Robot Workspace Optimization Based on a Novel Local and Global Performance Indices

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Robot Workspace Optimization Based on a Novel Local and Global Performance Indices

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Abstract—**In this paper, a novel performance index is introduced for the kinematics optimization of serial robot manipulators. The serial robot manipulators used in order to compare optimization results were classified as in [1]. The new performance index is a combination of a manipulability measure and condition number used by previous authors. To find the optimum link lengths and volumes of these robot manipulators, two design objectives are used: maximize the workspace area covered by the robot manipulator and maximize the new local index. As examples, two spherical three-link robot manipulators are examined based on above design objectives. Finally, optimization results of these robot manipulators are obtained and compared to each other.**

I. INTRODUCTION

Generally speaking, a robot manipulator structure can be subdivided into a regional structure and orientation structure. The regional structure consists of the arm (first three links), which moves the end-effector to a desired position in the workspace of the robot manipulator. The orientation structure, comprised of the last three links, rotates the end-effector to the desired orientation in the workspace. In this study, the regional structure of the robot manipulators is examined rather then the orientation structure.

The workspace, also called work volume or work envelope of a manipulator, is the volume of space, which the end-effector of the manipulator can reach. The size and shape of the workspace depends on the coordinate geometry of the robot arm, and also on the number of degrees of freedom. Some workspaces are quite flat, confined almost entirely to one horizontal plane. Others are cylindrical; still others are spherical. Some workspaces have very complicated shapes. When choosing a robot arm for a certain industrial purpose, it is important that the workspace is large enough to encompass all the points that the robot arm will need to reach, but it is wasteful to use a robot arm with a workspace much larger than necessary.

The workspace of a robot is an important criterion in comparing manipulator geometries. The reachable workspace and the dexterous workspace are two important characteristics used in specifying the workspace of a robot manipulator. The reachable workspace, in which the robot manipulator is able to arbitrarily move its end-effector, does not include any singular points at which the manipulator loses one or more degrees of freedom. The dexterous workspace is the volume of space in which the end-effector can be arbitrarily oriented. The reachable workspace is the volume of space, which the robot can reach in at least one orientation. In the dexterous workspace the robot has complete manipulative capability. The dexterous workspace is a subset of the reachable workspace and is a very important performance index of the robot manipulator. A good robotic design has the volume of reachable workspace as large as possible, and additionally provides a maximum dexterous workspace in this volume.

There is a close relationship between the kinematics performance and design of robot manipulators. Because of this, several kinematics-related criteria have been suggested for designing a well-conditioned robot manipulator that has a dexterous workspace. ‘‘Ref. [2]’’ presented a close relationship between kinematics design and manipulator workspace. Since then, many authors have studied the workspace and singularity analysis of robot manipulators as in [3-9]. A numerical approach to determining the workspace was formulated and solved by tracing boundary surfaces of a workspace as in [10]. They considered the dexterous workspace of manipulators when the manipulator wrist can rotate fully about three axes fully through any point. ‘‘Ref. [4]’’ studied accessible regions of planar manipulators. ‘‘Ref. [5], [11] and [12]’’ developed performance indices that could be used as an optimization and design criteria. ‘‘Ref. [13]’’ defined the determinant of the Jacobian as the manipulability of a robot manipulator and proposed it as a performance criterion. Obviously, the manipulability of a robot manipulator defined in this way is a measure of the manipulator’s inverse Jacobian. ‘‘Ref. [14]’’ showed that the absolute of the Jacobian determinant is not a robust measure of the Jacobian invertibility because the diagonal elements of a square matrix with high numeric values has very large determinant values and its inverse may produce unexpected round off errors. ‘‘Ref. [15]’’ realized this problem and defined a condition number of the Jacobian matrix that yields reasonable numerical values even when the determinant is very small. This performance index has been used extensively by many authors as in [10], [11] and [16]. However, the determinant is still a component of the condition number. This performance index fails at singular points and yields uncontrollable values for the robot manipulator. The new index, introduced here, eliminates the determinant part of the Jacobian matrix completely. Thus, unexpected and uncontrollable values for kinematics optimization of robot manipulators is eliminated.

This paper is written following manner. In Section II, two-letter code combinations and the serial chain mechanisms are introduced. In Section III, the new local and global performance indices are defined. In Section IV, The design

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objectives and optimization method used to optimize the reachable workspace volumes and link lengths are presented. In Section V, two spherical three-degree-of-freedom, robot manipulators are given as examples for workspace optimization. The results obtained from the workspace optimization of the sixteen fundamental robot manipulators were discussed in Section VI. Finally the conclusion of this study is given in Section VII.

II. Two-Letter Code Desciption of the Robot Configurations

‘‘Ref. [1]’’ used a two-letter code to classify robot configurations. The first letter characterizes the first joint and the first joint’s relationship to the second joint. The second letter identifies the third joint and third joint’s association to the second joint. The code letters and their meanings are:

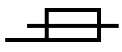
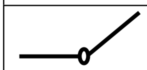
S :Slider
C :Rotary parallel to slider
N :Rotary perpendicular to rotary
R :Rotary perpendicular to rotary, or rotary parallel to rotary

The possible combinations of these rotary and prismatic joints comprise the sixteen robot configurations, which are named as

CC, CN, CR, CS, NC, NN, NR, NS, RC, RN, RR, RS, SC, SN, SR and SS.

The serial chain mechanisms are illustrated in Table I.

TABLE I.
The Serial Chain Mechanisms.

	A slider (prismatic joint) with its axis in the plane of the paper.
	A slider (prismatic joint) with its axis perpendicular to the plane of the paper.
	A rotary joint (revolute joint) with its axis perpendicular to the plane of the paper.
	A rotary joint (revolute joint) with its axis plane to the plane of the paper.

III. Definition of Performance Indices

Selection of a robot configuration depends on the task to be performed. The task determines structure and position of the robot mechanism. In order to analyze the efficiency of robots, it is needed to have some quantitative measure of their performance. The theory of kinematic synthesis has considerably furthered during last decades and various kinematic criteria have been developed to describe the manipulability and dexterity of robot manipulators. Most of these studies were derived from the definition of manipulability. A quantitative measure is required for the manipulation in order to compute capability of a robot manipulator. ‘‘Ref. [13]’’ proposed kinematics manipulability as a performance measure. Let $J(\theta)$ be velocity of Jacobian of the manipulator. When $J(\theta)$ loses its full rank, the manipulator loses one of its degrees of freedom; hence, the manipulability is defined as

$$w = \sqrt{\det(J(\theta)J^T(\theta))} \tag{1}$$

which for nonredundant manipulator reduces to

$$w = \left|\det J(\theta)\right| \tag{2}$$

where J is Jacobian matrix.

Considering in terms of Jacobian matrix of a robot manipulator, the condition number is an error amplifying factor of actuators, so affecting the accuracy of Cartesian velocity of the gripper. The precision of the control of the robot manipulators depends on the condition number of the Jacobian matrix as in [17]. The condition number based on the Jacobian matrix is given by

$$\kappa = \|J\|\left\|J^{-1}\right\| \tag{3}$$

where $\|\cdot\|$ is one of the matrix norms of the manipulator Jacobian, given by

$$\|J\| = \sqrt{tr(JNJ^T)} \tag{4}$$

where, tr stands for trace and N is a matrix defined by

$$1/n[I] \tag{5}$$

where n is the dimension of the square matrix and I is the identity matrix.

The new local index for nonredundant manipulator is defined here as the product of manipulability measure and condition number of the Jacobian matrix,

$$\rho_L = w\kappa \tag{6}$$

where w is given by (2) and κ is given by (3). This new performance index is independent of the Jacobian determinant. This can be shown as follows. The condition number can be expressed as,

$$\kappa = \sqrt{m}/\Delta \tag{7}$$

where m is the numerator value of the product of the Jacobian matrix and its transpose with the inverse Jacobian matrix and its transpose. Δ is the determinant of the Jacobian matrix of the robot manipulator. If ρ_L is extracted then the following equation is obtained.

$$\rho_L = w\kappa = \Delta\frac{\sqrt{m}}{\Delta} = \sqrt{m} \tag{8}$$

where m is always a positive value. Because the Jacobian matrix is configuration dependent, ρ_L gives a local property of the robot manipulator. To obtain a global property of the robot manipulator, the following adaptation is made.

$$\rho_G = A/B \tag{9}$$

where

$$A = \int_W m \, dW \quad \text{and} \quad B = \int_W dW$$

where, ρ_G is the new global performance index, m is the local index at a specific point of the robot manipulator workspace *W,* and *B* is the volume of the workspace.

IV. Workspace Optimization

One of the most complicated problem in manipulator kinematics is to find the optimal geometry. Mathematical equations that describe the behavior of robot kinematics are nonlinear, also have plenty of terms in general. The complexity of the optimal design problem urges to develop fast prototyping, which allows robot designers to expose structural defects of mechanism by studying the behavior of their prototypes instead of analyzing troublesome mathematical models. Modern mathematics does not posses generic techniques for having closed-form solutions to nonlinear equations. Hence iterative methods are still used for solving complicated system. In this work, a *minimax* algorithm which minimizes the worst-case value of a set of multivariable functions, starting at an initial estimate, is used for numerical optimization. This optimization is generally referred to as the *minimax* problem. It uses a sequential quadratic programming (SQP) method as in [18].

In our multi-objective design optimization problem, there are two objectives: the maximum workspace volume covered by the robot manipulator and the maximize ρ_L. The link lengths are the design variables, which are limited to upper, and lower bounds. Based on the robot configuration, there are a maximum of three optimized design variables or a minimum of one optimized design variable for three link robot manipulators. The consecutive link length ratios were constrained to an upper bound of 2 and to a lower bound of 1.1. The multi-objective design optimization problem is formulated as

$$\text{maximize } \rho_L \ (a,b,c)$$
$$\text{maximize } V \ (a,b,c) \quad \text{subject to}$$

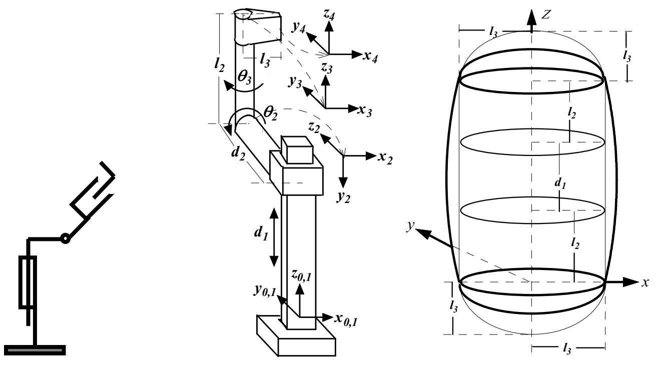
$$G_1 = \frac{a}{b} \geq 1.1,\ G_2 = \frac{a}{b} \leq 2,\ G_1 = \frac{b}{c} \geq 1.1,\ G_4 = \frac{b}{c} \leq 2 \tag{10}$$

where a, b and c (link lengths) are design variables, κ and V are objective functions and G_1, G_2, G_3 and G_4 are nonlinear inequality constraints. The link lengths of sixteen different robot manipulators were optimized according to the above multi-objective formulation.

V. Examples

A. SN Robot Manipulator

The SN robot manipulator in terms of the serial chain mechanisms is drawn in Fig. 1(a). The rigid body whose link parameters assigned according to the D-H method is given in Fig 1(b) as in [19]. The workspace volume is illustrated in Fig 1(c).



(a) (b) (c)

Figure 1(a) The serial chain mechanism, (b) Coordinate frames attached to the rigid body (c) Workspace volume of SN three-link robot manipulator.

The SN (spherical) robot manipulator has two revolute joints (θ_2 and θ_3) and one prismatic joint (d_1). The Jacobian matrix of the SN robot manipulator is given by

$$J = \begin{bmatrix} 0 & -l_3 \sin\theta_2 \cos\theta_3 - l_2 \cos\theta_2 & -l_3 \cos\theta_2 \sin\theta_3 \\ 0 & 0 & l_3(\cos\theta_3) \\ 1 & l_3 \cos\theta_2 \cos\theta_3 - l_2 \sin\theta_2 & -l_3 \sin\theta_2 \sin\theta_3 \end{bmatrix} \tag{11}$$

The SN robot manipulator is examined in terms of the new local and global indices as follows. The new local index of the SN robot manipulator is

$$\rho_{LSN} = \frac{\sqrt{(1 + l_2^2 + l_3^2 + l_3^2 \cos^2\theta_3) C}}{3} \tag{12}$$

where

$$C = ((l_2^2 + l_3)^2 (l_3^2 \cos^2\theta_3 + \cos^2\theta_2) + 2 l_3^2 \cos^2\theta_3 \sin^2\theta_2$$
$$+ 2 l_2 l_3 \cos\theta_2 \cos\theta_3 \sin\theta_2)$$

The approximate workspace volume covered by SN robot manipulator is

$$V_{SN} = \frac{4}{3}\pi l_3^3 + (d_1 + 2l_2)\pi l_3^2 \tag{13}$$

By optimizing the design objectives in (12) and (13), the maximum local index equals 13.7420 and the maximum workspace volume equals 87.491.

For the SN robot manipulator, The new local index versus the angle of rotation of the second joint, θ_2 and third joint, θ_3 is shown in Fig. 2.

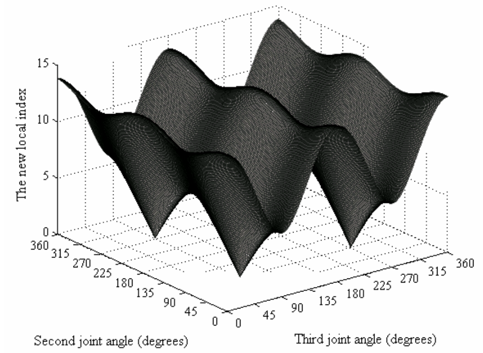


Figure 2. The new local index of SN robot manipulator versus the angle of rotation of the second joint, θ_2 and third joint, θ_3 .

The SN robot manipulator can achieve complete gross motion at the highest values of the new local index.

A contour analysis of the SN robot manipulator in terms of the new local index is shown in Fig. 3. The semi-vertical ellipses and whole-vertical ellipses are the best areas in which the SN robot manipulator has complete geometric dexterity. Also, the angles $\theta_3 = 90^\circ$, $\theta_2 = 90-270$ and $\theta_3 = 270^\circ$, $\theta_2 = 90-270$ are singular points of the SN robot manipulator.

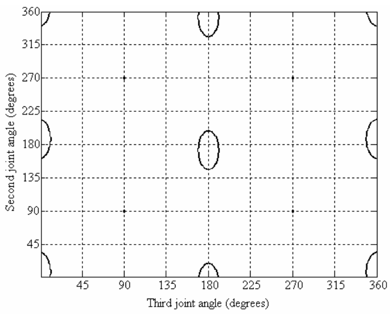


Figure 3. The contour analysis of SN robot manipulator .

The new global index for the SN robot manipulator can be calculated according to the definition in (9) by using the optimized l_2 and l_3 values in (12) and (13).

$$\rho_{G\,SN} = \frac{1}{37.3934}\int_0^{2\pi}\int_0^{2\pi}\frac{D}{3}d\theta_2 d\theta_3 \tag{14}$$

where

$$\begin{aligned} D &= 930\cos^2\theta_3 + 282\cos^2\theta_2 + 55\sin^2\theta_2\cos^2\theta_3 \\ &+ 61\cos\theta_2\cos\theta_3\sin\theta_2 + 112\cos^2\theta_2\cos^2\theta_3 + 370\cos^4\theta_3 \\ &+ 22\cos^4\theta_3\sin^2\theta_2 + 24\cos^3\theta_3\cos\theta_2\sin\theta_2 \end{aligned}$$

B. NR Robot Manipulator

The serial chain mechanism, rigid body and workspace volume of NR robot manipulator are illustrated in Fig. 4(a), Fig. 4(b) and Fig. 4(c), respectively.

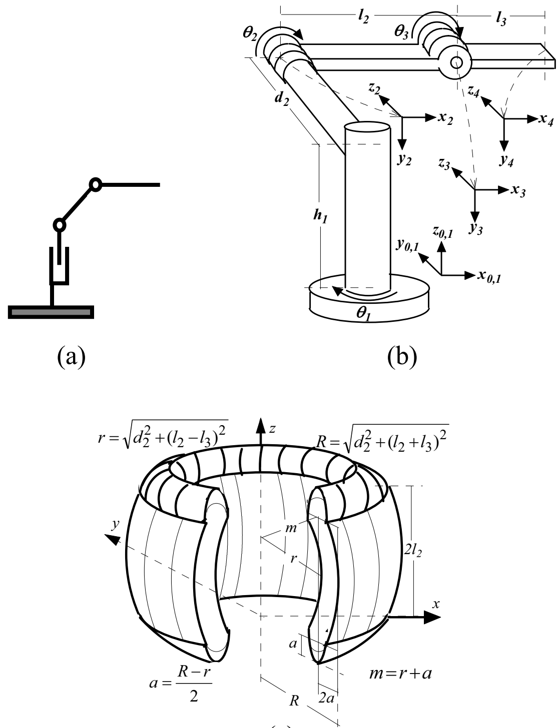


Figure 4 (a) The serial chain mechanism, (b) Coordinate frames attached to the rigid body (c) Workspace volume of SN three-link robot manipulator.

The NR (spherical) robot manipulator has three revolute joints (θ_1, θ_2 and θ_3). The Jacobian matrix of the NR robot manipulator is

$$J = \begin{bmatrix} -l_3\sin\theta_1\cos\theta_{23} - l_2\sin\theta_1\cos\theta_2 - d_2\cos\theta_1 & . & . \\ l_3\cos\theta_1\cos\theta_{23} + l_2\cos\theta_1\cos\theta_2 - d_2\sin\theta_1 & . & . \\ 0 & . & . \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} . & -l_3\cos\theta_1\sin\theta_{23} - l_2\cos\theta_1\sin\theta_2 & -l_3\cos\theta_1\sin\theta_{23} \\ . & -l_3\sin\theta_1\sin\theta_{23} - l_2\sin\theta_1\sin\theta_2 & -l_3\sin\theta_1\sin\theta_{23} \\ . & -l_3\cos\theta_{23} - l_2\cos\theta_2 & -l_3\cos\theta_{23} \end{bmatrix} \tag{15}$$

The new local index of the NR robot manipulator is

$$\rho_{L\,NR} = \sqrt{EF}/3 \tag{16}$$

where

$$\begin{aligned} E &= l_3^2\cos^2\theta_{23} + 4l_2l_3\cos\theta_2\cos\theta_{23} + l_2^2 + d_2^2 \\ &+ l_2^2\cos^2\theta_2 + l_3^2 + 2l_2l_3\sin\theta_2\sin\theta_{23} \end{aligned}$$

$$\begin{aligned} F &= -(2d_2^2l_3^2\cos^2\theta_{23} + 2l_2^2l_3^2\cos^2\theta_{23} + 4l_2l_3^3\cos\theta_2\cos\theta_{23} \\ &+ 2l_2^2l_3^2\cos^2\theta_2\cos^2\theta_{23} + 2l_2^3l_3\cos^3\theta_2\cos\theta_{23} \\ &+ 2l_2^2l_3^2\cos\theta_2\sin\theta_2\cos\theta_{23}\sin\theta_{23} + 2l_2l_3^3\cos\theta_2\cos^3\theta_{23} \\ &+ 2d_2^2l_2l_3\cos\theta_2\cos\theta_{23} + 3l_2^2l_3^2\cos^2\theta_2 + 2l_2^3l_3\cos\theta_2\cos\theta_{23} \\ &+ 2l_2^3l_3\cos^2\theta_2\sin\theta_2\sin\theta_{23} + 2l_2l_3^3\sin\theta_2\sin\theta_{23}\cos^2\theta_{23} \\ &+ 2l_3^4\cos^2\theta_{23} + l_2^4\cos^2\theta_2 + l_2^2d_2^2\cos^2\theta_2) \end{aligned}$$

where

$$\cos\theta_{23} = \cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3,$$
$$\sin\theta_{23} = \sin\theta_2\cos\theta_3 + \cos\theta_2\sin\theta_3.$$

The approximate workspace volume is

$$V_{NR} = 2a^2\pi^2 m + 8l_2\pi a(r+a) \tag{17}$$

For the NR robot manipulator, the new the local index versus the angle of rotation of the second joint, θ_2 and third joint, θ_3 is shown in Fig. 5.

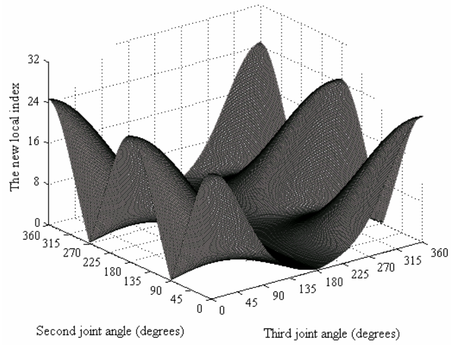


Figure 5. The new local index of NR robot manipulator versus the angle of rotation of the second joint, θ_2 and third joint, θ_3 .

The contour analysis of the NR robot manipulator in terms of the new local index is shown in Fig. 6.

The new global index for the NR robot manipulator can be calculated as follows:

$$\rho_{G\,NR} = \frac{1}{265.43}\int_0^{2\pi}\int_0^{2\pi}\frac{\sqrt{GH}}{3}d\theta_2 d\theta_3 \tag{18}$$

where

$$G = 2.73\cos^2\theta_{23} + 12\cos\theta_2\cos\theta_{23} + 3.3\cos^2\theta_2 + 6\sin\theta_2\sin\theta_{23} + 10$$

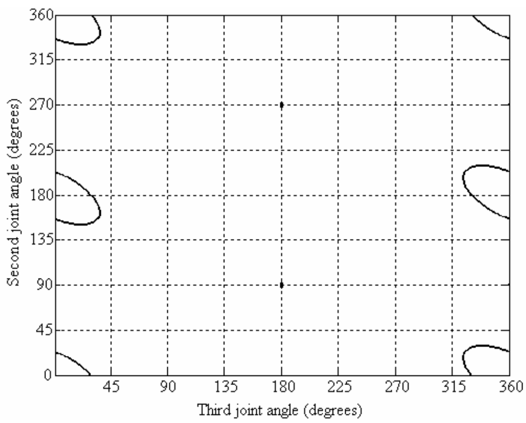


Figure 6. The contour analysis of NR robot manipulator.

$$\begin{aligned} H = -(&54.8\cos^2\theta_{23} + 76.7\cos\theta_2\cos\theta_{23} + 18\cos^2\theta_2\cos^2\theta_{23} \\ &+ 18\cos\theta_2\sin\theta_2\cos\theta_{23}\sin\theta_{23} + 16.4\sin\theta_2\sin\theta_{23}\cos^2\theta_{23} \\ &+ 16.4\cos\theta_2\cos^3\theta_{23} + 19.8\cos^3\theta_2\cos\theta_{23} + 51.2\cos^2\theta_2 \\ &+ 87\cos^2\theta_2\sin\theta_2\sin\theta_{23}) \end{aligned}$$

VI. Optimization Results

The new local and global performance indices and the workspace volume of the robot manipulators are given in Table II (M, J. T., R, P and *V* represent manipulator, joint type, revolute joint, prismatic joint and workspace volume of robot manipulators, respectively). The new global performance index of the sixteen manipulators is ordered from the highest value to the lowest value in Table II. The spherical robot manipulators (SN, RN, NR and NN) have better new global performance index values than the others. The RR and RC robot manipulators have high local index values, but the global performance index of these manipulators is not high compared to their local index values. A high local performance index does not necessarily produce a high global performance index.

TABLE II.
The New Local and Global Performance Indices and the Workspace Volume of the Robot Manipulators

M.	J. T.	ρ_G	ρ_L	V
SN	PRR	3.9476	13.742	87.491
RN	RRR	1.3203	24.083	258.92
NR	RRR	1.3144	25.855	265.43
N	RRR	1.0130	17.633	296.83
CN	PRR	0.9717	6.3700	29.503
NC	RRP	0.7863	16.009	91.944
NS	RRP	0.5977	22.922	137.89
CC	RPR	0.4230	9.6440	91.391
CR	RPR	0.3737	6.7420	221.89
SC	PPR	0.3284	2.4490	41.113
SR	PRR	0.2516	6.4580	95.961
RS	RRP	0.2477	6.4580	91.391
CS	RPP	0.1939	3.4640	22.275
RR	RPR	0.1745	20.668	266.74
RC	RPR	0.0359	12.284	109.86
SS	PPP	-	-	8.0000

The optimized link lengths of the robot manipulators are given in Table III. As can be seen in Table III, the optimized link lengths of the prismatic and revolute joints vary between 1.6529 and 2. Furthermore, the gross motion capability of the robot manipulator in Cartesian space is better as the number of revolute joints increase.

TABLE III.
The Optimized Link Lengths of the Robot Manipulators.

M.	l_1	l_2	l_3	d_1	d_2	d_3
SN	-	2.0000	1.8182	2	-	-
RN	2	1.8182	1.6529	-	-	-
NR	-	1.8182	1.6529	-	2.0000	-
NN	-	1.8182	1.6529	-	2.0000	-
CN	2	1.8182	-	2	-	-
NC	-	1.8182	-	-	2.0000	1.6529
NS	-	1.8182	-	-	2.0000	1.6529
CC	-	2.0000	1.8182	-	2.0000	-
CR	-	2.0000	1.8182		2.0000	-
SC	-	2.0000	-	2	2.0000	-
SR	2	2.0000	1.8182	2	-	-
RS	2	1.8182	-	-	-	2.0000
CS	2	-	-	-	2.0000	1.8182
RR	-	2.0000	1.6529	-	1.8182	-
RC	-	2.0000	1.6529		1.8182	-
SS	-	-	-	2	2.0000	2.0000

VII. Conculision

The sixteen fundamental robot manipulators were compared based on the new local and global performance indices. The comparison showed that spherical robot manipulators have higher global performance indices. Thus, these robot manipulators are better robot designs and achieve gross motion in all three spatial dimensions. On the other hand, the new local performance indices of some spherical robot manipulators are smaller then the new local indices of other robot configurations. This demonstrates that the high local performance index does not always produce high global performance index. The new local index can not be used for comparing the robot manipulators with each other as a global performance index. It just shows the gross motion ability of the robot manipulator itself.

REFERENCES

[1] V. Milenkovic, and B. Huang, "Kinematics of major robot linkages," *Robotics International of SME*, vol. 2, pp. 16–31, 1983.

[2] B. Roth, "Performance evaluation of manipulators from a kinematic view point,'' *NBS Special publication*, pp. 39-61, 1975.

[3] B. E. Shimano, and B. Roth, "Ranges of motion of manipulators,'' *Proc. CISM-IFToMM Symp. Theory Pract. Robots Manipulators,* pp. 18-27, Warsaw, Poland, 1976.

[4] Y. C. Tasai, and A. H. Soni, "Accessible region and synthesis of robot arms,'' *ASME J Mechanical Design*, 103(4) pp. 803-811, 1981.

[5] K. C. Gupta, and B. Roth, "Design consideration for manipulator workspace,'' ASME *Journal of Mechanism, Transmission and Automation in Design*, vol. 104, pp. 704–711, 1982.

[6] D. C. H. Yang, and Z. C. Lai, "On the dexterity of robotic manipulator service angle,'' ASME *Journal of Mechanism, Transmission and Automation in Design*, vol. 107, pp. 262–270, 1985.

[7] A. Bajpai, and B. Roth, "Workspace and mobility of closed-loop manipulator,'' *The International Journal of Robotics Research*, vol. 5, pp. 131–142, 1986.

[8] M. Cwikala, and T.W. Lee, "The optimum kinematic design of a spherical three-degree-of freedom parallel manipulator,'' ASME *Journal of Mechanism, Transmission and Automation in Design,* vol. 111, pp. 202–207, 1989.

[9] A. Kumar, and M. S. Patel, "Mapping the manipulator workspace using interactive computer graphics," *The International Journal of Robotics Research*, vol. 5, pp. 122–130, 1986.

[10] A. Kumar, and K. J. Waldron, "The workspace of a mechanical manipulator,'' ASME *Mechanical Design*, vol. 103, pp. 665–672, 1981.

[11] M. C. Gosselin, and J. Angeles, "A global performance index for the kinematic optimization of robotic manipulators," *Journal of Mechanical Design,* vol. 113, pp. 220–223, September 1991.

[12] R. P. Paul, and C. N. Stevenson, "Kinematics of robot wrists," *The International Journal of Robotics Research*, 2(1), pp. 31-38, 1983.

[13] T. Yoshikawa, ''Manipulability of robotic mechanism,'' *The International Journal of Robotics Research*, vol. 4, no. 2, MIT press, 1985.

[14] G. H. Forsythe, and C. B. Moler, *Computer solution of linear algebraic systems*, Englewood Cliffs, NJ: Prentice-Hall, 1967.

[15] J. K. Salisbury, and J. J. Craig, "Articulated hands: Force control and kinematic issues," *The International Journal of Robotics Research*, vol. 1, pp. 4-17, 1982.

[16] J. Angeles, and A. A. Rojas, "Manipulator inverse kinematics via condition number minimization and continuation," *The International Journal of Robotics and Automation*, vol. 2, pp. 61-69, 1987.

[17] G. H. Golub, and C. F. Van Loan, *Matrix computations*, The Johns Hopkins University Press: Baltimore, 1983, pp. 24-29.

[18] Han, S.P., "A Globally Convergent Method For Nonlinear Programming," *Journal of Optimization Theory and Applications*, Vol. 22, p. 297, 1977.

[19] J. Denavit, and R. S. Hartenberg, "A Kinematic Notation for lower-pair mechanisms based on matrices," *Journal of Applied Mechanics*, pp. 21, 5-221, June 1955.