1a) $5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^2 + 3n^2$ for $n \geq 1$

$5n^3 + 2n^2 + 3n^2 = 5n^3 + 5n^2$

$5n^3 + 5n^2 \leq 5n^3 + 5n^3$ for $n \geq 1$

$5n^3 + 5n^3 = 10n^3$

$5n^3 + 2n^2 + 3n \leq 10n^3$ for $n \geq 1$

Thus, we have found a c and n_0 where $cn^3 \geq 5n^3 + 2n^2 + 3n$ for all $n \geq n_0$

1b) $\sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n}$

$\sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n^2}$ for $n \geq 1$

$\sqrt{7n^2 + 2n^2} = 3n$

$\sqrt{7n^2 + 2n - 8} \leq 3n$ for $n \geq 1$

$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7n^2 + 2n - 8n}$ for $n \geq 1$

$\sqrt{7n^2 + 2n - 8n} = \sqrt{7n^2 - 6n}$

$\sqrt{7n^2 - 6n} \geq \sqrt{7n^2 - 6n^2}$ for $n \geq 1$

$\sqrt{7n^2 - 6n^2} = n$

$\sqrt{7n^2 + 2n - 8} \geq n$ for $n \geq 1$

Thus, $\sqrt{7n^2 + 2n - 8}$ is both $O(n)$ and $\Omega(n)$, making it $\Theta(n)$

1c) We have

$$d(n) = O(f(n)) \quad \text{and} \quad e(n) = O(g(n))$$

We can rewrite as

$$d(n) \leq c_1(f(n)) \quad \text{and} \quad e(n) \leq c_2(g(n))$$
for all $n \geq n_{01}$ for all $n \geq n_{02}$

Since we know all 4 terms are nonnegative, we can do

$$d(n) \cdot e(n) \leq c_1 \cdot f(n) \cdot c_2 \cdot g(n)$$
for all $n \geq \max(n_{01}, n_{02})$

Rewriting, we get

$$d(n)e(n) \leq (c_1 \cdot c_2) f(n) g(n)$$
for all $n \geq \max(n_{01}, n_{02})$

Since c_1 and c_2 are just arbitrary constants, we can rephrase as

$$d(n)e(n) \leq c(f(n)g(n)) \text{ for } n \geq n_0$$

where $c = c_1 \cdot c_2$ and $n_0 = \max(n_{01}, n_{02})$

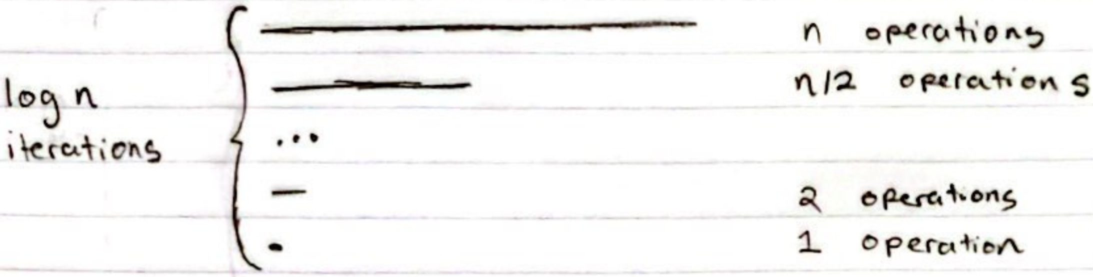
Thus, we have found a valid c and n_0, to satisfy the definition of big-O. Therefore...

$$d(n)e(n) = O(f(n)g(n))$$

2)

example 1 : $\Theta(n^2)$
example 2 : $\Theta(n)$
example 3 : $\Theta(\log(n))$
example 4 : $\Theta(n)$

Justification for example 4:



$n + \frac{n}{2} + \ldots 2 + 1 \approx 2n$, which is $\Theta(n)$, not $\Theta(n \log n)$

Justification for example 3:

$\log_2(n^2) = 2 \cdot \log_2(n)$, which is just $\Theta(\log(n))$

Justification for example 2:

only one loop, whose body iterates in constant time, n times

Justification for example 1:

$1 + 2 + 3 + 4 \ldots (n-2) + (n-1) + n$ is $\Theta(n^2)$