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Simultaneous plant-controller design optimization of a two-link planar manipulator

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Abstract

This paper presents a methodology based on numerical optimization techniques for simultaneously optimizing design parameters of a two-link planar rigid manipulator and a nonlinear gain PD controller designed for performing multiple tasks. The formulation of simultaneous plant-controller design optimization problem and the description of solution techniques based on a heuristic evolutionary algorithm for solving this optimization problem are presented. The gravity balanced two-link planar manipulator design is described and a class of nonlinear gain PD controllers is introduced for the set-point control of manipulator with counter-weights and different payloads. The results of the simultaneous design optimization involving a nonlinear PD controller and the two-link manipulator are given.

Keywords: Manipulators; Evolutionary algorithms; Nonlinear gain PD controller; Design optimization; Counter-weights

1. Introduction

In recent years, design philosophies based on a mechatronics approach have been explored to create high performance robotic systems. In a mechatronics approach, the mechanical structure, the actuators, the sensors, and the controller design of the robotic system cannot be separated but must be considered simultaneously. There are significant interactions and intricate relationships between the dynamic behaviors of these components which make up the overall system. To achieve high performance, a cohesive and comprehensive design methodology, considering these interactions and tradeoffs, is necessary.

Traditionally, it is a common practice to first design the mechanical structure of a robotic system with its drive elements, and then design the controller along with the necessary trajectory planning algorithms. This separation of mechanism design and control design is described as *optimum design vs optimal control* [1]. A vast number of methods have been reported in the literature for optimum design and optimal control, performed separately. A mechanism design which is optimal with respect to kinematics, kinetics and statics, is not necessarily optimal from the control standpoint [2]. On the other hand, a control design obtained using the optimal control approach can be improved by simultaneously modifying the mechanical structure [3]. A design optimization methodology for such a purpose would prove to be a valuable tool for designing the overall system by optimizing multiple design objectives subject to real world constraints.

Most of the existing work on integrated plant-controller design has been performed during the last two decades. Integrated structural/control design for space structures has been extensively studied by a number of researchers [4–6]. In most of these studies, the integrated structural/control design problem, formulated using linear models, is solved using optimal control theory. Recently, a number of studies have been reported on the integrated plant-controller approach for high performance robotic systems [2,7–9]. In Park and Asada [2], a concurrent design method

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was proposed for the design of mechanical structure and control for a two-link high speed robotic manipulator to achieve minimum settling time. This method performs the mechanical design first taking into consideration the control design aspects, and a gradient based optimization technique is used to optimize the design parameters. Rastegar et al. [7] presented a simultaneous kinematic, dynamic, and control design approach and demonstrated for the simultaneous design optimization of a two-link planar manipulator and a linear PD controller along with some desired trajectories. An integrated plant-controller design approach based on convex optimization theory and the linear models of the plant and controller was proposed by Encarnacao et al. [8] for the design of an autonomous underwater vehicle (AUV) to meet mission performance requirements with minimum energy expenditure. Zhang et al. [9] described an integrated design approach which adopted a negative mass redistribution by following a shaking-force/shaking-moment balancing scheme, for the modification of an existing four-bar mechanism, with the aim of obtaining a simpler dynamic model. The mechanical design was performed first followed by the controller design, and an optimization procedure was not used to determine the design parameters [9].

Gravity balanced design of robot manipulators can be used to improve their efficiency in terms of energy consumption. Counter-weights were used to create a gravity balanced manipulator design which minimizes motor torque requirements and simplifies the dynamic equations [10,11]. Chun and Cho [10] used counter-weights for the gravity balanced design of a two-link manipulator to simplify its dynamics, and then suggested control laws for this simplified dynamic model. In Koski and Osyczka [11], a multicriteria optimization approach was adopted to determine the optimal counter-weights required for the gravity balanced design by minimizing the joint torques and reaction forces. However, in this work, the aspects associated with the controller design were not considered in the design process.

For the design of mechatronics systems such as robotic manipulators, herein a plant-controller design optimization methodology is presented for *simultaneously* optimizing both the design parameters of the plant, represented by nonlinear dynamic equations, and the nonlinear feedback controller. This methodology incorporates a class of stochastic search methods called evolutionary algorithms for solving the highly nonlinear and non-convex optimization problems encountered during the simultaneous design optimization. Unlike gradient based optimization techniques or deterministic direct search methods adopted in the above mentioned literature for solving integrated plant-controller design optimization problems, evolutionary algorithms have better potential for locating global optimal solutions due to their population-based approach and the lack of dependency on the gradient information [12]. Using this optimization methodology, a two-link planar manipulator with counter-weights and a nonlinear PD controller are designed simultaneously for the tasks of carrying different payloads. The dynamic performance of the closed-loop system is optimized for set-point control tasks by searching for optimal counter-weights and controller parameters.

2. Simultaneous plant-controller design optimization methodology

In this section, a methodology for simultaneous plant-controller design optimization is described for closed-loop, nonlinear, mechatronics systems, like robotic manipulators. This design approach is used because the plant and controller should be designed simultaneously considering the interactions and tradeoffs between these two designs to achieve the best possible performance from a closed-loop system with constraints.

We start with a mathematical model of the closed-loop system which can be represented by ordinary differential equations without any time delays. The model of the *plant* is defined analytically by

$$\begin{aligned}\dot{\mathbf{x}}_p(t) &= \mathbf{f}_{p_1}(\mathbf{x}_p(t), \mathbf{u}(t), \mathbf{s}_p, t)\\ \mathbf{y}(t) &= \mathbf{f}_{p_2}(\mathbf{x}_p(t), \mathbf{u}(t), \mathbf{s}_p, t)\end{aligned} \tag{1}$$

where $t \in \mathbb{R}_+$ denotes time; $\mathbf{x}_p(t) \in \mathbb{R}^{n_{xp}}$ is the *state* vector of the plant; $\mathbf{u}(t) \in \mathbb{R}^{n_u}$ is the *control* vector for the plant; $\mathbf{y}(t) \in \mathbb{R}^{n_y}$ is the *output* vector of the plant; $\mathbf{s}_p \in \mathbb{R}^{n_p}$ denotes the parameter vector describing the plant. The model of the controller is defined analytically by

$$\begin{aligned}\dot{\mathbf{x}}_c(t) &= \mathbf{f}_{c_1}(\mathbf{x}_c(t), \mathbf{y}(t), \mathbf{r}(t), \mathbf{s}_c, t)\\ \mathbf{u}(t) &= \mathbf{f}_{c_2}(\mathbf{x}_c(t), \mathbf{y}(t), \mathbf{r}(t), \mathbf{s}_c, t)\end{aligned} \tag{2}$$

where $\mathbf{x}_c(t) \in \mathbb{R}^{n_{xc}}$ is the state vector of the controller; $\mathbf{r}(t) \in \mathbb{R}^{n_y}$ is the external reference input vector for the plant; $\mathbf{s}_c \in \mathbb{R}^{n_c}$ denotes the parameter vector describing the controller. The nonlinear function vectors $\mathbf{f}_{p_1}(\cdot), \mathbf{f}_{p_2}(\cdot)$, $\mathbf{f}_{c_1}(\cdot)$ and $\mathbf{f}_{c_2}(\cdot)$ are, in general, nonlinear and known. The closed-loop system defined above is shown in Fig. 1.

2.1. Formulation of simultaneous design optimization problem

The formulation of the design optimization problem for a plant and/or its controller involves the selection of *design variables* and *design objectives* along with the determination of *design constraints*. Typically, the design optimization of a plant and/or its controller involves multiple and conflicting objectives which should be considered simultaneously. Unlike single objective optimization problems, no concept

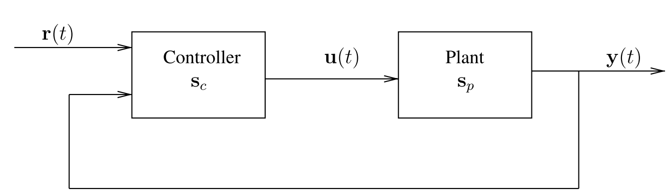

Fig. 1. Closed-loop dynamic system configuration.

of an optimal solution is universally accepted for multiobjective optimization. When the objectives are in conflict, the design optimization yields not a single optimal solution but a set of solutions called a *Pareto optimal set* [13] (a set of design solutions for which any single objective cannot be improved without compromising at least one of the other objectives). It should also be noted that due to the non-commensurable and non-convex nature of the design objectives, the benefits of the simultaneous design optimization methodology can be extended by presenting the entire set of Pareto optimal solutions to the decision maker. The rigorous way to generate the entire set of Pareto optimal solutions is to use vector optimization methods which can directly tackle the multiobjective optimization problem. Examining such vector optimization methods for generating the entire set of Pareto optimal solutions for the simultaneous plant-controller design optimization problem is beyond the scope of the present work but presents a worthwhile future research possibility.

Scalarization methods are more commonly adopted for handling multiple objectives by forming an amalgamation of multiple objectives into one single objective, which is then solved as a single objective optimization problem. One popular member of these scalarization methods is the weighted-sum approach which combines the multiple objectives into a single objective by pre-multiplying each objective with a subjective non-negative weight. It has been well recognized that the weighted-sum approach cannot be used to generate the entire set of Pareto optimal solutions when the conflicting objectives are non-convex [13]. However, for a given set of non-negative weights, there exists a Pareto optimal solution even when the conflicting objectives are non-convex. Within the scope of this paper, we consider the following problem formulation to obtain a single Pareto optimal solution by using the weighted-sum approach with a given set of non-negative weights to combine multiple objectives into one single objective.

The traditional design strategy for improving the closed-loop system performance of mechatronics systems involves solving the plant optimization problem and the controller optimization problem in sequence. Without loss of generality, in the following problem formulations we will assume that the design objective must be minimized to improve performance. The plant optimization problem is typically expressed in the form:

$$\min_{\mathbf{s}_p \in \Omega_p} \phi_{pobj}(\mathbf{s}_p, \mathbf{s}_c, t_f) = \min_{\mathbf{s}_p \in \Omega_p} \sum_{i=1}^{n_p^{\mathrm{O}}} w_{pi} \phi_{pi}(\mathbf{s}_p, \mathbf{s}_c, t_f) \tag{3}$$

subject to the inequality and equality constraints:

$$\begin{aligned} &\psi_{pj}^{\mathrm{I}}(\mathbf{s}_p, \mathbf{s}_c, t_f) \leqslant 0 \quad j = 1, \ldots, n_p^{\mathrm{I}} \\ &\psi_{pk}^{\mathrm{E}}(\mathbf{s}_p, \mathbf{s}_c, t_f) = 0 \quad k = 1, \ldots, n_p^{\mathrm{E}} \end{aligned} \tag{4}$$

where $\mathbf{s}_p$, ϕ_{pi}, ψ_{pj}^{I} and ψ_{pk}^{E} are plant design variables, objectives, inequality constraints, and equality constraints, respectively, and t_f is the evaluation time interval for the design objectives and constraints. The feasible solution set denoted as Ω_p consists of all those solutions which satisfy all the plant design constraints. The non-negative numbers w_{pi} are weights which assign the relative importance among the design objectives. The parameters n_p^{O}, n_p^{I} and n_p^{E} represent the number of plant design objectives, inequality constraints, and equality constraints, respectively. The plant design objectives and constraints are also influenced by the controller design variables $\mathbf{s}_c$, but the plant optimization is performed by changing only the plant design variables $\mathbf{s}_p$.

The controller optimization problem is generally described in the form:

$$\min_{\mathbf{s}_c \in \Omega_c} \phi_{cobj}(\mathbf{s}_p, \mathbf{s}_c, t_f) = \min_{\mathbf{s}_c \in \Omega_c} \sum_{i=1}^{n_c^{\mathrm{O}}} w_{ci} \phi_{ci}(\mathbf{s}_p, \mathbf{s}_c, t_f) \tag{5}$$

subject to the inequality and equality constraints of the usual form:

$$\begin{aligned} &\psi_{cj}^{\mathrm{I}}(\mathbf{s}_p, \mathbf{s}_c, t_f) \leqslant 0 \quad j = 1, \ldots, n_c^{\mathrm{I}} \\ &\psi_{ck}^{\mathrm{E}}(\mathbf{s}_p, \mathbf{s}_c, t_f) = 0 \quad k = 1, \ldots, n_c^{\mathrm{E}} \end{aligned} \tag{6}$$

where ϕ_{ci}, ψ_{cj}^{I} and ψ_{ck}^{E} are controller design variables, objectives, inequality constraints, and equality constraints, respectively. The feasible solution set denoted as Ω_c consists of all those solutions which satisfy all the controller design constraints. The non-negative numbers w_{ci} are weights which assigned to the controller design objectives. The parameters n_c^{O}, n_c^{I} and n_c^{E} represent the number of controller design objectives, inequality constraints, and equality constraints, respectively.

The simultaneous plant-controller design optimization can be formulated by combining the individual plant and controller design optimization problems as follows:

$$\begin{aligned} &\min_{[\mathbf{s}_p, \mathbf{s}_c]^{\mathrm{T}} \in \Omega_{pc}} \phi_{pcobj}(\mathbf{s}_p, \mathbf{s}_c, t_f) \\ &\quad = \min_{[\mathbf{s}_p, \mathbf{s}_c]^{\mathrm{T}} \in \Omega_{pc}} \{ w_{pobj} \phi_{pobj}(\mathbf{s}_p, \mathbf{s}_c, t) + w_{cobj} \phi_{cobj}(\mathbf{s}_p, \mathbf{s}_c, t_f) \} \\ &\quad = \min_{[\mathbf{s}_p, \mathbf{s}_c]^{\mathrm{T}} \in \Omega_{pc}} \sum_{i=1}^{n_{pc}^{\mathrm{O}}} w_{pci} \phi_{pci}(\mathbf{s}_p, \mathbf{s}_c, t_f) \end{aligned} \tag{7}$$

subject to the inequality and equality constraints:

$$\begin{aligned} &\psi_{pcj}^{\mathrm{I}}(\mathbf{s}_p, \mathbf{s}_c, t_f) \leqslant 0 \quad j = 1, \ldots, n_{pc}^{\mathrm{I}} \\ &\psi_{pck}^{\mathrm{E}}(\mathbf{s}_p, \mathbf{s}_c, t_f) = 0 \quad k = 1, \ldots, n_{pc}^{\mathrm{E}} \end{aligned} \tag{8}$$

where w_{pobj} and w_{cobj} are weights which assign the relative importance between plant and controller design objectives, and $w_{pci} \in \{w_{pobj} w_{pi}, w_{cobj} w_{ci}\}$, $\phi_{pci} \in \{\phi_{pi}, \phi_{ci}\}$, $\psi_{pcj}^{\mathrm{I}} \in \{\psi_{pj}^{\mathrm{I}}, \psi_{cj}^{\mathrm{I}}\}$, $\psi_{pck}^{\mathrm{E}} \in \{\psi_{pk}^{\mathrm{E}}, \psi_{ck}^{\mathrm{E}}\}$, $n_{pc}^{\mathrm{O}} = n_p^{\mathrm{O}} + n_c^{\mathrm{O}}$, $n_{pc}^{\mathrm{I}} = n_p^{\mathrm{I}} + n_c^{\mathrm{I}}$ and $n_{pc}^{\mathrm{E}} = n_p^{\mathrm{E}} + n_c^{\mathrm{E}}$. The feasible solution set denoted as Ω_{pc} consists of all those solutions which satisfy all the plant and controller design constraints. The above simultaneous design optimization formulation also represents the more general case where the plant and controller design objectives are not separable and in general, $n_{pc}^{\mathrm{O}} \neq n_p^{\mathrm{O}} + n_c^{\mathrm{O}}$, $n_{pc}^{\mathrm{I}} \neq n_p^{\mathrm{I}} + n_c^{\mathrm{I}}$ and $n_{pc}^{\mathrm{E}} \neq n_p^{\mathrm{E}} + n_c^{\mathrm{E}}$.

2.2. Evolutionary algorithms for simultaneous design optimization

The simultaneous plant-controller design optimization of mechatronics systems is a complex nonlinear optimization problem due to the presence of non-convex, non-differentiable, multiple, dynamic objectives and constraints. Therefore, it can be a mathematically and computationally challenging problem to solve using gradient based optimization techniques or deterministic direct search methods. Evolutionary algorithms are particularly well suited for this kind of optimization problem. Evolutionary algorithms can be considered as a broad class of stochastic optimization techniques motivated by the process of natural evolution found in biological organisms. An evolutionary algorithm maintains a population of candidate solutions for the problem at hand, and makes it evolve by iteratively applying a set of stochastic operators, known as *mutation*, *recombination* and *selection* [12]. The best known members in the class of evolutionary algorithms are [14]: genetic algorithms (GA), evolution strategies (ES), evolutionary programming (EP), and genetic programming (GP).

In this paper, to effectively solve the design optimization problems involving non-convex design objectives and continuous design variables, we consider a version of ES called $(\mu+\lambda)$-ES algorithm. It should be noted here that the use of this heuristic evolutionary algorithm for the optimization problems formulated in the previous subsection by itself does not guarantee global optimal solutions; however, the likelihood of locating global optimal solutions is increased due to its population-based approach and the lack of dependency on the gradient information as stated before. Like any other evolutionary algorithm, the basic components of $(\mu+\lambda)$-ES are representation of candidate solutions, initial population, genetic operators, fitness evaluation and termination condition. First, an initial population of μ parents is generated randomly and the fitness evaluation of each individual in this parent population is performed using design objectives and constraints. Then the generation cycle starts with the generation of an offspring population of λ ($>5\mu$) individuals using discrete/intermediate recombination and self-adaptive mutation operators [12], and the fitness evaluation is performed for each offspring individual. The best μ individuals are selected from the combined parent and offspring population of $\mu+\lambda$ individuals as a new parent population, and the generation cycle is repeated until some maximum number of generations are performed. Further details of the $(\mu+\lambda)$-ES algorithm can be found in [12].

3. Gravity balanced two-link manipulator design optimization

The manipulator consists of two revolute joints actuated by two direct drive motors to rotate the links in a vertical plane and carries a payload which is located at the distal end of the last link. The counter-weights are attached to each link as shown in Fig. 2. In the absence of friction and other disturbances, the dynamic equations of the two-link manipulator with counter-weights can be written as [15]

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{u} \tag{9}$$

where $\mathbf{u} \in \mathbb{R}^2$ is the vector of input torques and, $\ddot{\mathbf{q}}$ and $\dot{\mathbf{q}}$ are vectors of joint angular accelerations and velocities, respectively. The symmetric matrix $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{2\times 2}$ is called the inertia matrix of the manipulator, the vector $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} \in \mathbb{R}^2$ represents the Coriolis and centrifugal forces, and the vector $\mathbf{g}(\mathbf{q}) \in \mathbb{R}^2$ represents the gravitational torques. The entries of $\mathbf{M}(\mathbf{q})$, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ and $\mathbf{g}(\mathbf{q})$ can be found in [11].

In order to achieve gravity balancing for the two-link manipulator, we assume a stable controller (defined by a set of fixed parameters $\mathbf{s}_c$) which allows the manipulator to move from an initial point to the equilibrium point defined by the joint angular positions $\mathbf{q} = \mathbf{q}_e$ with $\dot{\mathbf{q}} = \ddot{\mathbf{q}} = \mathbf{0}$. Now, the counter-weights $\mathbf{s}_p = [m_{w1}, m_{w2}]^{\mathrm{T}} \in \Omega_p \subset \mathbb{R}^2$ required to gravity balance the manipulator can be found by minimizing the following normalized functional objective for each link:

$$\phi^n_{pi}(\mathbf{s}_p,\mathbf{s}_c) = \frac{|(\mathbf{g}(\mathbf{q}_e))_i|}{u_i^{\max}}, \quad i = 1,2 \tag{10}$$

where $u_i^{\max}$ is the allowed maximum input torque delivered by actuator i.

Using the weighted-sum objective formulation, the design problem for the gravity balanced manipulator with a payload m_{li} is mathematically stated as the following optimization problem: choose the counter-weights $\mathbf{s}_p = [m_{w1}, m_{w2}]^{\mathrm{T}} \in \Omega_p \subset \mathbb{R}^2$ to minimize

$$\min_{\mathbf{s}_p\in\Omega_p\subset\mathbb{R}^2} \phi_{pi}(\mathbf{s}_p,\mathbf{s}_c,m_{li}) = \min_{\mathbf{s}_p\in\Omega_p\subset\mathbb{R}^2} \left\{ w_1\phi^n_{p1}(\mathbf{s}_p,\mathbf{s}_c) + w_2\phi^n_{p2}(\mathbf{s}_p,\mathbf{s}_c) \right\} \tag{11}$$

subject to the design parameter bounds which constitute the feasible solution space $\Omega_p \subset \mathbb{R}^2$. The above functional objective is also influenced by the controller parameters

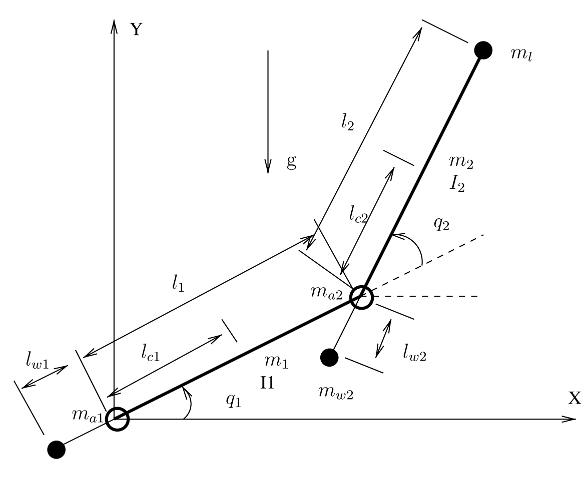

Fig. 2. Planar two-link robot manipulator.

$\mathbf{s}_c$, but the optimization is performed by changing only the counter-weights $\mathbf{s}_p$. The weights in (11) are chosen as $w_1 = w_2 = 1$ to give equal importance in minimizing the steady-state value of each joint torque.

For the manipulator design considered here with the given payload of m_l, by inspection of $\mathbf{g}(\mathbf{q})$, the following conditions are found to yield the minimum value of zero for the objective function in (11), thus canceling the gravity effects on both joints (with reference to Fig. 2):

$$m_1 l_{c1} + m_2 l_1 + m_{a2} l_1 + m_l l_1 + m_{w2} l_1 = m_{w1} l_{w1}, \tag{12}$$

$$m_2 l_{c2} + m_l l_2 = m_{w2} l_{w2} \tag{13}$$

where for link i ($i = 1, 2$), m_i denotes the mass, l_i denotes the length, l_{ci} denotes the distance from joint i to the center of mass of the link i, m_{ai} denotes the mass of the motor attached to link i, m_{wi} denotes the counter-weight attached to link i, l_{wi} denotes the distance from the joint to the counter-weight. Consequently, the elements of $\mathbf{g}(\mathbf{q})$ become zero for all configurations and this makes the manipulator perfectly gravity balanced for the given payload. Note that the counter-weights $\mathbf{s}_p$ obtained by solving Eqs. (12) and (13) for a single payload case will yield the global minimum of the objective function in (11) with any stable controller that can move the manipulator to an equilibrium state $\mathbf{q}_e$.

It can be noted from Eqs. (12) and (13), that for a different payload, the manipulator is not perfectly gravity balanced using these counter-weights. Indeed, different sets of counter-weights are required for different payloads to perfectly gravity balance the manipulator. However, if it is required to use one set of counter-weights to gravity "balance" the manipulator for different payloads with the objective of reducing actuator torque demand and lowering power consumption, the following optimization problem can be used to find the counter-weights. For the given payloads m_{li}, $i = 1 \dots m$, and the weight vector $\mathbf{w}_p = [w_{p1} \cdots w_{pm}]^{\mathrm{T}}$, choose the counter-weights $\mathbf{s}_p = [m_{w1}, m_{w2}]^{\mathrm{T}} \in \Omega_p \subset \mathbb{R}^2$ to minimize

$$\min_{\mathbf{s}_p \in \Omega_p \subset \mathbb{R}^2} \phi_{pobj}(\mathbf{s}_p, \mathbf{s}_c) = \min_{\mathbf{s}_p \in \Omega_p \subset \mathbb{R}^2} \sum_{i=1}^{m} w_{pi} \phi_{pi}(\mathbf{s}_p, \mathbf{s}_c, m_{li}) \tag{14}$$

subject to the design parameter bounds which constitute the feasible solution space $\Omega_p \subset \mathbb{R}^2$. The weight vector $\mathbf{w}_p$ in (14) is used to reflect the relative importance and/or the frequency of use of each payload. For example, if the manipulator is designed to carry three different payloads and the probability distribution of their expected usage is such that $p(m_{l1}) = 0.5$, $p(m_{l2}) = 0.4$ and $p(m_{l3}) = 0.1$. Then the weight vector $\mathbf{w}_p$ can be chosen as $\mathbf{w}_p = [0.5 \quad 0.4 \quad 0.1]^{\mathrm{T}}$.

4. Nonlinear gain PD controller design optimization

The problem of set-point control or trajectory tracking for robotic manipulators whose dynamics are given by (9) can be accomplished using the so-called PD+ controller [16]. Although the stability of closed-loop systems when using the PD+ control law is assured for symmetric and positive definite proportional and derivative gain matrices [16], the system performance is governed by the choice of controller gains which are assumed to be constant. To achieve high performance or to satisfy real constraints of actual manipulators such as actuator capabilities and joint friction, it may be necessary to have variable controller gains. In Kelly and Carelli [17], a class of nonlinear gain PD controllers was proposed for set-point position control of serial n-link rigid robotic manipulators whose dynamics are given by Eq. (9). The nonlinear gain PD controllers of this class are defined as

$$\mathbf{u} = \mathbf{K}_p(\tilde{\mathbf{q}})\tilde{\mathbf{q}} + \mathbf{K}_d(\mathbf{q}_d, \tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})\dot{\tilde{\mathbf{q}}} + \mathbf{g}(\mathbf{q}) \tag{15}$$

where $\mathbf{q}_d$ is the vector of desired joint positions, $\tilde{\mathbf{q}} = \mathbf{q}_d - \mathbf{q}$ is the vector of position errors, and $\dot{\tilde{\mathbf{q}}} = -\dot{\mathbf{q}}$ is the vector of velocity errors ($\dot{\mathbf{q}}_d = \mathbf{0}$ for set-point control). The derivative gain matrix $\mathbf{K}_d(\mathbf{q}_d, \tilde{\mathbf{q}}, \dot{\tilde{\mathbf{q}}})$ is symmetric and positive definite, and the proportional gain matrix $\mathbf{K}_p(\tilde{\mathbf{q}})$ is positive definite with the following structure:

$$\mathbf{K}_p(\tilde{\mathbf{q}}) = \mathrm{diag}[k_{p1}(\tilde{q}_1), k_{p2}(\tilde{q}_2), \dots, k_{pn}(\tilde{q}_n)] \tag{16}$$

It has been shown that the closed-loop system of the dynamic model (9) with the above controller is globally asymptotically stable for $k_{pi}(\tilde{q}_i) > 0$, $\forall \tilde{q}_i$ and $i = 1 \dots n$ [17].

In this paper, for set-point control of the two-link manipulator with counter-weights and different payloads, we consider the following class of nonlinear gain PD controllers without a gravity compensation term and expressed as

$$\mathbf{u} = \mathbf{K}_p(\tilde{\mathbf{q}})\tilde{\mathbf{q}} + \mathbf{K}_d(\tilde{\mathbf{q}})\dot{\tilde{\mathbf{q}}} \tag{17}$$

where the nonlinear gain matrices $\mathbf{K}_p(\tilde{\mathbf{q}})$ and $\mathbf{K}_d(\tilde{\mathbf{q}})$ are positive definite and have the following structure:

$$\begin{aligned} \mathbf{K}_p(\tilde{\mathbf{q}}) &= \mathrm{diag}\left[\frac{k_{p1}}{a_1 + |\tilde{q}_1|}, \frac{k_{p2}}{a_2 + |\tilde{q}_2|}\right] \\ \mathbf{K}_d(\tilde{\mathbf{q}}) &= \mathrm{diag}\left[\frac{k_{d1}}{b_1 + |\tilde{q}_1|}, \frac{k_{d2}}{b_2 + |\tilde{q}_2|}\right] \end{aligned} \tag{18}$$

where k_{pi}, k_{di}, a_i, and b_i are positive constants. As discussed in Section 3, the counter-weights computed using the optimization problem in (14) for multiple payloads can perfectly gravity balance the manipulator for at most one particular payload. For all other payloads, the gravity effects will not be exactly compensated using the controller in (17). Because of this, the proof of stability for the nonlinear PD controller in (15) [17] no longer applies to the controller in (17). When the manipulator is not perfectly gravity balanced for a given payload, the stability and robustness analysis for the class of controllers in (17) can be carried out following the approach proposed by Ravichandran et al. [18].

The elements of $\mathbf{K}_d(\tilde{\mathbf{q}})$ which depend on the angular position error $\tilde{q}_i$, provide *nonlinear damping* effects to the system. The choice of the nonlinear gain structure for $\mathbf{K}_d(\tilde{\mathbf{q}})$ is motivated by the following: we should be able to combine the vigor of the undamped system with the settling characteristics of the over-damped system by choosing a damping that is low for large errors and large for small errors. The choice of the nonlinear gain structure for $\mathbf{K}_p(\tilde{\mathbf{q}})$ tends to produce a bang–bang type of control effort for $a_i \to 0$, thus increasing the speed of the output response.

Now, we formulate the controller optimization problem in which nonlinear gain PD controller parameters are chosen to minimize the steady-state error, overshoot, rise time, and settling time of the output response. The aforementioned control objectives can be combined into a single performance measure by considering the integral of time multiplied by absolute error (ITAE) defined as

$$\phi_{\mathrm{ITAE}} = \int_0^{t_f} \rho(t)|\tilde{q}_i|\,\mathrm{d}t \tag{19}$$

where t_f is the evaluation time interval and $\tilde{q}_i(t) = q_{di} - q_i(t)$ is the error between the desired joint angle q_{di} and the actual joint angle $q_i(t)$. The weighting function $\rho(t)$ is chosen by trial and error as $\rho(t) = \{10t$ (if overshoot) or t (otherwise)$\}$ to reduce the overshoot. The following controller design objective which is normalized using the maximum value of $\phi_{\mathrm{ITAE}} (= \phi_{\mathrm{ITAE}}^{\max})$, is chosen for the output response of each link of the manipulator:

$$\phi_{ci}^n(\mathbf{s}_p, \mathbf{s}_c, t_f) = \frac{\phi_{\mathrm{ITAE}}}{\phi_{\mathrm{ITAE}}^{\max}} = \frac{\int_0^{t_f} \rho(t)|\tilde{q}_i|\,\mathrm{d}t}{\int_0^{t_f} \rho(t)|\tilde{q}_i|^{\max}\,\mathrm{d}t}, \quad i = 1,2 \tag{20}$$

where $\mathbf{s}_p$ is plant design parameters, $\mathbf{s}_c$ is controller design parameters, and $|\tilde{q}_i|^{\max}$ is the maximum error observed during the evaluation time interval t_f.

To avoid actuator saturation problems, the following constraints are imposed on the motor torques

$$|u_1(t)| \leqslant u_1^{\max}, \quad |u_2(t)| \leqslant u_2^{\max} \tag{21}$$

and these inequality constraints on the motor torques are included in the optimization problem using a penalty function formulation. To achieve this, normalized input torque violation measures ψ_i defined as

$$\psi_{ci}^n(\mathbf{s}_p, \mathbf{s}_c, t_f) = \max\left\{\frac{\max_{t\in[0,t_f]}|u_i(t)| - u_i^{\max}}{u_i^{\max}}, 0\right\}, \quad i = 1,2 \tag{22}$$

are included in the weighted-sum objective formulation whenever the motor torques exceed their allowed maximum values. Based on some initial simulations for the set-point control problem considered in this paper, it can be observed that the input torques reach their maximum at the initial moment of the control process. Since we consider $q_i(0) = 0$, at the initial moment we have $\tilde{q}_i = q_{di}$ and $\dot{\tilde{q}}_i = 0$. Therefore, from (17) and (18), we can assume the following constraints among the parameters of the nonlinear gain PD controller:

$$k_{pi} = \frac{u_i^{\max}(a_i + q_{di})}{q_{di}} \quad i = 1,2. \tag{23}$$

Using the weighted-sum objective formulation, the controller optimization problem for the manipulator with a payload m_{li} is mathematically stated as: choose the controller parameters $\mathbf{s}_c = [k_{d1}, k_{d2}, a_1, a_2, b_1, b_2]^{\mathrm{T}} \in \Omega_c \subset \mathbb{R}^6$ to minimize

$$\min_{\mathbf{s}_c\in\Omega_c\subset\mathbb{R}^6} \phi_{ci}(\mathbf{s}_p, \mathbf{s}_c, m_{li}, t_f) = \min_{\mathbf{s}_c\in\Omega_c\subset\mathbb{R}^6} \left\{\sum_{i=1}^{2} w_i\phi_{ci}^n(\mathbf{s}_p, \mathbf{s}_c, t_f) + \sum_{j=1}^{2} w_{2+j}\psi_{cj}^n(\mathbf{s}_p, \mathbf{s}_c, t_f)\right\} \tag{24}$$

subject to the design parameter bounds which constitute the feasible solution space $\Omega_c \subset \mathbb{R}^6$. The weights w_i in (24) are chosen as $w_1 = w_2 = 1$ and $w_3 = w_4 = 10$ by trial and error. To obtain robust dynamic performance for set-point control of the manipulator carrying different payloads, the following optimization problem can be used to find the nonlinear gain PD controller parameters. For the given payloads m_{li}, $i = 1\ldots m$, and the weight vector $\mathbf{w}_c = [w_{c1} \cdots w_{cm}]^{\mathrm{T}}$, choose the controller parameters $\mathbf{s}_c = [k_{d1}, k_{d2}, a_1, a_2, b_1, b_2]^{\mathrm{T}} \in \Omega_c \subset \mathbb{R}^6$ to minimize

$$\min_{\mathbf{s}_c\in\Omega_c\subset\mathbb{R}^6} \phi_{cobj}(\mathbf{s}_p, \mathbf{s}_c, t_f) = \min_{\mathbf{s}_c\in\Omega_c\subset\mathbb{R}^6} \sum_{i=1}^{m} w_{ci}\phi_{ci}(\mathbf{s}_p, \mathbf{s}_c, m_{li}, t_f) \tag{25}$$

subject to the design parameter bounds which constitute the feasible solution space $\Omega_c \subset \mathbb{R}^6$. The weight vector $\mathbf{w}_c$ in (25) is used to reflect the relative importance and/or the frequency of use of each payload.

5. Simultaneous design optimization results

In this section, simultaneous design optimization results for the optimal design of a two-link planar manipulator and a nonlinear gain PD controller are presented and compared against those obtained using a sequential design optimization. To achieve this optimal design, the simultaneous plant-controller design methodology along with the $(\mu + \lambda)$-ES algorithm as described in Section 2.2 is used to find optimal plant and controller parameters. The simultaneous design optimization problem for the gravity balanced manipulator and the nonlinear gain PD controller can be stated using the weighted-sum formulation in (7) as

$$\min_{[\mathbf{s}_p,\mathbf{s}_c]^{\mathrm{T}}\in\Omega_{pc}} \phi_{pcobj}(\mathbf{s}_p, \mathbf{s}_c, t_f) = \min_{[\mathbf{s}_p,\mathbf{s}_c]^{\mathrm{T}}\in\Omega_{pc}} \{w_{pobj}\phi_{pobj}(\mathbf{s}_p, \mathbf{s}_c, t) + w_{cobj}\phi_{cobj}(\mathbf{s}_p, \mathbf{s}_c, t_f)\} \tag{26}$$

subject to the design parameter bounds which constitute the feasible solution space $\Omega_{pc} \subset \mathbb{R}^8$. The plant and controller design objectives ϕ_{pobj} and ϕ_{cobj} in (26) are given by (14) and (25), respectively.

In the following design optimizations, the maximum values for the motor torques are chosen as $u_1^{\max} = 200$ N m and $u_2^{\max} = 80$ N m. The initial and desired joint angular positions for the manipulator are $\mathbf{q}(0) = [0, 0]^{\mathrm{T}}$ rad and $\mathbf{q}_d = [\frac{\pi}{4}, \frac{\pi}{2}]^{\mathrm{T}}$ rad. The following values are considered for the manipulator model parameters as shown in Fig. 2: $m_1 = m_2 = 1$ kg, $m_{a2} = m_l = 0.5$ kg, $l_1 = l_2 = 1.0$ m, $l_{c1} = l_{c2} = 0.5$ kg, $l_{w1} = l_{w2} = 0.4$ m, and $I_1 = I_2 = 0.0833$ kg m^2. The $(\mu + \lambda)$-ES algorithm, with $\mu = 15$ and $\lambda = 100$, is used to find the optimal plant and controller parameters. The initial population for the $(\mu + \lambda)$-ES algorithm is generated randomly by uniformly distributing the design parameter values within their respective ranges. The evaluation time interval t_f for design simulations is set to 4 s.

5.1. Sequential design optimization for single payload

In this design optimization, a payload of 0.5 kg is considered and the optimal counter-weights required to gravity balance the manipulator with this payload can be computed by solving Eqs. (12) and (13). Using these optimal counter-weights which are listed in Table 1 (in column P_{opt}), the controller optimization problem is solved by the $(\mu + \lambda)$-ES algorithm with the weighted-sum objective in (25) as its fitness function. By selecting this fitness function, the optimization algorithm searches for optimal $\mathbf{s}_{c\_npd}$ (nonlinear gain PD controller parameters) or $\mathbf{s}_{c\_pd}$ (linear PD controller gains) so that the desired control objectives are achieved. Results of the $(\mu + \lambda)$-ES optimization for the optimal values of the nonlinear gain PD controller parameters and the linear PD controller gains are given in Table 1 (in columns C_{npd_opt} and C_{pd_opt}).

5.2. Simultaneous design optimization for single payload

To verify the efficacy of the simultaneous design optimization framework based on evolutionary algorithms, simultaneous design optimization of the manipulator (carrying a payload of 0.5 kg) and controllers are performed using the $(\mu + \lambda)$-ES algorithm with the weighted-sum objective in (26) as its fitness function. The weights in (26) are chosen as $w_{pobj} = 0.2$ and $w_{cobj} = 0.8$ to give more importance in achieving the controller design objectives. By selecting this fitness function, the optimization algorithm searches for optimal $\mathbf{s}_p$ and $\mathbf{s}_{c\_npd}$ (or $\mathbf{s}_{c\_pd}$, in the case of linear PD controller) so that the desired design objectives are achieved.

Table 1
Plant-controller design optimization results for single payload

	P_{opt}	C_{npd_opt}	C_{pd_opt}	PC_{npd_opt}	PC_{pd_opt}
k_{d1}	–	34.59	96.16	34.51	96.19
k_{d2}	–	8.17	18.45	8.20	18.47
a_1	–	0.0643	–	0.0740	–
a_2	–	0.0581	–	0.0545	–
b_1	–	0.0947	–	0.0979	–
b_2	–	0.0478	–	0.0471	–
m_{w1} [kg]	12.5	(12.5)	(12.5)	12.5	12.5
m_{w2} [kg]	2.5	(2.5)	(2.5)	2.5	2.5
$\phi_{pcobj}(\cdot)$	–	0.0032	0.0106	0.0032	0.0106

Results of the $(\mu + \lambda)$-ES optimization for the optimal values of the counter-weights and the controller parameters (nonlinear gain PD as well as linear PD) are listed in Table 1 (in columns PC_{npd_opt} and PC_{pd_opt}). As shown in Table 1, the design parameters obtained from the sequential and simultaneous optimizations are in close agreement. This is due to the fact that for the chosen w_{pobj}, w_{cobj} and $\mathbf{q}_d$, the global minimum of ϕ_{pobj} and ϕ_{cobj} are obtained with the same set of plant and controller design parameters.

The closed-loop system output responses simulated using the counter-weights and the nonlinear gain PD controller parameters obtained from the simultaneous optimization are shown in Fig. 3, and compared against those responses simulated using the optimal counter-weights and the linear PD gains. The corresponding motor torques are shown in Fig. 4. It should be pointed out here that high values for the motor torques subject to available maximum torque constraints (which are chosen as $u_1^{\max} = 200$ N m and $u_2^{\max} = 80$ N m for the purpose of illustration) are used only during the transient phase to maximize the speed of the output response for both links as shown in Figs. 3 and 4. As discussed in Section 4, stable set-point control of the manipulator can be achieved by using a linear or nonlinear PD controller with any positive gain values. Hence, it is possible to lower the maximum motor torques available to any desired levels by allowing a reduction in the maximum achievable speed of the output response for both links accordingly.

It can be noted from Fig. 3 that the nonlinear gain PD controller achieves significant improvements in the closed-loop output responses as compared to those obtained from linear PD controller. One can also conclude that the nonlinear gain PD controller parameters are "optimum" by considering the efficient utilization of input power (see Fig. 4 where the control effort approaches a bang–bang type). The improvements in the transient performance are obtained with nonlinear gain PD controller

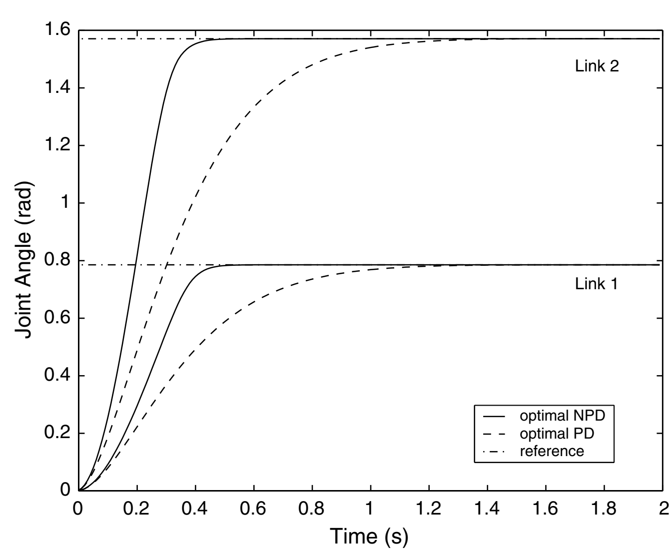

Fig. 3. Output responses using optimal NPD and PD gains.

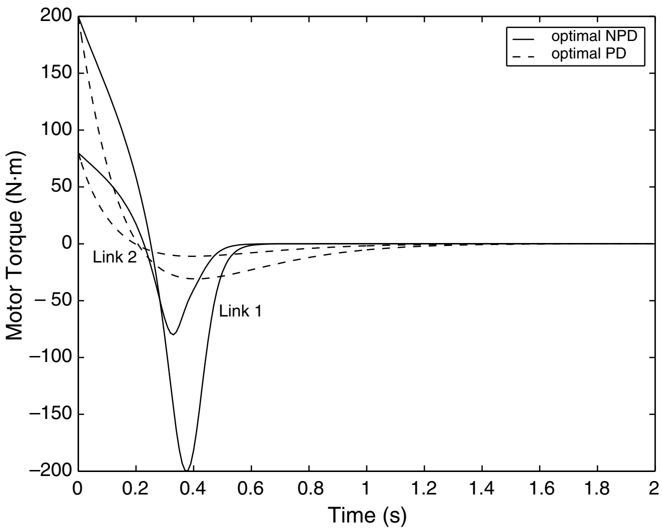

Fig. 4. Input torques using optimal NPD and PD gains.

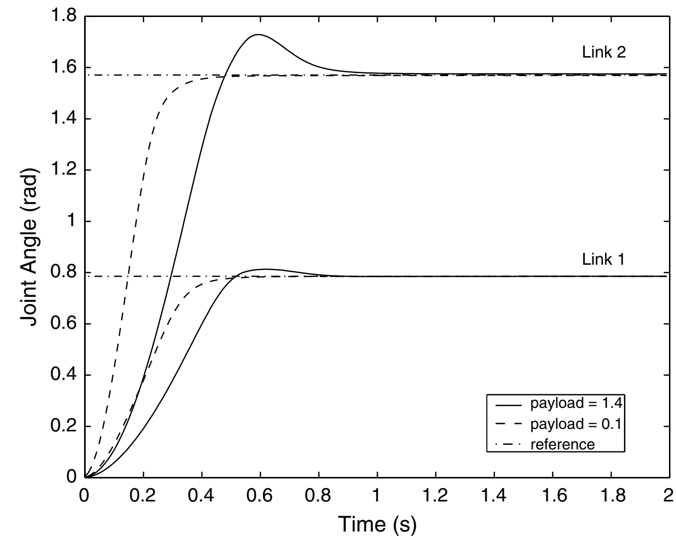

Fig. 5. Output responses using nominal NPD for different payloads.

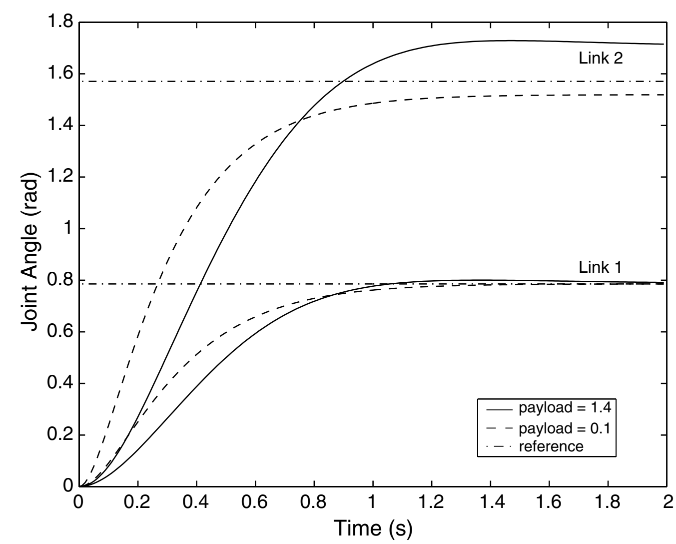

Fig. 6. Output responses using nominal PD for different payloads.

subject to the same maximum torque constraints as specified in the optimization problem. This will allow, while using nonlinear gain PD controller, one to increase the speed of the output response without saturating actuators or to use smaller actuators to obtain similar performance as compared to linear PD controller. However, it can be noted from Fig. 4 that more energy is consumed for improving the transient performance and this is not unexpected since the total energy consumption is not considered in the optimization problem formulation. If it is required to include the minimization of the total energy consumption, it can be readily incorporated within the formulation in (7) as a design objective with appropriate weight factor. Also, one can observe the effect of nonlinear damping in Fig. 4 where the input torques are reduced slowly when the errors are higher and they are reduced rapidly when the outputs are closer to the set points. This nonlinear damping effect contributed to the improvements in the transient performance of the closed-loop system.

The robustness of the optimal nonlinear gain PD and linear PD controllers are compared by simulating the closed-loop system output responses for the other payloads. The closed-loop system output responses simulated for different payloads using the counter-weights and nonlinear gain PD controller parameters obtained from the simultaneous optimization are shown in Fig. 5. Similarly, the output responses simulated for different payloads using the counter-weights and linear PD gains obtained from the simultaneous optimization are shown in Fig. 6. When the manipulator is not perfectly gravity balanced for the other payloads, the linear PD controller exhibits significant steady-state errors (see Fig. 6) as expected. In comparison, for the same payloads the nonlinear gain PD controller reduces the steady-state errors significantly as shown in Fig. 5. This nice feature of the nonlinear gain PD controller to handle steady-state errors when the manipulator is not perfectly gravity balanced is due to the freedom of increasing the nonlinear gain $k_{pi}(\tilde{q}_i)$ which compensates for the position error. However, the transient performance of the closed-loop system for higher payloads needs improvement and this can be accomplished by simultaneously optimizing the nonlinear gain PD controller parameters for different payloads as illustrated in the following subsection.

5.3. Sequential design optimization for multiple payloads

In this design optimization, three different payloads namely, $m_{l1} = 0.5$ kg, $m_{l2} = 1.4$ kg and $m_{l3} = 0.1$ kg are considered. Since it is required to create a gravity balanced manipulator design to handle more than one payload, the optimal counter-weights cannot be computed using (12) and (13). However, the required counter-weights to handle these payloads can be found by solving the optimization problem in (14). The associated weights in this optimization problem are taken as $w_{p1} = 0.5$, $w_{p2} = 0.4$, and $w_{p3} = 0.1$. Using the nonlinear gain PD controller parameters obtained from the simultaneous design optimization for the manipulator with the most highly weighted payload (in this case, the payload is m_{l1} and the corresponding controller parameters are listed in Table 1), the plant optimization problem is solved using the $(\mu + \lambda)$-ES algorithm with the weighted-sum objective in (14) as its fitness function. By selecting this fitness function, the optimization algorithm searches for optimal $\mathbf{s}_p$ so that the desired design objectives are achieved. Results of the $(\mu + \lambda)$-ES optimization for the optimal values of the counter-weights are shown in Table 2 (in column P_{opt}). The optimal counter-weights thus obtained are identical to the counter-weights required to perfectly gravity balance the manipulator with payload m_{l1}. These results can also be validated analytically using (12), (13) and the fact that the most highly weighted payload is the median of all the payloads considered.

Using the optimal counter-weights found in the above procedure, the controller optimization problem is solved using the $(\mu + \lambda)$-ES algorithm with the weighted-sum objective in (25) as its fitness function for improving set-point control performance of the manipulator carrying different payloads. Results of the $(\mu + \lambda)$-ES optimization for the optimal values of the nonlinear gain PD controller parameters are given in Table 2 (in column C_{npd_opt}). It can be noted from these results that by performing controller optimization for the manipulator to handle different payloads, the objective function ϕ_{pcobj} is improved by 25% of the value obtained from the controller optimization for single payload. The closed-loop system output responses simulated for three payloads using the counter-weights and the nonlinear gain PD controller parameters obtained from the controller optimization are shown in Fig. 7. In this figure, the output response simulated for the highest payload indicates that the overshoot is significantly reduced from that of the single payload case (see Fig. 5). However, to achieve these improvements, the speed of all the output responses are sacrificed slightly.

Table 2
Plant-controller design optimization results for multiple payloads

	P_{opt}	C_{npd_opt}	PC_{npd_opt}
k_{d1}	(34.51)	42.84	38.85
k_{d2}	(8.20)	13.41	11.02
a_1	(0.0740)	0.0204	0.0408
a_2	(0.0545)	0.0206	0.0204
b_1	(0.0979)	0.0773	0.0781
b_2	(0.0471)	0.0621	0.0484
m_{w1} [kg]	12.5	(12.5)	16.438
m_{w2} [kg]	2.5	(2.5)	3.625
$\phi_{pcobj}(\cdot)$	0.008485	0.006396	0.005896

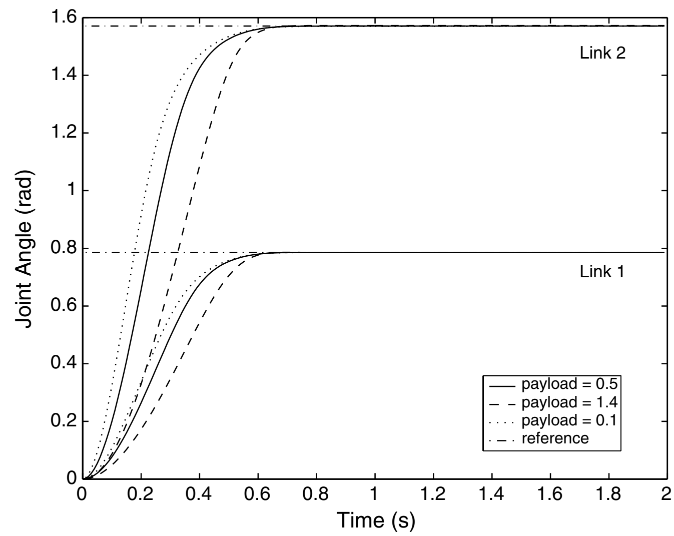

Fig. 7. Output responses using robust NPD for different payloads.

5.4. Simultaneous design optimization for multiple payloads

To enhance the dynamic performance of the closed-loop system consisting of the nonlinear gain PD controller and the manipulator with different payloads, simultaneous design optimization of the manipulator carrying different payloads (as defined in the previous subsection) and the nonlinear gain PD controller is performed using $(\mu + \lambda)$-ES algorithm with the weighted-sum objective in (26) as its fitness function. As before, the weights in (26) are chosen as $w_{pobj} = 0.2$ and $w_{cobj} = 0.8$ to give more importance in achieving controller design objectives. By selecting this fitness function, the optimization algorithm searches for optimal $\mathbf{s}_p$ and $\mathbf{s}_c$ so that the desired design objectives are achieved. Results of the $(\mu + \lambda)$-ES optimization for the optimal values of the counter-weights and the controller parameters are listed in Table 2 (in column PC_{npd_opt}) which indicate that the simultaneous design optimization yielded different design parameter values than those obtained with the sequential design optimization. These results also illustrate that the simultaneous plant-controller design optimization improves the objective function ϕ_{pcobj} by 8% of the value obtained from the sequential design optimization for the same tasks. The closed-loop system output responses simulated for three payloads using the counter-weights and the nonlinear gain PD controller parameters obtained from the simultaneous optimization are shown in Fig. 8. The output responses simulated for highly weighted payload using the optimal counter-weights and the nonlinear gain PD controller parameters obtained from various optimizations are compared in Fig. 9. As shown in Fig. 8, the simultaneous plant-controller design optimization reduces the overshoot (for the highest payload) without sacrificing too much the speed for all the output responses as compared to the sequential optimization case. This is clearly demonstrated for the most highly weighted payload m_{l1} in Fig. 9.

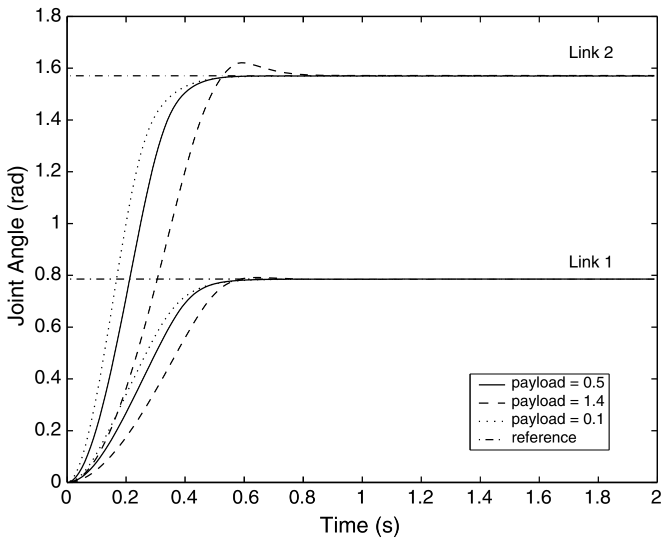

Fig. 8. Output responses using optimal NPD for different payloads.

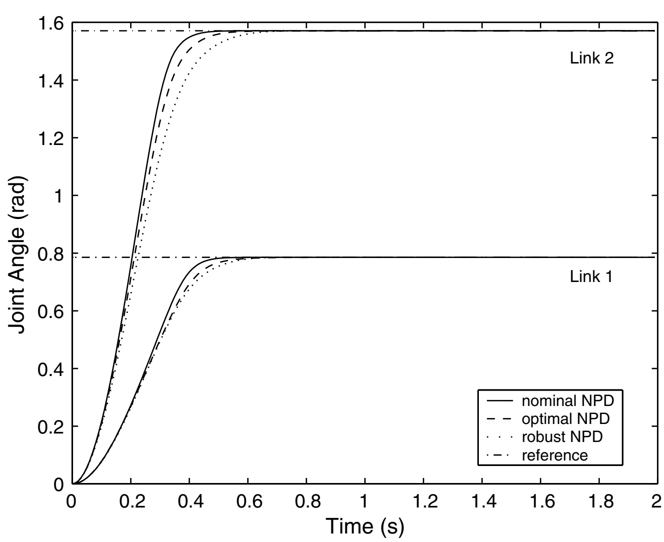

Fig. 9. Output responses using optimal NPD for nominal payload.

6. Conclusions

In this paper, a simultaneous plant-controller design optimization methodology has been investigated for the optimal design of a two-link rigid manipulator along with a nonlinear gain PD controller for performing multiple tasks. The multiple tasks considered in this paper are the set-point control of the manipulator for carrying different payloads. The simultaneous design optimization methodology has been developed using a heuristic search technique called evolution strategy, and a weighted-sum problem formulation to combine multiple objectives and to generate a single Pareto optimal solution. It has been demonstrated by the simulation results that the simultaneous design optimization method has the potential to yield more desirable designs of the manipulator and the controller for specified tasks. For future work, the proposed methodology can be extended by exploiting the population-based evolution strategy algorithm to adopt a vector optimization approach for generating the entire set of Pareto optimal solutions to illustrate the full benefits of the simultaneous design optimization methodology.

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