown in Fig. 1. There are n coordinate frames to specify the configuration. For any coordinate frame, say k, the Z_k axis is always the joint axis and the X_k axis is in the direction of the common normal between axes Z_k and Z_{k+1}. The Z_1 axis is attached to a fixed frame. The last link, link n, is associated with the hand or the end effector of the manipulator. The link k is connected to joint k. Three parameters are needed to specify the geometrical relation of two consecutive unlimited joints (the joint being of capable making a complete rotation). Referring to Fig. 2(a), these parameters are a_k, the common normal between two consecutive axes Z_k and Z_{k+1}; α_k, the twist angle of these two axes; and b_k, the axial offset of joint $k+1$ on the axis Z_{k+1}. For the limited revolute joint (the joint with physical rotational limit), an additional parameter β_k, the location angle of link $k+1$ with respect to link k, is needed as shown in Fig. 2(b). A position angle β_k is defined as the angle between the X_k axis and the X_{k+1} axis while both the link k and the link $k+1$ are in the midrange of their joint rotational limits.

The workspace of a manipulator is defined as the region that can be reached by the center of the manipulator's hand H and $W_k(H)$ denotes the workspace generated by the point H, holding the axis k as fixed while all the revolute joints $k, k+1, \cdots, n$ make rotations.

A general recursive formula for workspace $W_k(H)$ can therefore be formulated as follows [1]:

$$\left.\begin{aligned} x_k &= r_k \cos\theta_k \\ y_k &= r_k \sin\theta_k \\ z_k &= z_k^* \end{aligned}\right\} \tag{1}$$

where

$$r_k = \sqrt{x_k^{*2} + y_k^{*2}}$$

and

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_k^* = [A]_{k+1} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{k+1} \tag{2}$$

where $[A]_{k+1}$ denotes the homogeneous transformation that provides the geometrical relationship between two consecutive coordinate frames $k+1$ and k. Depending upon whether the revolute joint is unlimited or limited, expressions for $[A]_{k+1}$ are different. For the unlimited revolute joint it is given as

$$[A]_{k+1} = \begin{bmatrix} 1 & 0 & 0 & a_k \\ 0 & \cos\alpha_k & \sin\alpha_k & b_k \sin\alpha_k \\ 0 & -\sin\alpha_k & \cos\alpha_k & b_k \cos\alpha_k \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

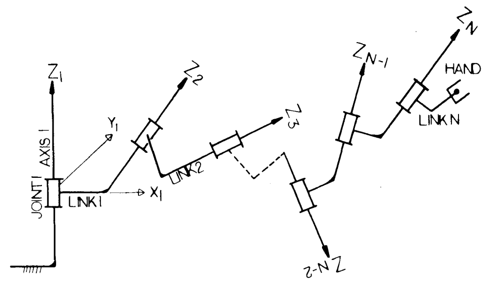
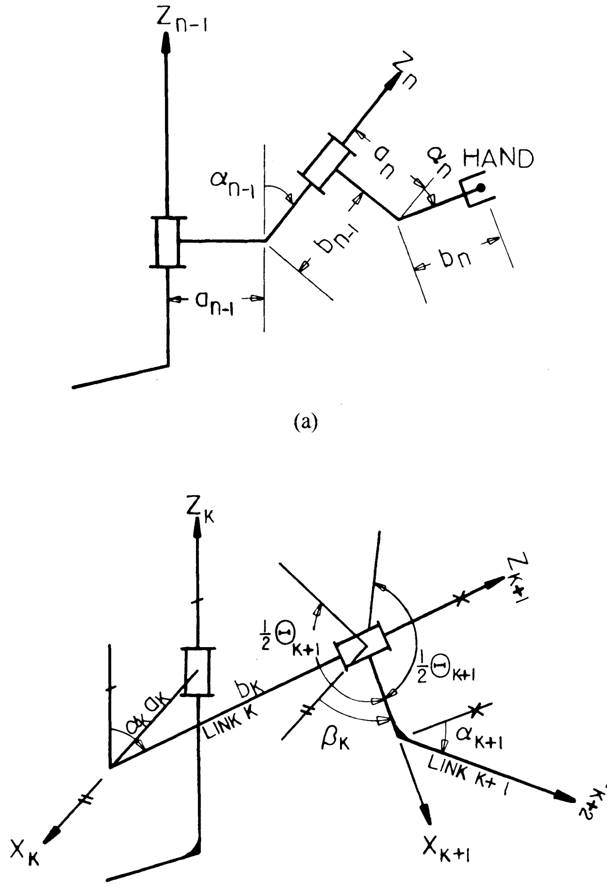


Fig. 1. A manipulator with n revolute joints in series.



(a)

(b)

Fig. 2. (a) Geometrical relationship between unlimited revolute joints $n-1$ and n. (b) Geometrical relationship between limited revolute joints k and $k+1$.

For the limited revolute joint we have

$$[A]_{k+1} = \begin{bmatrix} \cos\beta_k & -\sin\beta_k & 0 & a_k \\ \cos\alpha_k \sin\beta_k & \cos\alpha_k \cos\beta_k & \sin\alpha_k & b_k \sin\alpha_k \\ -\sin\alpha_k \sin\beta_k & -\sin\alpha_k \cos\beta_k & \cos\beta_k & \cos\alpha_k \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

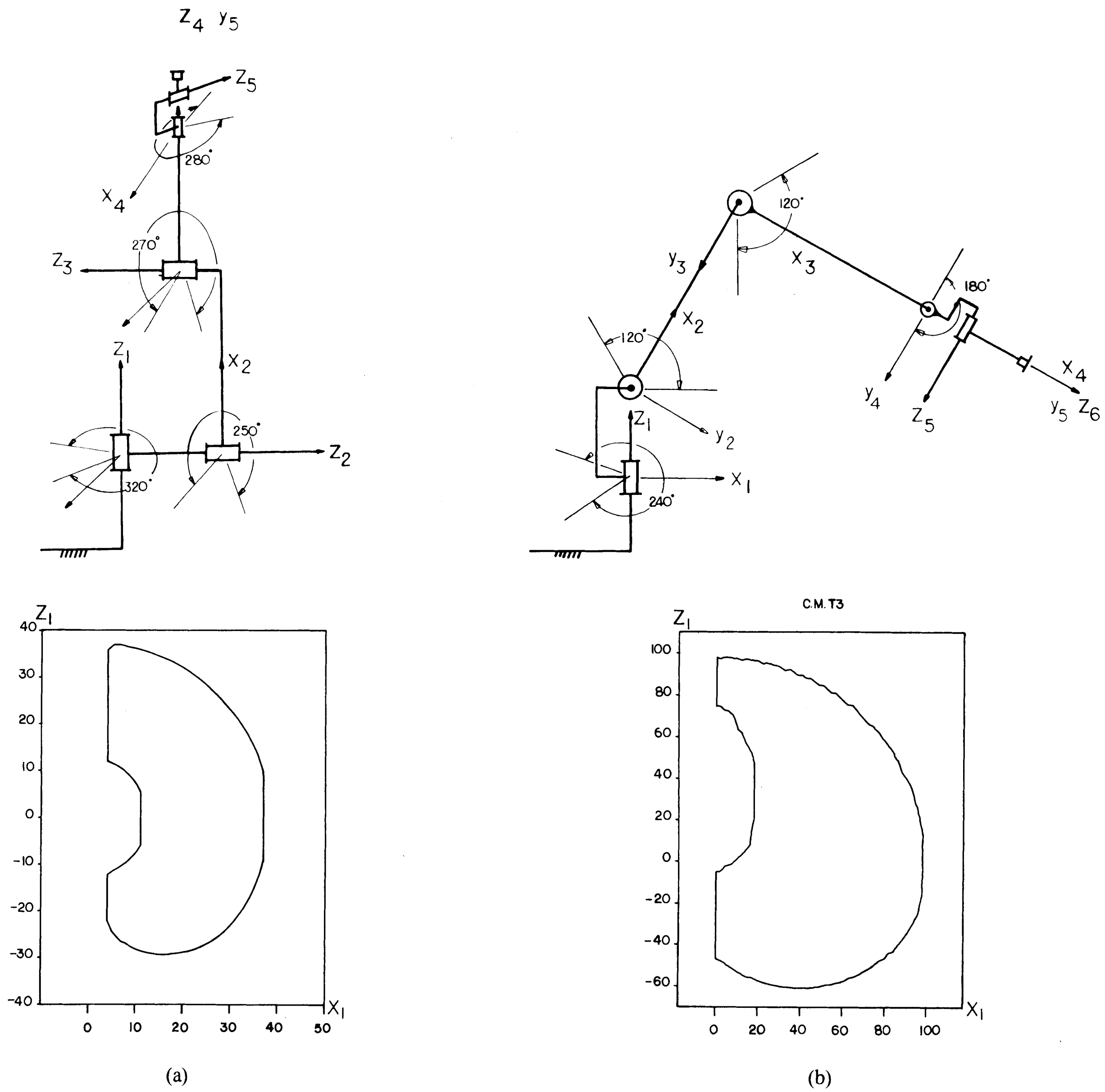


Fig. 3. (a) Structure and workspace of PUMA 600 robot. (b) Structure and workspace of T^3 robot.

B. Evaluation of Workspace Volume

Having an analytical representation, the boundary and the volume of the workspace can subsequently be determined by the construction of the cross section on $W_k(H)$ cut by the plane $X_k Z_k$ [2]. Mathematically this cross section may be expressed as

$$\left.\begin{aligned} x_k &= \pm r_k = f(\theta_{k+1}, \theta_{k+2}, \cdots, \theta_n) \\ z_k &= z_k^* = g(\theta_{k+1}, \theta_{k+2}, \cdots, \theta_n) \end{aligned}\right\}. \tag{5}$$

Because the $W_k(H)$ is symmetric with respect to the axis Z_k, (5) contains all geometrical information regarding the shape of the workspace, except at the small end portions of the workspace (caused by the rotational limits of joint 1) where variations of the shape of cross sections may exist. By scanning sequentially the joint angles $\theta_n, \theta_{n-1}, \cdots, \theta_{k+1}$ in (5) at a reasonably small interval according to their rotational limits, most of the points of $W_k(H)$ can thus be obtained. The volume of the workspace can subsequently be obtained by taking a numerical integration of the cross section along the rotational limits of the first joint.

A computer program, KAM, written in Fortran was developed based on the theory described here. Two commercial robots, PUMA 600 and Cincinnati Milacron T^3, are taken as examples to illustrate the cross sections given in (5). The corresponding results from KAM are given in Fig. 3.

C. An Optimality Criterion

An optimality criterion is proposed for the evaluation of the capability of a manipulator on its generation of workspace as follows.

For a given manipulator (i.e., the twist angles, rotation limits, location angles and the proportions of link length are

TABLE I
PERFORMANCE EVALUATION OF FIVE INDUSTRIAL ROBOTS BASED ON VOLUME INDEX

	d.o.f.	Total Link Length	Volume	VI	NVI
PUMA 600	6	51.0	183986	1.39	0.331
C.M.T^3	6	101.0	2295233	2.23	0.532
Pana-Roba	5	59.84	355937	1.66	0.397
AID 800	5	59.02	267350	1.30	0.310
CT V30	5	94.49	1299251	1.54	0.368

given), the ratio between the volume of the workspace (V) *and the cube of the total link length* (L) *is a constant.* That is

$$\frac{V}{(l_1 + l_2 + \cdots + l_{2n})^3} = \frac{V}{L_3} = \text{constant} = \text{VI}. \quad (6)$$

The proof of (6) is given in [2]. Notation VI, which stands for the volume index of manipulator workspace, gives an indication of the effectiveness of link length on the creation of reachable workspace. This index can be normalized by dividing its possible maximum value. The maximum workspace that a manipulator can possibly have is a sphere with radius L and centered at joint 1. Therefore, the maximum VI a manipulator can possibly have is

$$\frac{4\pi L^3/3}{L^3} = \frac{4\pi}{3}. \quad (7)$$

Using this value as a normalizing factor, a normalized volume index (NVI) can be defined as follows.

$$\text{NVI} = \frac{3V/(4\pi)}{(l_1 + l_2 + \cdots + l_{2n})^3}. \quad (8)$$

The value of this index is from zero to one.

Five commercially available manipulators, namely: PUMA 600 (Unimation, Inc.), T^3 (Cincinnati Milacron), Pana-robo (Matsushita, LTD), AID 800 (Automatix, Inc.), and V30 (Cybotech Corp.), are evaluated based on VI and NVI. The results are given in Table I.

III. THE OPTIMIZATION PROBLEM

The objective of the manipulator workspace problem specified in this investigation is the design of the proportions of a manipulator that would given maximum workspace volume for given constraints of geometrical properties. The problem may be formulated mathematically as an optimization problem as follows.

$$\min F = -\frac{3V/(4\pi)}{(l_1 + l_2 + \cdots + l_{2n})^3} \quad (9)$$

subject to

1) dimensional constraints on design parameters;

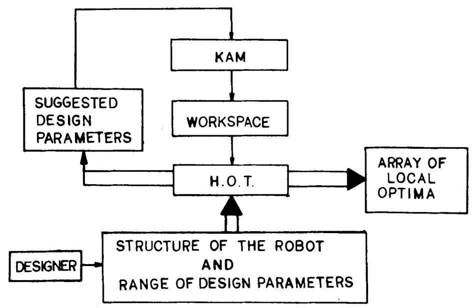


Fig. 4. A flow chart of the workspace design algorithm.

2) constraint on other kinematic, dynamic or stress quantities, such as limits on joint motion, stiffness of the manipulator systems, etc.

The objective function of the optimization problem uses the optimality criterion that is the normalized volume index (NVI) defined earlier in (8). Many optimization techniques are available. Because of its efficiency, the heuristic optimization techniques (HOT) of Lee and Freudenstein [3] is used. A brief qualitative introduction of the HOT is given in the next section.

A special-purpose computer program for optimum design of manipulator workspace was developed and was written in Fortran language. It combines HOT algorithm [3] and the KAM algorithm [2]. The design parameters for the optimization are those basic kinematical properties of a manipulator that include the common normals, twist angles, offset distances, and location angles among joints. Fig. 4 gives a flow chart of the workspace design algorithm.

IV. THE HEURISTIC OPTIMIZATION TECHNIQUE OF LEE AND FREUDENSTEIN

In large-scale mechanical design problems there are generally a substantial number of optimum or near-optimum solutions, but no procedure or formula is known that produces one of these quickly. A logical approach, then, would be to develop a relatively fast and simple method that yields acceptable answers within a reasonable time. This can be achieved by combining intelligent guessing

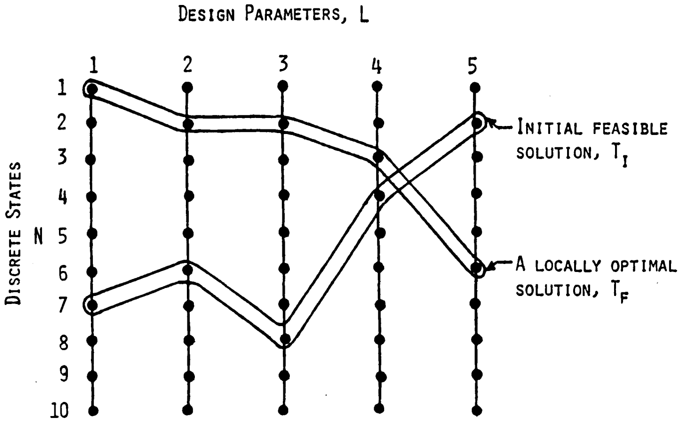


Fig. 5. An illustration of a selection-space matrix ($N \times L$): L = number of parameters to be optimized; and N = number of discrete states for each parameter.

with high-speed computation. The objective is to provide an intelligent partial and random search of the entire solution space for generating acceptable (i.e., optimum and near-optimum) solutions. The methods of combinatorial heuristics represent such a procedure and can be developed so as to utilize engineering knowledge and design experience.

In 1973 Lin and Kernighan [12] developed heuristic techniques in the communications field. Their heuristic principles have been extended and applied by Lee and Freudenstein [3] to the optimum design of large-scale mechanical systems. Subsequently a general-purpose computational algorithm entitled the heuristic optimization technique (HOT) was developed.

This heuristic algorithm differs from most of the convential optimization methods in many ways. It is not an exhaustive search method; rather it is a nonnumerical discrete technique and is probabilistic in nature. The attainability of a solution does not depend on continuity and differentiability, and it is essentially independent of the starting point of the search. An exact algorithm may require inordinate running time but the heuristic method of Lee and Freudenstein produces good answers in reasonable time, although one cannot guarantee that the optimum answer will appear. From a theoretical standpoint we cannot generally prove optimality of solutions, but we can obtain statistical confidence. For practical applications, frequently all that matters is that good answers be obtained in a reasonable computer time.

The method is a discrete one; therefore it requires the discretization of a problem. In a mechanical system this would naturally refer to the conversion of the solution space into a finite number of states of combinations called the selection space. The parameters are allowed only on certain prescribed discrete values or states, chosen in accordance with what seem to be reasonable ranges or proportions. This is advantageous, since the design engineer generally knows reasonable upper and lower bounds on key parameters such as the lengths of rigid bodies, spring stiffness, and damping coefficients. The selection space can be represented mathematically by a matrix ($N \times L$) in

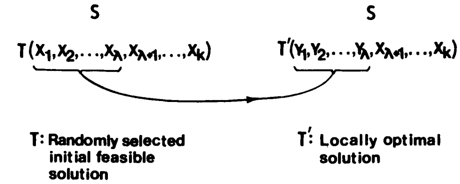


Fig. 6. λ-change transformation (S is a discrete set, where T and T^1 are its subsets, x_i and y_i are elements in the set S).

which there are N rows and L columns corresponding to N discrete states of each of the L design parameters. Fig. 5 illustrates a selection-space matrix ($N = 10$ and $L = 5$).

The task of combinatorial optimization was stated in general terms by Lin and Kernighan [12] as follows: "Find from a set S a subset T that satisfies some criterion C and minimizes an objective function F."

A basic approach to the problem is the iterative improvement of a set of randomly selected feasible solutions as follows:

Step 1: Generate a pseudorandom feasible solution, that is, a set T that satisfies criterion C.

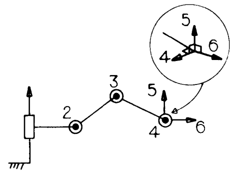
Step 2: Attempt to find an improved feasible solution T' by a transformation of T.

Step 3: If an improved solution is found, i.e. if $F(T') < F(T)$, replace T by T' and repeat from step 2.

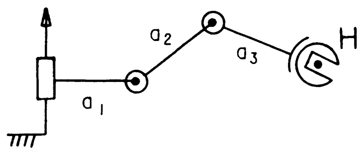
Step 4: If no improvement can be found, T is a locally optimum solution. Repeat from step 1, either until computation time runs out, or until the solution is satisfactory.

The heuristic procedure (the step 2 transformation) maps the random starting solutions of step 1 into locally optimum solutions, among which the global optimum will hopefully appear. The heart of the iterative procedure is step 2, which tries to improve a given solution. One transformation that has been applied to a variety of problems with success is the λ-change transformation [3], [12] (Fig. 6). It is based on an intelligent exchange of a fixed number of λ-elements from T, with λ-elements from $S - T$, such that the resulting solution T' is feasible and better. Heuristic principles [3] involving procedures such as "look ahead," "backtracking," "parameter reordering," and "semirandom search" are incorporated to determine the proper selection of elements. This interchange is repeated as long as exchanged groups from T to T' can be found. Eventually it will not be possible to further improve T by such exchanges, at which time we have a locally optimum solution. The method generates as many locally optimum solutions as computation time permits. The best of the locally optimum solutions may then be chosen as a solution to the problem.

To apply the HOT algorithm, the designer needs to supply only two subroutines: one defines the feasibility criteria, the other specifies the objective function. Once the designer selects N and provides the lower and upper bounds of each parameter, the solution space is defined by the input selection-space matrix, ($N \times L$). The algorithm then generates as many locally optimum solutions as computation time permits. The best of the locally optimum solutions may be chosen as a solution to the problem. The



(a)



(b)

Fig. 7. (a) A commonly adopted $6R$ robotic structure. (b) The equivalent RRRS structure.

principles of the combinatorial heuristics of Lee and Freudenstein and the algorithm are given in [3].

V. Example 1: Optimum Design of an RRRS Manipulator

A manipulator that has six revolute joints (R) and a special geometrical condition was chosen. If the link parameters, a_i, of the last three links are zero (i.e., the axes of the last three joints intersect at one point) and two pairs of them are orthogonal, then these last three revolute joints are equivalent to a spherical joint (S). The manipulator can then be considered as a three-link, RRRS manipulator as shown in Fig. 7. The advantage of this structure is that the entire manipulator workspace is primary workspace [5], [7]. A primary workspace is one in which every point within it is reachable by the manipulator's hand from all directions. Contrarily a workspace with points that cannot be reached by the manipulator's hand from all directions is referred to as secondary workspace.

We are interested in using this special manipulator to test our optimum design algorithm. Since the geometry of this manipulator is relatively simple, analytical determination of its workspace becomes possible. In addition, there is a good analogy between this manipulator and the human arm, which is supposed to be optimally proportioned. Therefore the task is to determine the manipulator proportions using the developed algorithm in order to obtain maximum workspace volume, and to then compare the proportions with the analytical result as well as the proportions of a human arm.

A. Numerical Optimization of Workspace Volume

Let us select certain geometrical configurations of the RRRS manipulator and then arbitrarily choose several of the design parameters to search for the optimum proportions that would give maximum workspace volume. Two manipulator structures that are commonly adopted in industry are optimized through the algorithm developed in this investigation. The result is then compared with the

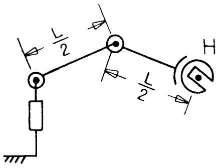


Fig. 8. The RRRS configuration with optimum workspace.

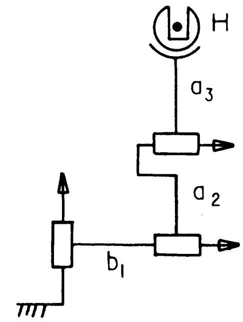


Fig. 9. Another commonly adopted RRRS robotic structure.

analytical solution. It is clear that usually for a limited revolute joint, the greater the rotation range the larger the corresponding workspace. It is therefore reasonable to consider all revolute joints unlimited in a general study.

The first kinematic structure, shown in Fig. 7(b), has the following value of design parameters: $a_4 = 0$, $a_5 = 0$, $a_6 = 0$; $b_1 = 0$, $b_2 = 0$, $b_3 = 0$, $b_4 = 0$, $b_5 = 0$, $b_6 = 0$; $\alpha_1 = 90°$, $\alpha_2 = 0°$, $\alpha_3 = 0°$, $\alpha_4 = 90°$, $\alpha_5 = 90°$, $\alpha_6 = 0°$; and a_1, a_2, and a_3 are dimensions not equal to zero. Considering the parameters a_1, a_2, and a_3 as the optimizing parameters, the result from the optimization algorithm is as follows.

$$a_1 : a_2 : a_3 = 0 : 1 : 1.$$

Fig. 8 shows the configuration of such a manipulator.

The second kinematic structure selected is shown in Fig. 9. The design parameters of this manipulator are $a_1 = 0$, $a_3 = 0$, $a_4 = 0$, $a_5 = 0$, $a_6 = 0$; $b_2 = 0$, $b_4 = 0$, $b_6 = 0$; $\alpha_1 = 90°$, $\alpha_2 = 0°$, $\alpha_3 = 90°$, $\alpha_4 = 90°$, $\alpha_5 = 90°$, $\alpha_6 = 0°$; and a_2, b_1, and b_3 are dimensions not equal to zero. This is also an RRRS structure. The parameters for optimization are now b_1, a_2, and b_3. The result of the optimization algorithm shows that the optimal proportion among b_1, a_2, and b_3 is given in the following.

$$b_1 : a_2 : b_3 = 0 : 1 : 1.$$

Exactly the same result as the previous illustration is obtained.

B. Analytical Determination of Workspace Volume

It is possible to analytically evaluate the workspace of an RRRS manipulator using the formula of a hollow sphere. Letting $a_1 = 0$, which is the case for most commercial robots, the workspace is therefore a solid or hollow sphere depending upon whether the parameters a_2 and a_3 are equal or unequal, respectively. Assuming that L is the total

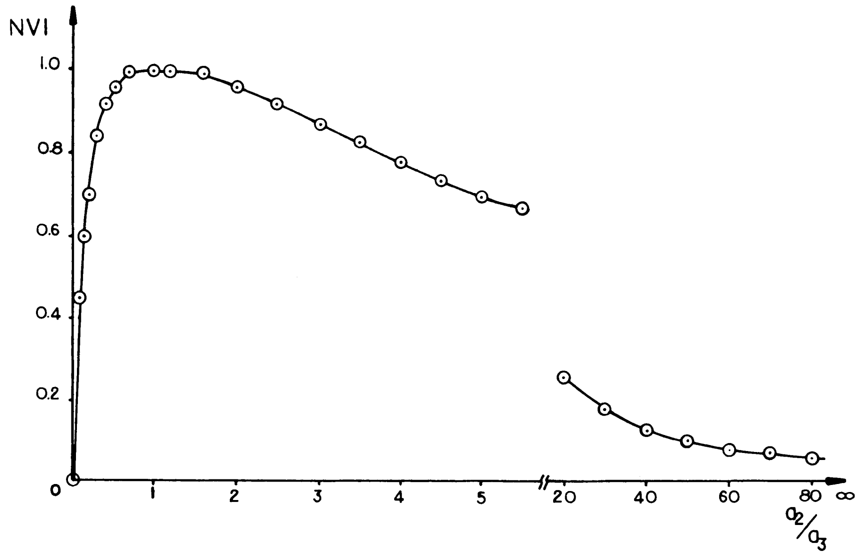


Fig. 10. Change of NVI with respect to the ratio of a_2/a_3.

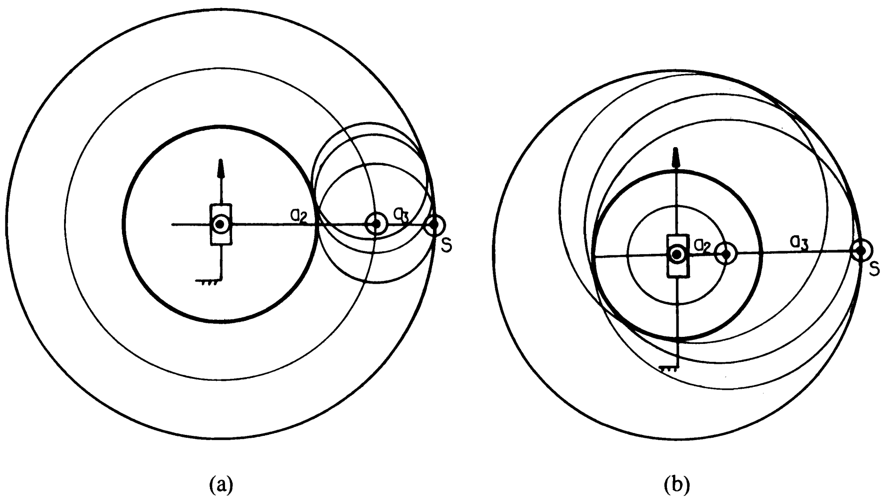


Fig. 11. (a) Workspace when $a_2 > a_3$. (b) Workspace when $a_2 < a_3$.

link length of the manipulator, the workspace volume, therefore, is

$$V = \begin{cases} \frac{4}{3}\pi\left[L^3 - |a_2 - a_3|^3\right], & \text{for } a_2 \neq a_3 \\ \frac{4}{3}\pi L^3, & \text{for } a_2 = a_3. \end{cases} \tag{10}$$

Obviously, the case $a_2 = a_3$ is the optimum. Fig. 10 shows the relationship between the NVI and the ratio of a_2/a_3. Figs. 11(a) and 11(b) show the workspace for $a_2 > a_3$ and $a_2 < a_3$, respectively.

In practice it is very difficult, if not impossible, to construct a manipulator whose last three joints, including the hand, could be exactly equivalent to a ball joint. Therefore it is reasonable to introduce a generalized hand size, say h. The term "generalized" means that in addition to the hand itself, the parameter h includes also the dimensions of the last three links, or the wrist. The effect of this generalized hand size h is therefore equivalent to a sphere with radius h and centered at S, as shown in the right side of Fig. 12. Because the first joint of the manipulator is

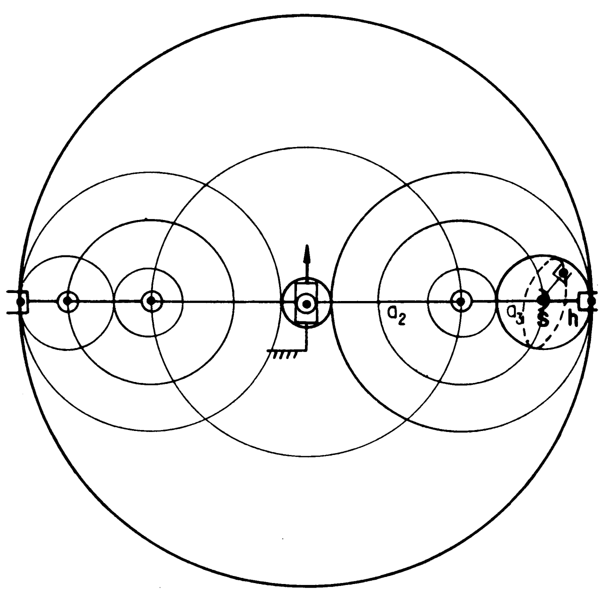


Fig. 12. RRRS robot with wrist/hand size h.

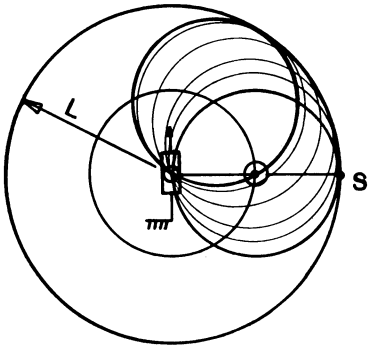


Fig. 13. Workspace of the optimum TRS structure is a sphere with radius L and centered at joint 1.

supposed to rotate in and out of the paper anyway, it is therefore convenient, on the evaluation of workspace, to consider this combination of wrist and hand structure as a revolute joint, as shown on the left side of Fig. 12. The volume of the workspace can be expressed as follows.

$$V = \begin{cases} \frac{4}{3}\pi(L+h)^3, \\ \quad \text{for } |a_2 - a_3| \leqslant h \\ \frac{4}{3}\pi\left[(L+h)^3 - (|a_2 - a_3| - h)^3\right]. \\ \quad \text{for } |a_2 - a_3| > h. \end{cases} \tag{11}$$

Equation (11) shows that as long as $|a_2 - a_3| \leqslant h$, the volume of the workspace is optimized. It should be pointed out that these results may sometimes be misleading, ie., one might misinterpret that the larger the h, the better the workspace. This conclusion is wrong because a large ratio of h to other link sizes will cause the manipulator to have poor dexterity and also bad dynamic behavior. Therefore, the following guideline on workspace design can be drawn. A manipulator should have not only a small difference between the second and third link length, but also a small ratio of the generalized hand size to other link sizes.

Finally it is noteworthy that the optimal model of the RRRS manipulator discussed in this section is kinematically equivalent to a TRS structure (T denotes a Hooke joint). Since $a_1 = b_1 = 0$ and $\alpha_1 = 90°$, the first two revolute joints can therefore be replaced by a Hooke joint. Referring to Fig. 13, the workspace created by this manipulator equals a sphere with radius L and centered at joint 1. Because the hand of a manipulator cannot reach any point beyond its total link length, theoretically, the possible maximum workspace will therefore be a sphere with radius equal to the total link length, and the sphere will be centered at the first joint. This proves that an RRRS manipulator with the following proportions is the global optimum based on the workspace volume: $a_1 = 0$, $a_2 = L/2$, $a_3 = L/2$; $b_1 = 0$, $b_2 = 0$, $b_3 = 0$; and $\alpha_1 = 90°$; $\alpha_2 = 0°$, $\alpha_3 = 0°$. It is worth noting that the structure will remain the same even through the following design parameters are changed: $a_3 = 0$, $b_3 = L/2$ and $\alpha_3 = 90°$.

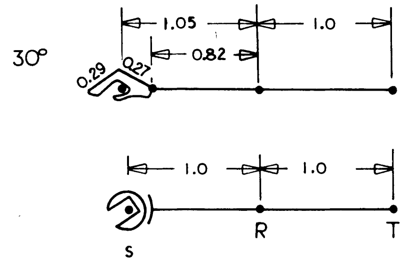


Fig. 14. A dimensional comparison between a human arm and the optimum TRS robot arm.

C. A Comparative Study of the Proportions of Human Arms with the Optimum RRRS Structure

Perhaps it is interesting and noteworthy to consider for a moment the structure of human arms. Is the human arm an "optimal design"? Is there any similarity between a human arm and the optimal RRRS structure discussed previously? There are many apparent basic differences between a human arm and the optimum TRS robot arm. For instance, usually anatomical joints of a human arm have six degs of freedom [13]; they are "designed" for two arms cooperation; and all anatomical joints are limited, etc. However, kinematically the proportions are surprisingly close.

The dimensional comparison between an average human arm [14] and the optimal TRS manipulator is shown in Fig. 14. The ratio between the upper arm and the forearm is 1 : 0.82. However, if the size for the hand is included in the consideration and the grasp position of the hand is estimated as $0.27 \cos 30°$ (referring to Fig. 14 where 0.27 is the average length proportion of the back of a hand and 30° refers to the average grasp angle), then the proportion between the upper arm and the distance between elbow and grasp point becomes 1 : 1.05. This ratio is very close to the proportions of the optimal TRS structure which equals 1 : 1. This finding is interesting! It suggests that perhaps a further analytical investigation on the subject of the workspace problem of the human arm is warranted.

VI. Example 2: A Design Improvement of Five Industrial Robots

In this example numerical experiments of the link lengths of five commercially available robots are performed using the optimum workspace design algorithm. The purpose is to search for the best proportions of the link lengths for the existing robots. To avoid major redesign, all non-zero link lengths are allowed to vary only within $\pm 20°$ of the original length, and other design parameters, such as twist angles, location angles, and zero length links remain unchanged.

The results of the optimal proportions obtained in this investigation are shown in Table II. From a comparison of the NVI values between the original and the computer-design models, one may conclude that for a small variation of link lengths, significant improvement on workspace volume can be achieved on PUMA 600 and CT V30 robots; there are improvements on other robots as well.

TABLE II
A DESIGN IMPROVEMENT[a] ON WORKSPACE VOLUME OF FIVE INDUSTRIAL ROBOTS

Robot	Design Parameters	Length Proporations	NVI	%Improvement of NVI
PUMA 600	b_1, a_2, b_2, b_3, b_5	0.56:1:0.22:0.94:0.11	0.331*	52.0
		0.37:1:0.05:0.91:0.14	0.503**	
C.M. T[3]	a_2, a_3, a_4, a_5	1:1:0.2:0.33	0.532*	2.4
		0.92:1:0.26:0.4	0.545**	
Pana-Robo	a_2, a_3, b_4	0.75:1:0.15	0.397*	5.5
		0.67:1:0.19	0.419**	
AID 800	a_2, a_3, b_4	0.75:1:0.13	0.310*	9.4
		0.58:1:0.14	0.339**	
CT V30	a_2, a_3, b_4	0.8:1:0.6	0.368*	20.0
		0.51:1:0.67	0.443**	

[a]Allowing a change within ±20 percent in link length proportions.
[b]Original design.
[c]This investigation.

TABLE III
A COMPARISON OF PUMA 600 ROBOT WITH AND WITHOUT SHOULDER

Robot	Volume	Total Link Length	NVI
PUMA 600	183986	51.0	0.331
PUMA 600 (No shoulder)	182549	41.0	0.632

Particularly in the case of the PUMA robot, a better ratio of volume to link length is obtained by reducing both design parameters b_1 and b_2, where b_1 and b_2 are offset distances of link 1 and 2, respectively. The parameter b_1 ($b_1 = 10$ in), which is commonly referred to as the shoulder of the PUMA arm, actually does not contribute much to the volume of the workspace. By eliminating the shoulder, the total link length in this case would be about 20 percent less than the original one, and the workspace volume would be only 0.8 percent less. The NVI value in this case would increase nearly 90 percent as compared to the original design (Table III). There may be reasons to include the shoulder in the PUMA structure that are not apparent to the authors. Nevertheless, from a kinematic viewpoint, eliminating the shoulder would optimize the workspace. It would also provide a better dynamic balancing for the robot.

VII. CONCLUSION

A numerically efficient procedure utilizing the heuristic optimization algorithm (HOT) to optimally design the manipulator workspace has been presented. The procedure is self-starting, does not require derivative evaluations, and is very useful for problems in which a reasonable initial estimate could be difficult. The method is designed to bypass or remove itself from local obstacles in the solution space; therefore it is particularly suitable for nonlinear problems such as the manipulator design problem. The method is simple and efficient to use. The selection of the solution-space matrix or data matrix can be based upon designer's experience and insight of the physical problem, or it can simply be based on the geometrical constraints of the systems. By intelligent manipulation of the data matrix, for instance, it is possible to refine a solution by using a finer grid, which would provide an improvement of the accuracy of that solution. Since it is a nonnumerical technique and is not gradient-dependent, instability does not arise. In the example problems we have illustrated that this algorithm can generate the known, exact solution in the case of designing the RRRS robot, and can improve existing industrial robots. This investigation deals with only one of many possible objectives for the optimization of manipulator design. Some other important engineering aspects, such as the dexterity of the workspace, the dynamics of the structure, and the controllability of the manipulator etc., need to be considered as well. As other optimality criteria become available, they may be easily incorporated into the computational algorithm presented here. Therefore it is believed that this study will broaden our capability for the optimization of manipulator design. Perhaps it will also pave the way for future research in this area.

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A Dynamic Approach to Nominal Trajectory Synthesis for Redundant Manipulators

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***Abstract*—The solution to the inverse manipulation problem for redundant manipulators has mostly been considered from geometric–kinematic standpoint so far. In this paper a procedure for the inverse problem solution is developed using the dynamic model of the manipulator and its actuators. Nominal trajectories in the space of joint coordinates are generated so as to be optimal with respect to total energy consumption of the actuators (hydraulic or electric DC motors). The treatment of constraints on joint coordinates and rates is also involved in the procedure. The algorithm is illustrated by two industrial robots.**

I. Introduction

Scientific research of dynamics and control synthesis for nonredundant manipulation systems has been extensively developed over the several past years. Modern, flexible, and adaptive manipulation imposes the use of active spatial mechanisms with an increased number of degrees of freedom (dof), in order to ensure successful completion of required tasks in real, industrial environments with various geometric and kinematic constraints. Moreover, manipulators that are nonredundant with respect to one part of a task, often become redundant for another part of the same task. Hence the investigation of the dynamics and control synthesis for redundant manipulators appears to be more and more important.

The extremely complex problem of redundant manipulator control can be substantially simplified, if the two-stage control concept (elaborated for nonredundant manipulators) is accepted [1], [2]. First, nominal trajectories in the space of joint coordinates $q(t)$ are determined, given an end effector motion and distribution of obstacles. In the second stage, tracking of the nominal trajectories should be provided in the presence of various disturbances. In this paper we are concerned with the first stage, in which the redundancy problem is completely resolved, thus enabling the application of all the control schemes elaborated for nonredundant robots [2], [3] to redundant manipulator control.

Manuscript received January 5, 1983; revised June 1983.

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0018-9472/84/0700-0700-0580$01.00