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Optimal Design of 6 DOF Parallel Manipulators Using Three Point Coordinates

Sung-Gaun Kim and Jeha Ryu*
Department of Mechatronics
Kwangju Institute of Science and Technology
1 Oryong-dong, Puk-gu, Kwangju 500-712, Korea
kimsg@kjist.ac.kr *ryu@kjist.ac.kr

Abstract-- **Development of optimal design methods for general 6 degree-of-freedom parallel manipulators is important in obtaining an optimal architecture or pose for the best kinematic performance. Use of performance index such as the condition number of the Jacobian matrix that is composed of nonhomogeneous physical units may lack in physical significance. In order to avoid unit mismatch in the Jacobian matrix, this paper presents a new Jacobian formulation by use of three point generalized coordinates, which results in a *(9x9)* dimensionally homogeneous Jacobian matrix. The condition number of the new Jacobian matrix is used to design an optimal architecture or pose of parallel manipulators for best dexterity. A design example with Gough-Stewart platform parallel manipulator by using the proposed formulation is shown to generate the same optimal configuration as from using the other Jacobian formulation methods.**

I. INTRODUCTION

Optimal selection of a kinematic configuration (mechanical architecture and operational pose) is an important consideration in the design and control of robot manipulators. For the purpose of optimally selecting kinematic design variables and manipulator pose (end-effector position and orientation), several measures of dexterity or manipulability such as a condition number or volume of manipulability ellipsoid of manipulator Jacobian matrix has been used for the open-chain serial robot manipulators [1-5] as well as for the closed-chain parallel manipulators [6-12].

However, as Lipkin and Duffy [13] pointed out, use of the condition number of the Jacobian matrix with non-homogeneous physical units in the optimal control or design may cause significant problems. When both the position and the orientation of the end-effector are included in the kinematic equations, it can be readily seen that the condition number of the Jacobian matrix will not be invariant under a scaling of the dimensions of the manipulator. Hence, the need for a formulation of the kinematic equations that would lead to a Jacobian matrix in which each of the entries would bear the same physical units so that the condition number would not to be affected by scaling of the manipulator.

In order to avoid this unit mismatch problem, Gosselin [14] defined a new Jacobian matrix that transforms actuator velocities into the linear velocities of two points on the end-effector. Tandirci et al. [15] used a "characteristic length (CL)" to normalize all translational elements. The CL that minimizes the condition number of the homogeneous Jacobian matrix is dubbed as the "Natural Length" (NL) by Ma and Angeles [9]. Angeles [16] and Angeles et al. [17] used NL as a design variable. Doty et al. [18] proposed a method of inverting nonsquare matrices with mixed physical units so that the solution is both unit and frame invariant. Stocco et. al.[12] presented a technique of pre and post-multiplying a design matrix by diagonal scaling matrices corresponding to a range of joint and task space variables. The CL or NL in the previous techniques for addressing the unit inconsistency problem can be chosen arbitrarily or through design optimization. Kim et al [19] proposed a new optimum quality index of a Stewart Platform based on a reciprocal product of two screws. This optimum index has dimensional homogeneity. Using this index, they obtained the same optimal Stewart platform design as that of Ma and Angeles [9].

In order to avoid unit mismatch problem in the Jacobian matrix, this paper presents a new Jacobian formulation by use of three point generalized coordinates of the parallel manipulator moving platform. The proposed formulation results in a *(9x9)* dimensionally homogeneous Jacobian matrix. The condition number of the new Jacobian matrix can be used to design an optimal architecture or pose of parallel manipulators for optimum dexterity. A design example with the Gough-Stewart Platform parallel manipulator by using the proposed formulation is shown to generate the same optimal configuration as that from the

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other Jacobian formulation methods by Ma and Angeles [9], Pittens and Podhorodeski [10], Kim et al. [19].

This paper is organized as follows; The following section introduces a new three point coordinate formulation of inverse Jacobian matrix of the Gough-Stewart platform parallel manipulator. The third section presents application of the proposed formulation to a design example of a Stewart platform manipulator. The last section presents discussions and conclusions of the current investigation.

II. Three Point Coordinate Formulation of Manipulator Jacobian

Consider a nearly general 6-6 parallel manipulator with a planar base and a planar mobile platform as shown in Fig. 1. The base joints are denoted by A_i and the platform joints by B_i (i = 1, ..., 6). An absolute coordinate frame is chosen with center O fixed to the base so that its x- and y-axes lie in the plane of the joints A_i. The coordinates of points A_i are x_{A_i}, y_{A_i}, and z_{A_i}, where $z_{A_i}=0$. The generalized coordinates describing the platform pose with respect to the base can be chosen as the 9 coordinates of 3 points T_j on the mobile platform, namely x_j, y_j, and z_j (j = 1, 2, 3). The points are selected in the plane of points B_i and the distance between each two of them is equal to k_i.

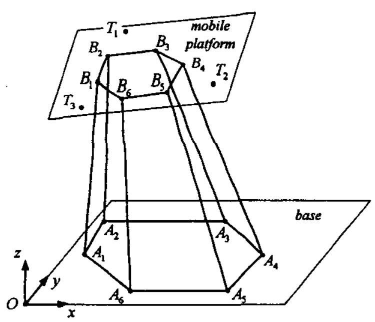


Fig. 1. A nearly general 6-6 parallel manipulator (with planar base and planar mobile platform).

Let $\mathbf{q}$ be the vector of generalized coordinates for describing the motion of the mobile platform:

$$\mathbf{q}=[x_1,y_1,z_1,x_2,y_2,z_2,x_3,y_3,z_3]^T \tag{1}$$

Since B_i and T_j points are on the same plane of a mobile platform, the coordinates of the platform joints B_i can be expressed in terms of the generalized coordinates (Fig. 2) as

$$\mathbf{OB}_i=\begin{bmatrix} k_{i,1}x_1+k_{i,2}x_2+k_{i,3}x_3 \\ k_{i,1}y_1+k_{i,2}y_2+k_{i,3}y_3 \\ k_{i,1}z_1+k_{i,2}z_2+k_{i,3}z_3 \end{bmatrix},\ i=1,\ \ldots,\ 6. \tag{2}$$

where $k_{i,j}$ (i = 1,2,..,6; j = 1,2,3) are dimensionless constants and $k_{i,1}+k_{i,2}+k_{i,3}=1$. Indeed, this is true because

$$\begin{aligned}\mathbf{OB}_i &= \mathbf{OT}_3+k_{i,1}\mathbf{T}_3\mathbf{T}_1+k_{i,2}\mathbf{T}_3\mathbf{T}_2 \\ &= k_{i,1}\mathbf{OT}_1+k_{i,2}\mathbf{OT}_2+(1-k_{i,1}-k_{i,2})\mathbf{OT}_3\end{aligned} \tag{3}$$

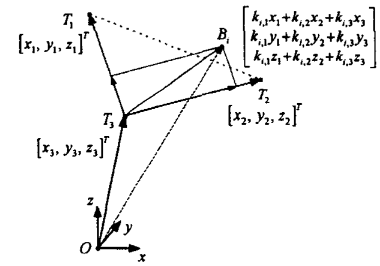


Fig. 2. Representing the coordinates of the platform joints in the generalized coordinates.

The coefficients $k_{i,j}$ (i= 1,2,...,6; j=1,2,3) in Eq. (3) are actually functions of the geometry of the mobile platform joints (B_i points) and the preselected three-points (T_j points). If the global vectors are transformed to the local moving reference frame, Eq. (3) can be written as

$$\mathbf{B}_i'=k_{i,1}\mathbf{T}_1'+k_{i,2}\mathbf{T}_2'+(1-k_{i,1}-k_{i,2})\mathbf{T}_3' \tag{4}$$

where $\mathbf{B}_i'$ and $\mathbf{T}_j'$ points are (2×1) constant vectors with x' and y' coordinates in the reference frame fixed on the mobile platform. Rewriting Eq. (4) gives

$$\mathbf{B}_i'-\mathbf{T}_3'=k_{i,1}(\mathbf{T}_1'-\mathbf{T}_3')+k_{i,2}(\mathbf{T}_2'-\mathbf{T}_3')\quad i=1,2,\ldots,6 \tag{5}$$

Then, for each i, the two unknowns ($k_{i,1}$ and $k_{i,2}$) in Eq. (5) can be obtained in terms of constant $\mathbf{B}_i'$ and $\mathbf{T}_j'$ coordinates.

The inverse kinematic relationship from the motion of the moving platform to actuator length can easily be derived as

$$\begin{aligned}\lambda_i\mathbf{n}_i &= k_{i,1}\mathbf{OT}_1+k_{i,2}\mathbf{OT}_2+k_{i,1}\mathbf{OT}_3-\mathbf{OA}_i \\ &= k_{i,1}\mathbf{t}_1+k_{i,2}\mathbf{t}_2+k_{i,1}\mathbf{t}_3-\mathbf{a}_i\end{aligned} \tag{6}$$

where λ_i is the magnitude of the actuating length and $\mathbf{n}_i$ is a unit vector along the actuation. In addition, because the three points on the moving platform are constrained on a rigid body, the following three constraint equations are derived as

$$
\begin{aligned}
k_1^{\,2} &= (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \\
k_2^{\,2} &= (x_2 - x_3)^2 + (y_2 - y_3)^2 + (z_2 - z_3)^2 \\
k_3^{\,2} &= (x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2
\end{aligned}
\tag{7}
$$

Time differentiation of Eq. (6) and (7) gives

$$\dot{\lambda}_i \mathbf{n}_i + \lambda_i \dot{\mathbf{n}}_i = k_{i,1}\dot{\mathbf{t}}_1 + k_{i,2}\dot{\mathbf{t}}_2 + k_{i,3}\dot{\mathbf{t}}_3 \tag{8}$$

where $\dot{\mathbf{t}}_i = [\ \dot{x}_i, \dot{y}_i, \dot{z}_i\ ]^T$ and

$$
\begin{aligned}
\dot{k}_1 &= \frac{(x_1 - x_2)}{k_1}\dot{x}_1 + \frac{(y_1 - y_2)}{k_1}\dot{y}_1 + \frac{(z_1 - z_2)}{k_1}\dot{z}_1 - \frac{(x_1 - x_2)}{k_1}\dot{x}_2 - \frac{(y_1 - y_2)}{k_1}\dot{y}_2 - \frac{(z_1 - z_2)}{k_1}\dot{z}_2 = 0 \\
\dot{k}_2 &= \frac{(x_2 - x_3)}{k_2}\dot{x}_2 + \frac{(y_2 - y_3)}{k_2}\dot{y}_2 + \frac{(z_2 - z_3)}{k_2}\dot{z}_2 - \frac{(x_2 - x_3)}{k_2}\dot{x}_3 - \frac{(y_2 - y_3)}{k_2}\dot{y}_3 - \frac{(z_2 - z_3)}{k_2}\dot{z}_3 = 0 \\
\dot{k}_3 &= \frac{(x_3 - x_1)}{k_3}\dot{x}_3 + \frac{(y_3 - y_1)}{k_3}\dot{y}_3 + \frac{(z_3 - z_1)}{k_3}\dot{z}_3 - \frac{(x_3 - x_1)}{k_3}\dot{x}_1 - \frac{(y_3 - y_3)}{k_3}\dot{y}_1 - \frac{(z_3 - z_1)}{k_3}\dot{z}_1 = 0
\end{aligned}
\tag{9}
$$

Since $\mathbf{n}_i$ is a unit vector, $\mathbf{n}_i^T \mathbf{n}_i = 1$ and $\mathbf{n}_i^T \dot{\mathbf{n}}_i = 0$. Therefore, multiplication of $\mathbf{n}_i^T$ with Eq.(8) gives

$$\dot{\lambda}_i = k_{i,1}\mathbf{n}_i^T \dot{\mathbf{t}}_1 + k_{i,2}\mathbf{n}_i^T \dot{\mathbf{t}}_2 + k_{i,3}\mathbf{n}_i^T \dot{\mathbf{t}}_3 \tag{10}$$

Equations (9) and (10) are combined into a matrix form as

$$\dot{\mathbf{\Lambda}} = \mathbf{M}\dot{\mathbf{\Pi}} \tag{11}$$

where $\dot{\mathbf{\Lambda}} = \left[\dot{\lambda}_1, \dot{\lambda}_2, \dot{\lambda}_3, \dot{\lambda}_4, \dot{\lambda}_5, \dot{\lambda}_6, 0, 0, 0\right]^T$ (12)

$$\dot{\mathbf{\Pi}} = [\dot{x}_1 \quad \dot{y}_1 \quad \dot{z}_1 \quad \dot{x}_2 \quad \dot{y}_2 \quad \dot{z}_2 \quad \dot{x}_3 \quad \dot{y}_3 \quad \dot{z}_3]^{\mathrm{T}} \tag{14}$$

and where unit vectors are defined as

$$\mathbf{n}_{i,x} = \frac{B_{i,x} - A_{i,x}}{\left\|B_{i,x} - A_{i,x}\right\|} \quad \mathbf{n}_{i,y} = \frac{B_{i,y} - A_{i,y}}{\left\|B_{i,y} - A_{i,y}\right\|} \quad \mathbf{n}_{i,z} = \frac{B_{i,z}}{\left\|B_{i,z}\right\|}$$

Note that all elements in the new *(9×9)* Jacobian matrix have non-dimensionalized unit because $k_{i,j}$ and unit vectors are dimensionless and because k_i (i=1,2,3) have the dimension of length. Note that k_i (i=1,2,3) must be kept in the **M** matrix for nondimensionalization even though they can not be zero.

III. Optimal Design of the Gough-Stewart Platform Manipulator

This section presents application of the proposed Jacobian formulation to optimal design of the Gough-Stewart platform manipulator configuration (architecture and pose).

A. Optimum Design Formulation

A 6-DOF Gough-Stewart platform manipulator consists of a mobile platform connected by six links to a base through spherical and universal joints. The base joints are fixed on the base while the links are of variable length. Changing the link lengths controls the position and orientation (the pose) of the mobile platform(see Fig. 3).

$$
\mathbf{M} = \begin{bmatrix}
k_{1,1}n_{1,x} & k_{1,1}n_{1,y} & k_{1,1}n_{1,z} & k_{1,2}n_{1,x} & k_{1,2}n_{1,y} & k_{1,2}n_{1,z} & k_{1,3}n_{1,x} & k_{1,3}n_{1,y} & k_{1,3}n_{1,z} \\
k_{2,1}n_{2,x} & k_{2,1}n_{2,y} & k_{2,1}n_{2,z} & k_{2,2}n_{2,x} & k_{2,2}n_{2,y} & k_{2,2}n_{2,z} & k_{2,3}n_{2,x} & k_{2,3}n_{2,y} & k_{2,3}n_{2,z} \\
k_{3,1}n_{3,x} & k_{3,1}n_{3,y} & k_{3,1}n_{3,z} & k_{3,2}n_{3,x} & k_{3,2}n_{3,y} & k_{3,2}n_{3,z} & k_{3,3}n_{3,x} & k_{3,3}n_{3,y} & k_{3,3}n_{3,z} \\
k_{4,1}n_{4,x} & k_{4,1}n_{4,y} & k_{4,1}n_{4,z} & k_{4,2}n_{4,x} & k_{4,2}n_{4,y} & k_{4,2}n_{4,z} & k_{4,3}n_{4,x} & k_{4,3}n_{4,y} & k_{4,3}n_{4,z} \\
k_{5,1}n_{5,x} & k_{5,1}n_{5,y} & k_{5,1}n_{5,z} & k_{5,2}n_{5,x} & k_{5,2}n_{5,y} & k_{5,2}n_{5,z} & k_{5,3}n_{5,x} & k_{5,3}n_{5,y} & k_{5,3}n_{5,z} \\
k_{6,1}n_{6,x} & k_{6,1}n_{6,y} & k_{6,1}n_{6,z} & k_{6,2}n_{6,x} & k_{6,2}n_{6,y} & k_{6,2}n_{6,z} & k_{6,3}n_{6,x} & k_{6,3}n_{6,y} & k_{6,3}n_{6,z} \\
\frac{(x_1 - x_2)}{k_1} & \frac{(y_1 - y_2)}{k_1} & \frac{(z_1 - z_2)}{k_1} & \frac{-(x_1 - x_2)}{k_1} & \frac{-(y_1 - y_2)}{k_1} & \frac{-(z_1 - z_2)}{k_1} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{(x_2 - x_3)}{k_2} & \frac{(y_2 - y_3)}{k_2} & \frac{(z_2 - z_3)}{k_2} & \frac{-(x_2 - x_3)}{k_2} & \frac{-(y_2 - y_3)}{k_2} & \frac{-(z_2 - z_3)}{k_2} \\
\frac{-(x_3 - x_1)}{k_3} & \frac{-(y_3 - y_1)}{k_3} & \frac{-(z_3 - z_1)}{k_3} & 0 & 0 & 0 & \frac{(x_3 - x_1)}{k_3} & \frac{(y_3 - y_1)}{k_3} & \frac{(z_3 - z_1)}{k_3}
\end{bmatrix}
\tag{13}
$$

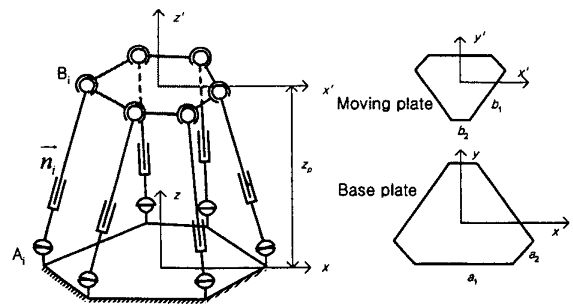


Fig. 3. Neutral pose and simplified architecture variables

In order to simplify the presentation of the essential idea of the proposed optimal design formulation, we limit our design to a simplified manipulator architecture whose base and moving plates are hexagons with three equal alternating long sides and three equal alternating short sides. The architecture of such a manipulator can then be fully defined by parameters a_1, a_2, b_1, b_2. Meanwhile, the pose of a manipulator is changing over the workspace, which means that the optimum architecture design must be sought over the entire workspace. Angeles [9] numerically monitored a number of poses over the workspace to find that neutral poses give a smaller condition number than other poses. He defined a neutral pose as a pose that has zero orientation and a center position ($[0, 0, z_p]^T$). At the neutral poses, the orientation of moving plate coincides with that of the base plate, as shown in Fig. 3. Note that, by definition, there are many neutral poses for a given manipulator because z_p can vary. Angeles [9] commented that the neutral poses might admit a minimum condition number and hence, might be the most kinematically stable configurations. Thus, the optimum design problem is to find architecture variables (a_1, a_2, b_1, b_2) and the pose height variable (z_p) such that the condition number of the corresponding Jacobian matrix is minimum.

Now, we can reduce our search space for optimization to neutral configurations only. Thus, an optimum design problem can be formulated as

Minimize the condition number

$$\kappa(\mathbf{M}(\mathbf{b})) \equiv \| \mathbf{M} \| \, \| \mathbf{M}^{-1} \| \tag{15}$$

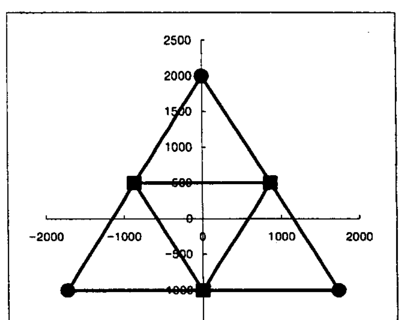
Subject to $a_1 \geq 0,\ a_2 \geq 0,\ b_1 \geq 0,\ b_2 \geq 0,\ z_p \geq 0.$ (16)

where $\| \bullet \|$ denotes the Euclidean norm of a matrix. The design variables are parameters of moving plate (b_1, b_2), base plate (a_1, a_2) and distance of them (z_p). The physical meaning of the first four constraints is to guarantee nonnegative architecture dimensions, the last inequality being to avoid the singular configuration at which both plates are coincident. The problem defined in Eq. (15-16) is highly nonlinear constrained optimization problem.

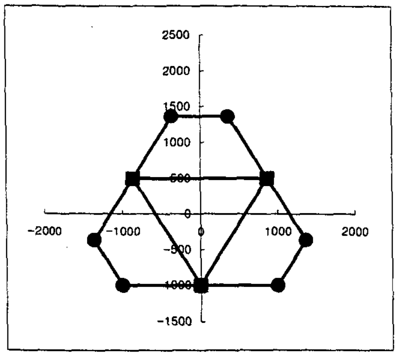
B. Numerical Examples

This section presents a numerical example of optimal design of the Gough-Stewart Platform by minimizing the condition number of new dimensionally homogeneous Jacobian matrix in order to investigate the validity of a new Jacobian formulation by use of three point generalized coordinates. Notice that since analytic derivatives of singular values with respect to the design variables are not available, the problem has to be solved using a numerical method. We used MATLAB constrained nonlinear optimizer '*constr*' function. This function uses a Sequential Quadratic Programming (SQP), in which a Quadratic Programming(QP) subproblem is solved at each iteration. An estimate of the Hessian of the Lagrangian is updated at each iteration using BFGS formula [20].

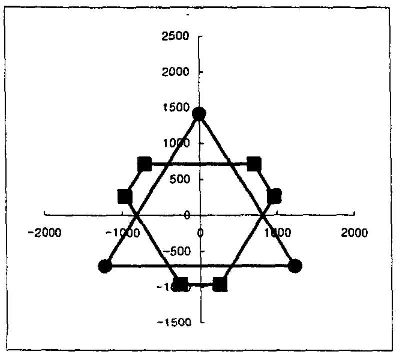
Depending on the initial guess of the design variables, several different local optimal configurations have been found. Three configurations typifying the family of optimal configurations are shown in Fig. 4. Case (a) shows an optimum configuration with two triangular plates, the base plate being twice as large as the moving plate. The other optimum configurations have a triangular moving plate with hexagonal base (case (b)) and have a hexagonal moving plate with a triangular base (case (c)). Notice that optimal configurations are functions of a pose variable(z_p) as well as architecture variables(a_1, a_2, b_1, b_2).



(a) $a_1 = 3.464,\ a_2 = 0,\ b_1 = 1.732,\ b_2 = 0,\ z_p = 1.732$



(b) $a_1 = 2,\ a_2 = 0.7321,\ b_1 = 1.7321,\ b_2 = 0,\ z_p = 1.$



(c) $a_1 = 2.4495,\ a_2 = 0,\ b_1 = 1.414,\ b_2 = 0.518,\ z_p = 1.$

Fig. 4. Optimal configurations (top views)

These local optimums are the same as those of the Pittens and Podhorodeski [10] and include a solution(case (a)) that was found by Angeles [9] and Kim [19]. Notice that the condition number values of the dimensionally nonhomogeneous (*6x6*) Jacobian matrix[9] are all the same(i.e. $\kappa = \sqrt{2}$) for all three case. These results show that the new Jacobian formulation by use of three point generalized coordinates can be used as another way of optimum design of parallel manipulators, which eliminates the unit mismatch problem in the Jacobian matrix. Note that changing units from meter to millimeter does not change the optimum results because of dimensional homogeneity in the new Jacobian. Note also that the new dimensionally homogeneous Jacobian formulation has a different dimensional scaling method from the characteristic length [15] or natural length [9].

We investigated other cost functions in order to see if we have different optimum configurations. The cost functions that have been considered are:

(i) Determinant of new Jacobian matrix

$$\text{Maximize } f_{obj} = \prod_{i=1}^{9} \sigma_i$$

(ii) Average singular value of new Jacobian matrix

$$\text{Maximize } f_{obj} = \frac{1}{9}\sum_{i=1}^{9} \sigma_i$$

(iii) Minimum singular value of new Jacobian matrix

$$\text{Maximize } f_{obj} = \sigma_{min}$$

where σ_i's are singular values of the **M** matrix.

Even though we used different cost functions, the optimum configurations are the same as in Fig. 4. This result is as expected [10].

IV. Conclusions and Discussions

By using a new Jacobian formulation based on the three non-collinear points of a mobile platform of a parallel manipulator, dimensionally homogeneous (*9x9*) Jacobian matrix is derived. Optimum configurations of the Gough-Stewart platform manipulator have been found by using local dexterity of the condition number based on the proposed Jacobian. The optimum configurations thus obtained are the same as previous designs. In addition, the same family of configurations were found to be optimal for other local performance criteria.

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