Dimensional Optimization Design of the Four-Cable-Driven Parallel Manipulator in FAST

Rui Yao, Xiaoqiang Tang, Jinsong Wang, and Peng Huang

***Abstract*—For the design of the five-hundred-meter aperture spherical radio telescope (FAST), a four-cable-driven parallel manipulator, which is long in span and heavy in weight, is adopted as the first-level adjustable feed-support system. The purpose of this paper is to optimize dimensions of the four-cable-driven parallel manipulator to meet the workspace requirement of constraint condition in terms of cable tension and stiffness. Accordingly, this optimization method adopts catenary simplification in order to set up the cable tension equilibrium equations, preliminarily optimizing three important dimensional parameters. Stiffness of the cable is also taken into consideration because of its effect on work performance of a cable-driven parallel manipulator. However, the stiffness value of a cable-driven parallel manipulator is not totally credible by traditional theoretical analysis. Therefore, an experimental method for stiffness analysis is presented in this paper. It applies Buckingham π theorem to set up an experimental stiffness similarity model to obtain a more credible stiffness value, which is used in the optimization process. In this way, dimensional optimization is realized with a set of optimized dimensions for building the feed-support system in the FAST. More importantly, the stiffness similarity model can be universally adopted in stiffness analysis of other large cable-driven parallel manipulator.**

***Index Terms*—Cable-driven parallel manipulator, optimization design, radio telescope, similarity model.**

I. Introduction

IN 1993, the large telescope was proposed by astronomers from ten countries at the General Assembly of the International Union Radio Science, which sponsored the development of a new generation of radio telescope with large receiving area of about 1 km^2. Chinese engineers began the conceptual design of a five-hundred-meter aperture spherical radio telescope (FAST) in 1994, and this large radio telescope will be built in a karst depression in Guizhou Province of southwest China [1], [2]. After more than ten years' efforts, the FAST team has already developed a complete layout design. Due to the largeness in FAST, no solid-support system is available for orientation of the feed. Therefore, a four-cable-driven parallel manipulator is introduced for the first adjustable feed-support system to provide a three-translational movement.

Since cable-driven parallel manipulator has advantages of simple configuration, high-load ability, large workspace, low price, and high speed, therefore, it is widely used [3], [4]. However, the cable-driven parallel manipulator can only bear tension, but not compression. Therefore, a cable system with j end-effector DOFs requires at least $i = j + 1$ cables [5], [6]. For three-translational movements of the feed in the FAST, a four-cable-driven parallel manipulator is developed.

The optimization target of the previous study on completely restrained positioning mechanisms (CRPMs) dimensional optimization design is workspace requirement and constraint condition is cable tension and stiffness [7]–[9], which will greatly influence the accuracy and vibration of a cable-driven parallel manipulator. Therefore, the purpose of this paper is to optimize dimensions of the four-cable-driven parallel manipulator to meet the workspace requirement of constraint condition in terms of cable tension and stiffness.

So far, great progress has been made in this field. However, two difficulties arose during the dimensional optimization of the cable-driven parallel manipulator: cable catenary algorithm and stiffness value of the four-cable-driven parallel manipulator.

Firstly, the weight of cable itself results in catenary. But most of previous research neglected the cable's catenary because the weight of the cable was not taken into consideration, while other researchers used precision catenary algorithm for cable catenary modeling [10], [11]. However, the precision catenary algorithm is complex, and therefore, slow for real-time control.

Over the past several decades, the precision catenary algorithm has been simplified and improved in the field of bridge and building construction [12]–[14]. Based on earlier improvements, the simplification of a catenary to a parabola is introduced for the research on the four-cable-driven parallel manipulator in this paper.

Secondly, there is no convincing theoretical method so far to achieve a totally credible stiffness value for a cable-driven parallel manipulator. A precision stiffness analysis must be taken into consideration for yield of drive, friction between the cables and feed system, and friction between cables and pulley. In general, there are two methods for stiffness solution. The first is, theoretical method to approximately obtain overall stiffness of cable-driven parallel manipulator [15], [16]; the second one is, experimental method to measure the stiffness [17], [18], which is more credible. Nevertheless, the four-cable-driven parallel manipulator is too large to build a real-size prototype to measure the stiffness. Therefore, a similarity experimental method for stiffness solution is worth exploring, which is also the focus of this paper.

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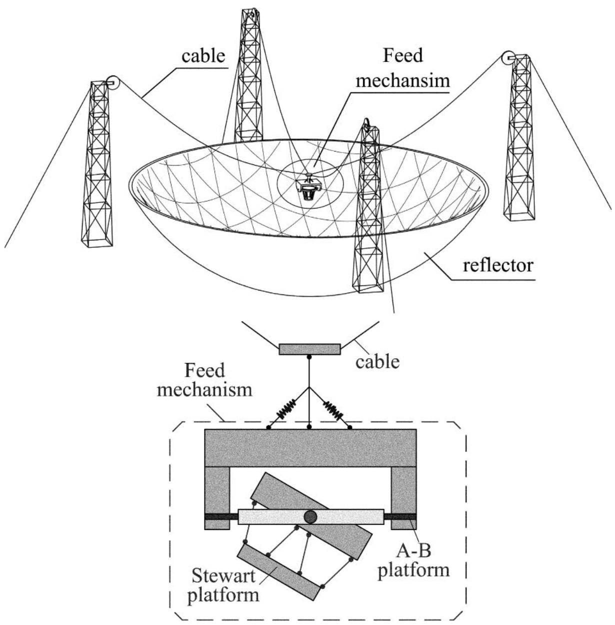

Fig. 1. Feed-support system of FAST.

At present, similarity method is widely applied in geological survey and architecture [19], [20]. But this method has not been used in complex mechanism. There are two main similarity methods for modeling: Buckingham π theorem and differential equations [21]–[23]. The Buckingham π theorem, which is the basis of most dimensional analyses, asserts that any complete physical relationship can be expressed in terms of a set of independent dimensionless products composed of the relevant parameters. Differential equations can describe the dynamic processes of a problem solution based on all relevant parameters.

It should be noted that similarity method has been used successfully in parallel manipulator analysis [24], [25]. It can also be applied to obtain the stiffness value of the four-cable-driven parallel manipulator. Since not all factors influencing stiffness can be expressed as a mathematical representation, Buckingham π theorem is more appropriate as the similarity method in this paper.

In this research on the four-cable-driven parallel manipulator, tension equilibrium equation is established by the way of simplified catenary equation. Then, workspace requirement of tension constraint conditions is put forward to preliminarily optimize the dimensional parameters. Based on Buckingham π theorem, a stiffness similarity model of the four-cable-driven parallel manipulator is set up. From stiffness experiments, real stiffness value, and the relationships between dimensional parameters and stiffness of the four-cable-driven parallel manipulator are obtained. Finally, a set of optimized dimensional parameters of the four-cable-driven parallel manipulator is available for building the feed-support system in FAST.

II. Description of the Four-Cable-Driven Parallel Manipulator in FAST

In Fig. 1, the feed-support system of the FAST includes two parts: a cable-driven parallel manipulator is the first-level feed-support system and a feed mechanism is the second-level feed-support system, which includes an A–B platform and a Stewart platform, with a feed on its moving platform.

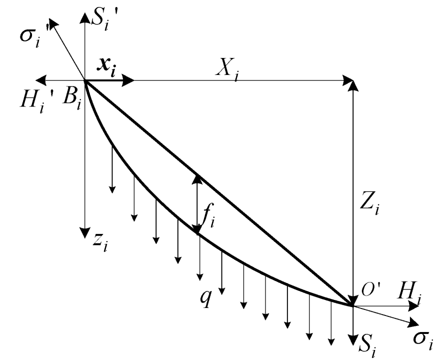

Fig. 2. Cable in its vertical section.

For the design of the cable-driven parallel manipulator in the FAST, several design proposals are made, one of which is the four-cable-driven parallel manipulator. Therefore, we will take this four-cable-driven parallel manipulator as an example to study the modeling and dimensional optimization method of a cable-driven parallel manipulator.

The four-cable-driven parallel manipulator in FAST draws the feed mechanism in three-translational movements. The A–B platform, as the moving platform of the four-cable-driven parallel manipulator, can provide proper pose angle of feed to track celestial bodies. The Stewart platform, fixed on the A–B platform, is to improve the orientation accuracy of the feed. Since the feed is slow in movement, all working positions of four-cable-driven parallel manipulator are supposed to be in the static balance.

The dimension parameters can greatly affect the workspace, cable tension, and stiffness of the four-cable-driven parallel manipulator, and therefore, we have to obtain a set of dimension parameters to meet workspace requirement, cable tension condition and get a better stiffness of the four-cable-driven parallel manipulator. The "optimization" in this paper aims to get a set of better dimension parameters for the four-cable-driven parallel manipulator.

III. Tension Equilibrium Equation of the Four-Cable-Driven Parallel Manipulator

In small cable-driven mechanism, due to the relatively inconsiderable weight of the cable, a cable is often analyzed as a two-force member. However, the cable span of the four-cable-driven parallel manipulator in FAST is more than 300 m and the cable weight is about 1/5 of the feed mechanism. Therefore, the catenary of the cable must be taken into account in the design of the four-cable-driven parallel manipulator.

Previous study has indicated that a cable can be seen as a parabola when the ratio of cable sag and span (i.e., sag ratio) is relatively small. As a result of the parabola simplification, a cable of the four-cable-driven parallel manipulator model in its vertical section is shown in Fig. 2. For analysis, the symbols used in Fig. 2 are defined as follows: σ_i is the tension of the cable, H_i is the horizontal component of tension σ_i, S_i is the vertical component of tension σ_i, X_i is the horizontal length of the cable's span, Z_i is the vertical length of the cable's span, q_o is the cable weight per meter, q is the cable weight distributed along the span length, m_0 is the mass of the feed mechanism, l_i is the real length of the cable, and f_i is the sag of the cable at the center of the span.

The condition of equilibrium for the cable is as follows:

$$H_i \frac{d^2 z_i}{dx_i^2} + q = 0. \tag{1}$$

By integrating the equation twice, and substituting the boundary conditions $z_i = 0$ at $x_i = 0$, and $z_i = Z_i$ at $x_i = X_i$, we obtain

$$q = \frac{q_0 l_i}{X_i} \tag{2}$$

$$z_i = \frac{q x_i}{2H_i}(X_i - x_i) + \frac{Z_i}{X_i} x_i \tag{3}$$

$$H_i = \frac{q X_i^2}{8 f_i}. \tag{4}$$

Then the length of cable can be expressed as follows:

$$l_i = \int_0^{X_i} \left[1 + \left(\frac{dz_i}{dx_i} \right)^2 \right]^{1/2} dx_i. \tag{5}$$

Expanding the expression $[1 + (dz_i/dx_i)^2]^{1/2}$ binomially, and substituting it into (3)–(5) yields

$$l_i = X_i \left(1 + \frac{8 f_i^2}{3 X_i^2} + \frac{1}{2} \frac{Z_i^2}{X_i^2} - \frac{Z_i^4}{8 X_i^4} \right). \tag{6}$$

The moment equilibrium equation is as follows:

$$S_i = \frac{q X_i}{2} - H_i \frac{Z_i}{X_i}. \tag{7}$$

From (3)–(7), the expression for σ_i can be derived as follows:

$$\sigma_i = (H_i^2 + S_i^2)^{1/2}. \tag{8}$$

As shown in Fig. 3, two coordinates are set up in the four-cable-driven parallel manipulator: an inertial frame $\Re$: O-XYZ is located at the center of the reflector's bottom. Another moving frame $\Re'$: O'-$X'Y'Z'$is located at the intersected node of the four cables. Four cable towers are distributed symmetrically in a circle. B_i $(i = 1, 2, \ldots, 4)$ are the connected points of the cables and cable towers.

For analysis, the symbols used in this paper are defined as follows: ${}^{\Re}\boldsymbol{O}'$ is the vector $\boldsymbol{O}'$expressed in the inertial frame, ${}^{\Re}\boldsymbol{B}_i$ is the vector $\boldsymbol{B}_i$ expressed in the inertial frame, D is the diameter of the cable towers' distributed circle, D_o is the aperture of the reflector, h is the height of the cable tower, d is the diameter of the cable, and m_0 is the weight of the feed mechanism. The symbols with subscript i express the parameters of the ith cable.

According to Fig. 3, the vector of the cables can be expressed as follows:

$${}^{\Re}\boldsymbol{L}_i = {}^{\Re}\boldsymbol{B}_i - {}^{\Re}\boldsymbol{O}' \tag{9}$$

$$L_i = \left\| {}^{\Re}\boldsymbol{L}_{\rm i} \right\|, \qquad i = 1, 2, \ldots, 4 \tag{10}$$

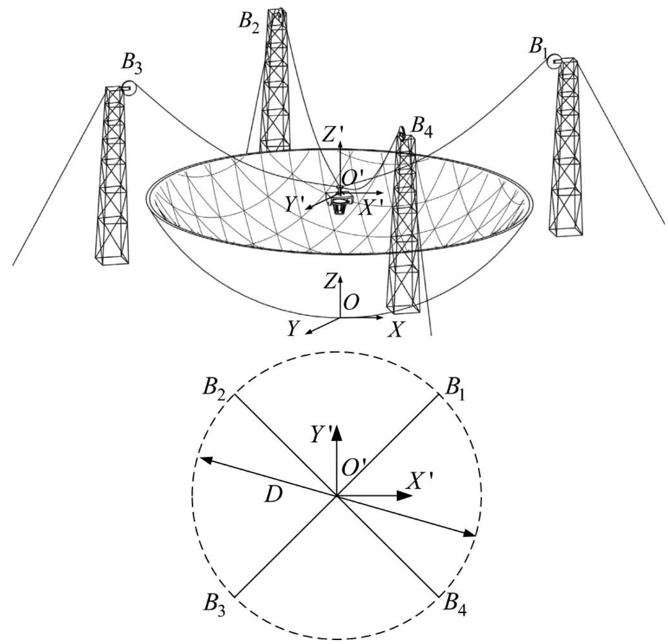

Fig. 3. Geometric parameter of the four-cable-driven parallel manipulator.

$$\boldsymbol{u}_i = {}^{\Re}\boldsymbol{L}_i / L_i, \qquad \boldsymbol{r}_i = {}^{\Re}\boldsymbol{O}'\boldsymbol{A_i} \tag{11}$$

where L_i is the span of the ith cable.

According to the virtual work principle, static equilibrium equations of the four-cable-driven parallel manipulator can be written as follows:

$$\boldsymbol{F} = J^{\boldsymbol{T}} \boldsymbol{\sigma} \tag{12}$$

where $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \ldots, \sigma_4]^{\rm T}$,

$$\boldsymbol{J}^{\rm T} = \begin{bmatrix} \boldsymbol{u}_1 & \cdots & \boldsymbol{u}_4 \\ \boldsymbol{r}_1 \times \boldsymbol{u}_1 & \cdots & \boldsymbol{r}_4 \times \boldsymbol{u}_4 \end{bmatrix}.$$

Considering the cable weight $\mathbf{F}$ should be expressed as follows:

$$\boldsymbol{F} = \left[0, 0, m_0 g + \sum_{i=1}^{4} q_0 l_i \right]^{\rm T} \tag{13}$$

where $\boldsymbol{J}^{\rm T} \in R^{3\times4}$ is the tension transmission matrix.

When $\boldsymbol{J}^{\rm T}$ is nonsingular, $\boldsymbol{\sigma} = \left(\boldsymbol{J}^{\rm T}\right)^{+} \boldsymbol{F}$ is the least 2-norm solution, $\left(\boldsymbol{J}^{\rm T}\right)^{+}$ is the Moore–Penrose general inverse matrix of $\boldsymbol{J}^{\rm T}$, then

$$\boldsymbol{\sigma} = \left(\boldsymbol{J}^{\rm T}\right)^{+} \boldsymbol{F}. \tag{14}$$

IV. Dimensional Design Based on the Workspace Requirement With Cable Tension Constraint Condition

A. *Tension Constraint Condition of the Four-Cable-Driven Parallel Manipulator*

In a general case, the four-cable-driven parallel manipulator has no singularity in its workspace [8]. However, cable-driven parallel manipulator is a tension redundant mechanism. If one of the cable tensions is negative or small, cable-driven parallel manipulator will be uncontrollable. Therefore, it is necessary to optimize the geometric parameters by adopting a cable tension constraint condition in the required workspace.

Two constraint conditions of cable tension are given as follows.

1) Each cable's tension is positive.

$$\sigma_i > 0 \qquad i = 1, 2, \ldots, 4. \tag{15}$$

2) The ratio η is in a definite range.

Assuming $\sigma_{\max} = \text{maximum}(\sigma_i)$ and $\sigma_{\min} = \text{minimum}(\sigma_i)$, η can be defined as follows:

$$\eta = \frac{\sigma_{\max}}{\sigma_{\min}}. \tag{16}$$

When η approaches to one, the cable tension performance is the best. If η becomes large enough, the cable will be uncontrollable. Therefore, η should be limited to

$$1 \leq \max(\eta) \leq \eta_0. \tag{17}$$

The workspace with tension constraint condition of the four-cable-driven parallel manipulator can be worked out by Monte Carlo method and an iterative program. The algorithm flow is shown in Fig. 4.

B. Dimensional Optimization Design Based on the Workspace Requirement in Terms of Tension Constraint Condition

According to the reference report of FAST provided by the National Astronomical Observatories Chinese Academy of Science [26], the aperture of the reflector is designated as 500 m, and the mass of the feed mechanism is about 30 000 kg. Fig. 5 shows the required workspace of the four-cable-driven parallel manipulator. It is a sphere crown with a radius of 160 m. The center of the sphere and the reflector are concentric. Height of the required workspace is from 140 to 220 m above the bottom of the reflector.

The purpose of the dimensional design is to optimize the three important geometric parameters for the four-cable-driven parallel manipulator: diameter of cable d, cable tower height h, and the diameter of cable tower's distributed circle D.

Considering the maximum observation scope, the cable tower height should be less than 290 m. For the sake of safety, the cable's diameter d is roughly calculated as 40 mm whose breaking load is 1460.8 kN.

According to the algorithm flow of the workspace with tension constraint condition, the 3-D parameters are optimized in Fig. 6.

Given the selective cable tower heights, Fig. 6(a) shows that cable tower height has almost no influence on workspace. It indicates that all the given cable tower heights satisfy the workspace requirement. In this situation, the cable tower is designed as 265 m, taking the construction cost into account.

As mentioned earlier, the cable tension ration η is put forth to avoid control invalidation. When the feed moves at the height of 220 m above the bottom of the reflector, the diameter of the boundary of the required workspace is $dw = 138$ m. The relationship of the η and diameter of required workspace is

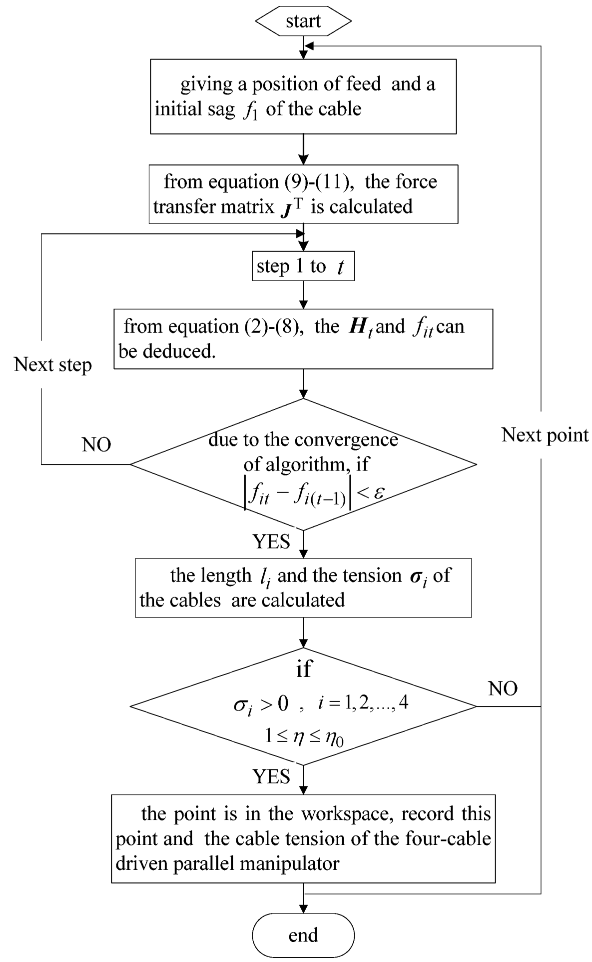

Fig. 4. Algorithm flow of the workspace solution with tension constraint condition.

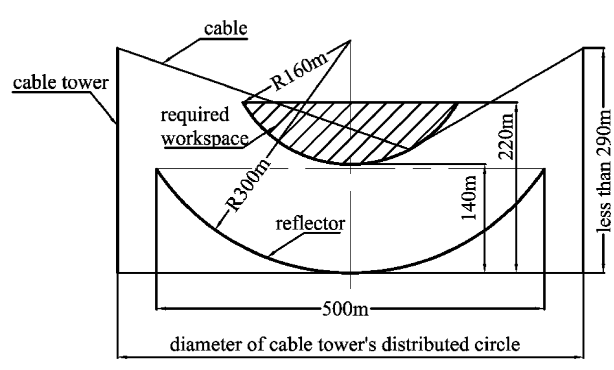

Fig. 5. Required workspace of feed.

shown in Fig. 6(b). The maximum η should be no less than 3. Therefore, maximum η can be designed as $\eta_0 = 3$.

Fig. 6(c) shows the influence of the diameter of cable tower's distributed circle D on workspace diameter. D should be more than 590 m to satisfy the required workspace. Considering the tension ratio and workspace requirement, D is designed as 600 m.

To sum up, a set of optimized geometric parameters of the four-cable-driven parallel manipulator is listed in Table I.

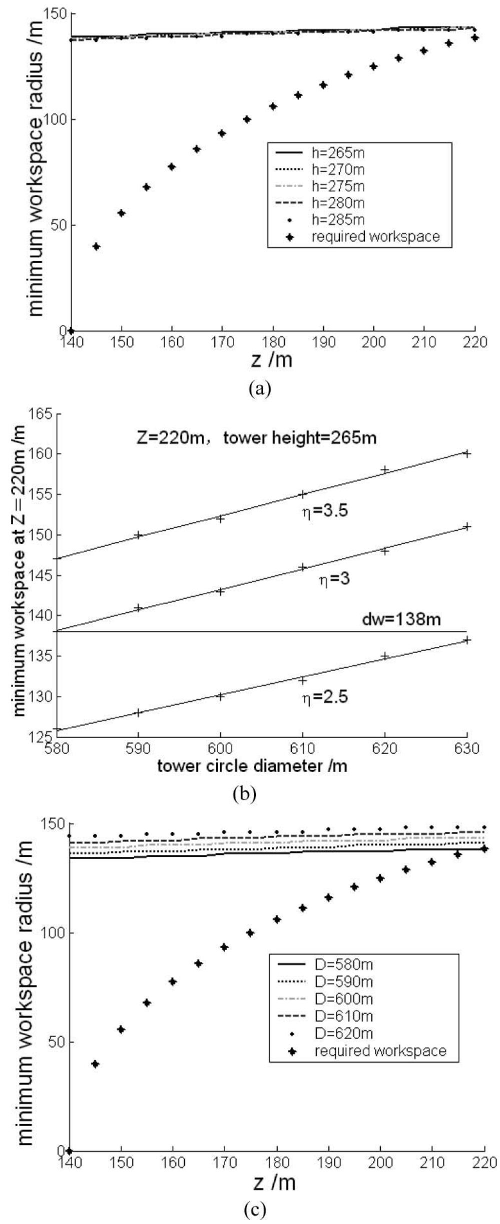

Fig. 6. Optimization of the dimensional parameters of the four-cable parallel manipulator. (a) Optimization of the cable tower height. (b) Tension ratio range of η. (c) Optimization of the cable tower's distributed circle.

TABLE I
PRELIMINARILY OPTIMIZED DIMENSIONAL PARAMETERS

Parameters	Symbol	Optimized dimension
Diameter of cable tower's distributed circle	D	600m
Height of cable tower	h	265m
Density of cable	ρ	7.716kg/m
Diameter of cable	d	40mm
Weight of the feed mechanism	m_0	30000kg
Force ratio	η_0	3

According to Table I, the required workspace and the workspace with cable tension constraint condition of the four-cable-driven parallel manipulator are shown in Fig. 7.

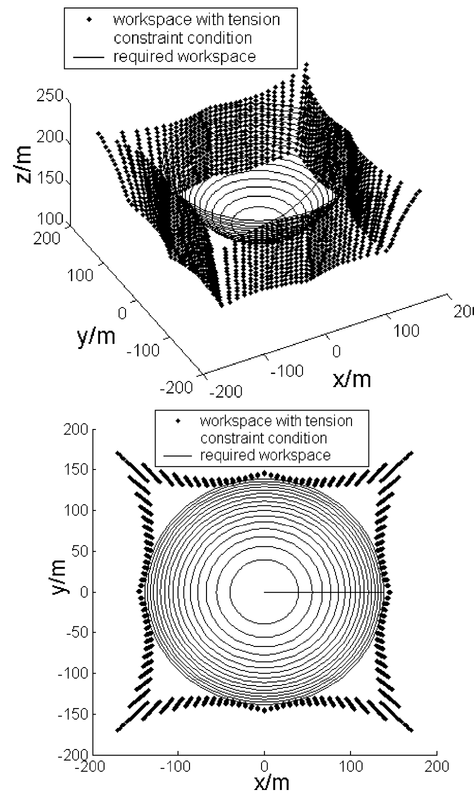

Fig. 7. Workspace of the four-cable-driven parallel manipulator.

V. Stiffness Analysis and Experiment of the Four-Cable-Driven Parallel Manipulator

Since cable-driven parallel manipulator is a wind-sensitive mechanism and stiffness will affect its work performance, cable deformation should be considered in optimization. The purpose of the stiffness analysis is to further optimize dimensional parameters by studying the relationship of dimensional parameters and overall stiffness of the four-cable-driven parallel manipulator. Therefore, this section aims to study by experiment the stiffness of the four-cable-driven parallel manipulator.

Former researchers have theoretically concluded that the main stiffness factor of a cable-driven parallel manipulator is yield behavior of cable. A precision stiffness analysis needs considering such factors as yield of drive, friction between the cables and feed system, and friction between cables and pulley. For this reason, we can only obtain approximate overall stiffness of a cable-driven parallel manipulator by theoretical method. In order to get the real stiffness of the four-cable-driven parallel manipulator, an experimental method is applied in three steps below.

Step 1: Get the geometric parameters influencing the stiffness through theoretical analysis of the overall stiffness of the four-cable-driven parallel manipulator.

Step 2: Set up stiffness similarity model of the four-cable-driven parallel manipulator by way of Buckingham π theorem and the geometric parameters got from Step 1.

Step 3: Obtain the real stiffness of the four-cable-driven parallel manipulator by stiffness experiment on the similarity model.

A. Theoretical Stiffness Analysis of the Four-Cable-Driven Parallel Manipulator

It is supposed that:

1) cable works in range of elastic deformation;
2) a flexible cable can only bear tension, but not compression;
3) cable weight is considered.

In a cable-driven parallel manipulator, its element stiffness is defined as k. Due to the sameness in diameter and material used for each cable, the four cables have the same element stiffness.

Supposing each cable's stiffness is $k_i\ (i = 1, 2, 3, 4)$, we derive

$$k_i = \frac{k}{l_i} \qquad (i = 1, 2, 3, 4) \tag{18}$$

$$k = EA \tag{19}$$

where l_i is the real length of the cables, A is the cross-sectional area of the cable, and E is Young's modulus of cable.

Then the stiffness matrix can be expressed as follows:

$$\boldsymbol{K} = \boldsymbol{J}_0^T \operatorname{diag}(k_1, k_2, k_3, k_4)\boldsymbol{J}_0 \tag{20}$$

where $\boldsymbol{J}_0$ is the tension transmission matrix.

$$\operatorname{diag}(k_1, k_2, k_3, k_4) = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix} \tag{21}$$

$$\boldsymbol{J}_0^{\mathrm{T}} = \begin{pmatrix} s\alpha_1 \cdot c\beta_1 & s\alpha_2 \cdot c\beta_2 & s\alpha_3 \cdot c\beta_3 & s\alpha_4 \cdot c\beta_4 \\ s\alpha_1 \cdot s\beta_1 & s\alpha_2 \cdot s\beta_2 & s\alpha_3 \cdot s\beta_3 & s\alpha_4 \cdot s\beta_4 \\ c\alpha_1 & c\alpha_2 & c\alpha_3 & c\alpha_4 \end{pmatrix} \tag{22}$$

where c and s represent cos and sin, respectively, and $\alpha_i\ (i = 1, 2, 3, 4)$ is the angle between S_i and σ_i. β_i is the angle between H_i and OX axis.

Therefore, the overall stiffness of the four-cable parallel manipulator can be calculated by stiffness matrix.

B. Setting up a Stiffness Similarity Model of the Four-Cable-Driven Parallel Manipulator with Buckingham π Theorem

As mentioned earlier, theoretical stiffness analysis can only provide an approximate solution, and experimental method is a more credible way to obtain real stiffness value. However, due to the large size of FAST, it is impossible to build a real size prototype. In this case, setting up a similarity model is worth exploring for analyzing the stiffness properties of the four-cable-driven parallel manipulator.

Due to the complexity of mechanical systems, the similarity method has not yet been in general used in parallel manipulator. Still, there have been some successful applications in parallel manipulator research. Ren [24] has built a similarity model of a parallel manipulator to study the vibration control. Compared with other parallel mechanisms, cable-driven parallel manipulator is a relatively simple mechanism, which can adopt similarity method. Similarity models are capable of predicting prototype response because homologous but not identical states of time, location, and mass are related. Wilfred *et al.* [21] indicates that Buckingham π theorem, based on dimensional analyses, is a useful theorem for setting up a similarity model.

TABLE II
MAIN PARAMETERS AND DIMENSIONAL FORMULA RELATED TO THE STIFFNESS

Parameters	Symbol	Dimensional formula
Length(geometry dimension)	l	L
Mass	m	M
Density of the cable	ρ	ML^{-1}
Density of the material	ρ_v	ML^{-3}
Tension	σ	MLT^{-2}
Acceleration of gravity	g	LT^{-2}
Young's modular	E	$ML^{-1}T^{-2}$

The geometrical similarity scale of the stiffness similarity model of the four-cable-driven parallel manipulator is 1:400. All the dimensional parameters will be written out as a dimensional formula in which the symbols for the units of length, mass, and time are denoted by L, M, and T, respectively. Considering the stiffness of the four-cable-driven parallel manipulator, the process of setting up the similarity model is expressed as follows.

It is supposed that the feed mechanism is rigid and cables are in the range of elastic deformation. The main geometric parameters and its dimensional formula related to the stiffness of the four-cable-driven parallel manipulator are shown in Table II.

According to the dimensional formula of the parameters in Table II, the dimensional equation of geometric parameters is expressed in a matrix as follows:

	m	l	ρ	ρ_v	σ	g	E
M	1	0	1	1	1	0	1
L	0	1	−1	−3	1	1	−1
T	0	0	0	0	−2	−2	2

After the analysis of this matrix, we find that the geometric parameters m and l themselves cannot make the other geometric parameters dimensionless. Then, by adopting $\sqrt{ml/\sigma}$ as an additional dimensionless parameter, the other parameters can be dimensionless.

	m	l	$\sqrt{ml/\sigma}$	pl/m	$\rho_v l^3/m$	mg/σ	El^2/σ
M	1	0	0	0	0	0	0
L	0	1	0	0	0	0	0
T	0	0	1	0	0	0	0

The similarity criterion of the dimensionless parameters can be described as follows:

$$\pi_1 = \frac{\rho l}{m} = \frac{ML^{-1}L}{M} = 1 \tag{23}$$

$$\pi_2 = \frac{\rho_v l^3}{m} = \frac{ML^{-3}L^3}{M} = 1 \tag{24}$$

$$\pi_3 = \frac{mg}{\sigma} = \frac{MLT^{-2}}{MLT^{-2}} = 1 \tag{25}$$

$$\pi_4 = \frac{EA}{\sigma} = \frac{El^2}{\sigma} = \frac{ML^{-1}T^{-2}L^2}{MLT^{-2}} = 1 \tag{26}$$

where A means the cross-sectional area of the cable.

According to the similarity criterion of the dimensionless parameters earlier, the similarity scale between the similarity model and prototype, which is expressed as λ, can be obtained. The subscripts μ and p represent a similarity model and a prototype, respectively.

Therefore the following apply.

Acceleration of gravity similarity scale is $\lambda_g = g_\mu / g_p = 1$.

Length and diameter similarity scale is $\lambda_l = l_\mu / l_p$, $\lambda_d = \lambda_l$.

Mass similarity scale is $\lambda_m = \lambda_\rho \lambda_l$.

Density of the cable similarity scale is $\lambda_\rho = \lambda_{\rho_v} \lambda_d$.

Tension similarity scale is $\lambda_\sigma = \lambda_{EA} = \lambda_\rho \lambda_l$.

Density of the material similarity scale is $\lambda_{\rho_v} = \rho_{v\mu} / \rho_{vp}$.

Young's modular similarity scale is $\lambda_E = E_\mu / E_p$.

Then, stiffness similarity scale is $\lambda_k = \frac{\lambda_{EA}}{\lambda_l} = \frac{\lambda_E \lambda_d^2}{\lambda_l}$.

However, due to the interdependence of the Young's modular and density, a problem came up that no cable could satisfy both the Young's modular similarity scale and density similarity scale at the same time. To solve this problem, we used a method of adjusting the density of the cable similarity scale without changing the Young's modular similarity scale. Therefore, the cables in the similarity model should use the same material as the prototype, and an additional counterweight cable is used to meet the density of the cable similarity scale. In this case, Young's modular similarity scale is $\lambda_E = 1$.

In the similarity model of the four-cable-driven parallel manipulator, two kinds of cables are to be used in the similarity model. One is load-carrying cable to satisfy the Young's modular similarity scale, the other one is counterweight cable to meet the density of the cable similarity scale. In the similarity scale, the diameter of the counterweight cable is defined as dc, and then, the similarity scale of cables' diameter is deduced as follows.

The load-carrying cable's diameter similarity scale is as follows:

$$\lambda_d = \sqrt{\lambda_\rho \lambda_l}.$$

The counterweight cable's diameter similarity scale is as follows:

$$\lambda_{\mathrm{dc}} = \sqrt{\lambda_\rho (1 - \lambda_l)}.$$

Then the density of the cable scale is as follows:

$$\lambda_\rho = \lambda_{\rho_v} (\lambda_d^2 + \lambda_{\mathrm{dc}}^2).$$

Therefore, the stiffness similarity scale of the four-cable-driven parallel manipulator can be expressed as follows:

$$\lambda_k = \frac{\lambda_{EA}}{\lambda_l} = \frac{\lambda_E \lambda_d^2}{\lambda_l} = \frac{\lambda_d^2}{\lambda_l}. \tag{27}$$

From the similarity method and similarity scale equations, the dimensional parameters of the similarity model of the four-cable-driven parallel manipulator are shown in Table III.

According to the parameters in Table III, the stiffness similarity model is set up in Fig. 8.

After calibration of the similarity model, the four-cable-driven parallel manipulator model is ready for stiffness experiment, which is shown in Fig. 9.

TABLE III
DIMENSIONAL PARAMETERS OF THE SIMILARITY MODEL OF THE FOUR-CABLE-DRIVEN PARALLEL MANIPULATOR

Parameters	Similarity scale	Dimension of the prototype	Dimension of the similarity model
Diameter of cable tower's distributed circle	$\lambda_l = 1:400$	600m	1.5m
Height of cable tower	$\lambda_l = 1:400$	265m	0.6625m
Density of cable	$\lambda_\rho = 1:400$	7.716kg/m	0.024723kg/m
Diameter of force cable	$\lambda_d = 1:400$	40mm	0.1mm
Diameter of counterweight cable	$\lambda_{dp} = 1:20$	0	2mm
Weight of the feedback system	$\lambda_m = 1:160000$	30000kg	0.1875kg
Young's modular scale	$\lambda_E = 1$	1.6×10^{11}Pa	1.6×10^{11}Pa

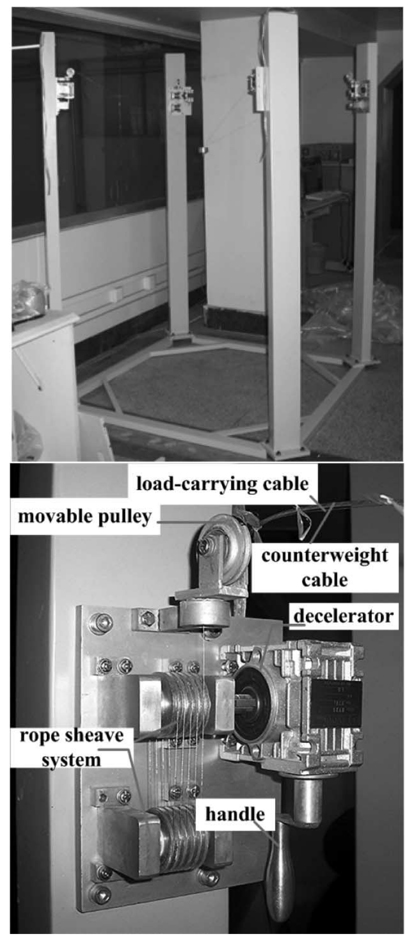

Fig. 8. Stiffness similarity model.

Fig. 9. Experiment on similarity model.

TABLE IV
RELEVANT PARAMETERS OF THE NINE EXPERIMENT POINTS

	A	B	C	D	E	F	G	H	I
Position at Z-axis in prototype (m)	140	150	160	170	180	190	200	210	220
Theoretical stiffness of the prototype at Z-direction (10^5 N/m)	4.7872	4.1929	3.6172	3.0529	2.5134	2.0067	1.5414	1.1252	0.76561
Theoretical stiffness of the similarity model at Z-direction (N/m)	1196.6	1049.3	904.19	763.16	628.35	501.65	385.32	281.28	191.4

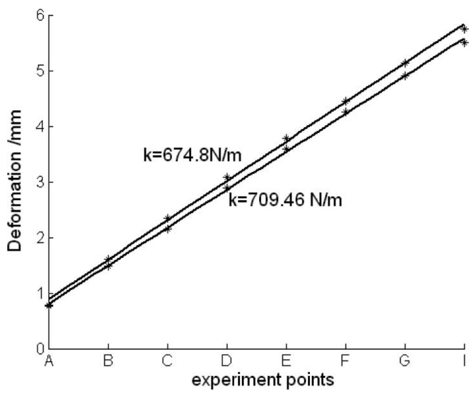

Fig. 10. Load-deformation graph at C point.

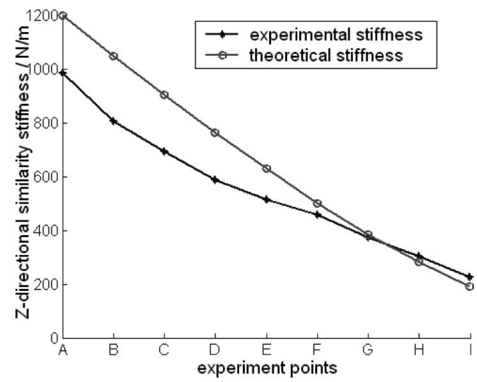

Fig. 11. Theoretical and experimental stiffness of the similarity model.

C. Stiffness Experiment on the Similarity Model of the Four-Cable-Driven Parallel Manipulator

Let us take a trajectory as an example. The experimental trajectory is a vertical line from 140 to 220 m up from the bottom of the reflector, which is at the center of the cable tower's distributed circle. Nine test points from A to I are selected, and the relevant parameters of the nine points are shown in Table IV.

At each test point, when a load is put on it, the deformation of the four-cable-driven parallel manipulator at Z-direction can be measured by using dial indicator. The experiment is done twice at each point, and we take the average value at each point as the final stiffness value.

For example, when a load is put on the feed mechanism at point C, the load-deformation graph is shown in Fig. 10. The final stiffness at C point of the similarity model is as follows:

$$k_c = \frac{709.46 + 674.8}{2} = 692.13 \text{ N/m}.$$

The experimental stiffness of the similarity model at each point is shown in Table V.

From Fig. 11, we can see that the experimental stiffness is weaker than theoretical stiffness while the change rate of the experimental stiffness is smaller than that of the theoretical stiffness.

Three reasons can explain the result as follows.

First of all, when the feed mechanism is drawn from higher to lower position, the cable becomes longer. From 18, the cable stiffness is in inverse proportional to the cable length. Hence, the stiffness of the four-cable-driven parallel manipulator becomes weaker.

Second, the theoretical cable length in stiffness analysis is from cable tower to feed mechanism. However, in the practical application, the cable length is from the endpoint of cable to the feed mechanism. Apparently, the cable in practical application should be longer. Therefore, the experimental stiffness is weaker than theoretical stiffness of the four-cable-driven parallel manipulator.

Finally, since stiffness of a cable-driven parallel manipulator is nonlinear, the cable tension can slightly influence the stiffness.

In conclusion, stiffness of the four-cable-driven parallel manipulator can be calculated on the basis of the experimental stiffness.

VI. DIMENSIONAL OPTIMIZATION BASED ON THE STIFFNESS OF THE FOUR-CABLE-DRIVEN PARALLEL MANIPULATOR

From stiffness similarity scale $\lambda_k = \lambda_d^2/\lambda_l$, we can derive

$$\frac{k_p}{k_m} = \frac{d_p^2/d_m^2}{l_p/l_m} \Rightarrow k_p = \frac{d_p^2}{l_p}\left(\frac{d_m^2}{l_m}k_m\right) \tag{28}$$

where $l_p = f(d, D)$.

In Section V, stiffness of the four-cable-driven parallel manipulator is obtained. According to (28), the dimensional parameters can be optimized in the following way.

3-D parameters will be optimized: diameter of cable d, diameter of cable tower's distributed circle D, and cable tower height h. The influence of the dimensional parameters on the stiffness of the four-cable-driven parallel manipulator is shown in Fig. 12.

As shown in Fig. 12(a) and (b), the stiffness of the four-cable-driven parallel manipulator is weakened in a proportionate and

TABLE V
EXPERIMENTAL STIFFNESS OF THE SIMILARITY MODEL

	A	B	C	D	E	F	G	H	I
Experimental stiffness of the similarity model (N/m)	984.87	803.64	692.13	586.69	512.55	456.62	372.19	302.64	226.18

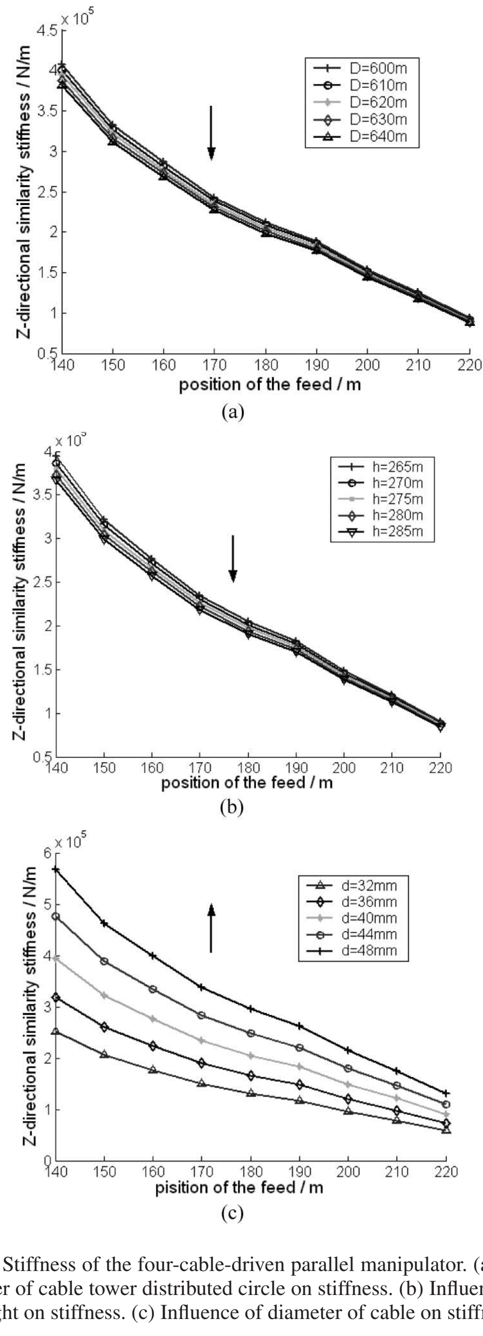

Fig. 12. Stiffness of the four-cable-driven parallel manipulator. (a) Influence of diameter of cable tower distributed circle on stiffness. (b) Influence of cable tower height on stiffness. (c) Influence of diameter of cable on stiffness.

slight way by increasing the diameter of the cable tower's distributed circle D and cable tower height h. In order to maximize the observation scope, D is designed as 600 m and h as 265 m.

As shown in Fig. 12(c), the diameter of the cable d has a great influence on its stiffness. The $\max(\eta)$ for cables of different diameter in the required workspace are shown in Table VI, and the maximum tension ratio of each case is almost the same. Therefore, the diameter of the cable is designed as $d = 44$ mm.

TABLE VI
$\max(\eta)$ BASED ON DIFFERENT DIAMETER OF CABLE

Diameter of the cable	$\max(\eta)$
$d = 32$mm	3.1
$d = 36$mm	3.1
$d = 40$mm	3
$d = 44$mm	3
$d = 48$mm	3.1

TABLE VII
OPTIMIZED DIMENSIONAL PARAMETERS OF THE FOUR-CABLE-DRIVEN PARALLEL MANIPULATOR

Parameters	Symbol	Final optimized dimension
Diameter of cable tower's distributed circle	D	600m
Height of cable tower	h	265m
Density of cable	ρ	9.32kg / m
Diameter of cable	d	44mm
Weight of the feed mechanism	m_0	30000kg
Tension ratio	η_0	3

According to the dimensional optimization with the workspace requirement in terms of the constraint condition of cable tension and stiffness, a set of final optimized dimensional parameters of the four-cable-driven parallel manipulator are obtained in Table VII. This set of optimized dimensional parameters can be applied in the construction of the feed-support system in FAST.

VII. CONCLUSION

In conclusion, we emphasize the following.

Firstly, in this paper, we have introduced the effective catenary simplification formula into the tension equilibrium equation to work out the workspace with tension constraint condition. Accordingly, a set of preliminary geometric parameters are optimized.

Secondly, we have realized a similarity model with a counterweight cable to meet the stiffness similarity of the four-cable-driven parallel manipulator. A more credible stiffness value of the four-cable-driven parallel manipulator is obtained by the stiffness experimental method. This method is quite effective for stiffness analysis in cable-driven parallel manipulator research and development.

Thirdly, based on the workspace requirement with constraint condition of cable tension and stiffness, a set of optimization dimensional parameters is obtained to build the feed-support system of FAST. The dimensional optimization method can avoid control invalidity and supply a better stiffness for a cable-driven parallel manipulator. More importantly, it provides a theoretical basis for further study.

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