Concept-based Evolutionary Exploration of Design Spaces by a Resolution-Relaxation-Pareto Approach

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***Abstract*— The set-based concept approach is suggested as a means to simultaneously explore different design spaces at both the conceptual and the particular design levels. The proposed technique employs dynamic relaxation of concept-based Pareto-optimality. Two types of exploration problems are defined. The first type aims to reveal the approximated concepts' fronts, within a relaxation zone, whereas the second type is about which of the concepts' front intersect with that zone. A unique benchmarking approach and measures are employed to test the suggested algorithm with respect to the aforementioned types of problems. The numerical study shows that the algorithm can cope with various numerical difficulties in a simultaneous way. The use of exploration tools, such as proposed here, is expected to enhance the understanding of conceptual design spaces by both experienced and novice designers.**

Keywords*— *conceptual design; computer-supported design; evolutionary multi-objective optimization; set-based concept; design space exploration; benchmark

I. Introduction

The term design space exploration has been used synonymously with searching for Pareto-optimal solutions (e.g. [1]). Here, the term is used in a much wider scope. Namely, in contrast to the motivation of studies such as [1], the motivation for the proposed search approach is not optimization but obtaining some general knowledge about the design space (as detailed in section II A). Our approach to design space exploration involves pre-defined design concepts that are used to explore the design space both at the level of the concepts and at the level of associated particular designs. This is achieved by the set-based concept approach. In this approach, a design concept (in short – a concept) is pre-defined by the designers as a set of potential solution alternatives, which possess some common features [2]. Such a representation has been termed Set-Based Concept (SBC). In contrast to the traditional way of evaluating concepts, the SBC approach allows concept evaluation to be based not only on optimality considerations, but also on performance variability, which is inherent to the SBC representation [3].

Fig. 1 illustrates the SBC approach. Three concepts of aircrafts are shown. The (generally different) design spaces of the concepts are marked by ellipses of different gray levels.

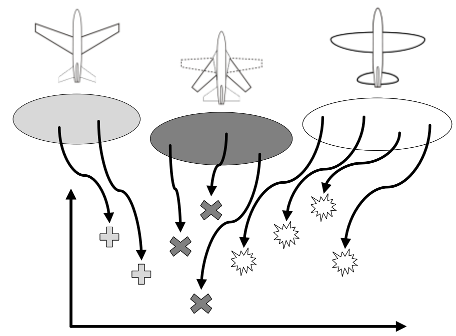

Fig. 1. Illustration of the SBC approach

The associated performance vectors of particular designs, from all concepts, are to be compared in a mutual objective space. The most studied SBC approach is known as the s-Pareto approach [4]. It involves finding which particular designs, of which concepts, are associated with the Pareto-front that is obtained by domination comparisons among all individual designs from all concepts.

In principle, existing multi-objective evolutionary algorithms can be easily used to find the front per each concept, and consequently a sorting procedure may be used to find the s-Pareto front and optimal set. However, such algorithms could also be tailored to simultaneously search all the concepts' spaces to find the same information. For example, NSGA-II, [5], served as a base for finding s-Pareto in [6], and ε-MOEA [7] was tailored for that purpose in [8]. The reader is referred to [6] and [8] for discussions on why the simultaneous search approach should be preferred over independent concept searches, and why tailoring of the algorithms is needed. Although these discussions are done in the context of an s-Pareto search, they are valid for the current study.

In [3], it was suggested that concepts should not be selected by the s-Pareto approach. The alternative of a concept-based relaxed-Pareto approach was first suggested in [9], and was developed into an evolutionary algorithm in [10], using relative (dynamic) relaxation. As detailed below, this paper follows the above studies, but extends the application of the SBC approach from concept selection to design space exploration.

The contribution of this paper is threefold including: a. the proposed general approach to design space exploration, b. the suggested search algorithm, and c. the proposed method to assess the algorithm. In contrast to previous studies that suggest the SBC approach for concept selection, this study highlights its possible utilization for design space exploration. In particular, here we suggest employing SBC representations and the concept-based relaxed-Pareto approach, as a means to explore design spaces. The difference between search for selection and exploration is elaborated in the next section on the methodology. The proposed exploration algorithm is primarily based on two past studies on the SBC approach. The first is the study in [10], which combined the use of a relative SBC relaxation approach with NSGA-II, to produce *Cr1-NSGA-II*. The second is the study in [8], which tailored ε-MOEA to the SBC approach to produce C-ε-MOEA. In [8], the phenomenon of premature concept convergence was studied using a unique benchmarking approach. It was demonstrated in [8] that while the tailored ε-MOEA (C-ε-MOEA) may cope with that phenomenon, the tailored NSGA-II, of [6], does not. Here, we adapt some ideas from [8] and [10], including the ε-resolution of C-ε-MOEA, and the relative relaxation of *Cr1-NSGA-II*. However, the current study substantially differs from works such as the above both by the use of a search based on mutation only, rather than using both crossover and mutations, and by aiming at exploration rather than selection. Two types of exploration problems are defined and the proposed algorithm is shown to handle both types. The algorithm testing is based on a complex benchmarking example involving a simultaneous search with ten test-function-based concepts, with each exhibiting a different kind of a numerical difficulty.

The rest of this paper is organized as follows. In section II the foundations for the search methodology are described and in section III, the pseudo-code of the proposed search procedure is presented. Section IV provides an explanation on the benchmarking example and the used measures, as well as the description and analysis of the results. Finally, section V outlines the conclusions of this paper.

II. METHODOLOGY

A. *Concept-based Design Space Exploration*

This paper proposes to use the SBC approach for Design Space Exploration using computer-based models of the design space. The proposed method is herby termed *Concept-based Design Space Exploration* (C-DSE). The primary difference between using the SBC approach for such a motivation, and using it for concept selection, is in the definition of the sought information, as further discussed below.

Fig. 2, which is adapted from [10], helps illustrating a possible motivation for C-DSE. It shows the individual Pareto-fronts of four concepts, which are designated by •,▫,⅄, and ▪ for the 1st, 2nd, 3rd, and 4th concept respectively. Using the s-Pareto approach would result in a "combined front" of concept 1 and 2 including the front of the 2nd concept and most of the front of the 1st concept. However, as claimed in [3], one should not ignore the front of the 3rd concept, since that its pair-wise comparisons, with the front of the 1st concept and independently with that of the 2nd concept, are not conclusive. One may declare that the 3rd concept is neither dominated by the 1st nor by the 2nd concept.

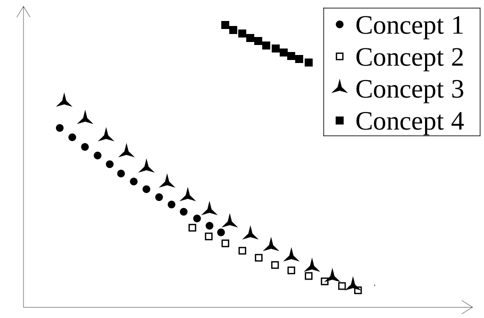

Fig. 2. Concepts' Fronts in Selection and in Exploration

On the other hand, the 4th concept is dominated by any of the others, therefore it may be viewed as of no relevance for selection.

One may conclude from the above discussion, that when the search aims at finding concepts for selection, the decision makers should be presented with the fronts of the 1st, 2nd and 3rd concept. Nevertheless, for C-DSE one may want to find also the front of the 4th concept. When the aim of the search is C-DSE, in general one would desire to unveil important information, which is not necessarily restricted to a particular selection problem. In general C-DSE can be used for a general extraction of meaningful conceptual knowledge about design spaces. This is explained in the following.

In principle, any multi-objective design optimization problem can be transformed into a C-DSE, with a very large number of meaningful concepts. To do that, one can divide the parameter space into subspaces using meaningful divisions of the parameters. As an example, consider a beam bending case, which involves continues parameters. The set of all feasible values of the beam's "length" parameter can be divided into three crisp sub-sets (sub-ranges) including "short-length," "medium-length" and "long-length." Such divisions can be amended by divisions according to discrete parameters. For example, if the beam material has to be selected from 5 different materials, these materials, inherently constitute meaningful divisions. Next, meaningful concepts can be constructed using such divisions. The concept of a long steel beam and the concept of a short aluminum beam are two such examples. Now, consider a bi-objective problem of minimum weight and minimum deflection for the design of a cantilever beam. One may think of space exploration, in which all the meaningful beam concepts are to be explored, with the ultimate goal of finding the fronts of all the involved concepts (see problem definition in subsection IIB). The advantage of such a complete C-DSE approach is that the results can be transformed into meaningful design rules, such as *"concept A is better than concept B."* Such a rule can be extracted when the Pareto-optimal set of concept A fully dominates the Pareto-optimal set of concept B. Rules that are more complex can be extracted in less conclusive situations, involving particular designs. For example, *"concept A has particular designs that are better than most of the particular designs of concept B."* Obtaining such design rules could be significant to both experienced and novice engineers. In the former case, the C-DSE may reassure or dispute the engineers' knowledge, whereas in the latter case it may help building the knowledge of the un-experienced engineers.

With the use of a meaningful division procedure, thousands of meaningful concepts may be constructed per real-life engineering problems. In general, an independent search, of the front of each concept, could be computationally prohibitive with an increasing number of concepts. In such a case, a compromise must be made to perform a restricted version of the complete C-DSE, based on the available computational resources. The alternative, which is considered here, is that of a simultaneous search that allows the designer to restrict the search, in order to save computational resources. It provides the designer with a means to address the tradeoff between the (possibly unachievable) desire to unveil accurate representation of the fronts of all concepts and the desire to achieve meaningful information within the restriction imposed by the available computational resources.

B. *Types of Concept-based Exploration Problems*

We propose to consider two types of C-DSE problems. The first aims at finding (good approximation of) the concepts' fronts within a relaxation zone in the objective space. The second type of a C-DSE problem does not aim to find the concepts' fronts. It has a more limited scope when compared with the first type. In the second problem type, the goal is to reveal which of the concepts' front intersect with the relaxation zone. The above descriptions involve the term relaxation zone, which is based on a relaxation distance from the s-Pareto front. The following provides a definition to the first problem type.

Let n_o be the dimension of the objective-space $\mathbb{R}^{n_o}$ and n_c be the number of concepts. Let $X_m \subseteq \mathbb{R}^{n_m}$ be the design-space of the m-th concept, and $f_m : X_m \to \mathbb{R}^{n_o}$ is the concept's objective-function. Let n_m be the dimension of X_m . Furthermore, let s be any particular design and let m_s and x_s represent the concept index and the design vector of s , respectively. Also, $x_{s,j}$ $(j = 1,\dots,n_m)$ represents the j-th element of x_s, and $y_s = f_{m_s}[x_s]$ represents the performance vector of s , with $y_{s,i}$ $(i = 1,\dots,n_o)$ representing the i-th element of y_s.

Without loss of generality, the *complete C-DSE*, which is based on a Pareto-approach, is hereby defined as finding all the feasible Pareto-optimal solutions and front, for each problem of the following n_c independent problems:

$$\min f_m[x] \quad \text{for} \quad m = 1,\dots,n_c \tag{1}$$

On the other hand, the s-Pareto search problem is to find the non-dominated set of the union of solutions from all the above problems. Let P^* be the set of all feasible particular designs from all concepts, then the s-Pareto set is defined as follows:

$$G^* = \left\{ s \in P^* \mid \nexists s' \in P^* : \quad y_{s'} \succ y_s \right\} \tag{2}$$

Where $y' \succ y$ stands for y' dominates y.

Let F^* be the union of all the solutions of the *complete C-DSE* problem (see eq. 1). Then the first C-DSE problem type, as described above, is to find the concept-based relaxed Pareto-optimal set, which is defined without loss of generality, for a minimization problem, as follows:

$$R^* = \left\{ s \in F^* \mid \exists s' \in G^* : \quad y_s \succ y_{s'} + r \right\} \tag{3}$$

where vector $r = \left(r_1,\dots,r_{n_o}\right) \in \mathbb{R}_+^{n_o}$ is the vector of relaxation. The second C-DSE problem type simply amounts to finding at least one solution from each concept that takes a part in R^*.

III. Concept-based Evolutionary Exploration

This section starts, in III.A, with several mathematical definitions that are used in the description of the algorithm. Next, in III.B, measures are defined concerning the utilization of the computational resources and the termination condition for the algorithm. These are followed, in III.C and III.D, by a pseudo-code description of the proposed evolutionary search method.

A. *Definition of sets used by the algorithm*

The Concept Population History (P_m), is the set of all evaluated designs from all past and present iterations, which are associated with concept m.

The Population History (P) is the union set of the combined population history from all the examined concepts:

$$P = \bigcup_{m=1}^{n_c} P_m \tag{4}$$

The Concept Non-dominated Set $\left(F_m\right)$, is the current set of all non-dominated designs of P_m :

$$F_m = \left\{ s \in P_m \mid \nexists s' \in P_m : \quad y_{s'} \succ y_s \right\} \quad (m=1,\dots,n_c) \tag{5}$$

The Integrated Non-dominated Sets (F) is the union set of all the current non-dominated sets from all the examined concepts:

$$F = \bigcup_{m=1}^{n_c} F_m \tag{6}$$

The Global Non-dominated Set (G), is the set of all non-dominated designs of P :

$$G = \left\{ s \in P \mid \nexists s' \in P: \quad y_{s'} \succ y_s \right\} \tag{7}$$

If converged, then G is an approximation of the s-Pareto set.

The Relaxed Non-dominated Set (*R*), contains G and all the designs of *F*, which satisfy the proximity condition:

$$R = \left\{ s \in F \mid \exists s' \in G: \quad y_s \succ y_{s'} + r \right\} \tag{8}$$

where vector $r = \left(r_1, ..., r_{n_o}\right) \in \mathbb{R}_+^{n_o}$ is the vector of relaxation. Only designs that are included in *R* might be selected for mutation. If converged, then *R* is an approximation of the concept-based relaxed Pareto-optimal set.

The Concept ε-Cells Set (M_m), is a set of all sub-sets of $R \bigcap P_m$ that are obtained by the ε-Sorting procedure, as follows.

Let vector $\varepsilon = \left(\varepsilon_1, ..., \varepsilon_{n_o}\right) \in \mathbb{R}_+^{n_o}$ and a related function $\mathrm{SortKey}_\varepsilon : \mathbb{R}^{n_o} \to \mathbb{Z}^{n_o+1}$ be used to define a lattice in the objective-space. The $\mathrm{SortKey}_\varepsilon$ is defined as follows:

$$\mathrm{SortKey}_\varepsilon\left[y\right] = \left(\left\lfloor \lambda \right\rfloor, \left\lfloor \frac{y_1}{\varepsilon_1} \right\rfloor, \left\lfloor \frac{y_2}{\varepsilon_2} \right\rfloor, \cdots, \left\lfloor \frac{y_{n_o}}{\varepsilon_{n_o}} \right\rfloor \right) \tag{9}$$

$$\text{Where :} \quad \lambda = \sum_{i=1}^{n_o} \frac{y_i}{\varepsilon_i}$$

The lattice-cell T_I is defined as the set of vectors, in the objective space, with the same SortKey (*I*):

$$T_I = \left\{ y \in \mathbb{R}^{n_o} \mid \mathrm{SortKey}_\varepsilon\left[y\right] = I \right\} \tag{10}$$

The performance vectors of the designs of the set $E_{m,I} \subseteq R \bigcap P_m$ share a common cell in the objective-space, i.e.:

$$E_{m,I} = \left\{ s \in R \bigcap P_m \mid y_s \in T_I \right\} \tag{11}$$

where *I* is the $\mathrm{SortKey}_\varepsilon$ value of $E_{m,I}$. The sets $E_{m,I}$ are hereby termed ε-Cells. As stated above, M_m is the set of all the non-empty ε-Cells $E_{m,I}$ of the m-th concept.

Integrated Concept ε-Cells Set (*M*) contains ε-Cells from all concepts:

$$M = \bigcup_{m=1}^{n_c} M_m \tag{12}$$

The Set of Selectable ε-Cells (*A*), contains all ε-Cells of *M* that meet the criteria $\nu_{m,I} < 1$, where $\nu_{m,I}$ is the ε-Cell resource utilization, as defined below.

B. *Measures of resource utilization*

The following measures of resource utilization are defined to support balancing the computational resources among the various concepts, and to provide a mean for the search termination condition.

Several measures of resource utilization are defined as follow:

- The resource utilization of the $E_{m,I}$ ε-Cell:

$$\nu_{m,I} = \frac{\mathrm{SelectionCount}\left[E_{m,I}\right]}{SelectionLimit} \tag{13}$$

  Where, the selection count is defined as the number of times any design *s* from the ε-Cell has been selected for mutation (during the evolutionary process). The selection count is limited with a predefined value (*SelectionLimit*).

- The resource utilization of the *m*-th concept:

$$\nu_m = \frac{1}{\left|M_m\right|} \cdot \sum_{E_{m,I} \in M_m} \nu_{m,I} \tag{14}$$

- The resource utilization of all concepts:

$$\nu = \frac{1}{\left|M\right|} \cdot \sum_{E_{m,I} \in M} \nu_{m,I} \tag{15}$$

C. *Main Procedure*

1 Perform *Initialization Procedure* (see subsection III.D.1).

2 While $A \neq \varnothing$:
(where *A* is defined at the end of subsection III.A)

  2.1 Select one ε-Cell ($E_{m,I}$) from set *A* according to *Cell Selection Procedure* (see subsection III.D.3).

  2.2 Randomly select one design *s* from $E_{m,I}$.

  2.3 Perform *Element Mutation* (see subsection III.D.5) on *s* to produce a new design *s′*.

  2.4 Evaluate *s′* and update all the sets that are defined in subsection III.A.

2.5 Update relaxation vector (r) according to *Relaxation Update Procedure* (see subsection III.D.4).

D. Sub-procedures

1) Initialization Procedure

1 Set the SelectionLimit.

2 Set ε and the final relaxation vector $\left(r_{\text{Final}} \in \mathbb{R}_{+}^{m}\right)$.

3 Randomly initialize a population of designs with equally sized subpopulations for each concept.

4 Evaluate each design.

5 Initialize all the sets defined in section III.A using infinite relaxation ($r = \infty$).

6 Initialize the relaxation vector (r) according to *Relaxation Initialization Procedure* (see below).

7 Set the initial vector of relaxation: $r_{\text{Initial}} = r$

2) Relaxation Initialization Procedure

1 Obtain the minimal and maximal values of each objective regarding all designs in *F*:

$$\begin{cases} y_i^{\max} = \max\left\{ y_{s,i} \mid s \in F \right\} & \left(i=1,\dots,n_{\text{o}}\right) \\ y_i^{\min} = \min\left\{ y_{s,i} \mid s \in F \right\} & \left(i=1,\dots,n_{\text{o}}\right) \end{cases} \tag{16}$$

2 Obtain the relaxation value for each objective as follows:

$$r_i = \max\left\{ y_i^{\max} - y_i^{\min}, r_{\text{Final},i} \right\} \quad \left(i=1,\dots,n_{\text{o}}\right) \tag{17}$$

3) Cell Selection Procedure

To ensure a fair selection, this procedure takes into consideration the resource utilization (see section III.B) of every concept and every selectable ε-Cell (from set *A*). The proposed rule is that the selected ε-Cell will be the ε-Cell with the lowest resource utilization from the concept with the lowest resource utilization.

Cell selection steps:

1 Select the concept *m* with the lowest resource utilization (v_m). If there is more than one concept with the same lowest resource utilization, then randomly select one of these concepts.

2 From the selected concept *m*, select the ε-Cell $E_{m,I}$ with the lowest resource utilization ($v_{m,I}$). If there is more than one ε-Cell with the same lowest resource utilization, then randomly select one of these ε-Cells.

4) Relaxation Update Procedure

The relaxation decay profile aims to retain wide relaxation at early stages of the evolutionary process, and then decay on a linear slope until the final relaxation value is achieved at an advanced stage of the evolutionary process. The profile also ensures that this final value is retained until the end of the evolutionary process.

This procedure updates the relaxation vector according to a predefined relaxation decay profile:

$$\begin{cases} r = \rho \cdot r_{\text{Final}} + \left(1-\rho\right) \cdot r_{\text{Initial}} \\ \rho = \max\left[0, \min\left[1, \alpha \cdot \nu - \beta\right]\right] \end{cases} \tag{18}$$

Where ν is the overall resource utilization (see subsection III.B), r_{Initial} and r_{Final} are the initial vector of relaxation and the final predefined vector of relaxation respectively. The coefficients α and β determine the steepness and saturation of the profile, respectively. We used the constant values: $\alpha = 1.5$ and $\beta = 0.3$ in the current study. It is noted that investigating the effect of changing these values is left for future research.

5) Element Mutation

This operator produces a new offspring design s' based on a parent design s. $x_{s'}$ will be identical to x_s except of one design parameter, which is randomly selected to be mutated.

We assume the design space of each concept is bounded and divided into hyper-rectangles, which are defined according to a desired resolution. Let $x_{m,j}^{\min}$, $x_{m,j}^{\max}$ and $N_{m,j}$ be the lower limit, the upper limit and the number of resolution intervals for the *j*-th component of the design vector of the *m*-th concept, respectively. The mutation is performed as follows:

1 Initialized $x_{s'}$ to be identical to x_s.

2 Randomly select one design parameter $j \in 1,\dots,n_m$ and a direction indicator $t \in \left\{+1,-1\right\}$.

3 Calculate the number of resolution intervals as available for the mutation step for the selected parameter and direction as follows:

$$\Delta = \begin{cases} \dfrac{x_{s,j} - x_{m,j}^{\min}}{x_{m,j}^{\max} - x_{m,j}^{\min}} \cdot N_{m,j} & t = -1 \\ \dfrac{x_{m,j}^{\max} - x_{s,j}}{x_{m,j}^{\max} - x_{m,j}^{\min}} \cdot N_{m,j} & t = +1 \end{cases} \tag{19}$$

If $\Delta < 1$ select a different direction and recalculate Δ.

4 Invoke random value drawn from the standard normal distribution Z, and calculate a normalized mutation step as follows:

$$\delta = \min\left[\Delta, 1+\left|Z\right|\right] \tag{20}$$

5 Set the new value of the selected design parameter of s' according to the mutation step:

$$x_{s',j} = x_{s,j} + t \cdot \frac{\delta}{N_{m,j}} \cdot \left(x_{m,j}^{\max} - x_{m,j}^{\min}\right) \tag{21}$$

IV. Experiments and Results

A. Test Problem

The devised test problem involves ten-concepts and a bi-objective minimization problem. Each concept's design-space and objective-function are based on one of the standard test-functions, which are commonly used for the development of multi-objective evolutionary algorithms (e.g., [5]). Some of the functions, such as KUR, have been transformed by translation and scaling in the objective space, as seen in TABLE I. Each transformation is presented, in Table I, as a row vector with the 1st and 2nd components corresponding to the 1st and 2nd objective, respectively.

The transformations are devised such that seven concepts, out of the ten, are included in the concept-based relaxed non-dominated set, with a relaxation vector, and only two are included in the s-Pareto. The number of intervals that are used for each component of the (design) decision vectors is kept constant for each concept, as detailed in the last column of Table I. It should be noted that ZDT3 is positioned here, on purpose, in a way that creates a numerical difficulty by having most of the ZDT3 front outside the relaxation zone.

TABLE I. Concepts' Details

Test Function	n_m	Objectives Scale	Objectives Offset	Design-Space bounds	N_m
FON	3	[1,1]	[0,0]	[−4,4]	40
KUR	3	[0.3,0.3]	[8.0,3.8]	[−5,5]	200
POL	2	[0.2,0.2]	[1,0]	[−π,π]	100
SCH1	1	[1.3,1.0]	[0,0]	$[-10^3,10^3]$	4×10^4
SCH2	1	[0.33,0.33]	[1,0]	[−5,10]	400
ZDT1	30	[1,1]	[1.5,2.0]	[0,1]	11
ZDT2	30	[1,1]	[0.3,0.3]	[0,1]	11
ZDT3	30	[1.6,1.6]	[0.5,2.0]	[0,1]	N_1=101
					$N_{2,\ldots,30}$=11
ZDT4	10	[1,1]	[0,0]	$x_1\in[0,1]$	21
				$x_{2,\ldots,10}\in[-5,5]$	
ZDT6	10	[1,1]	[2,1]	[0,1]	41

B. Performance Measures

We employ several performance measures. To define the measures, the following sets are used:

- The *Reference Pareto-optimal set* of the *m*-th concept (m=1,…,n_c) is denoted as $\hat{F}_m$.
- The *Integrated Reference Set* is the union of all the Reference Pareto-optimal sets of the examined concepts:

$$\hat{F} = \bigcup_{m=1}^{n_c} \hat{F}_m \tag{22}$$

- The *Approximated s-Pareto set*:

$$\hat{G} = \left\{ s \in \hat{F} \mid \nexists s' \in \hat{F}: \quad y_{s'} \succ y_s \right\} \tag{23}$$

- The *Relaxed Pareto-set* of the *Integrated Reference Set*:

$$\hat{R} = \left\{ s \in \hat{F} \mid \exists s' \in \hat{G}: \quad y_s \succ y_{s'} + r \right\} \tag{24}$$

The following two convergence measures are adaptations of the distance metric of [5]. Given a design s of concept m_s, let Υ_s be:

$$\Upsilon_s = \min\left\{ \left\| y_s - y_{s'} \right\| \mid s' \in \hat{F}_{m_s} \right\} \tag{25}$$

Using R , the obtained Relaxed Non-dominated Set, the convergence measure is defined as follows:

$$\Upsilon = \text{mean}\left\{ \Upsilon_s \mid s \in R \right\} \tag{26}$$

In addition, a concept convergence measure is defined. Given a set of designs F_m representing the obtained Concept Non-dominated Set, the concept convergence measure of the *m*-th concept, is defined as follows:

$$\Upsilon_m = \text{mean}\left\{ \Upsilon_s \mid s \in F_m \cap R \right\} \tag{27}$$

In order to assess the diversity of the resulted designs, we suggest the following two percentage-based measures. These are devised to account for the percentage of members of the relevant reference set that are discovered by the algorithm. The first is a global measure as follows:

$$\phi = \frac{\left|\left\{ s \in \hat{R} \mid \exists s' \in F_{m_s} : \left\| y_s - y_{s'} \right\| \le d \right\}\right|}{\left|\hat{R}\right|} \quad (28)$$

The second measure is for each concept as follows:

$$\phi_m = \frac{\left|\left\{ s \in \hat{F}_m \cap \hat{R} \mid \exists s' \in F_m : \left\| y_s - y_{s'} \right\| \le d \right\}\right|}{\left|\hat{F}_m \cap \hat{R}\right|} \quad (29)$$

The following discovery rank measure is devised to account for the amount of concepts that achieve a certain level of discovery within the relaxation zone. It is defined as follows:

$$\alpha_e = \left|\left\{ m \mid \phi_m > e \right\}\right| \quad e \in [0,1) \quad (30)$$

In addition to the above measures, the following analysis uses the total amount of evaluations at termination, as well as the number of evaluations per concept.

C. Results

Fig. 3 shows the obtained fronts when the algorithm is used with *Selection-Limit* = 40, $r = [1,1]$, and $\varepsilon = 0.1$. Using 30 independent runs, Table II shows the average and standard deviations of the results, as obtained for each concept within the relaxation zone, as well as the overall results. As seen from Table II, a reasonable number of evaluations is needed to reveal the seven concepts, within the relaxation zone. The required number of evaluations changes from concept to concept. ZDT3, as positioned here, is substantially more difficult to find than the other ones. First, as seen in Table II, it consumes, on the average, much more evaluations than the other concepts. Second, its obtained diversity is poor in comparison with the other concepts. This is in accordance with the note on ZDT3 following Table I.

Fig. 4 depicts the averaged discovery rank (from 30 runs) as a function of the selection-limit. It is clear that for discovering six out of the seven concepts, of the relaxation zone, it is sufficient to have the selection-limit to be about 20. On the other hand, if the exploration requires finding diversity above 90% , in at least 6 out of the 7 concepts, then it was found (not shown here) that the selection-limit must be increased substantially. Fig. 5 provides details about the relation between the selection-limit and the averaged total number of evaluations at termination. Fig. 6 and 7 provide the averaged convergence and diversity curves, respectively. It can easily be observed that these curves are in accordance with the above findings. Although ZDT3 was found to be, as expected, the most problematic concept, it is interesting to note that its convergence behavior, within the relaxation zone, was found (not shown here) to be similar to the averaged convergence of Fig. 6.

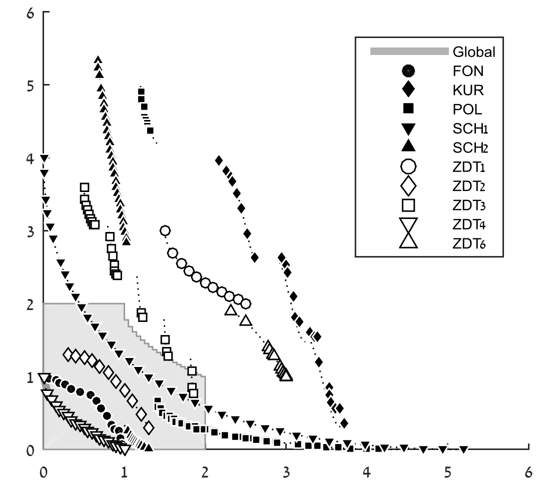

Fig. 3. Overview of Obtained Fronts

TABLE II. SUMMARY OF RESULTS FROM 30 RUNS

Concept	Convergence	Diversity	Evaluations
FON	0.025± 0.007	0.96±0.057	294±25
POL	0.015± 0.004	0.99±0.059	229±15
SCH1	0.017± 0.002	0.95±0.077	217±64
SCH2	0.002± 0	1.00±0.003	142±4
ZDT2	0± 0	0.99±0.010	1,973±267
ZDT3	0± 0	0.33±0.479	3,471±927
ZDT4	0.024± 0.044	0.95±0.046	961±124
Total	**0.017±0.010**	**0.94±0.028**	**11,110±1,050**

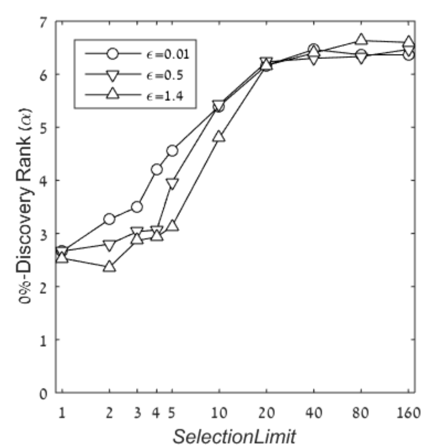

Fig. 4. Discovery Rank (α) vs. Selection-Limit

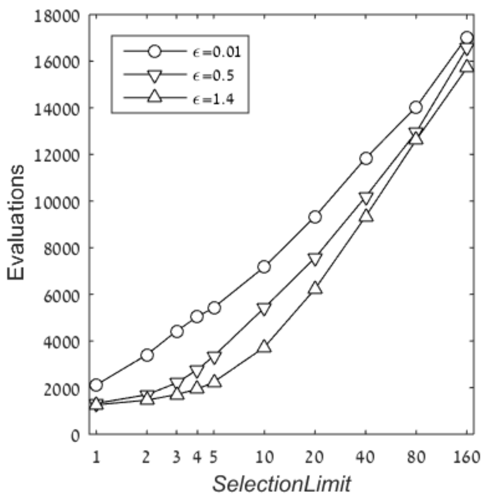

Fig. 5. Evaluations vs. Selection-Limit

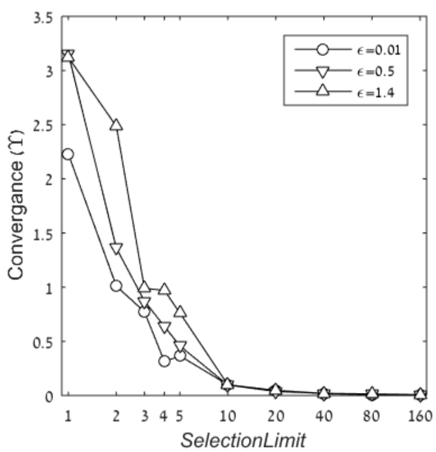

Fig. 6. Convergence vs. Selection-Limit

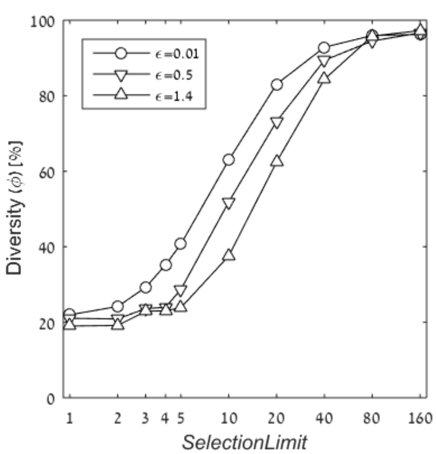

Fig. 7. Diversity vs. Selection-Limit

V. SUMMARY AND CONCLUSIONS

This paper employs a non-traditional set-based concept approach to simultaneously explore different design spaces as associated with design concepts. A relaxation zone is defined in a mutual objective space, and is used to devise two types of exploration problems. The first aims to reveal the approximated concepts' fronts within the relaxation zone, and the second is about which concepts have performances in that zone. A novel search algorithm is proposed and employed on a benchmarking problem, which is specially designed for the development of such algorithms. It is found that the suggested algorithm copes well with the devised problem and that some computational resources can be saved when addressing the second type of problem as compared with the first.

Future studies on the proposed algorithm should attempt to test it on additional benchmark problems, such as with different arrangements of the fronts of the various concepts, changing the relaxation values, and increasing the number of objectives. The proposed approach should also be tested with real-life problems. It is noted that the current algorithm was developed to mainly address the first type of problem, and therefore it is expected that for the second problem type a better algorithm be developed. Potential C-DSE approaches should not be restricted to an automated relaxed-Pareto approach, as done here. Other approaches can be developed. For example, a goal-based approach, a window-of-interest approach, and an interactive, rather than automatic search approach. Such a variety of potential problem definitions and solution approaches could serve to expand the available tools that may support both experienced and novice designers in exploring design spaces.

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