**Mutation Method Selection**

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( Methods and materials)

To find the balance between exploration and exploitation 4 methods were checked:

1. **Exploration -** Exploitation first method - Select random neighbor from the first neighbors for the first 100 generations and select random neighbor from second neighbors after 100 generations
2. **Exploitation -** Exploration first method - Select random neighbor from the second neighbors for the first 100 generations and select random neighbor first neighbors after 100 generations
3. Combined method - Select random neighbor from the second neighbors for the first 100 generations, then select random neighbor first neighbors for more 150 generations, and after 250 generations select random neighbor from the second neighbors again.
4. Random Method – Select a random configuration from the non-simulated configurations

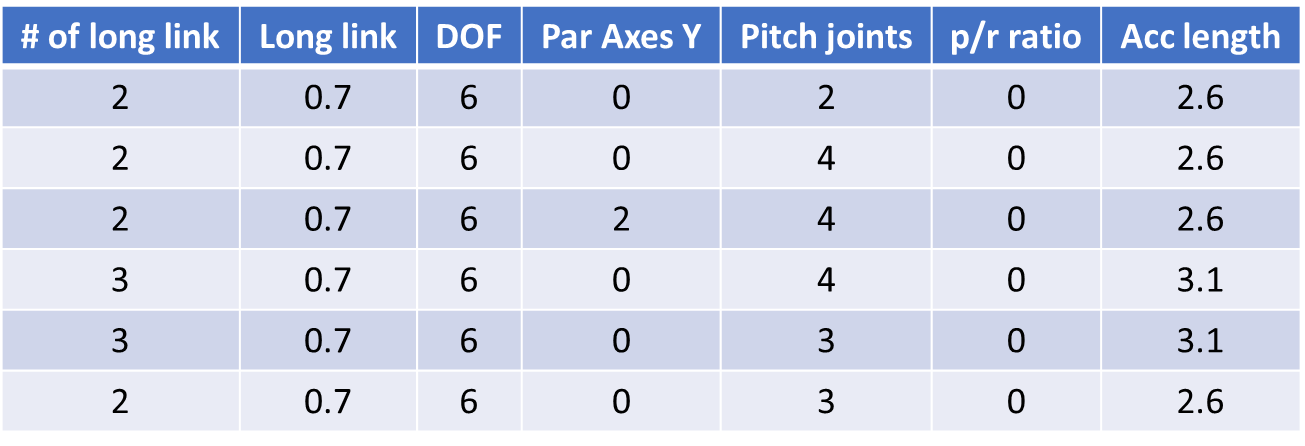
Those methods were tested in 4 different stop conditions:

1. Regular - global 1240 generations.
2. Aggressive - global 1240 generations and local if the concept for 30 generations in a row.
3. Medium - global 1240 generations and local if the concept for 50 generations in a row.
4. Ease - global 1240 generations and local if the concept for 100 generations in a row.

(Results)

To find the best mutation method 6 large concepts were chosen randomly (can be seen in Table 7).

Table 1- Checked Concepts



The selected concepts were fully sorted, all the configurations in those concepts were simulated and their results are known. To compare the methods, each method runs 30 times, for a maximum of 1240 generations. For each method, two indices were calculated: Hyper–Volume(HV) which bigger value means better result and minimum value of the manipulability, this index selected because it can be shown in figure 1 that big part of the results of the mid proximity joint is close to 0 and the manipulability is spreading more. Another metric used to compare the Inverse Generational Distance (IGD). IGD computes the average distance between the true set and the current set. The lower the IGD value, the better is the front.

~~In figure 2 it can be seen HV at each generation for each method.~~

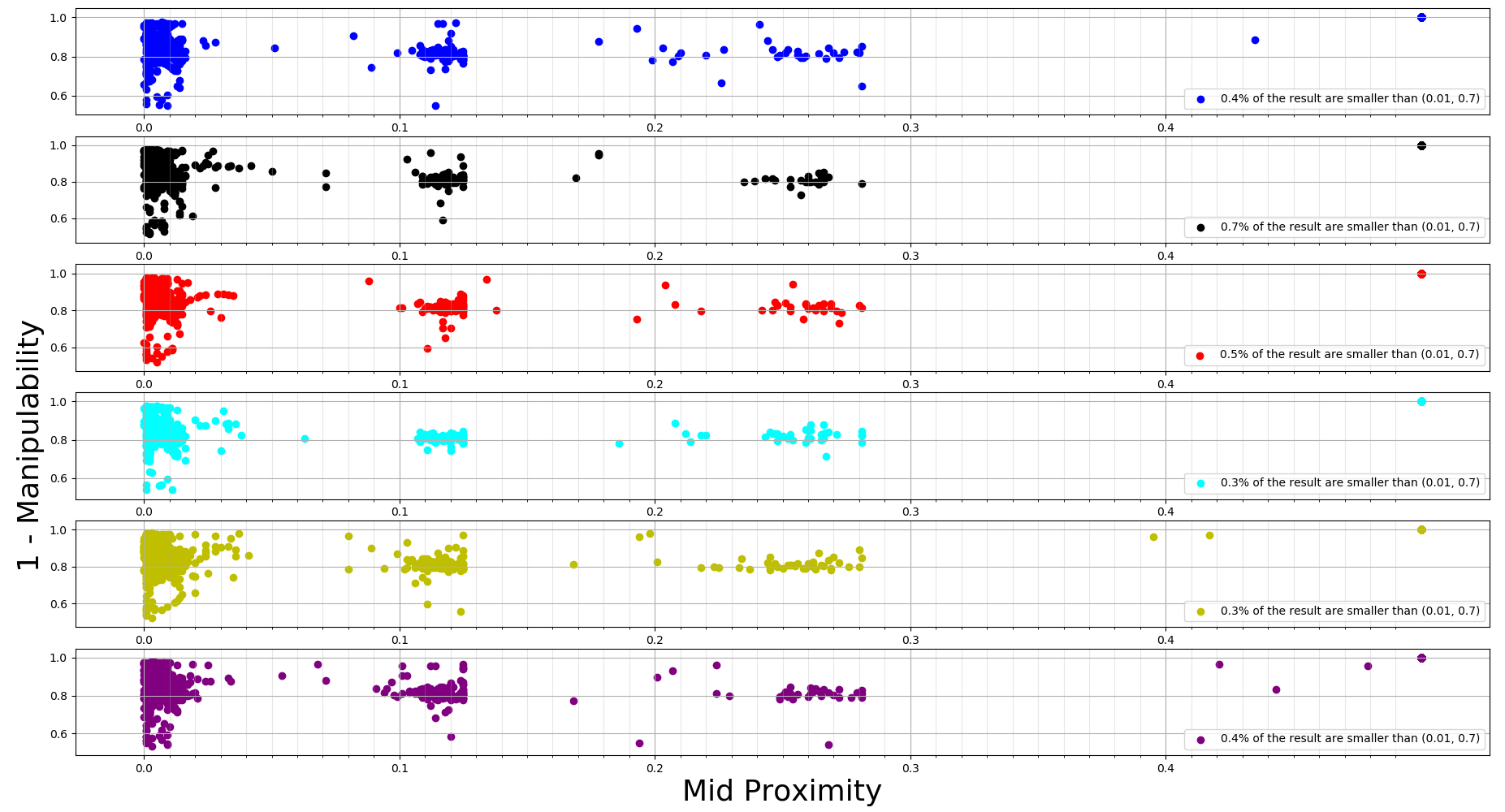


Figure - Concepts Full sorting

In figure 3 it can be seen minimum manipulability at each generation for each method.

From those figures, it can be seen that the most significant changes have occurred in the first 300 generations. Another thing that can be seen that except for the regular stop condition, all the stop conditions get their final position until generation number 200.

To compare the algorithms Wilcoxon test was applied over the results of each method of the 30 runs. Wilcoxon test return P-Value, which P-Value lower than 0.05 means there is a statistical significance that one method its better than another. from figures 5-7 it can be seen that methods with the same stop conditions are in the same population. The selected methods are with the highest median and the lowest Min-Manipulability median and IGD (Table 2).

In Table 2 there are the medians and variances of HV, IGD, Min Manipulabiliry, and the generation in which the WOI was last change over 30 runs on each method. The last four columns, under the Wilcoxon title, show if the method its less than (-) the reference method or there is uncertainty(~) according to the Wilcoxon test.

It can be seen that the regular stop condition (which simulate the Fair resource allocation method), gives the best results, but between the mutation methods, there is no statistical significance for any method. But because of this test compare only six fully sort concepts, the disadvantages of the fair method don’t take into account, such as the simulation time, and the number of concepts to optimize which will make the algorithm to stop early, after about 150-200 generations.

From figures 5 -7, it can be seen that Aggressive-Exploitation is statistical significance as not as good as Ease-Random in HV and Min-Manipulability metrics.

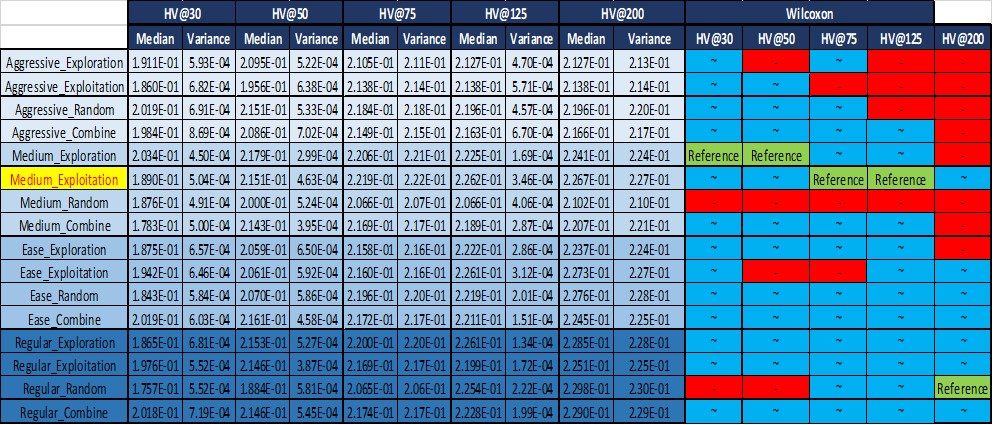
In Table 4 it can be seen a comparison of the HV at different generations. The selected numbers were taken from the median values in Table 2.

Table 2 - Summary of the results



Table 3 - Summary of the results without Regular

Table 4 - HV in specific Generation



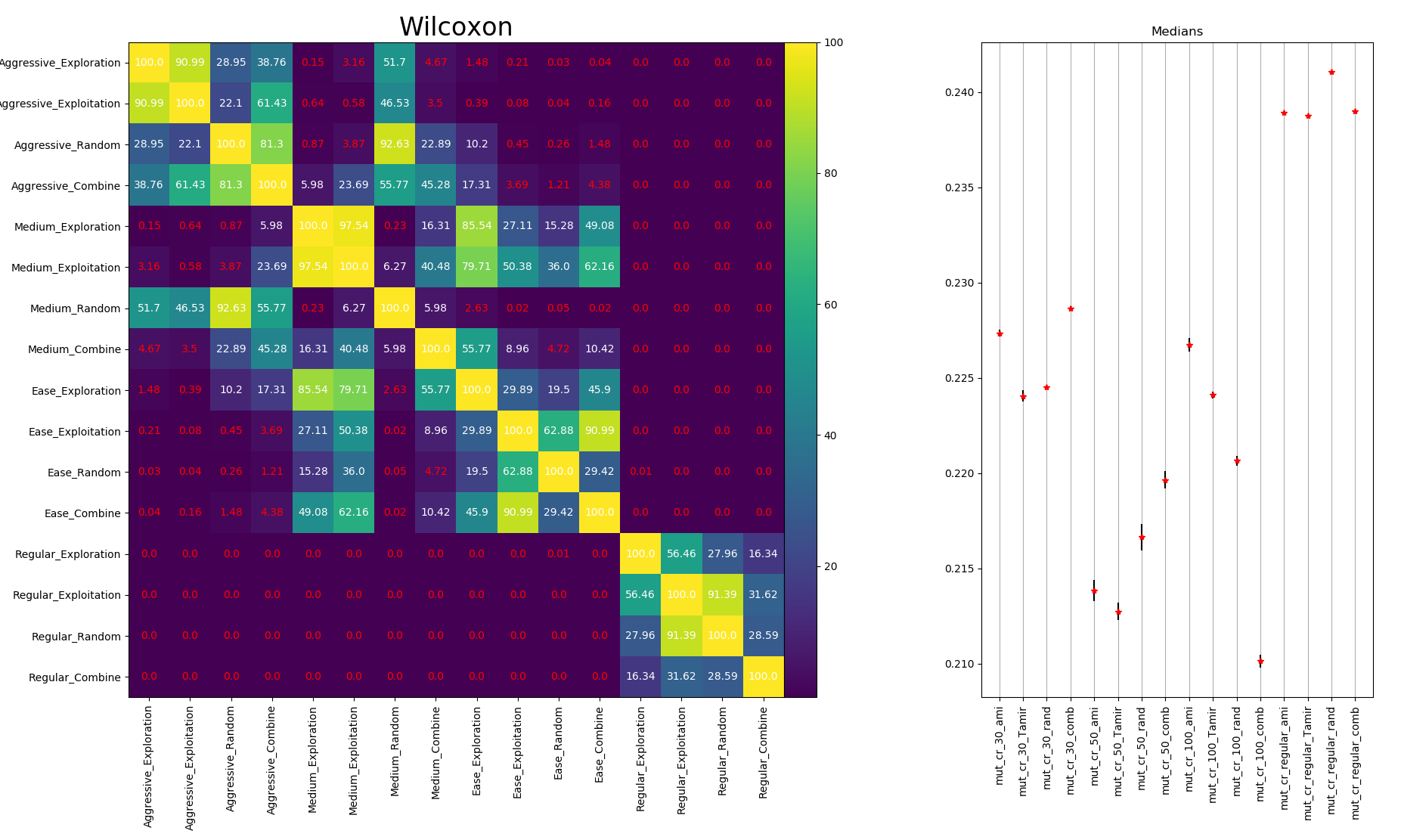


Figure 5- Wilcoxon: HV

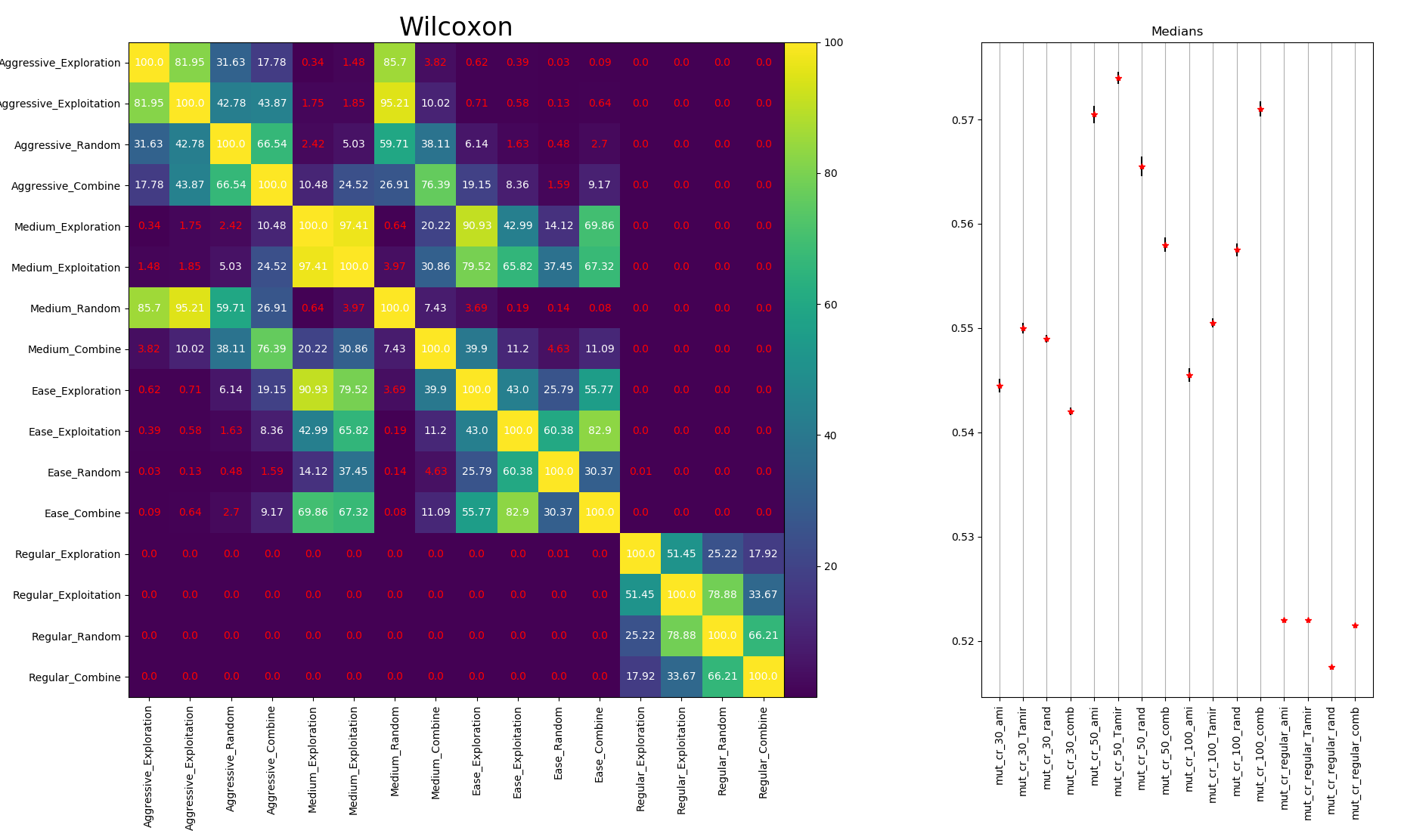


Figure 6 - Wilcoxon: Min Manipulability

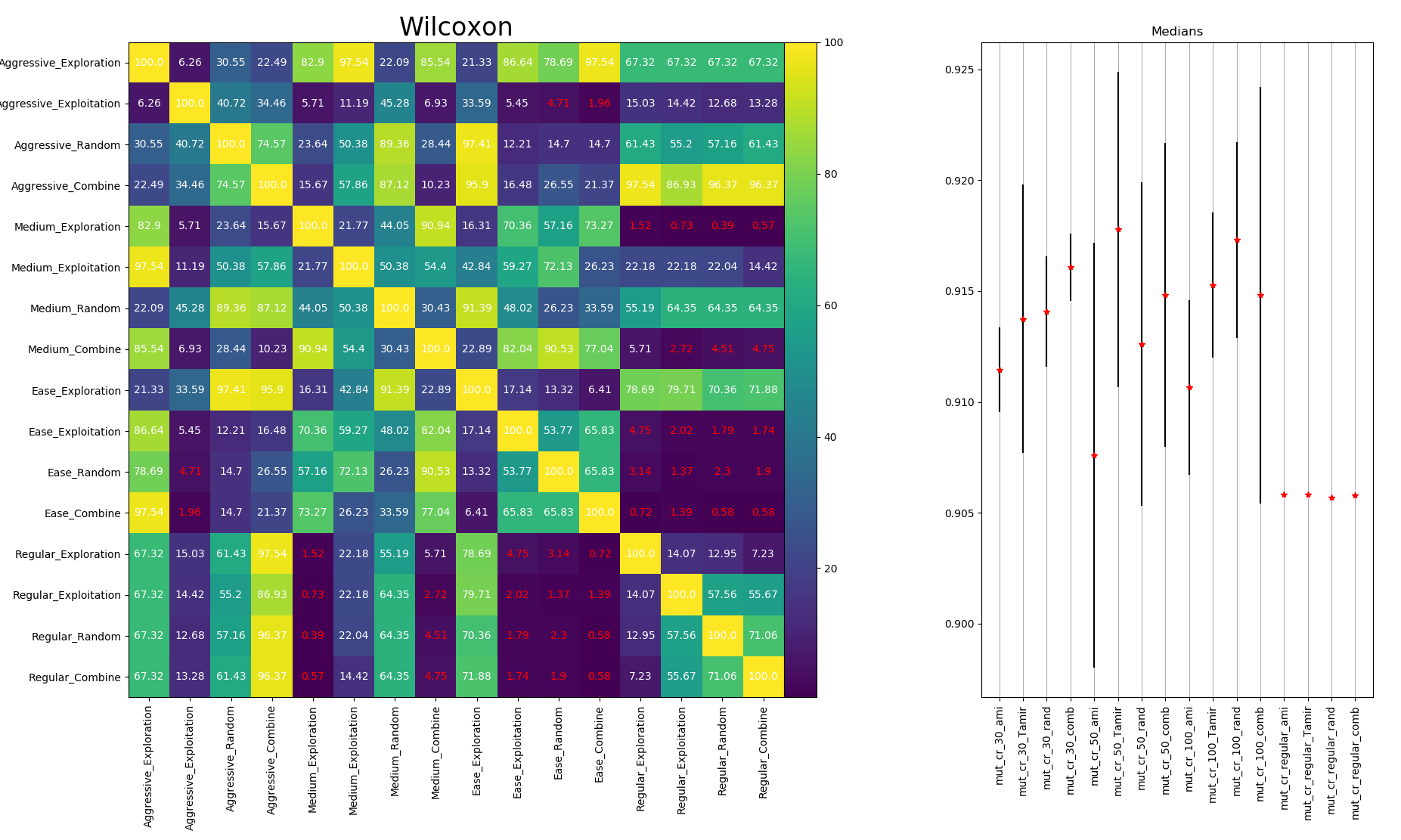


Figure 7 - Wilcoxon IGD

No clear answer was received from testing the 6 concepts. In Figure 2 it can be shown that the six concepts have a similar front, which may affect the results, therefore another five concepts with different front were selected also to make a comparison between the methods. The selected concepts are presented in Table 5.

Table 5 - New 5 Concepts



Those five selected concepts weren’t fully sorted, so in the comparison method, only the prior knowledge used. In figure 8 it can be shown the new concepts known fronts.

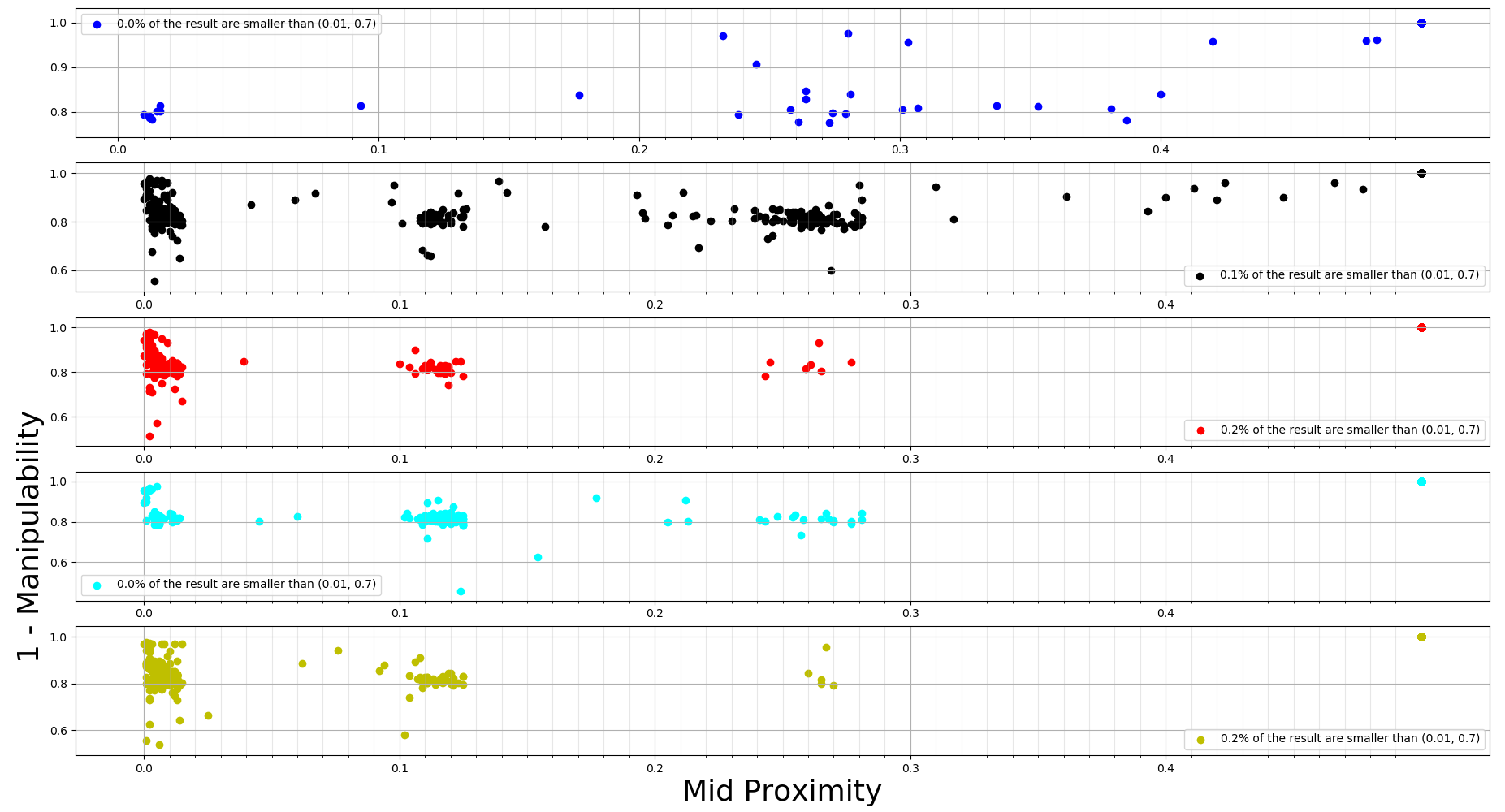


Figure 8 - New 5 Concepts Known Fronts

To do the comparison between the methods, several runs were done:

1. The 5 new concepts only
2. The 5 new concepts with 1 concept from the first 6 concepts
3. 4 concepts from the 5 new concepts with 2 concepts from the first 6 concepts
4. 3 concepts from the 5 new concepts with 3 concepts from the first 6 concepts
5. 2 concepts from the 5 new concepts with 4 concepts from the first 6 concepts
6. 1 concept from the 5 new concepts with 5 concepts from the first 6 concepts

For each run, there are 2 tables, one of the medians and variance of the HV, Min Manipulability, IGD metrics, and the number of generations. and the second table is the HV at several generations.

Table 6 - Run (a) results



Table 7 - HV @ Generation - run (a)



Table 8 - Run (b) results



Table 9 - HV @ Generation - run (b)



Table 10 - Run (c) results



Table 11 - HV @ Generation - run (c)



Table 12 - Run (d) results



Table 13 - HV @ Generation - run (d)



Table 14 - Run (e) results



Table 15 - HV @ Generation - run (e)

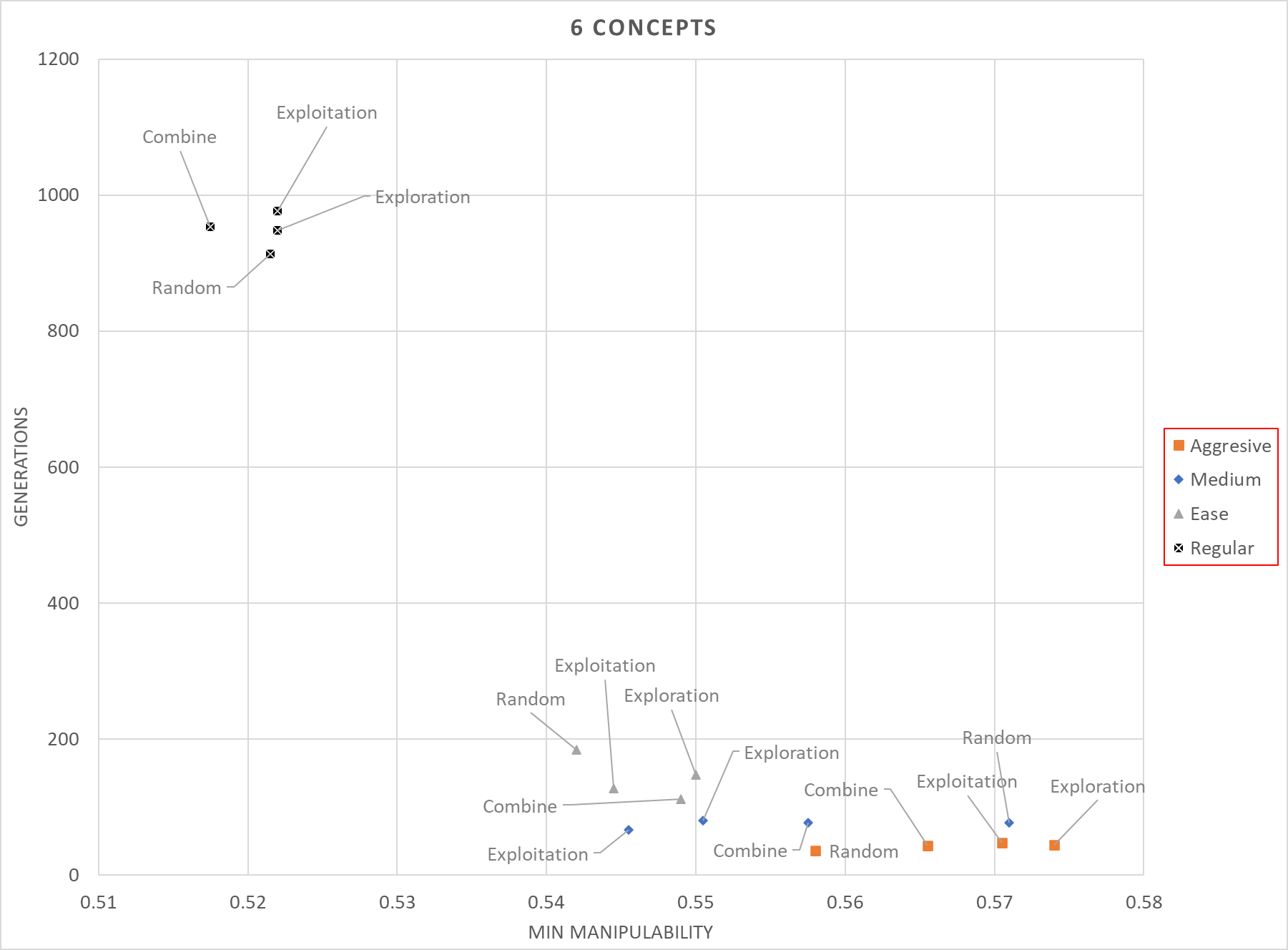


Table 16 - Run (f) results



Table 17 -HV @ Generation - run (f)

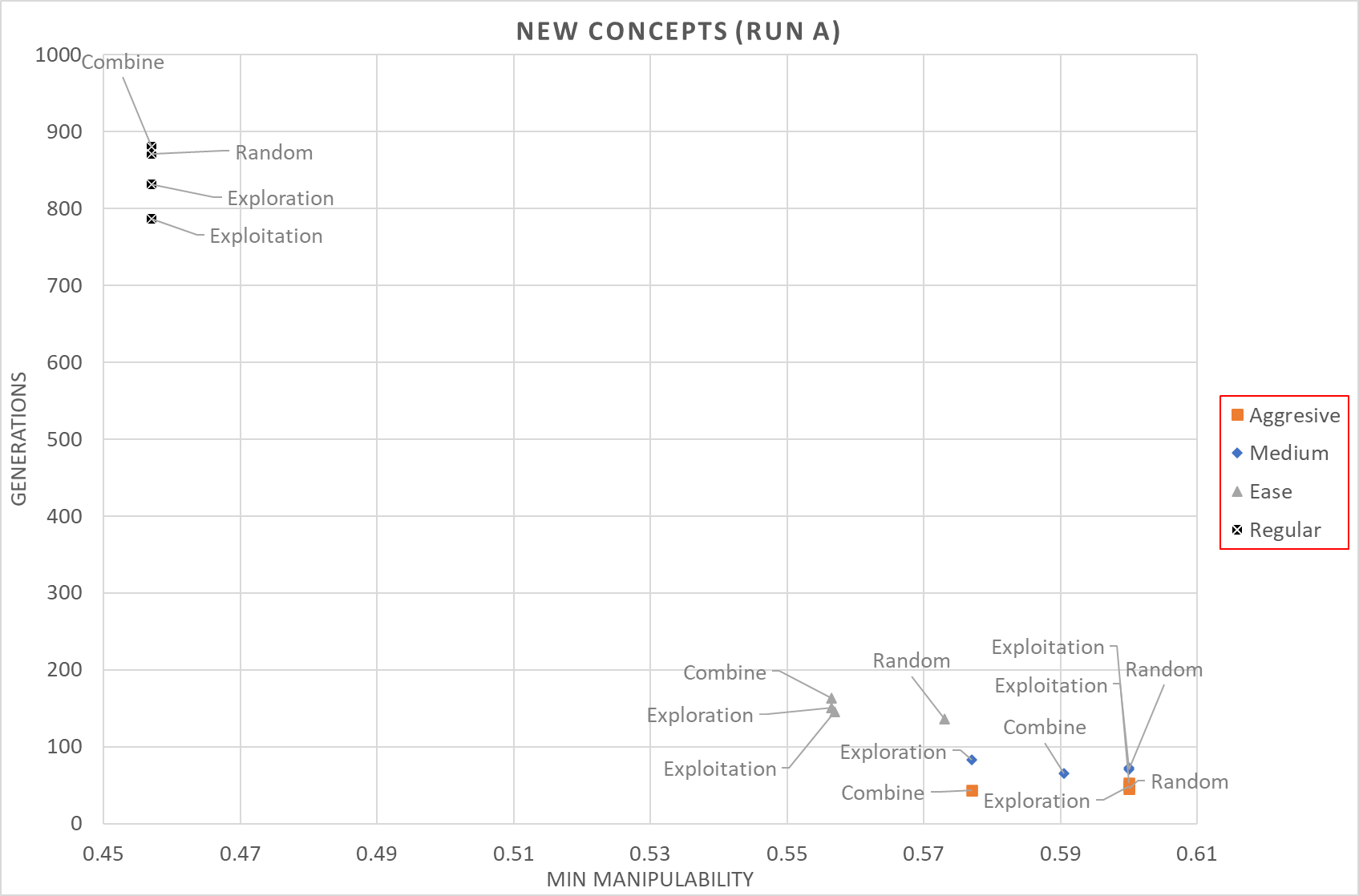




From fig\*\*, that show the Generations Vs Min Manipulability for the first 6 concepts, it can be shown that:

* In Regular: Combine and Random are dominating over Exploitation ו Exploration, but according to the Wilcoxon test, there is no statistical significance.
* In Ease: Exploration is dominated by the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Medium: Exploitation dominates over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Aggressive: Random dominates over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* Medium \_ Exploitation dominates over Ease\_ Combine and there is statistical significance only in the generations but not in minimum manipulability.
* 6 non-dominated methods left:

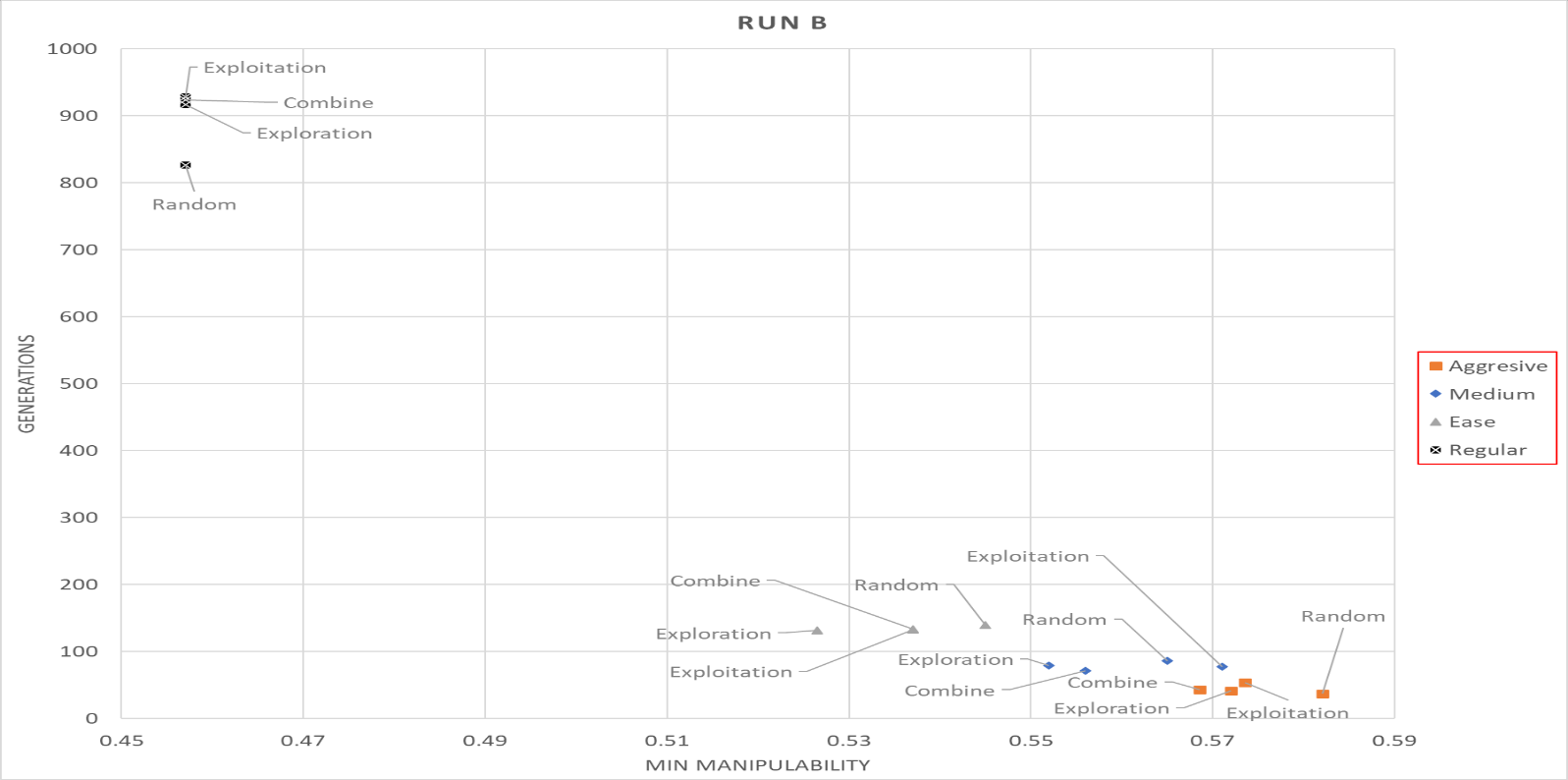
Regular\_ Combine, Regular\_ Random, Ease\_ Random, Ease\_ Exploitation, Medium\_ Exploitation, Aggressive\_ Random



From fig\*\*, that show the Generations Vs Min Manipulability for the new 5 concepts, it can be shown that:

* In Regular: there is no domination of one method over any other method.
* In EASE: there is no domination of one method over any other method.
* In MEDIUM: COMBINE and Exploration are dominating over the 2 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In AGGRESSIVE: COMBINE is dominant over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* AGGRESSIVE\_ COMBINE dominates over MEDIUM\_ COMBINE and there is statistical significance only in the generations but not in minimum manipulability.
* 10 non-dominated methods left:

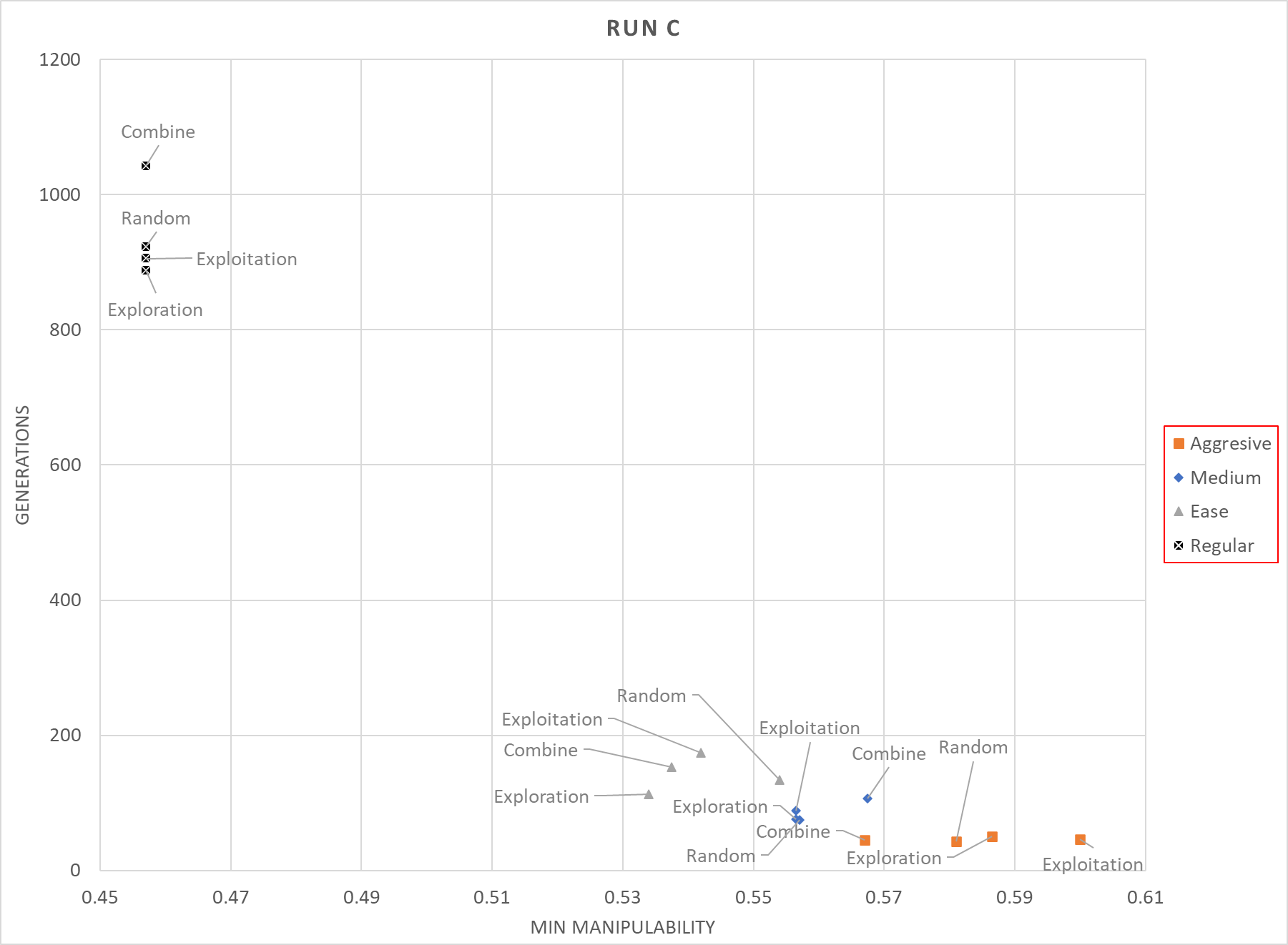
REGULAR\_COMBINE, REGULAR\_RANDOM, REGULAR\_ Exploitation, REGULAR\_ Exploration, EASE\_RANDOM, EASE\_ Exploitation, EASE\_ COMBINE, EASE\_ Exploration, MEDIUM\_ Exploration, AGGRESSIVE\_ COMBINE



From fig\*\*, that show the Generations Vs Min Manipulability run B, it can be shown that:

* In Regular: there is no domination of one method over any other method.
* In Ease: Exploration is dominate over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Medium: Combine and Exploration are dominating over the 2 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Aggressive: Exploitation is dominated by Combine and Exploration, but according to the Wilcoxon test, there is no statistical significance.
* 10 non-dominated methods left:

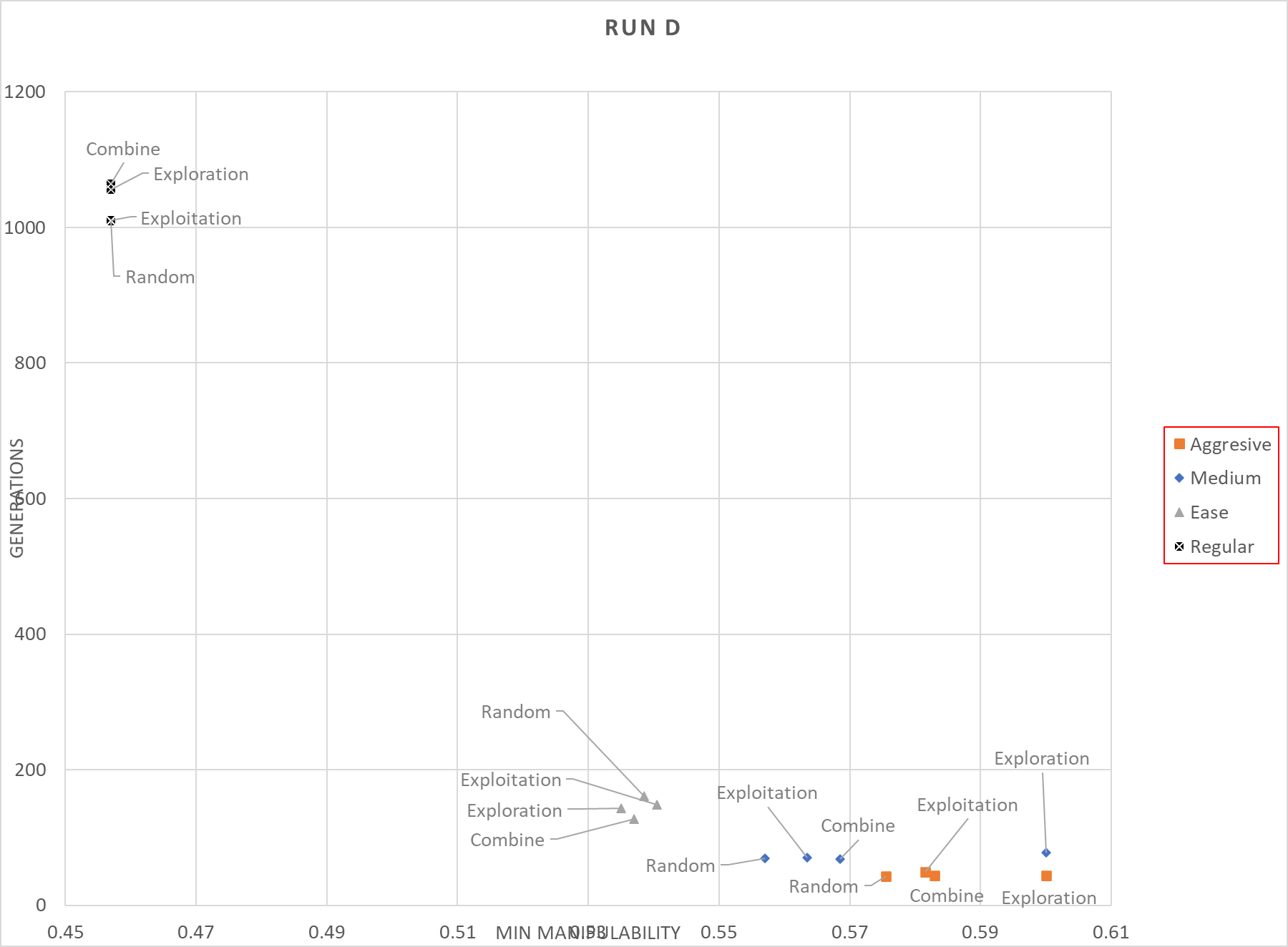
Regular\_ Combine, Regular\_ Random, Regular\_ Exploitation, Regular\_ Exploration, Ease \_ Exploration, Medium \_ Combine, Medium \_ Exploration, Aggressive \_ Combine, Aggressive \_ Exploration, Aggressive \_ Random



From fig\*\*, that show the Generations Vs Min Manipulability run C, it can be shown that:

* In Regular: there is no domination of one method over any other method.
* In Ease: Exploration dominates over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Medium: Combine is dominated by the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Aggressive: Exploitation and Exploration are dominated by Combine and Random, but according to the Wilcoxon test, there is no statistical significance.
* 10 non-dominated methods left:

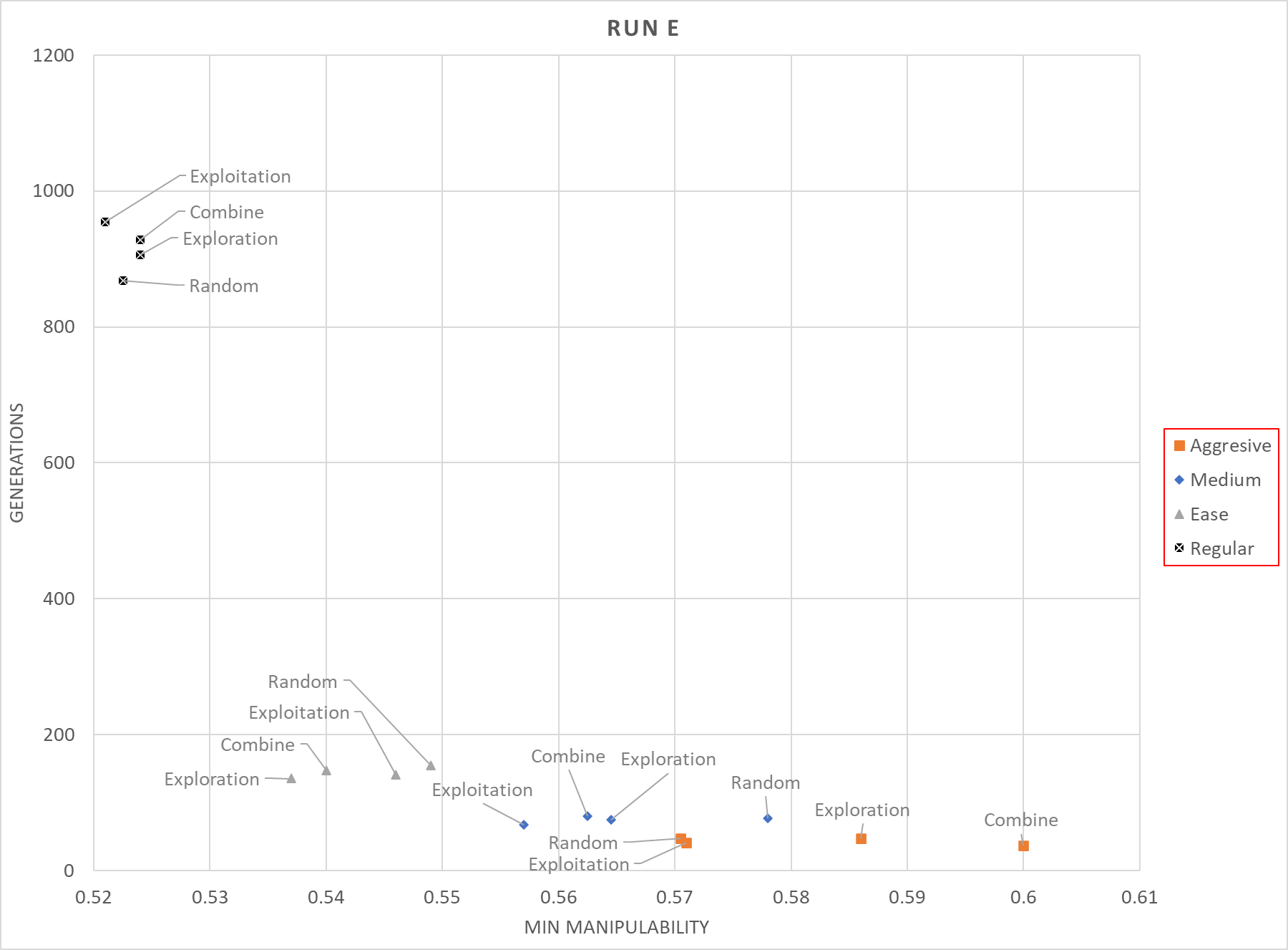
Regular\_ Combine, Regular\_ Random, Regular\_ Exploitation, Regular\_ Exploration, Ease \_ Exploration, Medium \_ Exploration, Medium \_ Exploitation, Medium \_ Random, Aggressive \_ Combine, Aggressive \_ Random



From fig\*\*, that show the Generations Vs Min Manipulability run D, it can be shown that:

* In Regular: there is no domination of one method over any other method.
* In Ease: Exploration and Combine dominate over the 2 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Medium: Combine and Random dominate over the 2 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Aggressive: Random dominates over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* 9 non-dominated methods left:

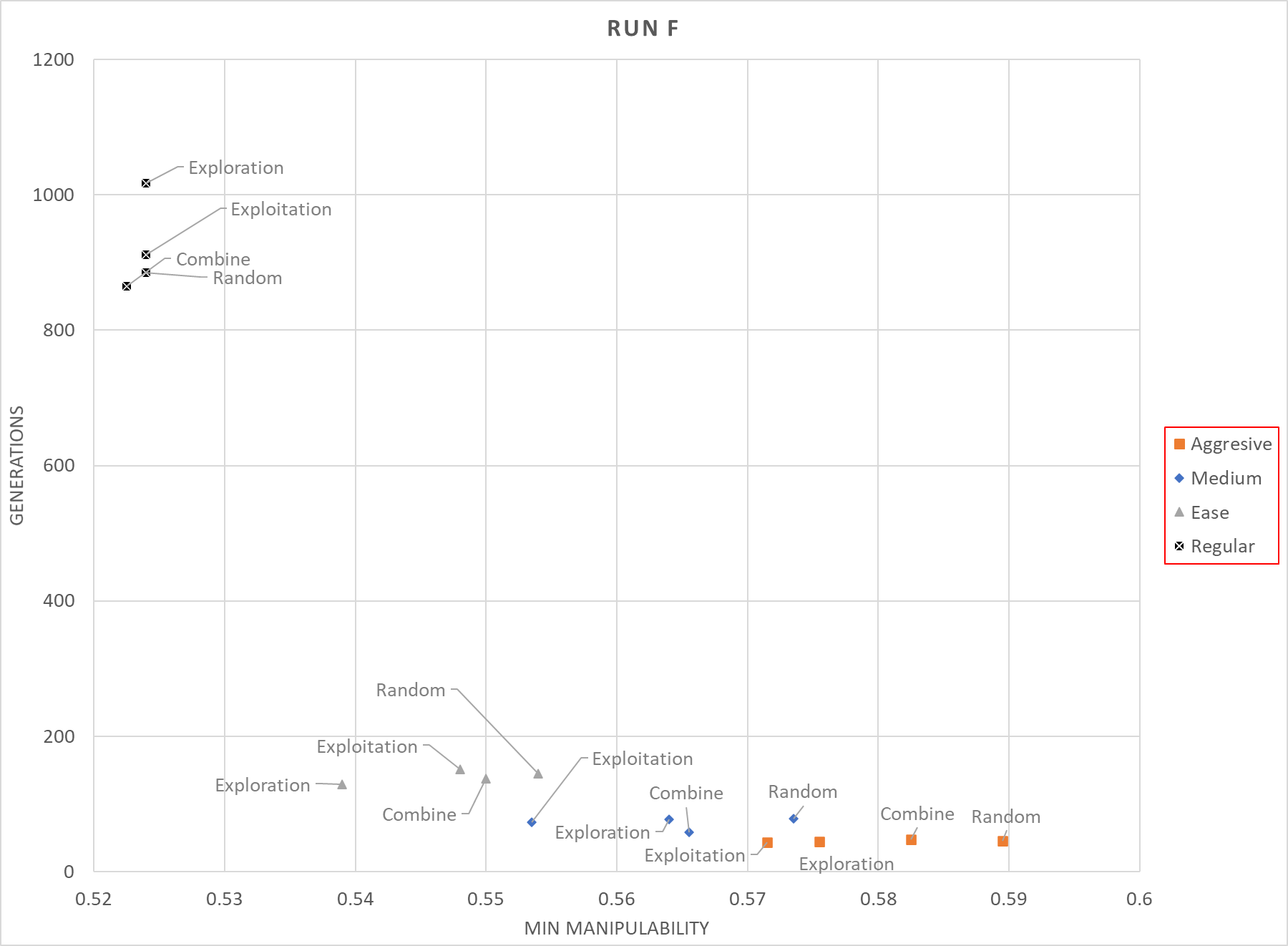
Regular\_Combine, Regular\_Random, Regular\_Exploitation, Regular\_Exploration, Ease\_Exploration, Ease\_Combine, Medium\_Random, Medium\_Combine, Aggressive\_Random



From fig\*\*, that show the Generations Vs Min Manipulability run E, it can be shown that:

* In Regular: Combine and Random dominated by the 2 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Ease: Exploration dominates over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Medium: Exploitation dominates over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Aggressive: Exploration dominated by the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* 7 non-dominated methods left:

Regular\_Random, Regular\_Exploitation, Ease\_Exploration, Medium\_ Exploitation, Aggressive \_Combine, Aggressive\_Random, Aggressive\_Exploitation.



From fig\*\*, that show the Generations Vs Min Manipulability run F, it can be shown that:

* In Regular: Combine dominates over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Ease: Exploration dominates over the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Medium: Exploitation and Combine dominate over the 2 other methods, but according to the Wilcoxon test, there is no statistical significance.
* In Aggressive: Exploitation dominated by the 3 other methods, but according to the Wilcoxon test, there is no statistical significance.
* 5 non-dominated methods left:

Regular\_ Combine, Ease\_Exploration, Medium\_ Exploitation, Medium \_Combine, Aggressive\_Exploitation.



In Table \*\*\* it can be seen how many times each method was in the set of the None-Dominated results. It can be seen there are 3 methods that appear 6 times: Regular\_Random, Regular\_Combine, Ease\_Exploration. There are 3 other methods that appear 5 times: Regular\_ Exploitation, Medium\_Exploitation, Aggressive\_Random.

In order to choose the right method, 3 methods examined(out of the 6 most appearing methods) and simulated over all the concepts for 2 days, to check how they perform overall the concepts.

In Table \*\* it can be seen the median result of the Generation and Min-Manipulability for each method and total.



From this table, it has seen that as the number of generations is increased the Min-Maniulability is decreased. Also, it can be seen that to Medium\_Exploitation, Aggressive\_Random there is the same Min-Manipulability, but Regular\_Random reach to this Min-Maniulability in the fewer generations.

Because the simulation time is also an important factor in the method selection, in addition to Regular\_Random, also Medium\_Exploitation selected, despite doesn’t have the lowest number of generations, have low enough generations.

Ease\_Exploration also selected because it of the 3 methods which arrived 6 times to the None-Dominated results.



In Table \*\*\* it can be seen the results of the 2 days run of each method. In the table, for each method, there are Total generations that suns during the simulation, the HV of the final DWOI, the generation number which the DWOI was last changed, and the value of the minimum manipulability in the DWOI.

In figure \*\*\* it can be seen the DWOI of each method in the same graph. From this graph, its seen that the major difference in the DWOI is were the Mid-Proximity Joint index is 0 and the manipulability is varied from 0.705 – 0.942.

