2016 IEEE 59th International Midwest Symposium on Circuits and Systems (MWSCAS), 16-19 October 2016, Abu Dhabi, UAE

Multi-objective Optimization of Planar 3PRR (Prismatic-Revolute-Revolute) Parallel Mechanism Using Genetic Algorithm

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***Abstract*—This paper proposes multi-objective optimization of planar 3PRR parallel kinematics mechanism which offers the advantages of lower degree of freedom parallel kinematics mechanisms. Workspace area, minimum eigenvalue across the workspace, and stiffness condition number across the workspace are chosen to be the objectives in the optimization in order to gain as large workspace area as possible while maintaining high stiffness in all directions under the defined kinematics constraints of the mechanism. The multi-objective optimization has been conducted by using multi-objective genetic algorithm. It is shown that the multi-objective optimization compromises the improvement of all objectives by providing non-dominated solutions. A decision maker can pick a preferred solution among those solutions.**

***Keywords*—3PRR, parallel kinematics mechanism, multi-objective optimization, genetic algorithm**

I. Introduction

Serial kinematics mechanisms (SKMs) have been widely used as manipulators. However, the serial kinematics mechanisms have many drawbacks such as lower stiffness, lower accuracy, less agility, higher inertia, and complicated inverse kinematics [1]. To overcome these drawbacks, parallel kinematics mechanisms (PKMs) have been proposed and developed particularly for some demanding applications such as high speed and high accuracy machining, positioning, and assembly [2-4]. Unfortunately, the PKMs have also some drawbacks such as less workspace area and complicated direct kinematics [5]. Furthermore, more drawbacks maybe resulted from the number of degrees of freedom (DOF) of the PKMs. High DOF PKMs such as hexapod (which has 6 DOF) have been extensively used, but they introduce more complexity [6]. Hence, some lower DOF PKMs with different topologies have been proposed and developed. Combination of a lower DOF PKM with an SKM or another lower DOF PKM may easily give the required higher DOF.

After the topology selection, optimization of the mechanism performances is usually conducted. Different performance criteria have been used as optimization objectives, depending on the application requirements. For example, Gao et al [7] used the stiffness and the kinematic dexterity as objectives to optimize Stewart manipulator. Lara-Molina et al [8] used global conditioning index, global payload index, and global gradient index as objectives to optimize Stewart manipulator. Furthermore, several optimization techniques have been applied. In general, those optimization techniques fall into two main categories: gradient-based optimization techniques and population-based optimization techniques. One of the most popular population-based techniques is genetic algorithm which is an evolutionary optimization technique and works based on the idea of natural selection or survival of the fittest. The genetic algorithm is a global search technique and therefore does not need initial values. It can be implemented for both single-objective and multi-objective optimization. Multi-objective genetic algorithm [9-12] is very convenient to conduct a posteriori multi-objective optimization where no preference information on the objectives is available prior to the optimization process.

This paper proposes and investigates the multi-objective optimization of a planar 3PRR parallel kinematics mechanism [13-14] as shown in Fig.1, by considering the workspace, the stiffness, and the stiffness condition number as objective functions. These chosen objective functions are very important

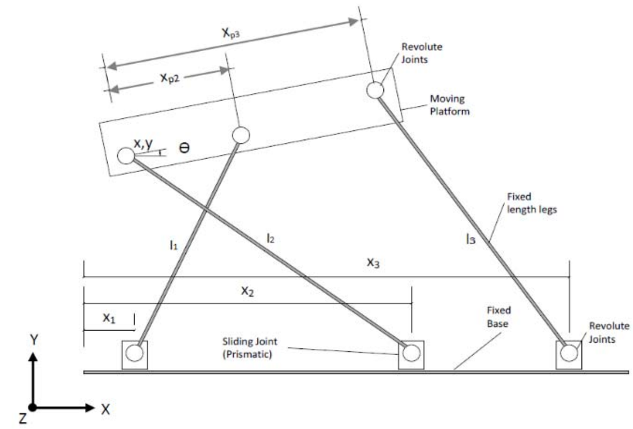

Fig. 1. The planar 3PRR parallel mechanism with two adjacent legs crossing each other to avoid singularity

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performance measures in some applications such as multi-axis machine tools where larger workspace area is expected to enable handling of larger workpiece, high stiffness is required to avoid excessive vibration during machining so that more accuracy can be achieved, and low stiffness condition number representing good stiffness in all directions is required since the machining loads might work in all directions.

II. Kinematics

The inverse kinematics for the proposed mechanism is defined as follows: given the position and orientation of the moving platform (x, y, and θ), obtain the position of all the sliders (x_1, x_2, and x_3). The equations of the inverse kinematics are given by:

$$x_1^2-(2x+2x_{p2}\cos\theta)x_1 \\ +(x^2+y^2+x_{p2}^2+2xx_{p2}\cos\theta+2yx_{p2}\sin\theta-l_1^2)=0 \quad (1)$$

$$x_2^2-2xx_2+(x^2+y^2-l_2^2)=0 \quad (2)$$

$$x_3^2-(2x+2x_{p3}\cos\theta)x_3 \\ +(x^2+y^2+x_{p3}^2+2xx_{p3}\cos\theta+2yx_{p3}\sin\theta-l_3^2)=0 \quad (3)$$

The kinematic constraints of the mechanism are given as follows:

$$0\le l_1\le y_{\max},0\le l_2\le y_{\max},0\le l_3\le y_{\max} \\ 0\le x_{p2}\le x_{\max},0\le x_{p3}\le x_{\max} \\ l_1+l_2-x_{p2}\le x_{\max},l_1+l_3-x_{p2}+x_{p3}\le x_{\max},x_{p2}-x_{p3}\le 0 \quad (4)$$

where y_{max} and x_{max} are the vertical and horizontal limits of the machine volume, respectively.

The Jacobian of the mechanism is formulated to analyze the stiffness of the mechanism while the prismatic joints (sliders) are assumed fixed. This is because only the stiffness of the legs and the platform is being analyzed. The Jacobian J is given by:

$$J=\begin{bmatrix} s_1^T & (Rp_1\times s_1)^T \\ s_2^T & (Rp_2\times s_2)^T \\ s_3^T & (Rp_3\times s_3)^T \end{bmatrix} \quad (5)$$

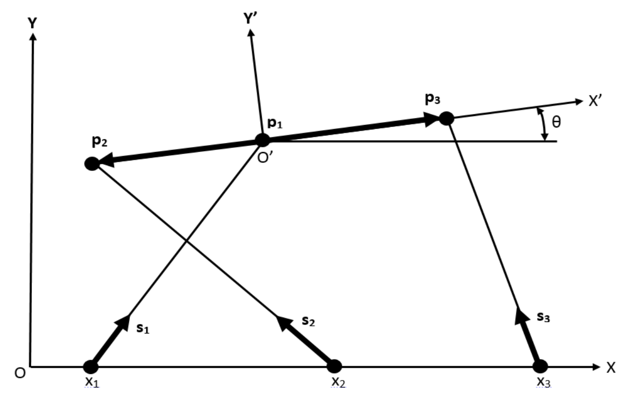

Fig. 2. The vectors used in the Jacobian calculation

where s_1, s_2, and s_3 are the leg unit vectors, R is the rotation matrix of the platform with respect to the inertial frame XY, and p_1, p_2, and p_3 are the position vectors of the platform in the local frame X'Y', as indicated in Fig.2.

III. Workspace Evaluation

Using the inverse kinematics equations provided in (1) to (3) as well as the kinematic constraints of the mechanism given in (4), the workspace of the planar 3PRR mechanism can be determined and plotted. The algorithm of the workspace determination is shown by the flowchart in Fig.3. The process involves iterations by varying the input value of test point (x,y). The test points should represent discrete points bounding possible workspace within the machine volume. Small increments in X and Y directions between these discrete points determines the density of the workspace plot. By providing the geometrical parameters of the mechanism which include l_1, l_2, l_3, x_{p2}, and x_{p3}, as well as the moving platform orientation θ, the inverse kinematics equations are solved for every input value (x,y). Whenever the solutions are all real numbers and meet the mechanism's constraints, the input value (x,y) is plotted.

Furthermore, the area of the workspace A is computed by considering that the workspace is comprised of a number of slender rectangles, the length of which is oriented vertically while the width of which is given by a small increment. The area of each rectangle is given by its length multiplied by the increment size. Summing the area of all the rectangles gives the area of the entire workspace.

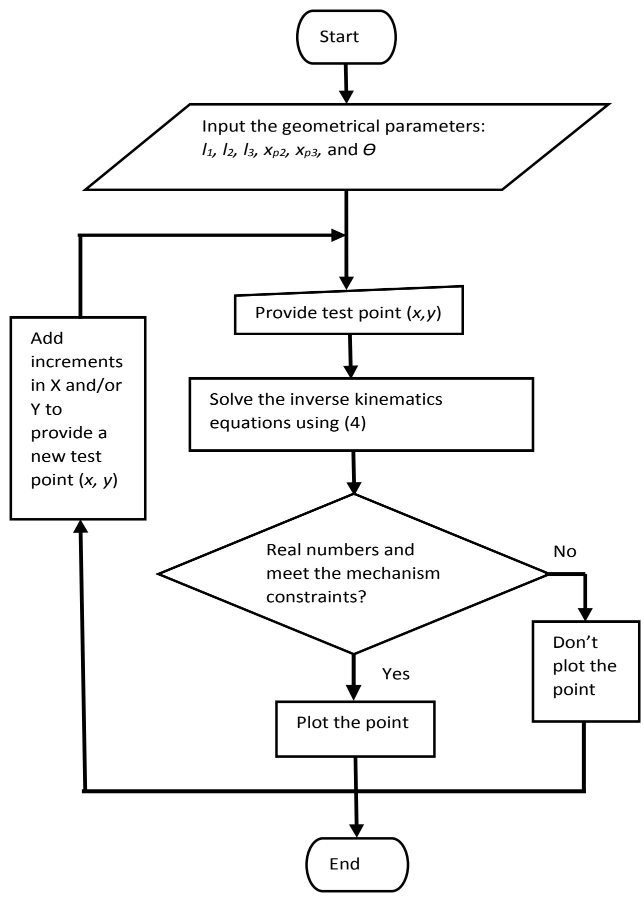

Fig. 3. Algorithm flowchart of the workspace determination

IV. Stiffness Evaluation

The stiffness of the mechanism is posture dependent. Each instant posture has its own stiffness values. In this work, only stiffness of the legs is taken into consideration. By utilizing the Jacobian of the mechanism, the Cartesian stiffness matrix of the mechanism can be mapped from the axial stiffness of the legs. The Cartesian stiffness matrix of the mechanism K_c is given by:

$$K_c = J_n^T K_a J_n \tag{6}$$

where J_n is the normalized version of the Jacobian as given by (5) and K_a is a diagonal matrix with the axial stiffness of the three legs as the diagonal elements:

$$K_a = \begin{bmatrix} \frac{A_1 E_1}{l_1} & 0 & 0 \\ 0 & \frac{A_2 E_2}{l_2} & 0 \\ 0 & 0 & \frac{A_3 E_3}{l_3} \end{bmatrix} \tag{7}$$

For simplicity, the cross section area and the modulus of elasticity are assumed identical among the three legs. The Cartesian stiffness matrix consists of direct and cross-coupling stiffness components. The direct stiffness maps in the three directions across the workspace can be plotted.

For optimization purpose, several criteria have been used in the literature to evaluate the stiffness of mechanisms, including the use of global stiffness index, the use of trace of the stiffness matrix, the use of weighted trace of the stiffness matrix, and the use of eigenvalues. In this paper, the last method is used since the eigenvalues are practical indicators of the minimum or maximum stiffness [15]. The minimum eigenvalues λ_{min} are used to represent the lowest stiffness in the workspace.

V. Stiffness Condition Number

The stiffness condition number κ_s measures how uniform the stiffness in different directions. The range of its values are from 1 to infinity. It also indicates whether the stiffness matrix is well-conditioned or ill-conditioned. An infinity stiffness condition number indicates singular stiffness matrix which should be avoided. Mathematically, the stiffness condition number is defined by:

$$\kappa_s = \|K_c\| \|K_c^{-1}\| \tag{8}$$

where K_c is given by (6).

To determine the stiffness condition number defined in (8), several possible definitions of the norms can be used. In this work, the norms are defined by using Frobenius norm definition, which is among the most commonly used definitions since it is analytic in terms of the posture parameters and does not require the costly evaluation of singular values, as follows:

$$\|K_c\| = \sqrt{trace(K_c K_c^T)} \tag{9}$$

$$\|K_c^{-1}\| = \sqrt{trace(K_c^{-1} (K_c^{-1})^T)} \tag{10}$$

VI. Multi-objective Optimization

To make a trade-off among the workspace area, the minimum eigenvalue, and the stiffness condition number, multi-objective optimization was conducted. It is worth noting that some of the objectives are conflicting with each other. For example, the workspace area is conflicting with the stiffness since larger workspace generally leads to lower stiffness, and vice versa. This implies that an improvement in an objective will sacrifice the other conflicting objective.

In the multi-objective optimization, the average values of the objectives are used. The optimization is stated as follows:

Maximize the average workspace area, and
Maximize the average minimum eigenvalue across the workspace, and
Minimize the average stiffness condition number
Subject to the kinematic constraints of the mechanism

The decision variables in the optimization are the geometric parameters of the mechanism which include the length of the legs (l_1, l_2, l_3) and the distances between the revolute joints at the moving platform (x_{p2}, x_{p3}). All of the objectives are functions of the decision variables. Furthermore, the objective values also vary with the platform orientation angle θ. To represent different objective values within the range of the platform orientation angles, average values of the objectives were used and calculated from several sample platform orientation angles within the range. The inclusion of more sample platform orientation angles will give more representative average value. However, it will increase the computation time since it involves more sequential iterations. At least, three sample orientation angles which include the most extreme angles and an intermediate angle should be used to get a representative average value.

The multi-objective optimization was conducted by using genetic algorithm tool provided in MATLAB Optimization Toolbox. The parameters of the algorithm are shown in Table I. The algorithm stops when the average relative change of the fitness function (i.e. objective function) value over the stall generations is less than the function tolerance.

The use of genetic algorithm for multi-objective optimization is preferred when no preference is available prior to the optimization. It gives several solution sets commonly called Pareto optimal sets that represent non-dominated values. From the Pareto optimal sets, the decision maker can pick any set which satisfy his/her preference. Some of the Pareto optimal sets are shown in Table III. For evaluation, these results are compared to the results of the single-objective optimization by using genetic algorithm summarized in Table II. It is shown that the single-objective optimization of the workspace gives the largest average workspace area (397,290 mm^2) while sacrificing the other two objectives. Similarly, the single-objective optimization of the minimum eigenvalue gives the highest average minimum eigenvalue (5.4×10^4) while sacrificing the other two objectives, and the single-objective optimization of the stiffness condition number gives the lowest average stiffness condition number (4.1×10^5) while sacrificing the other two objectives. On the other hand, the multi-objective optimization using genetic algorithm compromises the three objectives by providing non-dominated solution sets. No solution among them is superior in one objective than the solution of the single-objective optimization. A trade-off was made among all of the objectives.

Furthermore, which set to choose among the solution sets depends on the preference of the decision maker. For example, if the workspace area is considered as the most important objective while the average minimum eigenvalue already satisfies the minimum threshold, then the first row set in Table III might be the best choice among the available solution sets since it gives the largest average workspace area compared to the other solution sets.

TABLE I. PARAMETERS OF THE GENETIC ALGORITHM

Parameter	Value
Population size	50
Generations	500
Selection method	Stochastic uniform
Elite count	2
Crossover fraction	0.8
Stall generations	100
Function tolerance	10^{-6}

TABLE II. SINGLE-OBJECTIVE OPTIMIZATION RESULTS

l_1 (mm)	l_2 (mm)	l_3 (mm)	x_{p2} (mm)	x_{p3} (mm)	A (mm^2)	λ_{min}	κ_s
Optimization of the workspace							
514	575	504	327	337	397,290	7.3×10^3	3.0×10^7
Optimization of the minimum eigenvalue							
420	469	418	223	271	371,340	5.4×10^4	9.3×10^6
Optimization of the stiffness condition number							
436	549	404	200	216	308,070	1.5×10^4	4.1×10^5

TABLE III. SOME OPTIMAL SOLUTION SETS FROM MULTI-OBJECTIVE OPTIMIZATION

l_1 (mm)	l_2 (mm)	l_3 (mm)	x_{p2} (mm)	x_{p3} (mm)	A (mm^2)	λ_{min}	κ_s
412	719	580	251	508	296,100	2.1×10^4	1.7×10^6
360	396	428	274	430	293,680	5.7×10^4	5.7×10^6
378	653	506	290	536	287,700	4.1×10^3	2.0×10^6
373	654	502	383	547	281,625	3.4×10^4	2.5×10^6
396	631	587	270	538	260,240	2.9×10^4	8.2×10^5
362	488	488	380	561	259,950	5.3×10^4	8.6×10^6
400	677	603	264	562	255,390	2.9×10^4	1.6×10^6
404	613	636	245	539	242,450	5.3×10^4	5.8×10^5
393	323	471	212	507	200,700	6.1×10^4	7.4×10^5
443	324	438	213	497	189,310	6.3×10^4	9.4×10^5

CONCLUSIONS

Multi-objective optimization was conducted to compromise between the average workspace, the average minimum eigenvalues, and the average stiffness condition number of this mechanism. Knowledge on the priority order of the objectives is needed for decision making, particularly when the objectives are conflicting each other. The multi-objective optimization was conducted by using genetic algorithm which gives some non-dominant optimal solutions from which a decision maker can pick any preferred solution. In this case, the larger the diversity, the better it will be since more choices are available.

ACKNOWLEDGMENT

This research was supported by the Khalifa University Internal Research Fund.

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