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Parametric design optimization of 2-DOF R–R planar manipulator—A design of experiment approach

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Abstract

This work illustrates simulation approach for optimizing the parametric design and performance of a 2-DOF R–R planar manipulator. Using dynamic and kinematic models of a manipulator different performance measures for the manipulator are obtained for different combination of parameters with effect of noise incorporated to imitate the real time performance of the manipulator. A novel approach has been proposed to model, the otherwise difficult to model, noise effects. The data generated during simulation for various parameter combinations are utilized to analyze the statistical significance of kinematic and dynamic parameters on performance of manipulator using ANOVA technique. The parameter combinations, which give optimum performance measures obtained for different points in workspace, are compared and reported.

Keywords: Parametric design; Control factors; Noise factors; Positional error; Reliability

1. Introduction

Optimal design of manipulators, whose importance in manufacturing is increasing day by day, has been a challenge. Little success has been obtained in this area because of difficulties associated with geometrical constraints and complex models of the manipulator. Mostly industrial manipulators are required to perform tasks with a higher precision and speed than human beings. To perform a task, a robot is commanded to move its end-effector to a specified position but the actual position reached may be quite different from the desired one. This difference in the actual and desired position for the end-effector is termed as positional error of a manipulator and the average precision with which the manipulator moves its arm to the commanded position is termed as its positional accuracy [1]. The positional error for an industrial manipulator may be 0.1 mm and repeatability as high as 10 mm [2]. Some studies relevant to the stochastic analysis of positional error have been carried out. An overview of existing work on robot calibration and identified basic issues for robot precision was presented by Roth et al. [3] in 1987 but many developments have taken place since then. Whitney et al. [4] proposed a method for improving orientation and/or location accuracy of a programmable robot with respect to a target object. Their method consists of calibrating the position of a terminal control frame associated with a robot end-effector which is coupled to a robot distal link. Benhabib et al. [5] introduced direct and inverse error analyses while Jang et al. [6] had reported two types of positional errors, geometric and non-geometric for industrial robots. Geometric errors result from imprecise manufacturing and non-geometric errors result from gravity, joint compliances and gear transmission errors. Azadivar [7] investigated the effect of joint position errors on accuracy of performance in a process of inserting a pin into a hole. Bhatti and Rao [8] presented a concept of reliability as a probabilistic measure of manipulator performance. Menq and Borm [9] proposed five error measure indices to show the quantitative distribution of the positional error. Manoochehri and Seireg [10] developed a computer programme for form synthesis and optimal

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design of robot manipulator. The optimization strategy for synthesis and control utilized a dynamic programming approach that made it possible to select the optimum parameters in stage-wise manner without sacrificing the interactions inherent in the highly coupled nonlinear robotic systems. Offodile and Ugwu [11] investigated effect of various process variables such as speed of the tool and payload on the robot repeatability. They observed experimentally that speed of robot travel in work envelope has dominating effects on its repeatability.

Approaches incorporating statistical method in design exist but few have been included here. Parkinson et al. [12] and Chen et al. [13] indicated that the early design phase of a product has greatest impact on life cycle and its quality of performance. Welch et al. [14] observed that presence of systematic error rather than random error in statistical testing is inappropriate. Chen et al. [15] proposed robust concept exploration method, which facilitates quick evaluation of different design alternatives and helps in generation of top-level design specification in the early stages of design. The concept of robust design was applied, to the design of a high-speed civil transport by Chen et al. [16], to the design of family of general aviation aircraft by Simpson et al. [17], to manufacturing simulation by Peplinski et al. [18] and to the design of a turbine lift engine by Koch et al. [19,20]. Guinta et al. [21] used metamodelling techniques for deterministic computer experiments containing numerical noise. An extensive review of the methodologies to obtain a robust design that has less performance variation due to the variations of control factors and noise factors is given by Rout and Mittal [22]. The Taguchi method has been applied by Rout and Mittal [23] for finding the optimum parameters for reduction of performance variation, thereby increasing positional accuracy of a manipulator. Rout and Mittal [24] investigated the statistical significance of manipulator kinematic parameters using design of experiment approach. Parametric tolerance design of a manipulator using full factorial design of experiment approach has been attempted by Rout and Mittal [25] without considering noise factor. Lastly, Rout and Mittal [26] had attempted to design and optimize the performance of 2-degree of freedom manipulator using artificial neural network technique.

As it is very tedious, time consuming and uneconomical to build large number of prototypes and conduct physical experiments on manipulators by varying values of factors which can easily be changed by designer, i.e. control factors, a novel simulation method is proposed using control factors. Developed methodology helps in incorporating effect of noises in dynamic model of manipulator to have real time performance. Current work focuses on parametric design of manipulator using design of experiment (DOE) approach, to select a combination of control factors of a product or process in such a way that its performance will become insensitive to noise factors, that are difficult to control. DOE techniques are fairly standard approaches and commonly used in statistical quality control, but its application to robust robotic parameter design is rare. Experiments conducted using above technique helps in studying the combined effect of factors on performance. It allows the effect of a factor to be estimated at several levels of the other factors yielding conclusions that are valid over a range of experimental conditions [30]. This paper illustrates procedure to apply DOE technique to manipulator parameter design, to identify parameters responsible for performance variation and to find optimal combination, which deliver optimal performance. Discussed study is an offline strategy, which is novel and helps designer to select the parameters beforehand to reduce the performance variation, prior to actual manufacturing is carried out.

The rest of this paper is organized in six sections. In Section 2 steps required to apply DOE technique to manipulator design is discussed. In Section 3, developed novel simulation method to incorporate effect of noise factors and a search technique for simulating the performance is discussed. Data used for simulation and analysis of results of experiment are discussed in Sections 4 and 5. Finally, the investigation is concluded in Section 6.

2. Application of DOE technique to manipulator design

The robot parameters like its configuration, link dimensions, inertia, and actuators, etc. play vital role in its performance. Robotic system designer is required to make decisions regarding these parameters. Except in few specific applications, designer chooses a particular parameter or particular parameter combination by convenience, as no tools are available overlooking alternative solutions that may give optimal performance. By using the DOE approach for parametric design, it is possible to find the best combination of parameters for an optimal performance of the robotic manipulator. The DOE technique has been used successfully to optimize processes and designs for diverse non-robotic systems. A technique for application of DOE approach to a manipulator parameter design has been developed and presented next. A 2-DOF R–R planar manipulator is considered as an example to establish the application of parameter design technique to robot manipulators.

2.1. Statement of the problem

The main function for a robot manipulator is to accurately reach the commanded position. For the 2-DOF R–R planar manipulator, the target position is in the work-plane of the manipulator and is described by point $P(x, y)$ assuming xy-plane is the work-plane of the manipulator. The mathematical model to simulate the performance of manipulator and compute the position reached is presented in Section 2.1.1.

2.1.1. Kinematic and dynamic models of 2-DOF R–R planar manipulator

The kinematic and dynamic models of 2-DOF R–R planar manipulator are developed considering the manipulator shown in Fig. 1. The two links have lengths l_1, l_2, respectively. Let $P(x,y)$ be the target position of end-effector in the Cartesian workspace of the manipulator, the point of interest, for which the performance in terms of positional accuracy has to be modeled and optimized. The coordinates of P in Cartesian coordinate for joint angles θ_1 and θ_2 are given by [27]

$$x = l_1C_1 + l_2C_{12}, \tag{1}$$

$$y = l_1S_1 + l_2S_{12}, \tag{2}$$

where $C_i = \cos\theta_i$, $S_i = \sin\theta_i$, $C_{ij} = \cos(\theta_i+\theta_j)$ and $S_{ij} = \sin(\theta_i+\theta_j)$ with $i, j = 1, 2$ for the two links, link 1 and link 2, respectively.

From Eqs. (1) and (2), joint variables θ_1 and θ_2 are obtained as

$$\theta_1 = \tan^{-1}\left[\frac{y(l_1+l_2C_2) - xl_2S_2}{x(l_1+l_2C_2)+yl_2S_2}\right] \tag{3}$$

and

$$\theta_2 = \cos^{-1}\left[\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right]. \tag{4}$$

Differentiating Eqs. (1) and (2) and solving for joint velocities $\dot\theta_1$ and $\dot\theta_2$ gives

$$\dot\theta_1 = \frac{\dot xC_{12}+\dot yS_{12}}{l_1S_2} \tag{5}$$

and

$$\dot\theta_2 = -\frac{\dot x\,x+\dot y\,y}{l_1l_2S_2}, \tag{6}$$

where $(\dot x, \dot y)$ represent end-effector velocity $\vec v_e$ with $\dot x = v_e\cos\alpha$ and $\dot y = v_e\sin\alpha$, and α is angle made by $\vec v_e$ with positive x-axis of base frame.

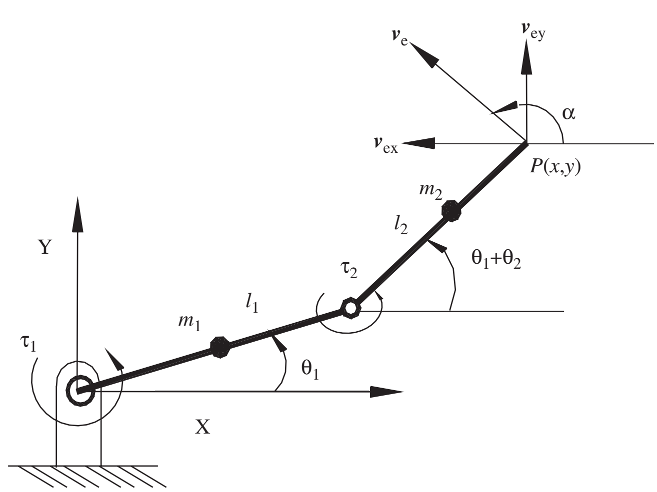

Fig. 1. 2-DOF R–R planar manipulator and its parameters.

Assuming link masses are m_1, m_2 and joint torques are τ_1, τ_2, respectively, and links as rigid members with mass concentrated at center of gravity, the dynamic model of 2-DOF R–R planar manipulator is [27]

$$\begin{aligned}\tau_1 = {} & \left[\left(\frac{m_1}{3}+m_2\right)l_1^2 + \frac{m_2}{3}l_2^2 + m_2l_1l_2C_2\right]\ddot\theta_1 \\ & + m_2\left(\frac{l_2^2}{3}+\frac{l_1}{2}l_2C_2\right)\ddot\theta_2 - m_2l_1l_2S_2\dot\theta_1\dot\theta_2 \\ & - \frac{m_2}{2}l_1l_2S_2\dot\theta_2^2 + \left(\frac{m_1}{2}+m_2\right)gl_1C_1 + \frac{m_2}{2}gl_2C_{12},\end{aligned} \tag{7}$$

$$\begin{aligned}\tau_2 = {} & \left[\frac{m_2}{3}l_2^2 + \frac{m_2}{2}l_1l_2C_2\right]\ddot\theta_1 + \frac{m_2}{3}l_2^2\ddot\theta_2 \\ & + \frac{m_2}{2}l_1l_2S_2\dot\theta_1^2 + \frac{m_2}{2}gl_2C_{12}.\end{aligned} \tag{8}$$

Eqs. (7) and (8) are solved for $\ddot\theta_1$ and $\ddot\theta_2$ and are written in a compact form as

$$\ddot\theta_1 = \frac{be-cd}{ad-b^2} \tag{9}$$

and

$$\ddot\theta_2 = \frac{bc-ae}{ad-b^2}, \tag{10}$$

where a, b, c, d and e, are given as

$$a = \left(\frac{m_1}{3}+m_2\right)l_1^2 + \frac{1}{3}m_2l_2^2 + m_2l_1l_2C_2, \tag{11}$$

$$b = m_2\left[\frac{l_2^2}{3}+\frac{1}{2}l_1l_2C_2\right], \tag{12}$$

$$\begin{aligned}c = {} & \left(\frac{m_1}{2}+m_2\right)gl_1C_1 + \frac{m_2}{2}gl_2C_{12} - m_2l_1l_2S_2\dot\theta_1\dot\theta_2 \\ & - \frac{m_2}{2}l_1l_2S_2\dot\theta_2^2 - \tau_1,\end{aligned} \tag{13}$$

$$d = \frac{m_2}{3}l_2^2, \tag{14}$$

$$e = \frac{m_2}{2}l_1l_2S_2\dot\theta_1^2 + \frac{m_2}{2}gl_2C_{12} - \tau_2. \tag{15}$$

Eqs. (1)–(15) are used to identify significant factors and compute the performance measures for the robotic manipulator.

2.2. Identification of factors and levels

Various parameters influencing the working of manipulator are identified with the help of a parameter diagram (P-Diagram) for manipulator as shown in Fig. 2. The parameters other than input and output are classified as control factor (CF) and noise factor (NF).

2.2.1. The control factors

A control factor is a product or process parameter whose values can be selected and controlled by the design or manufacturing engineer. The CFs for manipulator are identified from the kinematic and dynamic equations.

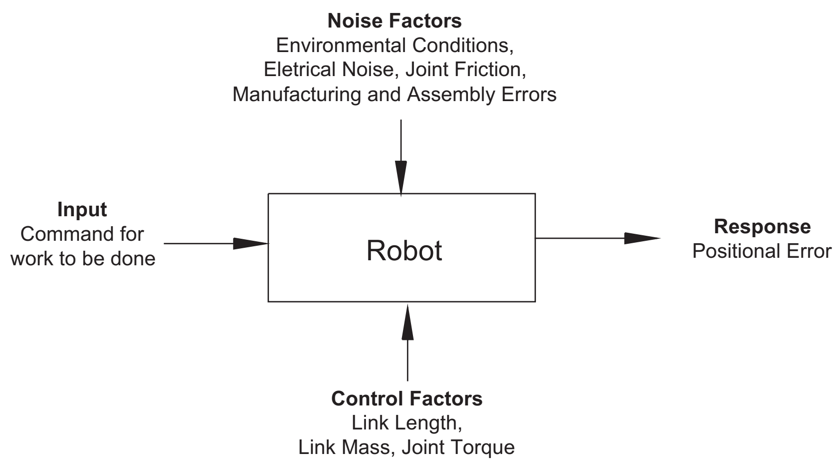

Fig. 2. Parameter diagram for robot performance.

From Eqs. (9) and (10), it is observed that $\ddot{\theta}_1$ and $\ddot{\theta}_2$ depend on following six independent parameters apart from the variables $\theta_1, \theta_2, \dot{\theta}_1$ and $\dot{\theta}_2$:

1. length of link 1, l_1;
2. length of link 2, l_2;
3. mass of link 1, m_1;
4. mass of link 2, m_2;
5. torque applied at joint 1, τ_1;
6. torque applied at joint 2, τ_2;

These six parameters are identified as CFs because designer can choose their values for optimal performance.

For design optimization using DOE technique, parameters can be considered at several levels. In this study, control factors are considered at two levels: high and low. Therefore, six control factors at two levels, results in a set of 2^6 (64) different combinations of six control factors.

2.2.2. Sources of noise in the manipulator—the noise factors

The function of the manipulator is to move its end-effector to a desired point accurately. However, the discrepancy exists between the desired and actual point reached because of noise factors. The NFs cause the end-effector to deviate from its target point. NFs are those factors that are difficult, expensive, or hard to control during production or operation. Some of the NFs that have direct influence on the performance of the manipulator are:

(a) environmental conditions in which manipulator operates,
(b) errors in manufacture and assembly,
(c) fluctuations in electricity supply, causing deviation in the joint actuator torque,
(d) friction at the manipulator joints, and
(e) joint compliance between the joint encoder and the actual angular output.

There can be several other noise factors, which may have influence on performance of the manipulator. As these noise factors are very difficult to quantify, their effects on performance are difficult to compute, a novel approach has been proposed to include the effects of NFs in the DOE to obtain a robust design of robotic manipulator. This approach is explained in Section 2.3.

2.3. Selection of performance measures for manipulator—the response variable

To investigate the impact of different parameters on performance variation of manipulator, several performance measures have been proposed by researchers. A performance measure for a manipulator is a parameter defined on the space of all postures of the manipulator that measures some general property of the manipulator that allows choosing the optimal combination of parameters [3,4]. For a robotic manipulator positional performance parameters are accuracy, repeatability, and resolution. A combination of these parameters is defined as positional error ε_i as distance between the actual point reached by the end-effector $P_i(x_i, y_i, z_i)$ in the ith experiment and desired point $P(x, y, z)$, that is

$$\varepsilon_i = \left((x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2\right)^{1/2}. \tag{16}$$

The following four performance measures have been considered in this work.

(a) *Positional error*: For the 2-DOF R–R planar manipulator with xy-plane as the workplane positional error ε_i from Eq. (16) will reduce to

$$\varepsilon_i = \left((x_i - x)^2 + (y_i - y)^2\right)^{1/2}. \tag{17}$$

The objective function to be optimized is to minimize the positional error considering the uncertainty.

(b) *Mean positional error*: Mean positional error $\bar{\varepsilon}$ is defined as

$$\bar{\varepsilon} = \frac{1}{n}\sum_{i=1}^{n} \varepsilon_i, \tag{18}$$

where n is number of experiments or replications and ε_i is positional error for ith experiment.

(c) *Signal to noise ratio*: The signal to noise (SN) ratio proposed by Taguchi, is used as the data transformation method to consolidate the repetitive data into one value, which reflects the mean value and amount of variation present in the data. For the robotic manipulator, the design objective is to minimize the positional error hence, it is always desired that it should be as small as possible. Therefore, as per Taguchi method, the target is of 'smaller-the-better' type and SN ratio for the smaller-the-better case is given as [28]

$$SN = -10\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i^2\right) \tag{19}$$

SN ratio is an essential indicator of the ability of the system to perform well in relation to the effect of noise and measure to carry out the analysis of experiment.

(d) *Reliability*: The reliability (R) of a manipulator is defined by Bhatti and Rao [8] as the probability of the end-effector reaching a point or in a close vicinity of it within specified range. If the end-effector reaches a point within the specified range, it is considered as a successful experiment. The reliability R is given as

$$R = \frac{\text{Number of successful experiments}}{\text{Total number of experiments}} = \frac{s}{n}. \tag{20}$$

The specified range around a target point is called the permissible error region and its shape and size depends on the intended use of the manipulator. The reliability as performance measure has been used to evaluate the overall performance of all control factor combinations. The results obtained are used for validating the outcome of DOE technique to manipulator design.

2.4. Design the experiment—choice of experimental design

Experiment is defined as a test in which purposeful changes are made to the input variables of a system so that reasons for changes in the output response observed can be identified. There are several factors of interest in an experiment therefore to deal with these factors a factorial experiment is used. Factorial experiment is an experimental strategy in which design factors are varied together and effects of all possible combinations of the levels of factors for the experiment are investigated. For a general case where there are a levels of factor A, b levels of factor B, c levels of factor C, arranged in a factorial experiment, there will be $a \times b \times c \times n$ total observations, where n is number of replications of the complete experiment. One of the special cases is that of k number of factors, each at two levels such a design requires 2^k observations and is called a 2^k factorial design. Hence, factorial design of six control factors, each at two levels leads to 2^6 runs and design matrix of these factors, are presented in Appendix A.

2.5. Performing and analyzing the experiment

To conduct the experiments, strategies adopted to simulate performances are discussed below in Section 3. To investigate the effect of parametric variation on performance of manipulator, an experiment is conducted with the help of design matrix. Each factorial combination is run for finite number of replications to capture the effect of noise. The developed methodology and data utilized to simulate the performance is provided in Sections 4 and 5. It is important to mention that while simulating the performance for the experiment no constraints such as maximum velocity and acceleration of links of manipulator are imposed.

From each replication positional error as outcome of the experiment is obtained and thereafter the performance measure SN ratio and mean positional error are computed for each CF combinations. For parameter optimization problem, optimal CF combination should have lowest performance variation and nearer to target mean performance. When both the conditions are satisfied SN ratio becomes highest. Therefore to obtain optimal parameter combination, SN ratio values are compared after the conduct of experiment.

To compute reliability of a particular parameter combination, simulation is conducted for more number of cycles. The Cartesian coordinates obtained from each simulation determine whether the point lies within the specified region or not. The reliability of the parameter combination is computed using Eq. (20).

Statistical analyses of performance of experiment are carried out using analysis of variance (ANOVA) technique, which is a powerful tool for understanding complex physical phenomenon. ANOVA is used to subdivide the total variation into variation due to main factors, variation due to interacting factors and variation due to error. After this statistical tests like **F**-test are used to study statistically significant CF and interacting factors, which helps in screening many factors to discover the vital few and how they interact. For current study statistically significant CF and interacting factors are determined using ANOVA, and its results are discussed in Section 6.

3. Strategy to incorporate effects of noise for parameter design

For given set of control factor values and target point $P(x, y)$, following six parameters are computed from the kinematic and dynamic model equations (3)–(6) and (9)–(10)

1. angular displacements θ_1, θ_2,
2. angular velocities $\dot{\theta}_1$, $\dot{\theta}_2$,
3. angular accelerations $\ddot{\theta}_1$, $\ddot{\theta}_2$.

The computed values of these six parameters are free from the effects of noise. In order to incorporate the effect of noise in the above parameters, the effect of noise, in form of individual errors for the six control factors is generated randomly. The randomly generated errors are assumed to follow normal distribution with zero mean and specified standard deviation. Using set of values of control factors with noise, the above six parameters with noise incorporated are obtained as: angular displacements (θ_1^n, θ_2^n) from Eqs. (3) and (4); angular velocities ($\dot{\theta}_1^n$, $\dot{\theta}_2^n$) from Eqs. (5) and (6); and angular accelerations ($\ddot{\theta}_1^n$, $\ddot{\theta}_2^n$) from Eqs. (9) and (10). To compute the point actually reached by end-effector with the presence of noise in control factors, a search technique has been developed and is described in Section 3.1.

3.1. Search technique

Taking computed $\ddot{\theta}_1^n$, $\ddot{\theta}_2^n$ as input and control factors at nominal values (without noise) the angular displacements of links θ_1^a and θ_2^a are obtained using Eqs. (9) and (10), where superscript 'a' indicates actual values with noise present. Since Eqs. (9) and (10) are nonlinear, transcedental equations, the values of θ_1^a and θ_2^a are obtained using a heuristic search algorithm.

The steps of the algorithm are given below:

Algorithm 1. Search algorithm

Step 1: Read nominal level values of six control factors l_1, l_2, m_1, m_2, τ_1 and τ_2.
Step 2: Read standard deviations σ_{l_1}, σ_{l_2}, σ_{m_1}, σ_{m_2}, σ_{τ_1} and σ_{τ_2} for the six control factors.
Step 3: Read the range and step size for θ_1, θ_2 variations and permissible error e.
Step 4: Read the manipulator target point $P(x, y)$.
Step 5: Obtain θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$ and $\ddot{\theta}_2$ for input (nominal values) of control factors from Eqs. (3)–(6) and (9)–(10).
Step 6: Generate random errors based on standard deviations for six control factors and obtain control factor values with noise. Using these values of control factors, compute θ_1^n, θ_2^n, $\dot{\theta}_1^n$, $\dot{\theta}_2^n$, $\ddot{\theta}_1^n$, and $\ddot{\theta}_2^n$ from Eqs. (3)–(6) and (9)–(10).
Step 7: Make starting guess for θ_1^a and θ_2^a, which is equal to θ_1 and θ_2 in Step 5
Step 8: Compute $\ddot{\theta}_1^a$, $\ddot{\theta}_2^a$ from Eqs. (9) and (10).
Step 9: Compare the values of $\ddot{\theta}_1^a, \ddot{\theta}_2^a$ obtained in Step 8 with values of $\ddot{\theta}_1^n, \ddot{\theta}_2^n$ obtained in Step 6. If differences are with in the specified permissible error e go to Step 10, else increment θ_1^a and θ_2^a for chosen step size within the range and go to Step 8.
Step 10: Terminate the search and return θ_1^a and θ_2^a.

For each factorial combination simulations are performed to obtain the individual performance measure of the experiment. The simulation is also run for required number of replications to compute combined performance measure. Likewise for other factorial combinations and replications, simulations are carried out to compute performance measure discussed.

4. Simulation

The numerical values used to simulate the performance are given below:

(a) Number of levels for each control factor = 2.
(b) Nominal values of six control factors at two levels and standard deviations are given in Table 1.
(c) Number of combinations in factorial design = $2^6 = 64$.
(d) Design matrix containing 64 CF combinations is provided in Appendix A.
(e) Chosen number of replications for each combination = 8 (for factorial design) = 100 (for reliability).
(f) Coordinates of target point in workspace: case I: $x = 0.40$ m, $y = 0.30$ m and case II: $x = 0.50$ m, $y = 0.40$ m.
(g) The step size of increment for search, range of search and permissible error value e for the search algorithm were chosen as: $\theta_{1(\text{incr})} = 0.01\theta_1$, $\theta_{2(\text{incr})} = 0.01\theta_2$ $0.5\,\theta_1 \leqslant \theta_1 \leqslant 1.5\,\theta_1$ and $0.5\,\theta_2 \leqslant \theta_2 \leqslant 1.5\,\theta_2$, $e \approx 0.05$.
(h) Tolerance selected for target point for computation of reliability $\Delta x = \pm 0.0005$ m, $\Delta y = \pm 0.0005$ m.

Using the above numerical values simulation were carried out. To simulate the performance for each factor combination is run for eight replications for cases I and II. The simulated performances are analyzed for statistical analysis and its results are provided in Tables 2 and 3, respectively. The performance measures, i.e. positional error and *SN* ratio are computed for each parameter combination and its trend are displayed in Figs. 3(a) and 3(b) for case I and Figs. 4(a) and 4(b) for case II. To validate the results of factorial design, performance measure reliability is computed after running the simulation for 100 cycles. The results for each combination are displayed in Figs. 3(c) and 4(c) for the two cases, respectively. After comparing the values of performance measure, optimal combinations having optimal performance are presented in Tables 4 and 5.

Table 1
Values for control factors at two different levels

Control factors	Low level	High level	Standard deviation
l_1 (m)	0.40	0.50	0.0001
l_2 (m)	0.25	0.35	0.0001
m_1 (kg)	5.5	6.5	0.01
m_2 (kg)	4.0	5.0	0.01
τ_1 (N m)	–500	–800	0.1
τ_2 (N m)	–100	–105	0.1

Table 2
Analysis of variance of factorial design for case I

Source	Sum of squares	DF	Mean square	**F**-value	Remark
l_1	11.52	1	11.52	3.67	Significant
l_2	47.33	1	47.33	58.72	Significant
m_1	3.154×10^{-5}	1	3.154×10^{-5}	3.913×10^{-5}	
m_2	30.19	1	30.19	37.46	Significant
τ_1	1.02	1	1.02	1.26	
τ_2	2.56	1	2.56	3.18	
l_1l_2	1.13	1	1.13	1.040	
$m_1\tau_1$	3.53	1	3.53	4.38	Significant
$m_2\tau_1$	4.03	1	4.03	5.00	Significant
$l_1l_2m_1\tau_1\tau_2$	3.56	1	3.56	4.41	Significant
Residual	385.27	478	0.81		
Corrected sum total	506.35	511			

Table 3
Analysis of variance of factorial design for case II

Source	Sum of squares	DF	Mean square	**F**-value	Remark
l_1	193.54	1	193.54	46.22	Significant
l_2	415.76	1	415.76	99.29	Significant
m_2	44.59	1	44.59	10.65	Significant
τ_1	39.04	1	39.04	9.32	Significant
l_1l_2	47.08	1	47.08	11.24	Significant
$l_1\tau_1$	147.30	1	147.30	35.18	Significant
$l_2\tau_1$	11.03	1	11.03	2.63	
$l_1l_2\tau_1$	80.47	1	80.47	19.22	Significant
Residual	2106.19	503	4.19		
Corrected sum total	3085.01	511			

5. Analysis of performance results

Analysis of the experimental results has been done using ANOVA technique and for computational work Design Expert (DX) Version-6 software [29] is used. Since each control factor has two levels, 8 replications of six-factor experiment required 512 simulations to obtain performance measure. Analysis of test cases mentioned is carried out and results are discussed below.

(i) The performance measure utilized for analysis of experiment is positional error (ε_i). For test case I, it is observed that control factors l_1, l_2, m_2, and interacting factors $m_1\tau_1, m_2\tau_1, l_1l_2m_1\tau_1\tau_2$ are statistically significant by comparing computed **F** statistic value with tabulated **F** statistic value for given factor and interaction factor combination. The results of ANOVA are presented in Table 2 and indicate that all control factors have statistically significant role to play in performance measure apart from the interactions. None of the control factors go out of contention for parameter design. Observing this *SN* ratio and reliability, are used to find the suitable control factor combinations for which performance is optimal. For case II, it is observed that control factors l_1, l_2, m_2, τ_1 and interacting factors l_1l_2, $l_1\tau_1$,$l_1l_2\tau_1$ are significant. The results of ANOVA are given in Table 3.

(ii) From the simulation results for mean positional error, *SN* ratio and reliability, for case I, in Figs. 3(a), 3(b) and 3(c), respectively, for 64 parameter combinations optimal parameters are selected. The basis for selection of optimum parameter combination is that it should have maximum value of *SN* ratio and reliability and minimum mean positional error. The optimum parameter combination using above performance measure are shown in Table 4. Similarly, from simulation of performance measures for case II as given in Figs. 4(a), 4(b) and 4(c), respectively, for 64 parameter combinations, the optimum parameter combinations are identified and presented in Table 5.

(iii) For case I, it is observed from Fig. 3(a) that mean positional error is maximum (2.2003×10^{-2} m) at combination number 42 and is minimum (0.07355×10^{-2} m) at combination number 29. For case II mean positional error is maximum (6.685×10^{-2} m) at combination number 4 and is minimum (0.88964×10^{-2} m) at combination number 55 as observed in Fig. 4(a). It is also observed that mean positional error is less than 2×10^{-2} m from combination numbers 47 to 64.

It is observed that statistically significant factors are different for different target points in workspace indicating that different factors have different contribution to performance variation as target position changes. It is important to notice that statistically significant factors are major contributor to performance variation of manipulator and even there is mathematical relationship and all parameters are utilized in simulating the performance. In addition to this optimum combination of control factors required to perform task are different for both the cases, which indicate that one set of parameters will behave differently for different type of task. It is seen that *SN* ratios of different combinations are increasing from combination no. 21st to 33rd and 53rd to 64th and maximum reliability appear at 16th and 33rd combination number of control factors for case I. Except few, all combination show poorer performance for case II as compared to case I.

The selection of suitable tolerance range for reliability computation is done by trial and error, because it is observed that a wider tolerance range gives higher reliability and a tighter tolerance gives poorer reliability for all factor combinations. Finally, the optimum factor combinations obtained using *SN* ratio and reliability do not agree in the both the cases but trends and performance measure peaks observed for different combination numbers

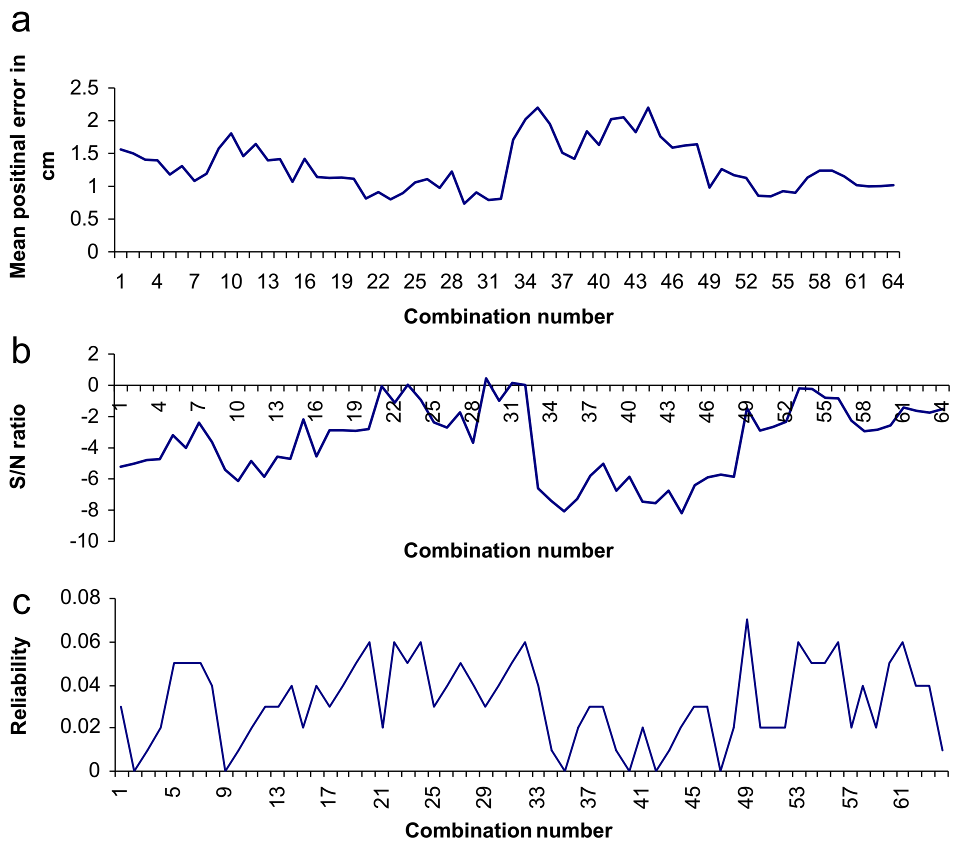

Fig. 3. Performance measures for case I: (a) mean positional error; (b) SN ratio; (c) reliability.

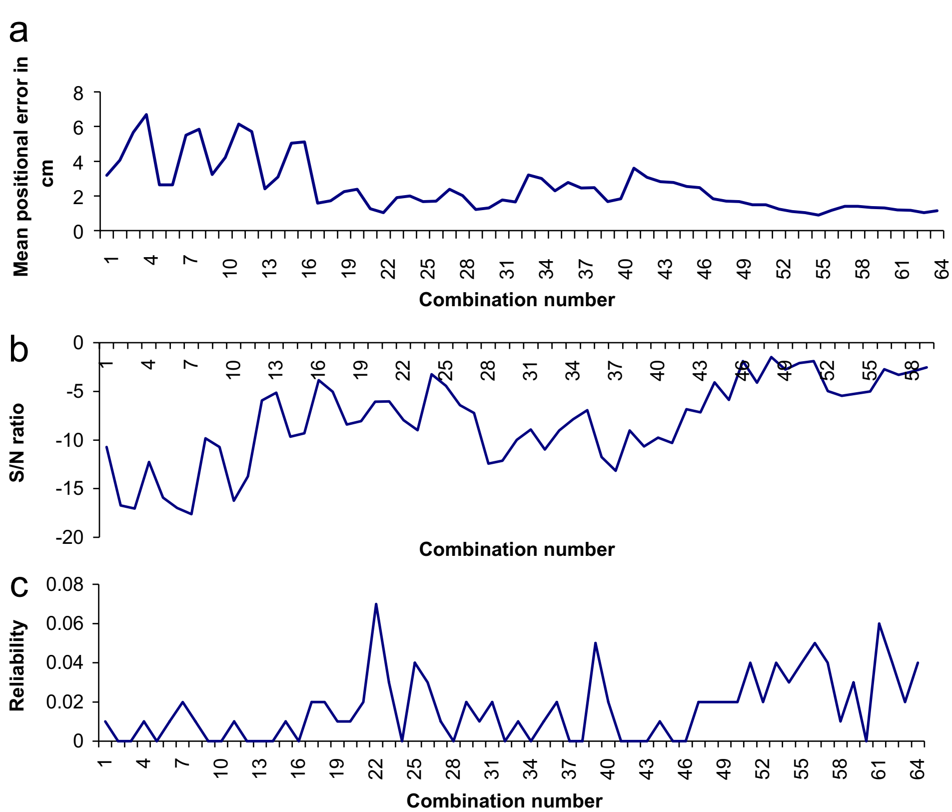

Fig. 4. Performance measures for case II.

Table 4
Optimum parameters for different performance measure for case I

Control factor	Target at $x = 0.40\,\mathrm{m}$, $y = 0.30\,\mathrm{m}$		
	SN ratio	Reliability	Mean positional error (m)
Value (combination no.)	0.446933 (29)	0.071 (49)	0.00739 (29)
l_1 (m)	0.40	0.50	0.40
l_2 (m)	0.35	0.35	0.35
m_1 (kg)	6.5	5.5	6.5
m_2 (kg)	5	4	5
τ_1 (N m)	–500	–500	–500
τ_2 (N m)	–105	–100	–105

Table 5
Optimum parameters for different performance measure for case II

Control factor	Target at $x = 0.50\,\mathrm{m}$, y = 0.40 m		
	SN ratio	Reliability	Mean positional error (m)
Value (combination no.)	–1.66023 (48)	0.072 (22)	0.00889 (55)
l_1 (m)	0.50	0.40	0.50
l_2 (m)	0.25	0.35	0.35
m_1 (kg)	6.5	5.5	5.5
m_2 (kg)	5	5	5
τ_1 (N m)	–800	–500	–800
τ_2 (N m)	–105	–105	–100

are equally comparable in both the cases. Possible reason for disagreement in optimal solution could be due to the transformation of positional error into the Taguchi's *SN* ratio which may not be same as the untransformed result obtained from reliability computation. It is observed that most of the combinations for cases I and II show poor simulation performance, possible reason may be attributed to the assumed retarding torque not being able to satisfy the requirement. Other reason may be the inherent round off error present in search solution computation.

6. Conclusions

Present work tries to look at the performance variation problem of manipulator in robot manufacturer and designer's perspective and its suitable solutions. It gives an insight in use of simulation method for modeling and optimizing the performance of robot manipulators. It illustrates a novel approach, i.e. search (heuristic) based method to obtain performance measures of manipulator. Followed by use of ANOVA technique to determine statistically significant parameter and parameter combinations. While simulating performance of manipulator no constraints on angular velocity and accelerations are applied. The approach presented will help in determining significant control factors responsible for performance variation and identification of the optimum parameters rather than spending effort in controlling performance of

Table A1

Combination number	l_1 (m)	l_2 (m)	m_1 (kg)	m_2 (kg)	τ_1 (N m)	τ_2 (N m)
1	0.40	0.25	5.5	4	−500	−100
2	0.40	0.25	5.5	4	−500	−105
3	0.40	0.25	5.5	4	−800	−100
4	0.40	0.25	5.5	4	−800	−105
5	0.40	0.25	5.5	5	−500	−100
6	0.40	0.25	5.5	5	−500	−105
7	0.40	0.25	5.5	5	−800	−100
8	0.40	0.25	5.5	5	−800	−105
9	0.40	0.25	6.5	4	−500	−100
10	0.40	0.25	6.5	4	−500	−105
11	0.40	0.25	6.5	4	−800	−100
12	0.40	0.25	6.5	4	−800	−105
13	0.40	0.25	6.5	5	−500	−100
14	0.40	0.25	6.5	5	−500	−105
15	0.40	0.25	6.5	5	−800	−100
16	0.40	0.25	6.5	5	−800	−105
17	0.40	0.35	5.5	4	−500	−100
18	0.40	0.35	5.5	4	−500	−105
19	0.40	0.35	5.5	4	−800	−100
20	0.40	0.35	5.5	4	−800	−105
21	0.40	0.35	5.5	5	−500	−100
22	0.40	0.35	5.5	5	−500	−105
23	0.40	0.35	5.5	5	−800	−100
24	0.40	0.35	5.5	5	−800	−105
25	0.40	0.35	6.5	4	−500	−100
26	0.40	0.35	6.5	4	−500	−105
27	0.40	0.35	6.5	4	−800	−100
28	0.40	0.35	6.5	4	−800	−105
29	0.40	0.35	6.5	5	−500	−100
30	0.40	0.35	6.5	5	−500	−105
31	0.40	0.35	6.5	5	−800	−100
32	0.40	0.35	6.5	5	−800	−105
33	0.50	0.25	5.5	4	−500	−100
34	0.50	0.25	5.5	4	−500	−105
35	0.50	0.25	5.5	4	−800	−100
36	0.50	0.25	5.5	4	−800	−105
37	0.50	0.25	5.5	5	−500	−100
38	0.50	0.25	5.5	5	−500	−105
39	0.50	0.25	5.5	5	−800	−100
40	0.50	0.25	5.5	5	−800	−105
41	0.50	0.25	6.5	4	−500	−100
42	0.50	0.25	6.5	4	−500	−105
43	0.50	0.25	6.5	4	−800	−100
44	0.50	0.25	6.5	4	−800	−105
45	0.50	0.25	6.5	5	−500	−100
46	0.50	0.25	6.5	5	−500	−105
47	0.50	0.25	6.5	5	−800	−100
48	0.50	0.25	6.5	5	−800	−105
49	0.50	0.35	5.5	4	−500	−100
50	0.50	0.35	5.5	4	−500	−105
51	0.50	0.35	5.5	4	−800	−100
52	0.50	0.35	5.5	4	−800	−105
53	0.50	0.35	5.5	5	−500	−100
54	0.50	0.35	5.5	5	−500	−105
55	0.50	0.35	5.5	5	−800	−100
56	0.50	0.35	5.5	5	−800	−105

Table A1 (*continued*)

Combination number	l_1 (m)	l_2 (m)	m_1 (kg)	m_2 (kg)	τ_1 (N m)	τ_2 (N m)
57	0.50	0.35	6.5	4	−500	−100
58	0.50	0.35	6.5	4	−500	−105
59	0.50	0.35	6.5	4	−800	−100
60	0.50	0.35	6.5	4	−800	−105
61	0.50	0.35	6.5	5	−500	−100
62	0.50	0.35	6.5	5	−500	−105
63	0.50	0.35	6.5	5	−800	−100
64	0.50	0.35	6.5	5	−800	−105

manipulator. This approach provides a powerful design tool for selecting combination of parameters for optimal performance of manipulator.

Appendix A. Design matrix for control factor combinations

The design matrix for control factor combinations is given in Table A1.

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